2.

APPROACHES AND STRATEGIES FOR LEARNING MATHEMATICS

2.1 INTRODUCTION

Approaches to teaching and learning strategiesinclude being a good classroom manager, focusing on academic skills, being enthusiastic, using effective strategies to keep students on task and using a variety of teaching approaches and resources.

In addition an effective teacher uses easy to follow presentation of material, is direct in teaching, explains and outlines instruction clearly, frequently observes what students are doing taking into account differences and re-designs teaching approaches when necessary.

research has consistently shown that giving poor instructions, or unclear directions, is perceived by students as ineffective teaching. In most instances when this happensstudents are lost, but the teacher is unaware and continues lecturing mostly to her/himself.

A link between teacher clarity and student achievement and satisfaction was identified by Hines, Cruickshank and Kennedy (1985).

They identified 12 behaviors that contribute to instructional clarity using relevant examples during explanation

1. reviewing material
2. asking questions to find out if students understood
3. answering student questions appropriately
4. repeating things when students did not understand
5. teaching in a step-by-step manner
6. providing students with sufficient examples of how to do the work
7. providing time for practice
8. teaching the lesson at a pace appropriate to students
9. explaining things and then stopping so that students could think about it
10. informing students of lesson objectives or what they were expected to be able to do on completion of instruction
11. Presenting the lesson in a logical manner.

Clarity involves:

- knowing the subject matter
- being able to see the information from a learner's perspective, and
- the ability to explain things in simple terms (Sotto, 1994).

2.2 SCENARIO FROM 1950'S AND 1980'S

Since the formal information in this period is inadequate, the data is collected from inter- views with experienced teachers and teacher educators whose ages were more than seventy.

So the mathematics teacher, as a professional must have a commendable degree of con- tent knowledge, must be conversant with various tasks like teaching & learning, testing, curricular development, text book writing, preparing remedial as well as enrichment materials, teaching aids etc. Teachers must be committed to their profession, on the one hand and to the welfare and service of the society on the other hand.

i. Content Development

After independence, mathematics curriculum in schools and colleges of teacher's training was based on colonial needs.

- School mathematics was restricted in three water tight compartments of Arithmetic, Algebra and Geometry.
- Teachers' competency was judged through presentation of contents using Chalk and Talk and lecture method.
- There were a few born teachers at that time who could do justice to teaching of mathematics through their traditional /indigenous methods.
- In most cases teachers gave dry lecture and worked out sums in black board and without much explanation to the 'Whys' and 'How's'.
- The teachers mostly advised students to work out sums on simplification, factorization etc. through repeated practice.
- Emphasis was given on memorization of theorems and solution of stereo typed problems with a view to get a 'pass' in examination.
- The percentage of trained teachers in schools was very small.
- Even in training colleges , the courses of teacher training in mathematics was on discussing aims of mathematics teaching and elaborating the features of analytic, synthetic methods without linking to the respective contents.

Mostly the discussions were theoretical not practical. Topics like

- History of mathematics and
- Examination reforms were included in the syllabus. Harbatian techniques i.e. preparation, presentation and application were followed in planning lessons of Mathematics. The weightage on mathematics method papers was about 10% (including theory and practical) of the total marks.

ii. Methods and Materials

In this period, the methodologies included in the theoretical description of teaching mathematics and examination reforms were as follows:

- ❖ Inductive,
- ❖ Deductive,
- ❖ Analytic and
- ❖ Synthetic methods and objectives.
- There was little linkage of these theories to practical problems.

Pedagogy of Mathematics - Sem-II (TS)

- Finding the correlation within and outside mathematics was left to the insightful teachers whose numbers were quite a few.
- Planning of lesson which followed traditional single-track approach of Herbartian steps was stereotyped and non-interactive.
- Discussion on reforms of examination was made in greater depth and details.
- Objective tests were also given in lesson planning and also at the application stage of teaching.
- In this period, contents part was examined by setting problems from different content areas of school mathematics up to XIIth standard.
- Different types of tests like objective, short answer and long answer were discussed Conclusion was drawn in preparing tests with a judicious mixture of such tests.

iii. Evaluation

- The questions were mostly long answer type.
- Marking scheme in all cases was to give full marks or Zero.
- Students were judged mainly on the basis of what they do not know, instead of what they do know.
- The merit of objective tests was discussed but not given proper emphasis.
- Discussion on reforms of examination was made in greater depth and details.
- Objective tests were also given in lesson planning and also at the application stage of teaching.

iv. Teacher Training

- The topics of the syllabus were taught by teacher in an isolated manner.
- Students were asked to answer questions in a systematic way.
- The courses of studies were mainly on educational & psychological theories, history of education, methods of teaching and school organization,
- It seems that much stress on theoretical aspect was given.
- A realistic and useful curriculum was the demand of the society at that time.

- Some obsolete topics in those papers were deleted.
- In each of those topics relevant needs of the society were reflected.
- In question papers, discussion of the topics together with relevancy in realistic situation were asked from the students.
- Consequently teaching in the classrooms was followed in the same way.

v. Out of School Activities

- There was no such item relating to out of school activities in Mathematics. However, students were allowed to prepare charts, models etc. in mathematics in their practice teaching.
- In the annual function, models, charts were presented and the students were asked to explain.
- Activities like discussion on puzzles and riddles, charts on history of mathemathemaics and life sketches of mathematician, tangrams and the like though not properly in the syllabus are sometime discussed in the classroom, or in wall magazine or college magazine.

2.3 POST 1980 SCENARIO

i. Content Development

The above approach of teaching mathematics was followed till 2000. But there large criticism among some teachers and teacher educators regarding such inclusion of contents which were that they believed that content should not be taught in isolation, without integrating them with methods and evaluations.

In the beginning of the 1st decade of this century, the syllabus was drastically changed. Instead of giving content separately, content- cum-methodology which is an integration of content and methods was introduced in the form of Pedagogical Analysis of mathematics contents. The salient features of theoretical syllabus are inclusion of

- Modern psychological theories (like contributions of the Piaget, Brunner, and Deniesetc.)
- The guidance /remedial measures of gifted and backward children in mathematics
- Modern techniques of evaluation

- Individualized and creative learning.

Again stress has also been given on simulated teaching, relating to exercise of special skills in teaching e.g. introducing the lesson, questioning, black board works, closure etc. Additional weightage has been given in the preparation of 'Achievement Test in mathematics'.

ii. Methods and Materials

The above approaches were by and large followed in 90's also. Additionally,

- Stress was given for guidance to the gifted as well as backward children.
- History of mathematics was also an item of discussion in the classroom.
- Drastic change occurred in the professional course of mathematics teaching as per NCERT & NCTE prescription.
- The courses were designed with the inclusion of content cum methodology in teaching.
- In fact modern contents /topics are included which are relatively new in teacher training
- Such discussion is being aimed to learn new contents as well as their methods of teaching. Some traditional topics which are included in school mathematics are taught with modern methodologies.
- Simulated teachings, development of teaching skills are being emphasized.
- Student teachers are encouraged to use computer Assisted Teaching through PowerPoint presentation and transparencies in Overhead projector.

The spread of such multimedia approach of teaching depends, however on the availability of materials, competent teachers and equipments.

iii. Evaluation

- Almost the same procedure as mentioned above was followed.
- Student teacher was asked to prepare objective, short answer and long answer type questions.

- In mathematical exhibitions, demonstration lesson and even in practice teaching, students are asked to use such tests in order to assess their background knowledge of students.
- The expected outcomes of students are again judged through questions involving knowledge, skill, understanding & application types.
- The approach of constructivist teaching and evaluation is accepted by teachers and educators.
- The assessment was through Unit teaching, classroom -interaction, oral tests in the classrooms, Comprehension test are now in vogue in evaluation process in mathematics. (Such approaches of evaluation are being given greater stress in recent time).
- Students are exposed to situation of learning through assessment and use of self-learning materials.
- Open ended assessments are also exposed to student's teachers.

At present students teacher are being taught to construct achievement test. Separate weightage on construction of 'Achievement Test' helps them to learn the techniques of assessment in a better way.

iv. Teacher Training

All most same pattern of syllabus was followed.

- But, there was criticism on teacher education in various seminars, symposia and conferences at state and national levels.
- The total weight-age of mathematics courses was the same as in the earlier stage .
- Instead of giving separate weight-age on theory and content in mathematics, the syllabus of mathematics has been grouped in different categories
- This arrangement of teachers' training is based on acquainting the teachers with contemporary theories of learning of mathematics, modern techniques of evaluation & curriculum planning and programmes of community activities.
- It is envisaged that the present arrangement would provide anall-round development of mathematics teaching-learning process.
- The planning and management of curriculum were mainly done by experts of curriculum planners in the Universities.

