The Photon Model Terms

Notes:
1) The pivot energy $E_{\rm piv}$ "normalizes" the energy at which the model is evaluated and should always be held fixed in the fitting process. Choosing a pivot energy in the middle of the fit energy range will reduce the cross-correlations of the other model parameters.
2) The photon model f is photon number flux in photons s^{-1} cm^{-2} keV^{-1}.
3) The total photon model is composed of the sum of the selected additive terms $f_i^{\rm A}$ times the product of the selected multiplicative terms $f_j^{\rm M}$,

$$f = \sum_i f_i^{\rm A} \times \prod_j f_j^{\rm M}, \tag{1}$$

4) You must select at least one additive term. If no multiplicative terms are selected, the product defaults to one.
5) All energies are in keV.

- Term 1, Power Law:

  1. A = amplitude in photons s^{-1} cm^{-2} keV^{-1},
  2. $E_{\rm piv}$ = pivot energy in keV,
  3. λ = index.

$$f_1^{\rm A} = A(E/E_{\rm piv})^{\lambda} \tag{2}$$

- Term 2, Broken Power Law:

  1. A = amplitude in photons s^{-1} cm^{-2} keV^{-1},
  2. $E_{\rm piv}$ = pivot energy in keV,
  3. λ_l = index below break,
  4. $E_{\rm b}$ = break energy in keV,
  5. λ_h = index above break.

$$f_2^{\rm A} = A \begin{cases} (E/E_{\rm piv})^{\lambda_l} & \text{if } E \leq E_{\rm b} \\ (E_{\rm b}/E_{\rm piv})^{\lambda_l}(E/E_{\rm b})^{\lambda_h} & \text{if } E > E_{\rm b} \end{cases} \tag{3}$$

- Term 3, Broken Power Law with Two Breaks:

    1. A = amplitude in photons s^{-1} cm^{-2} keV^{-1},
    2. $E_{\rm piv}$ = pivot energy in keV,
    3. λ_1 = index below first break,
    4. $E_{\rm b1}$ = first break energy in keV,
    5. λ_{12} = index between first and second break,
    6. $E_{\rm b2}$ = second break energy in keV,
    7. λ_2 = index above second break.

$$f_3^{\rm A} = A \begin{cases} (E/E_{\rm piv})^{\lambda_1} & \text{if } E \leq E_{\rm b1} \\ (E_{\rm b1}/E_{\rm piv})^{\lambda_1}(E/E_{\rm b1})^{\lambda_{12}} & \text{if } E > E_{\rm b1} \text{ \& } E \leq E_{\rm b2} \\ (E_{\rm b1}/E_{\rm piv})^{\lambda_1}(E_{\rm b1}/E_{\rm b2})^{\lambda_{12}}(E/E_{\rm b2})^{\lambda_2} & \text{if } E > E_{\rm b2} \end{cases} \tag{4}$$

- Term 4: Smoothly Broken Power Law: This model is derived as follows: we want $d(\log f)/d(\log E)$ to be a horizonal line at y=λ_1 for small E, then later a horizontal line at y=λ_2 for large E. This is done by making $d(\log f)/d(\log E)$ a line function (y = mx + b) applied to the hyperbolic cosine.

    1. A = amplitude in photons s^{-1} cm^{-2} keV^{-1},
    2. $E_{\rm piv}$ = pivot energy in keV,
    3. λ_l, lower index,
    4. $E_{\rm b}$, break energy in keV,
    5. Δ, break scale in decades of energy,
    6. λ_2, upper index.

$$m = \frac{\lambda_h - \lambda_l}{2} \tag{5}$$

$$b = \frac{\lambda_l + \lambda_h}{2} \tag{6}$$

$$\alpha_{\rm piv} = \frac{\log_{10}(E_{\rm piv}/E_{\rm b})}{\Delta} \tag{7}$$

$$\beta_{\mathrm{piv}} = m\Delta \log_{\mathrm{e}} \frac{\exp(\alpha_{\mathrm{piv}}) + \exp(-\alpha_{\mathrm{piv}})}{2} \tag{8}$$

$$\alpha = \frac{\log_{10}(E/E_{\mathrm{b}})}{\Delta} \tag{9}$$

$$\beta = m\Delta \log_{\mathrm{e}} \frac{\exp(\alpha) + \exp(-\alpha)}{2} \tag{10}$$

$$f_4^{\mathrm{A}} = A(E/E_{\mathrm{piv}})^b 10^{(\beta - \beta_{\mathrm{piv}})} \tag{11}$$

