1.
SCHOOL CURRICULUM IN MATHEMATICS

1.1 INTRODUCTION

School systems are a relatively new phenomenon in historical terms, having developed only during the past two hundred years or so. Before then, there existed schools in parts of the West, as an appendage to religious organisations. The purpose of these schools was to produce an educated cleric. Interest in mathematics was rudimentary-'the different kinds of numbers and the various shapes and sufficient astronomy to help to determine the dates of religious rituals'. However, in India the practice of education was a well-established phenomenon. Arithmetic and astronomy were core components of the course of study. Astronomy was considered essential for determining auspicious times for performing religious rituals and sacrifices. Geometry was taught because it was required for the construction of sacrificial altars and 'havan kunds' of various shapes and sizes. With the arrival of the British, the system of education underwent a major change. Western system of education was introduced to educate Indians on western lines for the smooth functioning of the Empire.

However, much of the curriculum development in mathematics has taken place during the past thirty/forty years. This is because of the new technological revolution which has an impact on society as great as the industrial revolution. Modern technology is therefore causing, and will increasingly cause educational aims to be rethought, making curriculum development a dynamic process. To a scanning eye, mathematics itself is being directly affected by the modern technology as new branches are developed in response to new technological needs, leaving some 'time-hallowed' techniques redundant. In addition, teaching of mathematics also gets affected in order to keep pace with new developments in

technology. Moreover, there exists a strong similarity of mathematics syllabi all over the world, with the result that any change which comes from the curriculum developers elsewhere is often copied or tried by others. India, for example, got swayed with the wave of new mathematics. Later, following the trends in other countries, new mathematics also receded here.

To conclude, the various trends in curriculum development we observe no longer remain a static process, but a dynamic one. Its focus from 'selection' and 'organisation' of the informational material shifts to the development of a curriculum that 'manifests life in its reality'.

1.2 HISTORY OF DEVELOPMENT OF CURRICULUM FRAME WORK

Mahatma Gandhi had visualised education as a means of awakening the nation's conscience to injustice, violence and inequality entrenched in the social order. Nai Talim emphasised the self-reliance and dignity of the individual, which would form the basis of social relations characterised by non-violence within and across society. Gandhiji recommended the use of the immediate environment, including the mother tongue and work, as a resource for socialising the child into a transformative vision of society. He dreamt of an India in which every individual discovers and realises her or his talents and potential by working with others towards restructuring the world, which continues to be characterised by conflicts between Nations, within society and between humanity and nature.

In 1937, when Gandhiji propounded the idea of basic education, the Zakir Husain committee was appointed to elaborate on this idea. It recommended:

'Knowledge of mathematics is an essential part of any curriculum.Every child is expected to work out the ordinary calculations required in the course of his craft work or his personal and community concerns and activities.'

The Secondary Education Commission appointed in 1952 also emphasised the need for mathematics as a compulsory subject in the schools.

In line with the recommendations of the National Policy on Education, 1968, when the NCERT published its *"Curriculum for the Ten Year School"*, it remarked that the- 'advent of automation and cybernatics in this century marks the beginning of the new scientific industrial revolution and makes it all the more imperative to devote special attention to the study of mathematics'. It stressed on an 'investigatory approach' in the teaching of mathematics.

After Independence, the concerns of education articulated during the freedom struggle were revisited by the National Commissions — the Secondary Education Commission (1952 - 53) and the Education Commission (1964 - 66). Both Commissions elaborated on the themes emerging out of Mahatma Gandhi's educational philosophy in the changed socio-political context with a focus on National development. Education under the Indian Constitution until 1976 allowed the state governments to take decisions on all matters pertaining to school education, including curriculum, within their jurisdiction. The Centre could only provide guidance to the States on policy issues. It is under such circumstances that the initial attempts of the National Education Policy of 1968 and the Curriculum Framework designed by NCERT in 1975 were formulated. In 1976, the Constitution was amended to include education in the Concurrent List,and for the first time in 1986 the country as a whole had a uniform National Policy on Education.

The NPE (1986) recommended a common core component in the school curriculum throughout the country. The policy also entrusted NCERT with the responsibility of developing the National Curriculum Framework, and reviewing the framework at frequent intervals. NCERT in continuation of its curriculum-related work carried out studies and consultations subsequent to 1975, and had drafted a curriculum framework as a part of its activity in 1984. This exercise aimed at making school education comparable across the country in qualitative terms and also at making it a means of ensuring national integration without compromising on the country's pluralistic character. Based on such experience, the Council's work culminated in the National Curriculum Framework for School Education, 1988. However, the articulation of this framework through courses of studies and textbooks in a rapidly changing

developmental context resulted in an increase in 'curricular load' and made learning at school a source of stress for young minds and bodies during their formative years of childhood and stress for young minds and bodies during their formative years of childhood and adolescence. This aspect has been coherently brought out in **Learning Without Burden**, 1993, the report of the Committee under the chairmanship of Professor Yash Pal.

The review of the National Curriculum Framework, 2000 was initiated specifically to address the problem of curriculum load on children. A committee appointed by the Ministry of Human Resource Development in the early 1990s had analysed this problem, tracing its roots to the system's tendency to treat information as knowledge. In its report, ***Learning Without Burden***, the committee pointed out that learning at school cannot become a joyful experience unless we change our perception of the child as a receiver of knowledge and move beyond the convention of using textbooks as the basis for examination. The impulse to teach everything arises from lack of faith in children's own creative instinct and their capacity to construct knowledge out of their experience. The size of textbooks has been growing over the years, even as the pressure to include new topics mounts and the effort to synthesise knowledge and treat it holistically gets weaker. Flabby textbooks, and the syllabi they cover, symbolise a systemic failure to address children in a child-centred manner. Those who write such encyclopaedic textbooks are guided by the popular belief that there has been an explosion of knowledge.

Therefore, vast amounts of knowledge should be pushed down the throats of little children in order to catch up with other countries. Learning Without Burden recommended a major change in the design of syllabi and textbooks, and also a change in the social ethos, which places stress on children to become aggressively competitive and exhibit precocity. To make teaching a means of harnessing the child's creative nature, the report recommended a fundamental change in the matter of organising the school curriculum, and also in the system of examination, which forces children to memorise information and to reproduce it. Learning for the sake of being examined in a mechanical manner takes away the joy of being young, and delinks school knowledge from everyday experience. To address this deep structural problem, the present document draws upon and elaborates on the insights of Learning Without Burden.

1.3 CURRICULUM FRAME WORK, CURRICULUM AND SYLLABUS

1.3.1 CURRICULUM FRAMEWORK

Curriculum usually consists of a statement of aims and objectives indicate the selection and organization of content, manifests certain patterns of learning and teaching.

Because the objective demand or because the content organization requires i.e. includes a program of evaluation of outcomes.

We need to plan and pay attention to systemic matters that will enable us to implement many of the good ideas that have already been articulated in the past.

Paramount among this is "The National Curriculum Framework ". The (NCF 2005) is one of four National Curriculum Frameworks published in 1975, 1988, 2000 and 2005 by the

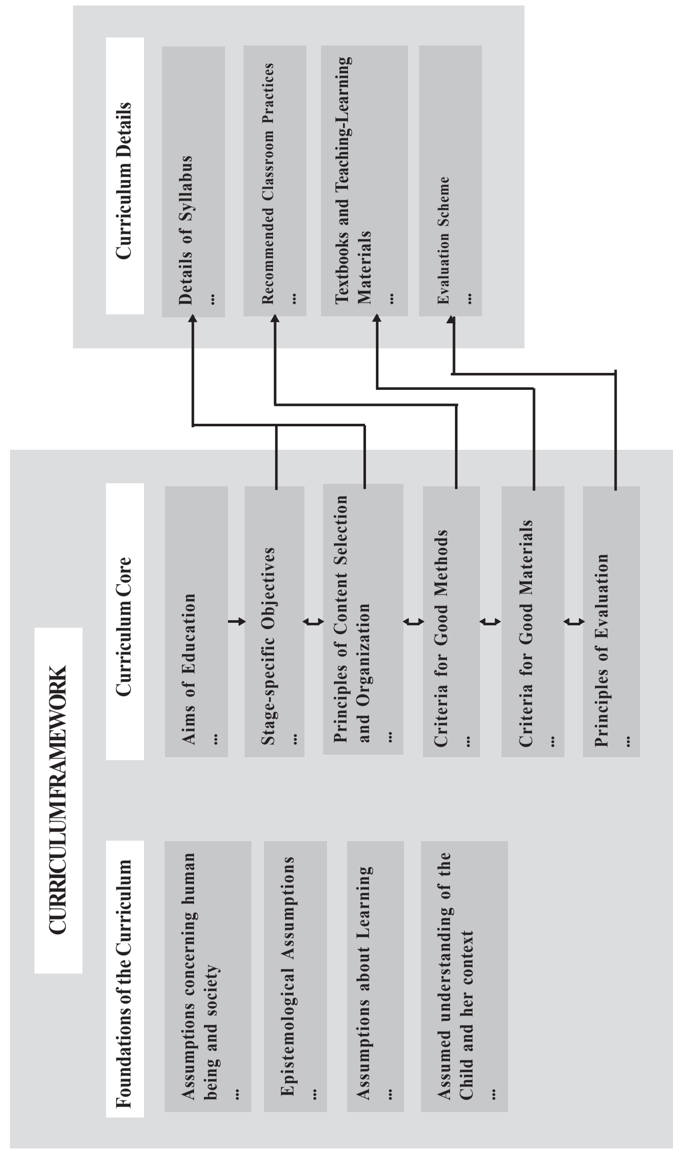
National Council of Educational Research and Training (NCERT) in India.

The Framework provides the framework for making syllabi, textbooks and teaching practices within the school education programmes in India. The NCF 2005 document draws its policy basis from earlier government reports on education as Learning Without Burden and National Policy of Education 1986-1992 and focus group discussion.

The NCF was framed considering the articulated ideas in the past such as

- connecting knowledge to life outside the school,
- ensuring that learning is shifted away from rote methods,
- enriching the curriculum to provide for overall development of children rather than remain textbook centric,
- making examinations more flexible and integrated into classroom life and,
- nurturing an over-riding identity informed by caring concerns within the democratic polity of the country

THE GRAPHICAL REPRESENTATION OF THE CURRICULUM FRAMEWORK



The emphasis for learning mathematics is that all students can learn and need to lean mathematics.Pedagogy and learning environment have to be made favourable for students to develop interest by going far beyond basic skills and include variety of mathematics models by pedagogy which devotes a greater percentage of instructional time to problem solving and active learning.

The National Policy on Education 1986 went further, Mathematics should be visualized as the vehicle to train a child to think, reason, analyze and to articulate logically. Apart from being a specific subject, it should be treated as a concomitant to any subject involving analysis and reasoning.

The National Curriculum Framework for School Education (NCFSE) 2000 document echoes such sentiments as well. Yet, despite this history of exhortations, mathematics education has remained pretty much the same, focussed on narrow aims.

Here, a quotation from William Thurston is appropriate:

"The long-range objectives of mathematics education would be better served if the tall shape of mathematics were de-emphasized, by moving away from a standard sequence to a more diversified curriculum with more topics that start closer to the ground. There have been some trends in this direction, such as courses in finite mathematics and in probability, but there is room for much more".

1.3.2 CURRICULUM

Etymologically, the term 'curriculum' which has been derived from the Latin root means 'race course'. The word race is suggestive of time and course - the path. Obviously, curriculum was seen as the prescribed course of study to be covered in a prescribed time frame. But, evolution of curriculum as a field of study began in 1890's only, albeit of the fact that thinkers of education were interested in exploring the field for centuries. Johann Friedrich Herbart (1776-1841), a German thinker, is generally associated with the evolution of curriculum- field. Herbart had emphasized the importance of 'selection' and 'organization' of content in his theories of teaching/ learning.

The first book devoted to the theme of curriculum entitled, *"The Curriculum"* was published in 1918 by Franklin Bobbitt followed by another book *"How to make Curriculum"* in 1924. In 1926, the National society for the study of education in America published the year book devoted to the theme of curriculum-The Foundation and Technique of Curriculum Construction. This way the curriculum development movement, from its beginning in 1890s, started becoming a vigorous educational movement across the world.

Definitions of Curriculum:

- It is the totality of experiences a child experiences in the school. It consists of that series of things which children and youth must do, aid experience by way of developing ability to do things well that make up the affairs of adult life.-**Bobbit**
- It is the total effort of the school to bring about desired outcomes in and out of school situations.-**Alexander & Saylor**
- It is the sequence of potential experiences set up in school for the purpose of disciplining children and youth in group ways of thinking and acting.-**Smith Stanley & Shores**
- Whatever the mode of expression the subject matter is the substantive hard core of the curriculum.-**Beauchamp**
- Curriculum is not concerned with what students will do in the learning situations, but what they will learn as a consequence of what they do.-**Johnson**
- Curriculum is a tool in the hands of the artist (teacher) to mould his material (student) in his studio (school).-**Cunnigham**
- It is a plan for action or a written document that includes strategies for achieving desired goals and ends. -**Ralph Taylor & Hilda Taba**
- As a plan for providing sets of learning opportunities for persons to be educated. **- J. Galen Saylon**

  After NPE 1986 and POA 1992 major reformation in school education was attempted in respect to Science and Mathematics.
- The curriculum was made a heavy and modernized at middle and secondary level and to handle the subject separate Science Graduate teachers were appointed at the middle and secondary stage.