Pedagogy of Mathematics - Sem-II (TS)

- In 1970's onwards, more teacher educators from constituent's teacher's training college were involved.
- 1980-90's – In service training courses in mathematics teaching in schools were and are being organized by boards of secondary education with the help of teachers in schools and teacher educators in training colleges.

v. Out of School Activities

The below practices were being followed in collaboration with mathematics association of mathematics teachers,

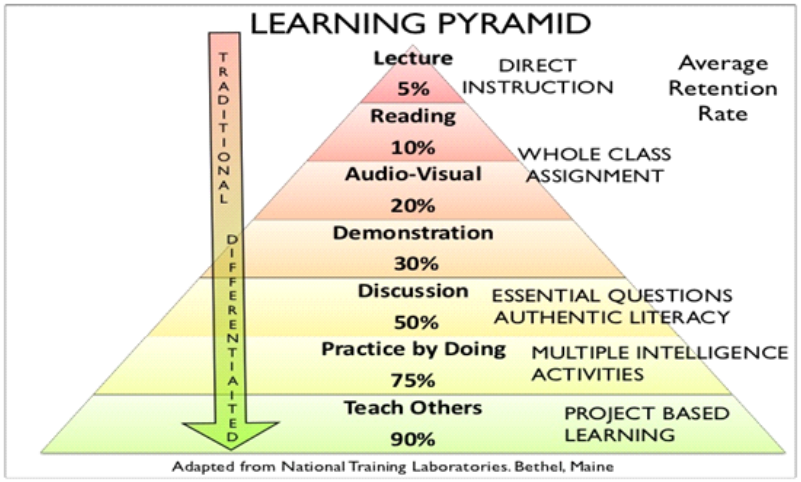
- to 'construct' mathematical concepts in a realistic way quiz & extempore talk competition were conducted.
- Such collaborative programme in mathematics learning and teaching helps student teachers to 'construct' mathematical concepts in a realistic way.
- Nature, Objectives and Approaches of teaching Arithmetic & Commercial mathematics, Algebra & Computing and Geometry & Trigonometry etc. were given importance.
- Knowledge of mathematics and understanding school curriculum are considered as essential.
- Students were engage with theory along with field experiences to help them to view knowledge not as external to the learner , but as something that is actively constructed during learning, integrate academic knowledge and professional knowledge.
- Lively methods like activity approach, open ended learning, self – learning, interactive lesson and the like are being experimented, such practices likely removed pitfalls and errors.
- future relevance of mathematics teaching to the society became the key factor.
- Methods and materials were to design accordingly by the teacher educators.
- Effectiveness of mathematics teaching of teachers got increasedwith these teaching skills.
- A network of co-curricular and out of school activities consisting of teacher educators of mathematics other educational institution, were further developed, fostered and encouraged.
- The focus of all such activities lies in the initiatives of teachers.
- Students got interest and joy in learning in a big way.

2.4 APPROACHES AND STRATEGIES FOR LEARNING MATHEMATICS

An approach to Learning refers to observable behaviours that indicate ways children become engaged in social interactions and learning experiences. Children's approaches to learning contribute to their success in school and influence their development and learning in all other domains. Children's ability to stay focused, interested, and engaged in activities supports a range of positive outcomes, including cognitive, language, and social and emotional development. It allows children to acquire new knowledge, learn new skills, and set and achieve goals for themselves. Many early learning experts view approaches to learning as one of the most important domains of early childhood development. In the domain of Approaches to Learning, programs need to ensure that children who are dual language learners can demonstrate their abilities, skills and knowledge in any language, including their home language.

2.4.1 DIFFERENCE BETWEEN APPROACH AND STRATEGY:

APPROACH	STRATEGY
1. Approach is a way of thinking and working in a set direction so as to accomplish certain goals.	1. Strategy is a proper and systematic plan to achieve the aims and objectives.
2. An approach gives rise to methods, the way of teaching something, which use classroom activities or techniques to help learners learn.	2. Strategy means selection of suitable pedagogical processes by means of using appropriate techniques.
3. Approaches to learning are the foundation that affects how children learn in every other content area.	3. Strategy is to facilitate learning, to motivate learners, to engage them in learning, and to help them focus in the classroom.
4. It encompasses children's engagement, motivation, and participation in the classroom.	4. Strategies can be changed and modified.



2.4.2 DIFFERENT APPROACHES AND STRATEGIES OF LEARNING

Teaching approaches are the set of principles, beliefs, or ideas about the nature of learning which is translated into the classroom. Teaching strategy is a long term plan of action designed to achieve a particular goal. Teaching method is a systematic way of doing something in an orderly logical arrangement of steps. It is more procedural. Whereas, teaching technique is a well-defined procedure used to accomplish a specific activity or task.

Examples of Teaching Approaches

- ***Teacher-Centred Approach:***

  The teacher is perceived to be the only reliable source of information in contrast to the learner-centred approach.

- **Learner-Centred Approach**:

  In which it is premised on the belief that the learner is also an important resource because he/she too knows something and is therefore capable of sharing something.

- ***Subject Matter-Centred Approach:***

  Subject matter gains primacy over that of the learner.

- ***Teacher Dominated Approach:***

In this approach, only the teacher's voice is heard. He/she is the sole dispenser of information.

- ***Interactive Approach:***

In this approach, an interactive classroom will have more student talk and less teacher talk. Students are given the opportunity to interact with teacher and with other students.

- ***Constructivist Approach***:

The students are expected to construct knowledge and meaning out for what they are taught by connecting them to prior experience.

- ***Banking Approach***:

The teacher deposits knowledge into the "empty" minds of students for students to commit to memory.

- ***Integrated Approach:***

It makes the teacher connects what he/she teaches to other lessons of the same subject (interdisciplinary) or connects his/her lessons with other subjects thus making his/her approach interdisciplinary and multidisciplinary.

- ***Disciplinal Approach:***

It limits the teacher to discussing his/her lessons within the boundary of his/her subject.

- ***Collaborative Approach:***

It will welcome group work, teamwork, partnerships, and group discussion.

- ***Individualistic Approach:***

It wants the individual students to work by themselves.

- ***Direct Teaching Approach:***

The teacher directly tells or shows or demonstrates what is to be taught.

- ***Indirect, Guided Approach***:

The teacher guides the learner to discover things for himself/herself. The teacher facilitates the learning process by allowing the learner to be engaged in the learning process with his/her guidance.

Other teaching approaches cited in education literature are:

- ***Research-Based Approach***:

As the name implies, teaching and learning are anchored on research findings.

- ***Whole Child Approach***:

The learning process itself takes into account not only the academic needs of the learners, but also their emotional, creative, psychological, spiritual, and developmental needs.

- ***Metacognitive Approach***:

The teaching process brings the learner to the process of thinking about thinking. The learner reflects on what he learned and on his/her ways of learning.

- ***Problem-Based Approach***:

As the name implies, the teaching- learning process is focused on problems. Time is spent on analysing and solving problems.

In summary, approaches vary in the degree of teacher and learner engagement, focus, number of learners involved in the teaching-learning process

7.4.3SELECTING APPROPRIATE APPROACH AND STRATEGIES

- How students approach a learning task will strongly influence the quality of their learning outcomes. A particular approach may be workable in one situation, but may fail in another situation.
- Before deciding a particular approach a teacher may question herself/ himself the following:
- Does it fulfill the aims and objectives of teaching-learning content and experience?
- Is it comfortable to work in it?
- Does it motivate the learners?
- What improvements will it bring in the learners?
- Does it have enough space for the learners to perform activities, experiments, observations, discussions and further readings and writings?

- Does it can be achieved in the schedule time and organize working space?
- To what extent is it workable in all the situations?
- Does it enhance the ego and self-esteem of the learner through competition to obtain high grades and other rewards?
- Does it match the level of the subject and the pace at which it is presented with student's prior knowledge?

7.4.4 ESSENTIAL COMPONENTS OF ALL APPROACHES AND STRATEGIES

The aim of the selection of the approach or strategies must be based on the following essential components, namely,

- ❖ Help the students to perceive clear goals and standards for learning;
- ❖ Make the concepts clear in its overall structure or the connections between topics,
- ❖ Encourage students' intrinsic interest
- ❖ Give students opportunities to gain qualitative feedback, especially but not only on their assessed work, rather than just giving marks or grades;
- ❖ Encourage active engagement with learning tasks,
- ❖ Engage students in inquiry or creative production to explore complex issues, problems or case studies of practice;
- ❖ Keep the workload to a level that allows students the wider exploration of ideas and the development of interest that characterizes deep approaches to learning;
- ❖ Gives students reasonable opportunities to make reasonable choices about what and how they will learn;
- ❖ Suits the available resources or the teaching aids.

7.4.5 APPROACHES AND METHODS OF TEACHING MATHEMATICS

What is the best method to teach a certain topic?

Or

How can I enable children tolearn mathematics?

Pedagogy of Mathematics - Sem-II (TS)

These are some of the questions for which every teacher wants to find a solution. Different methods of teaching mathematics have been proposed by different educators and the knowledge of these methods may help in working out a better teaching strategy. It is not appropriate for a teacher to commit to one particular method. A teacher should adopt a teaching approach after considering the nature of the children, their interests and maturity and the resources available. Every method has certain merits and few demerits and it is the work of a teacher to decide which method is best for the students. Some of the methods of teaching Mathematics are as follows:

- Lecture Method
- Inductive-Deductive Method
- Heuristic Method (Discovery/Inquiry Method
- Analytical-Synthetic Method
- Project Method
- Brain Storming
- Think-Pair-Share
- Learning by Doing
- Problem Solving Approach

All the above mentioned methods may not be equally appropriate and suitable for all levels of mathematics teaching. The teacher, after knowing about all these methods, their merits and demerits, should be able to make his/her own method by imbibing the good qualities of all the methods. The method finally adopted by the teacher must

- ensure maximum participation of the child,
- proceed from concrete to abstraction and
- provide knowledge at the understanding level

Some of the above mentioned methods are discussed as follows:

1. THE LECTURE METHOD

The lecture method is the most widely used form of presentation. Every teacher has to know how to develop and present a lecture. They also must understand the scopes and limitations of this method.

