

Algorithms Design and Analysis Lab

ETMA 351

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Semester: **4**



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MAHARAJA AGRASEN INSTITUTE OF TECHNOLOGY

COMPUTER SCIENCE & ENGINEERING DEPARTMENT

VISION

To Produce “Critical thinkers of Innovative Technology”

MISSION

To provide an excellent learning environment across the computer science discipline to inculcate professional behavior, strong ethical values, innovative research capabilities and leadership abilities which enable them to become successful entrepreneurs in this globalized world.

1. To nurture an **excellent learning environment** that helps students to enhance their problem solving skills and to prepare students to be lifelong learners by offering a solid theoretical foundation with applied computing experiences and educating them about their **professional, and ethical responsibilities**.
2. To establish **Industry-Institute Interaction**, making students ready for the industrial environment and be successful in their professional lives.
3. To promote **research activities** in the emerging areas of technology convergence.
4. To build engineers who can look into technical aspects of an engineering solution thereby setting a ground for producing successful **entrepreneur**.

VISION

To nurture young minds in a learning environment of high academic value and imbibe spiritual and ethical values with technological and management competence.

MISSION

The Institute shall endeavor to incorporate the following basic missions in the teaching methodology:

Engineering Hardware - Software Symbiosis

Practical exercises in all Engineering and Management disciplines shall be carried out by Hardware equipment as well as the related software enabling deeper understanding of basic concepts and encouraging inquisitive nature.

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The Institute strives to match technological advancements and encourage students to keep updating their knowledge for enhancing their skills and inculcating their habit of continuous learning.

Liberalization and Globalization

The Institute endeavors to enhance technical and management skills of students so that they are intellectually capable and competent professionals with Industrial Aptitude to face the challenges of globalization.

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The Engineering, Technology and Management disciplines have diverse fields of studies with different attributes. The aim is to create a synergy of the above attributes by encouraging analytical thinking.

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The Institute provides seamless opportunities for innovative learning in all Engineering and Management disciplines through digitization of learning processes using analysis, synthesis, simulation, graphics, tutorials and related tools to create a platform for multi-disciplinary approach.

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EXPERIMENT - 1

Algorithms Design and Analysis Lab

Aim

To implement following algorithm using array as a data structure and analyse its time complexity.

Syeda Reeha Quasar

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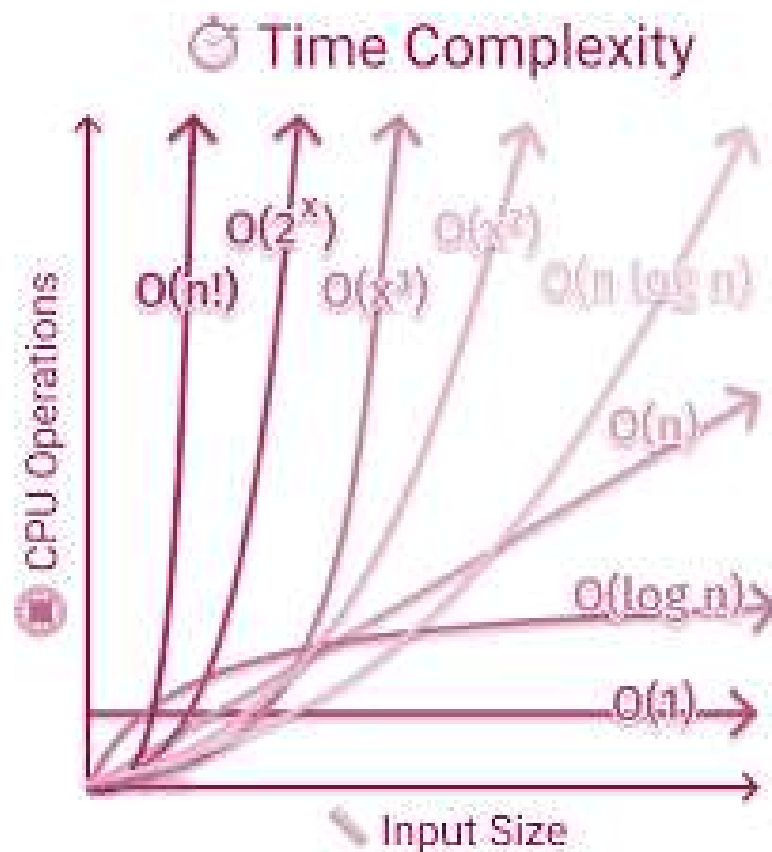
EXPERIMENT – 1

Aim:

To implement following algorithm using array as a data structure and analyse its time complexity.

Theory:

Time complexity is the amount of time taken by an algorithm to run, as a function of the length of the input. It measures the time taken to execute each statement of code in an algorithm.



Data Structure with their Time Complexity:



Data Structures

Data Structure	Time Complexity								Space Complexity
	Average				Worst				Worst
	Indexing	Search	Insertion	Deletion	Indexing	Search	Insertion	Deletion	
Basic Array	$O(1)$	$O(n)$	-	-	$O(1)$	$O(n)$	-	-	$O(n)$
Dynamic Array	$O(1)$	$O(n)$	$O(n)$	$O(n)$	$O(1)$	$O(n)$	$O(n)$	$O(n)$	$O(n)$
Singly-Linked List	$O(n)$	$O(n)$	$O(1)$	$O(1)$	$O(n)$	$O(n)$	$O(1)$	$O(1)$	$O(n)$
Doubly-Linked List	$O(n)$	$O(n)$	$O(1)$	$O(1)$	$O(n)$	$O(n)$	$O(1)$	$O(1)$	$O(n)$
Skip List	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	$O(n)$	$O(n)$	$O(n)$	$O(n)$	$O(n \cdot \log(n))$
Hash Table	-	$O(1)$	$O(1)$	$O(1)$	-	$O(n)$	$O(n)$	$O(n)$	$O(n)$
Binary Search Tree	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	$O(n)$	$O(n)$	$O(n)$	$O(n)$	$O(n)$
Cartesian Tree	-	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	-	$O(n)$	$O(n)$	$O(n)$	$O(n)$
B-Tree	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	$O(n)$
Red-Black Tree	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	$O(n)$
Splay Tree	-	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	-	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	$O(n)$
AVL Tree	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	$O(\log(n))$	$O(n)$

Clock/time analysis implementation

Code

```
#include <iostream>
#include <ctime>
using namespace std;

int main() {

    // use time() with NULL argument
    // cout << time(NULL);

    time_t current_time;

    // stores time in current_time
    time(&current_time);
```

```
cout << current_time;
cout << " seconds has passed since 00:00:00 GMT, Jan 1, 1970";
return 0;
}
```

Output

```
1635196484 seconds has passed since 00:00:00 GMT, Jan 1, 1970
...Program finished with exit code 0
Press ENTER to exit console.
```

Sorting implementation

Code

```
#include <bits/stdc++.h>
#include <chrono>
using namespace std;

void printArray(int arr[], int size)    {
    int i;
    for (i = 0; i < size; i++)
        cout << arr[i] << " ";
    cout << endl;
}

int main() {
    int arr[] = {64, 34, 25, 12, 22, 11, 90};
    int n = sizeof(arr)/sizeof(arr[0]);
    auto start = chrono::steady_clock::now();
    // unsync the I/O of C and C++.
```



```
ios_base::sync_with_stdio(false);

cout << "Array: ";
printArray(arr, n);
cout << endl;
cout << "Sorted array: ";
printArray(arr, n);
cout << endl;
auto end = chrono::steady_clock::now();

double time_taken = chrono::duration_cast<chrono::nanoseconds>(end -
start).count();
time_taken *= 1e-9;

cout << "Time difference is: " << time_taken << setprecision(6);
return 0;
}
```

Output

```
Array: 64 34 25 12 22 11 90
Sorted array: 64 34 25 12 22 11 90
Time difference is: 6.88e-05
...Program finished with exit code 0
Press ENTER to exit console.□
```

Merge Sort:

Conceptually, a merge sort works as follows:

1. Divide the unsorted list into n sublists, each containing 1 element (a list of 1 element is considered sorted).
2. Repeatedly merge sublists to produce new sorted sublists until there is only 1 sublist remaining. This will be the sorted list.

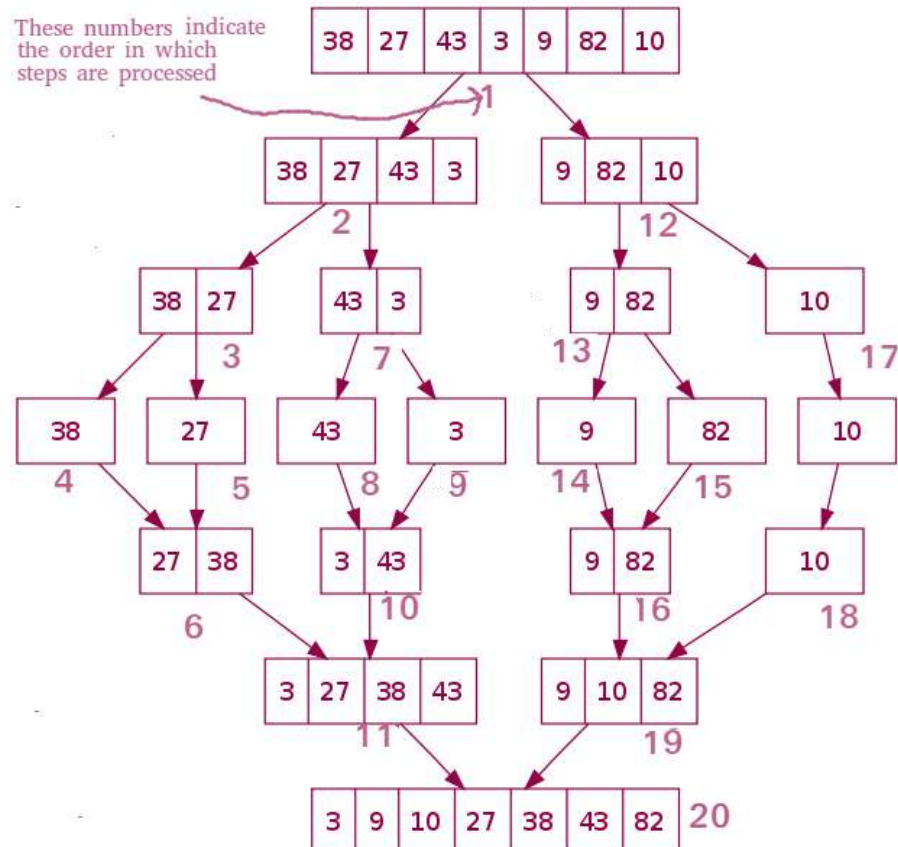
Pseudo code

MergeSort(arr[], l, r)

If $r > l$

1. Find the middle point to divide the array into two halves:
middle $m = l + (r-l)/2$
2. Call mergeSort for first half:
Call mergeSort(arr, l, m)
3. Call mergeSort for second half:
Call mergeSort(arr, m+1, r)
4. Merge the two halves sorted in step 2 and 3:
Call merge(arr, l, m, r)

Example:



Result and Analysis

The unordered list of elements gets sorted but additional space is used for merging.

The list of size N is divided into a max of $\log N$ parts, and the merging of all sub lists into a single list takes $O(N)$ time. Time complexity of Merge Sort is $O(n \log n)$ in all 3 cases (worst, average and best) as merge sort always divides the array in two halves and take linear time to merge two halves. The recurrence equation used is: $T(n) = 2T(n/2) + O(n)$

Source Code:

```
#include <bits/stdc++.h>
#include <chrono>

using namespace std;

// merging 2 arrays
void merge(int arr[], int const left, int const midIdx, int const right) {
    int temp[right - left + 1]; // sorted Arr

    int start = left, mid = midIdx + 1, tempIdx = 0;

    // sorted merging of left and right arrays
    while (start <= midIdx && mid <= right) {
        if (arr[start] <= arr[mid]) {
            temp[tempIdx] = arr[start];
            tempIdx++;
            start++;
        }
        else {
            temp[tempIdx] = arr[mid];
            tempIdx++;
            mid++;
        }
    }

    // check and add for remaining element in left arr
    while (start <= midIdx) {
        temp[tempIdx] = arr[start];
        tempIdx++;
    }
}
```

```
        start++;
    }

    // check and add for remaining element in right arr
    while (mid <= right) {
        temp[tempIdx] = arr[mid];
        tempIdx++;
        mid++;
    }

    for (int i = left; i <= right; i++) {
        arr[i] = temp[i - left];
    }
}

void mergeSort(int array[], int const begin, int const end) {
    if (begin >= end) {
        return; // Returns recursively
    }
    auto mid = begin + (end - begin) / 2;
    mergeSort(array, begin, mid);
    mergeSort(array, mid + 1, end);
    merge(array, begin, mid, end);
}

void printArr(int arr[], int size) {
    for (auto i = 0; i < size; i++) {
        cout << arr[i] << " ";
    }
    cout << "\n\n" << endl;
}

int main() {
    int n = rand() % 100;
    int arr[n];
    for (int i = 0; i < n; i++) {
        arr[i] = rand() % 100;
    }
    cout << "Given array: \n";
    printArr(arr, n);

    auto start = chrono::high_resolution_clock::now();
    // unsync the I/O of C and C++.
    ios_base::sync_with_stdio(false);
```

```
mergeSort(arr, 0, n - 1);

auto end = chrono::high_resolution_clock::now();

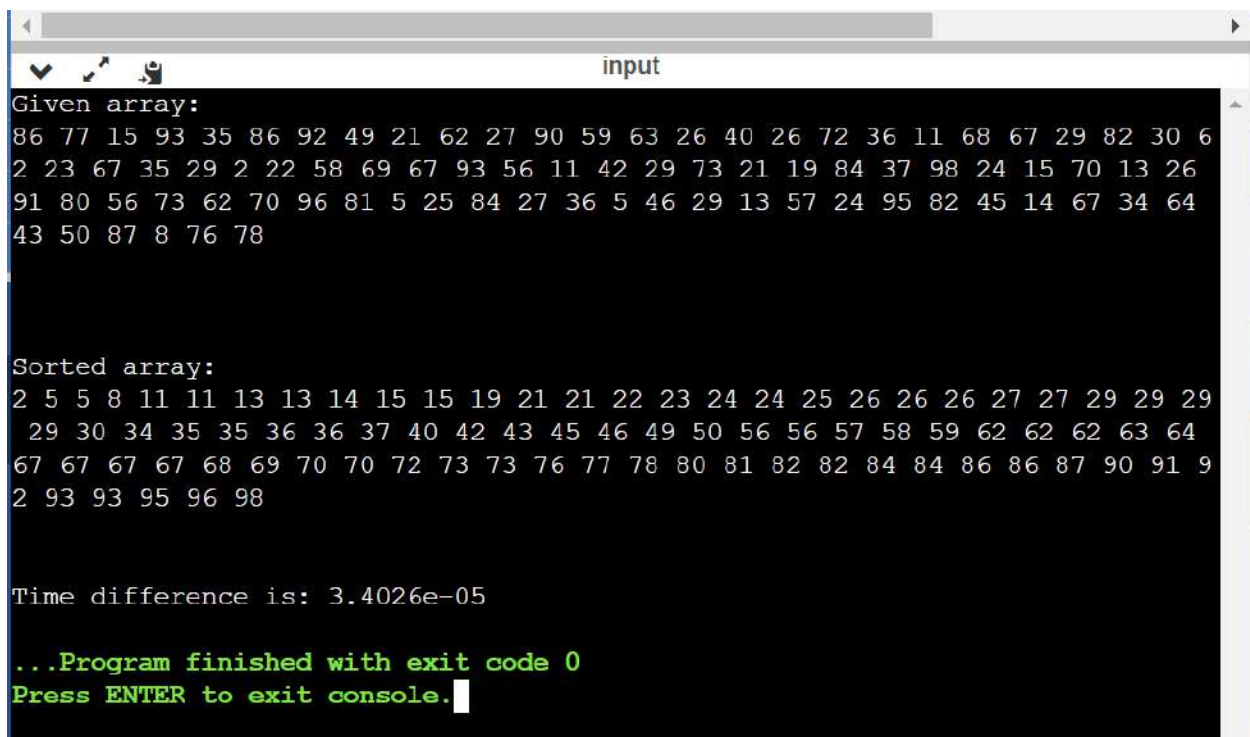
cout << "\nSorted array: \n";
printArr(arr, n);

double time_taken = chrono::duration_cast<chrono::nanoseconds>(end -
start).count();
time_taken *= 1e-9;

cout << "Time difference is: " << time_taken << setprecision(6);

return 0;
}
```

Output:



```
input
Given array:
86 77 15 93 35 86 92 49 21 62 27 90 59 63 26 40 26 72 36 11 68 67 29 82 30 6
2 23 67 35 29 2 22 58 69 67 93 56 11 42 29 73 21 19 84 37 98 24 15 70 13 26
91 80 56 73 62 70 96 81 5 25 84 27 36 5 46 29 13 57 24 95 82 45 14 67 34 64
43 50 87 8 76 78

Sorted array:
2 5 5 8 11 11 13 13 14 15 15 19 21 21 22 23 24 24 25 26 26 26 27 27 29 29 29
29 30 34 35 35 36 36 37 40 42 43 45 46 49 50 56 56 57 58 59 62 62 62 63 64
67 67 67 67 68 69 70 70 72 73 73 76 77 78 80 81 82 82 84 84 86 86 87 90 91 9
2 93 93 95 96 98

Time difference is: 3.4026e-05

...Program finished with exit code 0
Press ENTER to exit console.
```

Source Code:

```
#include <bits/stdc++.h>
#include <chrono>

using namespace std;

// merging 2 arrays
void merge(int arr[], int const left, int const mid, int const right) {
    auto const subArr1 = mid - left + 1;
    auto const subArr2 = right - mid;

    auto *leftArr = new int[subArr1],
        *rightArr = new int[subArr2];

    for (auto i = 0; i < subArr1; i++)
        leftArr[i] = arr[left + i];

    for (auto j = 0; j < subArr2; j++)
        rightArr[j] = arr[mid + 1 + j];

    auto indexOfSubArr1 = 0,
        indexOfSubArr2 = 0;
    int indexOfMergedArr = left;

    // sorted merging of left and right arrays
    while (indexOfSubArr1 < subArr1 && indexOfSubArr2 < subArr2) {
        if (leftArr[indexOfSubArr1] <= rightArr[indexOfSubArr2]) {
            arr[indexOfMergedArr] = leftArr[indexOfSubArr1];
            indexOfSubArr1++;
        }
        else {
            arr[indexOfMergedArr] = rightArr[indexOfSubArr2];
            indexOfSubArr2++;
        }
        indexOfMergedArr++;
    }

    // check and add for remaining element in left arr
    while (indexOfSubArr1 < subArr1) {
        arr[indexOfMergedArr] = leftArr[indexOfSubArr1];
        indexOfSubArr1++;
        indexOfMergedArr++;
    }
}
```

```
        // check and add for remaining element in left arr
        while (indexOfSubArr2 < subArr2) {
            arr[indexOfMergedArr] = rightArr[indexOfSubArr2];
            indexOfSubArr2++;
            indexOfMergedArr++;
        }
    }

void mergeSort(int array[], int const begin, int const end) {
    if (begin >= end) {
        return; // Returns recursively
    }
    auto mid = begin + (end - begin) / 2;
    mergeSort(array, begin, mid);
    mergeSort(array, mid + 1, end);
    merge(array, begin, mid, end);
}

void printArr(int arr[], int size) {
    for (auto i = 0; i < size; i++) {
        cout << arr[i] << " ";
    }
    cout << "\n\n" << endl;
}

int main() {
    int n = rand() % 100;
    int arr[n];
    for (int i = 0; i < n; i++) {
        arr[i] = rand() % 100;
    }
    cout << "Given array: \n";
    printArr(arr, n);

    auto start = chrono::high_resolution_clock::now();
    // unsync the I/O of C and C++.
    ios_base::sync_with_stdio(false);

    mergeSort(arr, 0, n - 1);

    auto end = chrono::high_resolution_clock::now();

    cout << "\nSorted array: \n";
```

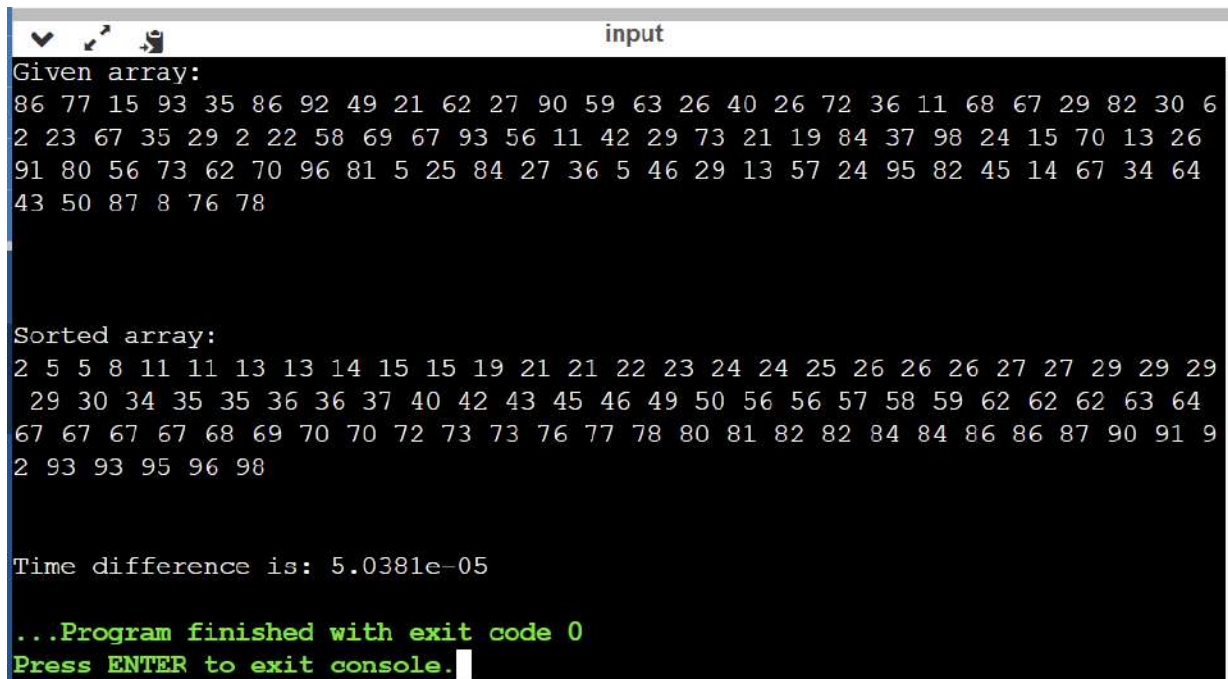
```
    printArr(arr, n);

    double time_taken = chrono::duration_cast<chrono::nanoseconds>(end -
start).count();
    time_taken *= 1e-9;

    cout << "Time difference is: " << time_taken << setprecision(6);

    return 0;
}
```

Output:



```
input
Given array:
86 77 15 93 35 86 92 49 21 62 27 90 59 63 26 40 26 72 36 11 68 67 29 82 30 6
2 23 67 35 29 2 22 58 69 67 93 56 11 42 29 73 21 19 84 37 98 24 15 70 13 26
91 80 56 73 62 70 96 81 5 25 84 27 36 5 46 29 13 57 24 95 82 45 14 67 34 64
43 50 87 8 76 78

Sorted array:
2 5 5 8 11 11 13 13 14 15 15 19 21 21 22 23 24 24 25 26 26 26 27 27 29 29 29
29 30 34 35 35 36 36 37 40 42 43 45 46 49 50 56 56 57 58 59 62 62 62 63 64
67 67 67 67 68 69 70 70 72 73 73 76 77 78 80 81 82 82 84 84 86 86 87 90 91 9
2 93 93 95 96 98

Time difference is: 5.0381e-05

...Program finished with exit code 0
Press ENTER to exit console.
```


Batch Analysis:

Source Code:

```
#include <bits/stdc++.h>
#include <chrono>

using namespace std;

// merging 2 arrays
void merge(int arr[], int const left, int const mid, int const right) {
    auto const subArr1 = mid - left + 1;
    auto const subArr2 = right - mid;

    auto *leftArr = new int[subArr1],
        *rightArr = new int[subArr2];

    for (auto i = 0; i < subArr1; i++)
        leftArr[i] = arr[left + i];

    for (auto j = 0; j < subArr2; j++)
        rightArr[j] = arr[mid + 1 + j];

    auto indexOfSubArr1 = 0,
        indexOfSubArr2 = 0;
    int indexOfMergedArr = left;

    // sorted merging of left and right arrays
    while (indexOfSubArr1 < subArr1 && indexOfSubArr2 < subArr2) {
        if (leftArr[indexOfSubArr1] <= rightArr[indexOfSubArr2]) {
            arr[indexOfMergedArr] = leftArr[indexOfSubArr1];
            indexOfSubArr1++;
        }
        else {
            arr[indexOfMergedArr] = rightArr[indexOfSubArr2];
            indexOfSubArr2++;
        }
        indexOfMergedArr++;
    }

    // check and add for remaining element in left arr
    while (indexOfSubArr1 < subArr1) {
        arr[indexOfMergedArr] = leftArr[indexOfSubArr1];
        indexOfSubArr1++;
    }
```

```
        indexOfMergedArr++;
    }

    // check and add for remaining element in left arr
    while (indexOfSubArr2 < subArr2) {
        arr[indexOfMergedArr] = rightArr[indexOfSubArr2];
        indexOfSubArr2++;
        indexOfMergedArr++;
    }
}

void mergeSort(int array[], int const begin, int const end) {
    if (begin >= end) {
        return; // Returns recursively
    }
    auto mid = begin + (end - begin) / 2;
    mergeSort(array, begin, mid);
    mergeSort(array, mid + 1, end);
    merge(array, begin, mid, end);
}

void printArray(double arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << ", ";
    cout << endl;
}

void printArray(int arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << ", ";
    cout << endl;
}

void printArray(float arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << ", ";
    cout << endl;
}

double sortApp(int n) {
    // int n = rand() % 100;
    int arr[n];
```

```
        for (int i = 0; i < n; i++) {
            arr[i] = rand() % 100;
        }
// cout << "Given array: \n";
// printArr(arr, n);

    auto start = chrono::high_resolution_clock::now();
    // unsync the I/O of C and C++.
    ios_base::sync_with_stdio(false);

    mergeSort(arr, 0, n - 1);

    auto end = chrono::high_resolution_clock::now();

// cout << "\nSorted array: \n";
// printArr(arr, n);

    double time_taken = chrono::duration_cast<chrono::nanoseconds>(end -
start).count();
    time_taken *= 1e-9;

    // cout << "Time difference is: " << time_taken << setprecision(6);

    return time_taken;
}

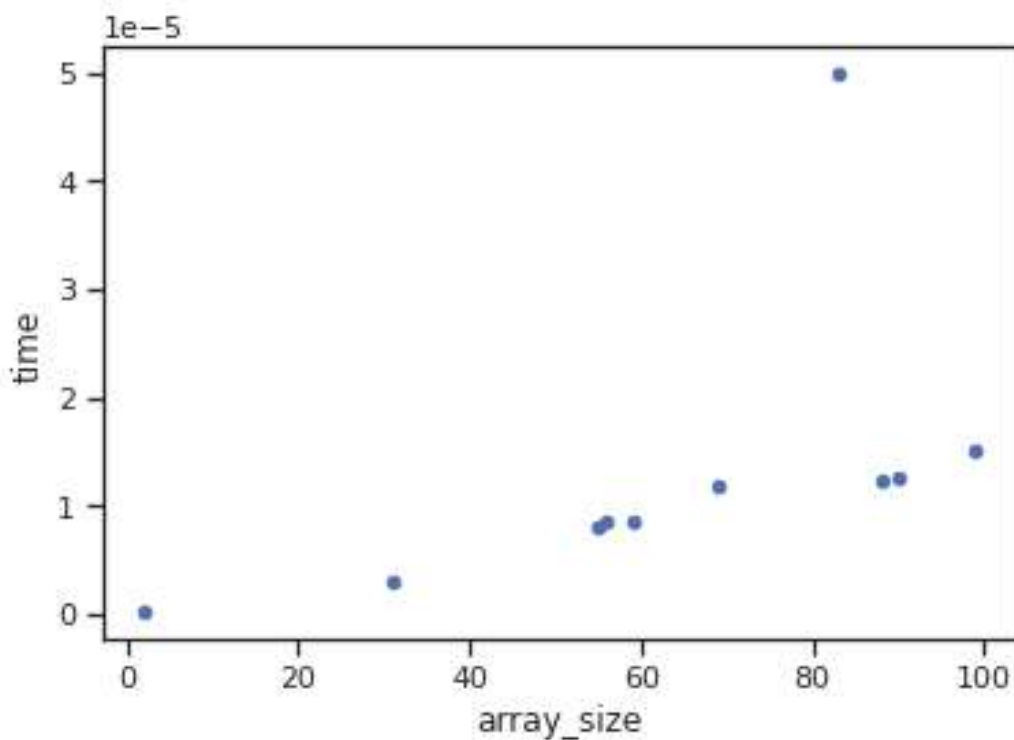
int main() {
    double times[10];
    int ns[10];
    for (int x = 0; x < 10; x++)
    {
        int n = rand() % 100;
        ns[x] = n;
        times[x] = sortApp(n);
    }
    cout << "value of n's: " << endl;
    printArray(ns, 10);
    cout << "time for each n: " << endl;
    printArray(times, 10);
}
```

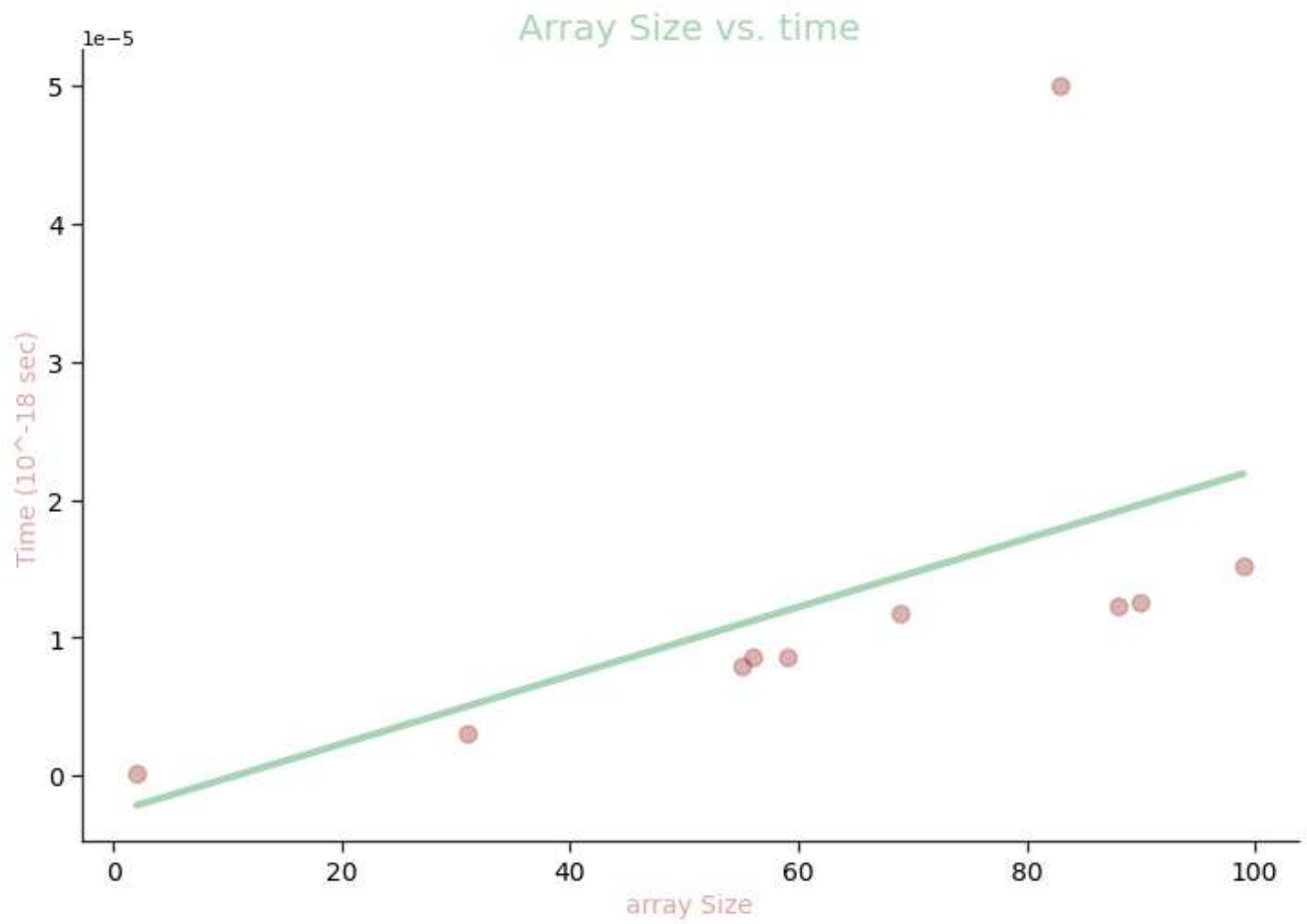
Output:

```
input
value of n's:
83, 88, 56, 69, 90, 59, 2, 99, 55, 31,
time for each n:
7.1799e-05, 2.4215e-05, 1.657e-05, 2.1988e-05, 2.472e-05, 1.6775e-05, 2.34e-
07, 2.9243e-05, 1.5824e-05, 6.567e-06,

...Program finished with exit code 0
Press ENTER to exit console.
```

```
arraySize = [83, 88, 56, 69, 90, 59, 2, 99, 55, 31]
time = [5.0004e-05, 1.2248e-05, 8.572e-06, 1.1836e-05, 1.2539e-05, 8.589e-
06, 1.34e-07, 1.5209e-05, 7.996e-06, 3.056e-06]
```





Merge Sort

Viva Questions

1. Why is time complexity of merge sort?

Ans.

$$T(n) = 2T(n/2) + \theta(n)$$

Time complexity of Merge Sort is $\theta(n \log n)$ in all 3 cases (worst, average and best) as merge sort always divides the array into two halves and takes linear time to merge two halves.

$$O(n \log n)$$

Auxiliary Space: $O(n)$

Algorithmic Paradigm: Divide and Conquer

Sorting In Place: No in a typical implementation

Stable: Yes

2. Is it possible to do merge sort in place?

Ans.

There do exist in-place merge sorts. They must be implemented carefully. First, naive in-place merge such as described here isn't the right solution. It downgrades the performance to $O(N^2)$.

3. What are the total number of passes in merge sort of n numbers?

Ans.

$$\log(n)$$

Suppose an array of n elements.

Now iterate through this array just one time. The time complexity of this operation is $O(n)$. This time complexity tells us that we are iterating one time through an array of length n.

Now iterate through this array n times. The time complexity of this operation is $O(n*n)$. This time complexity tells us that we are iterating n times through an array of length n .

Now the time complexity of merge sort is $O(n\log(n))$. This says that we are iterating through an n element array, $\log(n)$ times.

4. Given two sorted lists of size m , n then what are the number of comparisons needed in the worst case by merge sort?

Ans.

To merge two lists of size m and n , we need to do **$m+n-1$ comparisons** in worst case.

5. What is the output of merge sort after the 2nd pass given the following sequence of numbers: 3,41,52,26,38,57,9,49

Ans.

3,26,41,52,9,38,49,57

6. Give any application of merge sort?

Ans.

- Merge Sort is useful for sorting linked lists in $O(n \log n)$ time
- Merge sort can be implemented without extra space for linked lists
- Merge sort is used for counting inversions in a list
- Merge sort is used in external sorting

Quick Sort:

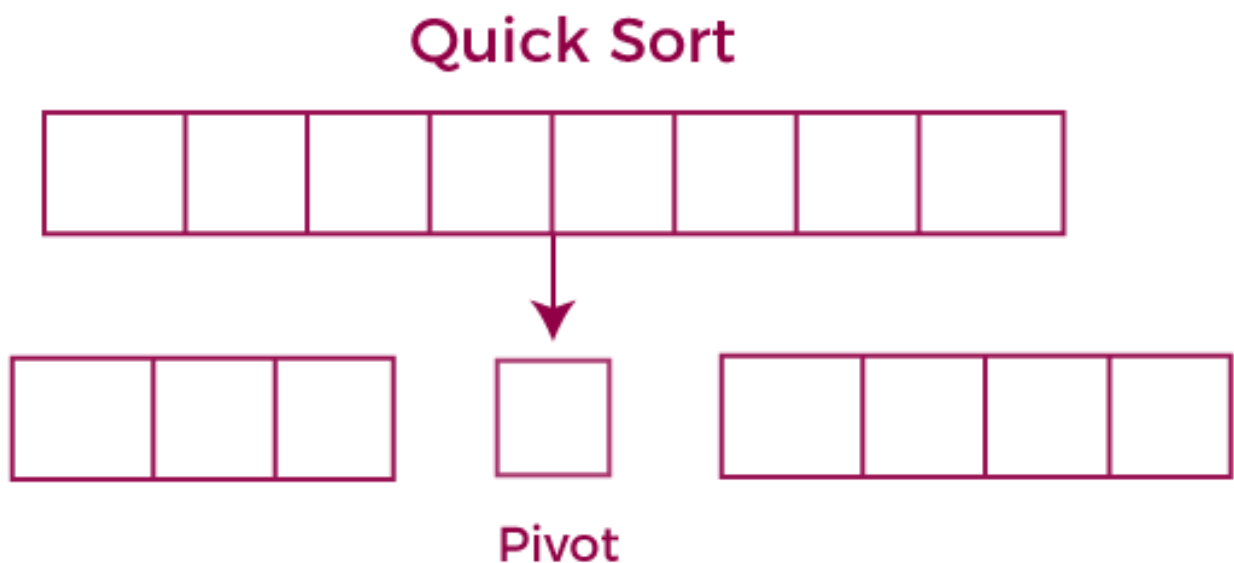
Sorting is a way of arranging items in a systematic manner. Quicksort is the widely used sorting algorithm that makes $n \log n$ comparisons in average case for sorting an array of n elements. It is a faster and highly efficient sorting algorithm. This algorithm follows the divide and conquer approach. Divide and conquer is a technique of breaking down the algorithms into subproblems, then solving the subproblems, and combining the results back together to solve the original problem.

Divide: In Divide, first pick a pivot element. After that, partition or rearrange the array into two sub-arrays such that each element in the left sub-array is less than or equal to the pivot element and each element in the right sub-array is larger than the pivot element.

Conquer: Recursively, sort two subarrays with Quicksort.

Combine: Combine the already sorted array.

Quicksort picks an element as pivot, and then it partitions the given array around the picked pivot element. In quick sort, a large array is divided into two arrays in which one holds values that are smaller than the specified value (Pivot), and another array holds the values that are greater than the pivot. After that, left and right sub-arrays are also partitioned using the same approach. It will continue until the single element remains in the sub-array.



Choosing the pivot

Picking a good pivot is necessary for the fast implementation of quicksort. However, it is typical to determine a good pivot. Some of the ways of choosing a pivot are as follows -

- Pivot can be random, i.e. select the random pivot from the given array.
- Pivot can either be the rightmost element or the leftmost element of the given array.
- Select median as the pivot element.

Pseudo code

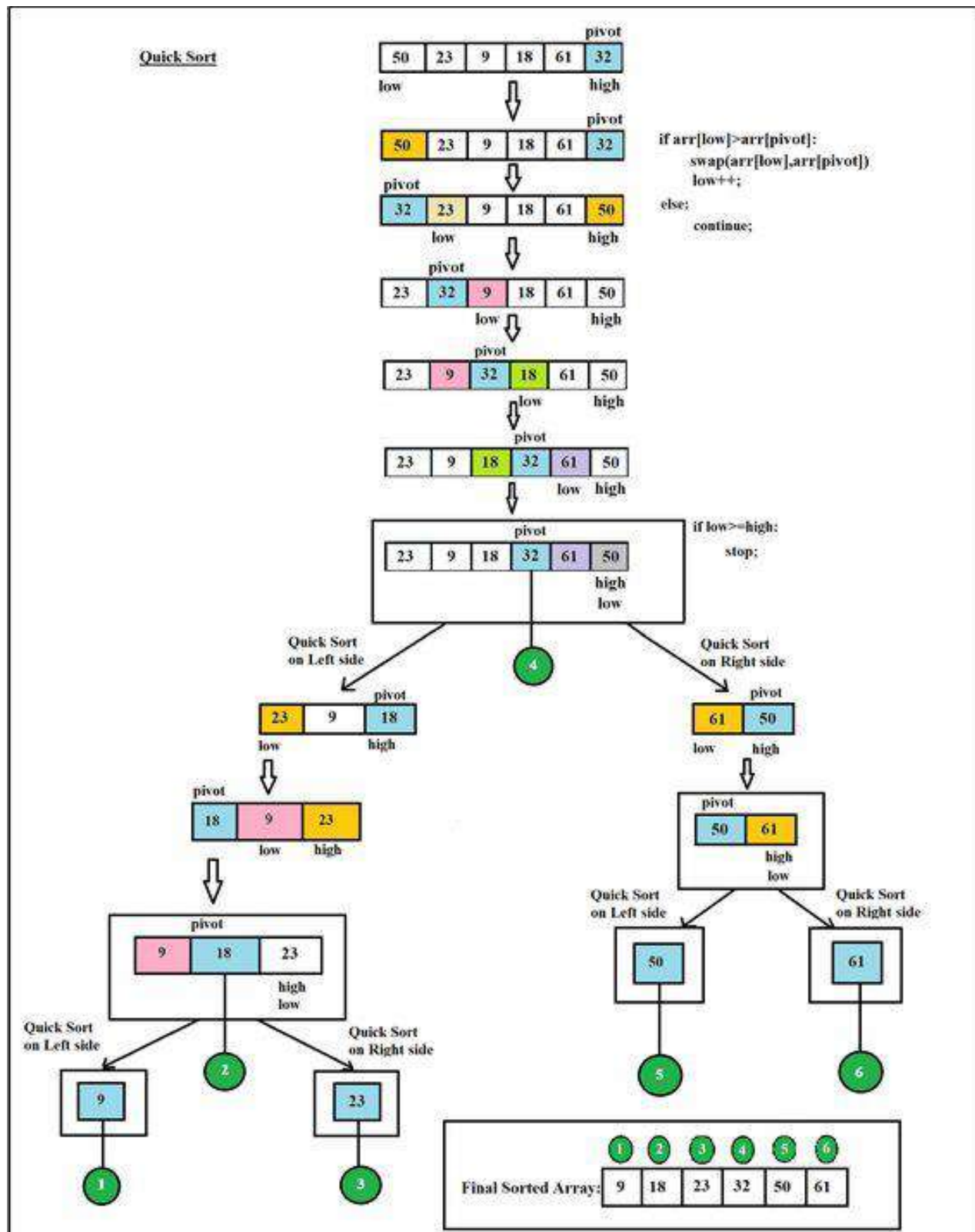
Algorithm:

```
1. QUICKSORT (array A, start, end)
2. {
3.   1 if (start < end)
4.   2 {
5.   3 p = partition(A, start, end)
6.   4 QUICKSORT (A, start, p - 1)
7.   5 QUICKSORT (A, p + 1, end)
8.   6 }
9. }
```

Partition Algorithm:

The partition algorithm rearranges the sub-arrays in a place.

```
1. PARTITION (array A, start, end)
2. {
3.   1 pivot ? A[end]
4.   2 i ? start-1
5.   3 for j ? start to end -1 {
6.   4 do if (A[j] < pivot) {
7.   5 then i ? i + 1
8.   6 swap A[i] with A[j]
9.   7 }}
10.      8 swap A[i+1] with A[end]
11.      9 return i+1
12. }
```

Example:

Result and Analysis

Time complexity

Case	Time Complexity
Best Case	$O(n \cdot \log n)$
Average Case	$O(n \cdot \log n)$
Worst Case	$O(n^2)$

- **Best Case Complexity** - In Quicksort, the best-case occurs when the pivot element is the middle element or near to the middle element. The best-case time complexity of quicksort is **$O(n \cdot \log n)$** .
- **Average Case Complexity** - It occurs when the array elements are in jumbled order that is not properly ascending and not properly descending. The average case time complexity of quicksort is **$O(n \cdot \log n)$** .
- **Worst Case Complexity** - In quick sort, worst case occurs when the pivot element is either greatest or smallest element. Suppose, if the pivot element is always the last element of the array, the worst case would occur when the given array is sorted already in ascending or descending order. The worst-case time complexity of quicksort is **$O(n^2)$** .

Space Complexity

Space Complexity	$O(n \cdot \log n)$
Stable	NO

Source Code:

```
#include <bits/stdc++.h>
#include <chrono>

using namespace std;

// A utility function to swap two elements
void swap(int *a, int *b)
{
    int temp = *a;
    *a = *b;
    *b = temp;
}

// last element pivot and lower in 1 half and greater in 1 half divide
int partition(int arr[], int low, int high)
{
    int pivot = arr[high];
    int i = (low - 1);    // Index of smaller element and indicates the right
    position of pivot found so far

    for (int j = low; j <= high - 1; j++)
    {
        // j == current element
        if (arr[j] < pivot)
        {
            i++; // increment index of smaller element
            swap(&arr[i], &arr[j]);
        }
    }
    swap(&arr[i + 1], &arr[high]);
    return (i + 1);
}

void quickSort(int arr[], int low, int high)
{
    if (low < high)
    {
        int p = partition(arr, low, high);

        // Separately sort elements before partition and after partition
        quickSort(arr, low, p - 1);
        quickSort(arr, p + 1, high);
    }
}
```

```
    }  
}  
  
void printArray(int arr[], int size)  
{  
    int i;  
    for (i = 0; i < size; i++)  
        cout << arr[i] << " ";  
    cout << "\n\n" << endl;  
}  
  
int main()  
{  
    int n = rand() % 100;  
    int arr[n];  
    for (int i = 0; i < n; i++)  
    {  
        arr[i] = rand() % 100;  
    }  
    cout << "Given array: \n";  
    printArray(arr, n);  
  
    auto start = chrono::high_resolution_clock::now();  
  
    quickSort(arr, 0, n - 1);  
  
    auto end = chrono::high_resolution_clock::now();  
  
    cout << "\nSorted array: \n";  
    printArray(arr, n);  
  
    double time_taken = chrono::duration_cast<chrono::nanoseconds>(end -  
start).count();  
    time_taken *= 1e-9;  
  
    cout << "Time difference is: " << time_taken << setprecision(6);  
  
    return 0;  
}
```

Output:

```
input
Given array:
86 77 15 93 35 86 92 49 21 62 27 90 59 63 26 40 26 72 36 11 68 67 29 82 30 6
2 23 67 35 29 2 22 58 69 67 93 56 11 42 29 73 21 19 84 37 98 24 15 70 13 26
91 80 56 73 62 70 96 81 5 25 84 27 36 5 46 29 13 57 24 95 82 45 14 67 34 64
43 50 87 8 76 78

Sorted array:
2 5 5 8 11 11 13 13 14 15 15 19 21 21 22 23 24 24 25 26 26 26 27 27 29 29 29
29 30 34 35 35 36 36 37 40 42 43 45 46 49 50 56 56 57 58 59 62 62 62 63 64
67 67 67 67 68 69 70 70 72 73 73 76 77 78 80 81 82 82 84 84 86 86 87 90 91 9
2 93 93 95 96 98

Time difference is: 3.4026e-05

...Program finished with exit code 0
Press ENTER to exit console.
```

Batch Analysis:

Source Code:

```
#include <bits/stdc++.h>
#include <chrono>

using namespace std;

// A utility function to swap two elements
void swap(int *a, int *b)
{
    int temp = *a;
    *a = *b;
    *b = temp;
}

// last element pivot and lower in 1 half and greater in 1 half divide
int partition(int arr[], int low, int high)
{
    int pivot = arr[high];
    int i = (low - 1);    // Index of smaller element and indicates the right
    position of pivot found so far

    for (int j = low; j <= high - 1; j++)
    {
        // j == current element
        if (arr[j] < pivot)
        {
            i++; // increment index of smaller element
            swap(&arr[i], &arr[j]);
        }
    }
    swap(&arr[i + 1], &arr[high]);
    return (i + 1);
}

void quickSort(int arr[], int low, int high)
{
    if (low < high)
    {
        int p = partition(arr, low, high);

        // Separately sort elements before partition and after partition
```

```
        quickSort(arr, low, p - 1);
        quickSort(arr, p + 1, high);
    }
}

void printArray(double arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << ", ";
    cout << endl;
}

void printArray(int arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << ", ";
    cout << endl;
}

void printArray(float arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << ", ";
    cout << endl;
}

double sortApp(int n) {
    int arr[n];
    for (int i = 0; i < n; i++) {
        arr[i] = rand() % 100;
    }

    auto start = chrono::high_resolution_clock::now();
    ios_base::sync_with_stdio(false);

    quickSort(arr, 0, n - 1);

    auto end = chrono::high_resolution_clock::now();

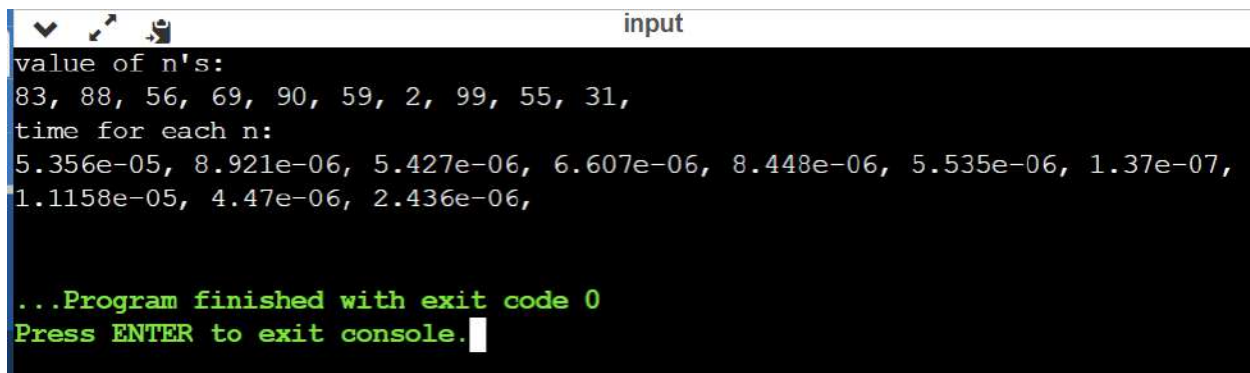
    double time_taken = chrono::duration_cast<chrono::nanoseconds>(end -
start).count();
    time_taken *= 1e-9;

    return time_taken;
}
```



```
int main() {
    double times[10];
    int ns[10];
    for (int x = 0; x < 10; x++)
    {
        int n = rand() % 100;
        ns[x] = n;
        times[x] = sortApp(n);
    }
    cout << "value of n's: " << endl;
    printArray(ns, 10);
    cout << "time for each n: " << endl;
    printArray(times, 10);
}
```

Output:

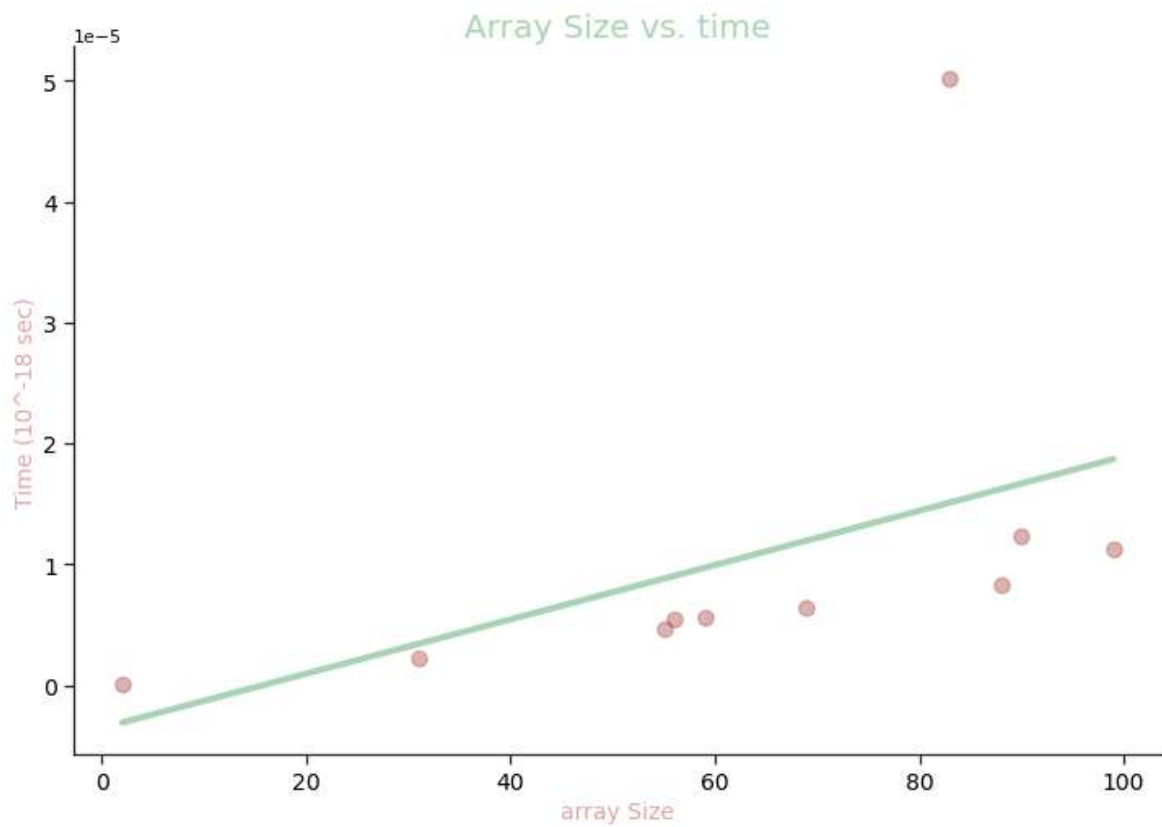
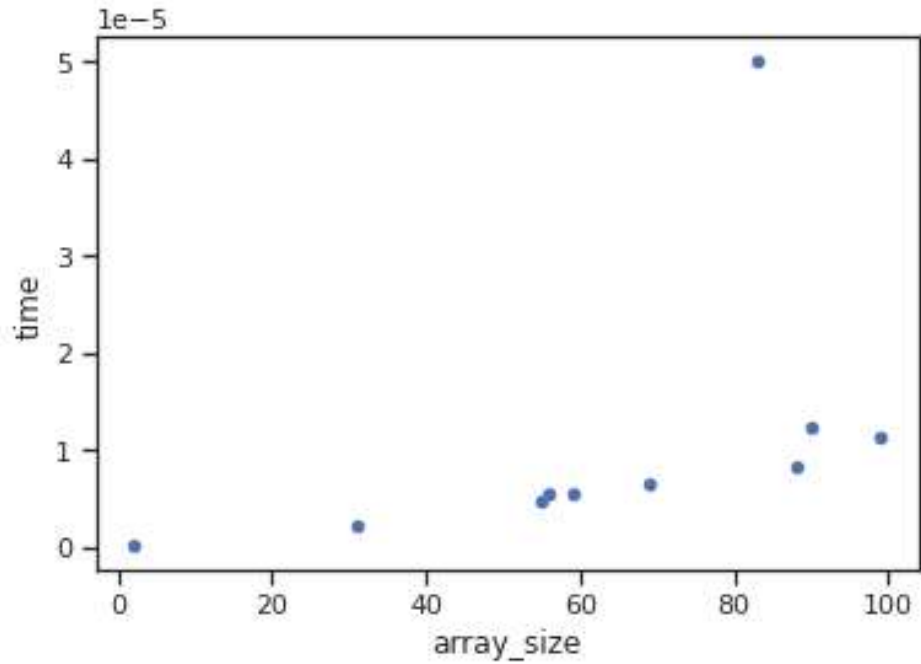


```
input
value of n's:
83, 88, 56, 69, 90, 59, 2, 99, 55, 31,
time for each n:
5.356e-05, 8.921e-06, 5.427e-06, 6.607e-06, 8.448e-06, 5.535e-06, 1.37e-07,
1.1158e-05, 4.47e-06, 2.436e-06,

...Program finished with exit code 0
Press ENTER to exit console.
```

```
arraySize = [83, 88, 56, 69, 90, 59, 2, 99, 55, 31]
```

```
time = [5.017e-05, 8.247e-06, 5.499e-06, 6.48e-06, 1.2334e-05, 5.57e-06,
1.35e-07, 1.1314e-05, 4.678e-06, 2.284e-06]
```



Quick Sort

Viva Questions

1. What value of q does PARTITION return when all elements in the array $A[p \dots r]$ have the same value?

Ans.

If all elements are equal, then when PARTITION returns, $q = r$ and all elements in $A[p \dots q-1]$ are equal. We get the recurrence $T(n) = T(n-1) + \Theta(n)$ for the running time, and so $T(n) = \Theta(n^2)$.

PARTITION(A, p, r)
 1 $x = A[r]$ 2 $i = p - 1$ 3 for $j = p$ to $r - 1$ 4 if $A[j] \leq x$ 5 $i = i + 1$ 6 exchange $A[i]$ with $A[j]$ 7 exchange $A[i + 1]$ with $A[r]$ 8 return $i + 1$
 Since the if test on the analogue of line 4 is thus successful when all entries are equal, in that case the value of k when the for loop terminates is $k = r-1$, so the value of q returned is $q = k+1 = r$.

2. Give a brief argument that the running time of PARTITION on a sub array of size n is $\Theta(n)$.

Ans.

There is a for statement whose body executes $r - 1 - p = \Theta(n)$ times. In the worst case every time the body of the if is executed, but it takes constant time and so does the code outside of the loop. Thus the running time is $\Theta(n)$.

3. How would you modify QUICKSORT to sort in non-increasing order?

Ans.

It is one of the fastest sorting algorithms known and is the method of choice in most sorting libraries. QUICKSORT is based on the divide and conquer strategy.

4. Why Quick sort is considered as best sorting?

Ans.

The cache efficiency argument has already been explained in detail. In addition, there is an intrinsic argument, why Quicksort is fast. If implemented like with two "crossing pointers", e.g. here, the inner loops have a very small body. As this is the code executed most often, this pays off.

5. What is randomized version of Quick sort?

Ans.

Randomized quick sort chooses a random element as a pivot. It is done so as to avoid the worst case of quick sort in which the input array is already sorted.

Bubble Sort:

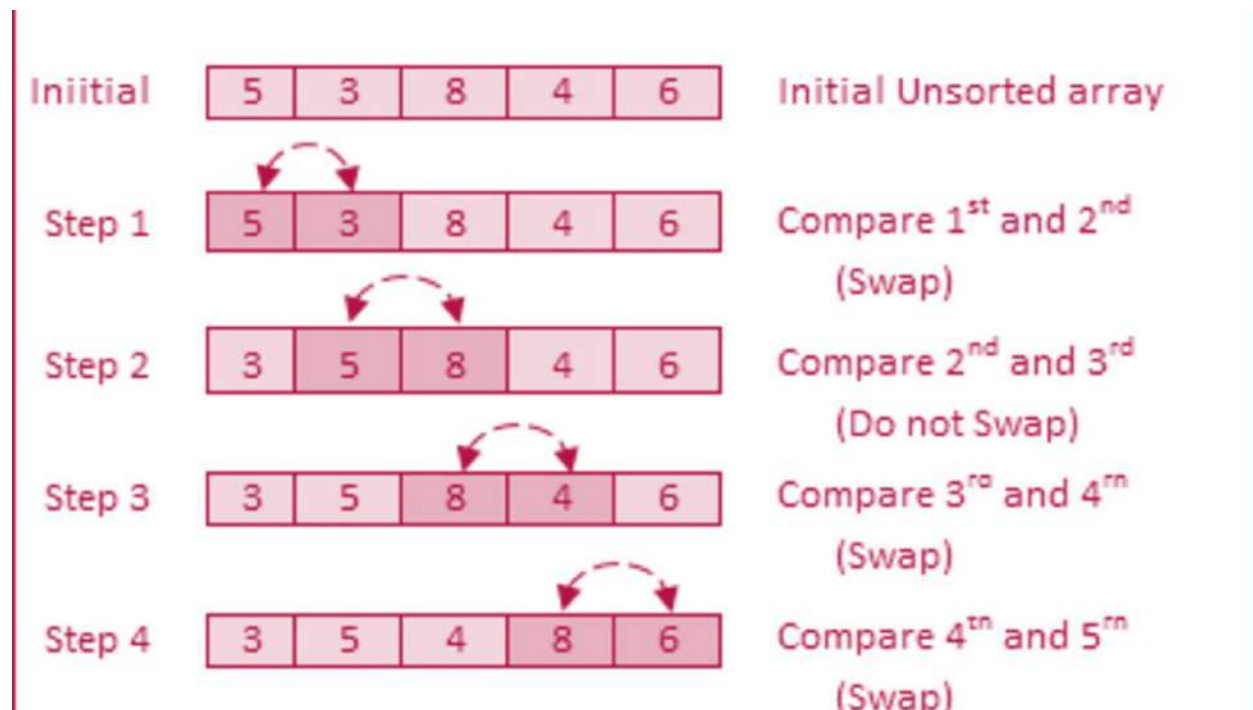
This sorting algorithm is comparison-based algorithm in which each pair of adjacent elements is compared, and the elements are swapped if they are not in order.

Pseudo code

We assume `list` is an array of `n` elements. We further assume that `swap` function swaps the values of the given array elements.

```
begin BubbleSort (list)
  for all elements of list
    if list[i] > list[i+1]
      swap (list[i], list [i+1])
    end if
  end for
  return list
end BubbleSort
```

Example:



Result and Analysis

Worst and Average Case Time Complexity: $O(n^2)$. Worst case occurs when array is reverse sorted and its time complexity is $O(n^2)$ and Best case occurs when array is already sorted and its time complexity is $O(n)$ (linear time). This sort takes just $O(1)$ extra space.

Source Code:

```
#include <bits/stdc++.h>
#include <chrono>
using namespace std;

void swap(int *xp, int *yp) {
    int temp = *xp;
    *xp = *yp;
    *yp = temp;
}

void bubbleSort(int arr[], int n) {
    int i, j;
    bool swapped;
    for (i = 0; i < n - 1; i++) {
        swapped = false;
        for (j = 0; j < n - i - 1; j++) {
            if (arr[j] > arr[j+1]) {
                swap(&arr[j], &arr[j + 1]);
                swapped = true;
            }
        }
        if (swapped == false) { // no swap means array sorted so break out
            break;
        }
    }
}

void printArray(int arr[], int size) {
    int i;
    for (i = 0; i < size; i++)
```

```
        cout << arr[i] << " ";
    cout << endl;
}

int main() {
    // int arr[] = {64, 34, 25, 12, 22, 11, 90};
    // int n = sizeof(arr)/sizeof(arr[0]);

    int n = rand() % 100;
    int arr[n];

    for (int i = 0; i < n; i++) {
        arr[i] = rand() % 100;
    }

    cout << "Array: ";
    printArray(arr, n);
    cout << endl;

    auto start = chrono::high_resolution_clock::now();

    // unsync the I/O of C and C++.
    ios_base::sync_with_stdio(false);

    bubbleSort(arr, n);

    auto end = chrono::high_resolution_clock::now();

    cout << "Sorted array: ";
    printArray(arr, n);
    cout << endl;

    double time_taken = chrono::duration_cast<chrono::nanoseconds>(end -
start).count();
    time_taken *= 1e-9;

    cout << "Time difference is: " << time_taken << setprecision(6);

    return 0;
}
```

Output:

```
input
Array: 86 77 15 93 35 86 92 49 21 62 27 90 59 63 26 40 26 72 36 11 68 67 29
82 30 62 23 67 35 29 2 22 58 69 67 93 56 11 42 29 73 21 19 84 37 98 24 15 70
13 26 91 80 56 73 62 70 96 81 5 25 84 27 36 5 46 29 13 57 24 95 82 45 14 67
34 64 43 50 87 8 76 78

Sorted array: 2 5 5 8 11 11 13 13 14 15 15 19 21 21 22 23 24 24 25 26 26 26
27 27 29 29 29 29 30 34 35 35 36 36 37 40 42 43 45 46 49 50 56 56 57 58 59 6
2 62 62 63 64 67 67 67 67 68 69 70 70 72 73 73 76 77 78 80 81 82 82 84 84 86
86 87 90 91 92 93 93 95 96 98

Time difference is: 5.3534e-05

...Program finished with exit code 0
Press ENTER to exit console.
```


Batch Analysis:

Source Code:

```
#include <bits/stdc++.h>

#include <chrono>

using namespace std;

void swap(int *xp, int *yp)
{
    int temp = *xp;
    *xp = *yp;
    *yp = temp;
}

void bubbleSort(int arr[], int n)
{
    int i, j;
    bool swapped;
    for (i = 0; i < n - 1; i++)
    {
        swapped = false;
        for (j = 0; j < n - i - 1; j++)
        {
            if (arr[j] > arr[j + 1])
            {
                swap(&arr[j], &arr[j + 1]);
                swapped = true;
            }
        }
    }
}
```

```
        if (swapped == false)
        { // no swap means array sorted so break out
            break;
        }
    }
}
```

```
void printArray(double arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << ", ";
    cout << endl;
}
```

```
void printArray(int arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << ", ";
    cout << endl;
}
```

```
void printArray(float arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << ", ";
    cout << endl;
}
```

```
double sortApp(int n)
{
```

```
int arr[n];
for (int i = 0; i < n; i++)
{
    arr[i] = rand() % 100;
}

auto start = chrono::high_resolution_clock::now();
ios_base::sync_with_stdio(false);

bubbleSort(arr, n);

auto end = chrono::high_resolution_clock::now();

double time_taken = chrono::duration_cast<chrono::nanoseconds>(end -
start).count();
time_taken *= 1e-9;

return time_taken;
}

int main()
{
    double times[10];
    int ns[10];
    for (int x = 0; x < 10; x++)
    {
        int n = rand() % 100;
        ns[x] = n;
        times[x] = sortApp(n);
    }
    cout << "value of n's: " << endl;
```

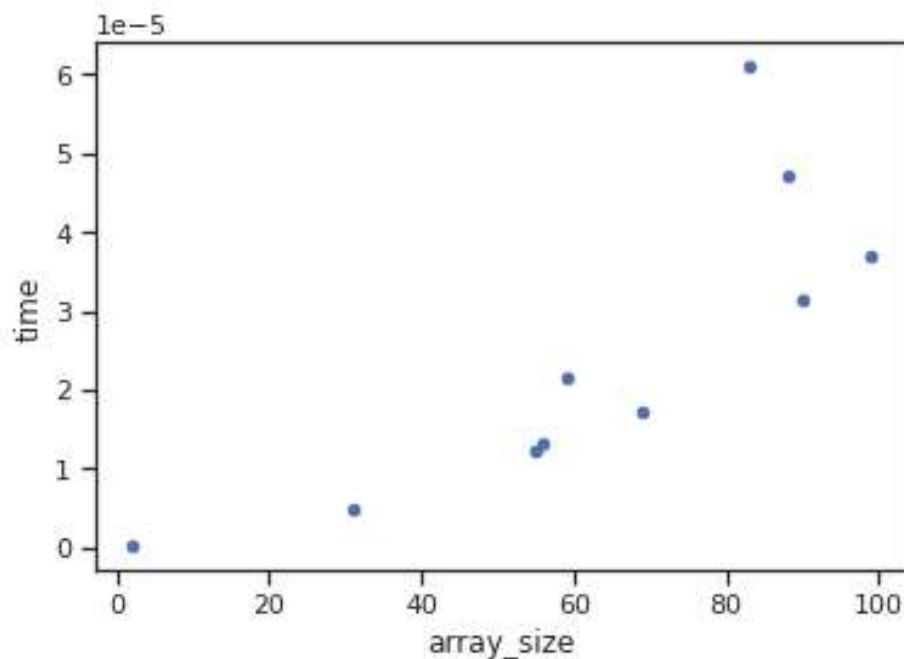
```
printArray(ns, 10);  
cout << "time for each n: " << endl;  
printArray(times, 10);  
}
```

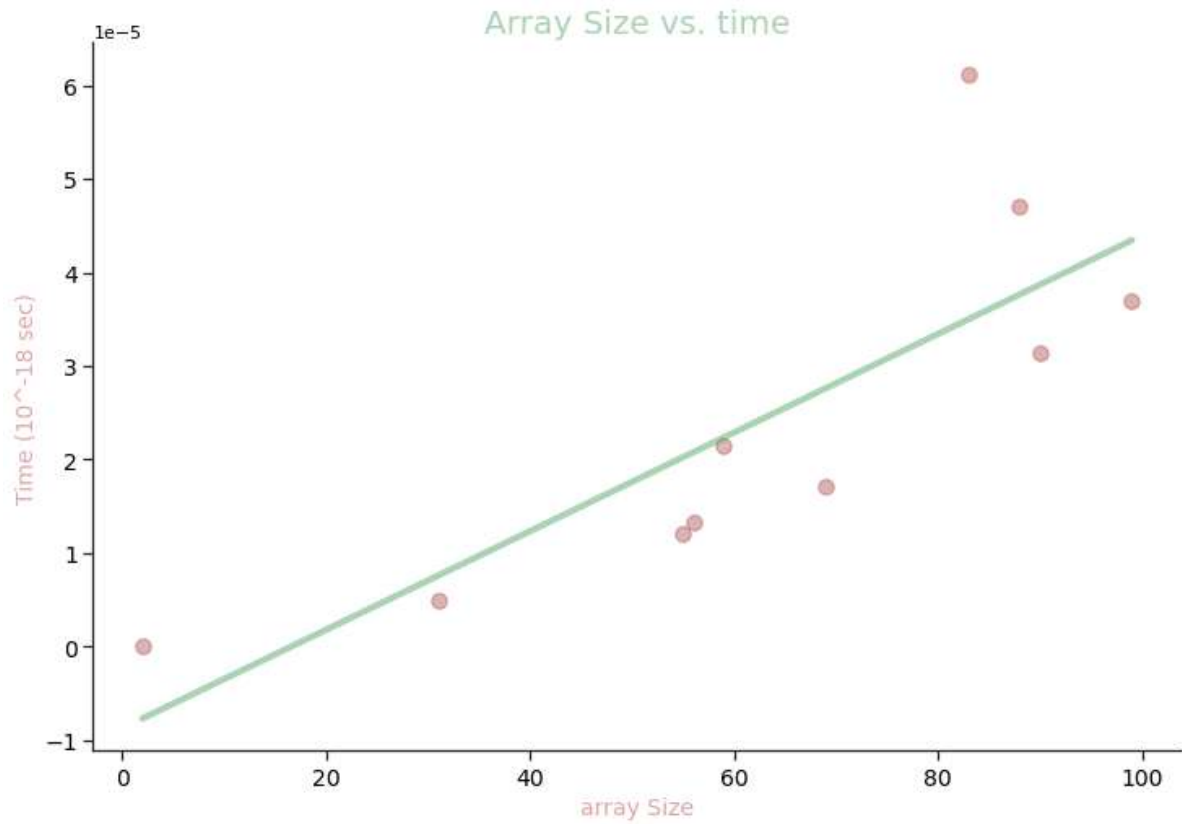
Output:

```
input  
value of n's:  
83, 88, 56, 69, 90, 59, 2, 99, 55, 31,  
time for each n:  
8.4473e-05, 3.8244e-05, 1.6018e-05, 2.1333e-05, 6.078e-05, 2.466e-05, 1.39e-  
07, 4.5619e-05, 1.4549e-05, 6.06e-06,  
  
...Program finished with exit code 0  
Press ENTER to exit console.
```

arraySize = [83, 88, 56, 69, 90, 59, 2, 99, 55, 31]

**time = [6.1129e-05, 4.7125e-05, 1.3298e-05, 1.7092e-05, 3.147e-05, 2.1538e-05,
8.6e-08, 3.6987e-05, 1.2126e-05, 4.909e-06]**





Bubble Sort

Viva Questions

1. How can the best-case efficiency of bubble sort be improved?

Ans.

Some iterations can be skipped if the list is sorted, hence efficiency improves to $O(n)$.

A better version of bubble sort, known as modified bubble sort, includes a flag that is set if an exchange is made after an entire pass over the array. If no exchange is made, then it should be clear that the array is already in order because no two elements need to be switched.

2. Is bubble sorting a stable sort?

Ans.

Yes

sorting algorithm is said to be stable if two objects with equal keys appear in the same order in sorted output as they appear in the input array to be sorted.

3. How much time will bubble sort take if all the elements are same?

Ans.

$O(n)$

Bubble sort has a worst-case and average complexity of $O(n^2)$, where n is the number of items being sorted. Most practical sorting algorithms have substantially better worst-case or average complexity, often $O(n \log n)$.

4. What would happen if bubble sort didn't keep track of the number of swaps made on each pass through the list?

Ans.

The algorithm wouldn't know when to terminate as it would have no way of knowing when the list was in sorted order.

5. List main properties to be considered in bubble sort?

Ans.

- Large values are always sorted first.
- It only takes one iteration to detect that a collection is already sorted.
- The best time complexity for Bubble Sort is $O(n)$
- The space complexity for Bubble Sort is $O(1)$, because only single additional memory space is required.