- Science and Mathematics kits were supplied to schools under OBB scheme to learn the subject by doing.
- Teachers were trained.
- Free textbooks were started distributed to the children by state at primary stage.

The curriculum mentioned in that stage was found very loaded especially for middle and secondary level. And also textbooks developed at that time was not so attractive for mass children especially at elementary stage due to which both teacher as well as children was facing difficulty to learn Mathematics. Teachers appointed were not professionally qualified and no continuous training for up gradation of teacher's proficiency was made. Only one time Junior Basic Training to all primary teachers appointed was done for regularize the services. At upper primary level though normal training was there but not compulsory for all.

In 1993 Yashpal committee report concerned over the curricular load. The report said that children's non learning area of the content is the major burden for school children. So in nineties some notable changes were made in respect to the following:

Especially at primary level:

1. MLL and competency achievement.
2. Reduction of curricular load.
3. Activity based teaching.
4. Child centric approach.
5. Joyful learning.

These developments made throughout the country. DPEP was launched. Then again major changes started with all new innovative ideas in the school curriculum especially at primary level mainly in language and mathematics. While looking in child's point of view it was observed specially for Mathematics at primary level:

1. The curriculum was over loaded
2. Content was not related to children's life and were not integrated to social and cultural life.
3. The approach in the textbook was extremely mechanical.

4. Problem in the textbook are usually unfamiliar and uninteresting and not relevant.
5. Teacher teaches the subject in a very mechanical manner without using any concrete object
6. TLM.
7. Traditional Mathematics teaching is not related to real life.
8. Rote memorization was more stressed than understanding.

In the year 1998:

In the year 1998 state with the outcomes of DPEP new curriculum was developed at primary level for the first time where the following were given much importance.

- Competency based approach.
- Child centric approach.
- Joyful approach. Activity based approach.

The primary Mathematics Curriculum was designed scientifically on the above where spiralling of the concept and day to day children life experience as well as child liking elements were given more importance. The content load was reduced as far as possible especially at primary level.

The primary curriculum developed in the year 1998 consisted of following:

1. Pre number concept
2. Number concept
3. Four operation (in spiralling order in accordance with the competency of number)
4. Measurement
5. Fraction
6. Time
7. Shape (Geometry)
8. Puzzle, riddle, rhythm, etc.

All the concepts were graded in 3 phases for a particular class not in one phase only so that children of all level can learn and achieve easily as well as learning in discrete in manner.

At I & II the mathematics was totally integrated with the other subjects with focusing major 7 competencies which are:

1. Listening and responding.
2. Speaking and conversation.
3. Reading and Writing.
4. Number and counting.
5. Working with materials
6. Problem solving
7. Exploration

Curriculum at U.P. level:

Learner friendly Mathematics curriculum was developed keeping the fact in mind the following points:

1. Practical or utilization value.
2. Discipline value
3. Economical saving value
4. Punctuality value
5. Cultural value
6. Patience etc.

In this stage (V to VII) Mathematics learning is not only for higher studies but also for use of the knowledge and skill of Mathematics in their life long process, to make one self-sufficient, self-efficient, self-confident and self-dependent. With these aim the curriculum was developed in our state by making the curriculum more realistic, practical, useful, suitable, justified up to the learner's mental ability of that stage rather than stereotype, theoretical and traditional.

- To make Mathematics more understandable enjoyable and permanently retainable in the mind, more use of TLM, practical work, project work were reflected in the curriculum specifically which need Mathematics Lab,
- Mathematics corner and Mathematics club in each school. For this teaching-learning strategies were suggested as activity based, child centred, load free, stress free, enjoyable, interesting and effective.

The content was designed in a way different from traditional one which was:

1. Computation- number concept, operation – 20%
2. Socially applicable Mathematics – 25%
3. Transaction of money and its maintenance – 10%
4. Geometry – 15%
5. Measurement – 10%
6. Preliminary Algebra (for verbal only) – 8%
7. Data Based Mathematics – 5%
8. Contribution of Mathematics – 2%
9. Use of Mathematics – 3%
10. Mathematics Magic/ Fun/ Puzzle/ Quiz, etc. – 2%

Much focus was given in the curriculum for making Mathematics teaching-learning meaningful and purposeful for the learners in linkage with their life skills as well as to create interest or get an enjoyment in learning so that they can develop love and affection for the subject.

The following were suggested in the curriculum for teaching-learning process:

1. Activity based teaching-learning.
   - Group activity
   - Individual activity
   - Whole class activity
2. Use of teaching-learning material.
3. Types of activities.
   - Songs, picture study, narration of events
   - Games, puzzles, quiz
   - Conversation, group discussion
   - Observation, exploration, making and studying of bills
   - Preparation of tables, charts, models, dices etc.
   - Field trips to market places, post office, bank, mela, picnic, etc.
   - Practical activities related to measurement of geometry (concept, measurement etc.) Collection, study and analysis of data

- Practice and application through logical thinking, problem solving, project work etc.

Textbooks:

- New textbooks were developed in the latest approaches in Mathematics for class V to VII in linkage with L.P. level textbooks in the year 2004-2005.
- The concepts were introduced with child's life activity so that the Mathematics can be meaningful for them.
- A totally different types of textbooks were developed which were made different from traditional Mathematics textbooks

e.g. railway journey, picnic, visited hotel and restaurant, games, history of Mathematics etc. are content of the lessons of Mathematics textbook besides rhymes, story, puzzles etc.

Development of Mathematics Curriculum as per NCF 2005 and RTE act 2009

NCF-2005 reiterate the values enshrined in our constitution, reduction of curricular burden on children, ensuing quality education for all and systematic changes as makers of curricular reform. It recognizes the primacy of children's experiences, their voices and their active involvement in the process of learning. Learning at school should be such that children can construct knowledge from experiences and environment.

Guiding principles of NCF 2005

- Connecting knowledge to life outside the school
- Ensuring that learning is shifted away from the rote methods
- Enriching the curriculum to provide for overall development of children rather than remain textbook centric
- Making examination more flexible and integrated into classroom life.

For Mathematics Vision of school Mathematics has been laid in NCF-2005 as follows:

- Children learn to enjoy Mathematics rather than fear it.
- Children learn important Mathematics: Mathematics is more than formulas and mechanical procedures.

- Children see Mathematics as something to talk about, to communicate through, to discuss among them, to work together on.
- Children pose and solve meaningful problems.
- Children use abstractions to perceive relation-ships, to see structures, to reason out things, to argue the truth or falsity of statements.
- Children understand the basic structure of Mathematics: Arithmetic, Algebra, Geometry and Trigonometry, the basic content areas of school Mathematics, all offer a methodology for abstraction, structuration and generalization.
- Teachers engage every child in class with the conviction that everyone can learn Mathematics.

In Mathematics, the new syllabi emphasize reasoning and conceptual grasp at every stage. In primary Mathematics weightage has been projected to areas like shapes, spatial understanding, pattern, measurement and data handling. And Mathematics modelling, data analysis and interpretation provided at secondary stage set the frame to perceive Mathematics as discipline.

At higher secondary stage constructivism and problem solving form the twin objectives of syllabus. Emphasis on activity rather than rote memorization of facts and formulae continue through all stage.

RTE Act.

In the year 2009- Right to children free and compulsory education act was made, where curriculum, teaching-learning process and assessment were laid down to ensure:

- Conformity to constitutional value
- All round development of the child
- Building up child's knowledge, potentiality and talent
- Development of physical and mental abilities to the fullest extent
- Learning through activities, discovery and exploration in a child-cantered and child friendly manner Mother tongue as medium of introductions as far as possible.
- Making child free from fear, trauma and anxiety and helping the child to express views freely. Provide for comprehensive and continuous assessment of ability to understand and apply knowledge.

1.3.3 MATHEMATICS CURRICULUM

At the pre-primary stage, all learning occurs through play rather than through didactic communication. Rather than the rote learning of the number sequence, children need to learn and understand, in the context of small sets, the connection between word games and counting, and between counting and quantity. Making simple comparisons and classifications along one dimension at a time, and identifying shapes and symmetries, are appropriate skills to acquire at this stage. Encouraging children to use language to freely express one's thoughts and emotions, rather than in predetermined ways, is extremely important at this and at later stages.

Having children develop a positive attitude towards, and a liking for, Mathematics at the primary stage is as important, if not more as the cognitive skills and concepts that they acquire. Mathematical games, puzzles and stories help in developing a positive attitude and in making connections between mathematics and everyday thinking. It is important to note that mathematics is not just arithmetic. Besides numbers and number operations, due importance must be given to shapes, spatial understanding, patterns, measurement and data handling. The curriculum must explicitly incorporate the progression that learners make from the concrete to the abstract while acquiring concepts. Apart from computational skills, stress must be laid on identifying, expressing and explaining patterns, on estimation and approximation in solving problems, on making connections, and on the development of skills of language in communication and reasoning.

At the upper primary stage, students get the first taste of the power of Mathematics through the application of powerful abstract concepts that compress previous learning and experience. This enables them to revisit and consolidate basic concepts and skills learnt at the primary stage, which is essential from the point of view of achieving universal mathematical literacy. Students are introduced to algebraic notation and its use in solving problems and in generalisation, to the systematic study of space and shapes, and for consolidating their knowledge of measurement. Data handling, representation and interpretation form a

significant part of the ability of dealing with information in general, which is an essential 'life skill'. The learning at this stage also offers an opportunity to enrich students' spatial reasoning and visualisation skills.

At the secondary stage, students begin to perceive the structure of Mathematics as a discipline. They become familiar with the characteristics of mathematical communication: carefully defined terms and concepts, the use of symbols to represent them, precisely stated propositions, and proofs justifying propositions. These aspects are developed particularly in the area of geometry. Students develop their facility with algebra, which is important not only in the application of mathematics, but also within mathematics in providing justifications and proofs? At this stage, students integrate the many concepts and skills that they have learnt into a problem-solving ability. Mathematical modelling, data analysis and interpretation taught at this stage can consolidate a high level of mathematical literacy. Individual and group exploration of connections and patterns, visualisation and generalisation, and making and proving conjectures are important at this stage, and can be encouraged through the use of appropriate tools that include concrete models as in Mathematics laboratories and computers.

The aim of the Mathematics curriculum at the higher secondary stage is to provide students with an appreciation of the wide variety of the application of Mathematics, and equip them with the basic tools that enable such application. A careful choice between the often conflicting demands of depth versus breadth needs to be made at this stage. The rapid explosion of Mathematics as a discipline, and of its range of application, favours an increase in the breadth of coverage. Such increase must be dictated by mathematical considerations of the importance of topics to be included. Topics that are more naturally the province of other disciplines may be left out of the Mathematics curriculum. The treatment of topics must have an objective, that is, the communication of mathematical insights and concepts, which naturally arouse the interest and curiosity of students.

Major Objectives of the Mathematics Curriculum

- The mathematics curriculum aims at the following objectives.

- Proficiency in fundamental mathematical skills.
- Comprehension of basis mathematical concepts.
- Appreciation of significant meanings.
- Development of desirable attitudes.
- Efficiency in making sound mathematical applications.
- Confidence in making intelligent and independent interpretation

Vision for School Mathematics Curriculum

- Children learn to enjoy mathematics rather than fear it.
- Children learn important mathematics: Mathematics is more than formulas and mechanical procedures.
- Children see mathematics as something to talk about, to communicate through, to discuss among them, to work together on.
- Children pose and solve meaningful problems.
- Children use abstractions to perceive relation-ships, to see structures, to reason out things, to argue the truth or falsity of statements.
- Children understand the basic structure of Mathematics: Arithmetic, algebra, geometry and trigonometry, the basic content areas of school Mathematics, all offer a methodology for abstraction, structuration and generalisation.
- Teachers engage every child in class with the conviction that everyone can learn mathematics.

1.3.4 PRINCIPLES OF CURRICULUM CONSTRUCTION

The content of curriculum is determined on the basis of some academic principles which are stated below:

1. Aims of education and objectivity:

Life is complex. A curriculum should reflect the complexities of life. In other words, in farming the curriculum one should take into consideration the aims and objectives of education.

2. Principle of Child-centeredness:

The curriculum should be framed according to the actual needs, interests and capacities of the child. That means a curriculum must be child-centric as modern education is child-cantered.

3. Principle of civic and social needs:

Man is a social being. He lives in the society. The child develops in the society. Modern education aims at both developments of the individuality of the child as well as the development of the society.

4. Principle of conservation:

Man has conserved experiences very carefully for better adaptability. Education is regarded as a means of deserving the cultural heritage of humanity. The school serves two-fold functions in this regard- preservation of the past experiences and transmission of experiences.