- Lectures are used to introduce new topics, summarizing ideas, showing relationships between theory and practice, reemphasizing main points, etc.
- This method is adaptable to many different settings (small or large).
- It may be used to introduce a unit or a complete course.
- Finally, lectures can be effectively combined with other teaching methods to give added meaning and direction.

The teaching lecture is favoured by most teachers because it allows some active participation by the students. The success of the teaching lecture depends upon the teacher's ability to communicate effectively with the class. However in this method the feedbackis not very obvious and thus the teacher must develop a keen perception for subtle responses from the class-facial expressions, manner of taking notes, and apparent interest or disinterest in the lesson. The successful teacher will be able to interpret the meaning of these reactions and adjust the lesson accordingly.

Preparing the Teaching Lecture Planning:

The following four steps are followed in the planning phase of preparation:

- Establishing the objective and desired outcomes;
- Researching the subject;
- Organizing the material; and
- Planning productive classroom activities.

In all stages of preparing for the teaching lecture, the teacher should support any point to be covered with meaningful examples, comparisons, statistics, or testimony. While developing the lesson, the teacher also should strongly consider the use of examples and personal experiences related to the subject of the lesson.

Rehearsing:

After completing the preliminary planning and writing of the lesson plan, the teacher should rehearse the lecture to build self-confidence. It helps to smooth out the use notes, visual aids, and other instructional devices.

Pedagogy of Mathematics - Sem-II (TS)

Delivering a lecture Suitable Language:

In the teaching lecture, simple rather than complex words should be used whenever possible. The teacher should not use substandard English. If the subject matter includes technical terms, the teacher should clearly define each one so that no student is in doubt about its meaning. Whenever possible, the teacher should use specific rather than general words.

Tone and Pace:

Another way the teacher can add life to the lecture is to vary his or her tone of voice and pace of speaking. In addition, using sentences of different length also helps. To ensure clarity and variety, the teacher should normally use sentences of short and medium length

Use of notes:

For a teacher notes are a must because they help keep the lecture on track. The teacher should use them modestly and should make no effort to hide them from the students. Notes may be written legibly or typed, and they should be placed where they can be consulted easily.

Advantages of the Lecture Method:

- Lecture method
- Gives the teacher the chance to expose students to all kinds of material.
- Allows the teacher to precisely determine the aims, content, organization, pace and direction of a presentation.
- Can be used to arouse interest in a subject.
- Can complement and clarify text material.
- Complements certain individual learning preferences.
- Facilitates large-class communication.

Disadvantages of Lecture Method

- Places students in a passive rather than an active role, which hinders learning.
- Encourages one-way communication, therefore, the lecture must make a conscious effort to become aware of student problems and student understanding of content without verbal feedback.

- Requires a considerable amount of unguided student time outside of the classroom to enable understanding and long-term retention of content.
- Requires the teacher to have effective speaking skills.

2. INDUCTIVE-DEDUCTIVE METHOD INDUCTION

Is that form of reasoning in which a general law is derived from a study of particular objects or specific processes? Students use measurements, manipulators or constructive activities and patterns etc. to discover a relationship. They later formulate a law or rule about that relationship based on their observations, experiences, inferences and conclusion.

Example 1:

Ask pupils to draw a number of triangles. Ask them to measure the three angles of each triangle and find their sum. They will find that the sum of the three angles of all triangles is 180º

Example 2:

Ask pupils to find the sum of two odd numbers like 3+5=8, 5+7=12, 9+11=20, etc. They will find that the sum of two odd numbers is an even number.

Deduction

is the method in which the law is accepted and then applied to a number of specific examples? The child does not discover the law but develops skills in applying

Steps in the inductive method:

1. The first step is clear recognition of the problem. It should be clearly understood and defined by the pupils.
2. Once the problem has been defined, the child should start searching for data from all possible sources like books, magazines, journals, making visits to certain places etc.
3. Under the guidance of the teacher, the pupils organize the data which they have collected from various sources. They select relevant data and discard irrelevant material.
4. By studying particular instances, the pupils frame possible solutions.

Pedagogy of Mathematics - Sem-II (TS)

5. These solutions are discussed, argued and judged. Thus tentative solutions are eliminated and only the probable solutions remain.
6. The solutions are applied to the situation and results are verified.

Merits of Inductive method

1. This method is psychological. The student feels interested in experiment, experiences and discoveries.
2. This method fosters independence and self-confidence in the pupil which proves very useful in later life.
3. In this method, children discover the solution themselves. Hence it develops and encourages initiative and creative thinking
4. All that is learnt using inductive method is remembered easily as it is self-acquired.
5. In this method, the pupils observe and analyse particular objects of similar and different nature and try to arrive at general truth.
6. Inductive method takes into consideration all the maxims of good teaching. The process of induction calls for perception, reasoning, judgment and generalization.

Steps involved in deductive method

1. Like the inductive method, the first step is the clear understanding of the problem.
2. It may involve the study of a particular thing and phenomenon.
3. Principles and generalizations are reviewed to find the one which may be applicable to find a solution.
4. In this step the rule, principle or generation is applied to a problem and inference is formulated that the problem falls under such rule, principle or generalization.
5. Verification of the inference is done by done by applying it to a case. If it solves the problem then it is accepted otherwise the procedure is repeated to find the correct one.

Merits of deductive method

1. Deductive method is short and time-saving. It takes little time to solve the problem by predetermined formulae.

2. In the deductive method, the teacher's work is very much simplified.
3. He/she simply gives a rule and asks the pupils to verify it by application to several concrete examples.

For example,

Students are told that the area of rectangle = Length x Breadth.

4. Then a few sums are solved before the students. The students apply these formulae to solve these problems and they memorize it for future use.
5. This method is very useful for small children because with small children we generally use story or telling method.
6. This method glorifies memory, as the students have to memorize a considerable number of formulae and definitions.
7. This method is adequate and advantageous during practice and revision stage.

Inductive Method Deductive Method – A Comparison

So it can be concluded that in inductive method we proceed from particular instances to general laws or formulae. Through this method, children discover many new things themselves and learning becomes very easy. It provides self-reliance and confidence in the students. Inductive method is slow and safe as the general law is reached step-by-step. Students climb up the stairs of thought leading to definitions, principles or rules. In deductive method, we start with general law or formulae and then solve particular problems by applying this law or formulae. It is a method of verification and explanation and provides instruction. Deduction can give us the formal validity because the rule is taken for granted. The aim of this method is to fit the pupil generally for the battle of life.

In actual practice of teaching, the combination of induction and deduction must be practiced. The laws should be discovered by pupils inductively and they must be further verified deductively through applications to new situations.

3. HEURISTIC METHOD (DISCOVERY / INQUIRY METHOD)

Discovery through Inquiry adapts thescientific method. It organizes investigation of real world phenomena into four steps. Each step has a series of tasks that lead to then next step or the conclusion of the

investigation. The process involves an inquiry strategy that uses questions and the seeking of answers to guide the investigation as it proceeds.

Step I: Wondering:

What do we want to discover?

- Make observations on real world phenomena related to topic of study.
- Connect observations to topic and possibly subtopic of study.
- Identify questions to be answered or problems to be solved.

Step II: Designing:

How can we find out?

- Create hypothesis.
- Design methods to test hypothesis.
- Develop means to collect and analyse data.

Step III: Investigating:

What are we finding out?

- Conduct experiments to test hypothesis.
- Record data.
- Organize and analyse data.
- Prove or revisit and revise hypothesis.

Step IV: Discovery

What did we discover?

- State conclusion(s) after analysing data.
- Determine validity of conclusion(s).
- Construct meaning by connecting to real world.

Heuristic Method (Discovery/Inquiry Method) of teaching is simply the process of allowing the students to take the leading role in their own learning experiences. Theteacher becomes a facilitator and guide, making it possible for the learner to reachmutually-agreed-upon goals. The teacher serves as a resource person to stimulate,motivate, clarify, and explain.

The atmosphere in which such teaching takes place must be informal and non-threatening. In order for discovery teaching to be effective, the environment (including the teacher's attitude) must contribute to rather than detract from the attaining of objectives.

Rather than forcing his idea of content, the teacher attempts to keep his hands off the learning process whenever and wherever the student can carryit on for himself.

Discovery teaching brings four basic components of the educational setting into interaction: the student,the teacher, the environment, and the content.

- The student is an active participant who solves problems which he/she understands through the process of structuring his/her own learning experiences.
- The teacher plays the role of resource person and a facilitator.
- The environment includes both freedom and structure with freedom having the upper hand.
- The content may very well be propositional truth in a general context, waiting in the proper
- place for the student to track it down, confront it, and capture it for his own.

An effective discovery leader must be a mature teacher who knows not only the subject matter of the current lesson, but has an in-depth understanding of the subject.

The students have to be a willing participant, ready to explore numerous avenues of information and to appreciate new findings in the light of previous information.

A number of diverse methods can be used within the framework of discovery learning,since any single student may approach his subject matter from different perspectives. Surely, numerous different approaches will be adopted within the total group.

Importance:

Heuristic Method (Discovery/Inquiry Method) of teaching

- Allows for individualistic accomplishments.
- Is highly adaptive and versatile, limited only by the imagination of the participants at both the teacher and student level.
- Allows for free expression of individual creativity. It is a concept of learning about which we talk much and do little.

Pedagogy of Mathematics - Sem-II (TS)

- Develops the relationship of students to teachers and of students to student.