Bucket Sort:

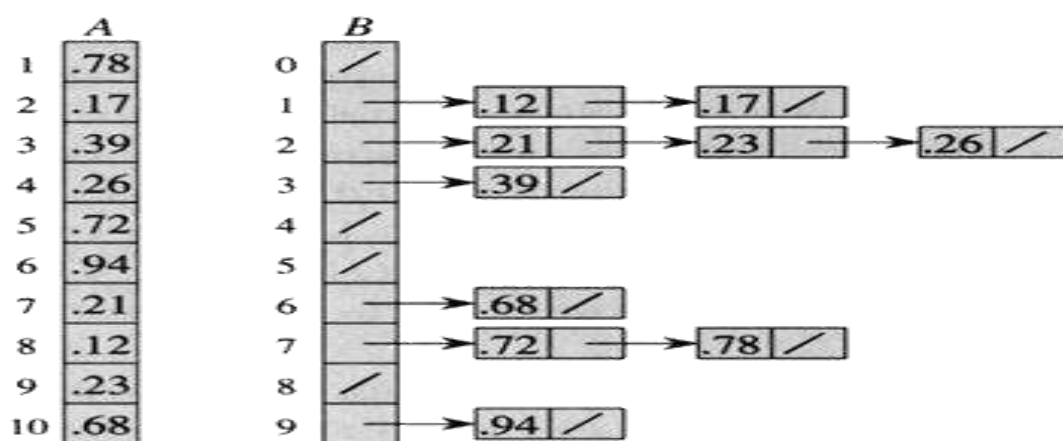
Bucket sort, or **bin sort**, is a sorting algorithm that works by distributing the elements of an array into a number of buckets. Each bucket is then sorted individually, either using a different sorting algorithm, or by recursively applying the bucket sorting algorithm.

It is a distribution sort, a generalization of pigeonhole sort, and is a cousin of radix sort in the most-to-least significant digit flavour. Bucket sort can be implemented with comparisons and therefore can also be considered a comparison sort algorithm.

It works as follows:

1. Set up an array of initially empty "buckets".
2. **Scatter**: Go over the original array, putting each object in its bucket.
3. Sort each non-empty bucket.
4. **Gather**: Visit the buckets in order and put all elements back into the original array.

. This will be the sorted list.



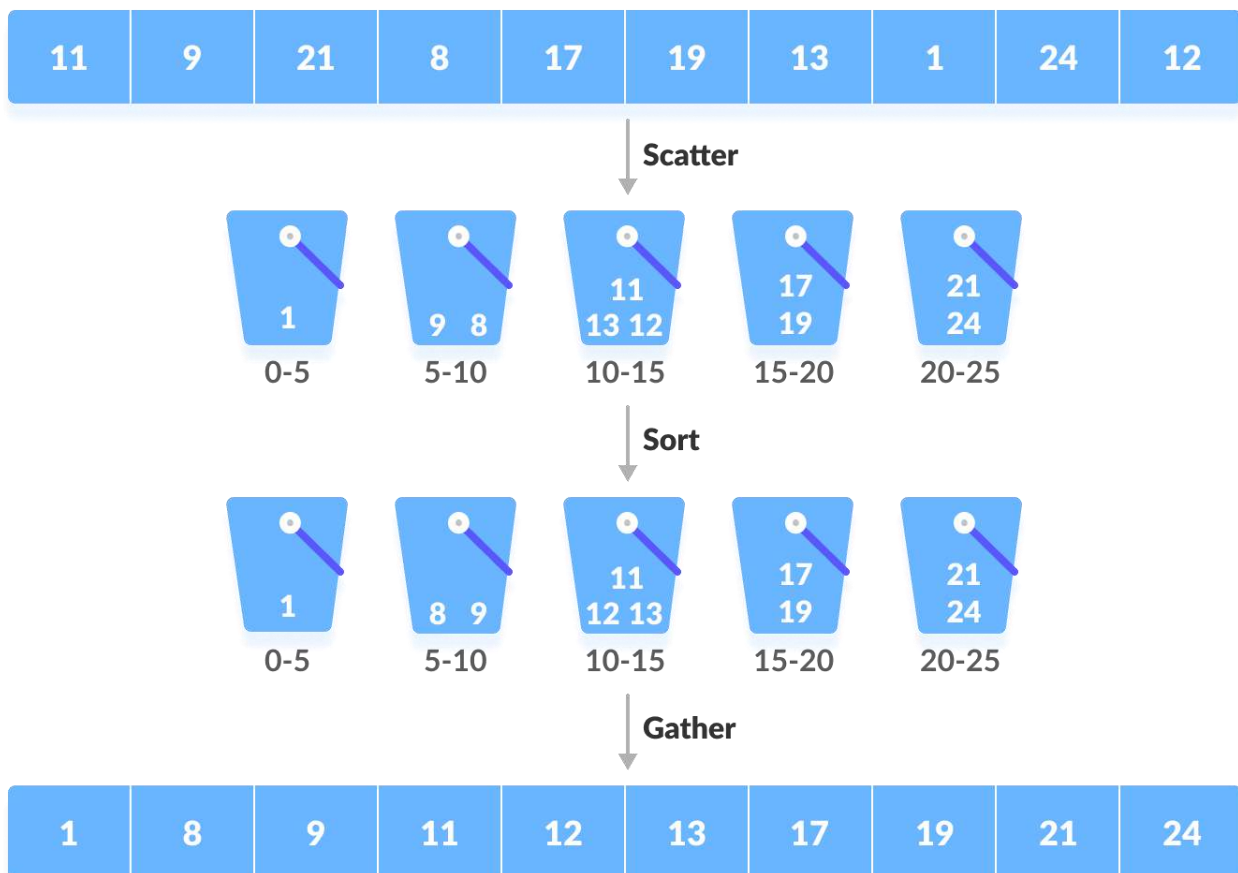
Pseudo code

The code assumes that input is an n -element array A and each element in A satisfies $0 \leq A[i] \leq 1$. We also need an auxiliary array $B[0 \dots n-1]$ for linked-lists (buckets).

BUCKET SORT (A)

1. $n \leftarrow \text{length}[A]$
2. For $i = 1$ to n do
3. Insert $A[i]$ into list $B[A[i]/b]$ where b is the bucket size
4. For $i = 0$ to $n-1$ do
5. Sort list B with Insertion sort
6. Concatenate the lists $B[0], B[1] \dots B[n-1]$ together in order.

Example:



Result and Analysis

In the above Bucket sort algorithm, we observe in the *best case*, the algorithm distributes the elements uniformly between buckets, a few elements are placed on each bucket and sorting the buckets is **$O(1)$** . Rearranging the elements is one more run through the initial list.

$$\begin{aligned}
 T(n) &= [\text{time to insert } n \text{ elements in array } A] + [\text{time to go through auxiliary array } B \\
 &\quad [0 \dots n-1] * (\text{Sort by INSERTION_SORT}) \\
 &= O(n) + (n-1) \cdot O(n) \\
 &= O(n^2)
 \end{aligned}$$

In the *worst case*, the elements are sent all to the same bucket, making the process take **$O(n^2)$** .

Source Code:

```

#include <bits/stdc++.h>
#include <chrono>
using namespace std;

void bucketSort(float arr[], int n) {
    vector<float> bucket[n];

    // put elements in respective buckets
    for (int i = 0; i < n; i++) {
        int bi = n * arr[i]; // Index in bucket
        bucket[bi].push_back(arr[i]);
    }

    for (int i = 0; i < n; i++) // sorting each buckets
        sort(bucket[i].begin(), bucket[i].end());

    int index = 0;
    for (int i = 0; i < n; i++) // Concatenate all buckets into arr[]
        for (int j = 0; j < bucket[i].size(); j++)
            arr[index++] = bucket[i][j];
}

```

```
void printArray(float arr[], int size)    {
    for (int i = 0; i < size; i++)
        cout << arr[i] << " ";
    cout << endl;
}

int main() {
    // int arr[] = {64, 34, 25, 12, 22, 11, 90};
    // int n = sizeof(arr)/sizeof(arr[0]);
    int n = rand() % 100;
    float arr[n];
    for (int i = 0; i < n; i++) {
        arr[i] = (float(rand())/float((RAND_MAX)));
    }
    cout << "Array: ";
    printArray(arr, n);
    cout << endl;

    auto start = chrono::high_resolution_clock::now();
    // unsync the I/O of C and C++.
    ios_base::sync_with_stdio(false);

    bucketSort(arr, n); // sort arr

    auto end = chrono::high_resolution_clock::now();

    cout << "Sorted array: ";
    printArray(arr, n);
    cout << endl;

    double time_taken = chrono::duration_cast<chrono::nanoseconds>(end -
start).count();
    time_taken *= 1e-9;

    cout << "Time difference is: " << time_taken << setprecision(6);
    return 0;
}
```

Output:

```
input
Array: 0.394383 0.783099 0.79844 0.911647 0.197551 0.335223 0.76823 0.277775
0.55397 0.477397 0.628871 0.364784 0.513401 0.95223 0.916195 0.635712 0.717
297 0.141603 0.606969 0.0163006 0.242887 0.137232 0.804177 0.156679 0.400944
0.12979 0.108809 0.998924 0.218257 0.512932 0.839112 0.61264 0.296032 0.637
552 0.524287 0.493583 0.972775 0.292517 0.771358 0.526745 0.769914 0.400229
0.891529 0.283315 0.352458 0.807725 0.919026 0.0697553 0.949327 0.525995 0.0
860558 0.192214 0.663227 0.890233 0.348893 0.0641713 0.020023 0.457702 0.063
0958 0.23828 0.970634 0.902208 0.85092 0.266666 0.53976 0.375207 0.760249 0.
512535 0.667724 0.531606 0.0392803 0.437638 0.931835 0.93081 0.720952 0.2842
93 0.738534 0.639979 0.354049 0.687861 0.165974 0.440105 0.880075

Sorted array: 0.0163006 0.020023 0.0392803 0.0630958 0.0641713 0.0697553 0.0
860558 0.108809 0.12979 0.137232 0.141603 0.156679 0.165974 0.192214 0.19755
1 0.218257 0.23828 0.242887 0.266666 0.277775 0.283315 0.284293 0.292517 0.2
96032 0.335223 0.348893 0.352458 0.354049 0.364784 0.375207 0.394383 0.40022
9 0.400944 0.437638 0.440105 0.457702 0.477397 0.493583 0.512535 0.512932 0.
513401 0.524287 0.525995 0.526745 0.531606 0.53976 0.55397 0.606969 0.61264
0.628871 0.635712 0.637552 0.639979 0.663227 0.667724 0.687861 0.717297 0.72
0952 0.738534 0.760249 0.76823 0.769914 0.771358 0.783099 0.79844 0.804177 0.
807725 0.839112 0.85092 0.880075 0.890233 0.891529 0.902208 0.911647 0.9161
95 0.919026 0.93081 0.931835 0.949327 0.95223 0.970634 0.972775 0.998924

Time difference is: 7.2901e-05

...Program finished with exit code 0
Press ENTER to exit console.
```

Batch Analysis:

Source Code:

```
#include <bits/stdc++.h>
#include <chrono>
using namespace std;

void bucketSort(float arr[], int n)
{
    vector<float> bucket[n];

    // put elemets in respective buckets
    for (int i = 0; i < n; i++)
    {
        int bi = n * arr[i]; // Index in bucket
        bucket[bi].push_back(arr[i]);
    }

    for (int i = 0; i < n; i++) // sorting each buckets
        sort(bucket[i].begin(), bucket[i].end());

    int index = 0;
    for (int i = 0; i < n; i++) // Concatenate all buckets into arr[]
        for (int j = 0; j < bucket[i].size(); j++)
            arr[index++] = bucket[i][j];
}

void printArray(double arr[], int size)
{
    for (int i = 0; i < size; i++)
```

```
        cout << arr[i] << " ";
    cout << endl;
}

void printArray(int arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << " ";
    cout << endl;
}

void printArray(float arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << " ";
    cout << endl;
}

int main()
{
    double times[10];
    int ns[10];
    for (int x = 0; x < 10; x++)
    {
        int n = rand() % 100;
        ns[x] = n;
        float arr[n];
        for (int i = 0; i < n; i++)
        {
            arr[i] = (float(rand()) / float((RAND_MAX)));
        }
    }
}
```

```
    }  
    cout << "Array: ";  
    printArray(arr, n);  
    cout << endl;  
  
    auto start = chrono::high_resolution_clock::now();  
    // unsync the I/O of C and C++.  
    ios_base::sync_with_stdio(false);  
  
    bucketSort(arr, n); // sort arr  
  
    auto end = chrono::high_resolution_clock::now();  
  
    cout << "Sorted array: ";  
    // printArray(arr, n);  
    cout << endl;  
  
    double time_taken = chrono::duration_cast<chrono::nanoseconds>(end -  
start).count();  
    time_taken *= 1e-9;  
  
    times[x] = time_taken;  
    cout << "Time difference is: " << time_taken << setprecision(6) << endl;  
}  
printArray(times, 10);  
printArray(ns, 10);  
return 0;  
}
```

Output:

```
input
0.611981 0.596899 0.645602 0.538557 0.148342 0.579022 0.0329634 0.70091 0.51
8151 0.832609 0.515049 0.112648 0.48981 0.510349 0.0484997

Sorted array:
Time difference is: 5.8125e-05
Array: 0.384658 0.637656 0.452122 0.143982 0.413078 0.247033 0.406767 0.0174
566 0.717597 0.573721 0.812947 0.582682 0.446743 0.477361 0.995165 0.0587232
0.0742604 0.640766 0.59728 0.222602 0.219788 0.630243 0.923513 0.737939 0.4
62852 0.438562 0.850586 0.952662 0.948911 0.899086 0.767014 0.333569 0.53674
3 0.219136 0.477551 0.94982 0.466169 0.884318 0.967277 0.183765 0.458039 0.7
80224 0.766448 0.904782 0.257585 0.761612 0.963505 0.331846 0.402379 0.56078
5 0.554448 0.622167 0.191028 0.477961 0.360105

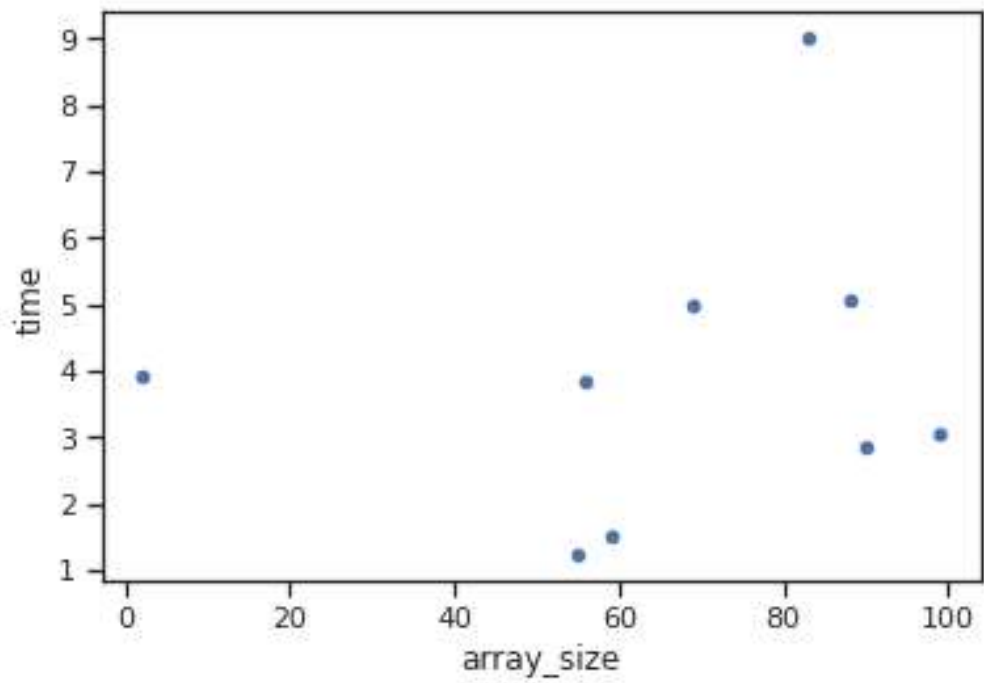
Sorted array:
Time difference is: 3.4668e-05
Array: 0.916523 0.210692 0.606542 0.865434 0.109778 0.373556 0.199003 0.6465
2 0.592692 0.676554 0.596341 0.0588605 0.560872 0.563617 0.242626 0.0189108
0.343841 0.00907344 0.923692 0.601427 0.770686 0.887197 0.933273 0.173065 0.
447982 0.487721 0.795231 0.639009 0.965682 0.155336 0.292889

Sorted array:
Time difference is: 1.3176e-05
7.9616e-05 3.5927e-05 5.5749e-05 4.1629e-05 5.2289e-05 3.5725e-05 1.955e-06
5.8125e-05 3.4668e-05 1.3176e-05
83 88 56 69 90 59 2 99 55 31

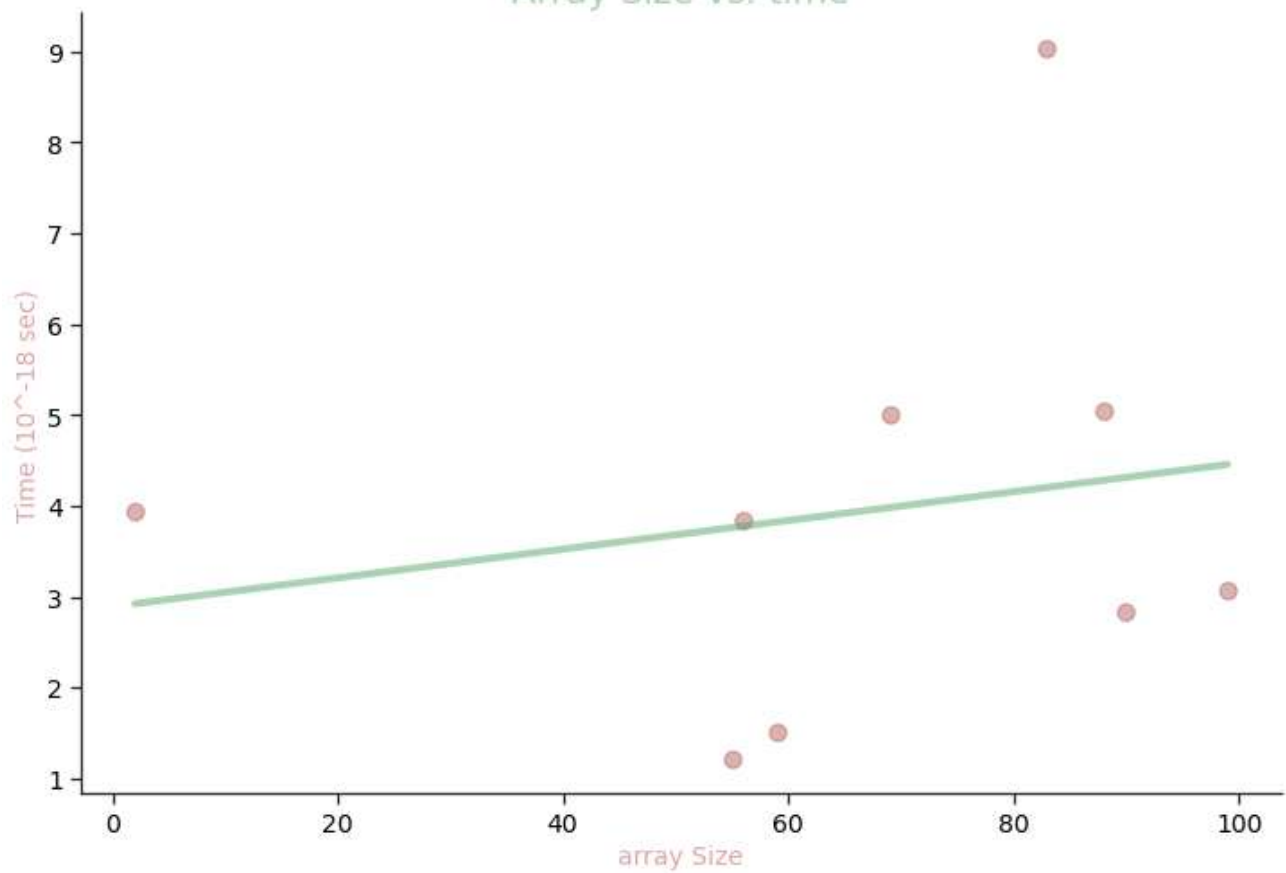
...Program finished with exit code 0
Press ENTER to exit console.
```

arraySize = [83, 88, 56, 69, 90, 59, 2, 99, 55, 31]

time = [9.0298, 5.0532, 3.8491, 5.0068, 2.8416, 1.505, 3.9323, 3.0733, 1.2236]



Array Size vs. time

**Bucket Sort**

Viva Questions

1. Is the bucket sort in place sort? Why or why not?

Ans.

No, it's not an in-place sorting algorithm. The whole idea is that input sorts themselves as they are moved to the buckets.

2. Is bucket sort a stable sort?

Ans.

Bucket sort is stable, if the underlying sort is also stable, as equal keys are inserted in order to each bucket. Counting sort works by determining how many integers are behind each integer in the input array A. Using this information, the input integer can be directly placed in the output array B.

3. Why bucket sort is good for large size arrays?

Ans.

Yes

The advantage of bucket sort is **that once the elements are distributed into buckets, each bucket can be processed independently of the others.** This means that you often need to sort much smaller arrays as a follow-up step than the original array.

Bucket sort works by distributing the array elements into a number of buckets. So bucket sort is most efficient in the case **when the input is uniformly distributed.**

4. State any disadvantage of using this sort.

Ans.

1. The problem is that if the buckets are distributed incorrectly, you may wind up spending a lot of extra effort for no or very little gain. As a result, bucket

sort works best when the data is more or less evenly distributed, or when there is a smart technique to pick the buckets given a fast set of heuristics based on the input array.

2. Can't apply it to all data types since a suitable bucketing technique is required. Bucket sort's efficiency is dependent on the distribution of the input values, thus it's not worth it if your data are closely grouped. In many situations, you might achieve greater performance by using a specialized sorting algorithm like radix sort, counting sort, or burst sort instead of bucket sort.
3. Bucket sort's performance is determined by the number of buckets used, which may need some additional performance adjustment when compared to other algorithms.

5. Can this sort be used for sorting negative numbers?

Ans.

Using Bucket sort for negative values simply **requires mapping each element to a bucket proportional to its** a distance from the minimal value to be sorted.

sortMixed(arr[], n)

- 1) Split array into two parts
create two Empty vector Neg[], Pos[]
(for negative and positive element respectively)
Store all negative element in Neg[] by converting into positive (Neg[i] = -1 * Arr[i])
Store all +ve in pos[] (pos[i] = Arr[i])
- 2) Call function bucketSortPositive(Pos, pos.size())
Call function bucketSortPositive(Neg, Neg.size())

bucketSortPositive(arr[], n)

- 3) Create n empty buckets (Or lists).
- 4) Do following for every array element arr[i].
 - a) Insert arr[i] into bucket[n*array[i]]
- 5) Sort individual buckets using insertion sort.
- 6) Concatenate all sorted buckets.

Radix Sort:

Radix Sort is a non-comparative sorting algorithm. It is one of the most efficient and fastest linear sorting algorithms. In radix sort, we first sort the elements based on last digit (least significant digit). Then the result is again sorted by second digit, continue this process for all digits until we reach most significant digit. We use counting sort to sort elements of every digit.

Pseudo code

```

Radix-Sort(A, d)
//It works same as counting sort for d number of passes.
//Each key in A [1...n] is a d-digit integer.
// (Digits are numbered 1 to d from right to left.)
  for j = 1 to d do
    //A[]-- Initial Array to Sort
    intcount[10] = {0};
    //Store the count of "keys" in count[]
    //key- it is number at digit place j
    for i = 0 to n do
      count[key of(A[i]) in pass j]++

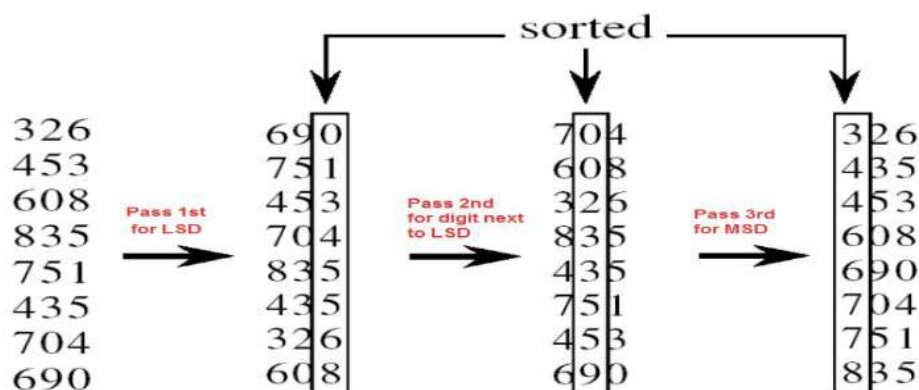
    fork = 1 to 10 do
      count[k] = count[k] + count[k-1]

    //Build the resulting array by checking
    //new position of A[i] from count[k]
    for i = n-1 down to 0 do
      result[ count[key of(A[i])] ] = A[j]
      count[key of(A[i])]--

    //Now main array A[] contains sorted numbers
    //according to current digit place
    for i=0 to n do
      A[i] = result[i]

  end for(j)
end func

```

Example:

In the above example:

For 1st pass: We sort the array on basis of least significant digit (1s place) using counting sort. Notice that 435 is below 835, because 435 occurred below 835 in the original list.

