

The Mathematical Association of Victoria
MATHEMATICAL METHODS (CAS)
SOLUTIONS: Trial Exam 2014

Written Examination 2

SECTION 1

1. C 2. D 3. C 4. D 5. B 6. A 7. B 8. A 9. A 10. E 11. E
 12. D 13. C 14. C 15. A 16. B 17. E 18. B 19. C 20. D 21. A 22. E

Question 1

$$2 \cos\left(\frac{x}{2}\right) + \sqrt{3} = 0$$

$$x = 4\pi n \pm \frac{5\pi}{3}, n \in \mathbb{Z} \quad \text{C}$$

Question 2

Range of $y = -\sin(a(x+b))$ is $[-1, 1]$

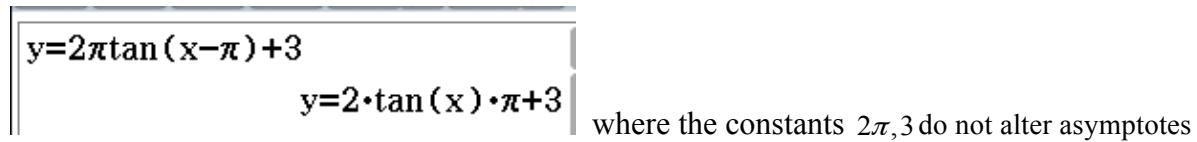
Range of $y = -4\sin(a(x+b))$ is $[-4, 4]$

Range of the graph of $f(x) = -4\sin(a(x+b)) - b$ is $[-4-b, 4-b]$ **D**

Question 3

Consider $f : [-\pi, 2\pi] \rightarrow \mathbb{R}$, where $f(x) = 2\pi \tan(x - \pi) + 3$.

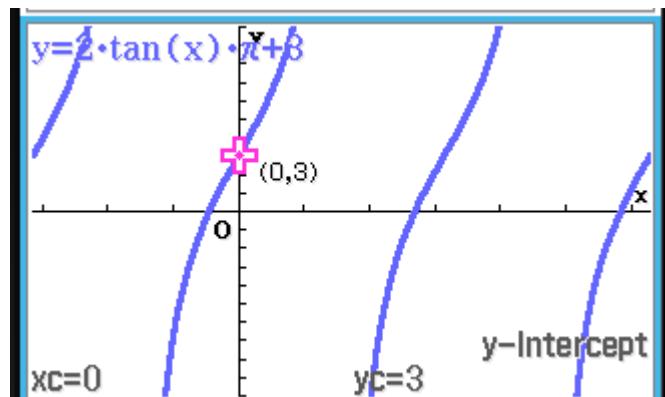
The horizontal translation in a positive direction of π will make no difference to the vertical asymptotes of the graph of $y = 2\pi \tan(x) + 3$.



The graph of $y = \tan(x)$ from $-\pi$ to π shows

expected asymptotes at $x = \pm\frac{\pi}{2}$.

For domain $[-\pi, 2\pi]$ we add the asymptote $x = \frac{3\pi}{2}$



The equations of asymptotes are $x = \frac{3\pi}{2}, x = \pm\frac{\pi}{2}$ C

Question 4

$$y = \sin(x)$$

$$\text{Then } y = \sin\left(x + \frac{\pi}{2}\right)$$

$$\text{Then } y = \sin\left(x + \frac{\pi}{2}\right) + \frac{1}{2}$$

Then $y = \sin\left(\frac{x}{2} + \frac{\pi}{2}\right) + \frac{1}{2}$

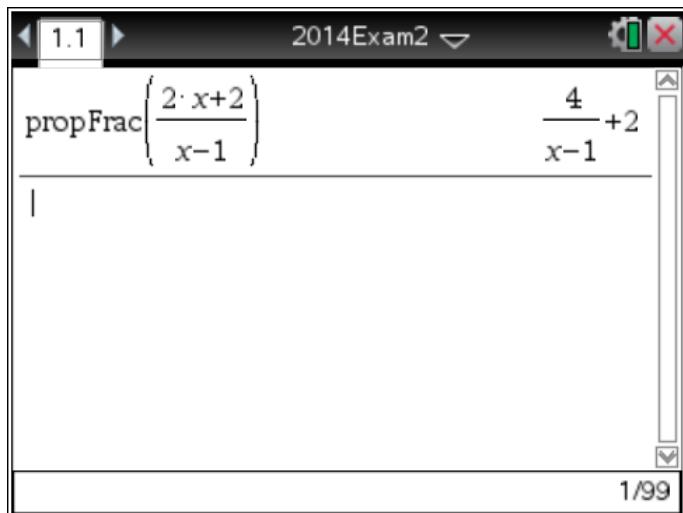
Using complementary functions this now becomes $y = \cos\left(\frac{x}{2}\right) + \frac{1}{2}$ **D**

Question 5

$$f(x) = \frac{2x+2}{x-1} = 2 + \frac{4}{x-1}$$

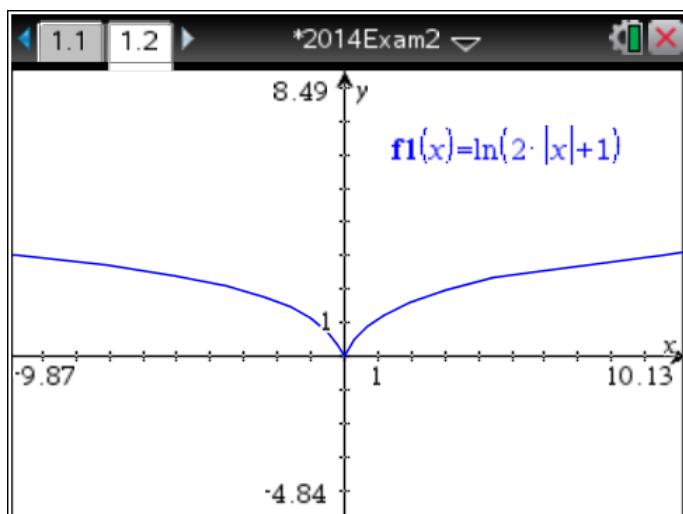
Domain $R \setminus \{1\}$, Range $R \setminus \{2\}$

B

**Question 6**

The graph of $f(x) = \ln(2|x|+1)$ has no asymptotes.

