A New Fast Algorithm to Estimate Real-Time Phasors Using Adaptive Signal Processing

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Abstract—**Phasor magnitude and angle of various harmonic and interharmonic components of the power signal are widely used as critical variables and performance indices for power system applications, such as protection, relaying, and state monitoring. This paper proposes a novel adaptive algorithm for estimating the phasor parameters in real time. It uses a faster quasi-second-order optimization technique to estimate amplitude and phase and does not require any matrix inversions. It features fast response, achieves high accuracy, and involves less computational complexity than many other models and window-based methods.**

Index Terms—**Adaptive signal processing, block least mean square, conjugate gradient-based search, decaying dc offset elimination, harmonic and interharmonic component estimation, phasor amplitude value, phasor angle.**

I. Introduction

POWER systems in many applications require real-time measurements of phasor magnitude and angle of the fundamental component and the harmonics present in the voltage and current signals of the power line. These are parameters of critical importance for the purpose of monitoring, control, and protection. Speedy and accurate estimations are required for a proactive response under abnormal conditions and to effectively monitor and preempt any escalation of system issues.

A variety of techniques for real-time estimation of phasors has been developed and evaluated in past two decades. They are either model based, least error squared (LES), recursive least square (RLS) [1], Kalman filtering [2], or other window-based methods. They all use the stationary signal sinusoidal model. LES, RLS, and Kalman filters are more suitable for online processing since they generate time trajectories of the evolved parameter but the complexity involved is significant and the matrix has to be fixed and accurate for the model to work. Any drift from the assumed nominal frequency will render the model highly inaccurate. Some artificial-intelligence techniques, such as genetic algorithms (GAs) [3] and neural networks [4], have been used to achieve precise frequency estimation and phasor estimations over a wide range with fast response. Although better performance can be achieved by these optimization techniques, the implementation algorithm is more complex and intense in computations.

Window-based methods, such as discrete Fourier transform (DFT), short time Fourier transform (STFT), and wavelets, are also applied extensively for real-time estimation of power system amplitude and phase parameters. DFT is desirable due to its low computational requirement and fast response. However, the implicit data window in the DFT approach requires a full cycle. To improve the performance of DFT-based approaches, some enhancements have been proposed. Adaptive methods based on the feedback loop by tuning the sample interval [5], adjusting the data window length [6], changing the nominal frequency used in DFT iteratively [7], correcting the gains of the orthogonal filters [8], and tuning the weighted factor [9] recursively, and compensation method to improve phasor estimation [10] are proposed. But due to the inherent limitations in such methods, at least one cycle of the analyzed signal is still required, which hardly meets the demand of high-speed response especially for protection schemes. A method using three consecutive samples of the instantaneous input signal is discussed in [11]. The noise and zero crossing issue may bring large errors to this method. The STFT-based approach has limitations in its accuracy and still requires half a cycle to respond [12]. Recursive wavelet transform (RWT), which is faster, can output phasor parameters in a quarter cycle and has been proposed recently [13]. In this method, inappropriately selecting the window length and sampling rate may cause the weighting matrix to go singular. Also, it has the inherent limitation of having more computational requirements and a higher sampling rate to achieve reasonable accuracy in short time. A combination of algorithms has also been used to overcome individual limitations [14] which can only improve the accuracy at the expense of increased complexity and additional delays.

Many techniques have been proposed to eliminate the imact of decaying dc components in phasor estimation. The conventional DFT algorithm achieves excellent performance when the signals contain only the fundamental frequency and interger harmonic frequency components since, in most cases, the currents containing decaying dc components may introduce fairly large errors in the phasor estimations [15], [16]. A digital filter that features high-pass frequency response can help filter the high-frequency noise. It performs well when its time constant matches the time constant of the exponentially decaying component. Theoretically, the decaying component can be completely removed from the original waveform once its parameters can be obtained. Based on this idea, [17] and [18] utilize additional samples to calculate the parameters of the decaying component. Reference [19] uses the simultaneous equations

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derived from the harmonics. The effect of dc components by DFT is eliminated by using the outputs of an even-sample-set and odd-sample-set [20]. Reference [21] hybridizes the partial sum-based method and least-squares-based method to estimate the dc offset parameters. A new Fourier algorithm and three simplified algorithms based on Taylor expansion were proposed to eliminate the decaying component in [22]. In [23], the author estimates the parameters of the decaying component by using the phase-angle difference between voltage and current. This method requires both voltage and current inputs. As a result, it is not applicable to the current-based protection schemes. The recursive wavelet approach was introduced in protective relaying for a long time [24]–[26]. The improved model with single direction recursive equations is more suitable for the application to real-time signal processing [25]. The band energy of any center frequency can be extracted through recursive wavelet-transform (RWT) with moderately low computational burden [13].

In this paper, the estimation is performed using linear adaptive filtering techniques. The phasor quantities to be determined are modeled as weights of the linear filter. This way, the model can be easily adapted to drifts in the nominal frequency, when it is known, for each separate component, easily tracking its amplitude and phase changes. Then adaptive block least mean square algorithm (B-LMS) with optimally derived step sizes using conjugate gradient search directions is used, rather than gradient-based directions, for minimizing the mean square error (MSE) or the cost function of the linear system. In simulations, the performance of this new algorithm is compared with a number of other popular published algorithms, both model based and window based.

