FOCUS
Foundation of Computing & Sciences
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Optimal Communication and Distillation Bounds for Multipartite Nonlocality

Multipartite Non-Adaptive Nonlocality Distillation

In nonlocality distillation, the goal is to increase the nonlocal value V of identical copies of a nonlocal box (NLB). We show that parity protocol is the optimal non-adaptive protocol for the class of XOR NLBs shared between n players.

$$V = \sum_{a=f(\hat{x})} p_{\hat{a}|\hat{x}} - \sum_{a\neq f(\hat{x})} p_{\hat{a}|\hat{x}}$$

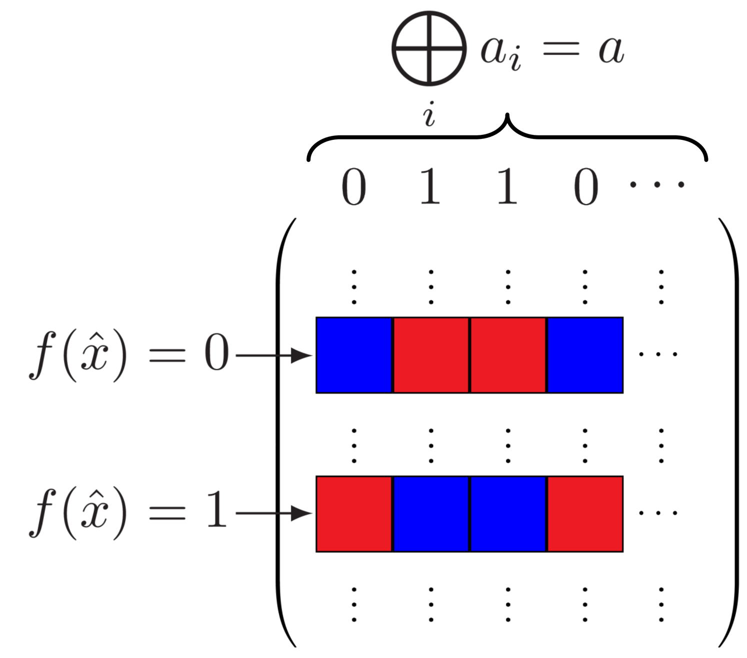
$$p_{\hat{a}|\hat{x}} = \begin{cases} \frac{1+\delta_{\hat{x}}}{2^n} & \text{if } \oplus_{i=1}^n a_i = a = f(\hat{x}) \\ \frac{1-\delta_{\hat{x}}}{2^n} & \text{otherwise} \end{cases}$$

Parity is Optimal for XOR Boxes

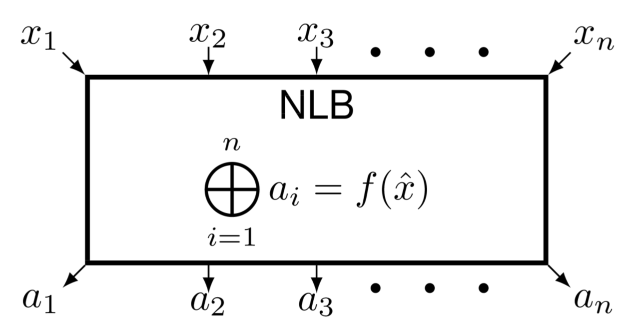
Theorem 1. The value of all non-adaptive distillation protocols, given m boxes is bounded by the value given by

$$\max_{1\leqslant k\leqslant m} \left| \sum_{\hat{x}\in\{0,1\}^n} (-1)^{f(\hat{x})}\delta_{\hat{x}}^k \right|.$$

The bound is tight since the value is attainable via the parity protocol.



The rows in the XOR NLBs map to two complementary probability distributions



Communication Bounds for Simulating Multipartite Nonlocality

$$p_{\hat{a}|\hat{x}} = \begin{cases} \frac{1-\epsilon}{2^{n-1}} & \text{if } \oplus_{i=1}^n a_i = x_1 \cdot x_2 \cdot \ldots \cdot x_n \\ \frac{\epsilon}{2^{n-1}} & \text{otherwise} \end{cases}$$

Given a multipartite generalization of the bipartite Popescu-Rohrlich-Box, we construct a protocol that achieves the exact communication necessary to simulate the probability distributions of the box. The construction is built upon a formulation of the problem as a linear program due to Pironio. The key observation is that every no-signalling distribution can be written as a convex combination of signalling strategies employed by the players. Identification of the optimal protocol boils down to carefully choosing the right weight for each signalling strategy such that the overall communication cost is minimized.

