

The Mathematical Association of Victoria

Trial Examination 2024

MATHEMATICAL METHODS

Trial Written Examination 2 - SOLUTIONS

SECTION A: Multiple Choice

Question	Answer	Question	Answer
1	C	11	A
2	C	12	D
3	A	13	C
4	B	14	A
5	B	15	B
6	D	16	D
7	B	17	A
8	D	18	C
9	C	19	B
10	D	20	A

Question 1 Answer C

$$f(x) = -\frac{3}{2} \sin(2x - \pi)$$

$$\text{Amplitude: } A = \frac{3}{2}$$

$$\text{Period: } P = \frac{2\pi}{2} = \pi$$

Question 2 Answer C

$$f(x) = \sqrt{x+2} \text{ and } g(x) = e^{2x}$$

Test range of $f \subseteq$ domain of g

$$[0, \infty) \subset R$$

domain of $g \circ f =$ domain of $f = [-2, \infty)$

Question 3 Answer A

$$0 = ax^2 + 4x + c$$

two unique solutions if $\Delta > 0$

$$4^2 - 4ac > 0$$

$$4ac < 16$$

$$ac < 4$$

Question 4**Answer B**

$$x + (m-1)y = 2 \Rightarrow y = \left(\frac{-1}{m-1} \right)x + \frac{2}{m-1}$$

$$(m+1)x + 3y = 8 - m \Rightarrow y = -\left(\frac{m+1}{3} \right)x + \frac{8-m}{3}$$

Equate gradients

$$-\frac{1}{m-1} = -\frac{m+1}{3}$$

Gives $m = \pm 2$

Test for infinite number of solutions

$$\begin{aligned} m = 2 & \quad x + y = 2 \\ & \quad 3x + 3y = 6 \\ m = -2 & \quad x - 3y = 2 \\ & \quad -x + 3y = 10 \end{aligned}$$

Answer: $m = 2$

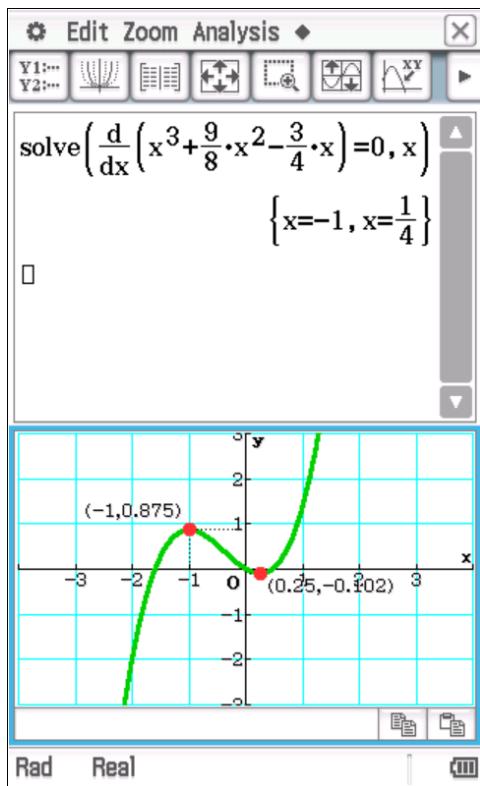
Question 5**Answer B**

$$g(x) = x^3 + \frac{9}{8}x^2 - \frac{3}{4}x$$

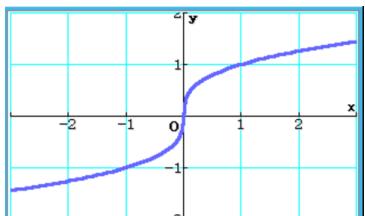
$$g'(x) = 3x^2 + \frac{9}{4}x - \frac{3}{4} = 0$$

$$\text{Gives } x = -1, x = \frac{1}{4}$$

strictly decreasing for $\left[-1, \frac{1}{4} \right]$

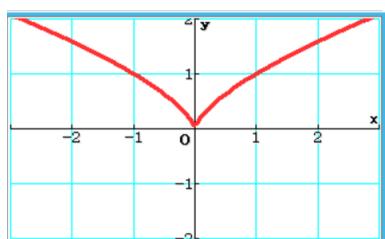
**Question 6****Answer D**

Option A $y = x^{\frac{1}{3}}$, $\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}} \neq 0$



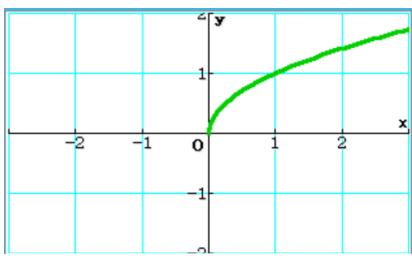
Gradient undefined at $x = 0$

Option B $y = x^{\frac{2}{3}}$, $\frac{dy}{dx} = \frac{2}{3}x^{-\frac{1}{3}} \neq 0$



Sharp point at $x = 0$

Option C $y = x^{\frac{1}{2}}$, $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} \neq 0$

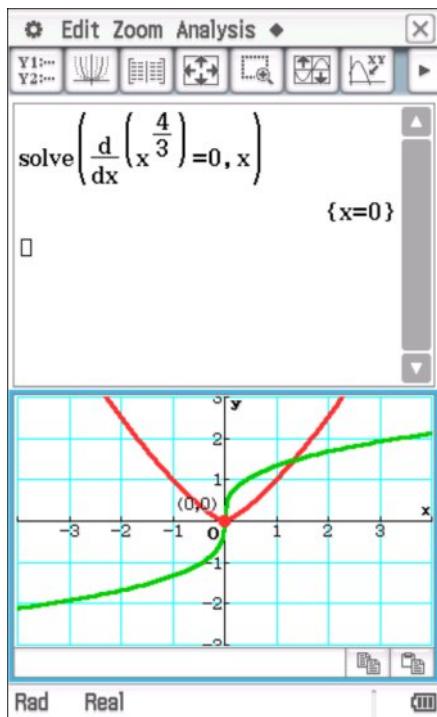


Endpoint at $(0,0)$

Option D $y = x^{\frac{4}{3}}$, $\frac{dy}{dx} = \frac{4}{3}x^{\frac{1}{3}} = 0$ for $x = 0$

Differentiable for all values over its maximal domain.

Gradient graph exists for all $x \in \mathbb{R}$.



Question 7

Answer B

Given $\int_1^3 f(x)dx = 4$ and $\int_3^1 g(x)dx = -2$.

$$\begin{aligned} &\text{Simplify } -\int_1^2 g(x)dx + \int_1^3 (2f(x) + 3)dx - \int_2^3 g(x)dx \\ &- \left(\int_1^2 g(x)dx + \int_2^3 g(x)dx \right) + 2 \int_1^3 f(x)dx + \int_1^3 (3)dx \\ &= -\int_1^3 g(x)dx + 2(4) + [3x]_1^3 \\ &= -2 + 8 + 6 \\ &= 12 \end{aligned}$$

Question 8 Answer D

Two balls of the same colour selected without replacement.

$$\text{Box A: } \left(\frac{1}{2} \times \frac{4}{7} \times \frac{3}{6} \right) + \left(\frac{1}{2} \times \frac{3}{7} \times \frac{2}{6} \right) = \frac{3}{14}$$

$$\text{Box B: } \left(\frac{1}{2} \times \frac{4}{7} \times \frac{3}{6} \right) + \left(\frac{1}{2} \times \frac{3}{7} \times \frac{2}{6} \right) = \frac{3}{14}$$

$$\text{Answer: } \frac{3}{14} + \frac{3}{14} = \frac{3}{7}$$

Question 9 Answer C

$$h: R \setminus \{1\} \rightarrow R, h(x) = \frac{1}{x-1} + 2.$$

$$\text{Average rate of change} = \frac{h(5) - h(2)}{5 - 2} = -\frac{1}{4}$$

$$h'(x) = -\frac{1}{4} \text{ at } x = -1 \text{ or } x = 3$$

```

Edit Action Interactive
define h(x)=1/(x-1)+2
done
h(5)-h(2)
5-2
-1/4
solve(d(h(x))/dx=-1/4,x)
{x=-1,x=3}

```

Question 10 Answer D

$$f(x) = \begin{cases} k \sin\left(\frac{1}{2}x\right) & 0 < x < \pi \\ 0 & \text{otherwise} \end{cases}$$

$$\int_0^\pi k \sin\left(\frac{1}{2}x\right) dx = 1 \text{ gives } k = \frac{1}{2}$$

$$\int_0^m \frac{1}{2} \sin\left(\frac{1}{2}x\right) dx = 0.5 \text{ gives } m = \frac{2\pi}{3}$$

Question 11 **Answer A**

$$f: R \setminus \{1\} \rightarrow R, f(x) = \frac{1}{(x-1)^2} - 2$$

The tangent line at $x = 0$ is $y = 2x - 1$.

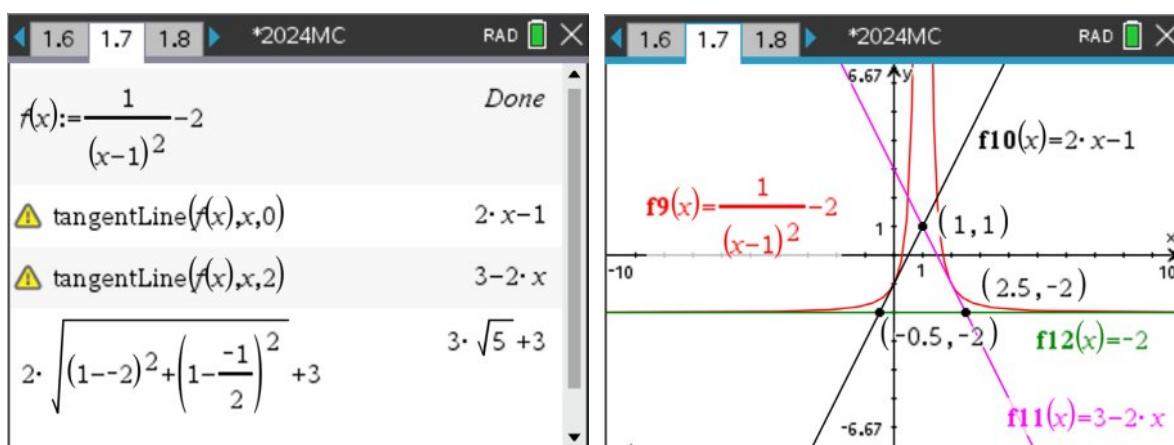
The tangent line at $x = 2$ is $y = 3 - 2x$.

The coordinates of the vertices of the triangle are $(1,1)$, $\left(-\frac{1}{2}, -2\right)$ and $\left(\frac{5}{2}, -2\right)$.

The length of the base is 3.