Problems:

- In Heuristic Method (Discovery/Inquiry Method) of teaching.
- Many students feel insecure in an unstructured environment of learning.
- It is much more comfortable to be able to listen to a lecture and take notes in orderly fashion than to be confronted with the haunting question, "What do you want to learn about this subject, and how do you propose to learn it?"
- If responsibilities are not taken seriously by the participants, the whole process could get out of control.
- It is also a time-consuming method.
- Develops the relationship of students to teachers and of students to student.

Principles for Effective Teaching by Heuristic Method. (Discovery / Inquiry Method)

To get the best results, a teacher must,

- Encourage the questioning mind, and equip students with skills for finding the answers.
- Suggest resources, but refrain from doing the research for the learner.
- Train the students to think
- Challenge the answers they suggest and not be satisfied with the easy answer they are 'supposed to get.'
- Insist the pupils to give evidence and make a convincing case for what they think and say
- Ensure that all resources are available and usable by the student.
- Teach the students how to use various resources as they track their solutions through books, articles, films, recordings, map, experiences, projects and most important other people.

4. PROJECT METHOD

This method aims to bring practically designed experience into the classroom.

Often conducted over a period of three to six months, the projects give students an opportunity to work in a team environment and apply theory learned in the classroom. There are some parts of the curriculum in which children are necessarily dependent on the teacher and others in which children can work more independently.

- Project work is more likely to constitute the more informal part of the program, the part where they have greater autonomy in the development of their work than when involved in teacher directed instruction.
- Project work can be seen as providing complementary learning opportunities to children
- in which they not only need to know how to use a skill but also when to use it.
- They need to learn to recognize for themselves the contexts in which the skill might be useful and the purposes which it can most appropriately serve.
- In project work they apply those skills in meaningful contexts.
- The project work can be seen as the part of the curriculum which is planned in negotiation with the children and which supports and extends the more formal and teacher directed instructional elements.

Scope and Strategies

This method is appropriate for any level, but is often employed for senior levels of education. Using projects usually requires a lot of preparation by the teacher. Some tips in this regard are as follows:

- Realize that the product of the project is not as important as the processes.
- It is not important that the students determine an optimal design. What is important, however, is that they experience the design process?
- It is important that the scope of the project is reasonable; care must be taken to ensure that the students are not overloaded.
- When possible, divide the project into sections and set dates for the submission of each section.

Pedagogy of Mathematics - Sem-II (TS)

- Assign projects to teams of two or three students to allow in-depth efforts, and to promote interaction among the students.

MATHEMATICS PROJECT IDEAS FOR HIGH SCHOOL

Here is a selection of ideas for projects

1. How is a Cricket Tournament schedule worked out? How would you do such a schedule bearing in mind distances between locations of games, home team advantage etc.? Could you devise a good schedule for one of your local competitions?
2. When possible, divide the project into sections and set dates for the submission of each section.
3. How do major hospitals schedule the use of operating theatres?
4. Are they doing it the best way possible so that the maximum number of operations is done each day?
5. Build a physical model to prove the Pythagorean Theorem.
6. Find as many triangles as you can with integer sides and a simple linear relation between the angles. What about the special case when the triangle is right-angled?
7. Build a true scale model of the solar system
8. Investigate the history of pi and find the ways in which it can be approximated.
9. Construct a Kaleidoscope.
10. Investigate its history and the mathematics of symmetry.
11. Explore the history and use of the Abacus.
12. Investigate card tricks and other magic tricks based in Mathematics.
13. Efficiency in Packing To investigate the efficiency of packing of objects of different shapes in a cuboid box. (Efficiency is the percentage of box space occupied by the objects.)
14. Geometry in real life In this project we try to find situations in daily life where geometrical notions can be effectively used. In particular, the student discovers situations in which properties of similar triangles learnt in the classroom are useful.
15. Experiments on Probability To appreciate that finding probability through experiment are different from finding probability by

calculation. Students become sensitive towards the fact that if they increase the number of observations, probability found through experiment approaches the calculated probability.

16. Displacement and rotation of a geometrical figure To study the distance between different points of a geometrical figure when it is displaced and / or rotated. Enhances familiarity with co-ordinate geometry.
17. Frequency of letters/ words in a language Analysis of a language text using graphical and pie chart techniques.

Analytic Method	Synthetic Method
1. The term "Analytic" is derived from word 'Analysis' which means to break or resolve a thing into its constituent elements.	1. The term 'synthetic' is derived from word "SYNTHESIS" which means to combine together.
2. This method includes breaking up the unknown problem into simpler parts which can be recombined to find solutions.	2. Child proceeds from known to unknown.
3. The procedure adopted is to go 'from unknown to known' and find out desired results.	3. Facts already known are applied to new situations so that the combination of known facts helps us to find new facts.
4. It is a method of unfolding of the statement in question or conducting its different operations to explain the different aspects minutely which are required for the presentation of pre-discovered facts.	4. It saves time and labour as the method is a short method.
5. This method includes breaking up the unknown problem into simpler parts which can be recombined to find solutions.	5. Majority of learners benefitted by this method.
6. The procedure adopted is to go 'from unknown to known' and find out desired results.	6. Most of the topics in mathematics are covered by this method.
7. This method is a Logical method and there is no room for doubts in the mind of learners.	7. There is systematic presentation of facts which makes the method a neat one.
8. Thinking and reasoning power of children is developed by this method.	8. The memory of the learner is glorified.
9. The Self-confidence of learners develop	9. Accuracy is developed by the method.
10. It encourages Scientific originality and creativity among learners	10. Child proceeds from known to unknown.
11. Logical approach to prove proposition and statements.	11. There is possibility of forgetting in this method.
12. Active participation in teaching/ learning process.	12. learners become passive listeners and they resort to cramming.
13. The learner gets clear understanding of the topic and helps him to go through the whole process himself.	13. Teacher is unable to carry all in the class with him.
14. Child proceeds from unknown to known.	14. Learners have least confidence.
15. This method is a lengthy method.	15. It increases the elements of doubt among learners.
16. It is not possible to learn all topics through this method.	16. Thinking and evolving a sense of discovery among learners.
17. This method is not suitable to all age groups students.	
18. The method is not useful at lower stage of mathematics learning which involve objective thinking and exploratory approach.	
19. The method has slow speed, Time taken for reasoning and thinking is long.	

Pedagogy of Mathematics - Sem-II (TS)

7.5 CONSTRUCTIVIST APPROACH – STATE DEVELOPED MODEL OF TEACHING MATHEMATICS STRATEGIES

7.5.1 CONSTRUCTIVIST APPROACH

Constructivist teaching approach is based on the belief that learning occurs as learners are actively involved in a process of meaning and knowledge construction as opposed to passively receiving information. Learners are the makers of meaning and knowledge.

One of the primary goals of using constructivist teaching is that students learn how to learn by giving them the training to take initiative for their own learning experiences.

According to Audrey Gray, the characteristics of a constructivist classroom are as follows:

- learners are actively involved
- environment is democratic
- activities are interactive and student-cantered
- teacher facilitates a process of learning in which students are encouraged to be responsible and autonomous

7.5.2 EXAMPLES OF CONSTRUCTIVIST ACTIVITIES

Furthermore, in the constructivist classroom, students work primarily in groups and learning and knowledge are interactive and dynamic. There is a great focus and emphasis on social and communication skills, as well as collaboration and exchange of ideas. This is contrary to the traditional classroom in which students work primarily alone, learning is achieved through repetition, and the subjects are strictly adhered to and are guided by a textbook. Some activities encouraged in constructivist classrooms are:

- ***Experimentation:*** students individually perform an experiment and then come together as a class to discuss the results.
- ***Research projects***: students research a topic and can present their findings to the class.

Pedagogy of Mathematics - Sem-II (TS)

- ***Field trips***: This allows students to put the concepts and ideas discussed in class in a real-world context. Field trips would often be followed by class discussions.
- ***Films:*** These provide visual context and thus bring another sense into the learning experience.
- ***Class discussions***: This technique is used in all of the methods described above. It is one of the most important distinctions of constructivist teaching methods.
- ***Online learning***:Constructivist approaches can also be used in online learning. For example, tools such as discussion forums, wikis and blogs can enable learners to actively construct knowledge.

7.5.3 DIFFERENCE BETWEEN TRADITIONAL AND CONSTRUCTIVIST

A contrast between the traditional classroom and the constructivist classroom is illustrated below:

The Traditional Classroom

- Begins with parts of the whole–Emphasizes basic skills
- Strict adherence to fixed curriculum
- Textbooks and workbooks
- Instructor gives/students receive
- Instructor assumes directive, authoritative role
- Assessment via testing / correct answers
- Knowledge is inert
- Students work individually

The constructivist Classroom

- Begin with the whole – expanding to parts
- Pursuit of student questions / interests
- Primary Sources / manipulative materials
- Learning is interaction – building on what students already know
- Instructor interacts / negotiates with students
- Assessment via student works observations, points of view, and tests. Process is as important as product

- Knowledge is dynamic / change with experiences
- Students work in groups

Because existing knowledge schemata are explicitly acknowledged as a starting point for new learning, constructivist approaches tend to validate individual and cultural differences and diversity.