For 2nd pass: We sort the array on basis of next digit (10s place) using counting sort. Notice that here 608 is below 704, because 608 occurred below 704 in the previous list, and similarly for (835, 435) and (751, 453).

For 3rd pass: We sort the array on basis of most significant digit (100s place) using counting sort. Notice that here 435 is below 453, because 435 occurred below 453 in the previous list, and similarly for (608, 690) and (704, 751).

Result and Analysis

In this algorithm running time depends on intermediate sorting algorithm which is counting sort. If the range of digits is from 1 to k, then counting sort time complexity is $O(n+k)$. There are d passes i.e. counting sort is called d time, so total time complexity is $O(nd+nk) = O(nd)$. As $k=O(n)$ and d is constant, so radix sort runs in linear time. It performs the same way for best case as well

Initial Array	10	21	17	34	44	11	654	123
Sorted based on One's Place	10	21	11	123	34	44	654	17
Sorted based on Ten's Place	10	11	17	21	123	34	44	654
Sorted based on Hundred's Place	010	011	017	021	034	044	123	654

Source Code:

```
#include <bits/stdc++.h>
#include <chrono>
using namespace std;

int getMax(int arr[], int n)    {
    int maxEle = arr[0];
    for (int i = 1; i < n; i++)
        if (arr[i] > maxEle)
            maxEle = arr[i];
    return maxEle;
}

void countSort(int arr[], int n, int exp)    {
    int output[n]; // output array
    int count[10] = { 0 };

    // Store count of occurrences in count[]
    for (int i = 0; i < n; i++)
        count[(arr[i] / exp) % 10]++;

    // Change count[i] so that count[i] now contains actual position of this
    digit in output[]
    for (int i = 1; i < 10; i++)
        count[i] += count[i - 1];

    for (int i = n - 1; i >= 0; i--) {
        output[count[(arr[i] / exp) % 10] - 1] = arr[i];
        count[(arr[i] / exp) % 10]--;
    }

    // Copy the output array to arr[], so that arr[] now contains sorted numbers
    according to current digit
    for (int i = 0; i < n; i++)
        arr[i] = output[i];
}

void radixsort(int arr[], int n)    {
    // Find the maximum number to know number of digits
    int m = getMax(arr, n);

    // Do counting sort for every digit. Note that instead of passing digit
    number, exp is passed. exp is 10^i where i is current digit number
```

```
        for (int exp = 1; m / exp > 0; exp *= 10)
            countSort(arr, n, exp);
    }

    void printArray(int arr[], int size)    {
        for (int i = 0; i < size; i++)
            cout << arr[i] << " ";
        cout << endl;
    }

    int main() {
        // int arr[] = {64, 34, 25, 12, 22, 11, 90};
        // int n = sizeof(arr)/sizeof(arr[0]);
        int n = rand() % 100;
        int arr[n];
        for (int i = 0; i < n; i++) {
            arr[i] = rand() % 100;
        }
        cout << "Array: ";
        printArray(arr, n);
        cout << endl;

        auto start = chrono::high_resolution_clock::now();
        // unsync the I/O of C and C++.
        ios_base::sync_with_stdio(false);

        radixsort(arr, n); // sort arr

        auto end = chrono::high_resolution_clock::now();

        cout << "Sorted array: ";
        printArray(arr, n);
        cout << endl;

        double time_taken = chrono::duration_cast<chrono::nanoseconds>(end -
start).count();
        time_taken *= 1e-9;

        cout << "Time difference is: " << time_taken << setprecision(6);
        return 0;
    }
```

Output:

```
input
Array: 86 77 15 93 35 86 92 49 21 62 27 90 59 63 26 40 26 72 36 11 68 67 29
82 30 62 23 67 35 29 2 22 58 69 67 93 56 11 42 29 73 21 19 84 37 98 24 15 70
13 26 91 80 56 73 62 70 96 81 5 25 84 27 36 5 46 29 13 57 24 95 82 45 14 67
34 64 43 50 87 8 76 78

Sorted array: 2 5 5 8 11 11 13 13 14 15 15 19 21 21 22 23 24 24 25 26 26 26
27 27 29 29 29 29 30 34 35 35 36 36 37 40 42 43 45 46 49 50 56 56 57 58 59 6
2 62 62 63 64 67 67 67 67 68 69 70 70 72 73 73 76 77 78 80 81 82 82 84 84 86
86 87 90 91 92 93 93 95 96 98

Time difference is: 3.7338e-05

...Program finished with exit code 0
Press ENTER to exit console.
```


Batch Analysis:

Source Code:

```
#include <bits/stdc++.h>

#include <chrono>

using namespace std;

int getMax(int arr[], int n)    {
    int maxEle = arr[0];
    for (int i = 1; i < n; i++)
        if (arr[i] > maxEle)
            maxEle = arr[i];
    return maxEle;
}

void countSort(int arr[], int n, int exp)    {
    int output[n]; // output array
    int count[10] = { 0 };

    // Store count of occurrences in count[]
    for (int i = 0; i < n; i++)
        count[(arr[i] / exp) % 10]++;

    // Change count[i] so that count[i] now contains actual position of this
    digit in output[]
    for (int i = 1; i < 10; i++)
        count[i] += count[i - 1];

    for (int i = n - 1; i >= 0; i--) {
        output[count[(arr[i] / exp) % 10] - 1] = arr[i];
        count[(arr[i] / exp) % 10]--;
    }
}
```

```
}

// Copy the output array to arr[], so that arr[] now contains sorted numbers
according to current digit
for (int i = 0; i < n; i++)
    arr[i] = output[i];
}

void radixsort(int arr[], int n)    {
    // Find the maximum number to know number of digits
    int m = getMax(arr, n);

    // Do counting sort for every digit. Note that instead of passing digit
    number, exp is passed. exp is 10^i where i is current digit number
    for (int exp = 1; m / exp > 0; exp *= 10)
        countSort(arr, n, exp);
}

void printArray(double arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << ", ";
    cout << endl;
}

void printArray(int arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << ", ";
    cout << endl;
}
```

```
void printArray(float arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << ", ";
    cout << endl;
}
```

```
double sortApp(int n) {
    int arr[n];
    for (int i = 0; i < n; i++) {
        arr[i] = rand() % 100;
    }

    auto start = chrono::high_resolution_clock::now();
    ios_base::sync_with_stdio(false);

    radixsort(arr, n); // sort arr

    auto end = chrono::high_resolution_clock::now();

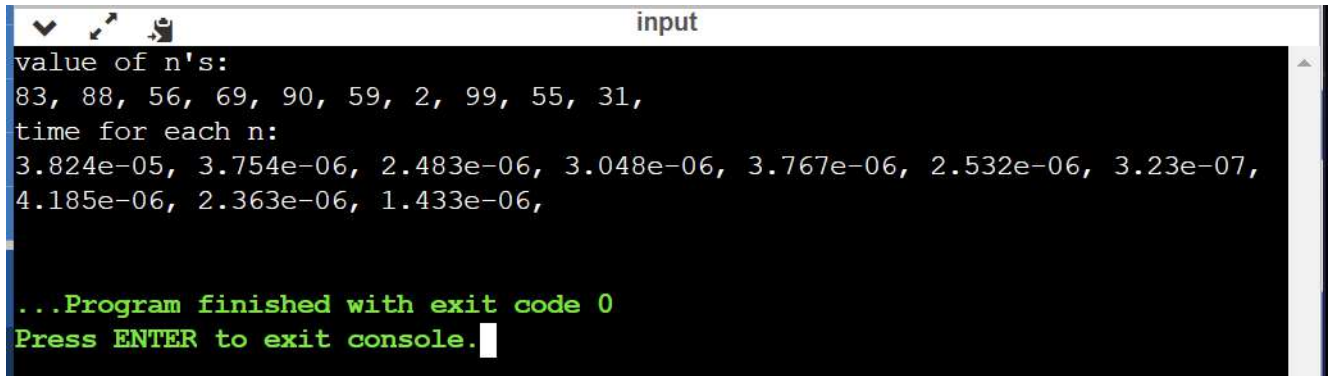
    double time_taken = chrono::duration_cast<chrono::nanoseconds>(end -
start).count();
    time_taken *= 1e-9;

    return time_taken;
}
```

```
int main() {
    double times[10];
    int ns[10];
```

```
for (int x = 0; x < 10; x++)
{
    int n = rand() % 100;
    ns[x] = n;
    times[x] = sortApp(n);
}
cout << "value of n's: " << endl;
printArray(ns, 10);
cout << "time for each n: " << endl;
printArray(times, 10);
}
```

Output:

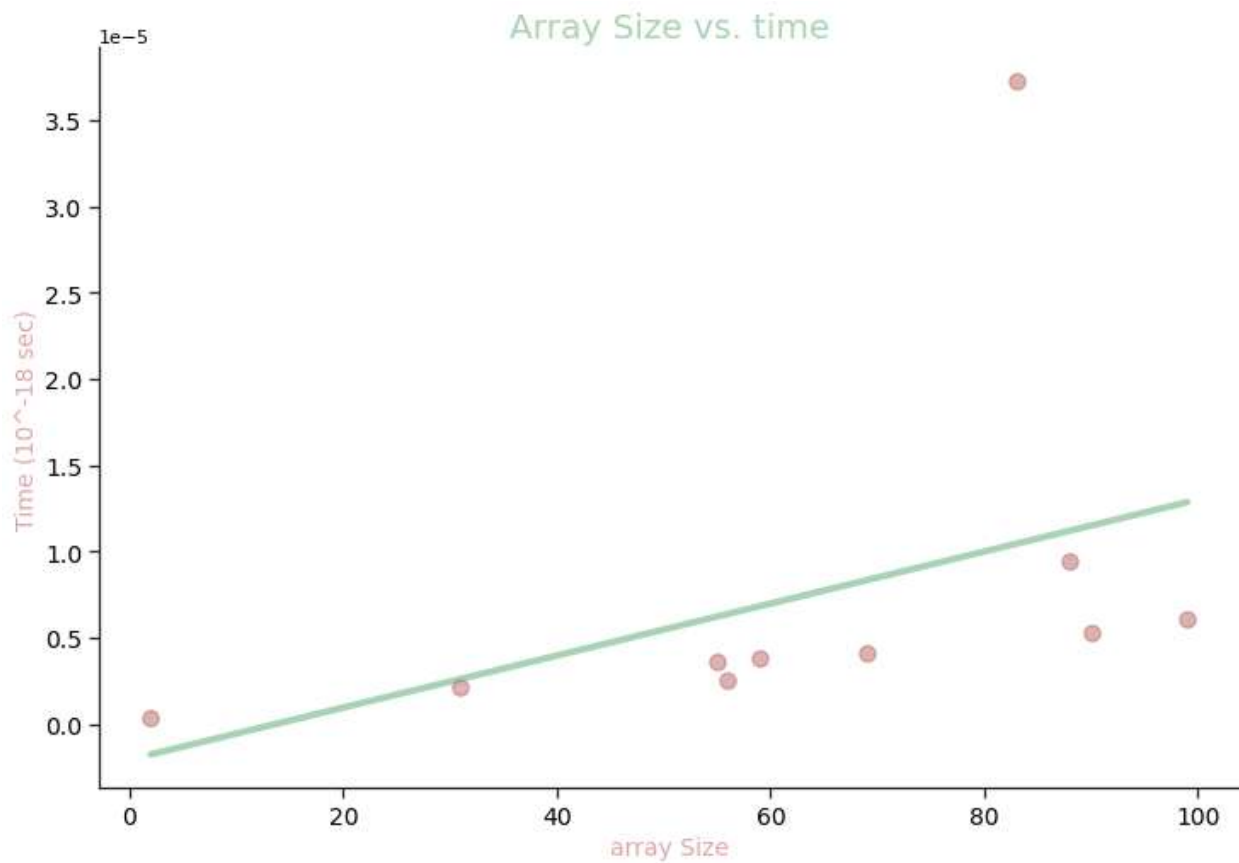
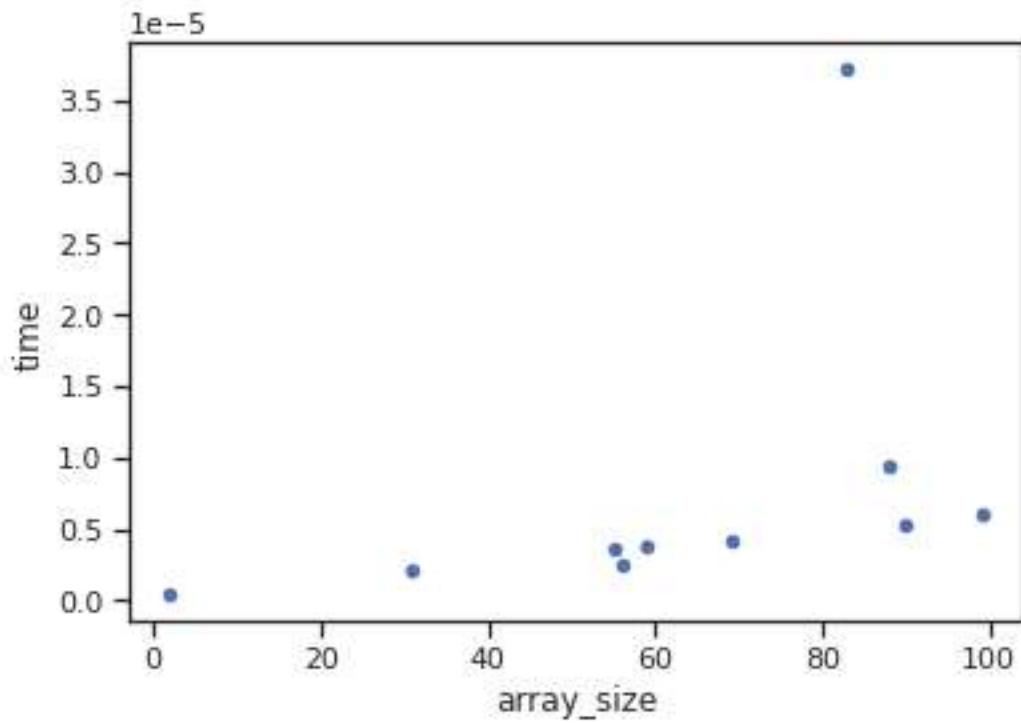


```
input
value of n's:
83, 88, 56, 69, 90, 59, 2, 99, 55, 31,
time for each n:
3.824e-05, 3.754e-06, 2.483e-06, 3.048e-06, 3.767e-06, 2.532e-06, 3.23e-07,
4.185e-06, 2.363e-06, 1.433e-06,

...Program finished with exit code 0
Press ENTER to exit console.
```

arraySize = [83, 88, 56, 69, 90, 59, 2, 99, 55, 31]

time = [3.7257e-05, 9.437e-06, 2.511e-06, 4.097e-06, 5.246e-06, 3.785e-06, 3.63e-07, 6.097e-06, 3.63e-06, 2.104e-06]



Radix Sort

Viva Questions

1. Is Radix Sort preferable to Comparison based sorting algorithms like Quick-Sort?

Ans.

If we have $\log_2 n$ bits for every digit, the **running time of Radix appears to be better than Quick Sort** for a wide range of input numbers. The constant factors hidden in asymptotic notation are higher for Radix Sort and Quick-Sort uses hardware caches more effectively.

Radix sort is slower for (most) real world use cases.

One reason is the complexity of the algorithm:

If items are unique, $k \geq \log(n)$. Even with duplicate items, the set of problems where $k < \log(n)$ is small.

Another is the implementation:

The additional memory requirement (which in it self is a disadvantage), affects cache performance negatively.

2. Is this sort stable?

Ans.

No

MSD sorts are not necessarily stable if the original ordering of duplicate keys must always be maintained. Other than the traversal order, MSD and LSD sorts differ in their handling of variable length input. LSD sorts can group by length, radix sort each group, then concatenate the groups in size order.

3. How much additional space is taken by this sort?

Ans.

$O(k + n)$

Radix sort's **space complexity is bound to the sort it uses to sort each radix**. In best case, that is counting sort. As you can see, counting sort creates multiple arrays, one based on the size of K , and one based on the size of N . B is the output array which is size n .

4. State any disadvantage of using this sort?

Ans.

- Since Radix Sort depends on digits or letters, Radix Sort is much less flexible than other sorts. Hence, for every different type of data it needs to be rewritten.
- The constant for Radix sort is greater compared to other sorting algorithms.
- It takes more space compared to Quicksort which is inplace sorting.

5. How can the performance of this sort be improved?

Ans.

Bucket sort is mainly useful when input is uniformly distributed over a range. For example, consider the following problem.

Sort a large set of floating point numbers which are in range from 0.0 to 1.0 and are uniformly distributed across the range. How do we sort the numbers efficiently?

A simple way is to apply a comparison based sorting algorithm. The lower bound for Comparison based sorting algorithm (Merge Sort, Heap Sort, Quick-Sort .. etc) is $\Omega(n \log n)$, i.e., they cannot do better than $n \log n$.

Can we sort the array in linear time? Counting sort can not be applied here as we use keys as index in counting sort. Here keys are floating point numbers.

The idea is to use bucket sort.

Shell Sort:

It is a generalized version of insertion sort. It is an in-place comparison sort. It is also known as **diminishing increment sort**; it is one of the oldest sorting algorithms invented by Donald L. Shell (1959.) This algorithm uses insertion sort on the large interval of elements to sort. Then the interval of sorting keeps on decreasing in a sequence until the interval reaches 1. These intervals are known as **gap sequence**.

Increment Sequences:

Shell's original sequence: $N/2, N/4, \dots, 1$ (repeatedly divide by 2);

Hibbard's increments: $1, 3, 7, \dots, 2^k - 1$;

Knuth's increments: $1, 4, 13, \dots, (3^k - 1) / 2$;

Sedgewick's increments: $1, 5, 19, 41, 109, \dots$

Here interval is calculated based on Knuth's formula as –

Knuth's Formula

$h = h * 3 + 1$, where h is interval with initial value 1

Pseudo code

Procedure shellSort ()

A: array of items

/ calculate interval*/*

while interval < A. length /3 do:

*interval = interval * 3 + 1*

end while

while interval > 0 do:

for outer = interval; outer < A. length; outer ++ do:

/ select value to be inserted */*

ValueToInsert = A[outer]

inner = outer;

*/*shift element towards right*/*

while inner > interval -1 && A [inner - interval] >=

valueToInsert do:


```

        A[inner] = A [inner - interval]
        inner = inner - interval
    end while
    /* insert the number at hole position */
    A[inner] = valueToInsert
end for
/* calculate interval*/
interval = (interval -1) /3;
end while
end procedure

```

Example:

	a ₁	a ₂	a ₃	a ₄	a ₅	a ₆	a ₇	a ₈	a ₉	a ₁₀	a ₁₁	a ₁₂
Input data	62	83	18	53	07	17	95	86	47	69	25	28
After 5-sorting	17	28	18	47	07	25	83	86	53	69	62	95
After 3-sorting	17	07	18	47	28	25	69	62	53	83	86	95
After 1-sorting	07	17	18	25	28	47	53	62	69	83	86	95

The first pass, 5-sorting, performs insertion sort on five separate sub arrays (a₁, a₆, a₁₁), (a₂, a₇, a₁₂), (a₃, a₈), (a₄, a₉), (a₅, a₁₀). For instance, it changes the sub array (a₁, a₆, a₁₁) from (62, 17, 25) to (17, 25, 62). The next pass, 3-sorting, performs insertion sort on the three sub arrays (a₁, a₄, a₇, a₁₀), (a₂, a₅, a₈, a₁₁), (a₃, a₆, a₉, a₁₂). The last pass, 1-sorting, is an ordinary insertion sort of the entire array (a₁... a₁₂).

Result and Analysis

Since in this algorithm insertion sort is applied in the large interval of elements and then interval reduces in a sequence, therefore the running time of Shell sort is heavily dependent on the gap sequence it uses. So in worst case time complexity is $O(n^2)$ and in Average Case time complexity depends on gap sequence while in Best Case Time complexity is $O(n \log n)$

Source Code:

```
#include <bits/stdc++.h>
#include <chrono>
using namespace std;

int shellSort(int arr[], int n) {
    for (int gap = n/2; gap > 0; gap /= 2) {
        for (int i = gap; i < n; i += 1) {
            int temp = arr[i];

            // shift gap sorted elements to find position for ith element
            int j = i;
            for (j; j >= gap && arr[j - gap] > temp; j -= gap)
                arr[j] = arr[j - gap];
            arr[j] = temp; // place i th element in correct position

        }
    }
    return 0;
}

void printArray(int arr[], int size) {
    for (int i = 0; i < size; i++)
        cout << arr[i] << " ";
    cout << endl;
}

int main() {
    // int arr[] = {64, 34, 25, 12, 22, 11, 90};
    // int n = sizeof(arr)/sizeof(arr[0]);

    int n = rand() % 100;
    int arr[n];

    for (int i = 0; i < n; i++) {
        arr[i] = rand() % 100;
    }

    cout << "Array: ";
    printArray(arr, n);
    cout << endl;

    auto start = chrono::high_resolution_clock::now();
```

```
// unsync the I/O of C and C++.
ios_base::sync_with_stdio(false);

shellSort(arr, n); // sort arr

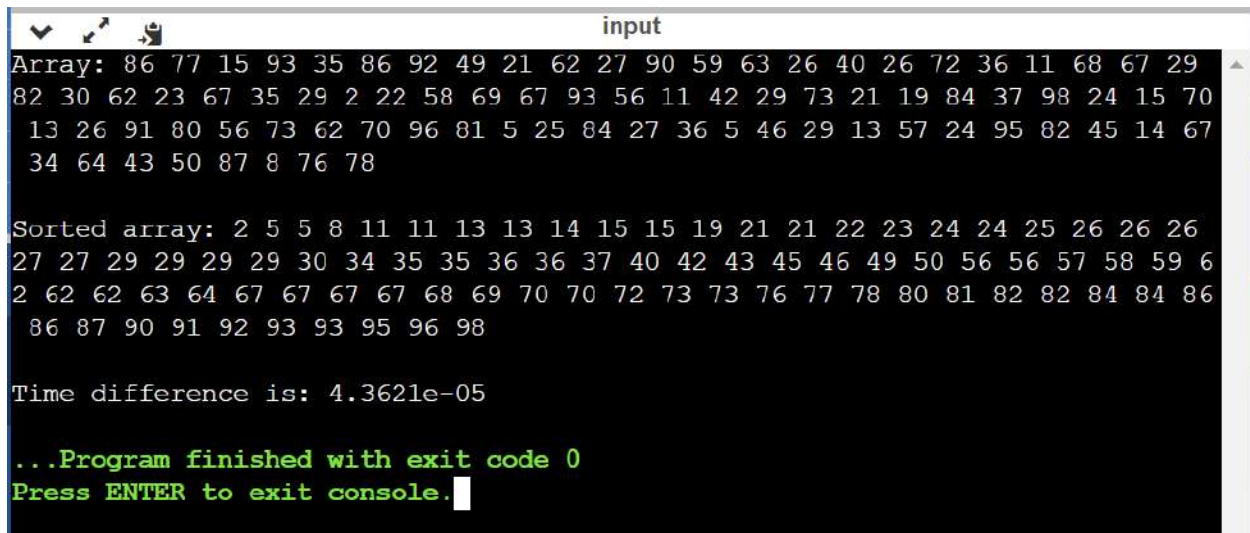
auto end = chrono::high_resolution_clock::now();

cout << "Sorted array: ";
printArray(arr, n);
cout << endl;

double time_taken = chrono::duration_cast<chrono::nanoseconds>(end -
start).count();
time_taken *= 1e-9;

cout << "Time difference is: " << time_taken << setprecision(6);
return 0;
}
```

Output:



```
input
Array: 86 77 15 93 35 86 92 49 21 62 27 90 59 63 26 40 26 72 36 11 68 67 29
82 30 62 23 67 35 29 2 22 58 69 67 93 56 11 42 29 73 21 19 84 37 98 24 15 70
13 26 91 80 56 73 62 70 96 81 5 25 84 27 36 5 46 29 13 57 24 95 82 45 14 67
34 64 43 50 87 8 76 78

Sorted array: 2 5 5 8 11 11 13 13 14 15 15 19 21 21 22 23 24 24 25 26 26 26
27 27 29 29 29 29 30 34 35 35 36 36 37 40 42 43 45 46 49 50 56 56 57 58 59 6
2 62 62 63 64 67 67 67 67 68 69 70 70 72 73 73 76 77 78 80 81 82 82 84 84 86
86 87 90 91 92 93 93 95 96 98

Time difference is: 4.3621e-05

...Program finished with exit code 0
Press ENTER to exit console.
```