The length of each of the other two sides is $\sqrt{\left(1 + \frac{1}{2}\right)^2 + (1+2)^2} = \frac{3\sqrt{5}}{2}$.

$$\text{Perimeter} = 3\sqrt{5} + 3$$



Question 12

$$y = g(x) = 4 \log_2(3x + 5), h = 1$$

$$\text{Area of the trapeziums} = \frac{1}{2}(g(0) + 2g(1) + 2g(2) + g(3))$$

$$= 2(\log_2(5) + \log_2(14) + \log_2(64) + \log_2(121))$$

$$= 2(\log_2(5 \times 14 \times 121) + \log_2(64))$$

$$= 2(\log_2(5 \times 14 \times 121) + 6)$$

$$= \log_2(5 \times 14 \times 121)^2 + 12 \\ \neq \log_2(5 \times 14 \times 121)^2 + 6^2$$

Question 13 Answer C

$$f(x) = 3 \tan\left(\frac{1}{2}\left(\frac{\pi}{3}x - 1\right)\right) + 5$$

$$\text{Solve } \frac{1}{2}\left(\frac{\pi}{3}x - 1\right) = \frac{\pi}{2}, \quad x = \frac{3(\pi+1)}{\pi}$$

OR

$$\frac{1}{2}\left(\frac{\pi}{3}x - 1\right) = -\frac{\pi}{2}, \quad x = \frac{-3(\pi-1)}{\pi}$$

$$\text{The period} = \frac{\pi}{\frac{\pi}{6}} = 6$$

$$\text{A general solution is } x = \frac{-3(\pi-1)}{\pi} + 6k, \quad k \in \mathbb{Z}$$

Question 14 Answer A

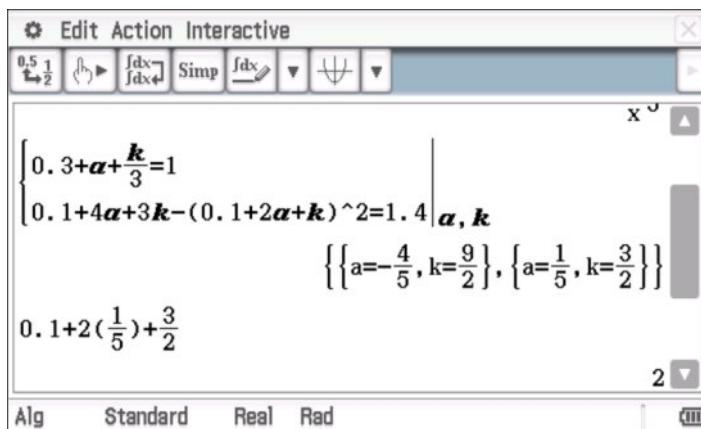
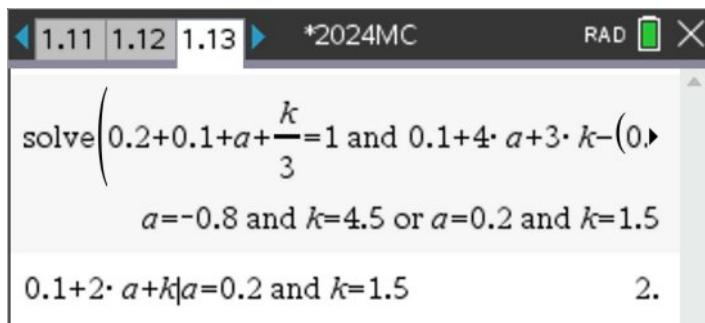
x	0	1	2	3
$\Pr(X = x)$	0.2	0.1	a	$\frac{k}{3}$

$$\text{Var}(X) = 0.1 + 4a + 3k - (0.1 + 2a + k)^2 = 1.4 \dots (1)$$

$$0.3 + a + \frac{k}{3} = 1 \dots (2)$$

$$a = 0.2, \quad k = 1.5$$

$$\text{E}(X) = 0.1 + 2a + k = 2$$

**Question 15 Answer B**

The domain of $s(x) = 1 - \log_e(1-x)$ is $(-\infty, -1)$.

The range $t(x) = 3 \cos(2x-1) + 1$ is $[-2, 4]$.

The domain of $s(x) + t^{-1}(x)$ is the intersection of $(-\infty, -1)$ and $[-2, 4]$ which is $[-2, 1)$.

Question 16 Answer D

$$f : R \setminus \left\{ \frac{a}{4} \right\} \rightarrow R, f(x) = \frac{2}{4x-a} + 3$$

$x_0 = \frac{a}{4}$ will fail as $x = \frac{a}{4}$ is an asymptote.

etwons method will also fail if the x -intercept of the tangent line at x_n is undefined.

ind the equation of the tangent line at any point on the curve. Let the x -coordinate be b .

$$y = \frac{3a^2 - 2a(12b+1) + 16b(3b+1)}{(a-4b)^2} - \frac{8x}{(a-4b)^2}$$

$$\text{Solve } y = \frac{3a^2 - 2a(12b+1) + 16b(3b+1)}{(a-4b)^2} - \frac{8x}{(a-4b)^2} = 0 \text{ when } x = \frac{a}{4}$$

$$b = \frac{3a-4}{12}, \text{ hence } x_0 = \frac{3a-4}{12} \text{ will fail.}$$

The x_0 values for any point on the S branch will fail as none of the x -intercepts of the

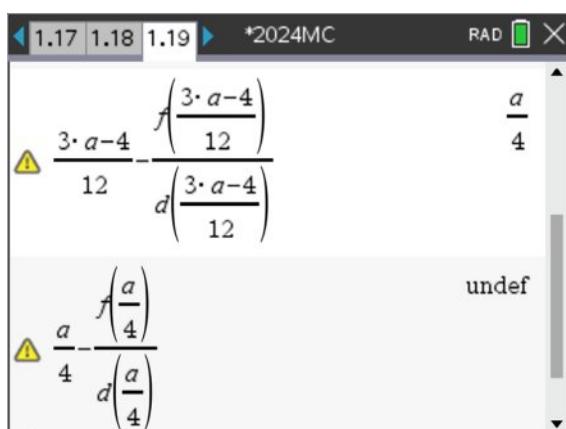
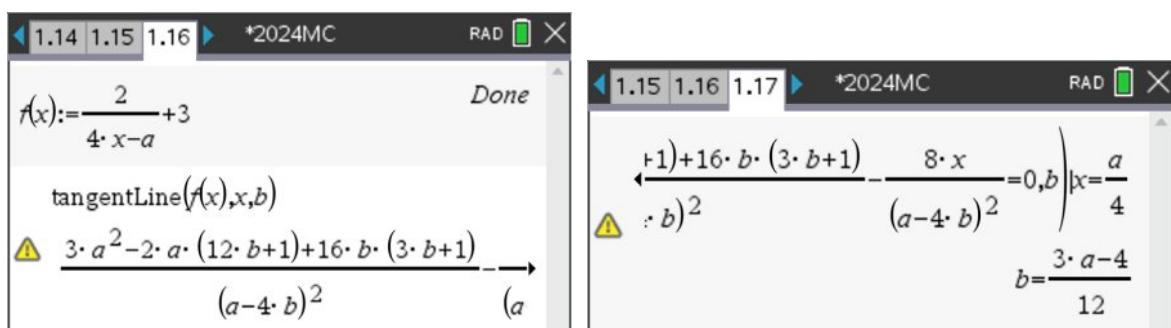
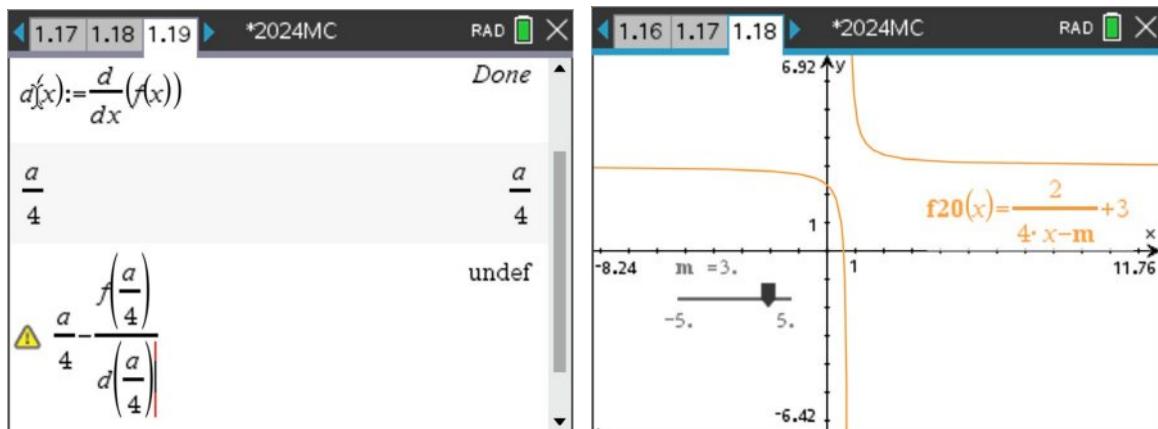
tangent lines are less than $\frac{a}{4}$.

$x_0 < \frac{3a-4}{12}$ will also fail as the x -intercept of the tangent lines are all greater than $\frac{a}{4}$.

So convergence will only occur if $\frac{3a-4}{12} < x_0 < \frac{a}{4}$.

ewtons method fails if $x_0 \in R \setminus \left(\frac{3a-4}{12}, \frac{a}{4} \right)$.

The answer can also be found by checking the values in the options to see if they fail when using ewtons method.



Question 17

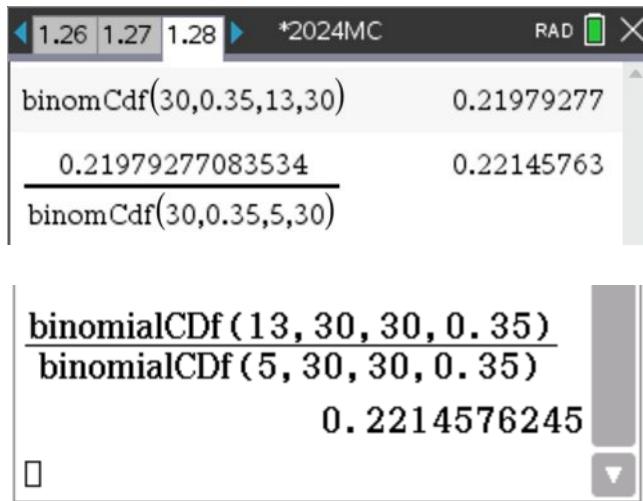
$$X \sim Bi(30, 0.35)$$

$$\Pr(X > 12 \mid X \geq 5)$$

$$\frac{\Pr(X \geq 13)}{\Pr(X \geq 5)}$$

Answer A

$$= \frac{0.2197...}{0.9925...} \\ = 0.2215 \text{ correct to four decimal places}$$

**Question 18 Answer C**

Let A_v be the average value of $f(x) = x^3 + x^2 - x + 1$ for the interval $[a, 1]$, where $a \in (-\infty, 1)$.

$$A_v = \frac{1}{1-a} \int_a^1 f(x) dx = \frac{3a^3 + 7a^2 + a + 13}{12}$$

A_v is a cubic function. $y = A_v$ will have 3 solutions between the two turning points.

$$\text{Solve } \frac{d}{da} \left(\frac{3a^3 + 7a^2 + a + 13}{12} \right) = 0 \text{ for } a.$$