This paper is organized as follows: Section II describes the formulation of phasor estimation as a linear filtering problem. Section III gives the overview of the conjugate gradient technique and B-LMS algorithm. Section IV has the proposed method along with convergence characteristics. In Section V, the simulations results are presented in comparison with the other algorithms followed by conclusions in Section VI.

II. Modeling Phasor Estimation as a Linear Filtering Problem

Linear filter: Consider a system with L inputs coming from sensors placed systematically in the environment that performs a weighted linear combination of inputs. Then, the output of the system is given by

$$y(n) = x_1(n)w_1(n) + x_2(n)w_2(n) + \ldots + x_L(n)w_L(n). \quad (1)$$

Such a system is called a linear filter, and the weights can be adaptively estimated with some algorithms.

A. Phasor Estimation Model With Fundamental and Harmonics

Let us consider a discrete input signal that contains a fundamental frequency with the sampling period ΔT. Also, without loss of generality, we will consider a harmonic h of the fundamental frequency in the signal; where h need not be an integer

$$\begin{aligned} y(n) = {} & V_{p0} sin[2\pi f_0 n\Delta T + \theta_0] \\ & + V_{ph} sin[2\pi f_h n\Delta T + \theta_h] \\ n = {} & 0, 1, 2, 3 \cdots \end{aligned} \quad (2)$$

where V_{p0}, f_0, θ_0 represents the amplitude, frequency, and phase of the fundamental component and V_{ph}, f_h, θ_h are the amplitude, frequency, and phase of hth harmonic or interharmonic if h is a real number such that $f_h = hf_0$ in the composite signal, respectively.

Then, using the trigonometric expansion for (2), we obtain

$$\begin{aligned} y(n) = {} & V_{p0} sin[2\pi f_0 n\Delta T] cos[\theta_0] \\ & + V_{p0} cos[2\pi f_0 n\Delta T] sin[\theta_0] \\ & + V_{ph} sin[2\pi f_h n\Delta T] cos[\theta_h] \\ & + V_{ph} cos[2\pi f_h n\Delta T] sin[\theta_h]. \end{aligned} \quad (3)$$

We use the following notations at this point:

$$\begin{aligned} w_1(n) &= V_{p0} cos[\theta_0] \\ w_2(n) &= V_{p0} sin[\theta_0] \\ w_3(n) &= V_{ph} cos[\theta_h] \\ w_4(n) &= V_{ph} sin[\theta_h] \\ x_1(n) &= sin[2\pi f_0 n\Delta T] \\ x_2(n) &= cos[2\pi f_0 n\Delta T] \\ x_3(n) &= sin[2\pi f_h n\Delta T] \\ x_4(n) &= cos[2\pi f_h n\Delta T]. \end{aligned}$$

Substituting above values in (3), we obtain

$$\begin{aligned} y(n) = {} & x_1(n)w_1(n) + x_2(n)w_2(n) \\ & + x_3(n)w_3(n) + x_4(n)w_4(n) \end{aligned}$$

which is simply a linear weighted combination of inputs.

Once the weights of the filters are estimated, the amplitude and phase of harmonics and interharmonics can be estimated with the following equations:

$$\begin{aligned} V_{p0} &= \sqrt{w_1(n)^2 + w_2(n)^2} \\ \theta_0 &= \arctan \frac{w_2(n)}{w_1(n)} \\ V_{ph} &= \sqrt{w_3(n)^2 + w_4(n)^2} \\ \theta_h &= \arctan \frac{w_4(n)}{w_3(n)}. \end{aligned}$$

Clearly, if the harmonic or the interharmonic we seek is absent in the signal, the corresponding weights describing it in the linear system will be close to zero.

B. Phasor Estimation Model With Decaying DC Component

Let us consider a discrete input signal that contains only the fundamental frequency with the sampling period ΔT and a single decaying dc component

$$y(n) = V_{p0} sin[2\pi f_0 n\Delta T + \theta_0] + V_{dc} e^{\frac{-n\Delta T}{\tau}} \quad n = 0, 1, 2, 3 \cdots \tag{4}$$

where V_{p0}, f_0, and θ_0 represent the amplitude, frequency, and phase of the fundamental component and V_{dc}, τ are the amplitude and time constant of the decaying dc component present in the composite signal.

Then, using Taylor's series expansion up to the first order for the second quantity in (4), we have

$$\begin{aligned} y(n) = & V_{p0} sin[2\pi f_0 n\Delta T] cos[\theta_0] \\ & + V_{p0} cos[2\pi f_0 n\Delta T] sin[\theta_0] \\ & + V_{dc} - \frac{V_{dc} \cdot n\Delta T}{\tau}. \end{aligned} \tag{5}$$

We use the following notations at this point:

$$\begin{aligned} w_1(n) &= V_{p0} cos[\theta_0] \\ w_2(n) &= V_{p0} sin[\theta_0] \\ w_3(n) &= V_{dc} \\ w_4(n) &= \frac{V_{dc}}{\tau} \\ x_1(n) &= sin[2\pi f_0 n\Delta T] \\ x_2(n) &= cos[2\pi f_0 n\Delta T] \\ x_3(n) &= 1 \\ x_4(n) &= -n\Delta T. \end{aligned}$$

Substituting the values in (5), we obtain

$$\begin{aligned} y(n) = & x_1(n)w_1(n) + x_2(n)w_2(n) \\ & + x_3(n)w_3(n) + x_4(n)w_4(n) \end{aligned}$$

which is simply a linear weighted combination of inputs.