- Term 5: David Band's Gamma-Ray Burst function (ApJ, **413**, 281, 1993), E_{peak} parameterization, a different parameterization than that given in the ApJ article. The relation is $E_{\mathrm{peak}} = E_0(2+\alpha)$.

  1. A = amplitude in photons s^{-1} cm^{-2} keV^{-1},
  2. E_{peak} in keV,
  3. low-energy index α,
  4. high-energy index β.

$$f_5^{\mathrm{A}} = A(E/100)^{\alpha} \exp(-E(2+\alpha)/E_{\mathrm{peak}}) \tag{12}$$

if

$$E < (\alpha - \beta)E_{\mathrm{peak}}/(2+\alpha), \tag{13}$$

and

$$f_5^{\mathrm{A}} = A\{(\alpha-\beta)E_{\mathrm{peak}}/[100(2+\alpha)]\}^{(\alpha-\beta)} \exp(\beta-\alpha)(E/100)^{\beta} \tag{14}$$

if

$$E \geq (\alpha - \beta)E_{\mathrm{peak}}/(2+\alpha) \tag{15}$$

- Term 6: David Band's Gamma-Ray Burst function (ApJ, **413**, 281, 1993), "old" parameterization:

  1. A = amplitude in photons s^{-1} cm^{-2} keV^{-1},
  2. E_0 in keV,
  3. low-energy index α,
  4. high-energy index β.

$$f_6^{\mathrm{A}} = A \begin{cases} (E/100)^{\alpha} \exp{(-E/E_0)} & \text{if } E < (\alpha-\beta)E_0 \\ ([\alpha-\beta]E_0/100)^{(\alpha-\beta)} \exp{(\beta-\alpha)} (E/100)^{\beta} & \text{if } E \geq (\alpha-\beta)E_0 \end{cases} \tag{16}$$

- Term 7, Comptonized, E_{peak} parameterization. The relation between the old and new parameterizations is $E_{\mathrm{peak}} = E_0(2+\alpha)$.

  1. A = amplitude in photons s^{-1} cm^{-2} keV^{-1},
  2. E_{peak} in keV,
  3. λ, index,
  4. E_{piv} = pivot energy in keV.

$$f_7^{\mathrm{A}} = A \exp{[-E(2+\lambda)/E_{\mathrm{peak}}]} (E/E_{\mathrm{piv}})^{\lambda} \tag{17}$$

- Term 8, Comptonized, "old" parameterization:

  1. A = amplitude in photons s^{-1} cm^{-2} keV^{-1},
  2. kT in keV,
  3. λ, index,
  4. E_{piv} = pivot energy in keV.

$$f_8^{\mathrm{A}} = A \exp{(-E/kT)} (E/E_{\mathrm{piv}})^{\lambda} \tag{18}$$

- Term 9, J. J. Brainerd's scattered power law (ApJ, **428**, 21-27, 1994). The parameterization has been revised to use a parameter ratio involving δ and τ instead of δ. This reduces the cross-correlation between the parameters.

  1. A = amplitude in photons s^{-1} cm^{-2} keV^{-1},
  2. Parameter ratio = $(\delta{+}2)/\tau$, where δ is the index of the unscattered power law,
  3. τ, optical depth,
  4. z, cosmological redshift,
  5. M, metallicity (solar = 1).

$$f_9^{\mathrm{A}} = A\nu_0^{-\delta} \exp(-\tau[\sigma_c \nu_0 + \sigma_p \nu_0]) \tag{19}$$

where

$$\nu_0 = \nu(1 + z), \tag{20}$$

σ_c is the Klein-Nishina cross-section, and σ_p is the photo-electric cross-section:

$$\sigma_p = \frac{0.3990711 + 5.071327M}{1.135962 + 0.01452396M} \nu_0^{-3.059308} \tag{21}$$