A



Question 7

$$(x-1)(x^2 + ax + b) = 0$$

$x = 1$ is one solution

$$x = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$
 will give another solution if

$$a^2 - 4b = 0, a^2 = 4b$$

$$x = -\frac{a}{2}$$
 is the solution

$$-\frac{a}{2} \neq 1, a \neq -2$$

B

Question 8

$$h(x) = g(f(x)) = (\sqrt{x+2})^2 + 5 = x+7$$

$$\text{Dom}(h) = \text{Dom}(f), [-2, \infty)$$

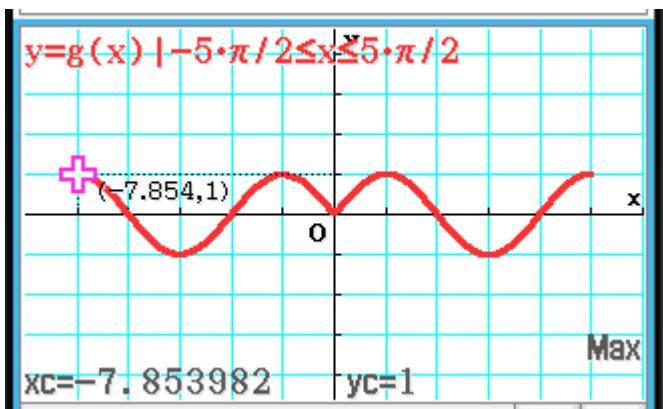
A

Question 9

$g : \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right] \rightarrow \mathbb{R}$, where $g(x) = \sin|x|$, is **not** differentiable at cusp and endpoints.

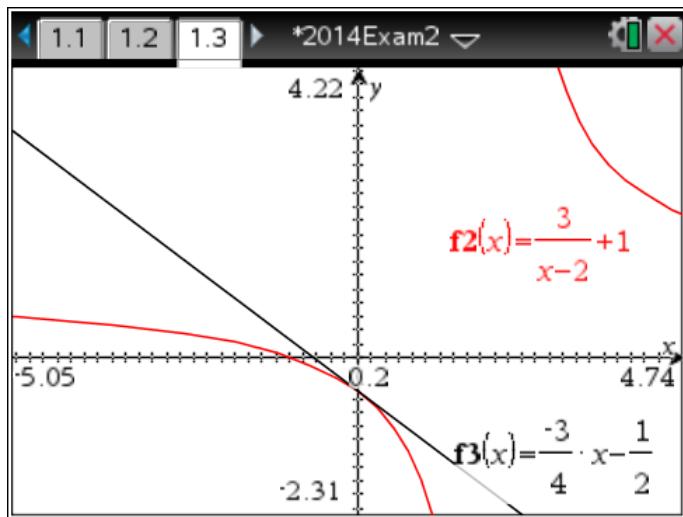
$$\text{At } x = \pm \frac{5\pi}{2}, x = 0$$

A



Question 10

The tangent line will be above the graph of $f(x) = \frac{3}{x-2} + 1$ when $x < 2$. E

**Question 11**

$$g : R \setminus \{-1\} \rightarrow R, g(x) = \frac{1}{x+1}$$

Using First Principles

$$\begin{aligned} & \lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right) \\ &= \lim_{h \rightarrow 0} \left(\frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} \right) \\ &= \lim_{h \rightarrow 0} \left(\frac{x+1 - (x+h+1)}{h(x+1)(x+h+1)} \right) \\ &= \lim_{h \rightarrow 0} \left(\frac{-1}{x^2 + xh + 2x + h + 1} \right) \end{aligned} \quad \text{E}$$

The calculator screen shows the following steps for differentiating $f(x) = \frac{1}{x+1}$:

- Define $f(x) = \frac{1}{x+1}$
- Calculate the difference quotient: $\frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h}$
- Simplify the difference quotient: $\text{comDenom}\left(\frac{1}{h \cdot (x+h+1)} - \frac{1}{h \cdot (x+1)}\right)$
- Simplify further: $\frac{-1}{x^2 + h \cdot x + 2 \cdot x + h + 1}$

Domain of the result might be larger than the... 1/3

Question 12

f is the graph of $h(x) = (x+1)^3(x-1)$ translated 3 units up.

$$f(x) = (x+1)^3(x-1) + 3 \quad \mathbf{D}$$

Question 13

$$g - f = -x^4 + x^2 + 7 - (x^2 + 2x + 3) = -x^4 + x^2 - 2x + 4$$

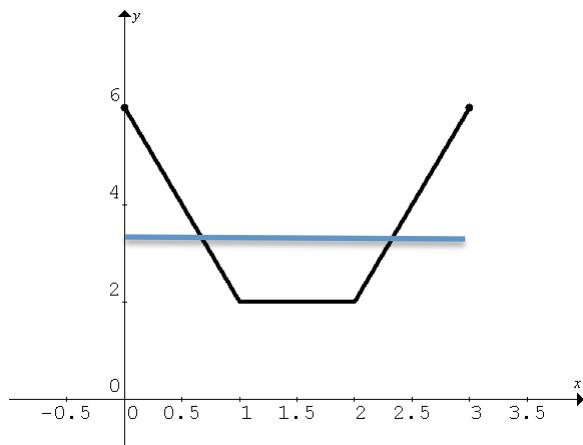
Solve $\frac{d}{dx}(-x^4 + x^2 - 2x + 4) = 0$ for x

Maximum value is $\frac{3}{2^3} + 4$ **C**

The calculator screen shows the following steps for solving the derivative equation:

- solve $\left(\frac{d}{dx} (-x^4 + x^2 + 7 - (x^2 + 2x + 3)) \right) = 0, x \right)$
- Result: $x = \frac{-2^3}{2}$
- Evaluating the function at $x = \frac{-2^3}{2}$: $-x^4 + x^2 + 7 - (x^2 + 2x + 3) \Big|_{x=\frac{-2^3}{2}} = \frac{3 \cdot 2^3}{4} + 4$

2/99

Question 14

The area above the line $y = \frac{10}{3}$ equals the area below the line.

