

Trial Examination 2016

MATHEMATICAL METHODS

Written Examination 2 - SOLUTIONS

SECTION A

1. A 2. C 3. E 4. C 5. A 6. D 7. E 8. B 9. B 10. A

11. E 12. D 13. A 14. E 15. B 16. D 17. D 18. C 19. C 20. B

SECTION A

Question 1

$$f : R \rightarrow R, f(x) = -2 - 3 \cos\left(\frac{x}{4} + 1\right)$$

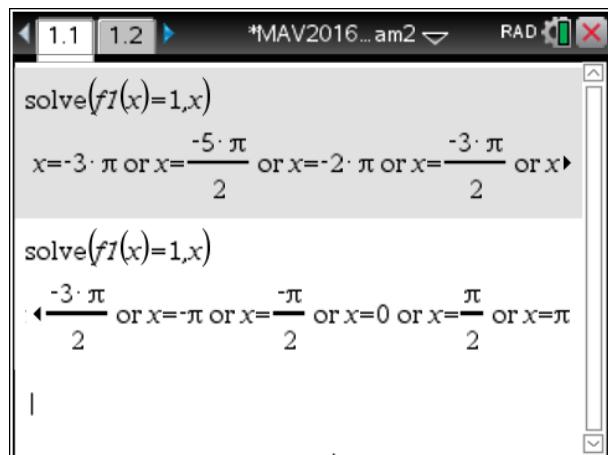
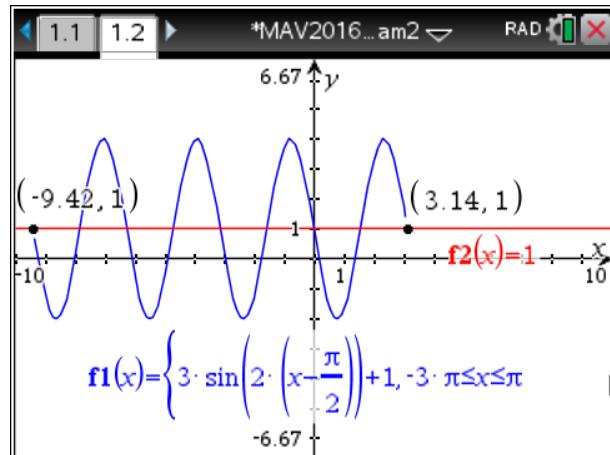
The range is $[-3 - 2, 3 - 2] = [-5, 1]$

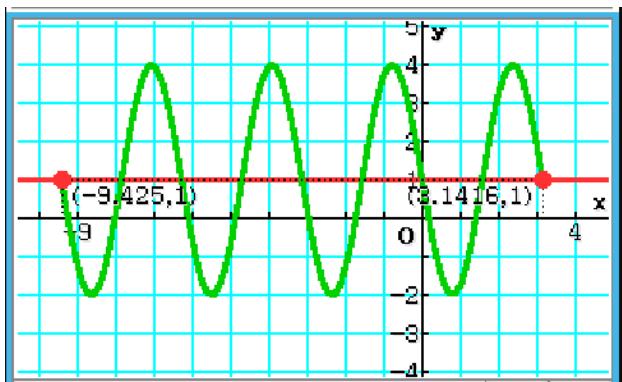
The Period is $\frac{2\pi}{\frac{1}{4}} = 8\pi$ A

Question 2

$$g : [-3\pi, \pi] \rightarrow R, g(x) = 3 \sin\left(2\left(x - \frac{\pi}{2}\right)\right) + 1$$

There are 9 intersections when $k = 1$. C





$$\text{define } f(x) = 3\sin(2(x - \frac{\pi}{2})) + 1$$

done

$$\text{solve}(f(x) = 1 \mid -3\pi \leq x \leq \pi, x)$$

$$\left\{ x=0, x=-3\pi, x=-2\pi, x=-\pi, x=\pi, x=\frac{-5\pi}{2}, x=\frac{-3\pi}{2} \right\}$$

Question 3

The domain of $f(g(x))$ is the same as the domain of g where $g(x) = \tan(\pi x)$.

The asymptotes of g have equations $y = \frac{1}{2} + k, k \in \mathbb{Z}$.

Hence the domain of $f(g(x))$ is $R \setminus \left\{ \frac{1}{2} + k \right\}, k \in \mathbb{Z}$ E

Question 4

$$\sin\left(\frac{3\pi}{2} - x\right) = -\cos(x)$$

$$\sqrt{15^2 - 2^2} = \sqrt{221}$$

$$\sin\left(\frac{3\pi}{2} - x\right) = -\frac{\sqrt{221}}{15} \quad \text{C}$$

Question 5

$$y = 2x^2 + 3x - 1 = 2\left(x + \frac{3}{4}\right)^2 - \frac{17}{8}$$

The graph of $y = x^2$ has been dilated by a factor of 2 from the x -axis and

Then translated $\frac{3}{4}$ units to the left and $\frac{17}{8}$ units down.

A possible rule for the transformation $T : R^2 \rightarrow R^2$ is

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} \frac{3}{4} \\ \frac{17}{8} \end{bmatrix} \quad \text{A}$$

The calculator screen shows the command `completeSquare(2·x^2+3·x-1,x)` and the resulting output: $2 \cdot \left(x + \frac{3}{4}\right)^2 - \frac{17}{8}$.

Question 6

$$f(x) = x^{\frac{3}{2}}$$

$$f(x^2y^2) = (x^2y^2)^{\frac{3}{2}} = x^3y^3$$

$$f(x^2)f(y^2) = (x^2)^{\frac{3}{2}}(y^2)^{\frac{3}{2}} = x^3y^3 \quad \mathbf{D}$$

The calculator screen shows the definition $f(x) = x^{\frac{3}{2}}$ and the judge command $\text{judge}(f(x^2y^2) = f(x^2)f(y^2))$ which returns `true`.

The calculator screen shows the definition $f(x) = x^{\frac{3}{2}}$ and the judge command $\text{judge}(f(x^2y^2) = f(x^2)f(y^2))$ which returns `TRUE`.