Once the weights of the filters are estimated, the amplitude and phase of the fundamental harmonic and the amplitude and time constant of the decaying dc can be estimated with the following equations:

$$\begin{aligned} V_{p0} &= \sqrt{w_1(n)^2 + w_2(n)^2} \\ \theta_0 &= \arctan \frac{w_2(n)}{w_1(n)} \\ V_{dc} &= w_3(n) \\ \tau &= \frac{w_3(n)}{w_4(n)}. \end{aligned}$$

III. B-LMS With Conjugate Gradient-Based Search Direction

A. B-LMS Algorithm

B-LMS or block least mean square error algorithm is extensively applied in numerous signal processing areas such as wireless communications, statistical, speech and biomedical signal processing [27]. The B-LMS algorithm provides a robust computational method for determining the optimum filter coefficients (i.e. weight vector w). The algorithm is basically a recursive gradient (steepest-descent) method that finds the minimum of the MSE and thus yields the set of optimum filter coefficients. The instantaneous error signal at instant n processed in blocks of length L and weights of size K for a linear system are given by

$$e_L(n) = y_L(n) - x_L^T(n)w(n)$$

where

$$\begin{aligned} y_L(n) &= [y(n), y(n-1) \ldots y(n-L)] \\ x_L(n) &= [X(n), X(n-1) \ldots X(n-L)] \\ X(n) &= [x_1(n), x_2(n-1) \ldots x_K(n)] \\ w(n) &= [w_1(n), w_2(n-1) \ldots w_K(n)] \end{aligned} \tag{6}$$

are the output, input, and weight vectors. For simplicity, henceforth the subscript L will be dropped from notation.

The mean square error (MSE) signal or the cost function is given by

$$E[w(n)] = \frac{1}{L} e^T(n)e(n). \tag{7}$$

Differentiating (7) with reference to weight vector w(n) yields:

$$\frac{\partial E[w(n)]}{\partial w(n)} = \frac{2}{L} e(n) \frac{\partial e(n)}{\partial w(n)}.$$

Hence, the negative gradient vector is given by

$$-\frac{\partial E[w(n)]}{\partial w(n)} = \frac{2}{L} x^T(n)e(n). \tag{8}$$

The update of the filter coefficients is done proportional to negative of the radient according to the following equation:

$$\hat{w}(n+1) = \hat{w}(n) + \eta \cdot \frac{2}{L} x^T(n)e(n) \tag{9}$$

where η is the step size or learning parameter and $\hat{w}(n), \hat{w}(n+1)$ represents the initial and the new estimates of the weight vectors, respectively.

The feedback loop around the estimate of the weight vector $\hat{w}(n)$ in the LMS algorithm acts like a *low-pass fitler*, passing the low-frequency components of the error signal and attenuating its high-frequency components. Also unlike other methods, the LMS algorithm does not require the knowledge of the statistics of the environment. Hence, it is sometimes called stochastic gradient algorithm. In precise mathematical terms, the LMS algorithm is optimal in accordance with the H^∞ (or *minimax*) criterion [28].

B. Conjugate Gradient Method

We know that the instanteneous error vector is linear function of weights from (6) and hence the error surface (MSE) is quadratic function of weight vector from (7). The gradient computed from (8) is clearly a first order partial differential w.r.t weights and if we use it in (9) for each iteration keeping the step size η constant, then the algorithm would be consider a first order optimization steepest descent method. It may need a large number of iterations leading to slow convergence for some quadratic problems. The conjugate gradient method tries to overcome this issue by generating orthogonal search vectors of gradient. It belongs to a class of second order optimization methods collectively known as *conjugate-direction methods* [29]. Consider minimization of a quadratic function $f(w)$

$$f(w) = \frac{1}{2}w^T Aw - b^T w + c$$

where w is a L-by-1 parameter vector, A is a L-by-L symmetric, positive definite matrix, b is a L-by-1 vector, and c is a scalar. Minimization of the quadratic function $f(w)$ is achieved by assigning w to the unique value

$$w^* = A^{-1}b.$$

Thus, minimizing $f(w)$ and solving the linear system of equations $Aw^* = b$ are equivalent problems. Given a matrix A, the set of nonzero vectors $s(1), s(2), .s(L)$ is A-*conjugate* if

$$s^T(n)As(j) = 0 \quad \text{for all } n \text{ and } j \text{ such that } n \neq j.$$

For a given set of vectors $s(1), s(2), \cdots s(L)$, the corresponding conjugate direction for the unconstrained minimization of the quadratic error function $f(w)$ is defined by

$$\begin{aligned} w(n+1) &= w(n) + \eta(n)s(n) \\ n &= 1, 2, 3 \cdots L \end{aligned} \tag{10}$$

where $s(n)$ is the gradient direction and $\eta(n)$ is a scalar defined by

$$f\left[w(n) + \eta(n)s(n)\right] = \min_{\eta} f\left[w(n) + \eta s(n)\right].$$