5. Principle of creativeness:

Education not only conserves that past experiences of humanity but also helps an individual to develop his innate potentialities.

6. Principle of forward-looking:

The aim of life-cantered education is not limited to the present life-situations in the family and society. Hence, education must prepare the child of shouldering future responsibilities. So in farming the curriculum we must take into consideration the future needs of the child as well as the needs of the society.

7. Principle of preparation for living:

The children should know the various activities of the environment around them and how these activities are enabling people to meet their basic needs of food, shelter, clothing, recreation, health and education.

8. Principle of integration and correlation:

Subjects should be arranged logically and psychologically in accordance with the child's developing interests.

9. Principle of learning ability:

Every item should be learnt. An item should not only be learnable, it should also have utility.

10. Principle of individual difference:

The curriculum should be framed in such a way that every individual can have opportunity for self-expression and development. The curriculum should be based on the psychology of individual difference, which can meet the complexities of modern democratic society.

11. Principle of social relevancy and utility:

Subjects should not be determined on the basis of their disciplinary value but on the basis of their intrinsic value, social relevancy and utility.

12. Principle for utilization of leisure:

Variety of subjects such as games and sports, fine arts, subjects of aesthetic value are to be introduced in the school program to utilize leisure.

13. Principle of variety and flexibility:

The curriculum should include such activities and experiences, which may facilitate his normal development. The curriculum for girls should naturally be different from that of boys; boys and girls have different needs and attitudes.

14. Principle of time:

Relative significance and importance of each subject in the curriculum has to be judged and determined in the light of the time available in the timetable, which is regarded as the mirror of the school programme.

1.3.5 APPROACHES TO CURRICULUM ORGANIZATION

There are different approaches to organize the mathematics curriculum. The important among them are:

a. Topical approach
b. Spiral approach
c. Logical and psychological Approach
d. Unitary Approach
e. Integrated Approach

a. Topical approach

In topical approach a topic once presented should be completely exhausted in the same class. This method demands that the entire topic, the portions easy as well as the difficult, should be covered in the same stage.

Disadvantages –

- This is not a psychological approach as the students are forced to learn many things for which they have no immediate need and relevance.

- Some parts of the topic will be more complex and difficult for the learner to understand at the stage.
- It does not take into account the mental development of the students.
- It introduces a large amount of irrelevant matter, the use of which cannot be appreciated by the learner at the stage.
- Topics once completed receive no attention at larger stages and there is every likelihood of them being forgotten.

a. Spiral approach

The Cambridge Report (1963) on Mathematics Curriculum emphasized the importance of interrelating and interweaving the different mathematical topics to be taken up throughout the school period and envisaged the progressive broadening and deepening of the child's mathematical knowledge and insight by what is called the "Spiral Approach".

Therefore, contrary to topical approach, spiral approach demands the division of the topic into a number of smaller independent units to be dealt with, in order of difficulty, suiting the mental capacities of children. For example, 'Set Theory' can be split up into various subunits and arranged in the increasing order of difficulty.

Advantages

- The spiral approach has the following advantages over topical approach.
- Subject matter is introduced in the increasing order of difficulty, in accordance with the needs and capacities of the students.
- It satisfies the psychological needs of the students
- The students are able to appreciate the relevance and significance of what they learn.
- It provides sufficient motivation for the students to learn.
- It provides opportunities to relate the topic with other topics, other branches and other subjects.

b. Logical and psychological Approach

The arrangement of subject matter based on the principles of psychology is known as psychological approach.in other words this approach is in tune with the principle of child-centeredness. In the logical approach stress is given to the logicalsequencing of the subject concerned. Content of every subject have been developed in tune with such a logical sequence.

Logical approach demands maintaining the logical sequence while developing a curriculum. At the same time, a good curriculum, if carefully developed can maintain the psychological approach without sacrificing the logical sequence of the subject. The only thing is that the logical continuity is taken care of at every stage of curriculum transaction done in tune with the psychological approach.

c. Unitary Approach

The students learn mathematics with its different branches and topics in watertight compartments. So in the last few decades, the teachers in mathematics have searched for broad unifying principles which could be made the core of the mathematical course.

Suppose that complete understanding of the function Y=ax+b be made one of the major division of first year algebra, all the other concepts can be developed through this because the attainment of an understanding of this important function will develop upon many experiences. Such an organization enables the pupils to see clearly the relationship between the various facts, processes and principles taught in the course. They know that each unit contributes to the course as a whole. The question of time and size need not enter in the organization of a unit. But for most high school pupils it may take more than a month to assimilate.

Experiences seem to show that a unit which can be studied in the four weeks is most suitable as to size. When it is not possible to finish it within that time the teacher should find a way of simplifying the unit either by transferring some of the materials to other units or by diving it into two smaller units.

Characteristics of the Unitary Organization

It organizes a body of facts, theorems or processes, closely related to one another and so organized as to contribute to the understanding of an important aspect of the course.

Pedagogy of Mathematics - Sem-II (TS)

It must be possible to present the theorems and process as a group in a form so definite that the learner may attain a conception of them before he undertakes the detailed study of the content of the unit.

It must be possible to set up outcomes of the study so definite that they are clear not only to the teacher but also to the pupils.

It should be clear that there is no single type of organization which may be set up to determines as “only acceptable” list of units for the course. Unitary organization makes teaching and learning purposeful and intelligent. Because the instructional materials are closely related to each other, they are easily retained. Economy of time and effort should be the result.

d. Integrated Approach

The main aim of education is acquisition of knowledge and the transfer of knowledge to study other subjects and to solve successfully the problems that arise in everyday life. Each subject in the curriculum aims at realizing these aims through different means. The study of every subject should highlight the unity of knowledge. While teaching any subject, the teacher can cite instances and examples to show that knowledge is a single integrated whole and the knowledge that one gains through courses like mathematics, physical/biological sciences, social sciences, language, arts and other constitute the whole. Such an integrated approach helps the students to get a holistic view of the school programme and thereby the study of each subject becomes more meaningful and significant.

1.3.6 SYLLABUS

Definition:

A syllabus is an outline and summary of topics to be covered in an education. It is descriptive (unlike the prescriptive or specific curriculum). A syllabus may be set out by an exam board or prepared by the professor who supervises or controls course quality.

Stage-wise Mathematics Syllabus

1. Primary stage
2. Upper Primary
3. Secondary

1. PRIMARY STAGE

Any curriculum for primary mathematics must incorporate the progression from the concrete to the abstract and subsequently a need to appreciate the importance of abstraction in mathematics. In the lowest classes, especially, it is important that activities with concrete objects form the first step in the classroom to enable the child to understand the connections between the logical functioning of their everyday lives to that of mathematical thinking.

Mathematical games, puzzles and stories involving number are useful to enable children to make these connections and to build upon their everyday understandings. Games – not to be confused with open-ended play - provide non- didactic feedback to the child, with a minimum amount of teacher intervention. They promote processes of anticipation, planning and strategy.

i. Mathematics is not just arithmetic

While addressing number and number operations, due place must be given to non-number areas of mathematics. These include shapes, spatial understanding, patterns, measurement and data handling. It is not enough to deal with shapes and their properties as a prelude to geometry in the higher classes. It is important also to build up a vocabulary of relational words which extend the child's understanding of space. The identification of patterns is central to mathematics. Starting with simple patterns of repeating shapes, the child can move on to more complex patterns involving shapes as well as numbers. This lays the base for a mode of thinking that can be called algebraic. A primary curriculum that is rich in such activities can arguably make the transition to algebra easier in the middle grades. Data handling, which forms the base for statistics in the higher classes, is another neglected area of school mathematics and can be introduced right from Class I.

ii. Number and number operations

Children come equipped with a set of intuitive and cultural ideas about number and simple operations at the point of entry into school. These should be used to make linkages and connections to number understanding rather than treating the child as a tabula rasa. To learn to

think in mathematical ways children need to be logical and to understand logical rules, but they also need to learn conventions needed for the mastery of mathematical techniques such as the use of a base ten system. Activities as basic as counting and understanding numeration systems involve logical understandings for which children need time and practice if they are to attain mastery and then to be able to use them as tools for thinking and for mathematical problem solving. Working with limited quantities and smaller numbers prevents overloading the child's cognitive capacity which can be better used for mastering the logical skills at these early stages.

Operations on natural numbers usually form a major part of primary mathematics syllabi. However, the standard algorithms of addition, subtraction, multiplication and division of whole numbers in the curriculum have tended to occupy a dominant role in these. This tends to happen at the expense of development of number sense and skills of estimation and approximation. The result frequently is that students, when faced with word problems, ask "Should I add or subtract? Should I multiply or divide?" This lack of a conceptual base continues to haunt the child in later classes. All this strongly suggests that operations should be introduced contextually. This should be followed by the development of language and symbolic notation, with the standard algorithms coming at the end rather than the beginning of the treatment.

iii. Fractions and decimals

Fractions and decimals constitute another major problem area. There is some evidence that the introduction of operations on fractions coincides with the beginnings of fear of mathematics. The content in these areas needs careful reconsideration. Everyday contexts in which fractions appear, and in which arithmetical operations need to be done on them, have largely disappeared with the introduction of metric units and decimal currency. At present, the child is presented with a number of contrived situations in which operations have to be performed on fractions. Moreover, these operations have to be done using a set of rules which appear arbitrary (often even to the teacher), and have to be memorized - this at a time when the child is still grappling with the rules for operating on whole numbers. While the importance of fractions in the conceptual structure of mathematics is undeniable, the above considerations seem to suggest that less emphasis on operations with fractions at the primary level is called for.

2. UPPER PRIMARY STAGE

Mathematics is amazingly compressible: one may struggle a lot, work out something, perhaps by trying many methods. But once it is understood, and seen as a whole, it can be filed away, and used as just a step when needed. The insight that goes into this compression is one of the great joys of mathematics. A major goal of the upper primary stage is to introduce the student to this particular pleasure.

The compressed form lends itself to application and use in a variety of contexts. Thus, mathematics at this stage can address many problems from everyday life, and offer tools for addressing them. Indeed, the transition from arithmetic to algebra, at once both challenging and rewarding, is best seen in this light.

i. Arithmetic and Algebra

A consolidation of basic concepts and skills learnt at primary school is necessary from several points of view. For one thing, ensuring numeracy in all children is an important aspect of universalisation of elementary education. Secondly, moving from number sense to number patterns, seeing relationships between numbers and looking for patterns in the relationships bring useful life skills to children. Ideas of prime numbers, odd and even numbers, tests of divisibility etc. offer scope for such exploration.

Algebraic notation, introduced at this stage, is best seen as a compact language, a means of succinct expression. Use of variables, setting up and solving linear equations, identities and factoring are means by which students gain fluency in using the new language.

The use of arithmetic and algebra in solving real problems of importance to daily life can be emphasized. However, engaging children's interest and offering a sense of success in solving such problems is essential.

ii. Shape, space and measures

A variety of regular shapes are introduced to students at this stage: triangles, circles, quadrilaterals, they offer a rich new mathematical experience in at least four ways. Children start looking for such shapes in nature, all around them, and thereby discover much symmetry and acquire a sense of aesthetics.

Secondly, they learn how many seemingly irregular shapes can be approximated by regular ones, which becomes an important technique in science. Thirdly, they start comprehending the idea of space: for instance, that a circle is a path or boundary which separates the space inside the circle from that outside it. Fourthly, they start associating numbers with shapes, like area, perimeter etc, and this technique of quantization, or arithmetization, is of great importance. This also suggests that mensuration is best when integrated with geometry.

An informal introduction to geometry is possible using a range of activities like paper folding and dissection, and exploring ideas of symmetry and transformation. Observing geometrical properties and inferring geometrical truth is the main objective here. Formal proofs can wait for a later stage.

iii. Visual learning

Data handling, representation and visualization are important mathematical skills which can be taught at this stage. They can be of immense use as "life skills". Students can learn to appreciate how railway time tables, directories and calendars organize information compactly.

Data handling should be suitably introduced as tools to understand process, represent and interpret day-to-day data. Use of graphical representations of data can be encouraged. Formal techniques for drawing linear graphs can be taught.

Visual Learning fosters understanding, organization, and imagination. Instead of emphasizing only two-column proofs, students should also be given opportunities to justify their own conclusions with less formal, but nonetheless convincing, arguments. Students' spatial reasoning and visualization skills should be enhanced. The study of geometry should make full use of all available technology. A student when given visual scope to learning remembers pictures, diagrams, flowcharts, formulas, and procedures.

3. SECONDARY STAGE

It is at this stage that Mathematics comes to the student as an academic discipline. In a sense, at the elementary stage, mathematics education is (or ought to be) guided more by the logic of children's psychology of learning rather than the logic of mathematics. But at the secondary stage, the student begins to perceive the structure of mathematics. For his, the notions of argumentation and proof become central to curriculum now.