Using counter examples

$$f(xy) = f(x)f(y), f(-1 \times -1) = 1, f(-1) \times f(-1) \text{ has no real solution}$$

$$f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}, f\left(\frac{-1}{-1}\right) = 1, \frac{f(-1)}{f(-1)} \text{ has no real solution}$$

The screenshot shows a TI-Nspire CX CAS calculator interface. The top menu bar includes 'Edit', 'Action', and 'Interactive'. Below the menu are several function keys: $\frac{0.5}{2}$, $\frac{1}{2}$, \int_{dx} , $\int_{\text{dx}}^{\text{d}}$, 'Simp', $\underline{\int_{\text{dx}}}$, and others. The main workspace contains the following text:

```

define f(x)=x^2
done
judge(f(x,y)=f(x)f(y)
Undefined
judge(f(x/y)=f(x)/f(y)
Undefined

```

$$f(x+y) = f(x) + f(y), \quad f(1+1) = 2^2, \quad f(1) + f(1) = 2$$

$$f(x-y) = f(x) - f(y), \quad f(2-1) = 1, \quad f(1) - f(1) = 0$$

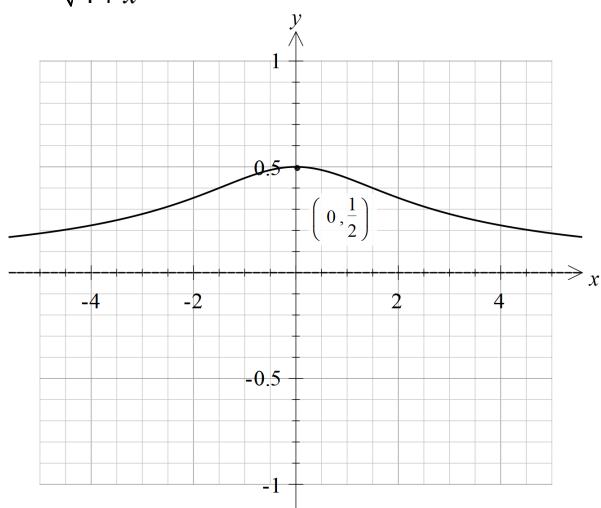
Question 7

When $x = 0, y = 0.5$.

Either **C** or **E**

When $x = 2, y \approx 0.35$

$$y = \frac{1}{\sqrt{4+x^2}} \quad \mathbf{E}$$



Question 8

$$f : (1, \infty) \rightarrow R, f(x) = \frac{3}{(x-1)^2} + 2$$

Range is $(2, \infty)$ which is the domain of the inverse function.

$$\text{Let } y = \frac{3}{(x-1)^2} + 2$$

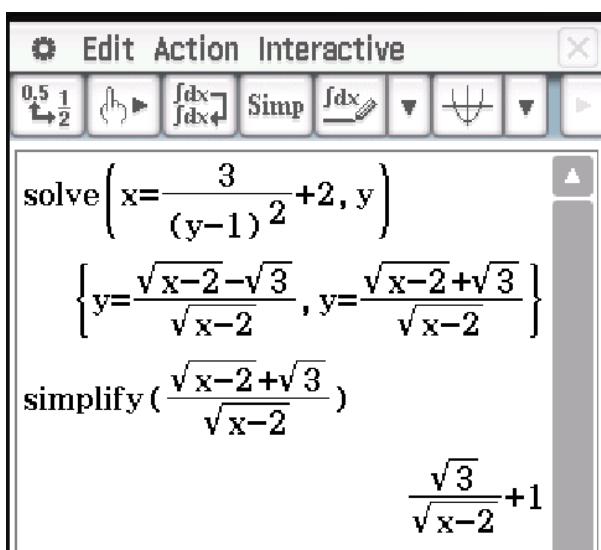
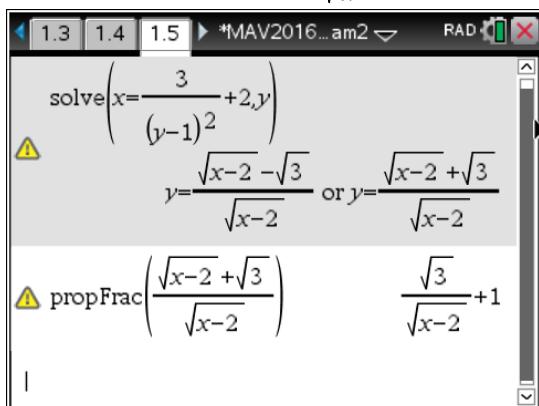
Inverse swap x and y .

$$x = \frac{3}{(y-1)^2} + 2$$

$$y = \pm \sqrt{\frac{3}{x-2}} + 1$$

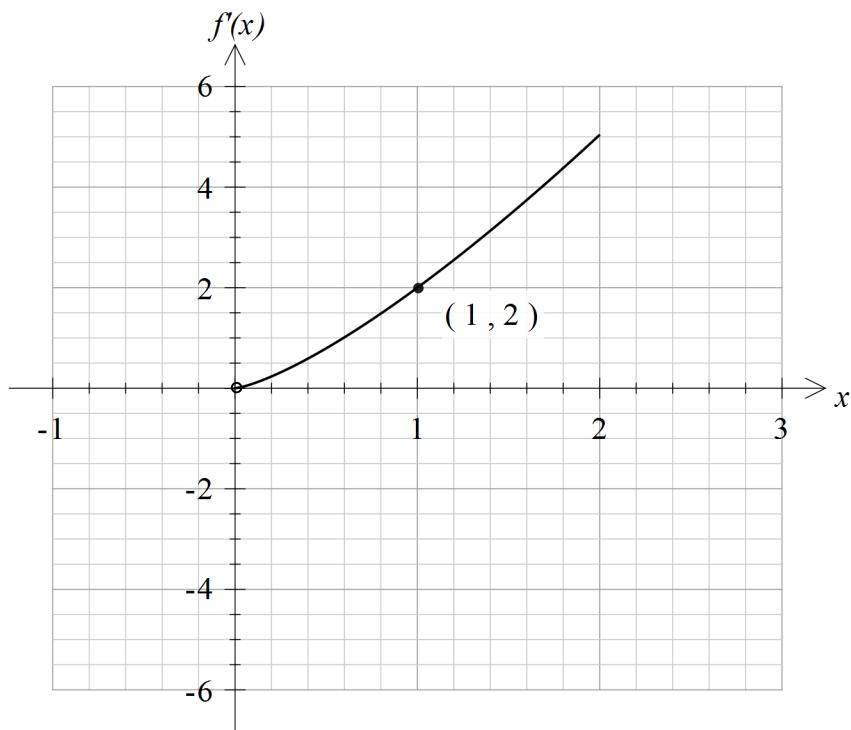
$$y = \sqrt{\frac{3}{x-2}} + 1, \text{ as the domain of } f \text{ is } (1, \infty).$$

$$f^{-1} : (2, \infty) \rightarrow R, f^{-1}(x) = \sqrt{\frac{3}{x-2}} + 1 \quad \mathbf{B}$$



Question 9

The point $(1, 2)$ is on the curve of f' .
Thus the gradient of the graph of f at $x = 1$ is 2.