- Term 10, Log Normal:
  1. A = amplitude in photons s^{-1} cm^{-2} keV^{-1},
  2. μ, $\log_e$ of energy,
  3. σ.

$$f_{10}^{\mathrm{A}} = \frac{A}{\sqrt{2\pi}\sigma E} \exp\left(-\frac{1}{2}\left[\frac{\log_e E - \mu}{\sigma}\right]^2\right) \tag{22}$$

- Term 11: Gaussian ($\log_{10} E$):
  1. A = amplitude in photons s^{-1} cm^{-2} keV^{-1},
  2. E_{cen}, centroid energy in keV,
  3. W, $\log_{10}$ FWHM, decades of energy.

$$\sigma = W/2.35482 \tag{23}$$

$$f_{11}^{\mathrm{A}} = \frac{A}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\left[\frac{\log_{10} E - \log_{10} E_{\mathrm{cen}}}{\sigma}\right]^2\right) \tag{24}$$

- Term 12: Gaussian ($\log_{10} E$) with linearly varying FWHM:
  1. A = amplitude in photons s^{-1} cm^{-2} keV^{-1},
  2. E_{cen}, centroid energy in keV,
  3. W, $\log_{10}$ FWHM at E_{cen}, decades of energy.
  4. s, slope of W w.r.t. $\log_{10} E$, decades per decade.

$$\mathcal{W} = W + s(\log_{10} E - \log_{10} E_{\rm cen}) \tag{25}$$

$$\sigma = \mathcal{W}/2.35482 \tag{26}$$

$$f_{12}^{\rm A} = \frac{A}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\left[\frac{\log_{10} E - \log_{10} E_{\rm cen}}{\sigma}\right]^2\right) \tag{27}$$

- Terms 13 and 14, Sunyaev-Titarchuk Comptonization spectra (references: Patrick Nolan's Ph.D thesis has the exact notation used herein; Sunyaev and Titarchuk, A&A, **86**, 121, 1980).

  1. A = amplitude in photons s^{-1} cm^{-2} keV^{-1},
  2. kT = electron energy in keV,
  3. τ = optical depth,
  4. G = geometry factor, which should be fixed at 3 for a spherical cloud and at 12 for a disk of electrons.

  $$f_{13}^{\rm A} = Ax^2 \exp{(-x)} \int_0^\infty t^{n-1} \exp -t \left(1 + \frac{t}{x}\right)^{n+3} {\rm d}t \tag{28}$$

  where

  $$n = -\frac{3}{2} + \sqrt{\gamma + \frac{9}{4}} \tag{29}$$

  $$\gamma = \frac{511\pi^2}{GkT(\tau + \frac{2}{3})^2} \tag{30}$$

- Terms 15 and 16, Optically-Thin Thermal Bremsstrahlung (OTTB):

  1. A = amplitude in photons s^{-1} cm^{-2} keV^{-1},
  2. kT = electron energy in keV,
  3. $E_{\rm piv}$ = pivot energy in keV.

  $$f_{15}^{\rm A} = A \exp{(-E/kT)} \exp{(E_{\rm piv}/kT)} (E/E_{\rm piv})^{-1} \tag{31}$$

- Terms 17 and 18, Black Body:

  1. A = amplitude in photons s^{-1} cm^{-2} keV^{-1},
  2. kT = temperature in keV.

$$f_{17}^{\mathrm{A}} = A\frac{E^2}{\exp{(E/kT)} - 1} \tag{32}$$

- Term 19, power law parameterized by photon number flux:
  1. F_{P} = Photon flux in photons s^{-1} cm^{-2} between E_1 and E_2,
  2. λ = index,
  3. E_1 = lower bound of energy range over which flux is integrated: leave fixed,
  4. E_2 = upper bound of energy range over which flux is integrated: leave fixed.

