



# **Victorian Certificate of Education 2022**

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

## STUDENT NUMBER

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Letter

# MATHEMATICAL METHODS

## Written examination 1

Wednesday 2 November 2022

**Reading time: 9.00 am to 9.15 am (15 minutes)**

**Writing time: 9.15 am to 10.15 am (1 hour)**

# **QUESTION AND ANSWER BOOK**

## Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
8	8	40



### **Materials supplied**

- Question and answer book of 13 pages
  - Formula sheet
  - Working space is provided throughout the book.

## Instructions

- Write your **student number** in the space provided above on this page.
  - Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
  - All written responses must be in English.

### **At the end of the examination**

- You may keep the formula sheet.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

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**DO NOT WRITE IN THIS AREA**

**Instructions**

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Question 1 (3 marks)**

- a. Let  $y = 3xe^{2x}$ .

Find  $\frac{dy}{dx}$ .

1 mark

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- b. Find and simplify the rule of  $f'(x)$ , where  $f : R \rightarrow R$ ,  $f(x) = \frac{\cos(x)}{e^x}$ .

2 marks

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**Question 2 (4 marks)**

- a. Let  $g : \left(\frac{3}{2}, \infty\right) \rightarrow R$ ,  $g(x) = \frac{3}{2x-3}$ .

Find the rule for an antiderivative of  $g(x)$ .

1 mark

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- b. Evaluate  $\int_0^1 (f(x)(2f(x)-3))dx$ , where  $\int_0^1 [f(x)]^2 dx = \frac{1}{5}$  and  $\int_0^1 f(x) dx = \frac{1}{3}$ .

3 marks

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**Question 3** (3 marks)

Consider the system of equations

$$\begin{aligned} kx - 5y &= 4 + k \\ 3x + (k + 8)y &= -1 \end{aligned}$$

Determine the value of  $k$  for which the system of equations above has an infinite number of solutions.

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**Question 4 (5 marks)**

A card is drawn from a deck of red and blue cards. After verifying the colour, the card is replaced in the deck. This is performed four times.

Each card has a probability of  $\frac{1}{2}$  of being red and a probability of  $\frac{1}{2}$  of being blue.

The colour of any drawn card is independent of the colour of any other drawn card.

Let  $X$  be a random variable describing the number of blue cards drawn from the deck, in any order.

- a. Complete the table below by giving the probability of each outcome. 2 marks

$x$	0	1	2	3	4
$\Pr(X=x)$	$\frac{1}{16}$		$\frac{6}{16}$		

- b. Given that the first card drawn is blue, find the probability that exactly two of the next three cards drawn will be red.

1 mark

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- c. The deck is changed so that the probability of a card being red is  $\frac{2}{3}$  and the probability of a card being blue is  $\frac{1}{3}$ .

Given that the first card drawn is blue, find the probability that exactly two of the next three cards drawn will be red.

2 marks

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**Question 5** (5 marks)

- a. Solve  $10^{3x-13} = 100$  for  $x$ .

2 marks

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- b. Find the maximal domain of  $f$ , where  $f(x) = \log_e(x^2 - 2x - 3)$ .

3 marks

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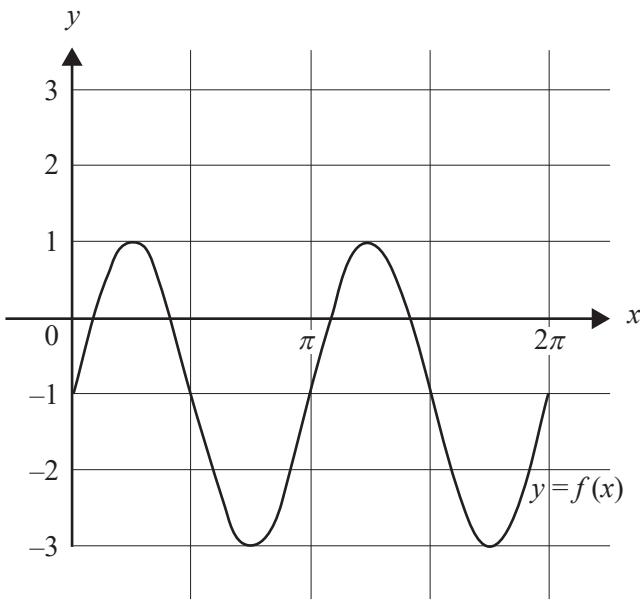
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**Question 6** (8 marks)

The graph of  $y = f(x)$ , where  $f: [0, 2\pi] \rightarrow R$ ,  $f(x) = 2 \sin(2x) - 1$ , is shown below.



- a. On the axes above, draw the graph of  $y = g(x)$ , where  $g(x)$  is the reflection of  $f(x)$  in the horizontal axis. 2 marks
- b. Find all values of  $k$  such that  $f(k) = 0$  and  $k \in [0, 2\pi]$ . 3 marks

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**Question 6 – continued**

- c. Let  $h : D \rightarrow R$ ,  $h(x) = 2 \sin(2x) - 1$ , where  $h(x)$  has the same rule as  $f(x)$  with a different domain. The graph of  $y = h(x)$  is translated  $a$  units in the positive horizontal direction and  $b$  units in the positive vertical direction so that it is mapped onto the graph of  $y = g(x)$ , where  $a, b \in (0, \infty)$ .