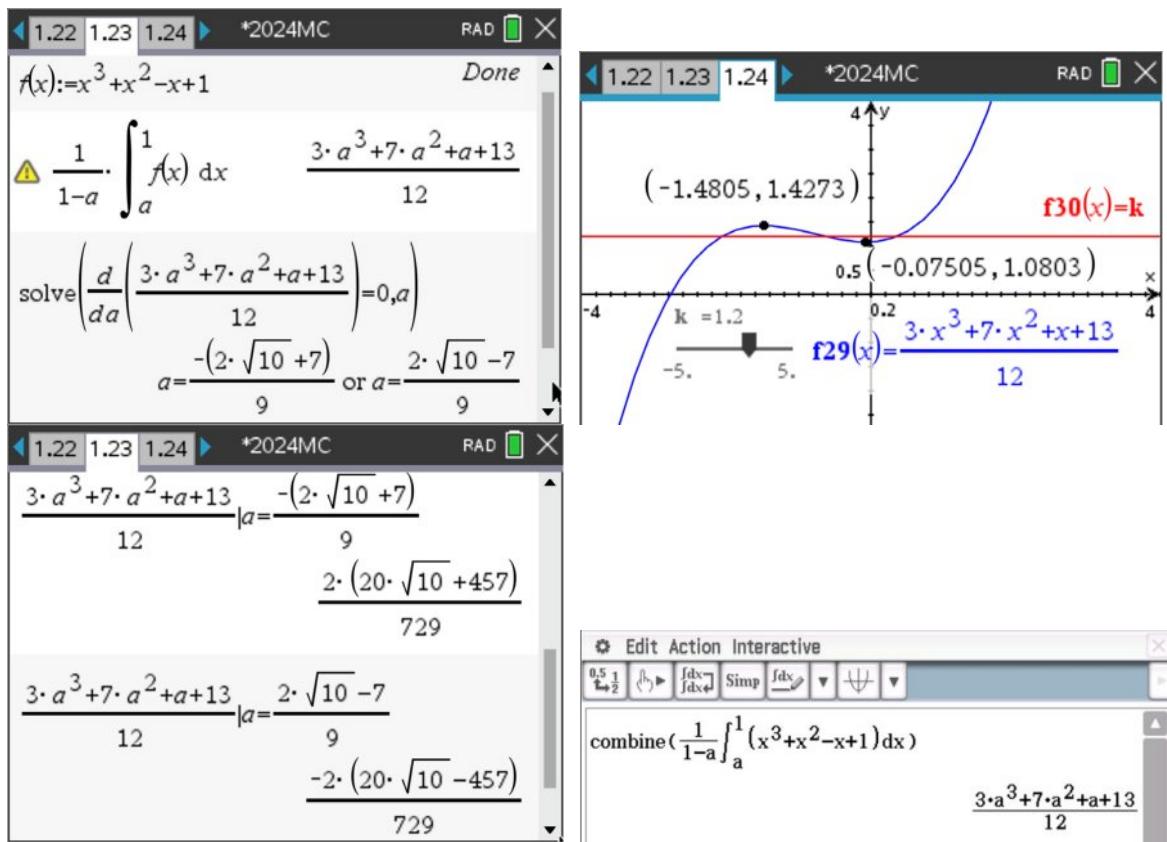
$$a = \frac{2\sqrt{10} - 7}{9}, \quad a = \frac{-2\sqrt{10} - 7}{9}$$

$$A_v \left(\frac{2\sqrt{10} - 7}{9} \right) = \frac{-2(20\sqrt{10} - 457)}{729}$$

$$A_v \left(\frac{-2\sqrt{10} - 7}{9} \right) = \frac{2(20\sqrt{10} + 457)}{729}$$

$$A_v \in \left(\frac{-2(20\sqrt{10} - 457)}{729}, \frac{2(20\sqrt{10} + 457)}{729} \right)$$

The answer can also be found by checking the values in the options to see if they give three a values.

**Question 19****Answer B**

$$f(x) = \frac{1}{\sqrt{18\pi}} e^{-\frac{1}{2}\left(\frac{2x-3}{6}\right)^2} = \frac{1}{3\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-3}{3}\right)^2}$$

$$X \sim N\left(\frac{3}{2}, 3^2\right)$$

- a dilation by a factor of 3 from the x-axis

$$f_1(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{2x-3}{6}\right)^2}$$

- a dilation by a factor of $\frac{1}{3}$ from the y-axis

$$f_2(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{6x-3}{6}\right)^2} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-1}{2}\right)^2}$$

- a translation of $\frac{1}{2}$ a unit left.

$$f_3(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

Check: $(x, y) \rightarrow (x, 3y) \rightarrow \left(\frac{x}{3}, 3y\right) \rightarrow \left(\frac{x}{3} - \frac{1}{2}, 3y\right)$

$$x' = \frac{x}{3} - \frac{1}{2}, x = 3x' + \frac{3}{2}$$

$$y' = 3y, y = \frac{y'}{3}$$

$$\frac{y'}{3} = \frac{1}{3\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{3x^3 + \frac{3}{2} - \frac{3}{2}}{3}\right)^2}$$

$$y' = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x')^2}$$

Question 20 **Answer A**

$$f : R \rightarrow R, f(x) = e^{x^3 + bx}$$

Solve $f''(x) = 0$ but does not work directly on the T or the CASO.

$$\text{So find } f''(x) = (9b^2x^4 + 6bx^2 + 6bx + 1)e^{x^3 + bx}.$$

There will be no points of inflection when $f''(x) \geq 0$ for all x .

$$\text{Solve } 9x^4 + 6bx^2 + 6x + b^2 = 0 \text{ and } \frac{d}{dx}(9x^4 + 6bx^2 + 6x + b^2) = 0$$

O

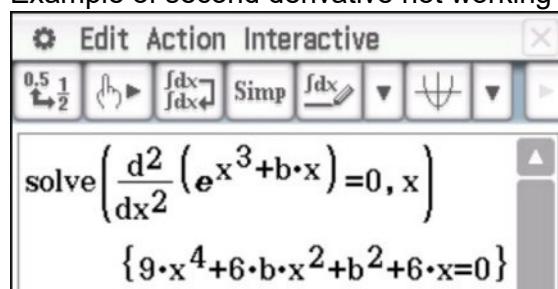
$$\text{Solve } (9b^2x^4 + 6bx^2 + 6bx + 1)e^{x^3 + bx} = 0 \text{ and } \frac{d}{dx}((9b^2x^4 + 6bx^2 + 6bx + 1)e^{x^3 + bx}) = 0$$

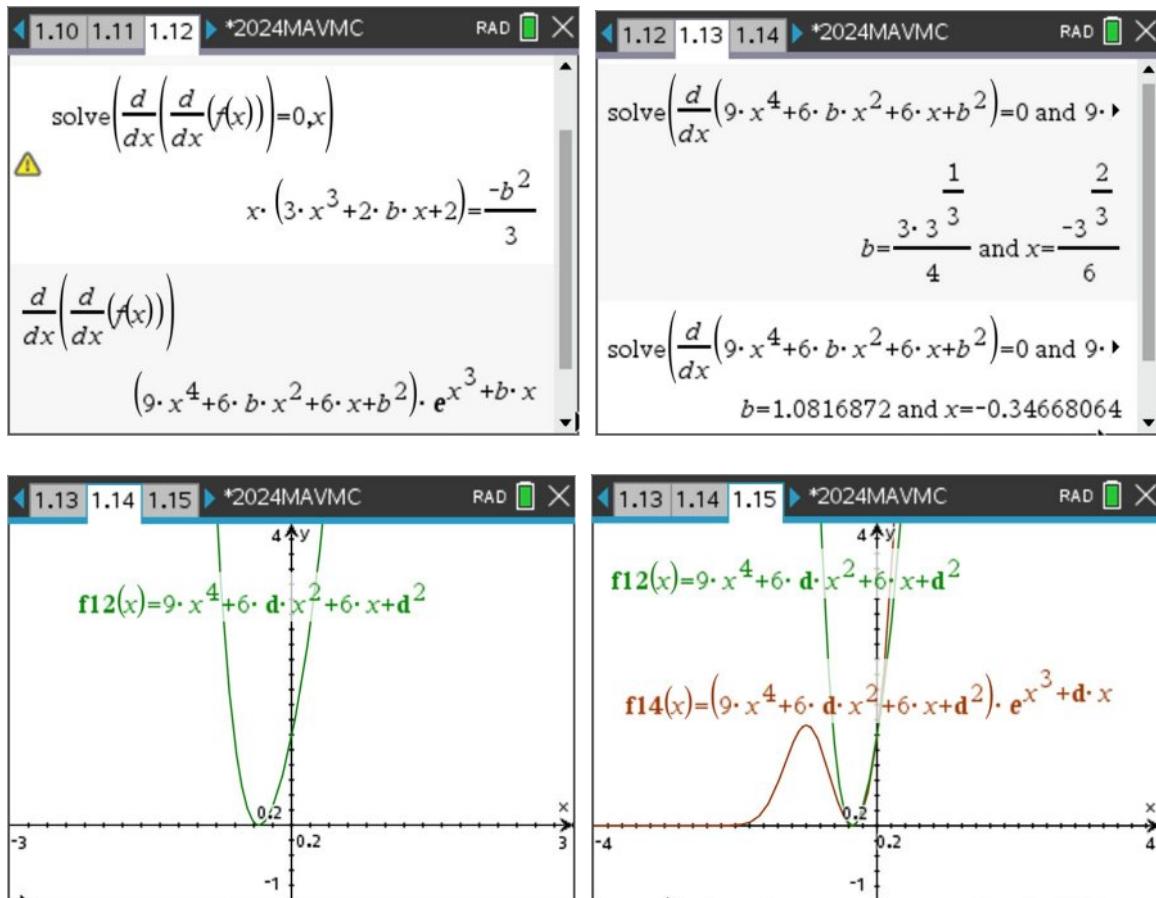
$$b = \frac{\frac{4}{3}}{4}$$

$$\text{There will be no points of inflection when } b \geq \frac{\frac{4}{3}}{4}.$$

The answer can also be found by checking the values in the options. The easiest way to do this is to graph the function and use a slider. Choose a value of b that gives two points of inflection and label them with their coordinates. Then use the slider to see when they disappear.

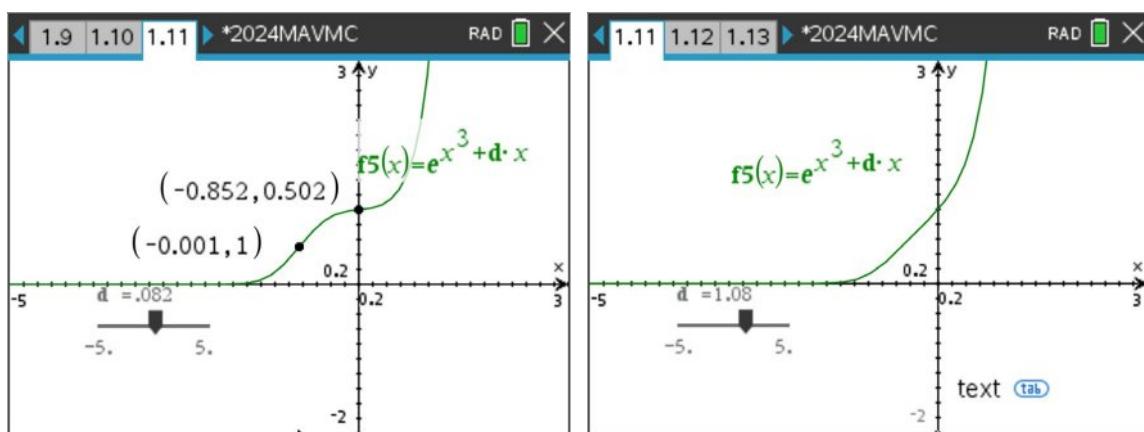
Example of second derivative not working on the CASO.