Mathematical terminology is highly stylised, self- conscious and rigorous. The student begins to feel comfortable and at ease with the characteristics of mathematical communication: carefully defined terms and concepts, the use of symbols to represent them, precisely stated propositions using only terms defined earlier, and proofs justifying propositions. The student appreciates how an edifice is built up; arguments constructed using propositions justified earlier, to prove a theorem, which in turn is used in proving more.

For long, geometry and trigonometry have wisely been regarded as the arena wherein students can learn to appreciate this structure best. In the elementary stage, if students have learnt many shapes and know how to associate quantities and formulas with them, here they start reasoning about these shapes using the defined quantities and formulas.

Algebra, introduced earlier, is developed at some length at this stage. Facility with algebraic manipulation is essential, not only for applications of mathematics, but also internally in mathematics. Proofs in geometry and trigonometry show the usefulness of algebraic machinery. It is important to ensure that students learn to geometrically visualise what they accomplish algebraically.

A substantial part of the secondary mathematics curriculum can be devoted to consolidation. This can be and needs to be done in many ways. Firstly, the student needs to integrate the many techniques of mathematics she has learnt into a problem solving ability. For instance, this implies a need for posing problems to students which involve more

than one content area: algebra and trigonometry, geometry and mensuration, and so on.

Secondly, mathematics is used in the physical and social sciences, and making the connections explicit can inspire students immensely. Thirdly, mathematical modelling, data analysis and interpretation, taught at this stage, can consolidate a high level of literacy. For instance, consider an environment related project, where the student has to set up a simple linear approximation and model a phenomenon, solve it, visualise the solution, and deduce a property of the modelled system. The consolidated learning from such an activity builds a responsible citizen, who can later intuitively analyse information available in the media and contribute to democratic decision making.

At the secondary stage, a special emphasis on experimentation and exploration may be worthwhile. Mathematics laboratories are a recent phenomenon, which hopefully will expand considerably in future.. Activities in practical mathematics help students immensely in visualisation. Indeed, Singh, Avtar and Singh offer excellent suggestions for activities at all stages. Periodic systematic evaluation of the impact of such laboratories and activities will help in planning strategies for scaling up these attempts.

1.4 FROM SUBJECT-CENTRED TO BEHAVIORIST TO CONSTRUCTIVIST APPROACH, TO CURRICULUM DEVELOPMENT

1.4.1 SUBJECT-CENTRED APPROACH

Definition

The oldest, and perhaps most obvious way to organize curriculum is through a subject-centered approach. This type of curriculum separates knowledge into various content areas. Modern schools, particularly middle and high schools, tend to operate in this fashion. Students take classes in English/language arts, math, science, social studies, fine arts, career and technical education, and so forth.

❖ **Focus of Approach**

In a subject-centered approach to curriculum, each content area contains its own set of skills and concepts for mastering that content. For example, in science, students learn about the scientific method and science-related vocabulary. This knowledge is then used when students conduct experiments and investigations. In math, students are taught logical and problem solving skills which they will need to produce while understanding various concepts... Teachers in these subject areas are specialists in their content.

❖ Types of Content

Schools that maintain a subject-centered approach categorize subjects into four different types. "Common content" represents subjects all students must study.

- In elementary schools, this consists of arithmetic, reading and writing (the three R's).
- In secondary schools, these subjects include: math, science, social studies/history and English/language arts.
- "Special content" describes classes that prepare students for specific professions. These might include vocational and technical education courses.
- "Elective content" refers to optional classes students can take to further their knowledge and skills. These might include college courses taken while still in high school, advanced auto mechanics courses, or special interest courses, such as photography or aeronautics.

❖ Objectives and Accountability

The central objective for any subject-centered approach to curriculum is student mastery of content knowledge. The teacher presents content and skills to students in a logical sequence. This step-by-step approach ensures that students gain all the information and skills needed to master this content area. There is little or no emphasis on the overlap of various subjects. Teachers only present the subject matter from their individual subject and are only accountable for student mastery of their content area.

❖ Types of curriculum design: subject centered

Curriculum design is about how a person envisions what a curriculum should be. There are several standard models of curriculum design. One of the most prominent is the subject-centered design.

Pedagogy of Mathematics - Sem-II (TS)

The subject-centered designer divides the curriculum into nice and neat subjects such as math, science, history, literature, etc. This structuring of the disciplines is for practical reasons. It organizes the curriculum into basic concepts that are combined based on what they have in common. The essential knowledge of each area is gathered together to be taught to students.

Where the division of the curriculum stops depends on its purpose. Any expert in education knows that subjects overlap and the division is often arbitrary. In addition, every subject can be further divide into smaller parts. For example, English can be broken down into writing, reading, speech, grammar, and more.

A major criticism of this design is the lack of integration or horizontal articulation. The learning is compartmentalized and the students often never see the connections across subjects. In addition, the subject-centered design does not take into account the needs and interest of the students. The textbook is made by experts in the field who already know what knowledge and even experiences a child requires.

Despite this, the subject design is by far the most popular approach. It is easy to do and practical. Its appropriateness needs to be left to the educator who is trying to help their students.

❖ Characteristics of Subject Centred Curriculum

Following are the characteristics of Subject Centred Curriculum,

1. Drill in specific skills is one of the typical characteristics of the subject curriculum.

Drill session, remedial work, review work, coaching classes are often devoted to such type of drill.This drill is gives in equal amount to all the pupils in the group.

2. Emphasis is placed upon acquiring information for future use

The subject matter selected for a course, is considered to be value in adult living rather child's immediate needs. Thus adult problems are given more weight than problems of children in youth.

3. Progress is measured by how much of the subject a student has learnt

as the subject matter is an important thing to be learnt, learning is measured by how well the subject matter has been mastered. Frequent tests are given to check the extent of achievement by the students.

4. Each subject is distinct entity (unit) with logical organisation of its own

Emphasis is placed on the acquisition of skills, facts and information in different logically organised subjects. The member of the staff, teaching different subjects do not plan courses together and they do not discuss common problems.

5. Subject matter is selected by adults previous to the teaching, learning situations

as the subject matter is taught in logically organised discipline, therefore, the content of the course is selected before it is taught. For this purpose they receive help from subject matter, specialist, supervisors, administrators and text book writers.

1.4.2 BEHAVIOURIST APPROACH

Whenever a mobile phone rings on any bus or train, people scramble to check whether it's theirs. This is a clear example of a near-automatic response to a stimulus. It illustrates perfectly one of the fundamental laws underlying behaviourism – the crucial bond between stimulus and response.

Behaviourism is the most influential and generalizable theory of learning that claims a scientific basis. This is because, like the most useful theories in any field, it is universal and underpinned by only a few principles. As its name suggests, it concentrates on behavioural changes in organisms. Thus, behaviorists define learning as a relatively permanent change in behaviour as the result of experience. This change in behaviour is always observable, with some behaviorists proposing that if no observable change happens, no learning has occurred. Although behaviourists do not deny that learners think, they mainly choose to ignore inaccessible mental processes and focus on observable behaviour.

Behaviourism focuses on observable learning events as demonstrated by stimulus and response relationships.

- Learning always involves a change in behaviour.
- Mental processes should be excluded from the scientific study of learning.
- The laws governing learning apply equally to all organisms, including human organisms.
- Organisms begin life as blank slates: there are no innate laws of behaviour.
- Learning results from external events in the environment.

Behaviourism is a deterministic theory: the subject has no choice but to respond to appropriate stimuli.

❖ **Educational implications of behaviourism**

Education has always had the modification of behaviour as one of its main purposes, and behaviourist principles operate at all educational levels – from the smile and approval of the infant's teacher to the award of credits and degrees at the highest levels. Although different educational sectors use behaviourist principles to different extents and in different ways – adults need less behavioural control than children, for example – we can see that behaviourism has influenced ideas about learner behaviour, curriculum planning, and the teacher's role in the classroom.

❖ **The teacher's –Learner's role in the classroom**

The sequencing of curricular events led to an interest in the correct sequencing of classroom events and the teacher's role in stimulating learners' behavioural responses. It focused on the importance of arranging stimuli to produce the most appropriate and desirable behavioural sequences. The specified nine 'internal processes and their corresponding instructional events' can be used to structure lesson plans, sessions or learning materials.

Table Internal processes and their corresponding instructional events

Teacher action	Learner response
1. Gaining learner's attention	1. Reception and attentiveness
2. Stating session objectives	2. Knowing what to expect
3. Reminding what was done before	3. Stimulation of long-term memory
4. Highlighting key features	4. Perceiving what is important
5. Structuring learning	5. Creating links and associations
6. Encouraging activity	6. Performing
7. Providing feedback	7. Learning awareness and satisfaction
8. Evaluating progress	8. Strengthening learning
9. Enhancing attention and signaling future learning	9. Gaining learning overview

❖ **Curriculum planning in behaviourist approach**

The following list outlines curriculum-planning steps commonly undertaken by teachers at different educational levels.

1. identify the need for the program;
2. determine the aims and instructional objectives of the program;
3. define the characteristics of the target group;
4. list the precise learning outcomes;
5. categorize learning outcomes according to Bloom's taxonomies;
6. break the material down into small units;
7. carefully sequence these units;
8. provide frequent practice to strengthen the stimulus–response bond;
9. ensure that the learner responds (does things);
10. observe and assess any behavioural changes;
11. provide opportunities for frequent learner feedback;
12. reinforce 'correct' behaviour with immediate rewards;
13. evaluate the effectiveness of the program;
14. modify and improve the program.

Pedagogy of Mathematics - Sem-II (TS)

1.4.3 CONSTRUCTIVIST APPROACH

Some teachers take a great interest when their pupils offer an incorrect answer to a question. In such a situation, these teachers proceed by asking pupils to explain how they arrived at that answer. It is likely that these teachers are operating from a constructivist perspective, which sees learners as constructors of meaning. By investigating the origins of a wrong answer, the teacher can uncover the learner's thinking processes, subsequently challenging and refining faulty mental constructs. It is difficult to draw a clear distinction between constructivism and cognitivism because constructivism is a natural progression from cognitivism and both are interested in cognitive processes. But whereas cognitivism focuses on how information is processed, constructivism focuses on what people do with information to develop knowledge. In particular, constructivism holds that people actively build knowledge and understanding by synthesizing the knowledge they already possess with new information.

For constructivists, learning is an active process through which learners 'construct' new meaning, such as,

- Knowledge is situated and constructed in social contexts.
- The learner is an active agent in the interpretation of the world.
- Constructivism focuses on meaning-making and the understanding of knowledge.
- Learning involves the interpretation of experience to construct meaning.
- Mental constructs may be modified as a result of confirmation or challenge.
- Other people are important in the formation and modification of mental constructs.

See the Table below for a comparison of behaviourism, cognitivism and constructivism.

Table

Theory	Mental activity	Learning process	Role of teacher
Behaviourism	Irrelevant	Stimulus–response Reinforcement External event	Controls environment and stimuli
Cognitivist	Perception Attention processing	Memory Surface and deep learning Encoding Internal event	Applies cognitive principles to facilitate cognitive processes
Constructivism	Meaning-making	Retuning schemata and mental constructs Internal event	Supports meaning-making Challenges existing ideas

❖ Educational implications of constructivism

Because constructivism is principally a theory about how people learn, we can draw many educational implications from the work of the key constructivist theorists, and those who have used constructivist principles. Some constructivist practices in education include:

- the diagnosis of learners' individual learning styles;
- the identification of learners' strengths or intelligences;
- curricular practices such as Individual Learning Plans (ILPs);
- attention to cultural inclusivity;
- innovative learning and teaching strategies such as problem-based learning;
- links between community-based learning and formal education;
- authentic assessment practices, which incorporate learners' views.

❖ Piaget

Jean Piaget (1896–1980) is commonly considered the pioneer and parent of constructivist thought. His theory of cognitive development, is based on principles which suggests that teachers should:

- nurture pupils' playfulness and natural curiosity about the world;
- use raw data and primary sources;

- provide physical, interactive and manipulative materials for pupils to work on;
- use cognitive terminology such as 'classify', 'analyse', 'predict' and 'create';
- encourage and accept learner autonomy and initiative;
- create opportunities for exploratory classroom discussion;
- engage pupils in experiences likely to engender cognitive conflict;
- Provide children an active engagement with their environment for the construction of meaning and learning;
- Teachers must not interfere and impose their ready-made solutions;

These views led to the 'discovery learning' school movement of the 1960s in which children were encouraged to discover the principles of subjects such as mathematics and science through processes of exploration.

❖ Bruner

According to the American psychologist Jerome Bruner, learning is goal-directed and driven by curiosity. Bruner adopted Piaget's ideas about active learning to form the basis of his principles of instruction and discovery learning (Bruner 1960).

Bruner believed that learning involves three processes:

- Knowledge acquisition, in which the learner asks, 'Does this confirm or refine my previous knowledge?' or 'Does this challenge my previous knowledge?'
- Knowledge transformation, in which the learner asks, 'What other things can this knowledge now do?'
- Knowledge review, in which the learner asks, 'Is the knowledge relevant?' and 'Is this knowledge adequate for the job in hand?'