The average value is $\frac{10}{3}$. C

Question 15

$$y = \log_e(x-1)$$

$$\text{Area} = \frac{1}{2}(\log_e(1.5) + \log_e(2) + \log_e(2.5))$$

$$= \frac{1}{2} \log_e \left(\frac{3}{2} \times 2 \times \frac{5}{2} \right)$$

$$= \frac{1}{2} \log_e \left(\frac{15}{2} \right)$$

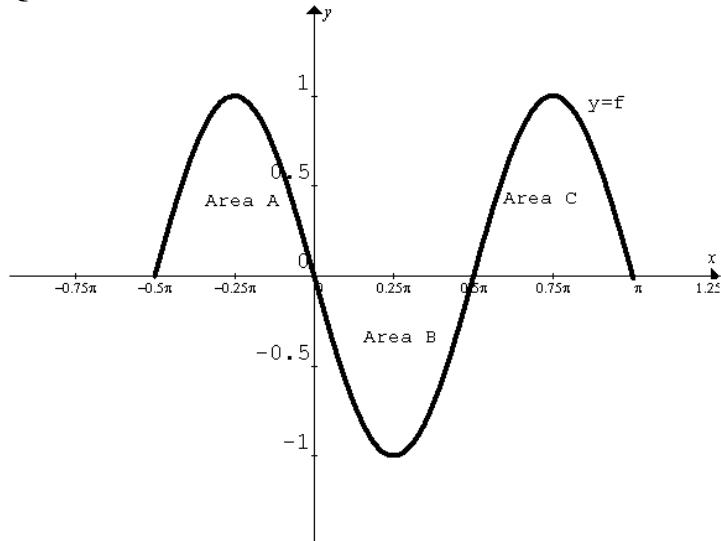
$$= \log_e \left(\sqrt{\frac{15}{2}} \right) \quad \text{A}$$

1.4 1.5 1.6 *2014Exam2

$$\frac{1}{2} \cdot \left(\ln\left(\frac{3}{2}\right) + \ln(2) + \ln\left(\frac{5}{2}\right) \right)$$

$$\frac{\ln\left(\frac{15}{2}\right)}{2}$$

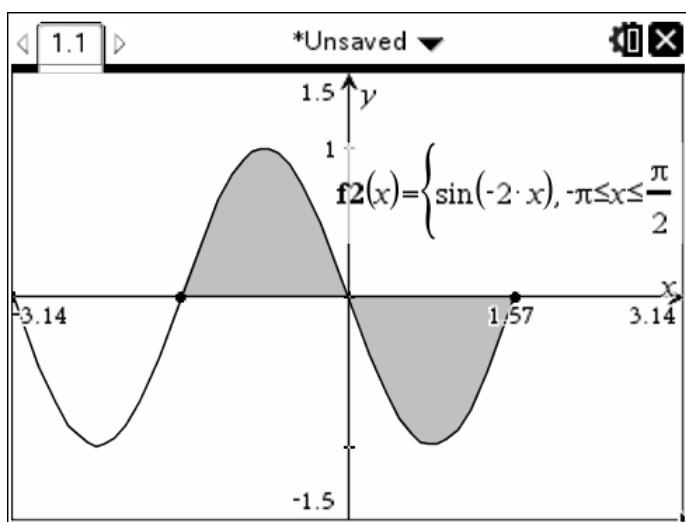
1/99

Question 16

The graph of f has been reflected in the x and y axes.

Area A = Area B

$$\text{Hence } \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (-f(-x)) dx = 0 \quad \mathbf{B}$$

**Question 17**

To find p .

Solve $2p^2 + 0.1 + 0.02 + p + p^2 = 1$ where $p > 0$

$$\left| \begin{array}{l} \text{CAS: } \{2 \cdot p^2 + 0.1 + 0.02 + p + p^2 = 1, p\} \\ \left\{ p = -\frac{11}{15}, p = \frac{2}{5} \right\} \end{array} \right| \text{ gives } p = \frac{2}{5} = 0.4$$

$$E(X) = 1.98$$

$$Var(X) = 2.4196$$

The standard deviation of X is closest to 1.5555 E

$1 \times 0.1 + 2 \times 0.02 + 3 \times 0.4 + 4 \times 0.4^2$	1.98
$0.1 + 4 \times 0.02 + 9 \times 0.4 + 16 \times 0.4^2$	6.34
$\text{ans} - 1.98^2$	2.4196
$\sqrt{2.4196}$	1.555506348

Question 18

$$\text{Bi}(n, p) = \text{Bi}(50, 0.05)$$

$$\Pr(X < 2) = \Pr(X \leq 1) = 0.2794 \quad \mathbf{B}$$

$\text{binomialCDf}(0, 1, 50, 0.05)$	
	0.2794317523

Question 19

$$S_0 = \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix}_{\leftarrow L}^{W}$$

$$T = \begin{bmatrix} 0.55 & 0.65 \\ 0.45 & 0.35 \end{bmatrix}$$

$$\text{Need } S_3 = T^3 S_0$$

$\begin{bmatrix} 0.55 & 0.65 \\ 0.45 & 0.35 \end{bmatrix} \wedge 3 \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix}$	
	$\begin{bmatrix} 0.5908 \\ 0.4092 \end{bmatrix}$

$$S_3 = \begin{bmatrix} 0.5908 \\ 0.4092 \end{bmatrix}_{\leftarrow L}^{W} \quad \mathbf{C}$$

Question 20

P and Q mutually exclusive when $\Pr(P \cap Q) = 0$

$$\text{Given } \Pr(P \cap Q) + \frac{1}{2} \Pr(P' \cap Q) = \frac{3}{10}$$

So

$$0 + \frac{1}{2} \Pr(P' \cap Q) = \frac{3}{10}$$

$$\therefore \Pr(P' \cap Q) = \frac{3}{5}$$

	P		P'	
Q	0	$\frac{3}{5}$	$\frac{3}{5}$	
Q'	$\Pr(P \cap Q')$	$\Pr(P' \cap Q')$	$\frac{2}{5}$	
	—	—	1	

$$\Pr(P \cap Q') + \Pr(P' \cap Q') = \frac{2}{5} \quad \mathbf{D}$$

Question 21

$N(\mu, \sigma^2)$

$$\Pr(X > 80) = \frac{85}{1000} \quad \text{and} \quad \Pr(X < 10) = 0.01$$

$$\Pr(Z > 1.37220) = \frac{85}{1000} \quad \text{and} \quad \Pr(Z < -2.32635) = 0.01$$