B**Question 10**

If $f(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + h$ has a stationary point of inflection at $(-3, 0)$
and a turning point at $(2, 0)$ then $f(x) = a(x + 3)^3(x - 2)^2$.

$$h = a \times 3^3 \times (-2)^2 = 108a$$

$$\frac{h}{a} = 108$$

A**Question 11**

$$f(x) = -2x^{\frac{7}{2}}, y = -4 \text{ to } y = -1$$

$$\text{Solve } f(x) = -4, x = 2^{\frac{2}{7}}$$

$$\text{and } f(x) = -1, x = 2^{-\frac{2}{7}}$$

Edit Action Interactive

0.5 1 $\frac{d}{dx}$ \int_{dx} Simp \int_{dx} ∇ ∇ ∇

```

define f(x)=-2x2
done
solve(f(x)=-4, x)
{x=4  $\frac{1}{7}$ }
solve(f(x)=-1, x)
{x= $\frac{1}{4 \cdot 7}$ }

```

$$\begin{aligned} \text{Average rate of change} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-1 + 4}{2^{\frac{2}{7}} - 2^{\frac{2}{7}}} \\ &\approx -7.5 \end{aligned} \quad \mathbf{E}$$

1.4 1.5 1.6 *MAV2016...am2 RAD $\frac{d}{dx}$

```

solve(-2*x2=-4, x) x=2  $\frac{1}{7}$ 
solve(-2*x2=-1, x) x= $\frac{1}{4 \cdot 7}$ 

```

$$\begin{aligned} \frac{-1+4}{2^{\frac{2}{7}}-2^{\frac{2}{7}}} &= \frac{5}{3 \cdot 2^{\frac{2}{7}}} \\ f(4^{\frac{1}{7}}) - f(\frac{1}{4^{\frac{1}{7}}}) &= -7.524864066 \end{aligned}$$

1.4 1.5 1.6 *MAV2016...am2 RAD $\frac{d}{dx}$

$$\frac{-1+4}{2^{\frac{2}{7}}-2^{\frac{2}{7}}} = \frac{5}{3 \cdot 2^{\frac{2}{7}}} = \frac{5}{3} \cdot 2^{-\frac{2}{7}} = -7.52486$$

Question 12

$$\log_a(b) = c \text{ and } \log_c(a) = b$$

$$\log_a(b) + \log_c(a) = b + c$$

$$\log_a(b) + \frac{\log_a(a)}{\log_a(c)} = b + c$$

$$\log_a(b) + \frac{1}{\log_a(c)} = b + c$$

$$\log_a\left(\frac{b}{c}\right) \neq b + c$$

D**Question 13**

$$f'(x) = 2x^3 - 2x$$

$$\begin{aligned} f(x) &= \int (2x^3 - 2x) dx \\ &= \frac{x^4}{2} - x^2 + c \end{aligned}$$

$$f(2) = \frac{9}{2}$$

$$\frac{9}{2} = \frac{2^4}{2} - 2^2 + c$$

$$c = \frac{1}{2}$$

$$\text{Solve } \frac{x^4}{2} - x^2 + \frac{1}{2} = 0 \text{ for } x.$$

$$x = \pm 1$$

A

The calculator screen shows the following steps:

- Step 1:** Integrate $2x^3 - 2x$ to get $\frac{x^4}{2} - x^2 + c$.
- Step 2:** Substitute $x=2$ into the antiderivative to find $c = \frac{1}{2}$.
- Step 3:** Solve the equation $\frac{x^4}{2} - x^2 + \frac{1}{2} = 0$ for x , resulting in $x = -1 \text{ or } x = 1$.

0.5 1 $\frac{1}{2}$ $\int \frac{d}{dx}$ $\int \frac{d}{dx}$ Simp $\int \frac{d}{dx}$ ∇ ∇

$$\int 2 \cdot x^3 - 2 \cdot x \, dx$$

$$\frac{x^4}{2} - x^2$$

define $f(x) = \frac{x^4}{2} - x^2 + c$
done

solve $\left(f(2) = \frac{9}{2}, c \right)$
 $\left\{ c = \frac{1}{2} \right\}$

solve $\left(\frac{x^4}{2} - x^2 + \frac{1}{2} = 0, x \right)$
 $\{x = -1, x = 1\}$

□

Alg Standard Real Rad |

Question 14

$$\begin{aligned} \int_2^3 (g(x)) \, dx &= -3 \\ &= 2 \int_3^2 (1 - 2g(x)) \, dx \\ &= 2 \int_3^2 (1) \, dx - 4 \int_3^2 (g(x)) \, dx \\ &= 2[x]_3^2 - 4 \times 3 \\ &= -14 \end{aligned}$$

E**Question 15**

$$\begin{aligned} \text{Area} &= \frac{1}{2}(e^0 + e + e^2 + e^3) \\ &= \frac{1}{2}(1 + e + e^2 + e^3) \end{aligned}$$

The rectangles are below the curve.

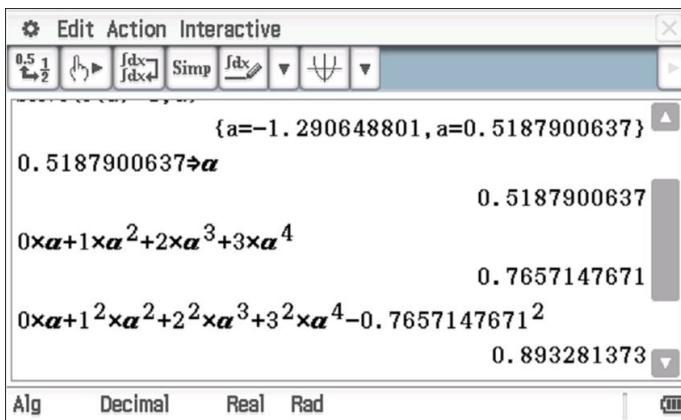
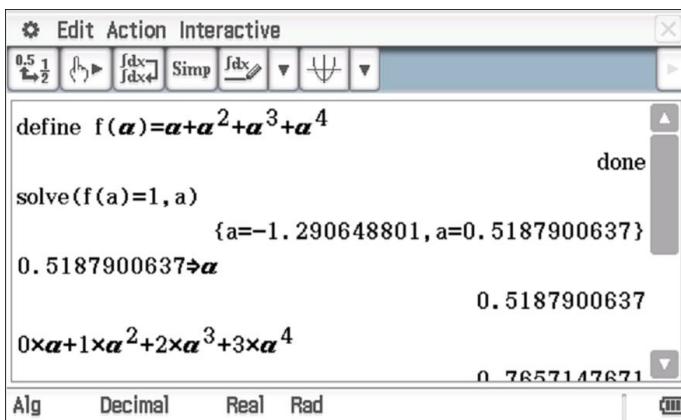
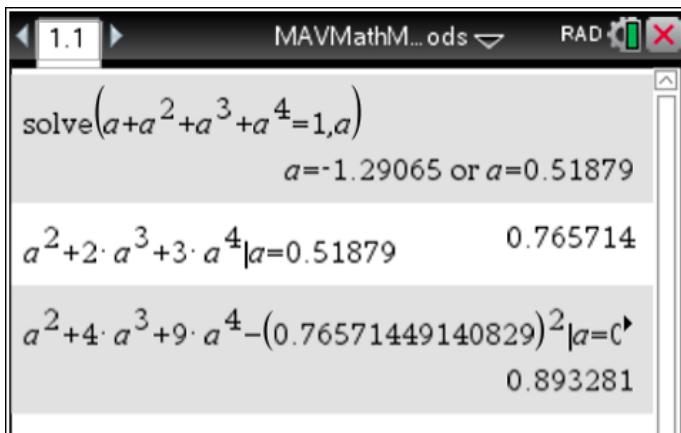
Underestimate