**Examples**

$$b < \frac{3^{\frac{4}{3}}}{4} \quad (\text{2 points of inflection})$$

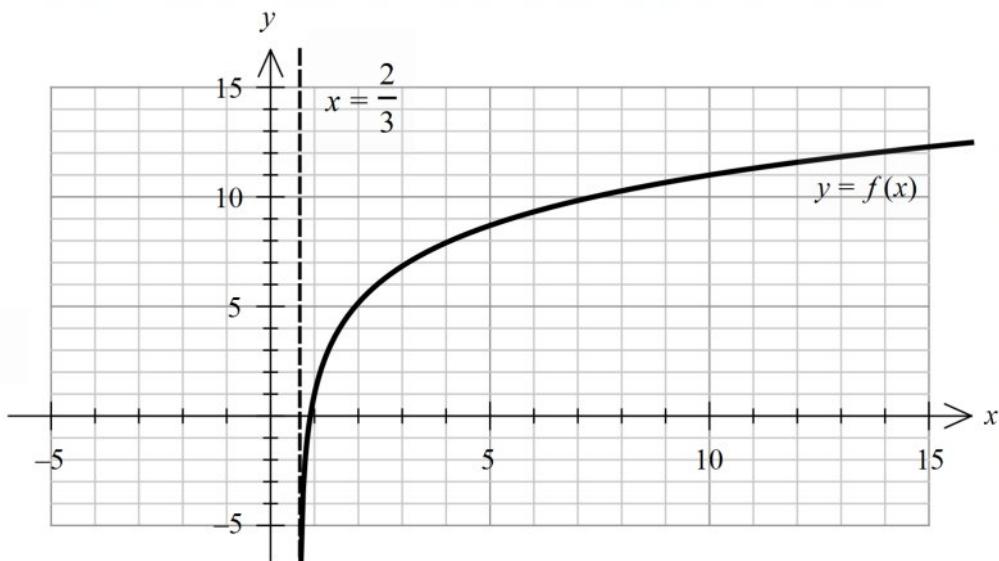
$$b = \frac{3^{\frac{4}{3}}}{4} \quad (\text{no points of inflection})$$

**END OF SECTION A SOLUTIONS**

SECTION B**Question 1**

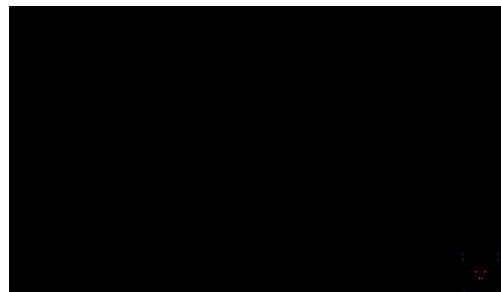
$$f: \left(\frac{2}{3}, \infty\right) \rightarrow R, f(x) = 3 \log_e(3x - 2) + 1$$

- a. Sketch and label asymptote $x = \frac{2}{3}$ 1A



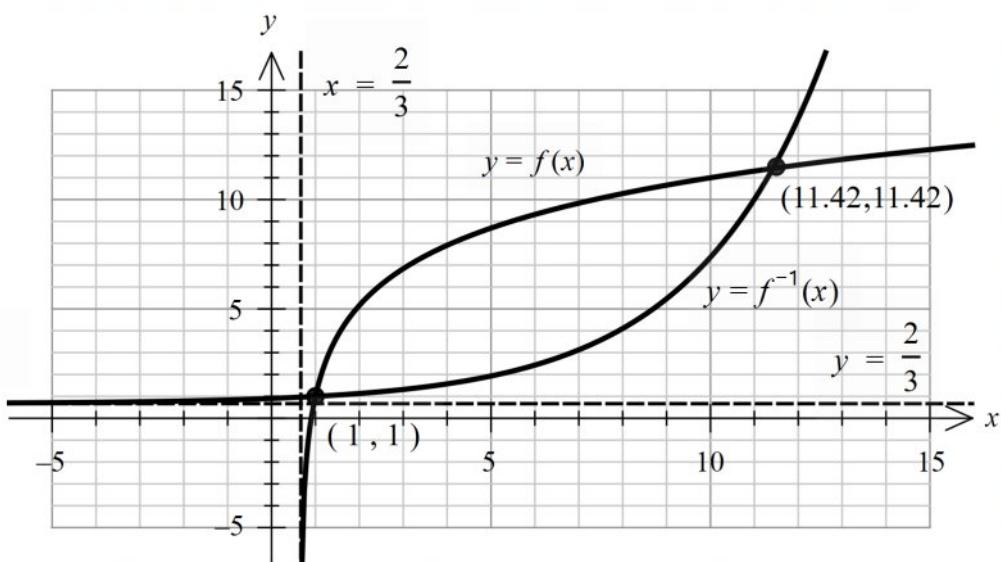
b. $f^{-1}(x) = \frac{1}{3} e^{\frac{x-1}{3}} + \frac{2}{3}$ 1A

Dom: $x \in R$ 1A



c. Sketch $y = f^{-1}(x)$. Shape and asymptote $y = \frac{2}{3}$ 1A

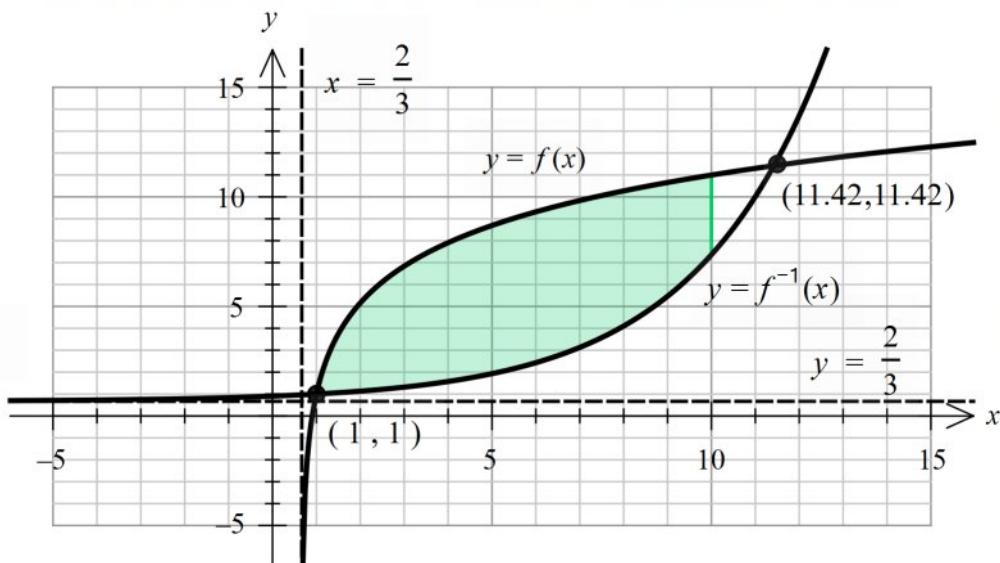
Points of intersection between $y = f(x)$ and $y = f^{-1}(x)$
 $(1, 1), (11.42, 11.42)$ 1A

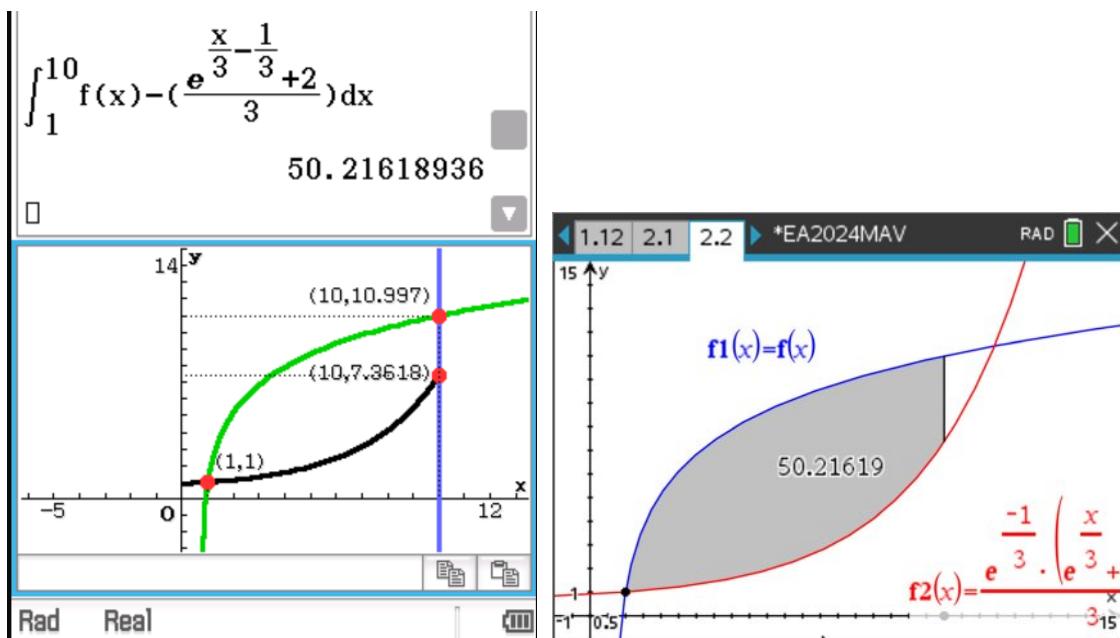


$$\left| \begin{array}{l} \text{solve} \left\{ f(x) = \frac{e^{\frac{x}{3}} - \frac{1}{3} + 2}{3}, x \right\} \\ \{x=1, x=11.42205843\} \end{array} \right|$$

d.i. $\int_1^{10} (f(x) - f^{-1}(x)) dx$ 1A

d.ii. Area = 50.22 sq units
Shading 1A





e.i. Average rate of change = $\frac{f(10) - f(1)}{10 - 1}$
 $= \frac{\log_e(28)}{3}$. n correct form $\frac{\log_e(a)}{b}$ **1A**

Edit Action Interactive

```
define f(x)=3ln(3x-2)+1
simplify(f(10)-f(1))
10-1
ln(28)
3
```

e.ii. Solve $f'(x) = \frac{\log_e(28)}{3}$ for x .
 $x = \frac{2}{3} + \frac{9}{\log_e(28)}$ **1A** (other forms)

The top two screenshots show the calculator's "Edit Action Interactive" window. The first shows the input and solution for the differential equation $\frac{d}{dx}(f(x)) = \frac{\ln(28)}{3}$, resulting in $x = \frac{\ln(784)+27}{3\cdot\ln(28)}$. The second shows the steps for simplifying this expression using the propFrac and simplify functions.

The bottom screenshot shows the "4.3" tab of the document *EA2024MAV. It displays the original differential equation and its solution $x = \frac{2\cdot\ln(28)+27}{3\cdot\ln(28)}$, along with the simplified form $\frac{9}{\ln(28)} + \frac{2}{3}$.

e.iii. The maximum value of the average rate of change will occur when the gradient of the line passing through $(a, f(a))$ and $(b, f(b))$ is steepest. This will occur when $a=1$ and $b=2$.

$$\text{Maximum average rate of change} = \frac{f(2) - f(1)}{2-1} = 6\log_e(2) \quad \mathbf{1M}$$

Solve $f'(x) = 6\log_e(2)$ for x .

$$x = \frac{2}{3} + \frac{1}{2\log_e(2)} \quad \mathbf{1A} \text{ (other forms)}$$

This screenshot shows the calculator's "Edit Action Interactive" window. It defines the function $f(x) = 3\ln(3x-2)+1$, calculates the average rate of change $\frac{f(2)-f(1)}{2-1} = 6\cdot\ln(2)$, and then solves the differential equation $\frac{d}{dx}(f(x)) = 6\cdot\ln(2)$ for x , resulting in $x = \frac{1}{2\cdot\ln(2)} + \frac{2}{3}$.

e.iv. f is continuous over the interval $[a,b]$ and smooth over the interval (a,b) but $f'(x) = \frac{9}{3x-2} \neq 0$ for any x . Hence, $f(a) \neq f(b)$. or the average value to equal zero, $f(a)$ must equal $f(b)$.