$\text{invNormCDf}\left("R", \frac{85}{1000}, 1, 0\right)$ 1.372203809
$\text{invNormCDf}\left("L", 0.01, 1, 0\right)$ -2.326347874

$$\text{Solve using } z = \frac{x - \mu}{\sigma}$$

$$\left| \begin{array}{l} 1.37220 = \frac{80-m}{s} \\ -2.32635 = \frac{10-m}{s} \end{array} \right|_{m,s}$$

{m=54.02928175, s=18.9263} ►

Giving $\mu = 54.029$ A

Question 22

$$f(x) = \begin{cases} |\sin(x)| & \frac{3\pi}{2} \leq x \leq 2\pi \\ 0 & \text{elsewhere} \end{cases}$$

$$\text{Variance} = \int_{\frac{3\pi}{2}}^{2\pi} x^2 f(x) dx - (E(X))^2$$

$$\text{Variance} = \int_{\frac{3\pi}{2}}^{2\pi} x^2 f(x) dx - \left(\int_{\frac{3\pi}{2}}^{2\pi} x f(x) dx \right)^2 = 0.14159 \quad E$$

f(x) = $\begin{cases} |\sin(x)|, & \frac{3\pi}{2} \leq x \leq 2\pi \\ 0, & \frac{3\pi}{2} \geq x \geq 2\pi \end{cases}$

done

$$\int_{\frac{3\pi}{2}}^{2\pi} x^2 \cdot f(x) dx$$

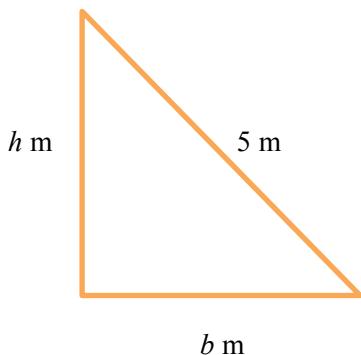
28.05363964

$$\text{ans} - \left(\int_{\frac{3\pi}{2}}^{2\pi} x \cdot f(x) dx \right)^2$$

0.1415926519

SECTION 2**Question 1** (10 marks)

a. i.



$$h^2 + b^2 = 25$$

$$h = \sqrt{25 - b^2}, h > 0$$

$$\frac{dh}{db} = \frac{-2b}{2\sqrt{25 - b^2}} = \frac{-b}{\sqrt{25 - b^2}} \quad \mathbf{1M}$$

$$\frac{db}{dt} = 2 \text{ m/s}$$

$$\frac{dh}{dt} = \frac{dh}{db} \times \frac{db}{dt} \quad \mathbf{1M}$$

$$\frac{dh}{dt} = \frac{-b}{\sqrt{25 - b^2}} \times 2$$

$$b = 3 \text{ m}$$

$$\frac{dh}{dt} = \frac{-3}{\sqrt{25 - 9}} \times 2 = \frac{-6}{\sqrt{16}} = \frac{-3}{2} \text{ m/s}$$

The ladder is moving $\frac{3}{2}$ m/s down the wall. **1A**

ii. $h = 2\sqrt{6} \text{ m}, h^2 = 24, b = 1 \text{ m}, \frac{db}{dt} = 2 \text{ m/s}, t = 2 \text{ s} \quad \mathbf{1A}$

b. i. $A = \int (2 \log_e(t+1)) dt$

$$A = 2(t+1) \log_e(t+1) - 2t + c \quad \mathbf{1M}$$

$$A(0) = c = 0$$

$$A = 2(t+1) \log_e(t+1) - 2t \quad \mathbf{1A}$$

The calculator screen shows the following input and output:

$$\int 2 \cdot \ln(t+1) dt \quad 2 \cdot ((t+1) \cdot \ln(t+1) - t)$$

$$\text{solve}\left(2 \cdot ((t+1) \cdot \ln(t+1) - t) + c = 0, c\right) |_{t=0} \quad c=0$$

Below the input area, there is a blank line for further calculations.

At the bottom right of the screen, it says "2/99".

ii. $A(30) = 152.9 \text{ m}^2$ correct to one decimal place **1M 1A**

OR

$$A = \int_0^{30} (2 \log_e(t+1)) dt = 152.9 \text{ m}^2 \text{ correct to one decimal place} \quad \mathbf{1M 1A}$$

The calculator screen shows the following input and output:

$$\int 2 \cdot \ln(t+1) dt \quad 2 \cdot ((t+1) \cdot \ln(t+1) - t)$$

$$\text{solve}\left(2 \cdot ((t+1) \cdot \ln(t+1) - t) + c = 0, c\right) |_{t=0} \quad c=0$$

$$2 \cdot ((t+1) \cdot \ln(t+1) - t) |_{t=30} \quad 152.907$$

$$\int_0^{30} (2 \cdot \ln(t+1)) dt \quad 152.907$$

Below the input area, there is a blank line for further calculations.