This is a 1-D line search for fixed n. The residual of the steepest descent direction is

$$r(n) = b - Aw(n).$$

Then, to proceed to the next step, we use a linear combination of $r(n)$ and $s(n-1)$, as shown by the following equation:

$$\begin{aligned} s(n) &= r(n) + \beta(n)s(n-1) \\ n &= 1, 2, 3 \cdots L \end{aligned} \tag{11}$$

where the scaling factor $\beta(n)$ is given by *Polak-Ribiere formula*, a highly efficient preconditioner, that yields drastic improvements in convergence [30]

$$\beta(n) = \frac{r^T(n)\left[r(n) - r(n-1)\right]}{r^T(n-1)r(n-1)}. \tag{12}$$

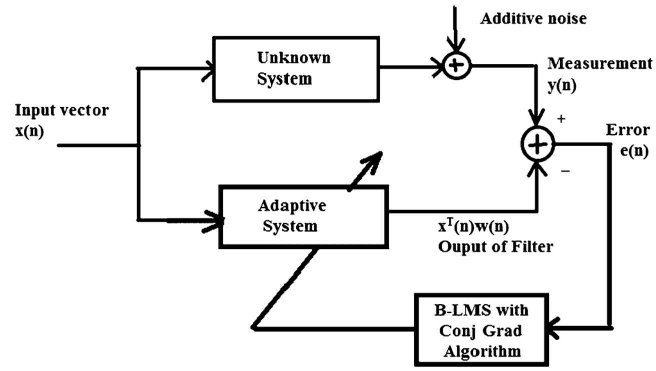

Fig. 1. Adaptive system model.

Thus, conjugate gradient methods do not require any matrix inversions for solving the linear system of equations and are faster than first-order approximation methods. In combination with the B-LMS algorithm, they provide a very efficient way to solve quadratic problems.

IV. Proposed B-LMS With Conjugate Direction Search-Based Adaptive System

The block diagram for a linear adaptive filter to estimate phasors quantities is shown in Fig. 1.

The weights w are defined such that they trace the unknown parameters, amplitude and phase of the frequency components. Thus the unknown linear system has weights reflecting the amplitude and phase of each component as shown by the formulations of phasor estimation problem in Section II. The method used here then helps the adaptive system incrementally to match the output of the unknown system corrupted with noise. The input vector x takes on the modeled values based on timestamp and frequencies while the desired vector y is the actual measurement vector of the composite signal. The B-LMS with conjugate gradient based search algorithm tries to track the unknown weight vector by matching the outputs of the adaptive and the unknown system through minimizing the error signal generated between the two in presence of noise. The noise need not be white Gaussin.

The steps of the B-LMS algorithm with conjugate gradient based search for the linear adaptive system are presented as follows.

Step 1) For $n = 0, 1, 2 \cdots$ and for a block size of length L, and weight vector size of K, we have:

Input signal $x(n) = [X(n)X(n-1)\ldots X_L(n-L)]^T$;

Measured signal $y(n) = [y(n)y(n-1)\ldots y_L(n-L)]^T$;

Error signal $e(n) = y(n) - x^T(n)w(n)$;

Initialize weights $\hat{w}(0) = [00\ldots 0]_K$;

Gradient search direction

$$s(0) = r(0) = -\frac{\partial E\left(\hat{w}(0)\right)}{\partial \hat{w}(0)} = \frac{2}{L}e(0)x(0).$$

Step 2) Find the optimal step-size scalar parameter using (14)

$$\eta(n) = \frac{e^T(n)x^T(n)s(n) + s^T(n)x(n)e(n)}{2s^T(n)x(n)x^T(n)s(n)}.$$

Step 3) Update the weight vector

$$\hat{w}(n+1) = \hat{w}(n) + \eta(n)s(n).$$

Step 4) Find the new gradient direction

$$r(n+1) = -\frac{\partial E\left(\hat{w}(n+1)\right)}{\partial \hat{w}(n+1)} = \frac{2}{L}e(n+1)x(n+1).$$

Step 5) Use the *Polak-Ribiere formula* to calculate $\beta(n+1)$:

$$\beta(n+1) = \max\left\{\frac{r^T(n)\left[r(n) - r(n-1)\right]}{r^T(n-1)r(n-1)}, 0\right\}.$$

Step 6) Update the direction vector

$$s(n+1) = r(n+1) + \beta(n+1)s(n).$$

Step 7) Set $n = n+1$, and go back to Step 2).

If the MSE $E(w)$ is a quadratic function of weights as shown in (7), the optimal value of weights (w^*) will reach in at most K iterations where K is the size of the weight vector w.

A. *Analysis of Convergence Characteristics*

The conjugate gradient method is the most prominent iterative method for solving sparse systems of linear equations. An intuitive reason of generating vectors to be orthogonal is that it gives the minimum error. Thus conjugate vectors $s(0), s(1) \ldots s(L-1)$ generated using (11) are linearly independent since they are orthogonal. They form a set of basis vectors that spans the vector space of w. So the optimum weight vector can be thought of as a linear combination of these vectors. And hence starting from an arbitrary point $w(0)$, the conjugate direction method is *guaranteed* to find the optimum w^* of the quadratic equation $E(w) = 0$ in at most K iterations where K is the size of weight vector w.