7.5.4 ROLE OF THE TEACHER

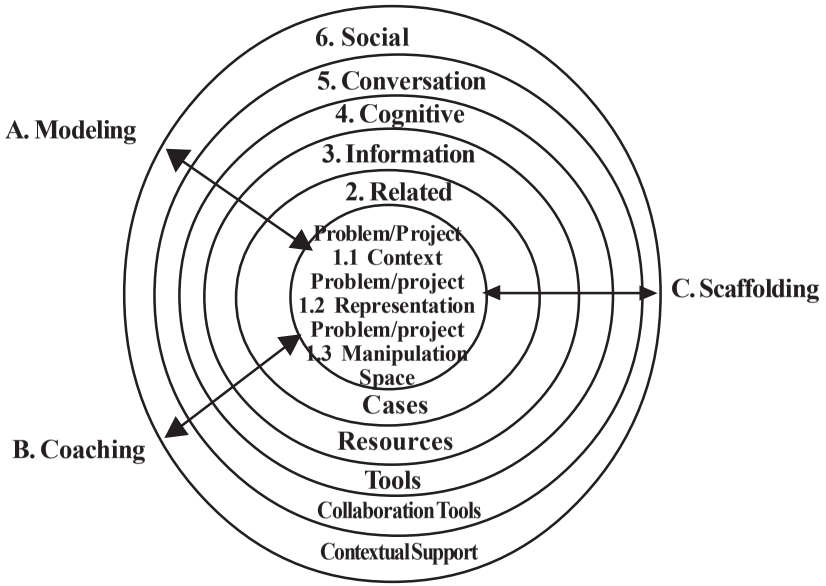
In the constructivist classroom, the teacher's role is to prompt and facilitate discussion. Thus, the teacher's main focus should be on guiding students by asking questions that will lead them to develop their own conclusions on the subject.

- Constructivist teachers uses cognitive terminology like "create", "classify", "predict" ,analyse"
- Constructivist teachers invite student's ideas and questions.
- Constructivist teachers encourage free discussions.
- Constructivist teachers help students to test their own ideas.
- Constructivist teachers encourage students to challenge the ideas of others
- Constructivist teachers allow students to drive the lessons, shifts instructional strategies and alter content.
- Constructivist teachers inquire about students understanding of concept before sharing their own understanding of concepts,
- Constructivist teachers nurture the curiosity of students.

7.5.5 STATE DEVELOPED MODEL OF TEACHING MATHEMATICS STRATEGIES

Parker J. Palmer (1997) suggests that "good teachers join self, subject, and students in the fabric of life because they teach from an integral and undivided self, they manifest in their own lives, and evoke in their students, a capacity for connectedness".

Pedagogy of Mathematics - Sem-II (TS)



David Jonassen identified three major roles for facilitators to support students in constructivist learning environments:

- Modelling
- Coaching
- Scaffolding

Brief descriptions of the major roles are:

1. Modelling

Modelling is the most commonly used instructional strategy in CLEs. Two types of modeling exist:

i. Behavioural modelling of the overt performance:

Behavioural modelling in Constructivist Learning Environments demonstrates how to perform the activities identified in the activity structure.

ii. Cognitivemodelling of the covert cognitive processes:

Cognitive modelling articulates the reasoning (reflection-in-action) that learners should use while engaged in the activities.

2. Coaching

The role of coach is complex and inexact. Coaching naturally and necessarily involves responses that are situated in the learner's task performance. Coaching may be unsolicited, when the coach,

- Motivates learners,
- Analysis their performance,
- provides feedback and
- advice on the performance, how to learn about how to perform, and
- Provokes reflection and articulation of what was learned.
- Provides diagnosis,
- Provides directions, and feedback.

3. Scaffolding

Scaffolding provides temporary frameworks to support learning and student performance beyond their capacities. The concept of scaffolding represents any kind of support for cognitive activity that is provided by an adult when the child and adult are performing the task together.

Hence, Scaffolding is a,

- systemic approach
- to supporting the learner,
- focusing on the task,
- the environment, the teacher, and the learner.

7.5.5.1CONSTRUCTIVIST LEARNING ENVIRONMENTS (CLEs)

Jonassen has proposed a model for developing constructivist learning environments (CLEs) around a specific learning goal. This goal may take one of several forms, from least to most complex:

- Question or issue
- Case study
- Long-term Project
- Problem (multiple cases and projects integrated at the curriculum level)

Jonassen recommends making the learning goals engaging and relevant but not overly structured. In CLEs, learning is driven by the problem to be solved;

- students learn content and theory in order to solve the problem.
- This is different from traditional objectivist teaching where the theory would be presented first and problems would be used afterwards to practice theory.
- Depending on student's prior experiences, related cases and scaffolding may be necessary for support.
- Instructors also need to provide an authentic context for tasks, plus information resources, cognitive tools, and collaborative tools.

7.5.5.2 CONSTRUCTIVIST ASSESSMENT

Traditionally, assessment in the classrooms is based on testing. In this style, it is important for the student to produce the correct answers. However, in constructivist teaching, the process of gaining knowledge is viewed as being just as important as the product. Thus, assessment is based not only on tests, but also on observation of the student, the student's work, and the student's points of view.Some assessment strategies include:

- ***Oral discussions***: The teacher presents students with a "focus" question and allows an open discussion on the topic.
- ***KWL (H) Chart:***(What we know, What we want to know, What we have learned, How we know it). This technique can be used throughout the course of study for a particular topic, but is also a good assessment technique as it shows the teacher the progress of the student throughout the course of study.
- ***Mind Mapping:*** In this activity, students list and categorize the concepts and ideas relating to a topic.
- ***Hands-on activities***: These encourage students to manipulate their environments or a particular learning tool. Teachers can use a checklist and observation to assess student success with the particular material.
- ***Pre-testing***: This allows a teacher to determine what knowledge students bring to a new topic and thus will be helpful in directing the course of study.

7.5.5.3 SPECIFIC APPROACHESBASED ON CONSTRUCTIVISM

Specific approaches to education that are based on constructivism include the following:

i. Constructionism

An approach to learning based on the constructivist learning ideologies presented by Jean Piaget. In this approach, the individual is consciously engaged in the construction of a product. The utilization of constructionism in educational settings has been shown to promote higher-order thinking skills such as problem-solving and critical thinking.

ii. Guided Instruction

A learning approach in which the educator uses strategically placed prompts, cues, questions, direct explanations, and modeling to guide student thinking and facilitate an increased responsibility for the completion of a task.

iii. Problem-Based Learning

A structured educational approach which consists of large and small group discussions.

Problem-based learning begins with an educator presenting a series of carefully constructed problems or issues to small groups of students.The problems or issues typically pertain to phenomena or events to which students possess limited prior knowledge.

- The first component of problem-based learning is to discuss prior knowledge and ask questions related to the specific problems or issues.
- Following the class discussion, there is typically time in which students individually research or reflect on the newly acquired information and/or seek out areas requiring further exploration.
- After a pre-determined amount of time (as outlined by the educator), students will meet in the same small groups that were composed prior to the class discussion.

Pedagogy of Mathematics - Sem-II (TS)

- In the first meeting, groups will spend between one to three hours further discussing the problems or issues from class in addition to presenting any new information collected during individual research.
- Following the first meeting, students will independently reflect on the group discussion, specifically in comparing thoughts regarding the problems or issues in question.
- Typically, groups will meet a second time to critically analyse individual and group thoughts and discussions and will attempt to synthesize the information in order to draw conclusions about the given problem or issue.
- Within the educational setting, problem-based learning has enabled students to actively construct individual understandings of a topic using both prior and newly acquired knowledge.
- Moreover, students also develop self-directed and group learning skills which ultimately facilitates the comprehension of the problems or issues.

iv. Inquiry-Based Learning

An educational approach associated with problem-based learning in which the student learns through investigating issues or scenarios.

In this approach, students pose and answer questions individually and/or collaboratively in order to draw conclusions regarding the specific issues or scenarios.

Within the educational setting, inquiry-based learning has been beneficial in developing student inquiry, investigation, and collaboration skills, in turn, increasing overall comprehension of the issue or scenario.

Effective essential questions include student thought and research, connect to student's reality and can be solved in different ways. There are no incorrect answers to essential questions, rather answers reveal student understanding.

v. Anchored Instruction

An educational approach associated with problem-based learning in which the educator introduces an 'anchor' or theme in which students will be able to explore. The 'anchor' acts as a focal point for the entire

task, allowing students to identify, define, and explore problems while exploring the topic from a variety of different perspectives.

vi. Cooperative Learning

A variety of educational approaches focusing on individuals working together to achieve a specific learning outcome.

vii. Reciprocal Peer Teaching

A cooperative learning approach wherein students alternate roles as teacher and learner. The utilization of Reciprocal Peer Teaching (RPT) in educational settings has been effective in the development of teamwork, leadership, and communication skills in addition to improving students' understanding of course content.

viii. Jigsaw

A highly structured cooperative learning approach which is implemented in four stages:

- ***Introduction:***

In the introduction stage, the class is divided into heterogeneous 'home' groups consisting of between three to seven students. Upon establishing the 'home' groups, the teacher will discuss the subtopics pertaining to the subject matter.

- ***Focused Exploration:***

In the focused exploration stage, each student within all 'home' groups selects one of the subtopics. Students from each 'home' group that have selected the same subtopic will form a 'jigsaw 'group.

- ***Reporting and Re-shaping:***

It is in the 'jigsaw' group that students will explore the material pertaining to the subtopic and will prepare for teaching it to their 'home' group, the reporting and re-shaping.

- ***Integration and Evaluation:***

The approach concludes in the fourth stage, integration and evaluation, wherein each of the 'home' groups combines the learning of each subtopic together to create the completed piece of work.