At the bottom right of the screen, it says "4/99".

iii. Solve $A = 2(t+1)\log_e(t+1) - 2t = 202.907\dots$ for t **1M**

$t = 37.0\dots \text{ min}$

It takes an extra 7 minutes **1A**

OR

$$\text{Solve } \int_{30}^a (2 \log_e(t+1)) dt = 50 \text{ for } a \quad \mathbf{1M}$$

$t = 37.0\dots \text{ min}$

It takes an extra 7 minutes

1A

$\int_0^{30} (2 \cdot \ln(t+1)) dt$

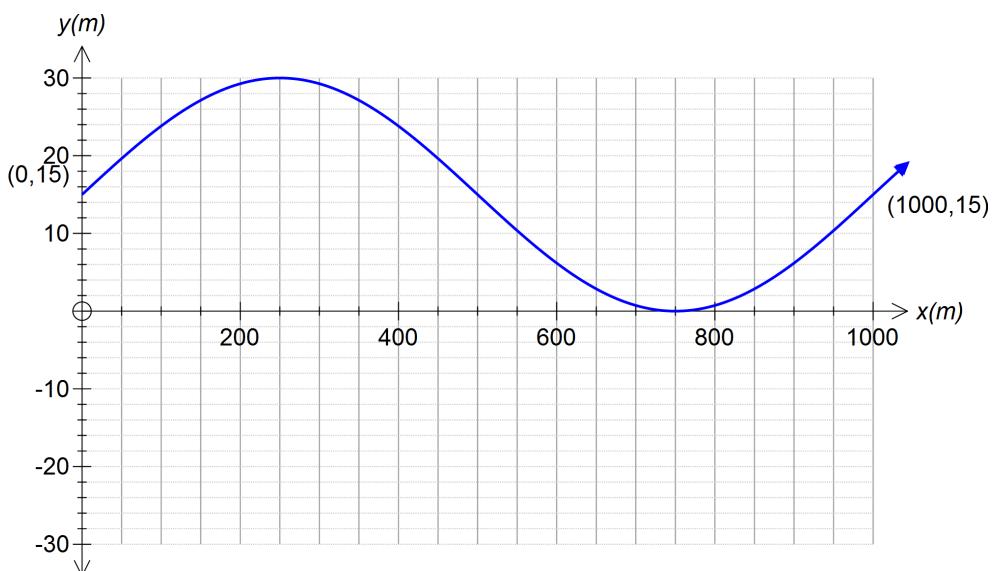
solve $(2 \cdot ((t+1) \cdot \ln(t+1) - t) = 202.9072, t)$

$t = 37.0619$

solve $\left(\int_{30}^a (2 \cdot \ln(t+1)) dt = 50, a \right)$

$a = 37.0619$

3/6

Question 2 (17 marks)

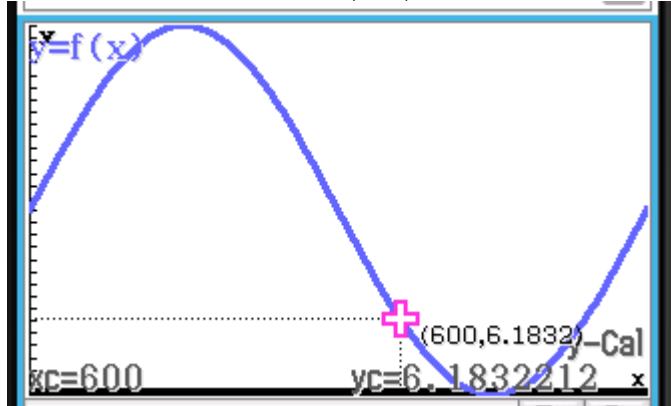
- a. i. a is amplitude so $a = 15$ **1A**
- ii. c is vertical translation so $c = 15$ **1A**
- iii. period = $1000 = \frac{2\pi}{\pi/b}$

giving $1000 = 2b$

$$b = 500. \quad \mathbf{1M}$$

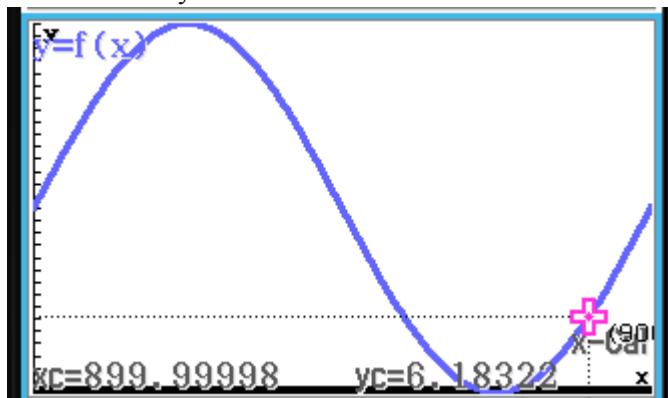
b. $y = 15 + 15 \sin\left(\frac{\pi x}{500}\right)$ 1A

c. i. $y(600) = 15 + 15 \sin\left(\frac{6\pi}{5}\right)$



Depth of the lake 600 metres from the café is 6.18 metres (or 618 centimetres) 1A

ii. Solve $y = 6.18322$ for the domain $0 \leq x \leq 1000$

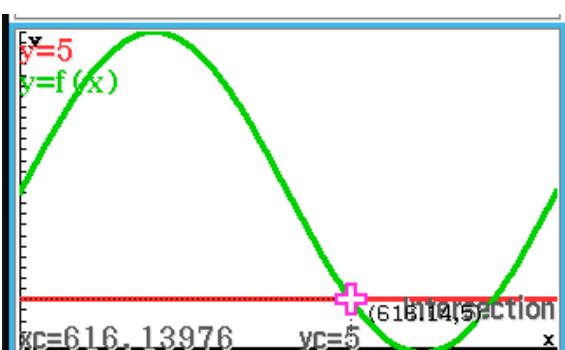


Depth of water found at $x = 600$ and $x = 900$ to nearest metre. 1A

Compete when $y \geq 5$

d. i. Solve $y \geq 5$ 1M

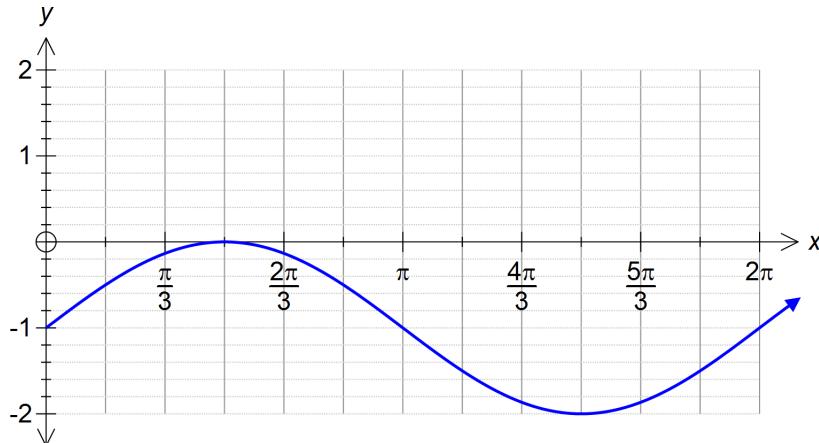
```
define f(x)=15+15sin(pi*x/500)
done
solve(f(x)=5 | 0≤x≤1000, x)
{x=616.1397636, x=883.860}
```