Since this method does not involve any matrix inversions it is superior to those model based methods where choosing inappropriate parameters for the algorithm could render some intermediate calculation matrix singular. Also since the search direction employed is conjugate gradient which is quasi-second order optimization technique, it is faster than first order (Jacobian) technique used in LES or RLS and comparably faster than other model based methods using nonlinear curve-fitting techniques, such as the Kalman filter, with fewer computations.

Once the problem of estimation is modeled as a linear filtering problem, a combined generic model containing harmonics, interharmonics, and decaying dc component could be used to compute any number of components of interest in the composite signal without any matrix inversions.

The use of such an approach also provides flexibility of adapting the model to changes in frequency in real time when the drift is known. Each individual component of interest can be finely tuned rather than whole matrix changes as required in model-based methods based on nonlinear curve fitting like Kalman filtering or RLS.

To analyze the convergence characteristics, we define the window length as the cycle of the nominal frequency l_s, which is independent of the signal sampling frequency f_s defined as N times the nominal frequency f_0 in Hertz. The variable l_s and f_s determine the number of samples N_s within a data block (i.e., $N_s = l_s \cdot f_s / f_0 = l_s / N$). There are many measures for analysis and classifications of power-quality disturbances [31]. But the standard one is the total vector error (TVE) that is used to measure the phasor accuracy [32]. Once the amplitude error ΔV_h (in percentage of real value) and the phase error $\Delta\theta_h$ (in degrees) are available, the expression is given by $TVE_h = \sqrt{(\Delta V_h)^2 + (\Delta\theta_h/0.573)^2}$, where 0.573 is the arcsine of 1% in degrees for the hth frequency component.

The signal model used for convergence analysis is given

$$y(n) = \sum_{h=1}^{5} \frac{V_0}{h} sin[2\pi h f_0 n \Delta T + h\theta_0] + z(n)$$
$$n = 0, 1, 2 \cdots$$

which is a discrete signal containing hth-order harmonics, ΔT is the sampling period, n is the sample number. V_0, θ_0, f_0 are fixed amplitude, phase, and frequency of the fundamental harmonic. $z(n)$ is white Gaussian noise of zero mean.

In this sample test case, we let $f_0 = 60$ Hz and harmonic components be up to 320 Hz. For high accuracy, we choose a higher sampling rate, here $N = 120$ samples per cycle (i.e., sampling frequency $=$ 7.2 kHz). Analysis results are given in Table I, where the window length used, convergence characteristic, response time, and TVE (maximum in 2 cycles of training) are compared for the fundamental component only. The total number of components is 5, so the minimum number of samples $=$ no of weights is $5 * 2 = 10$. That is, two weights for each frequency component. The number of iterations used is equal to the number of weights for all runs, that is, the optimal number of iterations per the B-LMS algorithm. We observe that as the data window is increased, the accuracy improves with each iteration, finally at a half cycle window, the response time saturates. Then, only the time needed to respond is the time it takes to acquire the next sample. Also, only for a very small data window length ($\ll$0.25 cycle), the convergence is uncertain.

V. Performance Evaluation

In this section, the performance of the algorithm is fully evaluated under various test conditions covering static state, dynamic state, and transient state and the results are compared with conventional DFT methods and latest published techniques in [5]–[7], [9], [11], [13], [21], and [22]. All tests are performed with sampling rate $N = 120$ samples per cycle (i.e.,

TABLE I
CONVERGENCE CHARACTERISTICS RESULTS

Window l_s =(No of samples in block)/(Total no of samples in 1 cycle)	Convergence of the algorithm	Response Time(ms) (after gathering one block of data)	TVE %
$1/12$	No	N/A	N/A
$2/12$	Yes	11.2	0.5441
$3/12$	Yes	9.8	0.3283
$4/12$	Yes	6.8	0.1660
$5/12$	Yes	4.61	0.1983
$6/12$	Yes	1.3889	0.0392
$8/12$	Yes	1.3889	0.0015
$10/12$	Yes	1.3889	0.0003
$12/12 = 1$	Yes	1.3889	0.0002

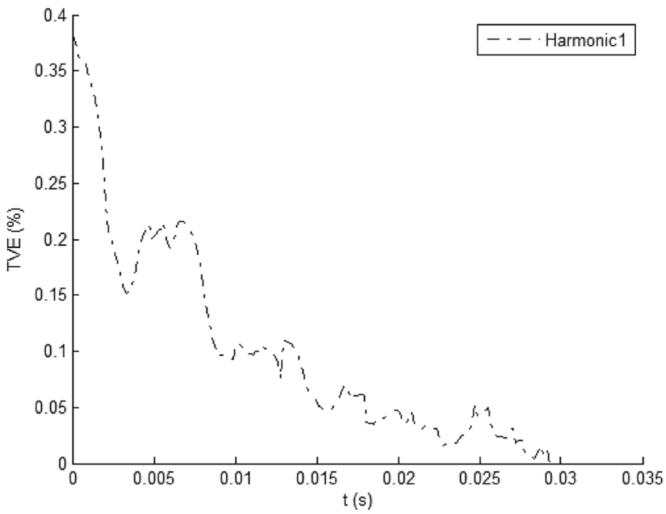

Fig. 2. Static test results using a quarter-cycle data window.