B**Question 16**Solve $a + a^2 + a^3 + a^4 = 1$ for a $a \approx 0.51879$

$E(X) = a^2 + 2 \times a^3 + 3 \times a^4 \approx 0.766$

$Var(X) = a^2 + 4a^3 + 9a^4 - (0.765714 \dots)^2 \approx 0.893$

D



Question 17

	J	J'	
R	$0.3p$	$0.7p$	p
R'	$0.3 - 0.3p$	p^2	$1 - p$
	0.3	0.7	1

Independent events $\Pr(J \cap R) = \Pr(J) \times \Pr(R)$

Let $a = \Pr(J \cap R) = 0.3p$

$$0.3 + p - a + p^2 = 1$$

Solve $p^2 + 0.7p - 0.7 = 0$ for p .

$$p = \frac{\sqrt{329} - 7}{20}$$

$$a = 0.3p$$

$$a = \frac{3(\sqrt{329} - 7)}{200}$$

D

solve $\left(p^2 + \frac{7}{10} \cdot p - \frac{7}{10} = 0, p\right)$
 $p = \frac{-(\sqrt{329} + 7)}{20}$ or $p = \frac{\sqrt{329} - 7}{20}$

solve $\left(p^2 + \frac{7}{10} \cdot p - \frac{7}{10} = 0, p\right)$
 $p = -1.256918$ or $p = 0.5569179$

solve $(0.7 \cdot p + p^2 = 0.7, p)$

$\left\{ p = \frac{-\sqrt{329}}{20} - \frac{7}{20}, p = \frac{\sqrt{329}}{20} - \frac{7}{20} \right\}$

Question 18

Solve $\int_1^a (\log_e(x)) dx = 1$ for a .

$$a = e$$

$$\text{Var}(X) = E(X^2) - (E(X))^2$$

$$\begin{aligned} \text{Var}(X) &= \int_1^e (x^2 \log_e(x)) dx - \left(\int_1^e (x \log_e(x)) dx \right)^2 \\ &= -\frac{e^4}{16} + \frac{2e^3}{9} - \frac{e^2}{8} + \frac{7}{144} \end{aligned}$$

C

The calculator screen shows the input: $\text{solve}\left(\int_1^a \ln(x) dx = 1, a\right)$. The result is displayed as:

$$\int_1^e \left(x^2 \cdot \ln(x)\right) dx - \left(\int_1^e (x \cdot \ln(x)) dx \right)^2$$

$$\frac{-e^4}{16} + \frac{2 \cdot e^3}{9} - \frac{e^2}{8} + \frac{7}{144}$$

The calculator screen shows the input: $\text{solve}\left(\int_1^a \ln(x) dx = 1, a\right)$. The result is displayed as $\{a=e\}$.

Below, the expanded form of the equation is shown:

$$\text{expand}\left(\int_1^e x^2 \cdot \ln(x) dx - \left(\int_1^e x \cdot \ln(x) dx \right)^2\right)$$

$$\frac{-e^4}{16} + \frac{2 \cdot e^3}{9} - \frac{e^2}{8} + \frac{7}{144}$$

Question 19

$n = 2000$ and $p = 0.2$. Since n is large, the distribution of \hat{P} is approximately normal with $\mu = 0.2$ and $\sigma = \sqrt{\frac{0.2 \times 0.8}{2000}}$.

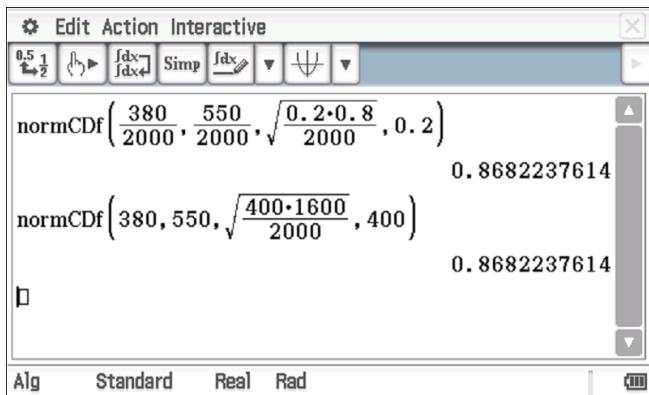
$$\Pr\left(\frac{380}{2000} < X < \frac{550}{2000}\right) \approx 0.868 \quad \text{C}$$

The calculator screen shows the input: $\text{normCdf}\left(\frac{380}{2000}, \frac{550}{2000}, 0.2, \sqrt{\frac{0.2 \cdot 0.8}{2000}}\right)$. The result is 0.8682237.

Below, another calculation is shown:

$$\text{normCdf}\left(380, 550, 400, \sqrt{\frac{400 \cdot 1600}{2000}}\right)$$

$$0.8682237$$

**Question 20**

If 200 such samples were taken, $0.9 \times 200 = 180$ of the confidence intervals would be expected to contain p . **B**

SECTION B
EXTENDED RESPONSE QUESTIONS

Question 1 (14 marks)

a. $f(x) = ax^{\frac{2}{3}}$

Substitute the point $\left(-8, \frac{16}{3}\right)$

$$\frac{16}{3} = a(-8)^{\frac{2}{3}}$$

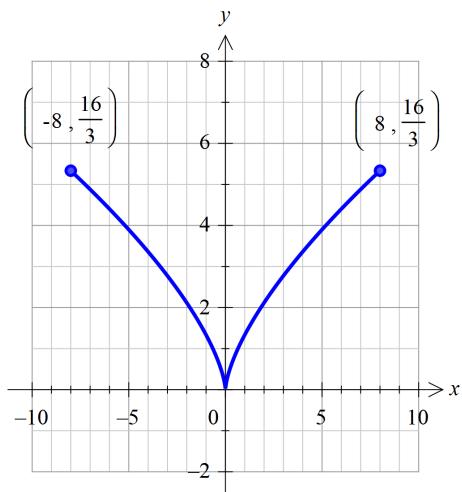
$$\frac{16}{3} = 4a$$

$$a = \frac{4}{3}$$

1M Show that

b. Shape 1A

Endpoints 1A



c. $f_1(x) = f(-x)$ reflects the graph over the y -axis.

As the function $f(x) = \frac{4}{3}x^{\frac{2}{3}}$ is an even function the transformation described makes no difference. 1A

d.