He also considered that, to become mature thinkers, people must acquire three major intellectual skills for representing the world. In children, these usually appear in the sequence shown in Table

Table Bruner's representational modes

Mode	Representation	Example
Enactive	This is direct knowledge of how to do something.	A child sees herself in a mirror.
Iconic	Knowledge is represented by internal images that stand for an idea.	An older child (5 to 7 years) may draw a mirror including a reflection.
Symbolic	More abstract and flexible thought occurs. Language is the main tool for reflective thinking.	An adolescent may describe the physics of reflection for a plane mirror.

❖ Bruner and education

For Bruner, the teacher's task is to develop children's skills at particular ages, in different modes of representation. Bruner claims that any subject can be taught in some form to anybody at any age. The skill, of course, is to present knowledge in forms that are appropriate to the various ages of child learners. Thus, Bruner suggests that the following activities may be appropriate to his three representational modes:

Table

Mode	Activity
Enactive	Children should handle things, actually or virtually.
Iconic	Children should see and imagine things.
Symbolic	Children should perform symbolic operations.

❖ Vygotsky

The Russian psychologist Lev Vygotsky emphasized social processes as the means by which all reasoning and understanding arises. In particular, interactions with parents and other important adults lead to the creation of knowledge, which is internalized by children.

Vygotsky argued that all higher mental functions are internalized social relations:

'Schools are another cultural tool with a function of providing theoretical or scientific knowledge as opposed to the empirical and unstructured knowledge that people acquire naturally. This scientific knowledge has been fashioned over centuries and does not have to be reinvented by every child unaided'.

Unlike Piaget, Vygotsky affirms the role of teachers and experts in guiding learning as well as passing on theoretical knowledge, teachers support learners in the learning process.

The concept for which Vygotsky is best known is the ***Zone of Proximal Development (ZPD)***.

This is an intellectual space where learner and teacher interact.

- The teacher can gauge intellectual development of the learner and provide the appropriate support to advance the learner's thinking.
- With teacher support, learners can achieve more than they would unaided.
- More knowledgeable peers can perform the same function as teachers.

Vygotsky's ideas were ahead of their time and have generated much later research into the way that experts support learners.

❖ Bandura

Whereas Vygotsky argued that people learn by means of language – that is, by discussing concepts – the Canadian psychologist Albert Bandura went further and proposed that imitation of others is a cognitively efficient means of learning.

According to him, modelling involves imitative rather than original behaviour, but it can be seen in a constructivist light – that is, people adapt modelled behaviour as a mental framework for their own purposes. From an educational point of view, it is clear that much learning occurs by observation. Modeling is part of all learning – for example, in most subjects teachers will present examples of good work that they would like students to use.

❖ Vygotsky and education

Vygotskian thinkers have developed his theories to include guided learning and scaffolding.

- ***Guided learning***: joint knowledge construction aided by skillful teacher-managed discussion.

- ***Scaffolding***: supports which help learners to construct new knowledge.

Scaffolding may consist of resources, challenging activities and mentoring provided by teachers or more experienced peers. Scaffolding is a powerful metaphor as it suggests supports that are gradually withdrawn when learners have constructed their understanding and can act independently.

The level of scaffolding required is also affected by how far into the ZPD learners have progressed.

Four stages have been identified in learners' progression through the ZPD:

1. 1 Scaffolding is provided by others.
2. 2 Scaffolding is provided by learners themselves – for example, by self-talk.
3. 3 Scaffolding becomes redundant as learners act automatically.
4. 4 Scaffolding is required again if there are changes in the task or context.

Scaffolding strategies for the classroom In order to scaffold learning, teachers should:

- provide time for pupils to construct relationships with each other;
- allow pupils' responses to drive lessons, determining the teaching methodology and content;
- inquire about pupils' understanding of concepts, including false understandings, before sharing their own understanding of these concepts;
- encourage pupils to engage in dialogue with the teacher and with each other;
- encourage inquiry by asking open-ended questions and encouraging peer questioning;
- seek elaboration of pupils' responses to questions;
- wait for a response after asking questions;

- create metaphors and use different teaching styles to aid mental representation;
- model the behaviour or the techniques to be acquired.

❖ **NCF 2005 has special focus on "Constructivism"**

It means active engagement of the learner. It involves the following:

- Enquiry
- Exploration
- Questioning
- Debates
- Applications and reflection
- Leading to meaningful understanding
- Arriving to meaningful understanding
- Creation of new ideas

❖ **Highlights the value of interaction with:**

- Environment
- Peers
- Older people to enhance learning
- Learning task must be designed to enable children to seek out knowledge from sites other than textbooks.
- Need therefore to move away from rigid lesson planning to planning and designing activities that challenge children to think and try out what they are learning.

❖ **Core of high quality education and learning**

- Processing information
- Construct new knowledge
- Gather and analyse data
- Applying knowledge in new situations
- Problem Solving
- Regulating our own intellectual processes

1.5 RECOMMENDATIONS OF NCF-2005 AND APSCF-2011 ON MATHEMATICS CURRICULUMNATIONAL FOCUS GROUP POSITION PAPER ON MATHEMATICS AND STATE POSITION PAPER (2011) ON MATHEMATICS

1.5.1 RECOMMENDATIONS OF NCF-2005 ON MATHEMATICS CURRICULUM

NCF-2005 reiterate the values enshrined in our constitution, reduction of curricular burden on children, ensuing quality education for all and systematic changes as makers of curricular reform. It recognizes the primacy of children's experiences, their voices and their active involvement in the process of learning. Learning at school should be such that children can construct knowledge from experiences and environment.

Guiding principles of NCF 2005

- Connecting knowledge to life outside the school
- Ensuring that learning is shifted away from the rote methods
- Enriching the curriculum to provide for overall development of children rather than remain textbook centric
- Making examination more flexible and integrated into classroom life.

For Mathematics Vision of school Mathematics has been laid in NCF-2005 as follows:

- Children learn to enjoy Mathematics rather than fear it.
- Children learn important Mathematics: Mathematics is more than formulas and mechanical procedures.
- Children see Mathematics as something to talk about, to communicate through, to discuss among them, to work together on.
- Children pose and solve meaningful problems.
- Children use abstractions to perceive relation-ships, to see structures, to reason out things, to argue the truth or falsity of statements.
- Children understand the basic structure of Mathematics: Arithmetic, Algebra, Geometry and Trigonometry, the basic content areas of school Mathematics, all offer a methodology for abstraction, structuration and generalization.
- Teachers engage every child in class with the conviction that everyone can learn Mathematics.

Recommendations of NCF

- Shifting the focus of Mathematics education from achieving 'narrow' goals of mathematical content to 'higher' goals of creating mathematical learning environments, where processes like
- Formal problem solving
  - Use of heuristics
  - Estimation and approximation
  - Representation
  - Reasoning and proof
  - Making connecting and mathematical communication take precedence.
- Engaging every student with a sense of success, while at the same time offering conceptual challenges to the emerging mathematician.
- Changing modes of assessment to examine student's mathematisation abilities rather than procedural knowledge.
- Enriching teachers with a variety of mathematical resources.

1.5.2 RECOMMENDATIONS OF APSCF-2011 ON MATHEMATICS CURRICULUM

As a part of development of State Curriculum Framework, the curriculum committee examined the major challenges and concern being faced by the school education system in the state. A high-powered Advisory Committee was constituted. It was decided to develop a Curriculum Framework document along with 20 Position Papers in different domains of knowledge. National and State level experts from different universities and institutions and teachers, teacher educators and NGOs were involved in the process.

State Curriculum Framework – Guiding Principles

- Keeping the potential of the child to learn always in focus.

- Respecting the systems of knowledge such as languages children bring to school.
- Connecting knowledge to life outside the school
- Children should not feel that what they are learning at school has no relevance to their lives.
- Ensuring that learning is shifted away from rote methods and the focus should be on interactions, project work, and analysis etc.
- Enriching the curriculum to provide for overall development of children rather than remain textbook centric.
- Making examinations more flexible and integrated into classroom life
- More focus on assessment for learning than assessment of learning.
- Promoting social constructivism, issue-based curriculum and critical pedagogy across curricular areas.
- Nurturing towards flora and fauna, respect for bio-diversity and social-diversity, respect respect to the work shall be prompted as a part of school curriculum.
- Locating classroom practices in the languages and cultures of children.

1.5.3 RECOMMENDATIONS OF NATIONAL FOCUS GROUP POSITION PAPER ON MATHEMATICS

The main goal of mathematics education in schools is the mathematisation of the child's thinking. Clarity of thought and pursuing assumptions to logical conclusions is central to the mathematical enterprise. There are many ways of thinking and the kind of thinking one learns in mathematics is an ability to handle abstractions, and an approach to problem solving. Universalization of schooling has important implications for mathematics curriculum. Mathematics being a compulsory subject of study, access to quality mathematics education is every child's right.

Recommendations of National Focus Group Position Paper on Mathematics

- Shifting the focus of Mathematics education from achieving 'narrow' goals of mathematical content to 'higher' goals.

- Engaging every student with a sense of success, while at the same time offering conceptual challenges to the emerging mathematician.
- Changing modes of assessment to examine student's mathematisation abilities rather than procedural knowledge.
- Enriching teachers with a variety of mathematical resources.

1.5.4 RECOMMENDATIONS OF STATE POSITION PAPER (2011) ON MATHEMATICS

Huge curriculum load in terms of information loaded textbooks, ineffective methods of teaching learning processes, memory based examinations etc. warranted for improving the existing situation by way of undertaking curricular and examination reforms. This document lays the foundations of a completely fresh perspective on the education of children keeoing their potential to learn at the heart of curriculum planning.

- From teacher cantered to learner cantered.
- From Teaching to Learning.
- From Textbook oriented to Experiential oriented.
- From Lecture method to activity based.
- From memorizing the knowledge to construction of knowledge.

The shift in focus from mathematical content to mathematical learning environments, where a whole range of processes take precedence: formal problem solving, use of heuristics, estimation and approximation, optimisation, use of patterns, visualisation, representation, reasoning and proof, making connections, mathematical communications. Giving importance to these processes also helps in removing fear of mathematics from children's minds. A crucial implication of such a shift lies in offering a multiplicity of approaches, procedures, solutions. Such learning environment invites participation, engage children, and offer a sense of success.

1.6 TRENDS IN MATHEMATICS

The three principal broad trends in mathematics can be characterise as:

i. A new unity in the mathematical sciences,
ii. Variety of Applications

iii. the presence of the computer technology.

i. Unity of Mathematical Sciences

Developing children's abilities for mathematisation is the main goal of mathematics education. The narrow aim of school mathematics is to develop 'useful' capabilities, particularly those relating to numeracy–numbers, number operations, measurements, decimals and percentages. The higher aim is to develop the child's resources to think and reason mathematically, to pursue assumptions to their logical conclusion and to handle abstraction. It includes a way of doing things, and the ability and the attitude to formulate and solve problems.

This calls for a curriculum that is ambitious, coherent and teaches important principles of mathematics.

- It should be ambitious in the sense that it seeks to achieve the higher aim mentioned above, rather than only the narrower aim.
- It should be coherent in the sense that the variety of methods and skills available piecemeal (in arithmetic, algebra, geometry) cohere into an ability to address problems that come from other domains such as science and social studies in high school.
- It should be important in the sense that students feel the need to solve such problems, that teachers and students find it worth their time and energy to address these problems.

The twin concerns of the Mathematics curriculum are:

- what can mathematics education do to engage the mind of every student, and
- how can it strengthen the student's resources?

As mathematics is a compulsory subject at the secondary stage, access to quality mathematics education is the right of every child. In the context of universalization ofeducation, the first question to ask is, -"what mathematics can be offered in eight years of schooling that will stand every child in good stead rather than be a preparation for higher secondary education alone?".

Most of the skills taught in primary school mathematics are useful. However, reorientation of the curriculum towards addressing the 'higher aims' mentioned above will make better use of the time that children spend in school in terms of the problem-solving and analytical skills that it builds, and in preparing children to better meet a wide variety of problems in life. Also, the tall shape of mathematics (where mastery of one topic is a prerequisite for the next) can be de-emphasised in favour of a broader-based curriculum with more topics that starts from the basics. This will serve the needs of different learners better.

Following general tactics of problem solving can be taught progressively during the different stages of school:

- abstraction,
- quantification,
- analogy,
- case analysis,
- reduction to simpler situations,
- even guess-and-verify exercises.

Moreover, when children learn a variety of approaches (over time), their toolkit becomes richer, and they also learn which approach is the best. Children also need exposure to the use of heuristics, or rules of thumb, rather than only believing that Mathematics is an 'exact science'. The estimation of quantities and approximating solutions is also an essential skill.School Mathematics can play a significant role in developing such useful skills.

Visualisation and representation are skills that Mathematics can help to develop. Modelling situations using quantities, shapes and forms are the best use of mathematics.

Mathematical concepts can be represented in multiple ways, and these representations can serve a variety of purposes in different contexts. All of this adds to the power of Mathematics.