1A

$$\left| \begin{array}{l} \frac{d}{dx}(f(x)) \\ \frac{9}{3 \cdot x - 2} \\ \square \end{array} \right|$$

f.i. $h : \left(\frac{2}{b}, \infty\right) \rightarrow R$, $h(x) = a \log_e(bx-2) + 1$ where $h(x) = 3f(5x)-2$

given $f(x) = 3 \log_e(3x-2) + 1$

$$h(x) = 3f(5x) - 2$$

$$h(x) = 9 \log_e(15x-2) + 1$$

$$a = 9, b = 15$$

1A

$$\begin{aligned} &\text{define } h(x)=3f(5x)-2 && \text{done} \\ &\text{expand}(h(x)) && 9 \cdot \ln(15 \cdot x - 2) + 1 \\ &\square && \end{aligned}$$

f.ii. Solve $h'_1(x) = f'(x)$

$$x = \frac{3k-1}{3k}$$

As $k \rightarrow \infty$, $x \rightarrow 1$

As $k \rightarrow -\infty$, $x \rightarrow 1$

$$x = 1$$

1A

The calculator screen shows the following steps:

- Input: $h(x):=3 \cdot f(k \cdot x) - 2$
- Solve: $\text{solve}\left(\frac{d}{dx}(f(x))=\frac{d}{dx}(h(x)), x\right)$ resulting in $x = \frac{3 \cdot k - 1}{3 \cdot k}$
- Limit as $k \rightarrow \infty$: $\lim_{k \rightarrow \infty} \left(\frac{3 \cdot k - 1}{3 \cdot k} \right) = 1$
- Limit as $k \rightarrow -\infty$: $\lim_{k \rightarrow -\infty} \left(\frac{3 \cdot k - 1}{3 \cdot k} \right) = 1$

Question 2

$$h(t) = a \sin(b(t-18)) + c$$

a. max 50, min 10, amp = 20

translation vertically: $-20 + 30 = 10$

$$a = 20, c = 30$$

1M (explanation)

b. One cycle = 18 hours

$$\frac{2\pi}{b} = 18$$

$$2\pi = 18b$$

$$\text{Gives } b = \frac{\pi}{9}$$

1M (show that)

$$\begin{aligned} \text{c. } & \frac{1}{18} \int_0^{18} h_A dx - \frac{1}{18} \int_0^{18} h_B dx & \text{1M} \\ &= \frac{5(\pi+2)}{\pi} \text{ m} & \text{1A} \end{aligned}$$

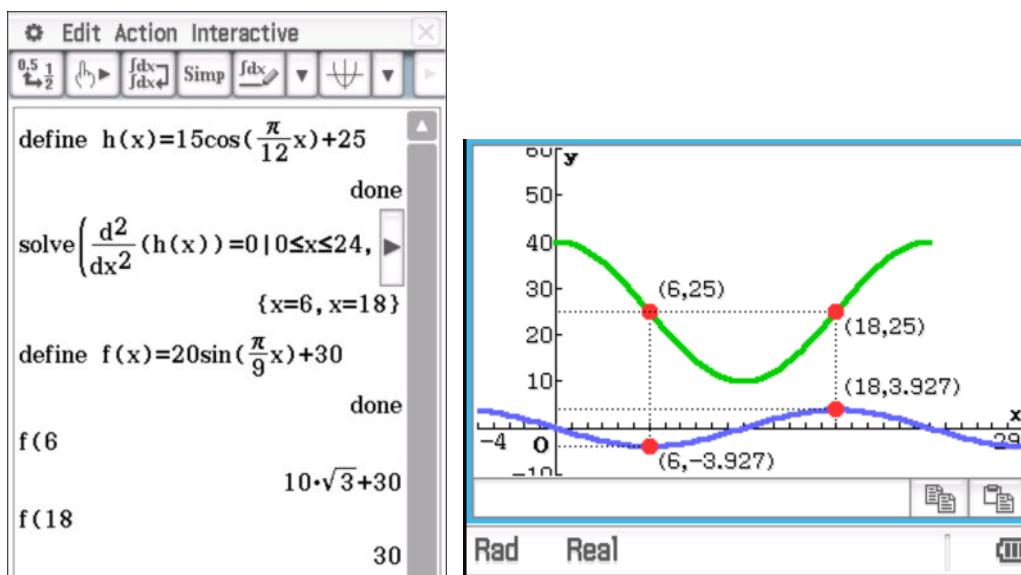
The calculator screen displays the following steps:

- Top menu: Edit Action Interactive
- Calculator view: $\frac{1}{18} \int_0^{18} f(x) dx$
- Result: 30
- Next step: $\text{combine}\left(\frac{1}{18} \int_0^{18} h(x) dx\right)$
- Result: $\frac{25\pi - 10}{\pi}$
- Next step: $\text{combine}\left(30 - \frac{25\pi - 10}{\pi}\right)$
- Final result: $\frac{5\pi + 10}{\pi}$

$$\text{d. } h_B(t) = 15 \cos\left(\frac{\pi t}{12}\right) + 25$$

The height of the river would be changing fastest at the points of inflection of the graphs of h_B . So when $t = 6$ and $t = 18$. **1M**

$$h_A(6) = 10\sqrt{3} + 30 \text{ and } h_A(18) = 30 \quad \text{1A}$$



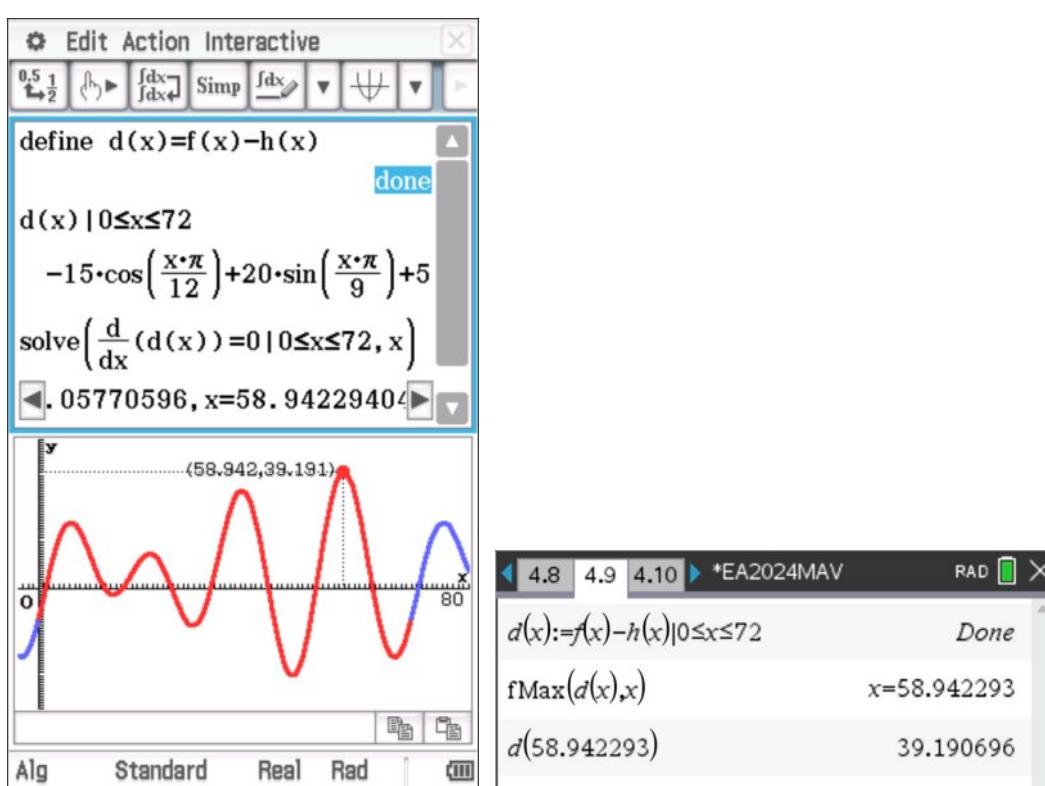
e. Let $d(x) = h_A(x) - h_B(x)$

The period of the graph of $d(x)$ is the lowest common multiple of 18 and 24 which is 72 hours.

1A

The maximum difference is 39.19 m.

1A



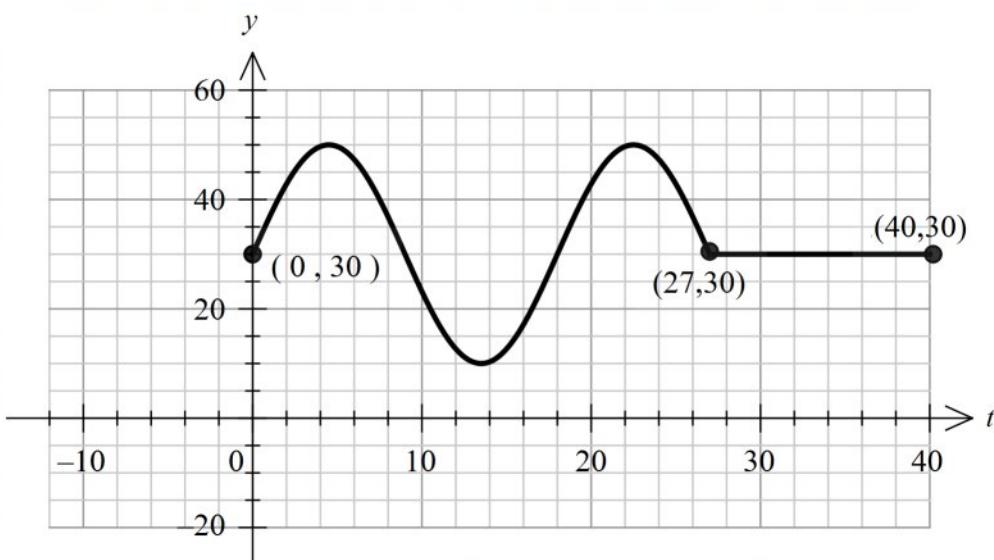
f. $w(t) = \begin{cases} h_A(t) & 0 \leq t \leq 27 \\ 30 & 27 < t \leq 40 \end{cases}$

Graph of piecewise function w

1A

Coordinates $(0, 30)$, $(40, 30)$, $(27, 30)$

1A



Graphing calculator interface:

- Top Bar:** Edit, Action, Interactive.
- Buttons:** 0.5, $\frac{1}{2}$, $\int_{\text{f}}^{\text{t}}$, $\int_{\text{f}}^{\text{d}}$, Simp, $\int_{\text{f}}^{\text{d}}$.
- Equation Input:** solve $\left(\frac{h(p)-h(0)}{p-0}=0.5, p\right)$
 $\{p=-25.06034947, p=-19.45\}$
- Equation Output:** $h(t) \mid 0 \leq t \leq 27$
 $20 \cdot \sin\left(\frac{(t-18) \cdot \pi}{9}\right) + 30$
- Equation Input:** $30 \mid 27 < t \leq 40$
- Equation Output:** 30
- Graph View:** Shows the function $h(t)$ plotted on a coordinate plane with x-axis from 0 to 43 and y-axis from 1 to 31. Points $(0, 30)$, $(27, 30)$, and $(40, 30)$ are marked with red dots and connected by a green line segment.
- Bottom Buttons:** Alg, Standard, Real, Rad, \int .

g. $t \in (0, 27) \cup (27, 40)$ **1A**

h. $p(t) = \begin{cases} w(t) & 0 \leq t \leq 40 \\ m \cos(n(t-r)) + s & 40 < t \leq k \end{cases}$

am Sunday to pm Tuesday is 0 hours.
 $k = 60$ **1A**

- i. Continuous and smooth at $t = 40$. So there is a turning point at $t = 40$.