- a dilation of a factor of 2 units from the x -axis gives $y = \frac{8}{3}x^{\frac{2}{3}}$

then

- a translation of 3 units up gives $f_2(x) = \frac{8}{3}x^{\frac{2}{3}} + 3$ 1A

e.

- a reflection in the x -axis gives $y_1 = -\frac{4}{3}x^{\frac{2}{3}}$
- a reflection in the y -axis gives $y_2 = -\frac{4}{3}(-x)^{\frac{2}{3}} = -\frac{4}{3}x^{\frac{2}{3}}$
- a translation of 1 unit in the positive direction of the y -axis gives $y_3 = -\frac{4}{3}x^{\frac{2}{3}} + 1$
- a translation of 3 units in the positive direction of the x -axis gives $y_4 = -\frac{4}{3}(x-3)^{\frac{2}{3}} + 1$

The image equation is $f_3(x) = -\frac{4}{3}(x-3)^{\frac{2}{3}} + 1$ **2A**

f. $T : R^2 \rightarrow R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

gives $\begin{bmatrix} 2 & 0 \\ 0 & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2x+1 \\ -\frac{1}{3}y+2 \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$

Rearrange to get

$$x = \frac{x_1 - 1}{2}$$

$$y = \frac{y_1 - 2}{-\frac{1}{3}} = -3(y_1 - 2)$$
 1M

Susbtitute to find the image equation for $f(x) = \frac{4}{3}x^{\frac{2}{3}}$.

$$-3(y_1 - 2) = \frac{4}{3}\left(\frac{x_1 - 1}{2}\right)^{\frac{2}{3}}$$
 1M

Image equation is

$$y_1 = -\frac{4}{9}\left(\frac{x_1 - 1}{2}\right)^{\frac{2}{3}} + 2$$

Giving $y = 2 - \frac{4}{9}\left(\frac{x-1}{2}\right)^{\frac{2}{3}}$ **1A**

g. $y = 2 - \frac{4}{9}\left(\frac{x-1}{2}\right)^{\frac{2}{3}}$ gives a dilation of a factor of 2 from y -axis and a translation of 1 unit in the positive

direction of the x -axis from the original function $f(x) = \frac{4}{3}x^{\frac{2}{3}}$ where the domain was $[-8, 8]$.

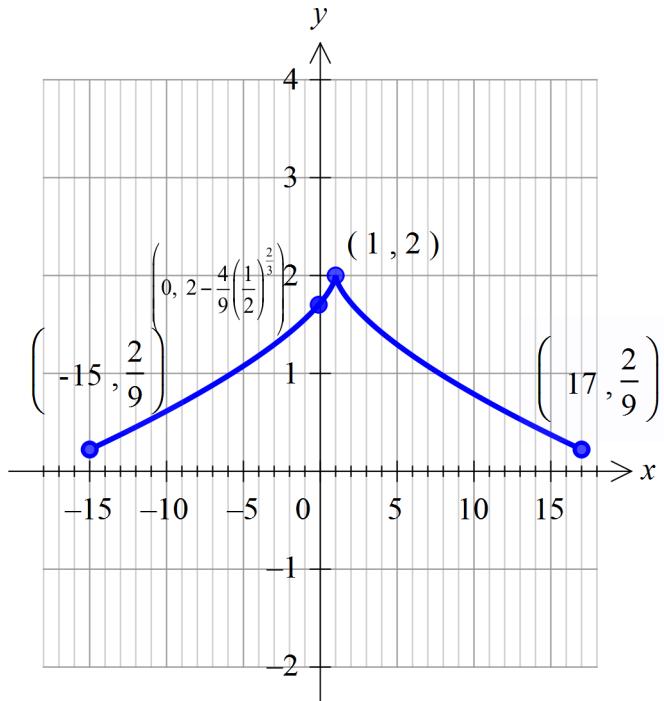
A dilation of a factor of 2 from y -axis gives the domain $[-16, 16]$.

Then a translation of 1 unit in the positive direction of the x -axis gives a domain $[-15, 17]$. **1A**

h. Shape 1A**Endpoints 1A**

Coordinates of Cusp and

$$y\text{-intercept } \left(0, 2 - \frac{4}{9} \left(\frac{1}{2} \right)^{\frac{2}{3}} \right) \quad \mathbf{1A}$$



Question 2 (15 marks)

a. $P(0) = 95 + 16 \sin\left(\frac{\pi}{15}(0 - 7.5)\right)$

$$= 95 - 16 \sin\left(\frac{\pi}{2}\right)$$

$$= 95 - 16$$

$$= 79$$

79 beats per minute

1M Show that

```

Edit Action Interactive
0.5 1 ∫ dx Simp Fdx
define f(t)=95+16sin(π/15(t-7.5))
f(0)
done
79

```

b. Period = $\frac{2\pi}{\frac{\pi}{15}} = 30$ **1A**

Amplitude = 16 **1A**

c. $P(t) = 95 + 16 \sin\left(\frac{\pi}{15}(t - 7.5)\right)$

$$P'(t) = 16 \times \frac{\pi}{15} \cos\left(\frac{\pi}{15}(t - 7.5)\right) \quad \text{1A}$$

$$P'(t) = \frac{16\pi}{15} \cos\left(\frac{\pi t}{15} - \frac{\pi}{2}\right)$$

$$= \frac{16\pi}{15} \cos\left(\frac{\pi}{2} - \frac{\pi t}{15}\right)$$

Giving $P'(t) = \frac{16\pi}{15} \sin\left(\frac{\pi t}{15}\right)$. **1M Show that**

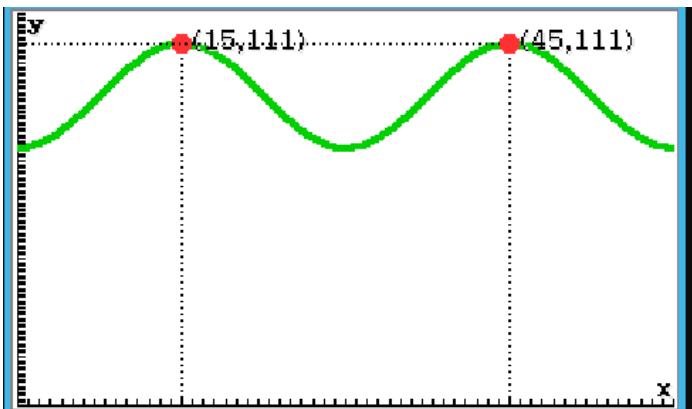
d. Solve $\frac{16\pi}{15} \sin\left(\frac{\pi t}{15}\right) = 0$ for $t \in [0, 60]$ 2 cycles **1M**