Depth of water suitable for $x \in [0, 616] \cup [884, 1000]$ to the nearest **metre** 1A

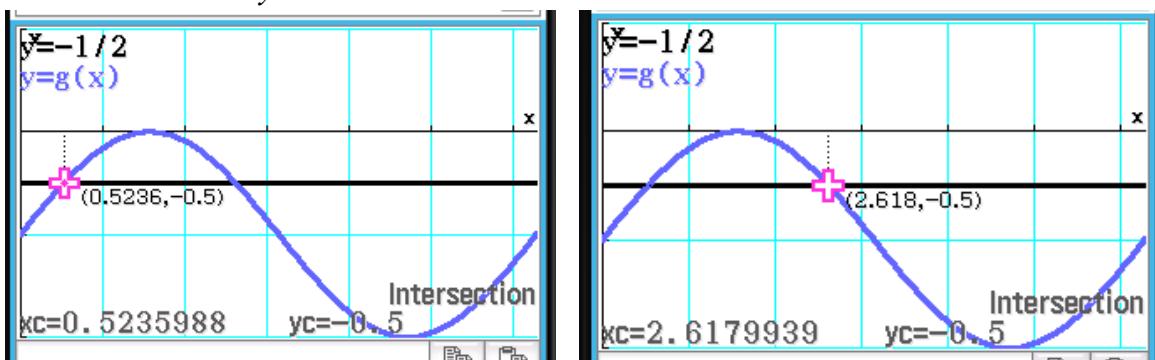
ii. Length of race, to nearest **metre**, before it is interrupted by shallow water is 616 metres. 1A

$$f(x) = \sin(x) - 1 \text{ for } 0 \leq x \leq 2\pi$$



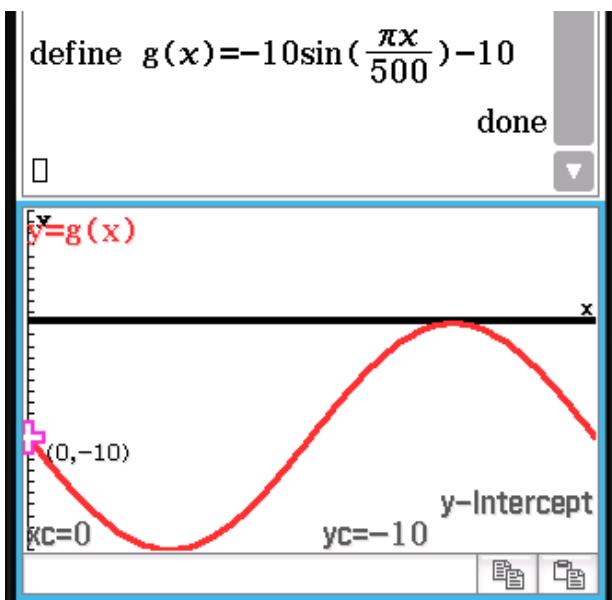
e. zero depth at $\left(\frac{\pi}{2}, 0\right)$ 1A

Stand in water when $y > -0.5$ 1M



f. Tourists can walk at distance from the café, to nearest **centimetre**, for $x \in (0.52, 2.62)$ 1A

$$g(x) = -10 \sin\left(\frac{\pi x}{500}\right) - 10 \text{ for } 0 \leq x \leq 1000$$



g. Straight line, between $x = 750$ and $x = 875$.

Using the equation of a straight line $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

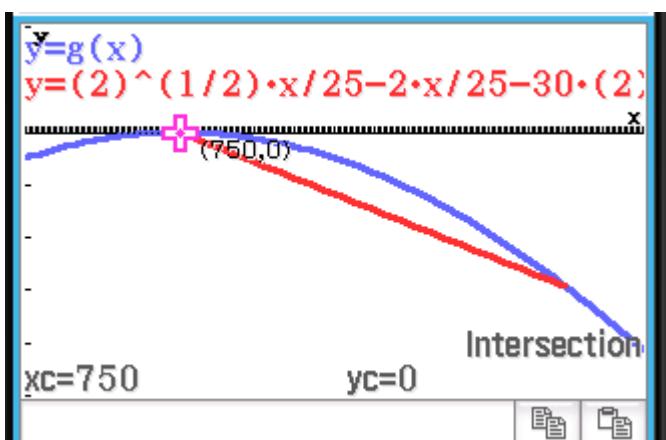
where $\frac{y - y(750)}{x - 750} = \frac{y(875) - y(750)}{875 - 750}$ 1M

$$\left(\frac{y - g(750)}{x - 750} = \frac{g(875) - g(750)}{875 - 750}, \right. \\ \left. \left\{ y = \frac{-(x - 750) \cdot (-\sqrt{2} + 2)}{25} \right\} \right)$$

1M

The equation of the line that represents the slide is,:;

$$\text{expand} \left(y = \frac{-(x - 750) \cdot (-\sqrt{2} + 2)}{25} \right) \\ y = \frac{\sqrt{2} \cdot x}{25} - \frac{2 \cdot x}{25} - 30 \cdot \sqrt{2} + 60$$



graph of the water slide

Simplified equation of water slide $y = -\frac{x}{25}(2 - \sqrt{2}) - 30\sqrt{2} + 60$ **1A**

h. Transform $y = 15 + 15 \sin\left(\frac{\pi x}{500}\right)$ to the image graph $g(x) = -10 \sin\left(\frac{\pi x}{500}\right) - 10$

Step 1: Dilate from the x -axis by a factor of $\frac{2}{3}$ units **1A**

Step 2: Reflect in the x -axis **1A**

Question 3 (16 marks)

Rowing Ergo (R), Cycle (C), Step (S).