TABLE II
NOISE TEST RESULTS

Noise Level (SNR)	Data block length (l_s)	B-LMS TVE %	RWT TVE %
$0.1\%(60dB)$	0.25	0.2295	0.36
	0.5	0.0197	0.12
	1.0	0.00007	0.036
$0.32\%(50dB)$	0.25	0.3582	0.94
	0.5	0.1137	0.40
	1.0	0.00057	0.064
$1\%(40dB)$	0.25	1.5025	1.60
	0.5	0.3984	1.08
	1.0	0.0023	0.26
$3.2\%(30dB)$	0.25	8.8573	2.73
	0.5	1.3426	1.59
	1.0	0.0086	0.68

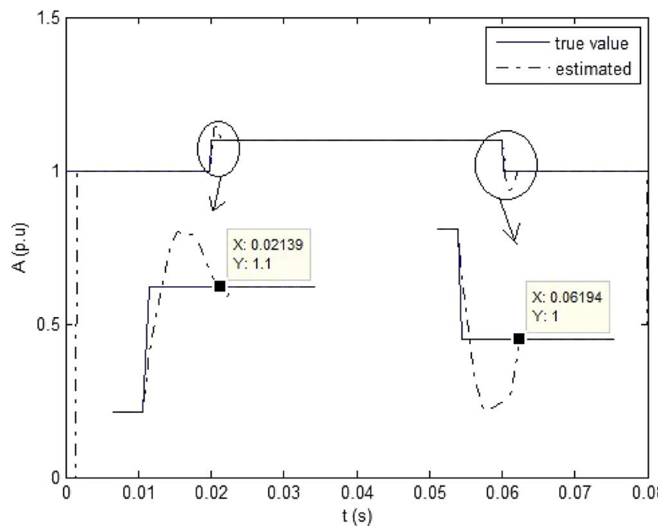

Fig. 3. Dynamic response for the amplitude step.

sampling frequency $= 7.2$ kHz), and a block size varies depending on the number of weights to be computed and whether the number of iterations used, if any, is equal to the number of weights. The higher sampling rate is useful for high accuracy.

A. Static Test

A signal model containing harmonics and 0.1% (signal-to-noise ratio $SNR = 60$ dB) zero mean white noise, is used similar to the one in convergence analysis. Let $V_0 = 1$ and $\theta_0 = 5^o$. The total vector error of the first component is plotted as a function of time. Compared to the DFT-based methods in [5]–[7] that require a full cycle (16.67 ms), the algorithm can output phasor parameters only in about less than 4 ms. Here, we used a block size of 30 (i.e., 25 cycle). The result is shown in Fig. 2.

B. Noise Test

The inherent noise rejection capability of the algorithm is investigated by this test. The signal model for the static test is used. Each run is for 2 cycles. We observe that even under a worst case condition of 30-dB signal-to-noise ratio, if the data window used is equal to one cycle, the algorithm can still give a very low TVE. For different noise levels, the TVE for the B-LMS and RWT is evaluated and compared as shown in Table II.

C. Dynamic Step and Ramp Change Test

To evaluate the dynamic response when exposed to an abrupt signal change, a positive step followed by a reverse step back to the starting value under various conditions is applied to the amplitude and phase angle of a sinusoidal signal, respectively. Studies indicate that under both types of steps the algorithm shows similar dynamic behavior. Here we used a block size of 12 samples since only two weights are to be computed (i.e..1 cycle). The results of the amplitude step (10% of normal value) and phase step ($\pi/18$ rad), are presented by Figs. 3–5, respectively without any iterations used for blocks of data. The steps occur at 0.02 and 0.06 s. One can observe that the outputs track the changes in the inputs extremely fast. It took 1.39 ms and 1.94 ms to fully track the amplitude step change and 1.67 ms and 1.81 ms to track phase step change that occurred at two different times respectively. To investigate the effect of prefiltering on the algorithm dynamic performance, a third order Butterworth low-pass filter with a cutoff frequency of 320 Hz is used

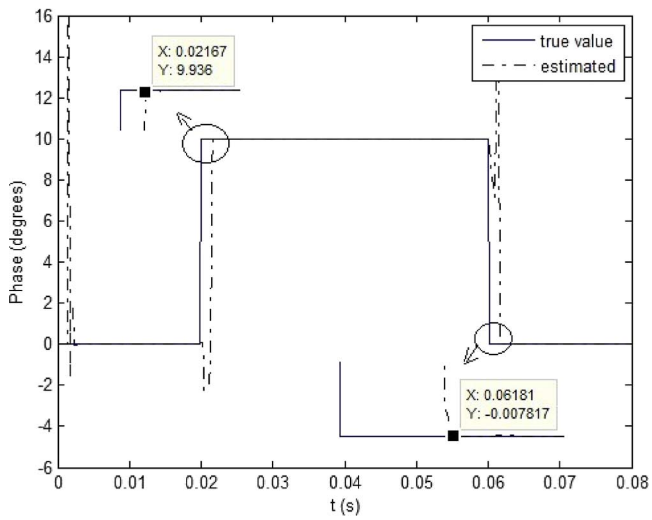

Fig. 4. Dynamic response for the phase step.