For example, a function may be represented in algebraic form or in the form of a graph. The representation p/q can be used to denote a fraction as a part of the whole, but can also denote the quotient of two numbers, p and q. Learning this about fractions is as important, if not more, than learning the arithmetic of fractions.

ii. Variety of Applications

There is also a need to make connections between Mathematics and other subjects of study.

When children learn to draw graphs, they should also be encouraged to think of functional relationships in the sciences, including geology. Our children need to appreciate the fact that Mathematics is an effective instrument in the study of science.

The importance of systematic reasoning in Mathematics cannot be over-emphasised, and is intimately tied to notions of aesthetics and elegance so dear to mathematicians.Proof is important, but in addition to deductive proof, children should also learn when pictures and constructions provide proof. Proof is a process that convinces a sceptical adversary; school mathematics should encourage proof as a systematic way of argumentation. The aim should be to develop arguments, evaluate arguments, make and investigate conjectures, and understand that there are various methods of reasoning.

Mathematical communication is precise and employs unambiguous use of language and rigour in formulation, which are important characteristics of mathematical treatment. The use of jargon in Mathematics is deliberate, conscious and stylised. Mathematicians discuss what appropriate notation is since good notation is held in high esteem and believed to aid thought. As children grow older, they should be taught to appreciate the significance of such conventions There is also a need to make connections between Mathematics and other subjects of study. When children learn to draw graphs, they should also be encouraged to think of functional relationships in the sciences, including geology. Our children need to appreciate the fact that Mathematics is an effective instrument in the study of science.

For instance, this means that setting up of equations should get as much coverage as solving them.

In discussing many of these skills and processes, we have referred to a multiplicity of approaches and procedures. These are all crucial for liberating school Mathematics from the tyranny of applying them only to those algorithms that are taught.

iii. Use of technology

Technology can greatly aid the process of mathematical exploration, and clever use of such aids can help engage students. Calculators are typically seen as aiding arithmetical operations; while this is true, calculators are of much greater pedagogic value. Indeed, if one asks whether calculators should be permitted in examinations, the answer is that it is quite unnecessary for examiners to raise questions that necessitate the use of calculators. On the contrary, in a non- threatening atmosphere, children can use calculators to study iteration of many algebraic functions..

If ordinary calculators can offer such possibilities, the potential of graphing calculators and computers for mathematical exploration is far higher. However, these are expensive, and in a country where the vast majority of children cannot afford more than one notebook, such use is luxurious. It is here that governmental action, to provide appropriate alternative low-cost technology, may be appropriate. Research in this direction will be greatly beneficial to school education.

It must be understood that there is a spectrum of technology use in mathematics education, and calculators or computers are at one end of the spectrum.

While notebooks and blackboards are the other end, use of graph paper, geo boards, abacus, geometry boxes etc. is crucial. Innovations in the design and use of such material must be encouraged so that their use makes school mathematics enjoyable and meaningful.

1.7 MOVING FROM TEXTBOOK TO TEACHING-LEARNING MATERIALS, GOING BEYOND THE TEXTBOOK

According to researcher, the Activity Based Approach is unique and effective to attract school children. The teachers who are involved in implementing this method have developed activities for each learning unit which facilitated readiness for learning. The aim of activity-based approach is for learners to construct process of self-learning and problem solving and transfer of information and skills. This method has brought out the potential of the learners in classroom situations.

Need of activity based approach:

According to Dhand (1995), "child-centered educational aids to foster self- learning and allows a child to study according to his/her aptitude and skill. Activities in each milestone include games, rhymes, drawing and songs to teach a letter or a word, or understand a concept."

- Encourages independence and team learning
- Provides a wide variety of manipulative open-ended and creative activities
- Provides students experience and active participation in the exploration of their environment
- Make students advance at their own rate –the rate that is with their abilities, interest and motivations
- Encourages self-reliance and development of initiative in an atmosphere of trust
- Encourages children to follow many of their own interests and desires to learn
- Problem-solving, critical and creative thinking and deep understanding are emphasized
- Learners are encouraged to explore the new knowledge independently.

Types of Teaching-Learning Aids in Mathematics

Equipments needed by learners to pursue their own individual studying comprise text books writing equipments, simple drawing and measuring instruments like geometry instruments like geometry instrument box, slid rule, calculator.

i. School Laboratory

The equipment should be provided by the school in the mathematics laboratory. Weights and measurement, drawing instruments (Black board apparatus real objects such as beads, ball frames abacus, coins (card board models), facsimile currency notes, toy money seeds, sticks, pebbles, coloured balls / solids pencils various work experience materials should be provided in the numerals and digits engraved on the cross roads square circular figures, models relating to the teaching of theorems should also be provided.

Pedagogy of Mathematics - Sem-II (TS)

Aids to teach angles, tangent, chord property board, strings nails etc, are also required.

Black board magic lanterns epidiascopes, over head projectors. Films strip projects Radio and Television sets, (Telecast) newspapers must also be made available.

ii. Pictures and Charts

- Pictures and charts coins geometrical figures and shapes are also needed we also require charts containing various units of weights and measures.
- Pictures of great mathematicians.
- Logarithm flow charts.
- Pictures related to the history of mathematics in every day life.
- Pictures charts showing use of mathematics in everyday life.

iii. Black Board and Geo Board

Black board is one of the necessary equipments of the class and is the commonest and the most indispensable aid for mathematics teacher. If used properly, it becomes the most valuable device of making teaching concrete and comprehensive. Geo boards are also used to draw fine geometrical figure, graph, drawing etc. Black board and Geo board are described as the second tongue of the mathematics teachers. But it is a pity that many teachers fail to use them. Generally in Geo board coloured chalks are used to draw geometrical.

iv. Transparencies

Overhead projector transparencies is a useful alternative to the black board. In that teachers can save time by preparing the material in advance. This device is a wonderful teaching device with the help of which it is possible to make vivid enlarged projections of objects from the size of a postage stamp to practically that of a quarto page. This can be used to project transparent as well as opaque picture. It can also be used to project content figures Nd diagrams of the printed or written pages. It is quite useful for complicated figures.

v. Models

Models are the important Visual teaching devises. Models concrete object some considerably larger than the real objects some small replicas of objects which are too larger.An object is s real thing, but a mode is just a recognisable three dimensional representation of it. Even a square peace of cardboard is a model. All other geometric farms can be cut out of cardboard or a thick paper and used as models.

vi. Flannel Cutting

Flannel cutting are the useful aids which can also be applied in teaching mathematics. The chief value of this is that the prepared diagrams, pictures are needed and can be used again and again over a period of years.Sometime it is difficult to build up a complicated diagram, graph, and picture. In such a case the teacher with the help of the students can collect the cutting and show on Flannel board. The cuttings of the pictures and diagrams can be displayed on flannel board. Children can observe them and learn the concepts. Flannel cutting help in making mathematics teaching real. They bring clarity and enrich meaning.

vii. Provision of Sunday

Special mathematical games riddles, and puzzles, visits to exhibitions, factories and fairs and exhibitions post offices, banks are also very useful.

viii. Using ICT

Many software programmes can be used at different levels within the one group or class. Valuable teacher time can be taken up in establishing the correct starting point for a particular student. Colour codes and symbolic representations taped to the front of software boxes can help, and a card with clear instructions can be given to the individual student. Once students have practiced this procedure, they will be able to locate and load the software themselves, and an older student can either help or supervise. It is helpful to keep symbols constant and, where possible, include the students in the choice of how information can be presented symbolically.

The following key benefits have been identified from the research evidence:

- Greater collaboration between pupils
- An increased focus on strategies and interpretation
- Fast and accurate feedback to pupils using ICT

- Increased motivation amongst pupils.
- Teachers can maximise the impact of ICT in maths teaching by:
- using ICT as a tool in working towards learning objectives
- developing a knowledge of the multimedia software available
- considering how to provide access to ICT resources for all
- incorporating the use of portable ICT equipment in teaching.

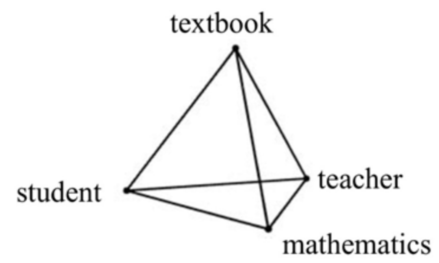
1.8 PRINT RESOURCES: TEXTBOOKS, POPULAR MATHEMATICS BOOKS, JOURNALS AND MAGAZINES

Materials in the typical general education classroom tend to be limited in scope. Commonly found supplies such as textbooks may be supplemented with student workbooks or worksheets. These tools functions as an add-ons to the curriculum rather than as an embedded tool for delivering the curriculum.

1.8.1 TEXT BOOKS

The mathematics textbook is one of the most important resources for teaching and learning mathematics. Whereas a number of studies have examined the use of mathematics textbooks by teachers there is a dearth of research into the use of mathematics textbooks by students.

Below is the tetrahedron model of the textbook use,where mathematics textbook is implemented as an instrument at all three sides of the triangle: teachers use textbooks in the lesson and to prepare their lessons, by using the textbook in the lesson teachers also mediate textbook use to students, and finally students learn from textbooks. Thus, each triangle of the tetrahedron-model represents an activity system on its own.



Uses of Text Books

- Textbooks are especially helpful for beginning teachers. The material to be covered and the design of each lesson are carefully spelled out in detail.
- Textbooks provide organized units of work. A textbook gives you all the plans and lessons you need to cover a topic in some detail.
- A textbook series provides you with a balanced, chronological presentation of information.
- Textbooks are a detailed sequence of teaching proceduresthat tell you what to do and when to do it. There are no surprises—everything is carefully spelled out.
- Textbooks provide administrators and teachers with a complete program. The series is typically based on the latest research and teaching strategies.
- Good textbooks are excellent teaching aids. They're a resource for both teachers and students.

Qualities of good text-book

The qualities of a good text book in mathematics can be broadly classified under the following criteria.

1. Mechanical Features
2. Author
3. Organisation and presentation of the subject matter

1. Mechanical Features

- Printing should be done on good quality paper with strong and durable binding.
- Font should be appropriate and easily readable.
- Cover page should be appealing and attractive.

2. Author

- It should be written by well-qualified and well-experienced author.

Pedagogy of Mathematics - Sem-II (TS)

- It should be revised by a committee constituted by the State Government.

3. Organisation and presentation of the subject matter

Following points should be considered in organisation and presentation of the subject matter,

- It should be based on the designed curriculum and syllabus
- Its language should be simple and clear.
- It should suit the age of learner.
- It should be according to according to the aims and objectives of teaching mathematics.
- It should contain all the definitions, concepts and principles with clear and definite of purpose.
- It should contain relevant matter.
- It should contain appropriate and attractive illustrations, pictures, figures, graphs and tables related to the content matter.
- It should consist of the international terminology in terms symbols, formulae and definitions.

The following table lists some of the most common weaknesses of textbooks, along with ways of overcoming those difficulties

Weakness	Student Difficulty	Ways of Overcoming Problem
The textbook is designed as a sole source of information.	Students only see one perspective on a concept or issue.	Provide students with lots of information sources such as trade books, CD-ROMS, websites, encyclopedias, etc.
Textbook is old or outdated.	Information shared with students is not current or relevant.	Use textbook sparingly or supplement with other materials.
Textbook questions tend to be low level or fact-based.	Students assume that learning is simply a collection of facts and figures.	Ask higher-level questions and provide creative thinking and problem-solving activities.
Textbook doesn't take students' background knowledge into account.	Teacher does not tailor lessons to the specific attributes and interests of students.	Discover what students know about a topic prior to teaching. Design the lesson based on that knowledge.
Reading level of the textbook is too difficult.	Students cannot read or understand important concepts.	Use lots of supplemental materials such as library books, Internet, CD-ROMs, etc.
The textbook has all the answer to all the questions.	Students tend to see learning as an accumulation of correct answers.	Involve students in problem-solving activities, higher-level thinking questions, and extending activities.

1.8.2 POPULAR MATHEMATICS TEXT BOOKS

1. Euclid's Element by Euclid
2. The Principles of Mathematics by Bertrand Russell
3. Fermat's Enigma: The Epic Quest to Solve the World's Greatest Mathematical Problems by Simon Singh
4. Journey through Genius: The Great Theorems of Mathematics by William Dunham
5. How to solve it: a new aspect of mathematical method by George Polya.
6. Mathematics Mysteries by Calvin C. Clawson.
7. Handbook of mathematics by R.P.Singh

1.8.3 MATHEMATICS JOURNALS

Many mathematics Institutions and associations publishes journals which are research based articles in various magazines on various concepts of mathematics. Following are the popular journals and magazines**:**

1. The Maths Teacher (Bi-monthly) - S.I.T.U. Colony, Madras - 28
2. Ghanita Bharati - Bulletin of the Indian Society for History of Mathematics
3. Mathematics Today - Green Park Extension, New Delhi
4. Resonance - Journal of Science Education, Indian Academy of Sciences, Bangalore
5. Telugu - (Additional Magazine) - Monthly, Telugu Academy l, Hyderabad.
6. Ganita Chandrika (Quarterly) Association for improvement of Maths Education, Vijayawada
7. The Maths Education - Station Road, Siwan, Bihar (Quarterly)
8. Mathematical Education - Macmillan India Ltd., Patullos Road, Chennai. (Quarterly)
9. Mathematics Teacher, N.C.T.M., N.Y. Washington D.C.
10. ArithmeticTeacher, N.C.T.M., N.Y. Washington D.C

11. Scripta Mathematics, Amsterdam Avenue, New York.
12. Utilitas Mathematica ISSN:0315-3681)
13. *American Mathematical Monthly* is designed to be accessible to a wide audience.
14. *The Mathematical Gazette* contains letters, book reviews and expositions of attractive areas of mathematics.
15. *The Mathematical Intelligencer* is a mathematical journal that aims at a conversational and scholarly tone.
16. *Notices of the AMS* - Each issue contains one or two expository articles that describe current developments in mathematical research, written by professional mathematicians. The Notices also carries articles on the history of mathematics, mathematics education, and professional issues facing mathematicians, as well as reviews of books, plays, movies, and other artistic and cultural works involving mathematics.