60 staff members, all take part in at least one class.

$$\Pr(R) = 0.25$$

$$\Pr(C) = 0.55$$

$$\Pr(S) = 0.75$$

R, C and S are independent.

a. i. Independent $\therefore \Pr(R \cap S) = \Pr(R) \times \Pr(S)$

$$\Pr(R \cap S) = 0.25 \times 0.75 = 0.1875 \quad \text{or } \frac{3}{16} \quad \mathbf{1A}$$

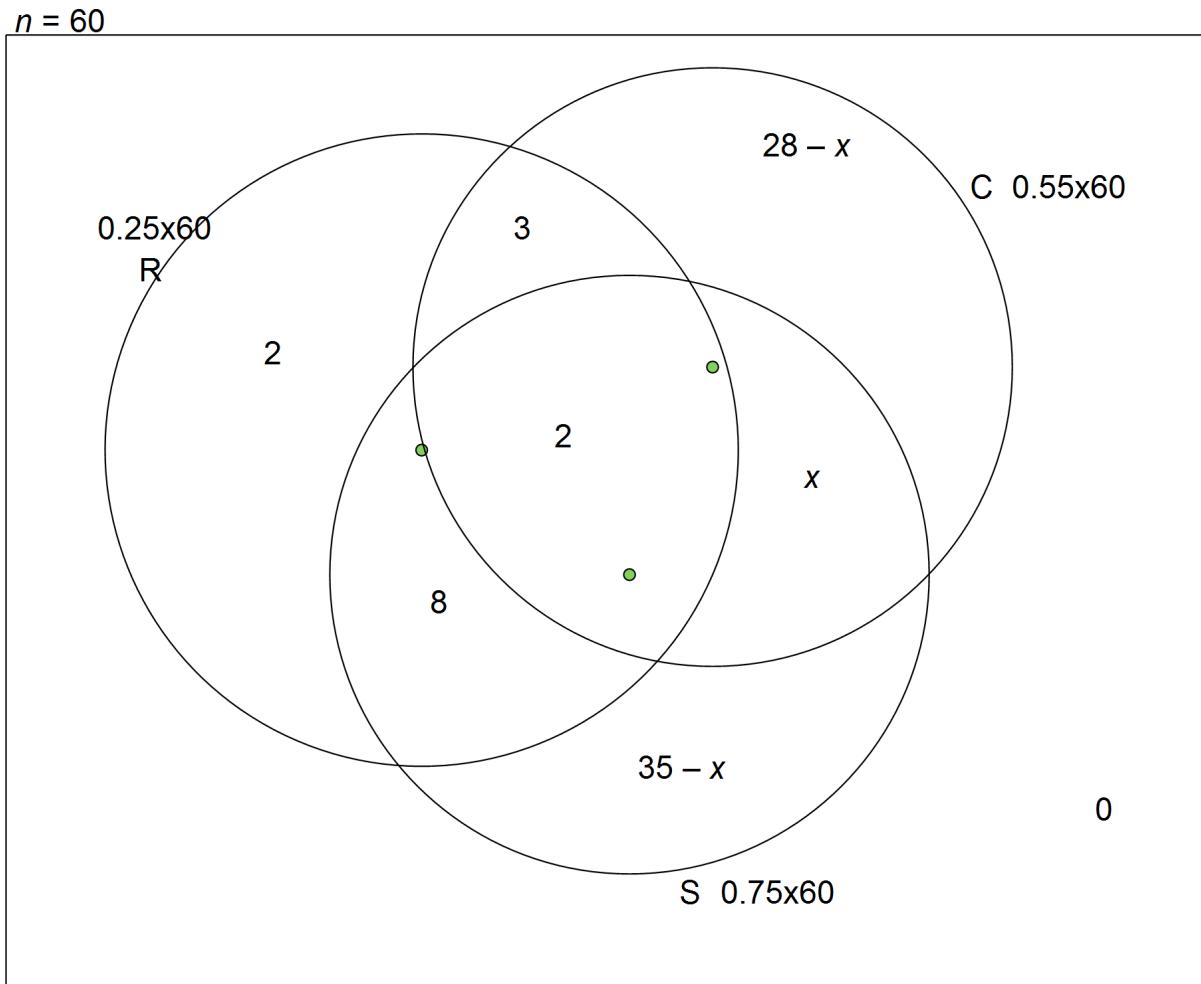
ii. $\Pr(R | S) = \frac{\Pr(R \cap S)}{\Pr(S)}$

$$\Pr(R | S) = \frac{0.1875}{0.75} = 0.25 \quad \text{or } \frac{1}{4} \quad \mathbf{1A}$$

b. i. $\Pr(R \cap S \cap C') = 0.25 \times 0.75 \times 0.45 \quad \mathbf{1M}$

$$\Pr(R \cap S \cap C') = 0.084375 \quad \text{or } \frac{27}{320} \quad \mathbf{1A}$$

ii. Venn diagram.



1M

From Venn diagram we get $2 + 3 + 2 + 8 + 28 - x + x + 35 - x = 60$

$$x = 18$$

10 Staff members take C only **1A**

60 staff members

If a staff member takes Step there is a probability of 20% of them taking Step the next day.
If a staff member takes Cycle there is a probability 35% of them taking Step Class the next day.

No R classes are offered.