Batch Analysis:

Source Code:

```
#include <bits/stdc++.h>

#include <chrono>

using namespace std;

int shellSort(int arr[], int n) {
    for (int gap = n/2; gap > 0; gap /= 2) {
        for (int i = gap; i < n; i += 1) {
            int temp = arr[i];

            // shift gap sorted elements to find position for ith element
            int j = i;
            for (j; j >= gap && arr[j - gap] > temp; j -= gap)
                arr[j] = arr[j - gap];
            arr[j] = temp; // place i th element in correct position
        }
    }
    return 0;
}

void printArray(double arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << ", ";
    cout << endl;
}

void printArray(int arr[], int size)
```

```
{  
    for (int i = 0; i < size; i++)  
        cout << arr[i] << ", ";  
    cout << endl;  
}
```

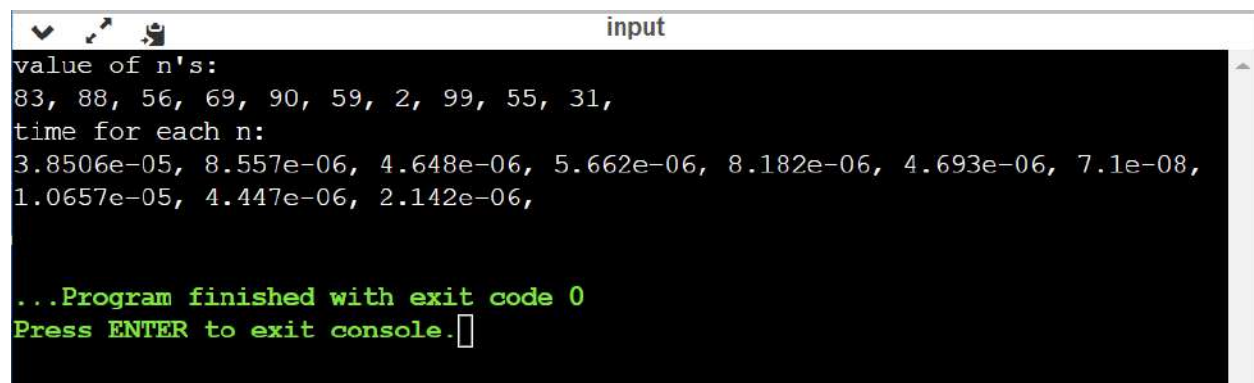
```
void printArray(float arr[], int size)  
{  
    for (int i = 0; i < size; i++)  
        cout << arr[i] << ", ";  
    cout << endl;  
}
```

```
double sortApp(int n) {  
    int arr[n];  
    for (int i = 0; i < n; i++) {  
        arr[i] = rand() % 100;  
    }  
  
    auto start = chrono::high_resolution_clock::now();  
    ios_base::sync_with_stdio(false);  
  
    shellSort(arr, n); // sort arr  
  
    auto end = chrono::high_resolution_clock::now();  
  
    double time_taken = chrono::duration_cast<chrono::nanoseconds>(end -  
start).count();  
    time_taken *= 1e-9;  
  
    return time_taken;
```

```
}

int main() {
    double times[10];
    int ns[10];
    for (int x = 0; x < 10; x++)
    {
        int n = rand() % 100;
        ns[x] = n;
        times[x] = sortApp(n);
    }
    cout << "value of n's: " << endl;
    printArray(ns, 10);
    cout << "time for each n: " << endl;
    printArray(times, 10);
}
```

Output:

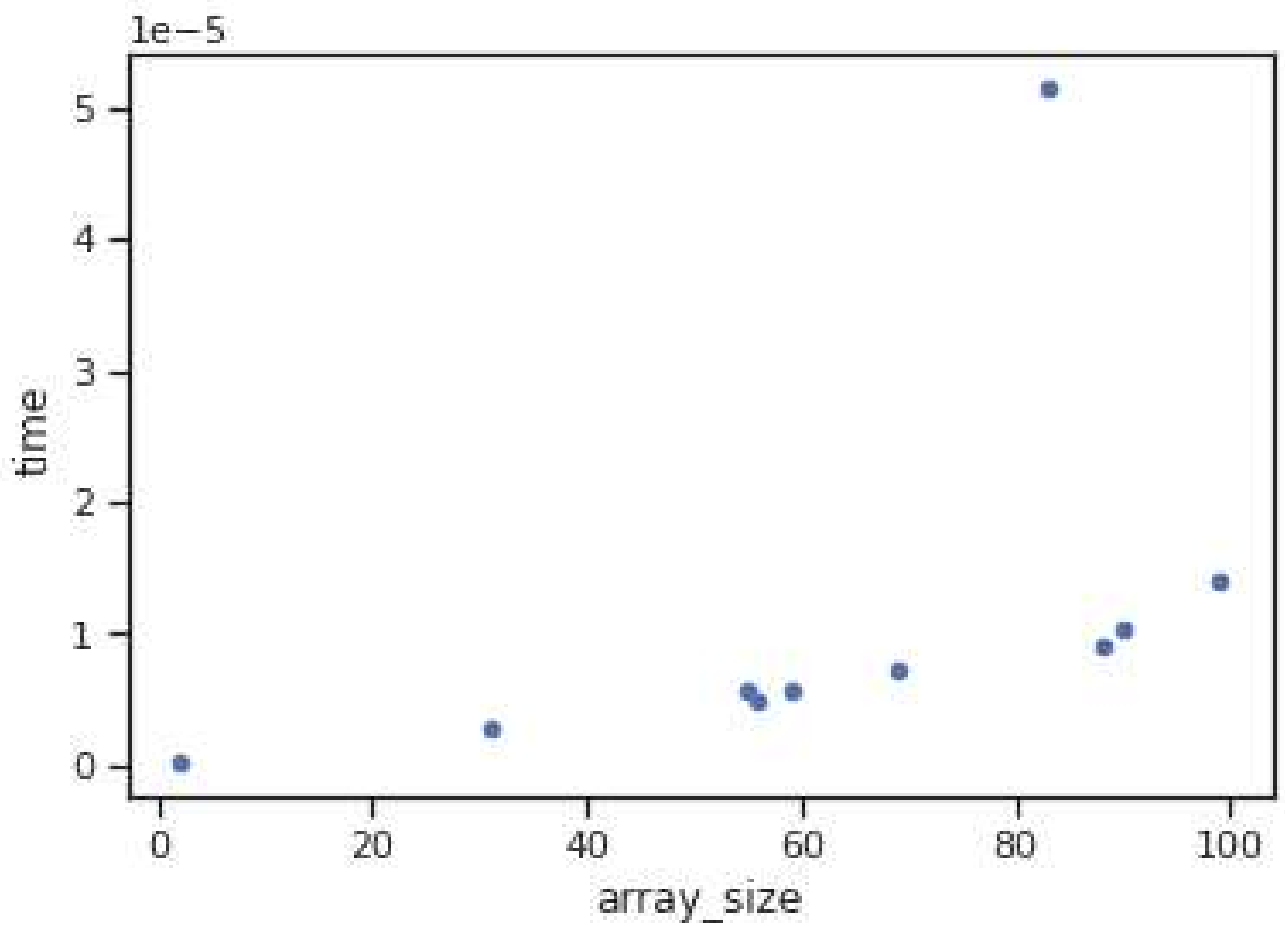


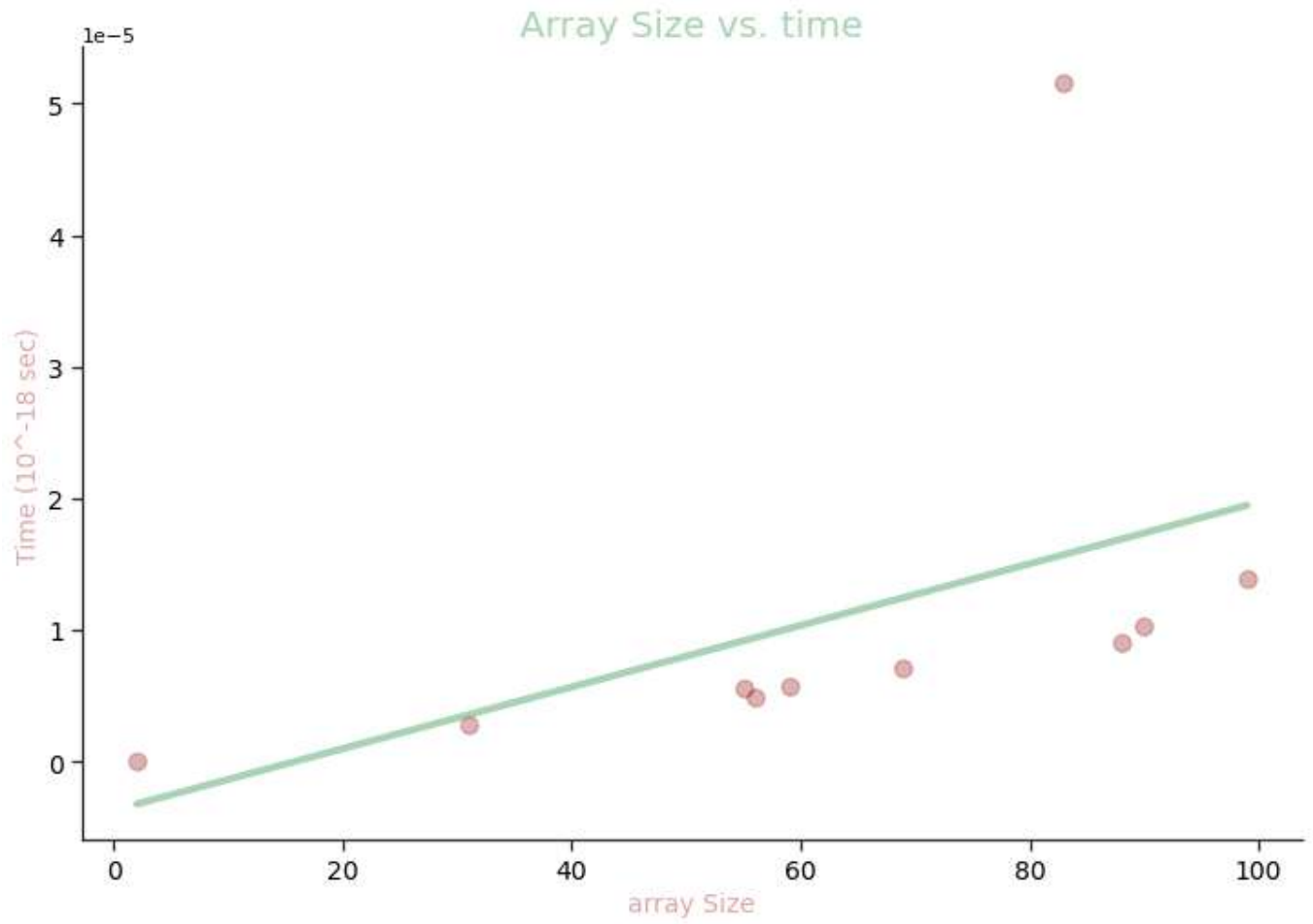
```
input
value of n's:
83, 88, 56, 69, 90, 59, 2, 99, 55, 31,
time for each n:
3.8506e-05, 8.557e-06, 4.648e-06, 5.662e-06, 8.182e-06, 4.693e-06, 7.1e-08,
1.0657e-05, 4.447e-06, 2.142e-06,

...Program finished with exit code 0
Press ENTER to exit console.
```

arraySize = [83, 88, 56, 69, 90, 59, 2, 99, 55, 31]

time = [5.1556e-05, 9.109e-06, 4.9e-06, 7.176e-06, 1.0281e-05, 5.658e-06, 9e-08, 1.3916e-05, 5.566e-06, 2.774e-06]





Shell Sort

Viva Questions

1. How can the time complexity of shell sort be improved?

Ans.

Shell Sort improves its time complexity by taking the advantage of the fact that using Insertion Sort on a partially sorted array results in less number of moves.

2. State any application of shell sort?

Ans.

- Shellsort performs more operations and has higher cache miss ratio than quicksort.
- However, since it can be implemented using little code and does not use the call stack, some implementations of the qsort function in the C standard library targeted at embedded systems use it instead of quicksort. Shellsort is, for example, used in the uClibc library. For similar reasons, an implementation of Shellsort is present in the Linux kernel.
- Shellsort can also serve as a sub-algorithm of introspective sort, to sort short subarrays and to prevent a slowdown when the recursion depth exceeds a given limit. This principle is employed, for instance, in the bzip2 compressor.

3. Given the following list of numbers: [5,16,20,12,3,8,9,17,19,7]. What is the content of the list after all swapping is complete for a gap size of 3?

Ans.

[5, 3, 8, 7, 16, 19, 9, 17, 20, 12]

4. Is this sort stable and does it takes extra additional space?

Ans.

Yes, Shell sort is a stable Algorithm.

No, No addition space is required. Shell sort is an in-place sorting algorithm as it requires **$O(1)$** auxiliary space.

Selection Sort:

Selection sort is a simple sorting algorithm. This sorting algorithm is an in-place comparison-based algorithm in which the list is divided into two parts, the sorted part at the left end and the unsorted part at the right end. Initially, the sorted part is empty and the unsorted part is the entire list. The smallest element is selected from the unsorted array and swapped with the leftmost element, and that element becomes a part of the sorted array. This process continues moving unsorted array boundary by one element to the right.

The selection sort algorithm sorts an array by repeatedly finding the minimum element (considering ascending order) from unsorted part and putting it at the beginning. The algorithm maintains two subarrays in a given array.

- 1) The subarray which is already sorted.
- 2) Remaining subarray which is unsorted.

Pseudo code

Procedure selection sort

list: array of items

n: size of list

for i = 1 to n - 1

/ set current element as minimum*/*

min = i

/ check the element to be minimum */*

for j = i+1 to n

if list[j] < list [min] then

min = j;

end if

end for

/ swap the minimum element with the current element*/*

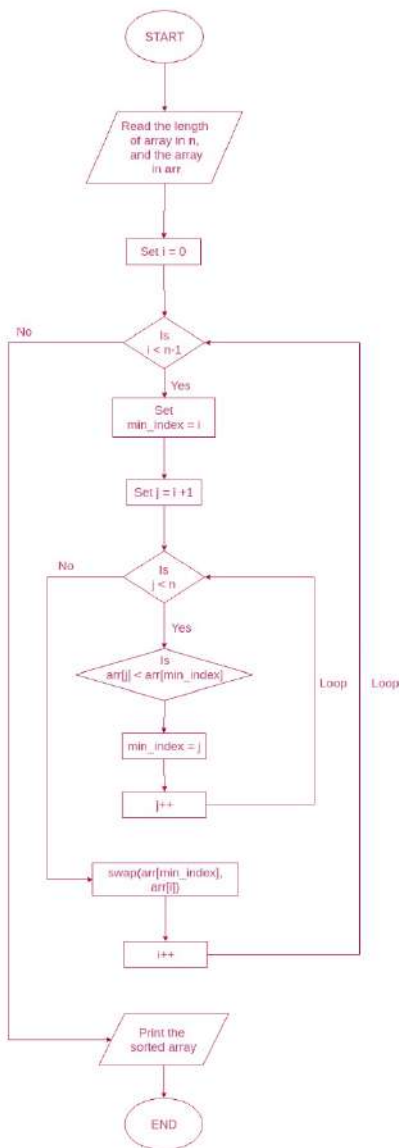
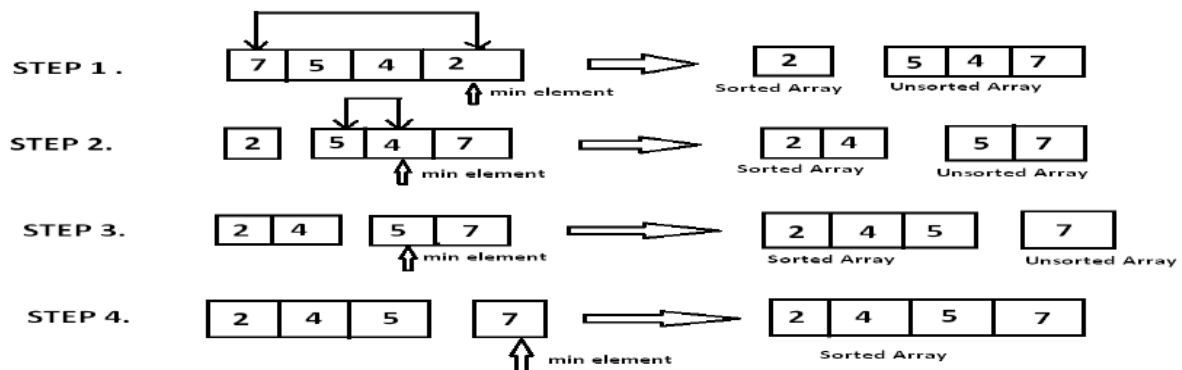
if indexMin != i then

swap list[min] and list[i]

end if

end for

end procedure:

Example:Flowchart for Selection Sort

Result and Analysis

To find the minimum element from the array of N elements, N-1 comparisons are required. After putting the minimum element in its proper position, the size of an unsorted array reduces to N-1 and then N-2 comparisons are required to find the minimum in the unsorted array. Therefore $(N-1) + (N-2) + \dots + 1 = (N(N-1))/2$ comparisons and N swaps result in the overall complexity of $O(N^2)$

Source Code:

```
#include <bits/stdc++.h>
#include <chrono>
using namespace std;

void swap(int *xp, int *yp)
{
    int temp = *xp;
    *xp = *yp;
    *yp = temp;
}

// keep finding minimum element and place it in beginning
void selectionSort(int arr[], int n)
{
    for (int i = 0; i < n - 1; i++)
    {
        int min_idx = i;
        for (int j = i + 1; j < n; j++)
            if (arr[j] < arr[min_idx])
                min_idx = j;
        swap(&arr[min_idx], &arr[i]); // swap min with first
    }
}

void printArray(int arr[], int size)
{
    int i;
    for (i = 0; i < size; i++)
        cout << arr[i] << " ";
    cout << endl;
```

```
}

int main()
{
    // int arr[] = {64, 34, 25, 12, 22, 11, 90};
    // int n = sizeof(arr)/sizeof(arr[0]);

    int n = rand() % 100;
    int arr[n];

    for (int i = 0; i < n; i++)
    {
        arr[i] = rand() % 100;
    }

    cout << "Array: ";
    printArray(arr, n);
    cout << endl;

    auto start = chrono::high_resolution_clock::now();

    // unsync the I/O of C and C++.
    ios_base::sync_with_stdio(false);

    selectionSort(arr, n);

    auto end = chrono::high_resolution_clock::now();

    cout << "Sorted array: ";
    printArray(arr, n);
    cout << endl;

    double time_taken = chrono::duration_cast<chrono::nanoseconds>(end -
start).count();
    time_taken *= 1e-9;

    cout << "Time difference is: " << time_taken << setprecision(6);

    return 0;
}
```

Output:

```
input
Array: 86 77 15 93 35 86 92 49 21 62 27 90 59 63 26 40 26 72 36 11 68 67 29
82 30 62 23 67 35 29 2 22 58 69 67 93 56 11 42 29 73 21 19 84 37 98 24 15 70
13 26 91 80 56 73 62 70 96 81 5 25 84 27 36 5 46 29 13 57 24 95 82 45 14 67
34 64 43 50 87 8 76 78

Sorted array: 2 5 5 8 11 11 13 13 14 15 15 19 21 21 22 23 24 24 25 26 26 26
27 27 29 29 29 29 30 34 35 35 36 36 37 40 42 43 45 46 49 50 56 56 57 58 59 6
2 62 62 63 64 67 67 67 67 68 69 70 70 72 73 73 76 77 78 80 81 82 82 84 84 86
86 87 90 91 92 93 93 95 96 98

Time difference is: 4.2556e-05

...Program finished with exit code 0
Press ENTER to exit console.
```

Batch Analysis:

Source Code:

```
#include <bits/stdc++.h>
#include <chrono>
using namespace std;

void swap(int *xp, int *yp)
{
    int temp = *xp;
    *xp = *yp;
    *yp = temp;
}

// keep finding minimum element and place it in beginning
void selectionSort(int arr[], int n)
{
    for (int i = 0; i < n - 1; i++)
    {
        int min_idx = i;
        for (int j = i + 1; j < n; j++)
            if (arr[j] < arr[min_idx])
                min_idx = j;
        swap(&arr[min_idx], &arr[i]); // swap min with first
    }
}

void printArray(double arr[], int size)
{
    for (int i = 0; i < size; i++)
```



```
        cout << arr[i] << ", ";
    cout << endl;
}

void printArray(int arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << ", ";
    cout << endl;
}

void printArray(float arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << ", ";
    cout << endl;
}

double sortApp(int n)
{
    int arr[n];
    for (int i = 0; i < n; i++)
    {
        arr[i] = rand() % 100;
    }

    auto start = chrono::high_resolution_clock::now();
    ios_base::sync_with_stdio(false);

    selectionSort(arr, n);
```

```
    auto end = chrono::high_resolution_clock::now();

    double time_taken = chrono::duration_cast<chrono::nanoseconds>(end -
start).count();
    time_taken *= 1e-9;

    return time_taken;
}

int main()
{
    double times[10];
    int ns[10];
    for (int x = 0; x < 10; x++)
    {
        int n = rand() % 100;
        ns[x] = n;
        times[x] = sortApp(n);
    }
    cout << "value of n's: " << endl;
    printArray(ns, 10);
    cout << "time for each n: " << endl;
    printArray(times, 10);
}
```

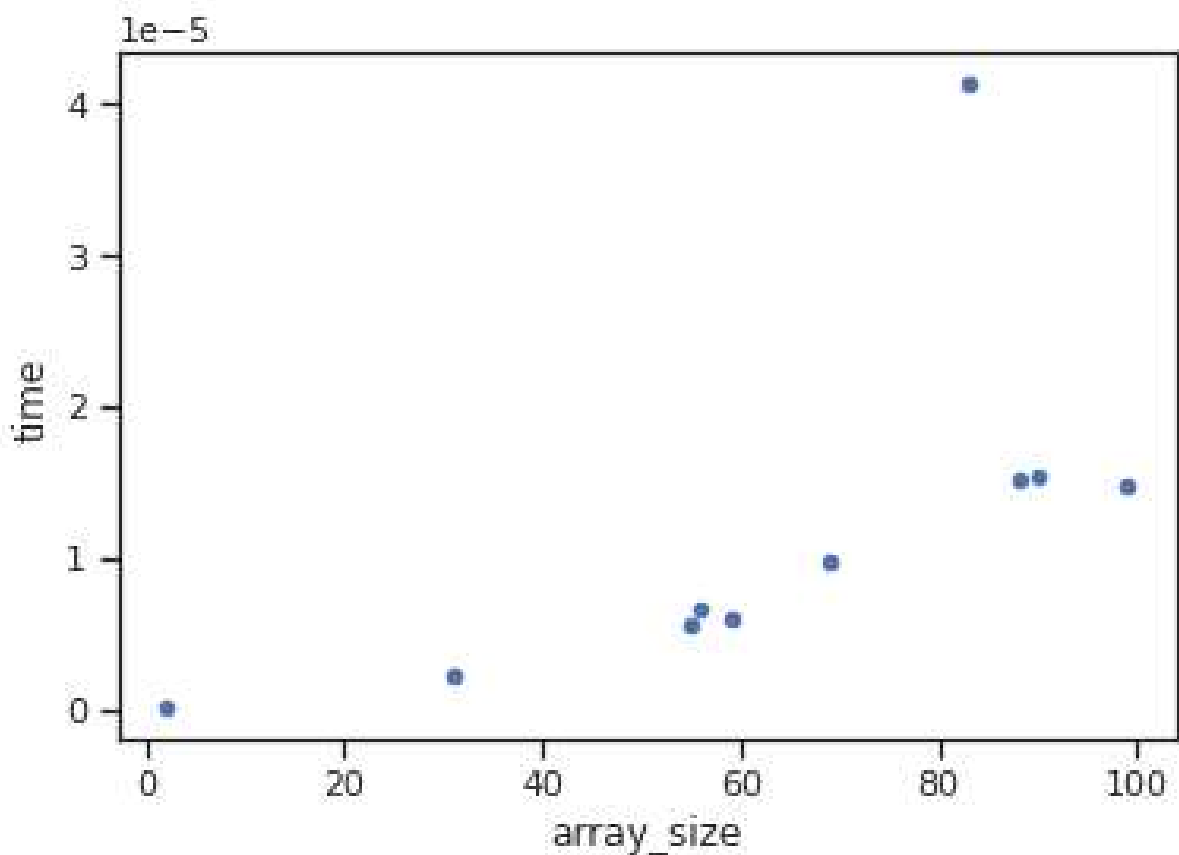
Output:

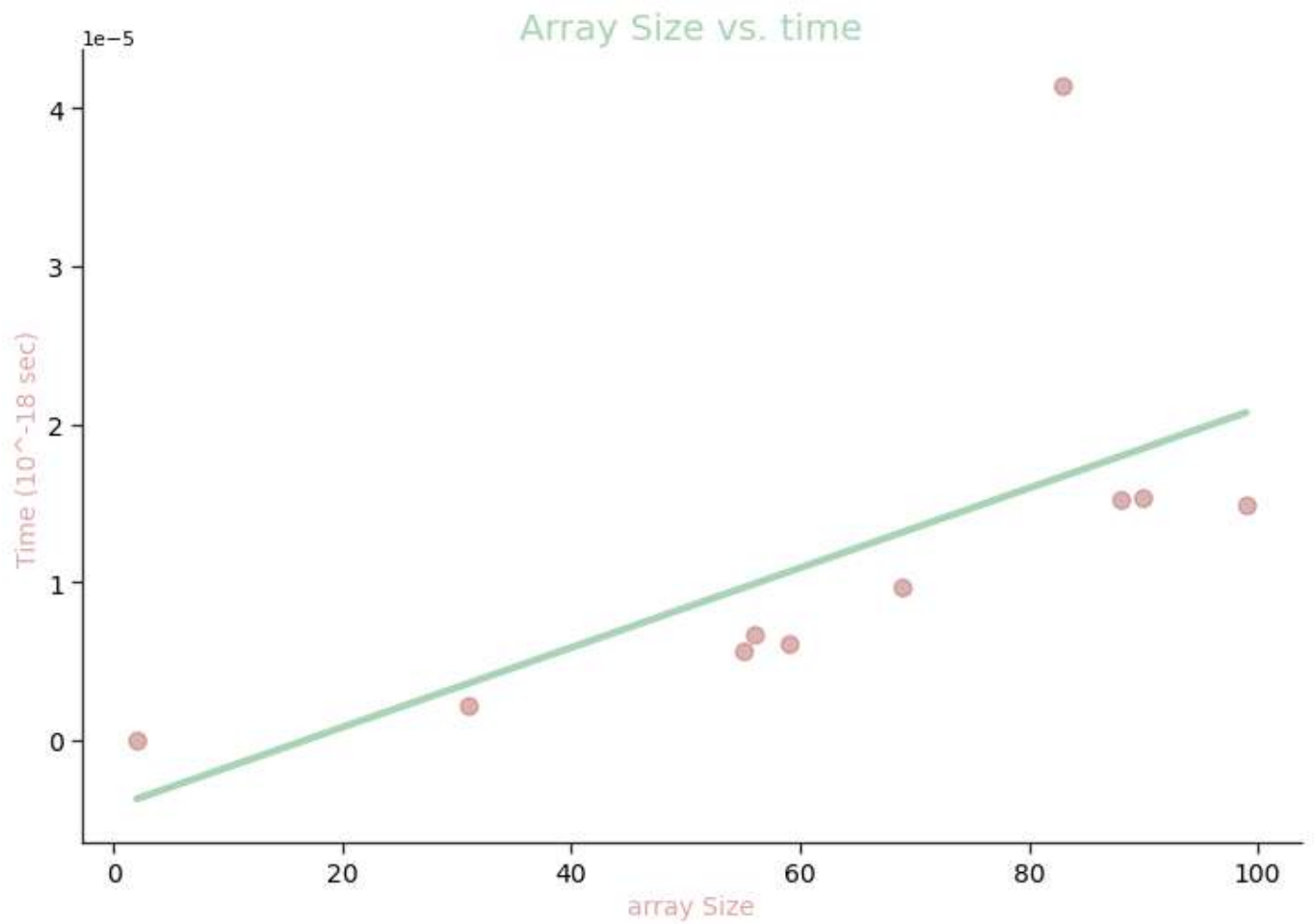
```
input
value of n's:
83, 88, 56, 69, 90, 59, 2, 99, 55, 31,
time for each n:
4.4366e-05, 1.6036e-05, 6.706e-06, 9.483e-06, 1.5256e-05, 7.488e-06, 8.3e-08
, 1.8461e-05, 6.788e-06, 2.525e-06,

...Program finished with exit code 0
Press ENTER to exit console.
```

arraySize = [83, 88, 56, 69, 90, 59, 2, 99, 55, 31]

time = [4.1399e-05, 1.5271e-05, 6.697e-06, 9.652e-06, 1.537e-05, 6.069e-06, 4.2e-08, 1.4875e-05, 5.673e-06, 2.172e-06]





Selection Sort

Viva Questions

1. What is the average case performance of Selection Sort?

Ans.

$\Theta(N^2)$

Average Case Time Complexity is: **$O(N^2)$**

2. For each i from 1 to $n-1$, how many comparisons are there in this sort?

Ans.

Selection sort finds the smallest element and swaps it into the first position. Then, from the remaining $N-1$ elements after the first one (which we know is the smallest), it finds the next smallest and swaps it into the 2nd position. And so on.

To find the smallest element from a group of N elements takes $N-1$ comparisons. Basically, compare item 1 to 2, remember the smallest. Compare item 3 to smallest and remember which ever is smaller, and so on. You end up doing 1 comparison for every element from 2 through N .

We have to find the smallest element $N-1$ times*, but each time we have a smaller list to look through.

For N elements it takes $1+2+3+\dots+N-1 = (N-1)N/2$ comparisons.

3. If all the input elements are identical then how it will affect the time taken by this sort?

Ans.

Selection sort will run in **$O(n^2)$** regardless of whether the array is already sorted or not.

4. What is the output of selection sort after the 2nd iteration given the following sequence of numbers: 16 3 46 9 28 14

Ans.

3 9 46 16 28 14

5. What is straight selection sort?

Ans.

Selection sorting refers to a class of algorithms for sorting a list of items using comparisons. These algorithms select successively smaller or larger items from the list and add them to the output sequence. This is an improvement of the Simple Selection Sort and it is called Straight Selection Sort. Therefore, instead of replacing the selected element by a unique value in the i -th pass (as happens in Simple Selection Sort), the selected element is exchanged with the i -th element.

Heap Sort:

This algorithm is divided into two basic parts:

- Creating a Heap of the unsorted list.
- Then a sorted array is created by repeatedly removing the largest/smallest element from the heap, and inserting it into the array. The heap is reconstructed after each removal.

What is a heap?

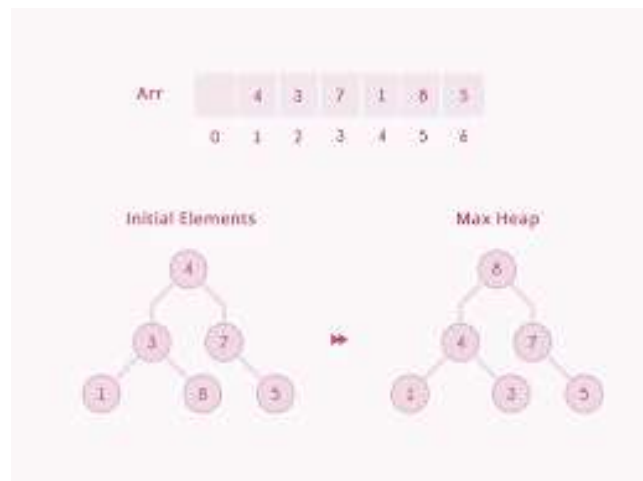
Heap is a special tree-based data structure, which satisfies the following special heap properties:

Shape Property: Heap data structure is always a Complete Binary Tree, which means all levels of the tree are fully filled.

Heap Property: All nodes are either *[greater than or equal to]* or *[less than or equal to]* each of its children. If the parent nodes are greater than their children, heap is called a **Max-Heap**, and if the parent nodes are smaller than their child nodes, heap is called **Min-Heap**.

Initially on receiving an unsorted list, the first step in heap sort is to create a Heap data structure (Max-Heap or Min-Heap). Once heap is built, the first element of the Heap is either largest or smallest (depending upon Max-Heap or Min-Heap), so we put the first element of the heap in our array. Then we again make heap using the remaining elements, to again pick the first element of the heap and put it into the array. We keep on doing the same repeatedly until we have the complete sorted list in our array.

Heap sort is a comparison-based sorting technique based on Binary Heap data structure. It is similar to selection sort where we first find the minimum element and place the minimum element at the beginning. We repeat the same process for the remaining elements.



Pseudo code

```

Heapsort (A) {
    BuildHeap(A)
    for i <- length (A) downto 2 {
        exchange A [1] <-> A[i]
        heapsize <- heapsize -1
        Heapify(A, 1)
    }
}

```

```

BuildHeap (A) {
    heapsize <- length (A)
    for i <- floor (length/2) downto 1
    Heapify (A, i)
}

```

```

Heapify (A, i) {
    le <- left (i)
    ri <- right(i)
    if (le<=heapsize) and (A[le]>A[i])

```

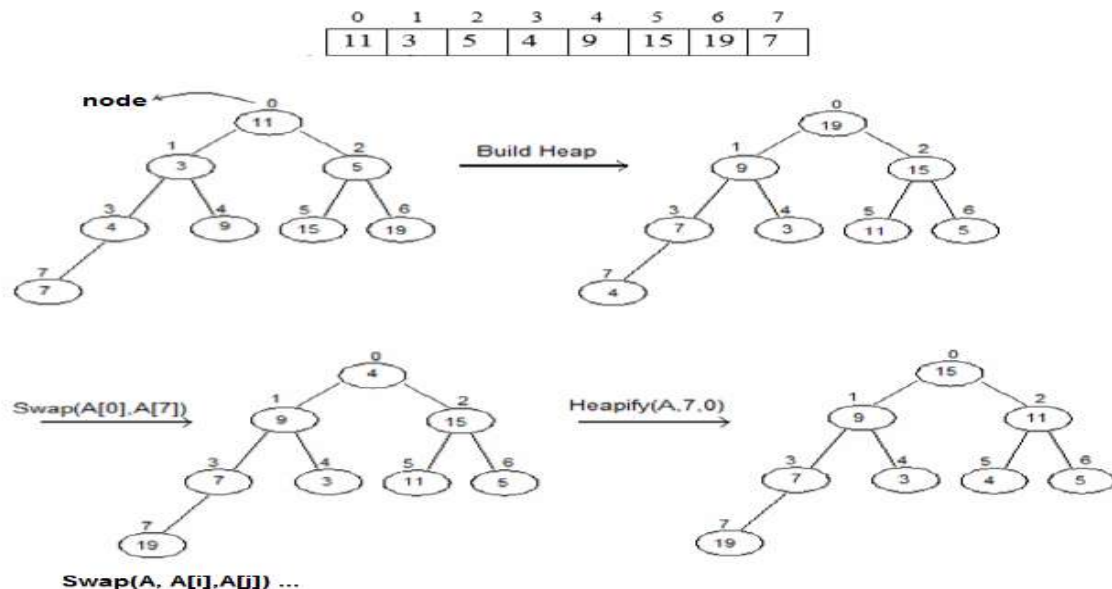


```

    largest <- le
else
    largest <- i
    if (ri <= heapsize) and (A[ri] > A[largest])
        largest <- ri
    if (largest != i) {
        exchange A[i] <-> A[largest]
        Heapify(A, largest)
    }
}

```

Example:



Result and Analysis

Heap sort has the best possible worst case running time complexity of $O(n \log n)$. It doesn't need any extra storage and that makes it good for situations where array size is large. Heapify runs in time $O(h)$ and there are at most $n = 2^{h+1} - 1$ nodes in an almost complete binary tree of height h , so Heapify runs in time $O(\ln n)$. Build-Heap calls Heapify $n/2$ times, so it takes $O(n \ln n) = O(n \log n)$.

Source Code:

```
#include <bits/stdc++.h>
#include <chrono>
using namespace std;

void heapify(int arr[], int n, int root)
{
    int largest = root;
    int leftChild = 2 * root + 1;
    int rightChild = 2 * root + 2;

    if (leftChild < n && arr[leftChild] > arr[largest])
        largest = leftChild;

    if (rightChild < n && arr[rightChild] > arr[largest])
        largest = rightChild;

    if (largest != root)
    {
        swap(arr[root], arr[largest]);
        heapify(arr, n, largest);
    }
}

void heapSort(int arr[], int n)
{
    // Build heap
    for (int i = n / 2 - 1; i >= 0; i--)
        heapify(arr, n, i);

    for (int i = n - 1; i > 0; i--)
    {
        // extract elements from heap
        swap(arr[0], arr[i]); // Move current root to end
        heapify(arr, i, 0);   // call max heapify on the reduced heap
    }
}

void printArray(int arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << " ";

    cout << endl;
}
```

```
}

int main()
{
    // int arr[] = {64, 34, 25, 12, 22, 11, 90};
    // int n = sizeof(arr)/sizeof(arr[0]);

    int n = rand() % 100;
    int arr[n];

    for (int i = 0; i < n; i++)
    {
        arr[i] = rand() % 100;
    }

    cout << "Array: ";
    printArray(arr, n);
    cout << endl;

    auto start = chrono::high_resolution_clock::now();

    // unsync the I/O of C and C++.
    ios_base::sync_with_stdio(false);

    heapSort(arr, n); // sort arr

    auto end = chrono::high_resolution_clock::now();

    cout << "Sorted array: ";
    printArray(arr, n);
    cout << endl;

    double time_taken = chrono::duration_cast<chrono::nanoseconds>(end -
start).count();
    time_taken *= 1e-9;

    cout << "Time difference is: " << time_taken << setprecision(6);

    return 0;
}
```

Output:

```
input
Array: 86 77 15 93 35 86 92 49 21 62 27 90 59 63 26 40 26 72 36 11 68 67 29
82 30 62 23 67 35 29 2 22 58 69 67 93 56 11 42 29 73 21 19 84 37 98 24 15 70
13 26 91 80 56 73 62 70 96 81 5 25 84 27 36 5 46 29 13 57 24 95 82 45 14 67
34 64 43 50 87 8 76 78

Sorted array: 2 5 5 8 11 11 13 13 14 15 15 19 21 21 22 23 24 24 25 26 26 26
27 27 29 29 29 29 30 34 35 35 36 36 37 40 42 43 45 46 49 50 56 56 57 58 59 6
2 62 62 63 64 67 67 67 67 68 69 70 70 72 73 73 76 77 78 80 81 82 82 84 84 86
86 87 90 91 92 93 93 95 96 98

Time difference is: 5.123e-05

...Program finished with exit code 0
Press ENTER to exit console.
```

Batch Analysis:

Source Code:

```
#include <bits/stdc++.h>
#include <chrono>
using namespace std;

void heapify(int arr[], int n, int root)
{
    int largest = root;
    int leftChild = 2 * root + 1;
    int rightChild = 2 * root + 2;

    if (leftChild < n && arr[leftChild] > arr[largest])
        largest = leftChild;

    if (rightChild < n && arr[rightChild] > arr[largest])
        largest = rightChild;

    if (largest != root)
    {
        swap(arr[root], arr[largest]);
        heapify(arr, n, largest);
    }
}

void heapSort(int arr[], int n)
{
    // Build heap
    for (int i = n / 2 - 1; i >= 0; i--)
```

```
        heapify(arr, n, i);

    for (int i = n - 1; i > 0; i--)
    {
        // extract elements from heap
        swap(arr[0], arr[i]); // Move current root to end
        heapify(arr, i, 0);    // call max heapify on the reduced heap
    }
}

void printArray(double arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << ", ";
    cout << endl;
}

void printArray(int arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << ", ";
    cout << endl;
}

void printArray(float arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << ", ";
    cout << endl;
}
```

```
double sortApp(int n)
{
    int arr[n];

    for (int i = 0; i < n; i++)
    {
        arr[i] = rand() % 100;
    }

    auto start = chrono::high_resolution_clock::now();
    ios_base::sync_with_stdio(false);

    heapSort(arr, n); // sort arr

    auto end = chrono::high_resolution_clock::now();

    double time_taken = chrono::duration_cast<chrono::nanoseconds>(end -
start).count();
    time_taken *= 1e-9;

    return time_taken;
}

int main()
{
    double times[10];
    int ns[10];

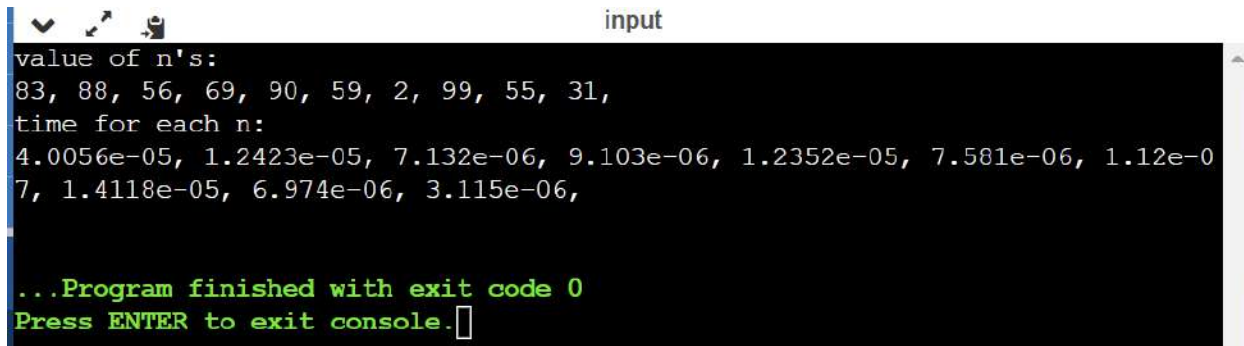
    for (int x = 0; x < 10; x++)
    {
        int n = rand() % 100;
```

```
        ns[x] = n;
        times[x] = sortApp(n);
    }

    cout << "value of n's: " << endl;
    printArray(ns, 10);

    cout << "time for each n: " << endl;
    printArray(times, 10);
}
```

Output:

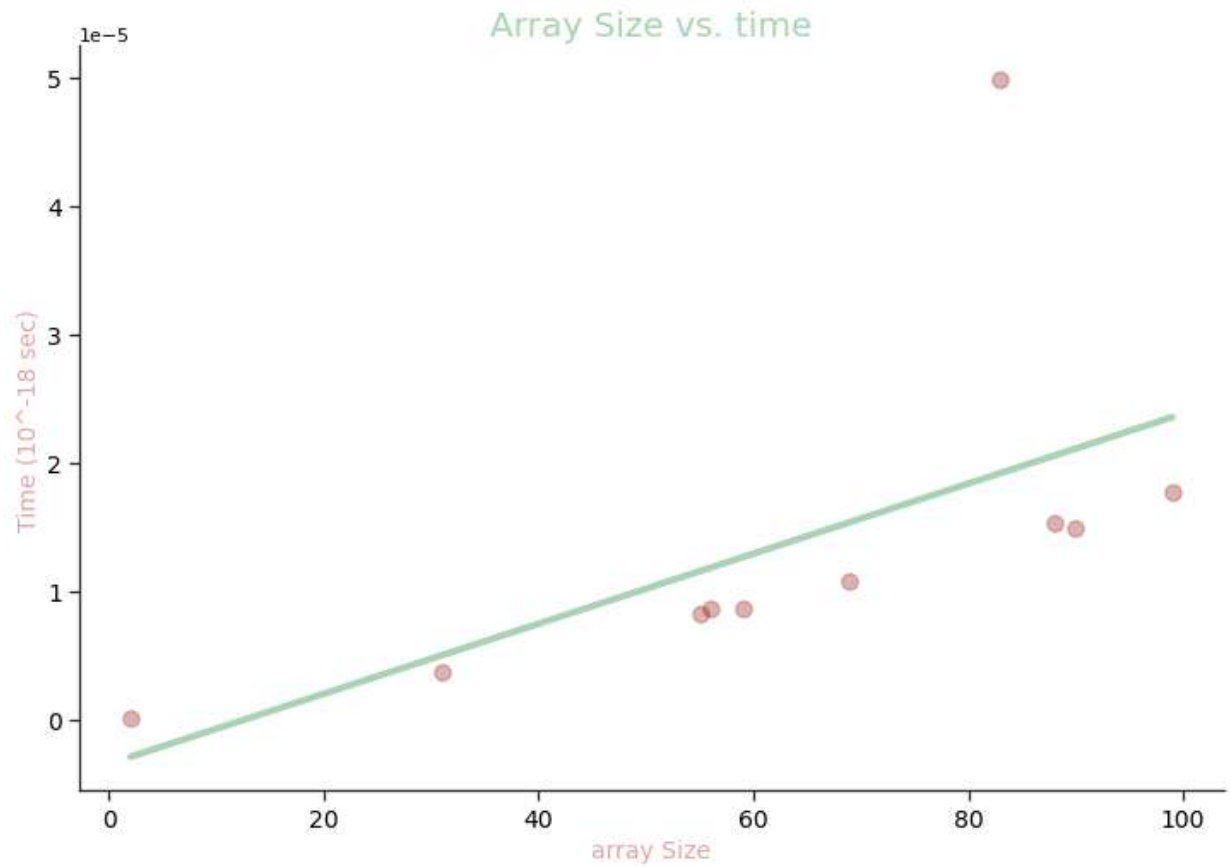
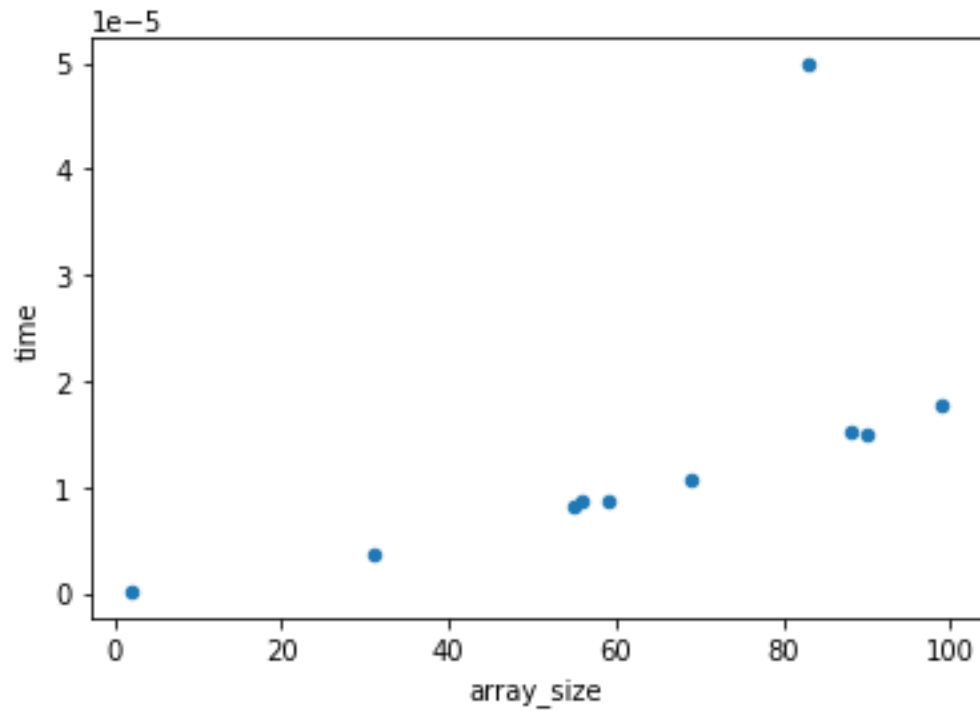


```
input
value of n's:
83, 88, 56, 69, 90, 59, 2, 99, 55, 31,
time for each n:
4.0056e-05, 1.2423e-05, 7.132e-06, 9.103e-06, 1.2352e-05, 7.581e-06, 1.12e-0
7, 1.4118e-05, 6.974e-06, 3.115e-06,

...Program finished with exit code 0
Press ENTER to exit console.
```

arraySize = [83, 88, 56, 69, 90, 59, 2, 99, 55, 31]

time = [4.9828e-05, 1.5332e-05, 8.655e-06, 1.0828e-05, 1.4977e-05, 8.714e-06, 1.48e-07, 1.7703e-05, 8.224e-06, 3.716e-06]



Heap Sort

Viva Questions

1. What are the minimum and maximum numbers of elements in a heap of height h ?

Ans.

2^h

A heap of height h is complete up to the level at depth $h-1$ and needs to have at least one node on level h .

Therefore the minimum total number of nodes must be at least $\sum_{i=0}^{h-1} 2^i + 1 = 2^h - 1 + 1 = 2^h$. $\sum_{i=0}^{h-1} 2^i + 1 = 2^h - 1 + 1 = 2^h$., and this tight since an heap with 2^h nodes has height h .

2. Where in a heap might the smallest element reside?

Ans.