$$\frac{16\pi}{15} \sin\left(\frac{\pi t}{15}\right) = 0$$

$$\sin\left(\frac{\pi t}{15}\right) = 0$$

$$\frac{\pi t}{15} = 0, \pi, 2\pi, 3\pi, 4\pi$$

$$t = 0, 15, 30, 45, 60$$



From the graph the local max occurs at $t = 15$ and $t = 45$

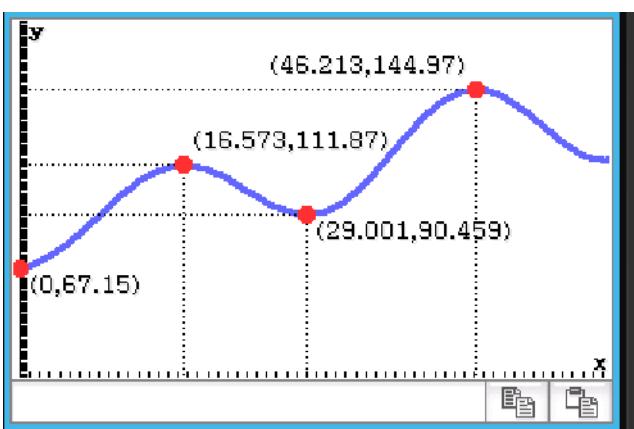
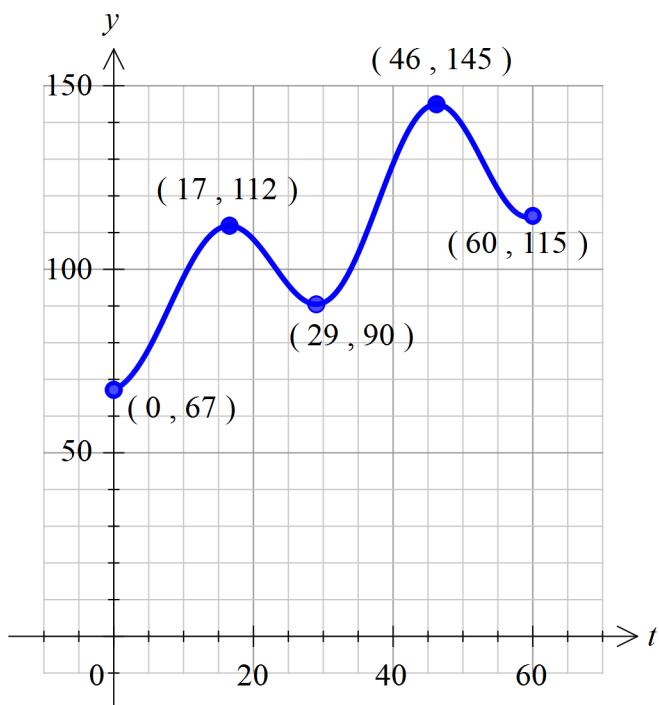
1A

e. $P_N : [0, 60] \rightarrow R, P_N(t) = 0.01(t + 85) \times P(t)$

Shape **1A**

Turning points **1A**

Endpoints **1A**

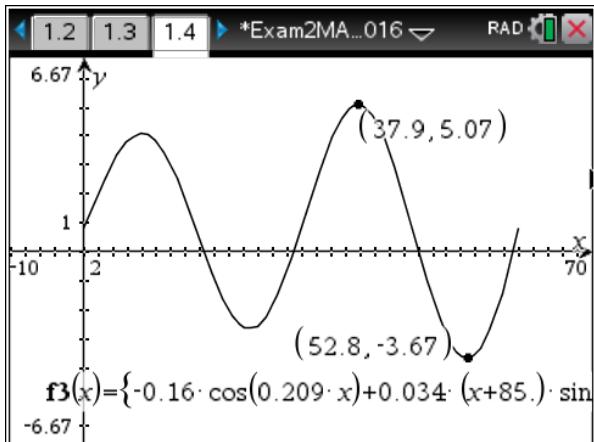


f. Maximum heart rate ≈ 144.97 beats /minute

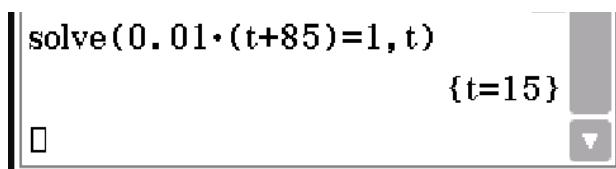
Maximum heart rate rounded to the nearest whole number = 145 beats /minute **1A**

g. Find when $P'_N(t)$ has its greatest magnitude **1M**

$t = 37.9$ minutes **1A**



h. Solve the equation $0.01(t+85) = 1$ gives $t = 15$ **1A**

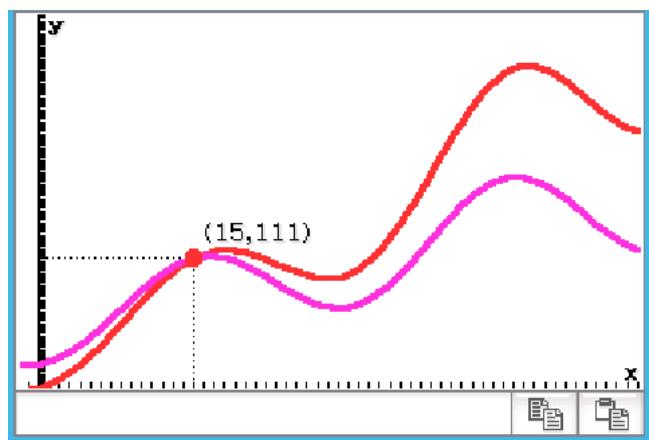


$$\text{Solve } 95 + 16 \sin\left(\frac{\pi}{15}(t - 7.5)\right) = 0.01(t + 85) \times P(t)$$

$$95 + 16 \sin\left(\frac{\pi}{15}(t - 7.5)\right) = 0.01(t + 85) \times \left(95 + 16 \sin\left(\frac{\pi}{15}(t - 7.5)\right)\right)$$

Gives the equation $1 = 0.01(t + 85)$ where previously found solution $t = 15$

So Sarah's pulse equals Stephen's pulse at $t = 15$.



Looking at the graph

Sarah's pulse can be measured to be lower than her twin brother Stephen's

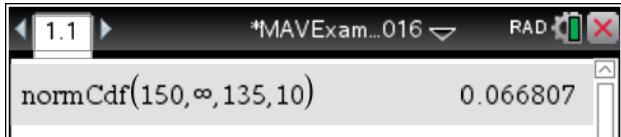
for $t \in (15, 60]$ **1A**

Question 3 (17 marks)

a. $X : N(135, 100)$

$\Pr(X > 150) = 0.0668\dots = 7\%$ to the nearest whole percent.