$p = m \cos(n(t-r)) + s$ completes two cycles before recording capacities break, reaching zero height twice. So the range is $[0,30]$.

Amplitude = 15, $m = 15$

$$\text{Period} = 10, n = \frac{2\pi}{10} = \frac{\pi}{5} \quad \mathbf{1H}$$

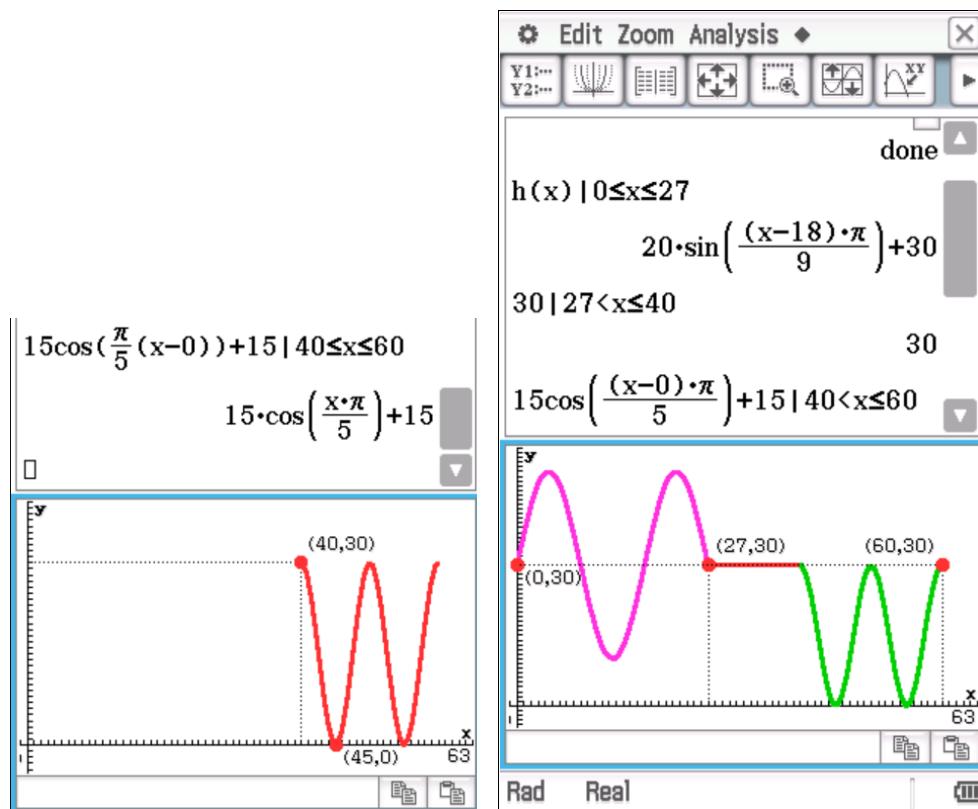
$$m = 15, s = 15. \quad \mathbf{1A}$$

$$r = 10q \text{ where } q \in \mathbb{Z} \quad \mathbf{1A}$$

OR

$$m = -15, s = 15. \quad \mathbf{1A}$$

$$r = 5q \text{ where } q \in \mathbb{Z} \quad \mathbf{1A}$$



Question 3

a. $X_{AF} \sim N(\mu, \sigma^2)$

Solve $\frac{5079 - \mu}{\sigma} = -0.841\dots$ and $\frac{6141 - \mu}{\sigma} = 1.281\dots \quad \mathbf{1M}$

$$\mu = 5500.0 \text{ kg and } \sigma = 500.2 \text{ kg} \quad \mathbf{1A}$$

The calculator screen shows the following steps:

- invNorm(0.2,0,1) = -0.84162123
- invNorm(0.9,0,1) = 1.28155156
- solve($\frac{5079-a}{b} = -0.84162123$ and $\frac{6141-a}{b} = 1.28155156$)
- $a = 5499.9746$ and $b = 500.1948$

b. $X_A \sim N(4085, 445^2)$, $X_{AB} \sim N(5375, 225^2)$

$$\Pr(X_A > 5079) = 0.0127\dots, \quad \Pr(X_{AB} > 5079) = 0.9058\dots, \quad \Pr(X_{AF} > 5079) = 0.8 \quad \mathbf{1M}$$

$$\Pr(X_A > 5079 | (X_A > 5079 + X_{AB} > 5079 + X_{AF} > 5079))$$

$$\begin{aligned} &= \frac{\frac{1}{3} \times 0.0127\dots}{\frac{1}{3} \times 0.0127\dots + \frac{1}{3} \times 0.9058\dots + \frac{1}{3} \times 0.8} \\ &= \frac{0.0127\dots}{0.0127\dots + 0.9058\dots + 0.8} \\ &= 0.0074 \end{aligned}$$

1A

The calculator screen shows the following steps:

$$\frac{\text{normCdf}(5079, \infty, 4085, 445) + \text{normCdf}(5079, \infty, 5375, 225)}{0.8 + \text{normCdf}(5079, \infty, 4085, 445) + \text{normCdf}(5079, \infty, 5375, 225)}$$

Output from the right panel:

normCdf(5079, ∞ , 4085, 445)	0.01275111
normCdf(5079, ∞ , 5375, 225)	0.90583831
normCdf(5079, ∞ , 445, 4085)	0.01275115058
normCdf(5079, ∞ , 225, 5375)	0.9058383704

c. $X_{AB} \sim N(5375, 225^2)$

$$\Pr(X_{AB} > 5450) = 0.3694\dots$$

$$X \sim Bi(20, 0.3694\dots) \quad \mathbf{1M}$$

$$\Pr(X > 5) = \Pr(X \geq 6) = 0.8076 \quad \mathbf{1A}$$

normCdf(5450,∞,5375,225) 0.3694414
binomCdf(20,0.3694414,6,20) 0.80759475

d. $X_2 \sim \text{Bi}(n, 0.3694..)$

Trial and error **1M** (other methods)

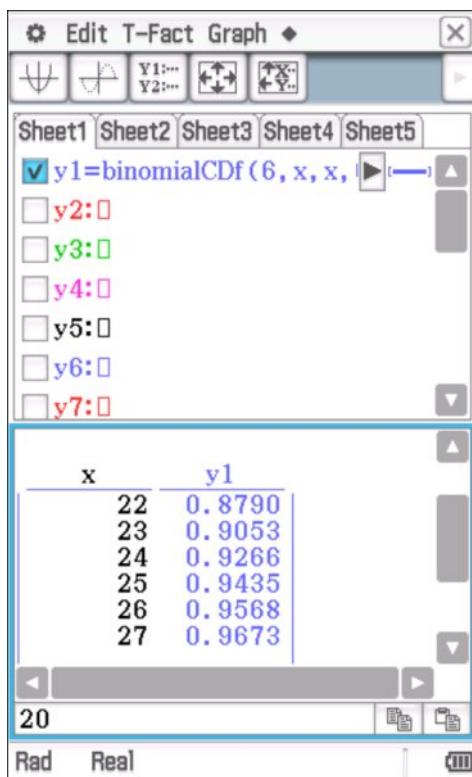
n	$\Pr(X_2 \geq 6)$
25	0.9435...
26	0.9568...

$n = 26$

1A

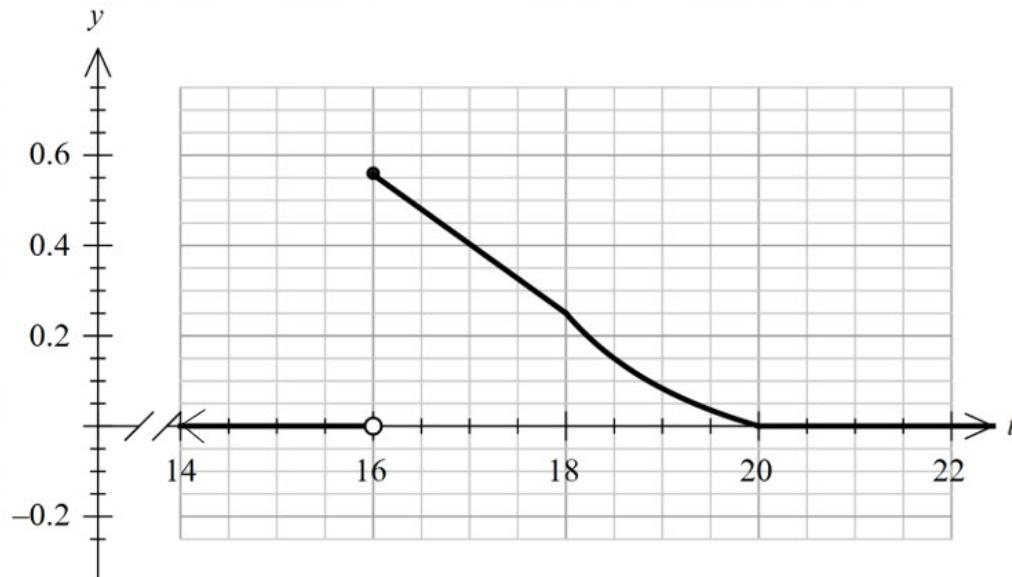
binomCdf($n, 0.3694414, 6, n$)| $n=25$ 0.94350065
binomCdf($n, 0.3694414, 6, n$)| $n=26$ 0.95683961

invBinomN(0.05, 0.3694414, 5, 1)
 $\begin{bmatrix} 25 & 0.05649935 \\ 26 & 0.04316039 \end{bmatrix}$

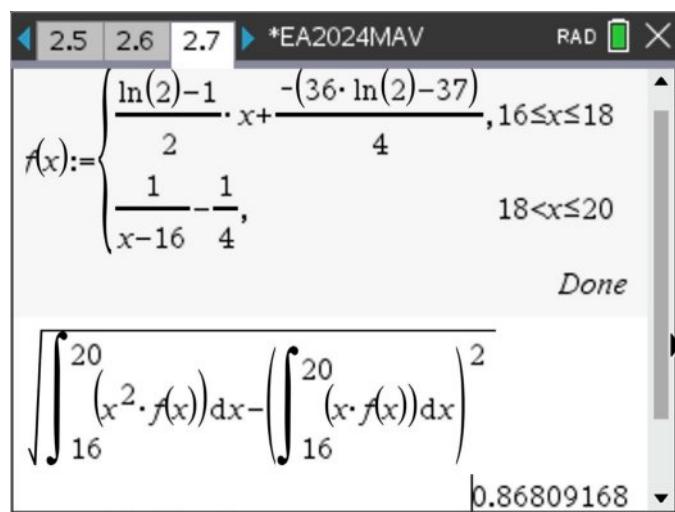


e. Shape, open circle, must draw along the x -axis **1A**

$$f(t) = \begin{cases} \frac{\ln(2)-1}{2}t + \frac{37-36\ln(2)}{4} & 16 \leq t \leq 18 \\ \frac{1}{t-16} - \frac{1}{4} & 18 < t \leq 20 \\ 0 & \text{elsewhere} \end{cases}$$



f. $\text{sd}(T) = \sqrt{\int_{16}^{20} (t^2 \times f(t)) dt - \left(\int_{16}^{20} (t \times f(t)) dt \right)^2}$ 1M
 $= 0.868$ 1A



g. $(0.0117, 0.2550)$ 1A

zInterval_1Prop 4,30,0.95: *stat.results*

"Title"	"1-Prop z Interval"
"CLower"	0.01169152
"CUpper"	0.25497514
" \hat{p} "	0.13333333
"ME"	0.12164181
"n"	30.