7.6 5 E LEARNING MODEL

Constructivism is a learning strategy that draws on student's existing knowledge, beliefs, and skills. With a constructivist approach, students synthesize new understanding from prior learning and new information. The constructivist teacher sets up,

- problems and monitors student exploration,
- guides student inquiry, and
- promotes new patterns of thinking.

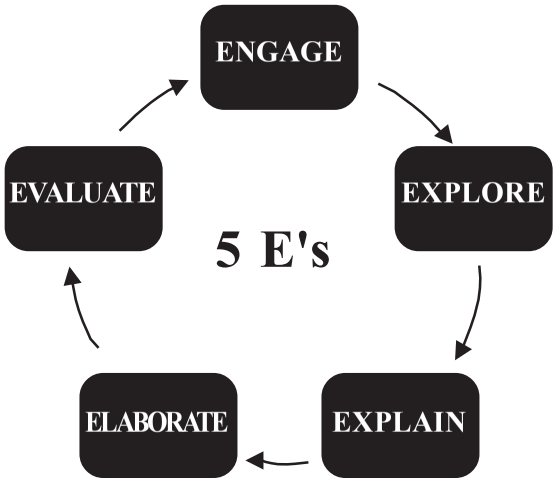
Working mostly with raw data, primary sources, and interactive material, constructivist teaching asks students to work with their own data and learn to direct their own explorations. Ultimately, students begin to ***"think of learning as accumulated, evolving knowledge"***. Constructivist approaches work well with learners of all ages, including adults.

The 5 E's is an instructional model based on the constructivist approach to learning, which says that-"learners build or construct new ideas on top of their old ideas". The 5 E's can be used with students of all ages, including adults.

Each of the 5 E'sdescribes a phase of learning, and each phase begins with the letter "E":

- Engage,
- Explore,
- Explain,
- Elaborate, and
- Evaluate.

- *The 5 E's allows students and teachers to,*
- experience common activities,
- use and build on prior knowledge and experience,
- construct meaning, and to continually assess their understanding of a concept.



i. **Engage**:

This phase of the 5 E's starts the process. An "engage" activity should do the following:

- Make connections between past and present learning experiences.
- Anticipate activities and focus students' thinking on the learning outcomes of current activities.
- Students should become mentally engaged in the concept, process, or skill to be learned.

ii. **Explore**:

- This phase of the 5 E's provides students with a common base of experiences.
- They identify and develop concepts, processes, and skills.
- During this phase, students actively explore their environment or manipulate materials.

iii. **Explain**:

- This phase of the 5 E's helps students explain the concepts they have been exploring.
- They have opportunities to verbalize their conceptual understanding or to demonstrate new skills or behaviours.
- This phase also provides opportunities for teachers to introduce formal terms, definitions, and explanations for concepts, processes, skills, or behaviours.

iv. **Elaborate**:

- This phase of the 5 E's extends student's conceptual understanding and allows them to practice skills and behaviours.
- Through new experiences, the learners develop deeper and broader understanding of major concepts.
- Obtain more information about areas of interest, and refine their skills.

v. **Evaluate**:

This phase of the 5 E's encourages learners to assess their understanding and abilities and lets teachers evaluate student's understanding of key concepts and skill development.

7.7 COLLABORATIVE LEARNING APPROACH (CLA)

Collaborative learning is an educational approach to teaching and learning that involves groups of learners working together to solve a problem, complete a task, or create a product. Collaborative learning is based on the idea that learning is a naturally social act in which the participants talk among themselves. It is through the talk that learning occurs.

7.7.1STEPS OF COLLABORATIVE LEARNING APPROACH

- Brief overview of the activity—short abstract that summarizes the activity's strategy.
- Desired learning outcomes—identify the learning objectives for the activity and consider (how these will be communicated to students.
- Strategy for "setting up" the activity, e.g., pre-work, team building required, and so forth— What work might students be asked to do so that they are prepared to begin the collaborative activity, i.e., reading, quiz, writing? What tools can the instructor use to determine if students are ready?
- Strategy for assessing the activity—How will student work be assessed: individually, by team, by role, by work product?

- Anticipated issues or challenges—What difficulties might the instructor or students encounter while working on a collaborative project: workload issues, meeting deadlines, combining their individual pieces into one, work distribution, and so forth? What can be done to address these issues proactively?
- To conclude this activity, you might ask a few participants to share on each of the areas above or to share in groups of two to three. Depending on the size of the group, you might also post a summary of the activities to a wiki so that the group can share ideas, teaching strategies, and solutions.

7.7.2 ENSURING MEANINGFUL LEARNING THROUGH CLA

There are many approaches to collaborative learning:

- Learning is an active process whereby learners assimilate the information and relate this new knowledge to a framework of prior knowledge.
- Learning requires a challenge that opens the door for the learner to actively engage his/her peers, and to process and synthesize information rather than simply memorize and regurgitate it.
- Learners benefit when exposed to diverse viewpoints from people with varied backgrounds.
- Learning flourishes in a social environment where conversation between learners takes place. During this intellectual gymnastics, the learner creates a framework and meaning to the discourse.
- Learners are challenged both socially and emotionally as they listen to different perspectives, and are required to articulate and defend their ideas. In so doing
- Learners begin to create their own unique conceptual frameworks and not rely solely on an expert's or a text's framework.
- Thus, in a collaborative learning setting, learners have the opportunity to converse with peers, present and defend ideas, exchange diverse beliefs, question other conceptual frameworks, and are actively engaged.

Pedagogy of Mathematics - Sem-II (TS)

7.7.3 WAYS OF APPLYING COLLABORATIVE LEARNING APPROACH

i. Stump your partner

- Students take a minute to create a challenging question based on the lecture content up to that point.
- Students pose the question to the person sitting next to them.
- To take this activity a step further, ask students to write down their questions and hand them in. These questions can be used to create tests or exams. They can also be reviewed to gauge student understanding.

ii. Think-pair-share/ Write-pair-share

- The instructor poses a question that demands analysis, evaluation, or synthesis.
- Students take a few minutes to think through an appropriate response.
- Students turn to a partner (or small groups) and share their responses. Take this a step further by asking students to find someone who arrived at an answer different from their own and convince their partner to change their mind.
- Student responses are shared within larger teams or with the entire class during a follow-up discussion.

iii. Catch-up

- Stop at a transition point in your lecture.
- Have students turn to a partner or work in small groups to compare notes and ask clarifying questions.
- After a few minutes, open the floor to a few questions.

iv. Fishbowl debate

- Ask students to sit in groups of three.
- Assign roles. For example, the person on left takes one position on a topic for debate, the person on right takes the opposite position, and the person in the middle takes notes and decides which side is the most convincing and provides an argument for his or her choice.
- Debrief by calling on a few groups to summarize their discussions.

v. Case study

- Create four to five case studies of similar difficulty.
- Have students work in groups of four or five to work through and analyze their case study.
- Provide 10-15 minutes (or adequate time to work through the cases).
- Walk around and address any questions.
- Call on groups randomly and ask that students share their analysis. Continue until each case study has been addressed.

vi. Team-based learning

- Start a course unit by giving students some tasks to complete, such as reading or lab assignments. Consider assigning these to be completed before class.
- Check students' comprehension of the material with a quick multiple-choice quiz. Have students submit their answers.
- Assign students to groups and have them review their answers with group members to reach consensus. Have each group submit one answered quiz.
- Record both the individual student assessment scores and the final group assessment score (both of which are used toward each student's course grade).
- Deliver a lecture that specially targets any misconceptions or gaps in knowledge the assessments reveal.
- Give groups a challenging assignment, such as solving a problem or applying a theory to a real world situation.

vii. Group problem solving

There are many instructional strategies that involve students working together to solve a problem, including inquiry based learning, authentic learning, and discovery learning. While they each have their own unique characteristics, they all fundamentally involve:

- Presenting students with a problem.
- Providing some structure or guidance toward solving the problem. Note, however, that they are all student-centered activities in which the instructor may have a very minimal role.
- Reaching a final outcome or solution.

7.8 PROBLEM SOLVING

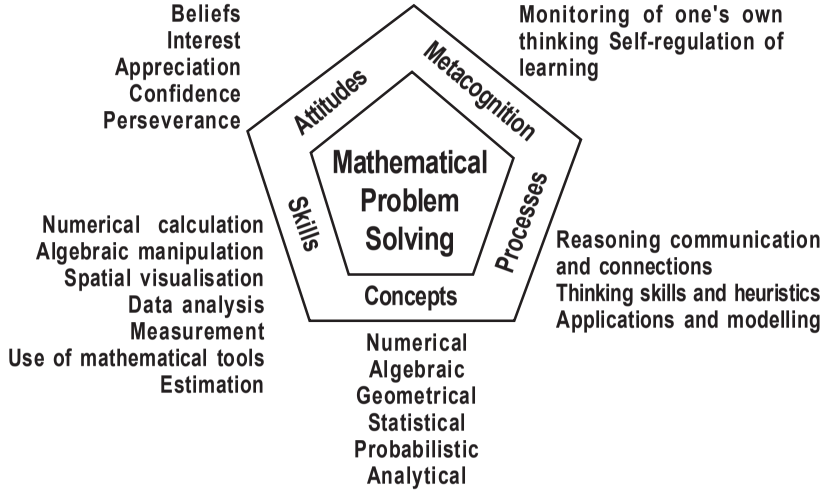
Introduction to Problem Solving:

Problem-solving is a process—an on-going activity in which we take what we know to discover what we don't know. It involves overcoming obstacles by generating hypo-theses, testing those predictions, and arriving at satisfactory solutions.