1A

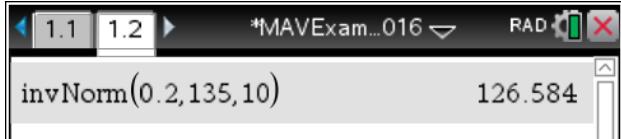


b. $\Pr(X > x) = 0.8$

$\Pr(X < x) = 0.2$

$x = 127$ days to the nearest whole number

1A

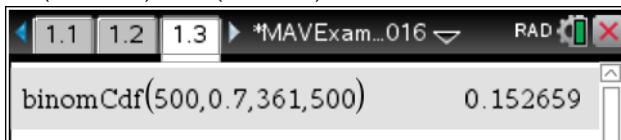


c. $Y : Bi(500, 0.7)$

1A

$\Pr(Y > 360) = \Pr(Y \geq 361) = 0.1527$ correct to 4 decimal places.

1A

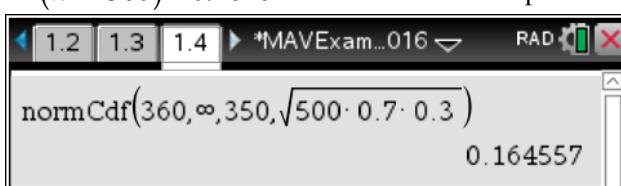


d. $W : N(350, 500 \times 0.7 \times 0.3)$

1M

$\Pr(W > 360) = 0.1646$ correct to 4 decimal places.

1A



e. $\mu = 0.7$

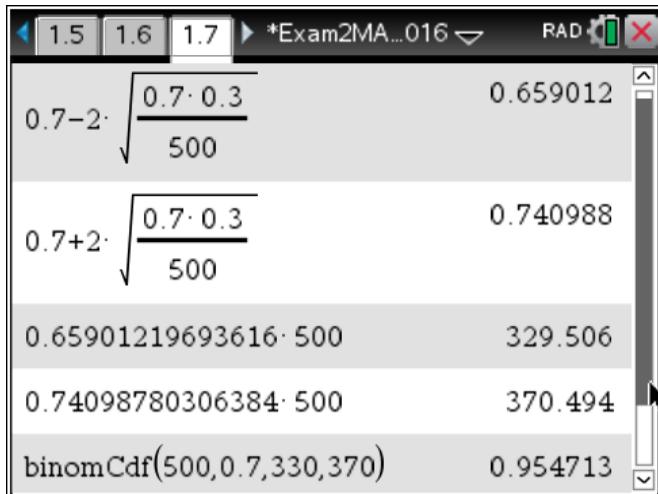
$$\sigma = \sqrt{\frac{0.7 \times 0.3}{500}} = \frac{\sqrt{105}}{500}$$

$$\left(0.7 - 2 \times \frac{\sqrt{105}}{500}, 0.7 + 2 \times \frac{\sqrt{105}}{500}\right) \quad 1M$$

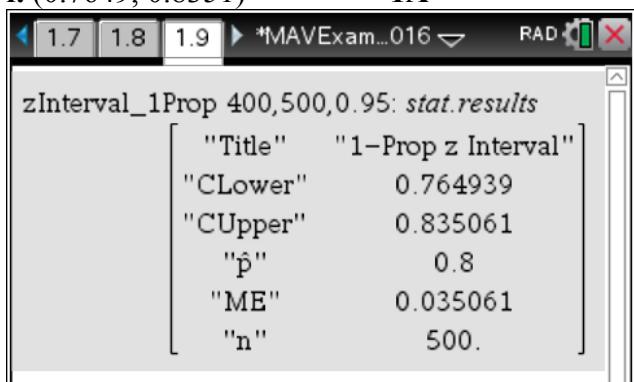
$$\Pr(0.6590\dots \leq \hat{P} \leq 0.7409\dots) = \Pr(329.5\dots \leq X \leq 370.4\dots) \quad 1M$$

$$= \Pr(330 \leq X \leq 370)$$

$$= 0.9547 \text{ correct to four decimal places.} \quad 1A$$



f. (0.7649, 0.8351)

1A

g.

Proportion of yellow balls \hat{p}	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
$\Pr(\hat{P} = \hat{p})$	$\frac{1}{22}$	$\frac{10}{33}$	$\frac{5}{11}$	$\frac{2}{11}$	$\frac{1}{66}$

Without replacement

$$\frac{6}{11} \times \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} = \frac{1}{22} \quad \text{1A}$$

$$4 \times \frac{5}{11} \times \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} = \frac{10}{33} \quad \text{1A}$$

1.5 1.6 1.7				*MAVExam...016	RAD	
$\frac{6}{11} \cdot \frac{5}{10} \cdot \frac{4}{9} \cdot \frac{3}{8}$				$\frac{1}{22}$		
$4 \cdot \frac{5}{11} \cdot \frac{6}{10} \cdot \frac{5}{9} \cdot \frac{4}{8}$				$\frac{10}{33}$		
$6 \cdot \frac{5}{11} \cdot \frac{4}{10} \cdot \frac{6}{9} \cdot \frac{5}{8}$				$\frac{5}{11}$		
$4 \cdot \frac{5}{11} \cdot \frac{4}{10} \cdot \frac{3}{9} \cdot \frac{6}{8}$				$\frac{2}{11}$		
				$\frac{5}{11}$		
				$\frac{2}{66}$		

h. $\Pr\left(\hat{P}=1 \mid \hat{P} > \frac{1}{2}\right) = \frac{\Pr\left(\hat{P}=1 \cap \hat{P} > \frac{1}{2}\right)}{\Pr\left(\hat{P} > \frac{1}{2}\right)}$ 1M

$$= \frac{\frac{1}{66}}{\frac{13}{66}} = \frac{1}{13}$$
 1A

i. $\Pr(X > 200) = 0.1, \Pr(X < 185) = 0.05$ 1M

$$\Pr\left(Z > \frac{200-\mu}{\sigma}\right) = 0.1, \Pr\left(Z < \frac{175-\mu}{\sigma}\right) = 0.05$$

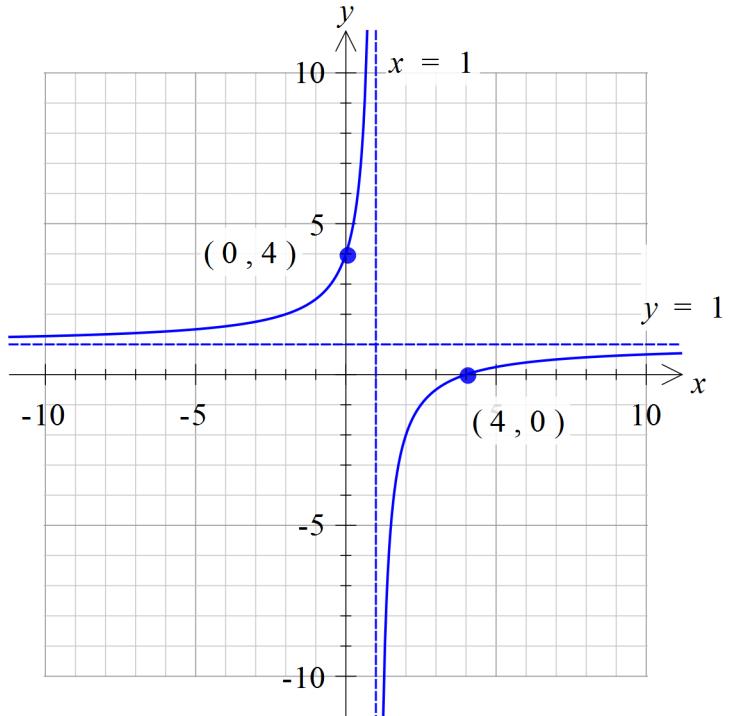
$$\frac{200-\mu}{\sigma} = 1.2815..., \frac{175-\mu}{\sigma} = -1.6448...$$
 1M