C-Level **0.95**
 \bar{x} **4**
n **30**

Lower **0.0116915**
Upper **0.2549751**
 \hat{p} **0.1333333**
n **30**

<< Back Help Next >>

OnePropZInt

h. Solve $1.96\sqrt{\frac{\frac{4}{30} \times \frac{26}{30}}{n}} < 0.1$ for n .

$n = 45$ **1A**

solve $1.96 \cdot \sqrt{\frac{\frac{4}{30} \cdot \frac{26}{30}}{n}} < 0.1, n$

$n > 44.391822$

i. Let AB be the African bush elephant and AF be the African forest elephant.

$$\Pr(AB \cap AF) = \Pr(AB) \times \Pr(AF) = \frac{2-k^2}{2} \text{ independent events}$$

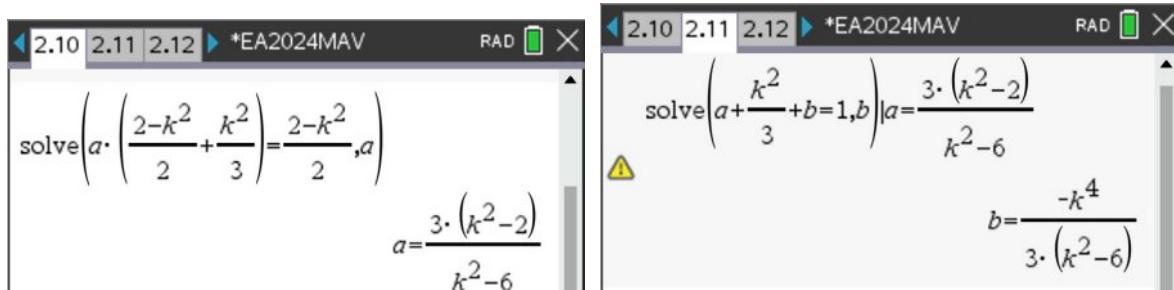
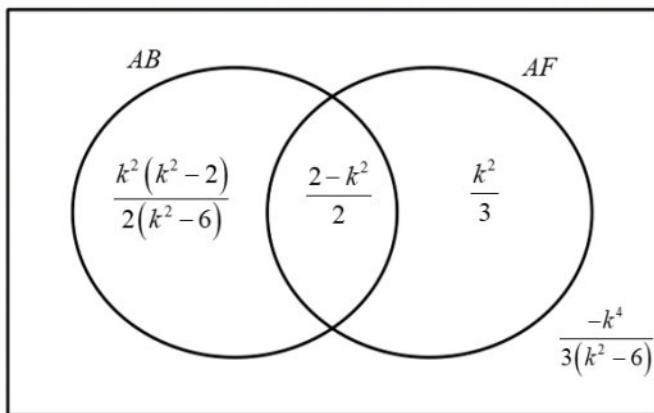
$$\Pr(AB) \times \left(\frac{2-k^2}{2} + \frac{k^2}{3} \right) = \frac{2-k^2}{2} \quad \mathbf{1M}$$

$$\Pr(AB) = \frac{3(k^2 - 2)}{k^2 - 6}$$

$$\Pr(AB \cup AF) + \Pr(AB' \cap AF') = 1$$

$$\frac{3(k^2 - 2)}{k^2 - 6} + \frac{k^2}{3} + \Pr(AB' \cap AF') = 1$$

$$\Pr(AB' \cap AF') = \frac{-k^4}{3(k^2 - 6)} \quad \text{1A}$$

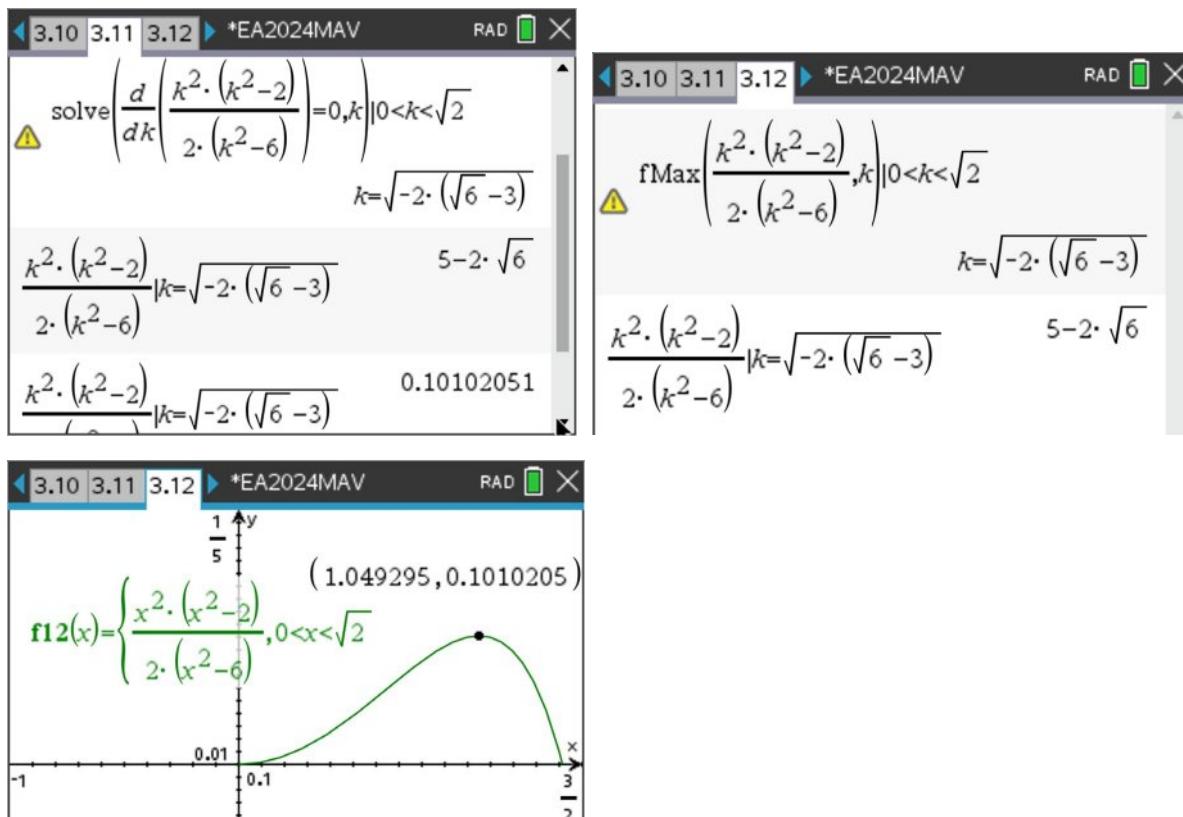


j. $\Pr(AB \cap AF') = \frac{3(k^2 - 2)}{k^2 - 6} - \frac{2 - k^2}{2} = \frac{k^2(k^2 - 2)}{2(k^2 - 6)}$

Solve $\frac{d}{dk} \left(\frac{k^2(k^2 - 2)}{2(k^2 - 6)} \right) = 0$ or use fmax

$$k = \sqrt{-2(\sqrt{6} - 3)}$$

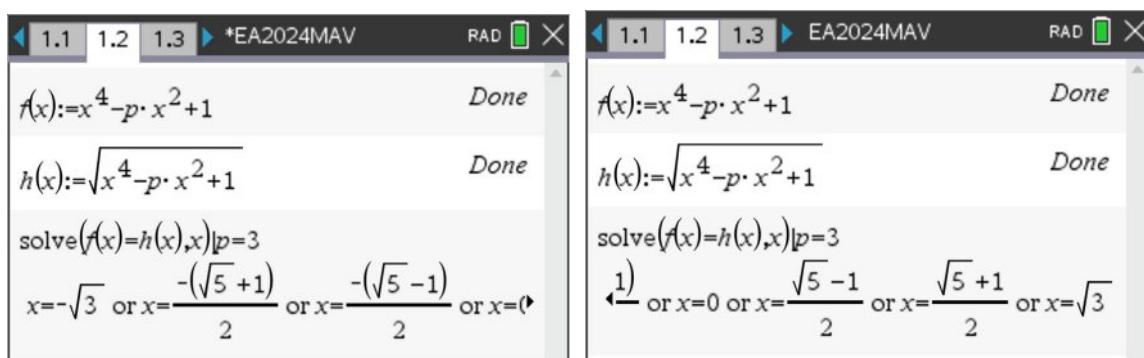
Maximum probability is $5 - 2\sqrt{6}$ 1A

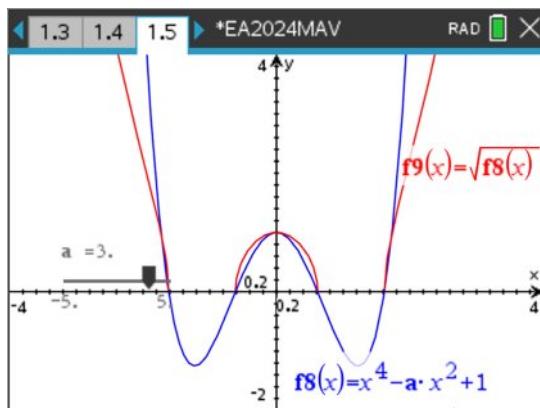
**Question 4**

$h(x) = \sqrt{x^4 - px^2 + 1}$ and $f(x) = x^4 - px^2 + 1$, and $p \in R$

a. Solve $h(x) = f(x)$ when $p = 3$

$$x = \pm\sqrt{3}, 0, \frac{-\sqrt{5} \pm 1}{2}, \frac{\sqrt{5} \pm 1}{2} \quad 1A$$



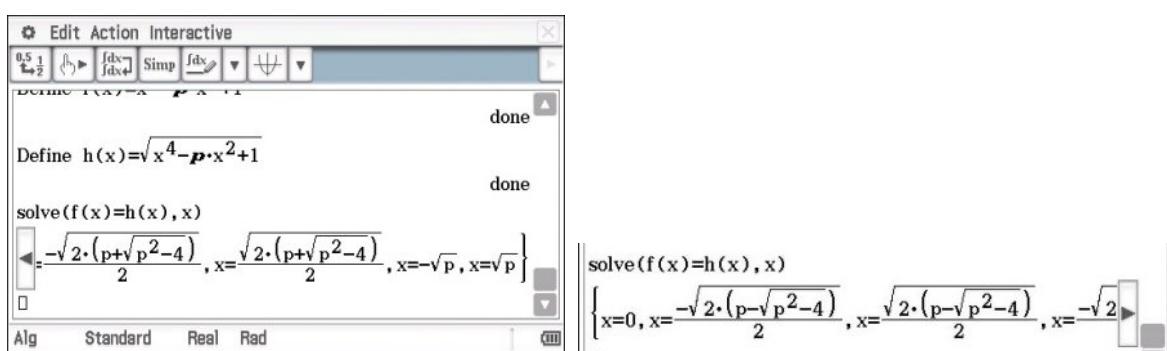
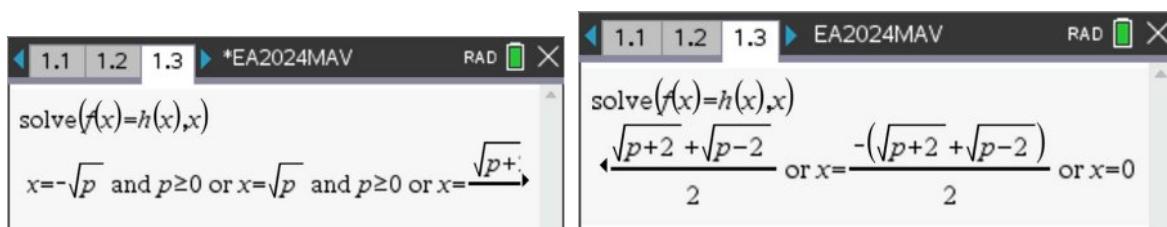


b. Solve $h(x) = f(x)$ for x .