Problem-solving involves three basic functions:

1. Seeking information
2. Generating new knowledge
3. Making decisions

Problem-solving is, and should be, a very real part of the curriculum. It presupposes that students can take on some of the responsibility for their own learning and can take personal action to solve problems, resolve conflicts, discuss alternatives, and focus on thinking as a vital element of the curriculum. It provides students with opportunities to use their newly acquired knowledge in meaningful, real-life activities and assists them in working at higher levels of thinking.



Here is a five-stage model that most students can easily memorize and put into action and which has direct applications to many areas of the curriculum as well as everyday life:

Here are some techniques that will help students understand the nature of a problem and the conditions that surround it:

- List all related relevant facts.
- Make a list of all the given information.
- Restate the problem in their own words.
- List the conditions that surround a problem.
- Describe related known problems.

For younger students, illustrations are helpful in organizing data, manipulating information, and outlining the limits of a problem and its possible solution(s). Students can use drawings to help them look at a problem from many different perspectives.

1. **Understand the problem.** It's important that students understand the nature of a problem and its related goals. Encourage students to frame a problem in their own words.
2. **Describe any barriers.** Students need to be aware of any barriers or constraints that may be preventing them from achieving their goal. In short, what is creating the problem? Encouraging students to verbalize these impediments is always an important step.
3. **Identify various solutions.** After the nature and parameters of a problem are understood, students will need to select one or more appropriate strategies to help resolve the problem. Students need to understand that they have many strategies available to them and that no single strategy will work for all problems. Here are some problem-solving possibilities:

- **Create visual images.** Many problem-solvers find it useful to create "mind pictures" of a problem and its potential solutions prior to working on the problem. Mental imaging allows the problem-solvers to map out many dimensions of a problem and "see" it clearly.
- **Guesstimate.** Give students opportunities to engage in some trial-and-error approaches to problem-solving. It should be understood, however, that this is not a singular approach to problem-solving but rather an attempt to gather some preliminary data.

Pedagogy of Mathematics - Sem-II (TS)

- **Create a table.** A table is an orderly arrangement of data. When students have opportunities to design and create tables of information, they begin to understand that they can group and organize most data relative to a problem.
- **Use manipulative.** By moving objects around on a table or desk, students can develop patterns and organize elements of a problem into recognizable and visually satisfying components.
- **Work backward.** It's frequently helpful for students to take the data presented at the end of a problem and use a series of computations to arrive at the data presented at the beginning of the problem.
- **Look for a pattern.** Looking for patterns is an important problem-solving strategy because many problems are similar and fall into predictable patterns. A pattern, by definition, is a regular, systematic repetition and may be numerical, visual, or behavioural.
- **Create a systematic list.** Recording information in list form is a process used quite frequently to map out a plan of attack for defining and solving problems. Encourage students to record their ideas in lists to determine regularities, patterns, or similarities between problem elements.

4. **Try out a solution.** When working through a strategy or combination of strategies, it will be important for students to …

- **Keep accurate and up-to-date records of their thoughts, proceedings, and procedures.**Recording the data collected, the predictions made, and the strategies used is an important part of the problem solving process.
- **Try to work through a selected strategy or combination of strategies until it becomes evident that it's not working, it needs to be modified, or it is yielding inappropriate data.** As students become more proficient problem-solvers, they should feel comfortable rejecting potential strategies at any time during their quest for solutions.
- **Monitor with great care the steps undertaken as part of a solution.** Although it might be a natural tendency for students to

"rush" through a strategy to arrive at a quick answer, encourage them to carefully assess and monitor their progress.

- **Feel comfortable putting a problem aside for a period of time and tackling it at a later time.** For example, scientists rarely come up with a solution the first time they approach a problem. Students should also feel comfortable letting a problem rest for a while and returning to it later.

5. **Evaluate the results.** It's vitally important that students have multiple opportunities to assess their own problem-solving skills and the solutions they generate from using those skills. Frequently, students are overly dependent upon teachers to evaluate their performance in the classroom. The process of self-assessment is not easy, however. It involves risk-taking, self-assurance, and a certain level of independence. But it can be effectively promoted by asking students question such as "How do you feel about your progress so far?" "Are you satisfied with the results you obtained?" and "Why do you believe this is an appropriate response to the problem?"

7.7.4 GEORGE POLYA (1887 - 1985)

Over the years, many have thought about the question whether the art of problem solving can be taught or is it a talent possessed by only a few? An effective and definite answer was given by the late George Polya. He maintained that the skill of problem solving can be taught. Polya was born in Hungary in 1887 and received his Ph.D. in mathematics from the University of Budapest. He taught for many years at the Swiss Federal Institute of Technology in Zurich. Among the numerous books that he wrote he seemed most proud of 'How to Solve It' (1945) which has sold nearly one million copies and has been translated into 17 languages.

Polya's four principles of Problem solving

I. Understand the problem

This principle seems so obvious that it need not be mentioned. However students are often stymied in their efforts to solve a problem because they don't understand it fully or even in part. Teachers should ask students such questions as

- Do you understand all the words used in stating the problems? If not, look them up in the index, in a dictionary or wherever they can be found.
- What are you asked to find or show can you restate the problem in your own words.
- Is there yet another way to state the problem O What does (key word) really mean?
- Could you work out some numerical examples that would help make the problem clear?
- Could you think of a picture or diagram that might help you to understand the problem.
- Is there enough information to enable you to find a solution. O Is there extraneous information? O What do you really need to know to find a solution.

II. Devise a plan

Devising a plan for solving a problem once it is fully understood may still require substantial effort. But don't be afraid to make start you may be on the right track. There are often many reasonable ways to try to solve a problem and the successful idea may emerge only gradually after several unsuccessful trials.

A partial list of strategies include.

- guess and check O look for a pattern
- make an orderly list O draw a picture
- thick of the problem as particularly solved
- think of a similar problem already solved
- eliminate possibilities
- solve simpler problem
- solve an equivalent problem
- solve an analogous problem
- use symmetry
- use a model
- consider special cases
- work backward O use direct reasoning
- use a formula
- solve an equation

- be ingenious

III. Carryout the plan

Carrying out the plan is usually easier than devising the plan. In general all you need is care and patience, given that you have the necessary skills. If a plan does not work immediately be persistent. If it still does not work, discard it and try a new strategy. Don't be misled this is the way mathematics is done, even by professionals.

IV. Look back

Much can be gained by looking back a completed solution to analyze your thinking and ascertain just what was the key to solving the problem. This is how we gain "Mathematical power", the ability to come up with good ideas for solving problems never encountered before.

7.8 CONCEPT MAPPING

Introduction

The concept map is a relatively new way to visualize complex subject matter. The technique of concept mapping was first developed in the 1960s and 1970s by American educator and research scientist Joseph D. Novak (1930–) while at Cornell University, in Ithaca, New York.

The concept map itself is founded in a learning theory called constructivism, which states that humans learn from previously acquired knowledge. Swiss developmental psychologist Jean Piaget (1896–1980) is generally recognized as the first scientist to formalize constructivism into scientific structure.

Later, the theory of concept mapping developed by Novak, and first published in 1977, helped to guide educational research and instruction. Since that time, concept mapping has been widely applied to science, education, business, and government.

Characteristics of Concept Mapping:

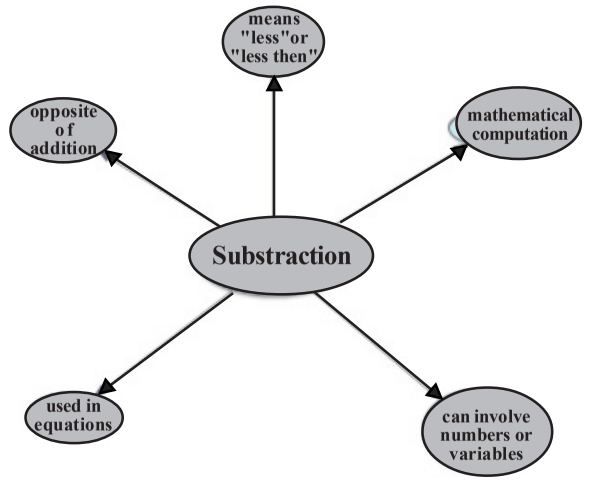
- A concept map or conceptual diagram is a diagram that depicts suggested relationships between concepts.
- It is a graphical tool that instructional designers, engineers, technical writers, and others use to organize and structure knowledge.
- A concept map typically represents ideas and information as boxes or circles, which it connects with labelled arrows in a downward-branching hierarchical structure.

Pedagogy of Mathematics - Sem-II (TS)

- The relationship between concepts can be articulated in linking phrases such as causes, requires, or contributes to.
- The technique for visualizing these relationships among different concepts is called concept mapping.
- A concept map is a way of representing relationships between ideas, images, or words in the same way that a sentence diagram represents.
- In a concept map, each word or phrase connects to another, and links back to the original idea, word, or phrase.
- Concept maps are a way to develop logical thinking and study skills by revealing connections and helping students see how individual ideas form a larger whole.
- Concept mapping is also sometimes used for brain-storming.
- Although they are often personalized and idiosyncratic, concept maps can be used to communicate complex ideas.

Example:

Below is an example of exploration and development cycle emphasizing the use of concept maps for subtraction.When students struggle to create a concept map representative of their conceptual understanding, they are personally engaging in the development of knowledge. A learner who has spent most of his or her life learning by rote is now required to develop a means for organizing current knowledge along with the assimilation of new information.