$$\mu = 189 \text{ days}, \sigma = 9 \text{ days}$$
 1A

1.1 1.2 1.3		*MAVExam...016	RAD	
invNorm(0.9, 0, 1)		1.28155		
invNorm(0.05, 0, 1)		-1.64485		
solve($\frac{200-a}{b} = 1.2815515665787$ and $\frac{175-a}{b} = -1.64485$)				
$a = 189.052$ and $b = 8.5429$				

Question 4 (14 marks)

- a. shape 1A
asymptotes with equations 1A
coordinates of axial-intercepts 1A



b. Let $y = \frac{x-4}{x-1}$

Inverse swap x and y .

Solve $x = \frac{y-4}{y-1}$ for y .

$$f^{-1}(x) = \frac{x-4}{x-1}$$

Hence $f(x) = f^{-1}(x)$ 1A



c. $g : R \setminus \{-n\} \rightarrow R, g(x) = \frac{x+m}{x+n}$

$$g^{-1}(x) = \frac{-nx + m}{x - 1}$$

$n = -1$ 1A

$m \in R \setminus \{-1, 0\}$ 1A

OR

$$g(x) = 1 + \frac{m+n}{x+n}$$

Horizontal asymptote at $y = 1$

Vertical asymptote at $x = -n$

$$n = -1 \quad \text{1A}$$

$$m \in R \setminus \{-1, 0\} \quad \text{1A}$$

d. $h: R \setminus \left\{-\frac{b}{n}\right\} \rightarrow R, h(x) = \frac{ax+m}{bx+n}$

$$h^{-1}(x) = \frac{-nx+m}{bx-a}$$

$$ax+m = -nx+m, \quad bx-a = bx+n$$

$a = -n$ **1M Show that**

e. $h_1: R \setminus \left\{-\frac{n}{4}\right\} \rightarrow R, h_1(x) = \frac{3x+2}{4x+n}$

$$n = -3 \quad \text{1A}$$

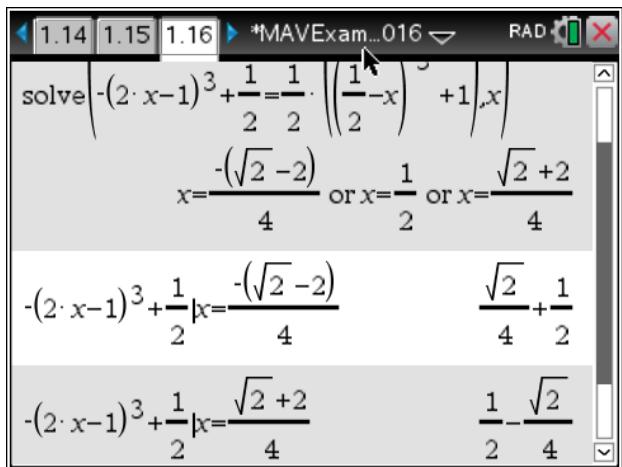
f. $w: R \rightarrow R, w(x) = -(2x-1)^3 + \frac{1}{2}$

$$w^{-1}(x) = \frac{1}{2} \left(\sqrt[3]{\left(\frac{1}{2} - x \right)} + 1 \right)$$

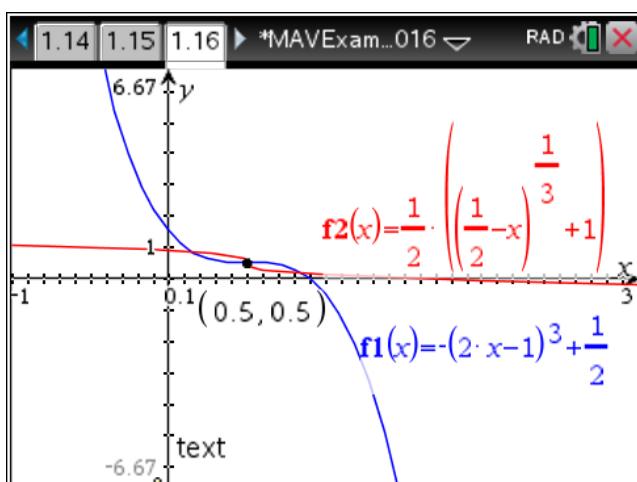
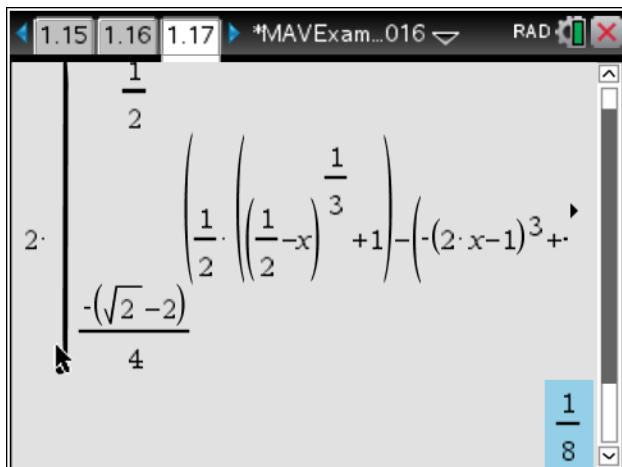
Solve $w(x) = w^{-1}(x)$ for x . **1M**

$$x = \frac{2-\sqrt{2}}{4}, x = \frac{1}{2} \text{ or } x = \frac{2+\sqrt{2}}{4} \quad \text{1A}$$

$$\left(\frac{2-\sqrt{2}}{4}, \frac{2+\sqrt{2}}{4} \right), \left(\frac{1}{2}, \frac{1}{2} \right), \left(\frac{2+\sqrt{2}}{4}, \frac{2-\sqrt{2}}{4} \right) \quad \text{1H}$$



g. Area = $2 \int_{\frac{2-\sqrt{2}}{4}}^{\frac{1}{2}} \left(\frac{1}{2} \left(\sqrt[3]{\frac{1}{2} - x} + 1 \right) - \left(-(2x-1)^3 + \frac{1}{2} \right) \right) dx$ **1M**
 $= \frac{1}{8}$ **1A**



h. $w_k : R \rightarrow R, w_k(x) = -(rx-1)^3 + s$

$$s = \frac{1}{r}$$
 1A