Optimal Protocol for Simulating PR-Boxes

Theorem 2. The amount of communication required by our protocol to simulate a given isotropic multipartite PR-Box is

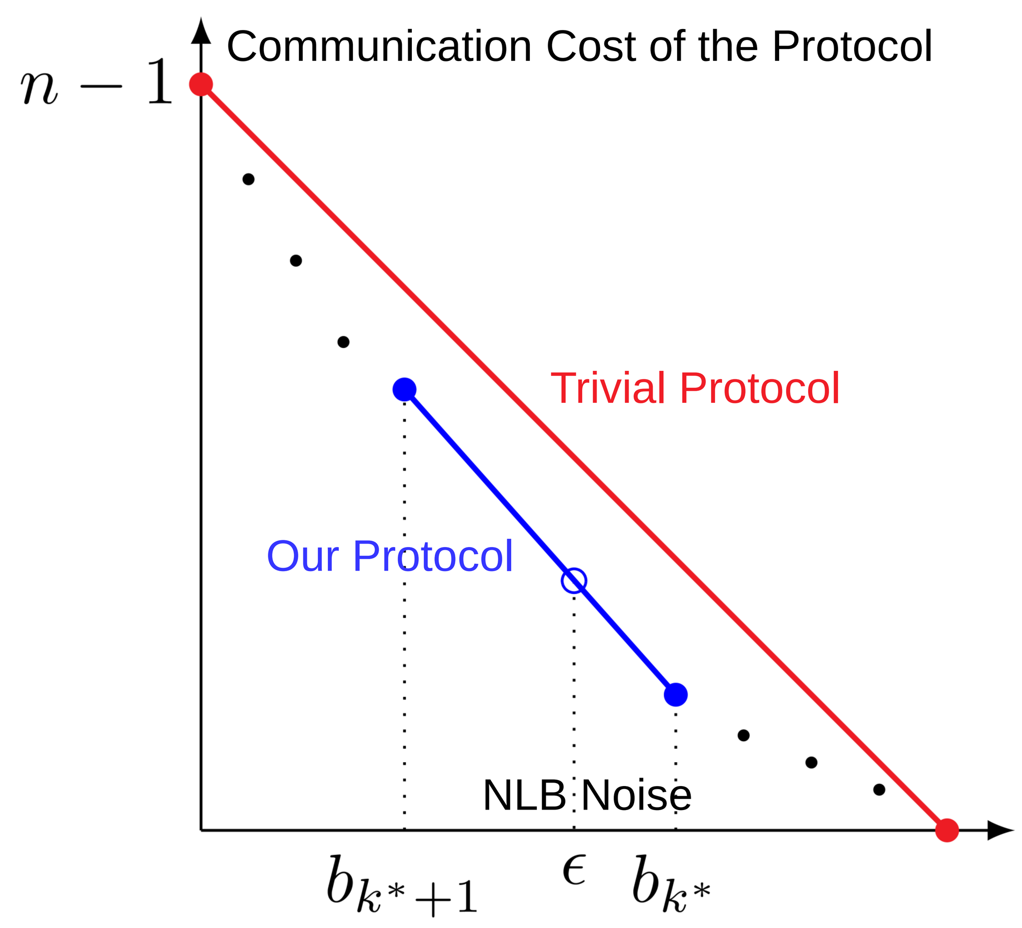
$$\max_{1\leqslant k\leqslant n} \frac{c_k - \epsilon(2c_k + N(k^2+1))}{D}, \text{ where}$$

$$N = 2^{n-1} \quad K = 2^{k-1}$$

$$D = N\left(k - 1 + \frac{1}{K}\right)$$

$$c_k = (k^2+1)(N-1) - n(2K-k-1).$$

Optimality of the protocol follows from matching solution of the primal and dual linear programs.



The general form of the communication bound resulting from our Protocol

Primal Program

$$\min \sum_i \sum_{\lambda_i} C_i p_{\lambda_i}$$

$$\text{subject to } \sum_i \sum_{\lambda_i} p_{\lambda_i} \mathsf{D}_{\lambda_i} = \text{NLB}$$