  Amplitude A in photons s^{-1} cm^{-2} keV^{-1} is

  $$A = F_{\mathrm{P}}(\lambda + 1)/(E_2^{\lambda+1} - E_1^{\lambda+1}), \tag{33}$$

  so that

  $$f_{19}^{\mathrm{A}} = AE^{\lambda} \tag{34}$$

- Term 20, power law parameterized by energy flux:
  1. F_{E} = Energy flux in keV s^{-1} cm^{-2} between E_1 and E_2,
  2. λ = index,
  3. E_1 = lower bound of energy range over which flux is integrated: leave fixed,
  4. E_2 = upper bound of energy range over which flux is integrated: leave fixed.

  Amplitude A in photons s^{-1} cm^{-2} keV^{-1} is

  $$A = F_{\mathrm{E}}(\lambda + 2)/(E_2^{\lambda+2} - E_1^{\lambda+2}), \tag{35}$$

  or, for $\lambda = -2$,

  $$A = F_E/\log{(E_2/E_1)}, \tag{36}$$

  so that

  $$f_{20}^{\mathrm{A}} = AE^{\lambda} \tag{37}$$

- Term 21, Optically-Thin Thermal Bremsstrahlung (OTTB) parameterized by photon number flux:

  1. $F_{\rm P}$ = Photon number flux in photons s^{-1} cm^{-2} between E_1 & E_2,
  2. kT,
  3. E_1 = lower bound of energy range over which flux is integrated: leave fixed,
  4. E_2 = upper bound of energy range over which flux is integrated: leave fixed.

  Amplitude A in photons s^{-1} cm^{-2} keV^{-1} is

  $$A = F_{\rm P} \ / \int_{E_1}^{E_2} \exp\left(-E/kT\right)/E \; dE. \tag{38}$$

  The integral must be evaluated numerically. Then

  $$f_{21}^{\rm A} = A \exp\left(-E/kT\right)/E \tag{39}$$

- Term 22, Optically-Thin Thermal Bremsstrahlung (OTTB) parameterized by energy flux:

  1. $F_{\rm E}$ = energy flux in keV s^{-1} cm^{-2} between E_1 and E_2,
  2. kT,
  3. E_1 = lower bound of energy range over which flux is integrated: leave fixed,
  4. E_2 = upper bound of energy range over which flux is integrated: leave fixed.

  Amplitude A in photons s^{-1} cm^{-2} keV^{-1} is

  $$A = \frac{F_{\rm E}}{kT[\exp\left(-E_1/kT\right) - \exp\left(-E_2/kT\right)]} \tag{40}$$

  so that

  $$f_{22}^{\rm A} = A \exp\left(-E/kT\right)/E \tag{41}$$

- Term 23, Yang Soong's pulsar spectral form (references: Y. Soong, Ph.D thesis, Univ. of Calif., San Diego, 1988; Y. Soong, et al., ApJ, **348**, 641, 1990):

    1. A = amplitude in photons s^{-1} cm^{-2} keV^{-1},
    2. power law index α,
    3. break energy E_{b} in keV,
    4. E-folding energy E_{F} in keV,
    5. equivalent width of line, E_{W} in keV,
    6. line centroid E_{cen} in keV,
    7. Full-Width at Half-Max of line, FWHM, in keV,
    8. pivot energy E_{piv} in keV,

$$f_{23}^{\mathrm{A}} = C(1 - E_{\mathrm{W}}G), \tag{42}$$

where

$$C = A \begin{cases} (E/E_{\mathrm{piv}})^{-\alpha} & \text{if } E \le E_{\mathrm{b}} \\ \mathrm{MF} \exp{(-E/E_{\mathrm{piv}})}/E & \text{if } E > E_{\mathrm{b}} \end{cases} \tag{43}$$

$$\mathrm{MF} = \exp{(E_{\mathrm{b}}/E_{\mathrm{F}})} E_{\mathrm{b}} (E_{\mathrm{b}}/E_{\mathrm{piv}})^{-\alpha}, \tag{44}$$

and

$$G = \frac{0.94}{\mathrm{FWHM}} \exp{\{-2.76[(E - E_{\mathrm{cen}})/\mathrm{FWHM}]^2\}}. \tag{45}$$

- Term 24, Tanaka's pulsar model (reference: J. Groove, et al., ApJ, **438**, L25–L28, 1995):