- i. Find the value for  $b$ . 1 mark

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- ii. Find the smallest positive value for  $a$ . 1 mark

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- iii. Hence, or otherwise, state the domain,  $D$ , of  $h(x)$ . 1 mark

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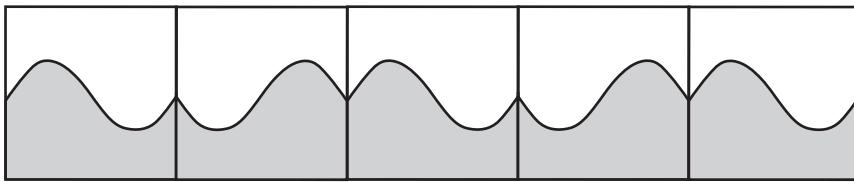
**Question 7 (7 marks)**

A tilemaker wants to make square tiles of size  $20 \text{ cm} \times 20 \text{ cm}$ .

The front surface of the tiles is to be painted with two different colours that meet the following conditions:

- Condition 1 – Each colour covers half the front surface of a tile.
- Condition 2 – The tiles can be lined up in a single horizontal row so that the colours form a continuous pattern.

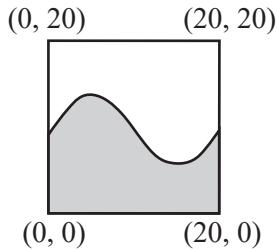
An example is shown below.



There are two types of tiles: Type A and Type B.

For Type A, the colours on the tiles are divided using the rule  $f(x) = 4 \sin\left(\frac{\pi x}{10}\right) + a$ , where  $a \in R$ .

The corners of each tile have the coordinates  $(0, 0)$ ,  $(20, 0)$ ,  $(20, 20)$  and  $(0, 20)$ , as shown below.



- a. i. Find the area of the front surface of each tile.

1 mark

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- ii. Find the value of  $a$  so that a Type A tile meets Condition 1.

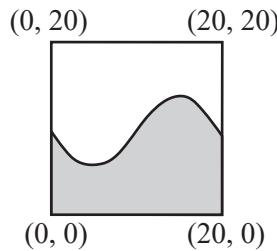
1 mark

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Type B tiles, an example of which is shown below, are divided using the rule  $g(x) = -\frac{1}{100}x^3 + \frac{3}{10}x^2 - 2x + 10$ .



- b. Show that a Type B tile meets Condition 1.

3 marks

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- c. Determine the endpoints of  $f(x)$  and  $g(x)$  on each tile. Hence, use these values to confirm that Type A and Type B tiles can be placed in any order to produce a continuous pattern in order to meet Condition 2.

2 marks

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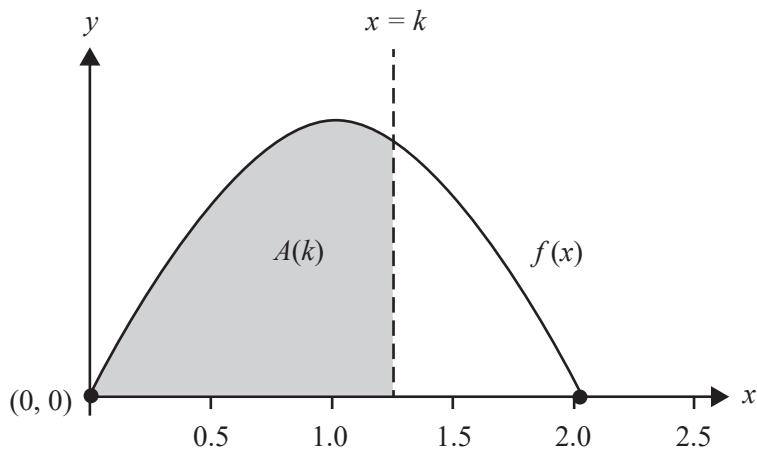
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**Question 8 (5 marks)**

Part of the graph of  $y = f(x)$  is shown below. The rule  $A(k) = k \sin(k)$  gives the area bounded by the graph of  $f$ , the horizontal axis and the line  $x = k$ .



- a. State the value of  $A\left(\frac{\pi}{3}\right)$ . 1 mark

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- b. Evaluate  $f\left(\frac{\pi}{3}\right)$ . 2 marks

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**Question 8 – continued**

- c. Consider the average value of the function  $f$  over the interval  $x \in [0, k]$ , where  $k \in [0, 2]$ .

Find the value of  $k$  that results in the maximum average value.

2 marks

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**Victorian Certificate of Education  
2022**

# **MATHEMATICAL METHODS**

## **Written examination 1**

### **FORMULA SHEET**

#### **Instructions**

This formula sheet is provided for your reference.  
A question and answer book is provided with this formula sheet.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

## Mathematical Methods formulas

### Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc \sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

### Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$
$\frac{d}{dx}((ax+b)^n) = an(ax+b)^{n-1}$	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c, x > 0$
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$	
product rule	$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

## Probability

$\Pr(A) = 1 - \Pr(A')$	$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$		
$\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = E(X)$	variance	$\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$

## Sample proportions

$\hat{P} = \frac{X}{n}$	mean	$E(\hat{P}) = p$
standard deviation	$\text{sd}(\hat{P}) = \sqrt{\frac{p(1-p)}{n}}$	approximate confidence interval $\left( \hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$