On Monday 15 take Cycle Class.

$$T = \begin{bmatrix} 0.2 & 0.35 \\ 0.8 & 0.65 \end{bmatrix}$$

$$S_0 = \begin{bmatrix} 45 \\ 15 \end{bmatrix}$$

c. i. Cycle class on Tuesday using $S_1 = T^1 S_0$

$$\begin{bmatrix} 0.2 & 0.35 \\ 0.8 & 0.65 \end{bmatrix} \hat{\ }^1 \begin{bmatrix} 45 \\ 15 \end{bmatrix} = \begin{bmatrix} 14.25 \\ 45.75 \end{bmatrix}$$

$$\begin{bmatrix} 0.2 & 0.35 \\ 0.8 & 0.65 \end{bmatrix} \begin{bmatrix} 45 \\ 15 \end{bmatrix} = \begin{bmatrix} 14.25 \\ 45.75 \end{bmatrix} \quad \textbf{1M}$$

46 staff take Cycle class on Tuesday **1A**

ii. Step class on Friday using $S_4 = T^4 S_0$

$$\begin{bmatrix} 0.2 & 0.35 \\ 0.8 & 0.65 \end{bmatrix} \hat{\ }^4 \begin{bmatrix} 45 \\ 15 \end{bmatrix} = \begin{bmatrix} 18.27440625 \\ 41.72559375 \end{bmatrix}$$

18 staff take Step class on Friday **1A**

iii. Cycle and Step using $S_\infty = T^\infty S_0$

$$\begin{bmatrix} 0.2 & 0.35 \\ 0.8 & 0.65 \end{bmatrix} \hat{\ }^{10} \begin{bmatrix} 45 \\ 15 \end{bmatrix} = \begin{bmatrix} 18.26086972 \\ 41.73913028 \end{bmatrix}$$

18 staff take Step class and 42 take Cycle class long term **2A**

10 Mathematics staff members. Independent events

$$\Pr(C) = 0.2$$

$$\Pr(S) = 0.6$$

d. $\Pr(S \cap S \cap C \cap C \cap C) = 0.6^2 \times 0.2^3$

$$\Pr(S \cap S \cap C \cap C \cap C) = 0.00288 \quad \text{or} \quad \frac{9}{3125} \quad \textbf{1A}$$

e. $\text{Bi}(n, p) = \text{Bi}(10, 0.6)$ 1M

<code>binomialCDf(7, 10, 10, 0.6)</code>
0.3822806016

$\Pr(X > 6) = \Pr(X \geq 7) = 0.3823$ 1A

f. Now $\Pr(C) = 0.3$

$\text{Bi}(n, p) = \text{Bi}(n, 0.3)$

$\Pr(X \geq 2) > 0.65$

Let $\Pr(X \geq 2) = 0.65$

$1 - [\Pr(X = 0) + \Pr(X = 1)] = 0.65$

$\therefore \Pr(X = 0) + \Pr(X = 1) = 0.35$ 1M

${}^nC_0 q^n p^0 + {}^nC_1 q^{n-1} p^1 = 0.35$

Where $q = 0.7, p = 0.3$.

${}^nC_0 0.7^n 0.3^0 + {}^nC_1 0.7^{n-1} 0.3^1 = 0.35$

Giving

$0.7^n + n \times 0.7^{n-1} \times 0.3^1 = 0.35$. Solve for n .

We need 7 Science staff members.

1A

Question 4 (15 marks)

a. $A = -1$

Expand $-(x-B)^4 + C = -x^4 + 4Bx^3 - 6B^2x^2 + 4B^3x - B^4 + C$ 1M

Equate coefficients

$$4B = 8, B = 2$$

$$-B^4 + C = -15, -16 + C = -15, C = 1$$

$$f(x) = -(x-2)^4 + 1 \quad 1A$$

expand $(-(x-b)^4+c)$
 $-x^4+4 \cdot b \cdot x^3-6 \cdot b^2 \cdot x^2+4 \cdot b^3 \cdot x-b^4+c$

1/99

b. $f(1) = -(1-2)^4 + 1 = 0$ 1M

Could use the turning point $(2, 1)$

Solve $f(x) = -(x-2)^4 + 1 = 0$ for $x, x = 3, b = 3$ 1A

solve $(-(x-2)^4+1=0,x)$
 $x=1 \text{ or } x=3$

1/99

c. i. $[1, 3]$ 1A

ii $(1, 3)$ 1A

d. $c = 2$

1Ae. g is not differentiable at $x = 3$ and therefore is not differentiable for all values of x . **1A**

f. i. $k = \frac{1}{2} \int_1^3 (f(x)) dx$ **1M**
 $= \frac{4}{5}$ **1A**

The calculator screen shows the integral $\frac{1}{2} \cdot \int_1^3 ((x-2)^4 + 1) dx$ being evaluated. The result is displayed as $\frac{4}{5}$.

ii. $\int_m^n (f(x) - k) dx = 2 \int_1^m (k - f(x)) dx$

Solve $f(x) = \frac{4}{5}$ for x **1M**

 $m = 1.3313$ and $n = 2.6687$ correct to four decimal places**1A**

The calculator screen shows the integral $\frac{1}{2} \cdot \int_1^3 ((x-2)^4 + 1) dx$ and the solve command $\text{solve}\left((x-2)^4 + 1 = \frac{4}{5}, x\right)$. The result is $x = 1.33126$ or $x = 2.66874$.

g. $f_1 : [1, 3] \rightarrow R, f_1(x) = -x^4 + 8x^3 + ax^2 + bx + d$

Find a and b in terms of d .

Solve $f_1(1) = 0$ and $f_1(3) = 0$ for a and b

$$a = \frac{d-57}{3} \text{ and } b = -\frac{4(d-9)}{3} \quad 1\mathbf{M}$$

$$\text{Solve } \frac{d}{dx} \left(-x^4 + 8x^3 + \frac{d-57}{3}x^2 - \frac{4(d-9)}{3}x + d \right) = 0 \text{ for } x \quad 1\mathbf{M}$$

$$x = 2 \text{ or } x = \frac{12 \pm \sqrt{6d+90}}{6}$$

For three solutions $6d+90 > 0$, $d > -15$ and $\frac{\sqrt{6d+90}}{6} < 1$, $d < -9$ as $1 < x < 3$

$$-15 < d < -9$$

1A

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Define f(x)=-x^4+8·x^3+a·x^2+b·x+d      Done
solve(f(1)=0 and f(3)=0,a,b)
a=(d-57)/3 and b=-4·(d-9)/3
solve(d/dx(-x^4+8·x^3+(d-57)/3·x^2+-4·(d-9)/3·x)=0,x)
x: (d+15)/12 or x=(-(sqrt(6·(d+15))-12)/6 or x=2
1/8

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