Uses of Concept Maps:

Concept maps can be used;

- To stimulate the generation of ideas, and are believed to aid creativity.
- For knowledge construction: how students construct their knowledge.
- New knowledge creation: e.g., transforming tacit knowledge into an organizational resource, mapping team knowledge.
- Collaborative knowledge modelling and the transfer of expert knowledge.
- Facilitating the creation of shared vision and shared understanding within a concept.
- Instructional design: concept maps "advance organizers" that provide an initial conceptual frame for subsequent information and learning.
- Increasing meaningful learning for example through writing activities where concept maps automatically generated from an essay is shown to the writer.
- Assessing learner understanding of learning objectives, concepts, and the relationship among those concepts.
- Examining the symmetry of complex ideas and arguments and associated terminology.
- To explore the complex roles and definitions.
- Knowledge elicitation.
- Communicating complex ideas and arguments.
- Enhancing metacognition (learning to learn, and thinking about knowledge)
- evaluation (to evaluate how students organize their knowledge) for record of understanding
- application
- For integration of various concepts.
- To ease Instruction.

7.9 EXPERIENTIAL LEARNING

Experiential learning is the process of learning through experience of and is more specifically defined as "learning through reflection on

doing".Hands-on learning is a form of experiential learning but does not necessarily involve students reflecting on their product. Experiential learning is distinct from rote or didactic learning, in which the learner plays a comparatively passive role. It is related to but not synonymous with other forms of active learning such as action learning, adventure learning , free choice learning, cooperative and service learning.

Experiential learning is often used synonymously with the term "experiential education" ,but while experiential education is a broader philosophy of education, experiential learning considers the individual learning process. As such, compared to experiential education, experiential learning is concerned with more concrete issues related to the learner and the learning context.

The general concept of learning through experience is ancient. Around 350 BCE, Aristotle wrote in Nicomachean Ethics for the things we have to learn before we can do them, we learn by doing them". But as an articulated educational approach, experiential learning is of much more recent vintage. Beginning in the 1970s, David A. Kolb helped to develop the modern theory of experiential learning, drawing heavily on the work of John Dewey, Kurt Lewin, and Jean Piaget.

Kolb - Learning Styles

David Kolb published his learning styles model in 1984 from which he developed his learning style inventory.

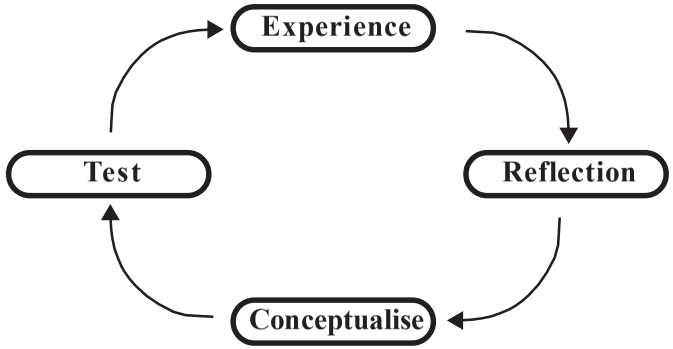
Kolb's experiential learning theory works on two levels: a four stage cycle of learning and four separate learning styles. Much of Kolb's theory is concerned with the learner's internal cognitive processes.

Kolb states that learning involves the acquisition of abstract concepts that can be applied flexibly in a range of situations. In Kolb's theory, the impetus for the development of new concepts is provided by new experiences.

"Learning is the process whereby knowledge is created through the transformation of experience" (Kolb, 1984, p. 38).

The Experiential Learning Cycle

Kolb's experiential learning style theory is typically represented by a four stage learning cycle in which the learner 'touches all the bases':



1. ***Concrete Experience*** - a new experience of situation is encountered, or a reinterpretation of existing experience is done.
2. ***Reflective Observation***–reflective observation of the new experience is of a particular importance and are any inconsistencies between experience and understanding.
3. ***Abstract Conceptualization*** - Reflection gives rise to a new idea, or a modification of an existing abstract concept.
4. ***Active Experimentation*** - the learner applies them to the world around them to see what results.

Abilities of an experiential learner*:*

Effective learning is seen when a person progresses through a cycle of four stages:

1. having a concrete experience followed by
2. observation of and reflection on that experience which leads to
3. the formation of abstract concepts (analysis) and generalizations (conclusions) which are then
4. used to test hypothesis in future situations, resulting in new experiences.

Kolb (1974) views learning as an integrated process with each stage being mutually supportive of and feeding into the next. It is possible to enter the cycle at any stage and follow it through its logical sequence.

However, effective learning only occurs when a learner is able to execute all four stages of the model. Therefore, no one stage of the cycle is an effective as a learning procedure on its own.

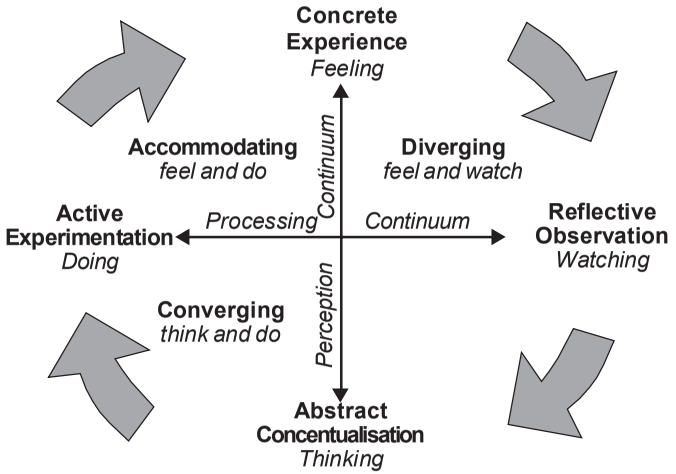
Learning Styles

Kolb's learning theory (1974) sets out four distinct learning styles, which are based on a four-stage learning cycle (see above).

Kolb explains that different people naturally prefer a certain single different learning style. Various factors influence a person's preferred style. For example, social environment, educational experiences, or the basic cognitive structure of the individual.

Whatever influences the choice of style, the learning style preference itself is actually the product of two pairs of variables, or two separate 'choices' that we make, which Kolb presented as lines of axis, each with 'conflicting' modes at either end:

A typical presentation of Kolb's two continuums is that the east-west axis is called the Processing Continuum (how we approach a task), and the north-south axis is called the Perception Continuum (our emotional response, or how we think or feel about it).



Kolb believed that we cannot perform both variables on a single axis at the same time (e.g. think and feel). Our learning style is a product of these two choice decisions.

It's often easier to see the construction of Kolb's learning styles in terms of a two-by-two matrix. Each learning style represents a combination of two preferred styles. The diagram also highlights Kolb's

terminology for the four learning styles; diverging, assimilating, and converging, accommodating:

Learning Styles Descriptions

Knowing a person's (and your own) learning style enables learning to be orientated according to the preferred method. That said, everyone responds to and needs the stimulus of all types of learning styles to one extent or another - it's a matter of using emphasis that fits best with the given situation and a person's learning style preferences.

Here are brief descriptions of the four Kolb learning styles:

Diverging (feeling and watching - CE/RO)

These people are able to look at things from different perspectives. They are sensitive. They prefer to watch rather than do, tending to gather information and use imagination to solve problems. They are best at viewing concrete situations at several different viewpoints.

Kolb called this style 'diverging' because these people perform better in situations that require ideas-generation, for example, brainstorming. People with a diverging learning style have broad cultural interests and like to gather information.

They are interested in people, tend to be imaginative and emotional, and tend to be strong in the arts. People with the diverging style prefer to work in groups, to listen with an open mind and to receive personal feedback.

Assimilating(watching and thinking - AC/RO)

The Assimilating learning preference is for a concise, logical approach. Ideas and concepts are more important than people. These people require good clear explanation rather than practical opportunity. They excel at understanding wide-ranging information and organizing it in a clear logical format.

People with an assimilating learning style are less focused on people and more interested in ideas and abstract concepts. People with this style are more attracted to logically sound theories than approaches based on practical value.

This learning style is important for effectiveness in information and science careers. In formal learning situations, people with this style prefer readings, lectures, exploring analytical models, and having time to think things through.

Converging (doing and thinking - AC/AE)

People with a converging learning style can solve problems and will use their learning to find solutions to practical issues. They prefer technical tasks, and are less concerned with people and interpersonal aspects.

People with a converging learning style are best at finding practical uses for ideas and theories. They can solve problems and make decisions by finding solutions to questions and problems.

People with a converging learning style are more attracted to technical tasks and problems than social or interpersonal issues. A converging learning style enables specialist and technology abilities. People with a converging style like to experiment with new ideas, to simulate, and to work with practical applications.

Accommodating(doing and feeling - CE/AE)

The Accommodating learning style is 'hands-on', and relies on intuition rather than logic. These people use other people's analysis, and prefer to take a practical, experiential approach. They are attracted to new challenges and experiences, and to carrying out plans.

They commonly act on 'gut' instinct rather than logical analysis. People with an accommodating learning style will tend to rely on others for information than carry out their own analysis. This learning style is prevalent within the general population.

Educational Implications

Both Kolb's (1984) learning stages and cycle could be used by teachers to critically evaluate the learning provision typically available to students, and to develop more appropriate learning opportunities.

Educators should ensure that activities are designed and carried out in ways that offer each learner the chance to engage in the manner that suits them best. Also, individuals can be helped to learn more effectively by the identification of their lesser preferred learning styles and the