1.8.4.MAGAZINES

1. *Mathematics Magazine C/o Dept of Maths, Purdue University,* offers lively, readable, and appealing exposition on a wide range of mathematical topics
2. Popular science magazines such as New Scientist and Scientific Americansometimes carry articles on mathematics.
3. Plus Magazine is a free online magazine run under the Millennium Mathematics Project at the University of Cambridge.

1.9 DALE'S CONE OF EXPERIENCE- USING THE CONE OF EXPERIENCE

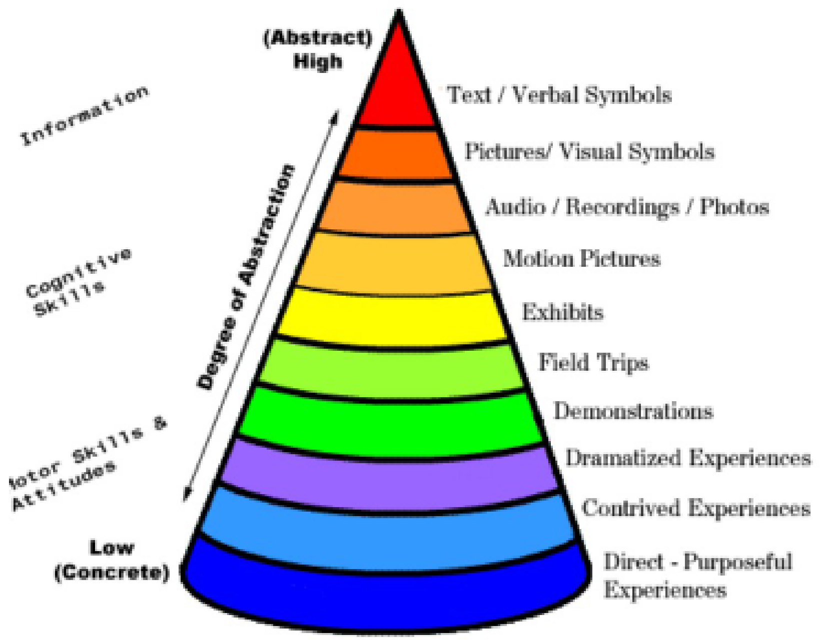
Edgar Dale (April 27,1900 in Benson, Minnesota, - March 8, 1985 in Columbus, Ohio) was an American educationist who developed the Cone of Experience.

Dale's Cone of Experience is a visual model that is composed of eleven (11) stages starting from concrete experiences at the bottom of the cone then it becomes more and more abstract as it reach the peak of the cone. Also, according to Dale, the arrangement in the cone is not based on its difficulty but rather based on abstraction and on the number

of senses involved. The experiences in each stages can be mixed and are interrelated that fosters more meaningful learning.

According to one of the principles in the selection and use of teaching strategies, the more senses that are involved in learning, the more and the better the learning will be but it does not mean that concrete experience is the only effective experience that educators should use in transferring knowledge to the learner. Like what was mentioned above, the experiences in each stages can be mixed and are interrelated thus, a balance must be achieved between concrete and abstract experiences in order to cater the and address all the need of the learner in all the domains of development and in order to help each learner in their holistic development.

Moreover, the generalisation about the Cone of Experience that was presented above is not enough. Actually, we should try to go deeper in each of the component of the cone since Educational Technology basically revolves around the Cone of Experience. By going one-by-one, starting from concrete to abstract, we will understand more the different components of the cone that will help us in grasping the real meaning of educational technology.To expand on each of the components, let us begin with the Direct Purposeful Experiences. These are first hand experiences which serve as the foundation of learning.



Graphic courtesy of Edward L. Counts, Jr.

1.9.1 LEVELS OF EDGAR DALE OF THE CONE OF EXPERIENCE ARE AS FOLLOWS:

Edgar Dale divided his of the Cone of Experience into three levels to explain students the learning experience. His cone of experience is as follows:

Level –I: Enactive – Direct Experiences

i. Direct Purposeful Experiences
ii. Contrived Experiences
iii. Dramatized experiences

Level-II: Iconic – Pictorial Experiences

iv. Demonstrations
v. Field Trips
vi. Exhibits
vii. Motion pictures

viii. Still Pictures / Educational Television
ix. Audio/Recording/Photos

Level-II: Symbolic – High Abstract Experiences

x. **Visual symbols**
xi. **Verbal Symbols**

i. Direct Purposeful Experiences

In this level, more senses are used in order to build up the knowledge. Also, in this level, the learner learned by doing things by him/herself. Learning happens through actual hands-on experiences. This level explains and proves one of the principles in the selection and use of teaching strategies, the more senses that are involved in learning, the more and the better the learning will be. This level also proves that educational technology is not limited to the modern gadgets and software that are commercially available nowadays. This shows that even the simple opportunity that you give to each child could help them learn. For example usage of the mathematical kit effectively etc.

ii. Contrived Experiences

The next level would be the Contrived Experiences. In this level, representative models and mock-ups of reality are being used in order to provide an experience that as close as reality. This level is very practical and it makes learning experience more accessible to the learner. In this stage, it provides more concrete experiences, even if not as concrete as direct experiences, that allows visualization that fosters better understanding of the concept. With this level of experience teacher can easily explain the concept of mensuration.

iii. Dramatized experiences

On the other hand, the next level would be the Dramatised experiences. In this level, learners can participate in a reconstructed experiences that could give them better understanding of the event or of a concept. Through dramatised experiences, learners become more familiar with the concept as they emerge themselves to the "as-if" situation. For example the concept of banking where the students can practically learns the concepts of profit and loss, discount and interest etc.

iv. Demonstrations

The next level would be the Demonstrations. It is a visualize explanation of important fact, idea, or process through the use of pictures, drawings, film and other types of media in order to facilitate clear and effective learning. In this level, things are shown based on how they are done. With this geometrical concepts like properties of triangles or the other various shapes can be taught easily and effectively.

v. Field Trips

Another level would be the Study Trips. This level extends the learning experience through excursions and visits on the different places that are not available inside the classroom. Through this level, the learning experience will not be limited to the classroom setting but rather extended in a more complex environment. For example visiting shops, museums etc.

vi. Exhibits

The level of study trips is followed by exhibits. It is a somewhat a combination of some of the first levels in the cone. Actually, exhibits are combination of several mock ups and models. Most of the time, exhibits are experiences that is “for your eyes” only but some exhibits includes sensory experiences which could be related to direct purposeful experiences. In this level, meanings ideas are presented to the learners in a more abstract manner. This experience allows student to see the meaning and relevance of things based on the different pictures and representations presented.

vii. Motion pictures

The next levels would be the level of television and motion pictures. Because of the rapid development of the modern technology, a lot of people believe that Educational technology is limited to these stages. They are not aware that these sages are only a small portion of EdTech. For television and motion pictures, it implies values and messages through television and films.

viii. Still Pictures / Educational Television

These are one dimensional slide, film strips and micro projections are classified as one dimensional. They can be either projected on to a screen by means of projection machines to provide visual experience.

ix. Audio/Recording/Photos

Recordings and radio are visual and auditory devices that can be used by a learner/group of learner that could enhance and extend learning experience. Using this level graphs, geometrical constructions, trigonometry etc. concepts can be taught.

x. Visual symbols

The most complex and abstract among all the components of the Cone of Experience. In the visual symbolic level, charts, maps, graphs, and diagrams are used for abstract representations. It helps in the clarification of the concept like fractions, place value, computation etc.

xi. Verbal Symbols

The verbal symbolic level does not involve visual representation or clues to their meanings. Mostly, the things involved in this level are words, ideas, principles, formula etc.

1.9.2 USES OF EDGAR DALE OF EXPERIENCE

The cone of experience helps the teacher in the following ways:

- Facilitates teaching-learning in complex concepts
- Teacher can use variety of materials and medium in order to maximize the learning experience.
- Gives clear and complete understanding of the concepts
- Helps the students in gaining real life experiences
- Improves retention rate of teaching-learning
- Student cantered approach
- Inculcates creativity and thoughtfulness among students
- Develops high level of mathematical thinking that includes, where appropriate, making comparisons, conjectures, interpretations, predictions, or generalizations

After going through the different components of the Cone of Experience, it could be said that itone medium is not enough thus if we can take take advantage of the other media. There’s nothing wrong with trying to combine several medium for as long as it could benefit the learners. Also, through the levels provided by the Cone of Experience, it could be said that concrete experiences must be provided first in order to support abstract learning.

Lastly, staying on the concrete experiences is not even ideal because through providing abstract experiences to the learner, the more he/she will develop his/her higher order thinking skills which is important for more complex way of thinking and for dealing with more complex life situations. Through understanding each component of the Cone of Experience, it could be said that Educational Technology is not limited to the modern gadgets that we have right now but rather it is a broad concept that includes all the media that we can use to attain balance as we facilitate effective and meaningful learning.

1.10 TEACHER AS CURRICULUM DEVELOPER- LOCALIZED CURRICULUM, PLACE FOR ARTISANS, KNOWLEDGE SYSTEMS IN CURRICULUM, LOCAL INNOVATORS AND INNOVATIVE PRACTICES IN MATHEMATICS

1.10.1 TEACHER AS CURRICULUM DEVELOPER

While curriculum specialists, administrators and outside educational companies spend countless hours developing curriculum, it is the teachers who know best what the curriculum should look like. After all, they work directly with the students meant to benefit from the curriculum. In order to create a strong curriculum, teachers must play an integral role in every step of the process.

The systemic changes that we have advocated require substantial investments of time, energy, and support on the part of teachers. Professional development, affecting the beliefs, attitudes, knowledge, and practices of teachers in the school, is central to achieving this change. In order for the vision described in this paper to become a reality, it is critical that professional development focuses on mathematics specifically. Generic 'teacher training' does not provide the understanding of content, of instructional techniques, and of critical issues in mathematics education that is needed by classroom teachers.

There are many mechanisms that need to be ensured to offer better teacher support and professional development, but the essential and central requirement is that of a large treasury of resource material which teachers can access freely as well as contribute to. Further, networking of teachers so that expertise and experience can be shared is important. In addition, identifying and nurturing resource teachers can greatly help the process. Regional mathematics libraries may be built to act as resource centres.

An important area of concern is the teacher's own perception of what mathematics is, and what constitute the goals of mathematics education. Many of the processes we have outlined above are not considered to be central by most mathematics teachers, mainly because of the way they were taught and a lack of any later training on such processes.

1.10.2 LOCALIZED CURRICULUM INNOVATIONS IN TEACHING MATHEMATICS

The Education Commission (1964-66) points out that "In the teaching of Mathematics emphasis should be more on the understanding of basic principles than on the mechanical teaching of mathematical computations".

Innovations in teaching of mathematics can be diversified in terms of Methods, Pedagogic Resources and Mastery Learning Strategy used in teaching-learning process.

1. Mastery Learning Strategy

Teaching Strategy is a generalized plan for a lesson and includes a specific structure to be followed. B.S. Bloom has developed Mastery Learning Strategy. It consists of different steps: division of content into units, formulation of objectives related to each unit, teaching and instruction are organized for realizing objectives of each unit, administering unit test to evaluate the mastery level and diagnose the learning difficulties, remedial instructions are given to remove the difficulties and attain mastery level by every student. This strategy plays an important role for learning of basics and fundamentals e.g. operations in different number systems – Natural numbers, Integers, Rational numbers, Real numbers.