    1. A = amplitude in photons s^{-1} cm^{-2} keV^{-1},
    2. α,
    3. kT,
    4. optical depth of first line, τ_1,
    5. optical depth of second line, τ_2,
    6. number of lines, N_{L}; fix at either 0, 1 or 2,
    7. line centroid E_{L} of first line in keV,
    8. line width W, HWHM in keV,

9. $E_{\rm piv}$ in keV.

$$f_{24}^{\rm A} = A\frac{C}{C_{\rm piv}}\exp\left(L_{\rm piv} - L\right) \tag{46}$$

where

$$C = AE^{-\alpha}\exp\left(-E/kT\right) \tag{47}$$

$$C_{\rm piv} = E_{\rm piv}^{-\alpha}\exp\left(-E_{\rm piv}/kT\right) \tag{48}$$

$$L = \sum_{i=1}^{N_{\rm L}} \frac{\tau_i(iW)^2[E/(iE_{\rm L})]^2}{(E - iE_{\rm L})^2 + (iW)^2} \tag{49}$$

and

$$L = \sum_{i=1}^{N_{\rm L}} \frac{\tau_i(iW)^2[E_{\rm piv}/(iE_{\rm L})]^2}{(E_{\rm piv} - iE_{\rm L})^2 + (iW)^2} \tag{50}$$

- Terms 25, improved Titarchuk Comptonization spectra (reference: Hua and Titarchuk, ApJ, 1995).

  1. A = amplitude in photons s^{-1} cm^{-2} keV^{-1},
  2. $E_{\rm E}$ = electron energy in keV,
  3. τ = optical depth; Function is currently correct only for $\tau < 2$!
  4. G = geometry factor, which should be fixed at 3 for a spherical cloud and at 12 for a disk of electrons.

- Terms 26, Optically-Thin Thermal Synchrotron (e.g., Liang et al., ApJ, **271**, p. 776, eq. A2).

  1. A = amplitude in photons s^{-1} cm^{-2} keV^{-1},
  2. $E_{\rm C}$ = energy scale. See Liang et al. for physical relations. Note lack of factor of 4.5

$$f_{26}^{\rm A} = A\exp\left[E/E_{\rm C}\right]^{1/3} \tag{51}$$

- Term 32, User Defined Function A
- Term 33, User Defined Function B

- Terms 34, 35 & 36, Gaussian Lines: These terms are actually evaluated using the error function (the complementary error function erfcc of Numerical Recipes by Press et al.) so that the mean of the function over the photon bin is used, rather than its value at the center. [For the other terms, if the photon bin is wide, the mean over the photon bin is evaluated numerically]. The energy edges of the photon bins are designated $E_{\rm l}$ and $E_{\rm r}$.

  1. A = amplitude in photons/s-cm^2,
  2. $E_{\rm cen}$, centroid energy in keV,
  3. FWHM in keV.

$$\sigma = \mathrm{FWHM}/2.35482 \tag{52}$$

$$a_l = \frac{E_{\rm l} - E_{\rm cen}}{\sqrt{2}\sigma} \tag{53}$$

$$a_r = \frac{E_{\rm r} - E_{\rm cen}}{\sqrt{2}\sigma} \tag{54}$$

$$f_{34}^{\rm A} = A\frac{\mathrm{erfcc}(a_l) - \mathrm{erfcc}(a_r))}{2(E_{\rm r} - E_{\rm l})} \tag{55}$$

- Term 40, (Relative) Effective Area Correction. This term is a crude model to empirically improve the detector response model. The detector response model is assumed to be incorrect only in the effective area assigned each detector. The detectors are brought into better agreement by dividing the input photon spectrum for each detector by an ad hoc correction factor; this is equivalent to multiplying the effective area by the same factor. Each detector has a different factor, which are all normalized with respect to the zero*th* detector. The factors are assumed to be independent of energy.