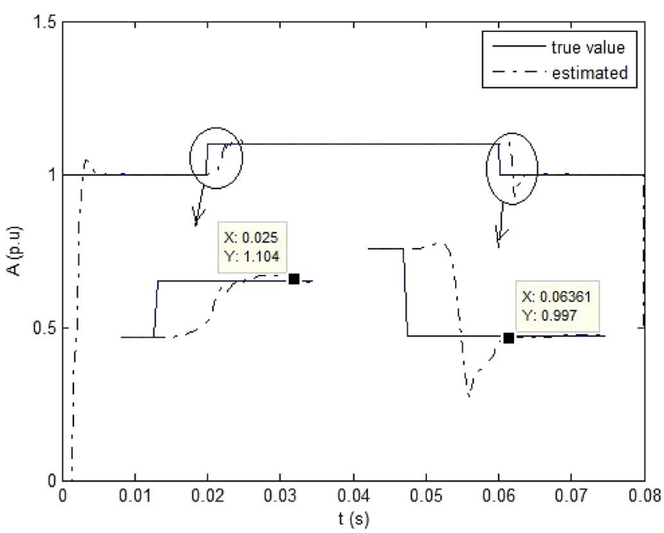

Fig. 5. Dynamic response for the amplitude step with prefiltering.

to process the input signals. Fig. 5 shows the result of amplitude step test. Compared to Fig. 3, which shows the transient behavior without signal prefiltering, one can see that the low pass just slows the response from 3 to 5 ms with no significant overshoot and undershoot and it is still less complex than the RWT-based method [13] that takes about a quarter cycle of fundamental component time period, DFT-based methods [5]–[7] and faster than instantaneous sample-based methods [9], [11] that require full cycle of fundamental component about 16.66 ms. A dynamic amplitude response for a ramp change is also done. The ramp occurs at 0.02 s and continues to steadily increase until 0.06 s, then it drops back to the original value as shown in Fig. 6. It is clear that the algorithm tracks the constant changes steadily, and the drop is tracked just in 1.67 ms. The performance of the algorithm showed similar result in the phase test.

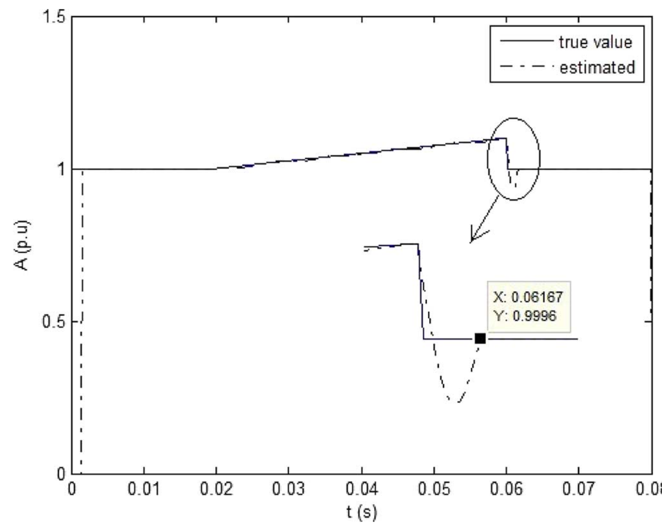

Fig. 6. Dynamic response for the amplitude ramp change.

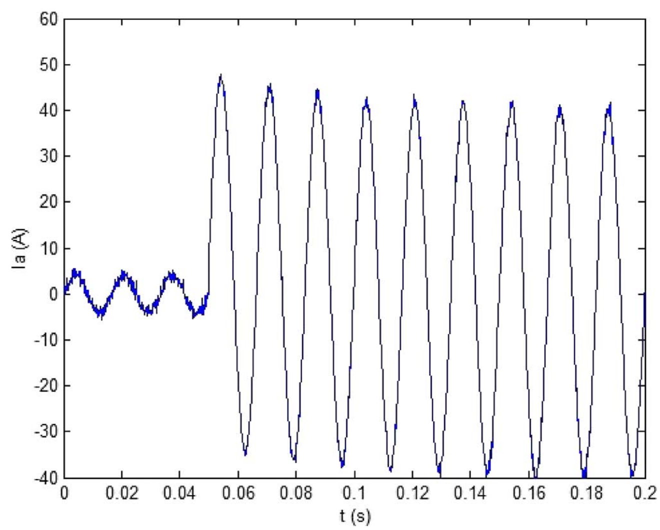

Fig. 7. Simulated current I_A waveform for the transient test.