$$x = \pm\sqrt{p}, 0, \frac{\pm\sqrt{p+2} + \sqrt{p-2}}{2}, \frac{\pm\sqrt{p+2} - \sqrt{p-2}}{2} \quad 1A$$

OR

$$x = \pm\sqrt{p}, 0, \frac{\pm\sqrt{2(p + \sqrt{p^2 - 4})}}{2}, \frac{\pm\sqrt{2(p - \sqrt{p^2 - 4})}}{2} \quad 1A$$



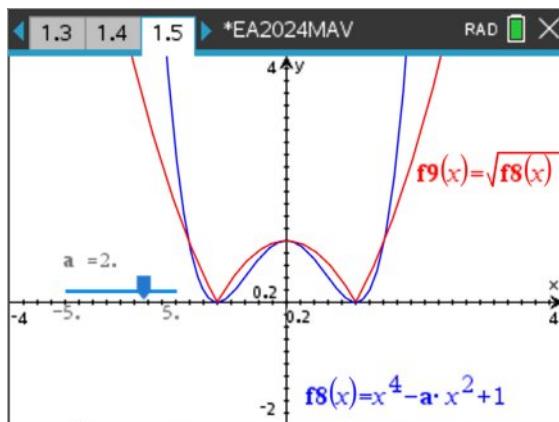
c. If $p = 2$, $\sqrt{p-2} = 0$, so $\frac{\pm\sqrt{p+2} + \sqrt{p-2}}{2} = \frac{\pm\sqrt{p+2} - \sqrt{p-2}}{2} = \frac{\pm\sqrt{p+2}}{2}$

OR

$$\text{If } p = 2, p^2 - 4 = 0, \text{ so } \frac{\pm\sqrt{2(p + \sqrt{p^2 - 4})}}{2} = \frac{\pm\sqrt{2(p - \sqrt{p^2 - 4})}}{2}$$

ence onl five solutions.

1A

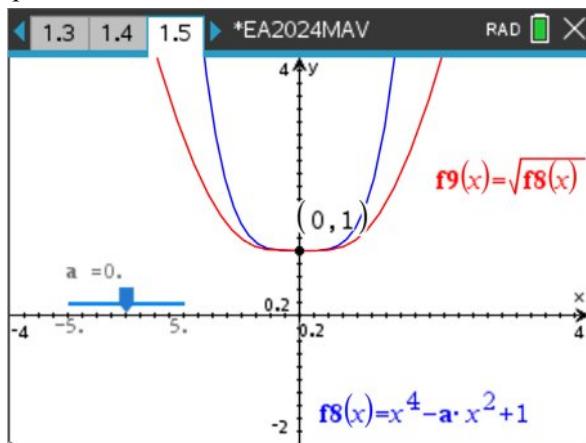


- d. 2 correct **1A**
All correct **2A**

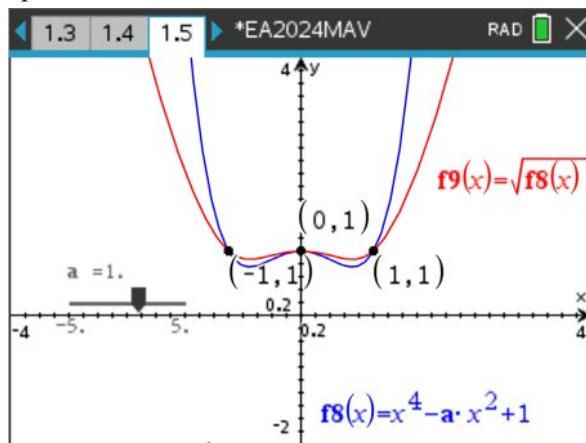
Number of points of intersection	1	3
p values	$p \leq 0$	$0 < p < 2$
x-coordinates of points of intersection	0	$0, \pm\sqrt{p}$

Examples

$p = 0$



$p = 1$



e. $g(x) = \sqrt{x}$, $f(x) = x^4 - px^2 + 1$

or $g(f(x))$ to exist the range of f has to be a subset of, or equal to, the domain of g .

The domain of g is $[0, \infty)$, the range of f will be $[0, \infty)$ for $p = 2$ and a subset of $[0, \infty)$ for $p < 2$.

The range of f , $\left[1 - \frac{p^2}{4}, \infty\right)$, is not a subset of $[0, \infty)$ for $p > 2$. **1A**

solve $\left(\frac{d}{dx}(f(x))=0, x\right)$
 $x=\frac{\sqrt{2 \cdot p}}{2} \text{ and } p \geq 0 \text{ or } x=\frac{-\sqrt{2 \cdot p}}{2} \text{ and } p \geq 0 \text{ or } \dots$

⚠️ $f\left(\frac{\sqrt{2 \cdot p}}{2}\right)$ $1-\frac{p^2}{4}$

f. Using the bounded area on the graph

$$\begin{aligned} \text{Area} &= 2(0.0196238\dots + 0.076381\dots) & \mathbf{1M} \\ &= 0.192 & \mathbf{1A} \end{aligned}$$

OR**Using definite integrals**

$$\begin{aligned} \text{Area} &= 2 \left(\int_{-\sqrt{3}}^{\frac{-\sqrt{5}-1}{2}} (h(x) - f(x)) dx + \int_{\frac{-\sqrt{5}+1}{2}}^0 (h(x) - f(x)) dx \right) \quad \mathbf{OR} \\ &= 2 \left(\int_0^{\frac{\sqrt{5}-1}{2}} (h(x) - f(x)) dx + \int_{\frac{\sqrt{5}+1}{2}}^{\sqrt{3}} (h(x) - f(x)) dx \right) \quad \mathbf{1M} \text{ (either form)} \\ &= 0.192 & \mathbf{1A} \end{aligned}$$

1.4 | 1.5 | 1.6 *EA2024MAV RAD X

y

0.0196238 0.076381 $(0, 1)$

$a = 3.$

$f8(x) = x^4 - a \cdot x^2 + 1$ $f9(x) = \sqrt{f8(x)}$

1.6 | 1.7 | 1.8 *EA2024MAV RAD X

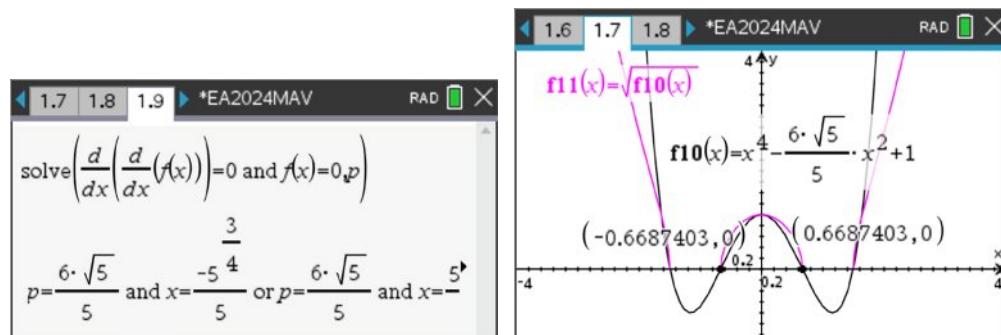
$2 \cdot \left(\int_{-\frac{\sqrt{5}-1}{2}}^{-\sqrt{3}} (h(x) - f(x)) dx + \int_{\frac{\sqrt{5}+1}{2}}^0 (h(x) - f(x)) dx \right) = 0.19200976$

$$\begin{aligned} \mathbf{g. Area} &= 2 \left(\int_{-\sqrt{p}}^{\frac{-\sqrt{p+2}-\sqrt{p-2}}{2}} (h(x) - f(x)) dx + \int_{\frac{-\sqrt{p+2}+\sqrt{p-2}}{2}}^0 (h(x) - f(x)) dx \right) \quad \mathbf{OR} \\ &= 2 \left(\int_0^{\frac{\sqrt{p+2}-\sqrt{p-2}}{2}} (h(x) - f(x)) dx + \int_{\frac{\sqrt{p+2}+\sqrt{p-2}}{2}}^{\sqrt{p}} (h(x) - f(x)) dx \right) \quad \mathbf{OR} \end{aligned}$$

$$= 2 \left(\int_0^{\frac{\sqrt{p}}{2}} (h(x) - f(x)) dx + \int_{\frac{\sqrt{p}}{2}}^{\frac{\sqrt{p} + \sqrt{p^2 - 4}}{2}} (h(x) - f(x)) dx \right) \quad \mathbf{1A}$$

h. Solve $f''(x) = 0$ and $f(x) = 0$ for p . **1M**

$$p = \frac{6\sqrt{5}}{5} \quad \mathbf{1A}$$



i. $h_v(x) = \sqrt{x^4 - 3x^2 + 1}$ and $f_v(x) = x^4 - 3x^2 + 1$

$$\text{Cross-sectional area} = \int_{-1.817...}^{1.817...} (2 - f_v(x)) dx - 0.1920... = 7.517... \quad \mathbf{1M}$$

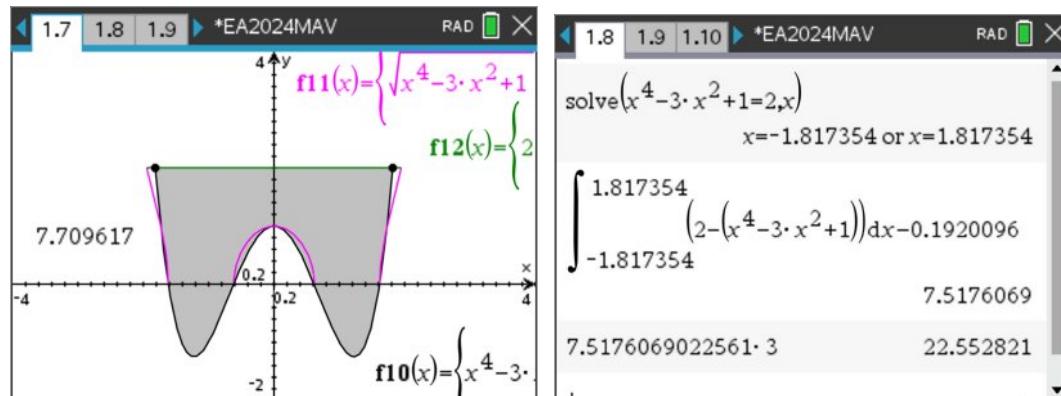
Volume = $7.517... \times 3 = 22.552...$

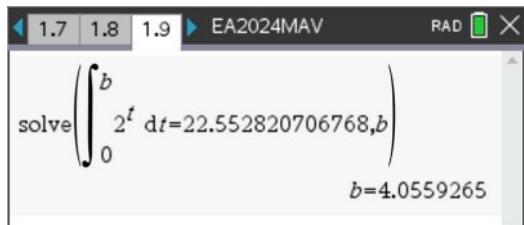
$$\frac{dv}{dt} = 2^t$$

$$\text{Solve } \int_0^b 2^t dt = 22.552... \text{ for } b. \quad \mathbf{1H}$$

$$b = 4.06 \text{ seconds} \quad \mathbf{1A}$$

The shaded area on the graph is $\int_{-1.817...}^{1.817...} (2 - f_v(x)) dx = 7.709...$ and then you need to subtract the bound area found in part f.





END OF SOLUTIONS