2. Methods

Method is a style of the presentation of content in classroom. The following are the innovative methods that I used to make teaching-learning process of Mathematics effective.

i. Inducto -Deductive Method

(Inductive method is to move from specific examples to generalization and deductive method is to move from generalization to specific examples). In classroom usually the instructions directly start with the abstract concepts and are being taught in a way that does not bring understanding on the part of majority of the students. Formulas, theorems, examples, results are derived, proved and used. But I start with specific examples and concrete things and then move to generalizations and abstract things. Then I show how generalization can be derived and it holds true through specific examples. This method helps students for better understanding, students don't have to cram the things and will have long lasting effect. Example: Pythagoras Theorem - In a right-angle D ABC right angled at B,*AB2 + BC2 = AC2* (Considering right angle triangles of different measurement leading to generalization and then establishing it through the theoretical proof).

ii. Analytic-Synthetic Method

(Analytic is breaking down and moving from unknown to known and Synthetic is putting together known bits of information and moving from known to unknown.) These methods are basically used in proving the results and solving sums. In textbooks mostly synthetic method is used, to prove something unknown we start with a certain known thing, but that leaves doubt in mind of students why we have started with that step and using this particular known thing. we use combination in order to explain and relate each step logically.

iii. Problem-Solving Method

This method aims at presenting the knowledge to be learnt in the form of a problem. It begins with a problematic situation and consists of continuous meaningful well-integrated activity. I choose a problem and give it to my students and engage them without spendingtime in going over the things. As they struggle with the problem to get solution, meanwhile it helps them in developing divergent thinking.

Example: There is a problem of finding the amount of water in a given container instead of deriving the formula of volume (cylinder filled with water).

iv. Play-Way Method

Try to use the activities that include a sort of fun or play and give joy to the students in my class room teaching. As the students don't realize that they are learning but in a way they are gaining knowledge through participating in different activities. This way helps to develop interest in mathematics, motivates them to learn more and reduces the abstract nature of the subject to some extent inherently.

Example: Mathematical games and puzzles.

v. Laboratory Method

Practice the teaching in class through the way of "learning by doing" and "learning by observation" and proceeding from concrete to abstract. All students do not just listen to the information given but do something practically also. They learn through hands on experience. This way leads them to discover mathematical facts. After discovering something by their own efforts, the they start taking pride in his achievement, it gives them in return happiness, mental satisfaction and encourages them towards further achievements.

Example: Making and observing models, paper folding, paper cutting, and construction work in geometry.

3. Pedagogic Resources

We can use pedagogic resources in teaching practice to integrate in a method for the transaction of a particular content and draw upon to advance the students' learning.

Teaching Aids

I am confirmed of the view that Teaching aids are the materials used for effective teaching and enhancing the learning of students. It can be anything ready- made or made by the teacher or made by students. Different teaching aids are used by me in teaching mathematics like

Charts, Manipulative , Programmed Learning Material (PLM), computers etc.

a. ***Charts*** – are used in class to display formulae, symbols, mathematical and geometrical figures. Charts are used for making students familiar to the symbols and for memorization of basic formulae. Even it is used to bring to the students two-dimension geometry and the graphical representation in a better way.

b. ***Manipulative*** – are such objects or materials that involve mathematics concepts, appealing to several senses, that can be touched and moved around by the students (not demonstrations of materials by the teacher). Each student needs material to manipulate independently. With students actively involved in manipulating materials, interest in mathematics will be aroused. Canny (1984) has shown that mathematics instruction and students' mathematics understanding will be more effective if manipulative materials are used. Models can be used to make things concrete like three dimension figures in geometry.

c. ***Programmed Learning Material (PLM)*** – It is a self-learning material in which learner can proceed at his own pace. It has the characteristics of all sequential steps, learner's response, self-pacing, immediate feedback, reinforcement and self- evaluation. It is helpful in acquisition of concepts like fractions, number systems, etc. and can be used as a remedy for slow learners for a specific content.

d. ***Computers and Television*** – Computer can be used for multimedia presentation for the concepts that requires visualization and imagination. Computer can also be used for providing Computer Assisted Instruction (CAI), it is similar to PLM i.e. it is a computerized PLM. Television can be used to show some good mathematics education show.

e. ***E- activities***

Activities here include all such work where in students play an active role,

Have to interact with different resources and generate knowledge. It includes Quiz competition, Projects, Role play, Seminars, Discussion, Mathematics club, Assignment, Field trips, etc.

Name of the Activity	Examples/Situations where Activity can be used
Quiz Competition	Logic, Properties of Numbers, Mathematical Rules and Results
Projects	Contribution by Different Mathematicians
Role Play	Arithmetical concepts like Profit & Loss, Simple & Compound Interest
Seminars	Shortcuts through Vedic Mathematics, Application of Mathematics in other Disciplines
Discussion	Properties of 'Zero', Difference between Rational and Irrational Numbers, Relating Different Concepts in Mathematics
Mathematics Clubs	Application of the concept studied, Preparing Models, Paper Folding (Origami)
Assignment	Self-Study, Extension of Knowledge
Field Trips	Experiencing the Functional use of Mathematics in Bank, Insurance Company

In any curriculum, content and presentation of content are the two most important and inseparable components. It is difficult to say anything definitely about which method and pedagogic resource is going to be most effective for presentation of a particular type of content. Selection of method and pedagogic resource depends on many factors like type of content, objectives to be achieved, level of the students, entry behaviour, availability of resources. Also acceptance of innovative methods and positive attitude of teachers towards it, is an important factor for the selection of method and pedagogic resource. The things included under innovations are existing in books, also there are researches which shows that some innovations are carried out in the classroom and has shown the positive effect on teaching learning process but their practical usage and implementation in classroom is not seen to the expected level.

1.10.3 PLACE OF ARTISANS

Mathematics and art are related in a variety of ways. Mathematics is described as an art motivated by beauty. Certain principles of geometry practices are discerned in arts as music dance architecture sculpture and textiles. The two-dimensional and three dimensional geometric constructions are used in the pattern of ornaments. The various concepts of geometry similar triangles, co-ordinate geometry, trigonometry etc. has got a great scope for artisans, architects, engineers and archaeologists.

1.10.4 KNOWLEDGE SYSTEM IN CURRICULUM

Offering a range of material to teachers that enriches their understanding of the subject, provides insights into the conceptual and historical development of the subject and helps them innovate in their classrooms is the best means of teacher support. For this, providing channels of communication with college teachers and research mathematicians will be of great help. When teachers network among themselves and link up with teachers in universities, their pedagogic competence will be strengthened immensely. Such systematic sharing of experience and expertise can be of great help.

Planning

Teachers know their students' needs better than others involved in the curriculum process. While state or federal standards often dictate the skills covered by the curriculum, a teacher can provide insight into the types of materials, activities and specific skills that need to be included. Teachers from multiple grade-levels may collaborate to identify skills students need at each level and ensure that the curriculum adequately prepares students to advance to the next grade-level and to meet the standards.

Creation

Because teachers must use the curriculum, they should have input in its creation. A teacher can gauge whether an activity will fit into a specified time frame and whether it will engage students. If multiple teachers will use the curriculum, allow as many of them as possible to

provide input during the creation stage. As teachers provide input, they will gain ownership in the final product and feel more confident that the curriculum was created with their concerns and the needs of their particular students in mind.

Implementation

Teachers must implement the curriculum in their own classrooms, sticking to the plan that has taken so much time, careful planning and effort to create. When a teacher fails to properly implement a strong curriculum, she risks not covering standards or failing to implement effective practices in the classroom. That does not mean a teacher cannot make minor changes. In fact, a strong curriculum is designed to allow a teacher to be flexible and to insert a few personalized components or choose from among a selection of activities.

Reflection

Reflecting on a curriculum allows teachers and others involved in the process to find any weaknesses in the curriculum and attempt to make it better. Teachers reflect on curriculum in multiple ways, such as keeping a journal as they implement the curriculum, giving student's surveys and reviewing the results or analyzing assessment data and individual student performance. Not only can reflection serve to improve a specific curriculum, it may guide the creation of new curriculum.

1.10.5 LOCAL INNOVATORS AND INNOVATIVE PRACTICES IN MATHEMATICS

Some of the innovative practices followed

1. Self-composed poems for important concepts.
2. Mathematical stories to create interest in the Subject.
3. Use of innovative teaching aids in classroom.
4. Self-framed worksheets from simple to complex to reteach the difficult concepts for slow learners.
5. Some interesting games and puzzles designed to motivate the students to love mathematics.
6. Some innovative techniques used in class-room teaching-learning process to remove math-phobia.

7. Self-written hand-outs are provided to students to remove common misconceptions
8. Innovative Remedial Measures are adopted according to the type of errors committed by the students in their Formative Assessments.

For example:

9. For computational errors :- drill work is practiced.
10. For language comprehensive errors: - suitable questions withsolutions are provided & explained in detail.
11. For careless errors:- common misconcepts are listed out from the past experience so that students recall them while solving questions.
12. For incorrect formulae error: hand-outs with correct formulae & figures are provided and wherever possible explained through activities.
13. Suitable innovative and interesting activities are carried out in the class room
14. With the help of Mathematics Kits maintained by every child from class VI to class X , suitable and appropriate activities are carried out by the students under the guidance of the teacher. The activities end up with a number of questions so that the aim of the activity is clearly understood by each and every child in the class.
15. The students perform some of the activities in groups too, for example while playing mathematical games a lot of peer learning takes place.
16. Instead of giving the exact or accurate solution for the problems many times open-ended questions are preferred which ensures a wider scope for thinking and reasoning
17. Thorough reading of NCERT text book in the class-room is followed to emphasize on the concepts.
18. Special attention is given to the solved examples of NCERT which simplify the concepts.
19. For homework or assignment questions are picked up from NCERT exemplar.

    Problems for classes VI, IX & X.

Need for innovative practices in teaching mathematics:

In view of the foregoing aims of teaching mathematics more focus should be laid in class room to the higher level of objectives underlying the mathematics subject, ***like critical thinking, analytical thinking, logical reasoning, and decision-making, problem-solving.*** Such objectives are difficult to be achieved only through verbal and mechanical methods that are usually used in the class of mathematics. As one of the verbal methods of instruction give all importance to speech and texts, to the book and to the teacher who used to be simply satisfied with giving the mathematical rules to pupils and having them memorize it,

E.g. the rule of signs and formulas in algebra, students memorize this and remember it! Another verbal method involves explanation. Teachers who use this method assume that the mental structure of the child is same as the adult's. This method leads to series of explanations and students at the initial steps of logical explanations trying to understand and grasp but slowly the gap is created between the explanations transmitted by teacher and received by students which lead to the poor understanding on part of students and they develop a fear of the subject-Math phobia.

Guidelines for a Teacher in Incorporating Innovations in Teaching Mathematics

1. For effective transaction of the curriculum and achievement of curricular objectives appropriate method and pedagogic resources should be used in providing learning experiences to the students.
2. A number of factors need to be considered while making use of a particular method and pedagogic resource: learners' capabilities, availability of resources, entry behavior, school environment, objectives to be achieved, the nature of content and the teacher's own preparation and mastery.
3. Decide on and plan in advance the innovative idea that the teacher would be incorporating to transact a particular concept so that loss of instructional time is prevented or minimized.
4. The immediate environment of the learner both natural and human should be used when and where possible for making learning concrete and meaningful.

5. Involve the students in the process of learning by taking them beyond the process of listening to that of thinking, reasoning and doing.
6. In order to promote self-study skills use of library and resource center needs to be encouraged.
7. Receiving regular feedback for teaching and learning should be an inbuilt component of teaching-learning process. Continuous and comprehensive evaluation has to be ensured as it plays an important role for the required modification in teaching-learning process.
8. Mathematics-teachers' organizations at different levels should be formed where sharing of ideas and experiences, developing resources in a collaborative manner and the mechanisms that enable teachers to carry out innovations is being discussed. Mathematics-teachers' organizations can be instrumental in establishing a climate of confidence in carrying out innovations and a positive attitude to new approaches in teaching mathematics.
9. Properly instruct and guide the students for carrying out different activities and precautionary measures should be taken so that students are not misguided.
10. Study mathematical journals and modern books of professional interest. Any facilities of in-service training should be availed of for improving teaching of mathematics.

SUGGESTIONS:

The teacher can always ask himself/herself two questions,

1. 'Is there some new way in which I can present this material in order to make it more meaningful and more interesting?'
2. 'What activities, demonstrations, teaching aids, etc. would enrich the classroom presentation and direct attention of students to the important elements?' Once the teacher discovers innovative ways to arouse interest and enthusiasm in the class, he will be able to use these ideas again the following year, since those will be new and fascinating to a different class. But teacher should keep in mind that as time passes, the world undergoes a change, the environment surrounding students changes and their needs also changes, so one has to continuously go on modifying and discovering new ways of teaching which proves him a better teacher.

Pedagogy of Mathematics - Sem-II (TS)

EVALUATION

1. Explain the following terms

i. Curriculum Framework

ii. Curriculum Core

iii. Curriculum

iv. Mathematics Curriculum

2. Write briefly about Recommendations of NCF-2005 and APSCF-2011 on Mathematics Curriculum.
3. Explain the Dale's Core Experience.
4. Explain briefly about Subject-centredBehaviouristConstructivist Approach, to Curriculum Development.
5. Discuss role of teacher as curriculum developer.
6. Write briefly about the importance of Print Resources in school curriculum.
7. What is the criteria of evaluation of text book?
8. Explain the following:
   - i. Knowledge Systems in Curriculum
   - ii. Local Innovators and Innovative practices in Mathematics..