D. Transient Test

Synthetic three-phase waveforms based on the model in (4) are used for testing the performance for eliminating the decaying dc offset. A change in the amplitude is performed, beginning with 0.05 s, similar to a three-phase fault generated in a real system. Fig. 7 shows the current waveform of Phase A used for simulation. One can see that the signal is contaminated with a decaying dc component, and high-frequency noise is present at the beginning and during the fault. The third-order Butterworth low-pass filter with a cutoff frequency of 320 Hz is used to attenuate the high-frequency components. Parameters estimation for steady state (20 cycles after the fault occurs) is used as a reference to measure the TVEs. As shown in Table III, the results are compared with the conventional full-cycle DFT(FCDFT), half-cycle DFT(HCDFT or STFT), least error square (LES), hybrid method [21], and simplified algorithm (SIMS) in [22]. In Table III, t_s is the time (in cycles) when the TVEs are measured. For high accuracy, the algorithm was adjusted to a full-cycle

TABLE III
TEST RESULTS FOR THE DECAYING DC OFFSET

Algorithm	ts(cycle)	IA TVE(%)	IB TVE(%)	IC TVE(%)
FCDFT	5	0.9474	0.9587	0.9559
HCDFT	12.5	1.0148	1.0196	1.0158
LES	1.0	0.1036	0.1073	0.1058
SIM3	1.0	0.1285	0.1089	0.1153
HM	1.0	0.1273	0.1168	0.1132
B-LMS	1.0	0.1023	0.1034	0.1046

window span. The results show that the accuracy is clearly comparable to those of LES. But it is superior in its convergence characteristics to LES, SIM3, and HM using the first-order optimization (Jacobian) technique.

VI. CONCLUSION

This paper introduces a new adaptive filtering approach to solve the problem of phasor estimation when the frequencies of components are known. The algorithm features very fast response and high accuracy over varied conditions in noise. It uses less than a quarter cycle of the fundamental component signal to estimate amplitude and phase for a signal contaminated with harmonics or interharmonics. A high-frequency resolution could be achieved using this model-based technique by appropriately modeling weights. The decaying dc component can be completely removed using this technique. The performance of the algorithm is evaluated under a variety of conditions that includes static test, noise test, dynamic test, and transient test. A comparison with other techniques demonstrates the advantage of using this approach. The computational burden is minimal when compared to nonlinear or second-order methods or wavelet-based methods; accuracy is high and response is very rapid to satisfy time-critical demand of the real-time applications in power systems. This model can be easily adapted to drifts in nominal frequency when it is known.

This is one of the most efficient time-domain methods. So only accurate information of amplitude and phase can be extracted using this method, not the frequency information. If one method tries to extract time- and frequency-domain information from a signal, for instance, RWT, its complexity goes high, its response time is reduced, and its accuracy is usually limited due to a tradeoff. In a high noise environment, difficulty increases further. But a combination of the two best methods, each extracting time- and frequency-domain information separately, would easily be better. Once this B-LMS technique is combined with a fast frequency-domain method, whose computations can precede this algorithm, then it can be used for frequency-distorted signals too. In power systems, as is well known, frequency is a much more tightly regulated parameter than the amplitude and phase of various signal components, where this technique can be productively employed. Further research is required to estimate the nominal frequencies and its drift using some efficient approaches and combining it with this filtering algorithm.

APPENDIX
DERIVATION OF OPTIMAL STEP SIZE FOR THE B-LMS ALGORITHM

From (10), we know that for conjugate gradient based method the incremental update for the weight vector is given by

$$\Delta w(n) = \hat{w}(n+1) - \hat{w}(n) = \eta(n)s(n). \tag{13}$$

Now, consider the Taylor series expansion of the instantaneous error surface given by (6) which is linear in the weight vector at time $n+1$

$$e(n+1) = e(n) + \sum_{l=0}^{L} \frac{\partial e(n)}{\partial w_l(n)} \Delta w_l(n) + \frac{1}{2!} \sum_{l=0}^{L} \sum_{m=0}^{L} \frac{\partial^2 e(n)}{\partial w_l(n) \partial w_m(n)} \Delta w_l(n) \Delta w_m(n) + \cdots$$

When the error is linear in weight w vector, the series reduces to the terms only up to the first order.
Hence

$$e(n+1) = e(n) + \sum_{l=0}^{L} \frac{\partial e(n)}{\partial w_l(n)} \Delta w_l(n)$$

which, in vector form, is given by

$$e(n+1) = e(n) - x^T(n)\Delta w(n).$$

Substituting the value of $\Delta w(n)$ from (13), we obtain

$$e(n+1) = e(n) - x^T(n)\eta(n)s(n).$$

Substituting the value in (7), we obtain

$$E[\eta(n)] = \frac{1}{L} e^T(n+1)e(n+1).$$

To find the minimum $\eta(n)$, we use the line search, that is, setting the first-order partial derivative to zero

$$\frac{\partial E\eta(n)}{\partial \eta(n)} = 0.$$

Then, we obtain the following expression for, optimal scalar $\eta(n)$:

$$\eta(n) = \frac{e^T(n)x^T(n)s(n) + s^T(n)x(n)e(n)}{2s^T(n)x(n)x^T(n)s(n)} \tag{14}$$

which is the optimal step-size parameter for the conjugate gradient search direction $s(n)$.

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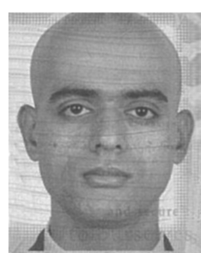

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