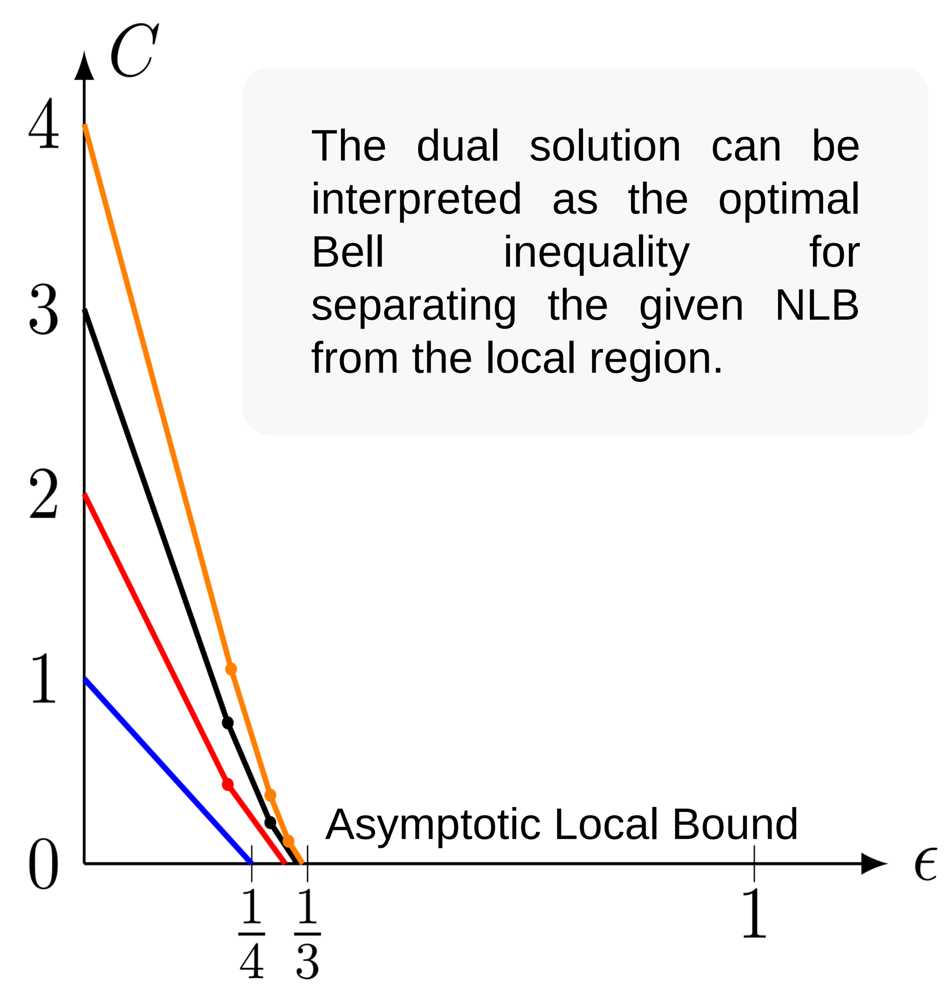
$$\sum_{\lambda_i} p_{\lambda_i} = 1$$

$$p_{\lambda_i} \geq 0$$

Dual Program

$$\max \hat{b} \cdot \hat{p}$$

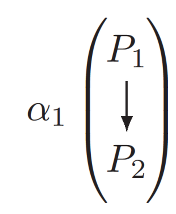
$$\text{subject to } \hat{b} \cdot \mathsf{D}_{\lambda_i} \leqslant C_i$$



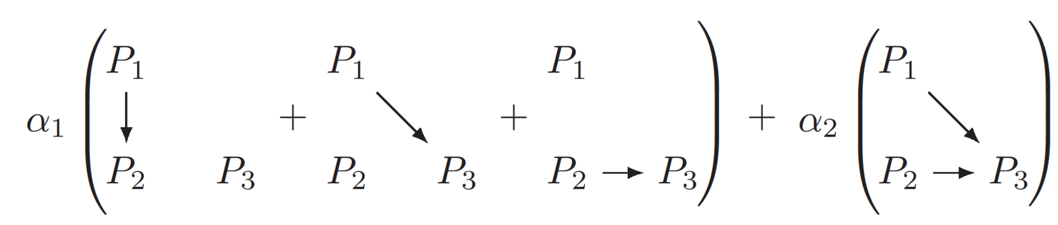
Communication bounds achieved by our Protocol for 2, 3, 4 and 5 players

The dual solution can be interpreted as the optimal Bell inequality for separating the given NLB from the local region.

The two breakpoints for each line determine how many parties communicate in the protocol. The number of parties that need to communicate increases as the noise level passes each breakpoint. This can be reformulated to detect k-partite nonlocality.

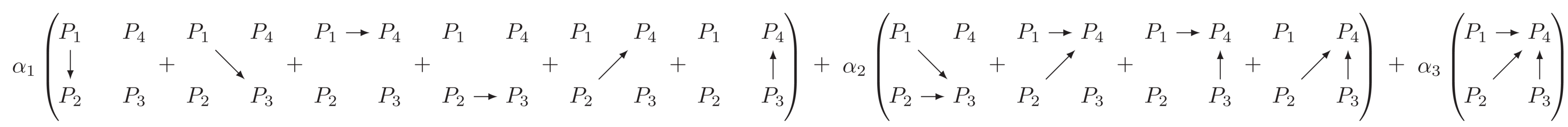


Communication structure of our Protocol for 2 players



Communication structure of our Protocol for 3 players

Communication structure of our Protocol for 4 players



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