  1. F_1, correction factor $1st$ detector w.r.t $0th$ detector,
  2. F_2, correction factor $2nd$ detector w.r.t $0th$ detector,
  3. F_3, correction factor $3rd$ detector w.r.t $0th$ detector,
  4. F_4, correction factor $4th$ detector w.r.t $0th$ detector,
  5. F_5, correction factor $5th$ detector w.r.t $0th$ detector,

6. F_6, correction factor $6th$ detector w.r.t $0th$ detector,
7. F_7, correction factor $7th$ detector w.r.t $0th$ detector,
8. F_8, correction factor $8th$ detector w.r.t $0th$ detector,
9. F_9, correction factor $9th$ detector w.r.t $0th$ detector,
10. F_{10}, correction factor $10th$ detector w.r.t $0th$ detector,

If there are N detectors numbered $k = 0, 1, ...N$, then for the $0th$ detector

$$f_{40}^{\mathrm{M}} = 1, \tag{56}$$

while for the kth detector, $k > 0$,

$$f_{40}^{\mathrm{M}} = 1/F_k \tag{57}$$

- Term 41, low-energy cutoff:
  1. cutoff energy E_{cut},
  2. folding energy E_{F},

$$f_{41}^{\mathrm{M}} = \begin{cases} (E/E_{\mathrm{cut}})^{(E_{\mathrm{cut}}/E_{\mathrm{F}})} \exp\left[(E_{\mathrm{cut}} - E)/E_{\mathrm{F}}\right] & \text{if } E \leq E_{\mathrm{cut}} \\ 1 & \text{if } E > E_{\mathrm{cut}} \end{cases} \tag{58}$$

- Term 42, high-energy cutoff:
  1. cutoff energy E_{cut},
  2. folding energy E_{F},

$$f_{42}^{\mathrm{M}} = \begin{cases} 1 & \text{if } E \leq E_{\mathrm{cut}} \\ (E/E_{\mathrm{cut}})^{(E_{\mathrm{cut}}/E_{\mathrm{F}})} \exp\left[(E_{\mathrm{cut}} - E)/E_{\mathrm{F}}\right] & \text{if } E > E_{\mathrm{cut}} \end{cases} \tag{59}$$

- Term 43, Multiplicative Power Law:
  1. index λ,
  2. pivot energy E_{piv}.

$$f_{43}^{\mathrm{M}} = (E/E_{\mathrm{piv}})^{\lambda} \tag{60}$$

- Term 44, Interstellar absorption (reference: Morrison and McCammon, ApJ, **270**, 119–122, 1983). Above 10 keV, function is an extrapolation of that of Morrison and McCammon; accuracy decreases due to neglect of Compton scattering. Scattering is by all elements according to solar (?) abundances even though function is specified in hydrogen column depth.

  1. Hydrogen column depth in units of 10^{24} Hydrogen atoms per cm^2.

- Terms 45 and 46, multiplicative Gaussian lines:

  1. Intensity I,
  2. Centroid E_{cen} in keV,
  3. FWHM in keV,

$$\sigma = \mathrm{FWHM}/2.35482 \tag{61}$$

$$G = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[\frac{-(E - E_{\mathrm{cen}})^2}{2\sigma^2}\right] \tag{62}$$

$$f_{45}^{\mathrm{M}} = \exp(IG) \tag{63}$$

- Term 47, multiplicative Lorentzian line:

  1. Intensity I,
  2. Centroid E_{cen} in keV,
  3. width W in keV,

$$f_{47}^{\mathrm{M}} = \exp\left[\frac{I}{W^2 + (E - E_{\mathrm{cen}})^2}\right] \tag{64}$$

- Term 48, Multiplicative Broken Power Law. This model may be used as a low-energy or high-energy cutoff by fixing λ_h or λ_l at zero, respectively.

  1. λ_l = index below break,
  2. E_{b} = break energy in keV,
  3. λ_h = index above break.

$$f_{47}^{\mathrm{M}} = \begin{cases} (E/E_{\mathrm{b}})^{\lambda_l} & \text{if } E \leq E_{\mathrm{b}} \\ (E/E_{\mathrm{b}})^{\lambda_h} & \text{if } E > E_{\mathrm{b}} \end{cases} \tag{65}$$

- Term 49, Redshifted Interstellar Absorption
- Term 54, User Defined Function C
- Term 55, User Defined Function D