Smallest element will be at the last level of the max heap.

3. Is an array that is in reverse sorted order a heap?

Ans.

The correct version of this statement is "An array sorted in ascending order is can be treated **as min- heap**" and its complementary statement is "An array sorted in descending order can be treated as max heap"

4. Does heap sort uses extra space for storage?

Ans.

No, The Heap sort algorithm can be implemented as an in-place sorting algorithm. This means that its memory usage is minimal because apart from what is necessary to hold the initial list of items to be sorted, it needs no additional memory space to work.

5. What is the effect of calling HEAPIFY (A, i) when the element $A[i]$ is larger than its children?

Ans.

No effect. The comparisons are carried out, $A[i]$ is found to be largest and the procedure just returns.

Insertion Sort:

Insertion sort is a simple sorting algorithm that works similar to the way you sort playing cards in your hands. The array is virtually split into a sorted and an unsorted part. Values from the unsorted part are picked and placed at the correct position in the sorted part.

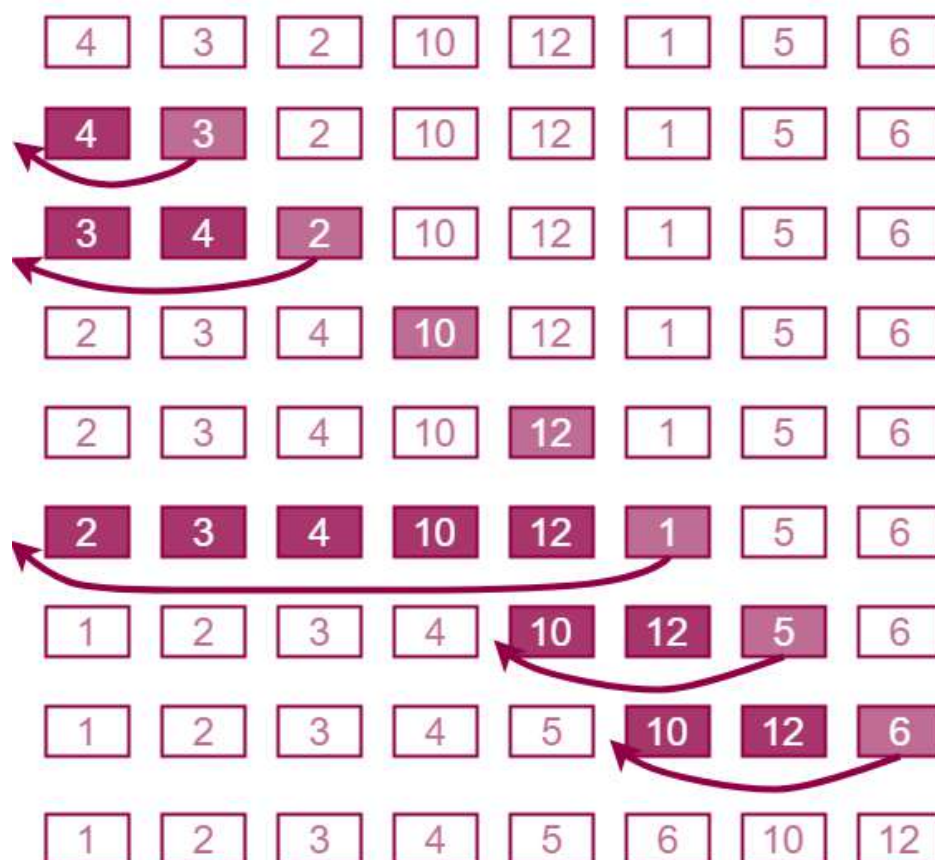
Algorithm

To sort an array of size n in ascending order:

- 1: Iterate from $arr[1]$ to $arr[n]$ over the array.
- 2: Compare the current element (key) to its predecessor.
- 3: If the key element is smaller than its predecessor, compare it to the elements before. Move the greater elements one position up to make space for the swapped element.

Example:

Insertion Sort Execution Example



Result and Analysis

Time Complexity: $O(n^2)$

Auxiliary Space: $O(1)$

Boundary Cases: Insertion sort takes maximum time to sort if elements are sorted in reverse order. And it takes minimum time (Order of n) when elements are already sorted.

Algorithmic Paradigm: Incremental Approach

Sorting In Place: Yes

Stable: Yes

Online: Yes

Uses: Insertion sort is used when number of elements is small. It can also be useful when input array is almost sorted, only few elements are misplaced in complete big array.

Source Code:

```
#include <bits/stdc++.h>
#include <chrono>
using namespace std;

// place current element in right order as we move forward
void insertionSort(int arr[], int n)
{
    for (int i = 1; i < n; i++)
    {
        int key = arr[i];
        int j = i - 1;
        // compare to all predecessors
        while (j >= 0 && arr[j] > key)
        {
            arr[j + 1] = arr[j];
            j = j - 1;
        }
        arr[j + 1] = key;
    }
}

void printArray(int arr[], int size)
```

```
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << " ";
    cout << endl;
}

int main()
{
    // int arr[] = {64, 34, 25, 12, 22, 11, 90};
    // int n = sizeof(arr)/sizeof(arr[0]);
    int n = rand() % 100;
    int arr[n];
    for (int i = 0; i < n; i++)
    {
        arr[i] = rand() % 100;
    }
    cout << "Array: ";
    printArray(arr, n);
    cout << endl;

    auto start = chrono::high_resolution_clock::now();
    // unsync the I/O of C and C++.
    ios_base::sync_with_stdio(false);

    insertionSort(arr, n);

    auto end = chrono::high_resolution_clock::now();

    cout << "Sorted array: ";
    printArray(arr, n);
    cout << endl;

    double time_taken = chrono::duration_cast<chrono::nanoseconds>(end -
start).count();
    time_taken *= 1e-9;

    cout << "Time difference is: " << time_taken << setprecision(6);
    return 0;
}
```

Output:

```
input
Array: 86 77 15 93 35 86 92 49 21 62 27 90 59 63 26 40 26 72 36 11 68 67 29
82 30 62 23 67 35 29 2 22 58 69 67 93 56 11 42 29 73 21 19 84 37 98 24 15 70
13 26 91 80 56 73 62 70 96 81 5 25 84 27 36 5 46 29 13 57 24 95 82 45 14 67
34 64 43 50 87 8 76 78

Sorted array: 2 5 5 8 11 11 13 13 14 15 15 19 21 21 22 23 24 24 25 26 26 26
27 27 29 29 29 29 30 34 35 35 36 36 37 40 42 43 45 46 49 50 56 56 57 58 59 6
2 62 62 63 64 67 67 67 67 68 69 70 70 72 73 73 76 77 78 80 81 82 82 84 84 86
86 87 90 91 92 93 93 95 96 98

Time difference is: 5.155e-05

...Program finished with exit code 0
Press ENTER to exit console.
```

Batch Analysis:

Source Code:

```
#include <bits/stdc++.h>

#include <chrono>

using namespace std;

// place current element in right order as we move forward
void insertionSort(int arr[], int n)
{
    for (int i = 1; i < n; i++)
    {
        int key = arr[i];
        int j = i - 1;
        // compare to all predecessors
        while (j >= 0 && arr[j] > key)
        {
            arr[j + 1] = arr[j];
            j = j - 1;
        }
        arr[j + 1] = key;
    }
}

void printArray(double arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << ", ";
    cout << endl;
}
```



```
void printArray(int arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << ", ";
    cout << endl;
}
```

```
void printArray(float arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << ", ";
    cout << endl;
}
```

```
double sortApp(int n)
{
    int arr[n];
    for (int i = 0; i < n; i++)
    {
        arr[i] = rand() % 100;
    }

    auto start = chrono::high_resolution_clock::now();
    ios_base::sync_with_stdio(false);

    insertionSort(arr, n);

    auto end = chrono::high_resolution_clock::now();
```

```
        double time_taken = chrono::duration_cast<chrono::nanoseconds>(end -
start).count();

        time_taken *= 1e-9;

        return time_taken;
}

int main()
{
    double times[10];
    int ns[10];

    for (int x = 0; x < 10; x++)
    {
        int n = rand() % 100;
        ns[x] = n;
        times[x] = sortApp(n);
    }

    cout << "value of n's: " << endl;
    printArray(ns, 10);

    cout << "time for each n: " << endl;
    printArray(times, 10);
}
```

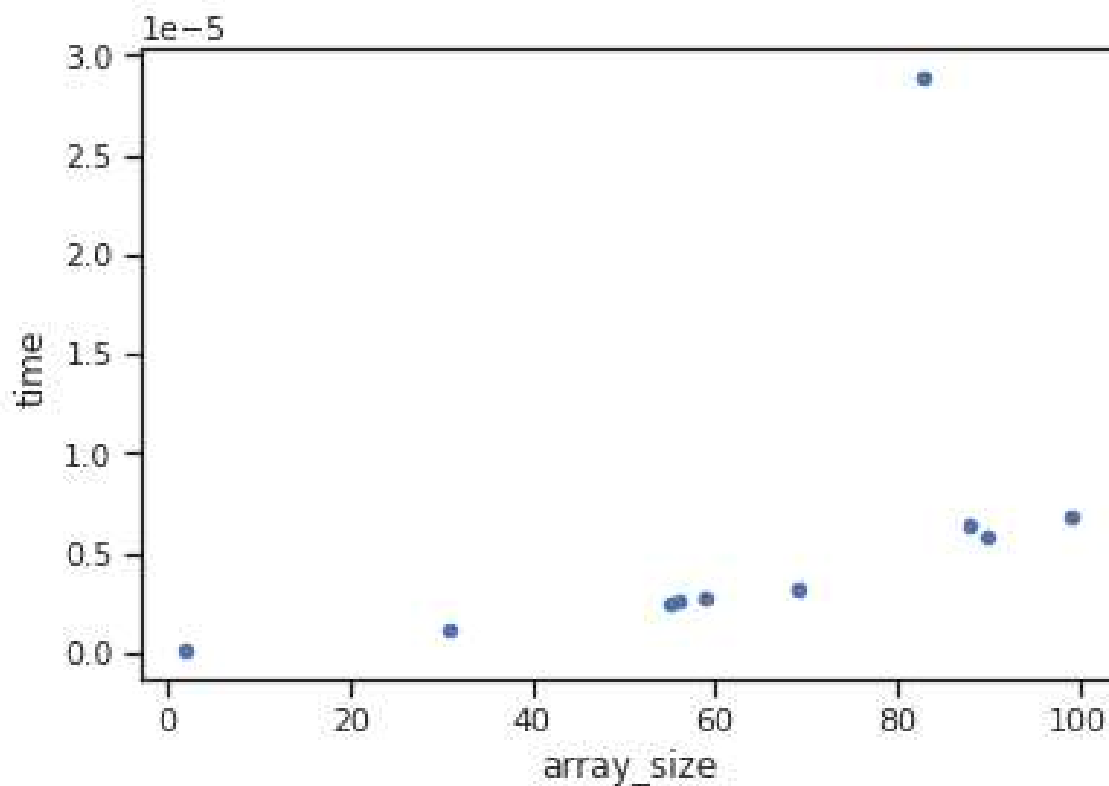
Output:

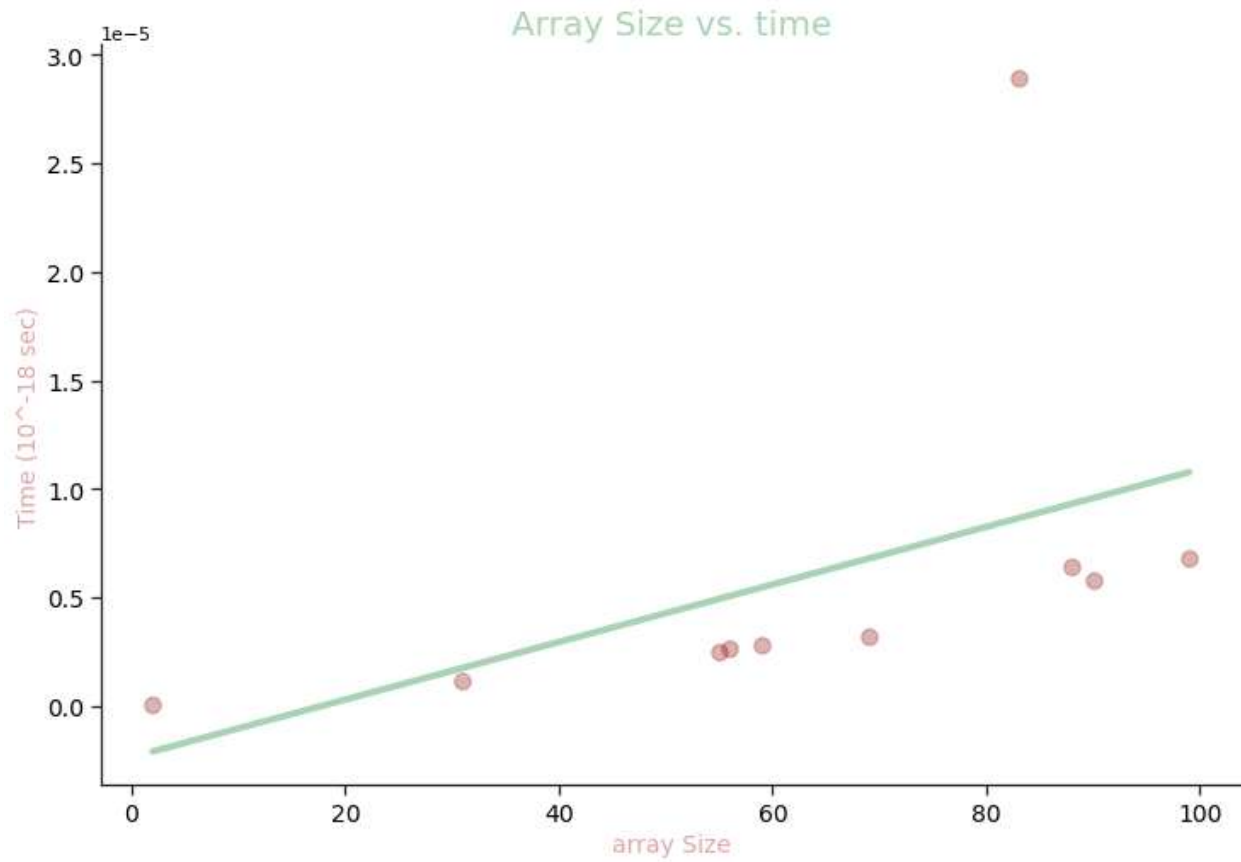
```
input
value of n's:
83, 88, 56, 69, 90, 59, 2, 99, 55, 31,
time for each n:
4.1828e-05, 7.931e-06, 3.27e-06, 3.856e-06, 7.223e-06, 3.512e-06, 9.7e-08, 8
.715e-06, 3.045e-06, 1.513e-06,

...Program finished with exit code 0
Press ENTER to exit console.
```

arraySize = [83, 88, 56, 69, 90, 59, 2, 99, 55, 31]

time = [2.8917e-05, 6.425e-06, 2.608e-06, 3.153e-06, 5.76e-06, 2.767e-06, 5.5e-08, 6.81e-06, 2.451e-06, 1.186e-06]





Insertion Sort

Viva Questions

1. Why is time complexity of insertion sort?

Ans.

Even though insertion sort is efficient, still, if we provide an already sorted array to the insertion sort algorithm, it will still execute the outer for loop, thereby requiring n steps to sort an already sorted array of n elements, which makes its best case time complexity a linear function of n .

Wherein for an unsorted array, it takes for an element to compare with all the other elements which mean every n element compared with all other n elements. Thus, making it for $n \times n$, i.e., n^2 comparisons. One can also take a look at other sorting algorithms such as *Merge sort*, *Quick Sort*, *Selection Sort*, etc. and understand their complexities.

Worst Case Time Complexity [Big-O]: $O(n^2)$

Best Case Time Complexity [Big-omega]: $O(n)$

Average Time Complexity [Big-theta]: $O(n^2)$

2. Is it possible to do insertion sort in place?

Ans.

Yes, It is. Insertion sort is used when number of elements is small. It can also be useful when input array is almost sorted, only few elements are misplaced in complete big array.

3. Is insertion sort the worst?

Ans.

Insertion sort has a fast best-case running time and is a good sorting algorithm to use if the input list is already mostly sorted. For larger or more unordered lists, an algorithm with a **faster worst** and average-case running time, such as mergesort, would be a better choice. Now iterate through this array just one time. The time

4. What is Binary Insertion Sort?

Ans.

We can use binary search to reduce the number of comparisons in normal insertion sort. Binary Insertion Sort uses binary search to find the proper location to insert the selected item at each iteration. In normal insertion, sorting takes $O(i)$ (at i th iteration) in worst case. We can reduce it to $O(\log i)$ by using binary search. The algorithm, as a whole, still has a running worst case running time of $O(n^2)$ because of the series of swaps required for each insertion.

5. Is insertion sort stable and in-place?

Ans.

Insertion sort is an online stable in-place sorting algorithm that builds the final sorted list one item at a time. It works on the principle of moving an element to its correct position in a sorted array. Advantages of Insertion Sort: Stable: it does not change the relative order of elements with equal keys.

6. Give any application of insertion sort?

Ans.

Insertion sort is used when number of elements is small. It can also be useful when input array is almost sorted, only few elements are misplaced in complete big array.



EXPERIMENT - 2

Algorithms Design and Analysis Lab

Aim

To implement Linear search and Binary search and analyse its time complexity.

Syeda Reeha Quasar

14114802719

4C7

EXPERIMENT – 2

Aim:

To implement Linear search and Binary search and analyse its time complexity.

Theory:

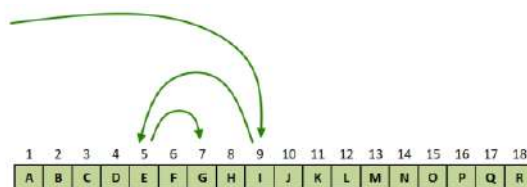
The search operation can be done in the following two ways:

Linear search:

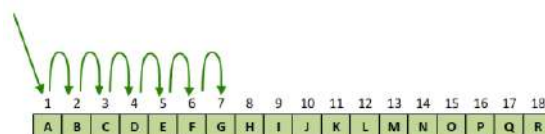
It's a sequential search is made over all items one by one. Every item is checked and if a match is found then that particular item is returned, otherwise the search continues till the end of the data collection.

Binary search:

This search algorithm works on the principle of divide and conquers. For this algorithm, the data collection should be in the sorted form. Binary search looks for a particular item by comparing the middle most item of the collection. If a match occurs, then the index of item is returned. If the middle item is greater than the item, then the item is searched in the sub-array to the left of the middle item. Otherwise, the item is searched for in the sub-array to the right of the middle item. This process continues on the sub-array as well until the size of the sub array reduces to zero.



Binary Search - Find 'G' in sorted list A-R



Linear Search - Find 'G' in sorted list A-R

Pseudo code for linear search:

Procedure *linear_search* (*list*, *value*)

 for each item in the list

 if match item == value

 return the item's location

 end if

 end for

end procedure

Pseudo code for Binary search:

BinarySearch (*A* [0...*N*-1], *value*)

{

 low = 0

 high = *N* - 1

 while (low <= high) {

 // invariants: value > *A*[*i*] for all *i* < low

 value < *A*[*i*] for all *i* > high

 mid = (low + high) / 2

 if (*A*[mid] > value)

 high = mid - 1

 else if (*A*[mid] < value)

 low = mid + 1

 else

 return mid

 }

 return not_found // value would be inserted at index "low"



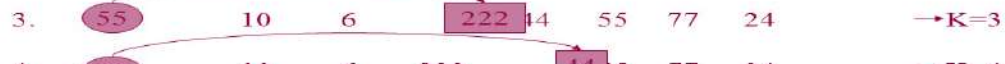


}

Sample Example:**Example of Linear Search**

List : 10, 6, 222, 44, 55, 77, 24

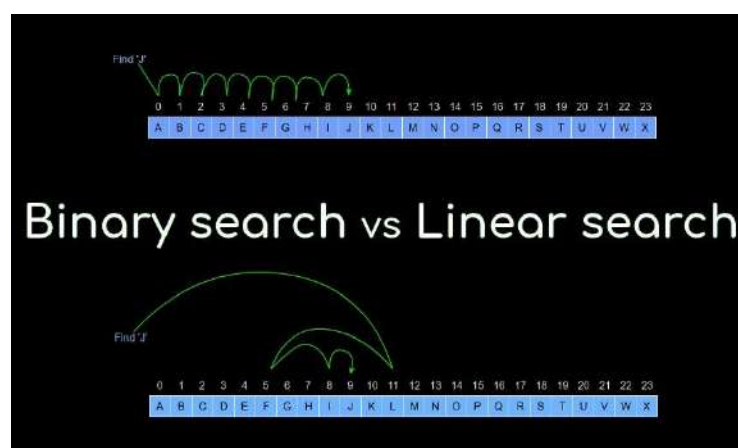
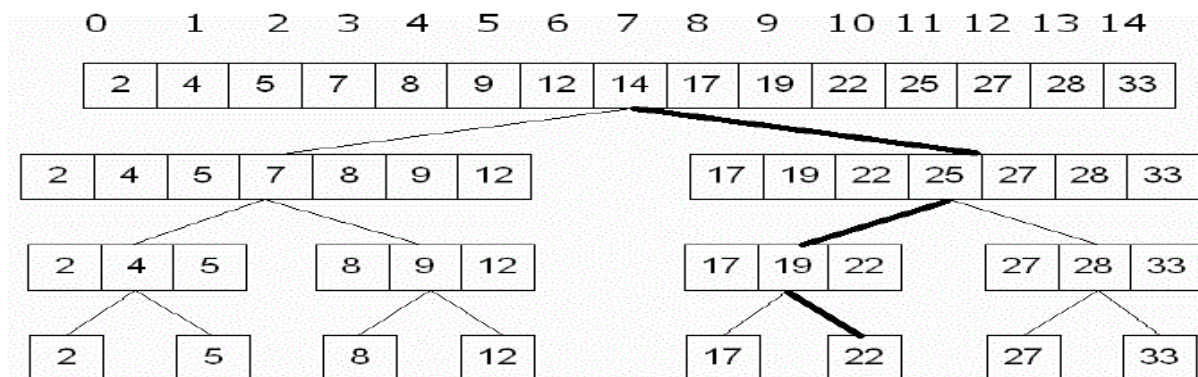
Key: 55

Steps:

1.  $\rightarrow K=1$
2.  $\rightarrow K=2$
3.  $\rightarrow K=3$
4.  $\rightarrow K=4$
5.  $\rightarrow K=5$

So, Item 55 is in position 5.**Example of Binary Search:**

A sorted array is taken as input. The mid value is found out recursively.



Result and Analysis

Input data needs to be sorted in Binary Search and not in Linear Search and it does the sequential access whereas Binary search access data randomly. So, time complexity of linear search is $O(n)$ while Binary search has time complexity $O(\log n)$ as search is done to either half of the given list.

Base of comparison	Linear search	Binary search
Time complexity	$O(N)$	$O(\log_2 N)$
Best case time	$O(1)$ first element	$O(1)$ center element
Prerequisite of an array	No prerequisite	Array must be sorted in order
Input data	No need to be sorted	Need to be sorted
Access	Sequential	random

LINEAR SEARCH	BINARY SEARCH
An algorithm to find an element in a list by sequentially checking the elements of the list until finding the matching element	An algorithm that finds the position of a target value within a sorted array
Also called sequential search	Also called half-interval search and logarithmic search
Time complexity is $O(N)$	Time complexity is $O(\log_2 N)$
Best case is to find the element in the first position	Best case is to find the element in the middle position
It is not required to sort the array before searching the element	It is necessary to sort the array before searching the element
Less efficient	More efficient
Less complex	More complex

Linear search:

Source Code:

```
#include <bits/stdc++.h>
using namespace std;

int linearSearch(int arr[], int n, int x)
{
    for (int i = 0; i < n; i++)
        if (arr[i] == x)
            return i;
    return -1;
}

void printArray(int arr[], int size)
{
    int i;
    for (i = 0; i < size; i++)
        cout << arr[i] << " ";
    cout << endl;
}

int main()
{
    // int arr[] = { 2, 3, 4, 10, 40 };
    // int n = sizeof(arr) / sizeof(arr[0]);

    int n = rand() % 100;
    int arr[n];

    for (int i = 0; i < n; i++)
    {
        arr[i] = rand() % 100;
    }

    cout << "Array: ";
    printArray(arr, n);

    int x;

    cout << "Enter element you want to search in the array: ";
```

```
    cin >> x;

    auto start = chrono::high_resolution_clock::now();
    // unsync the I/O of C and C++.
    ios_base::sync_with_stdio(false);

    int result = linearSearch(arr, n, x);

    auto end = chrono::high_resolution_clock::now();

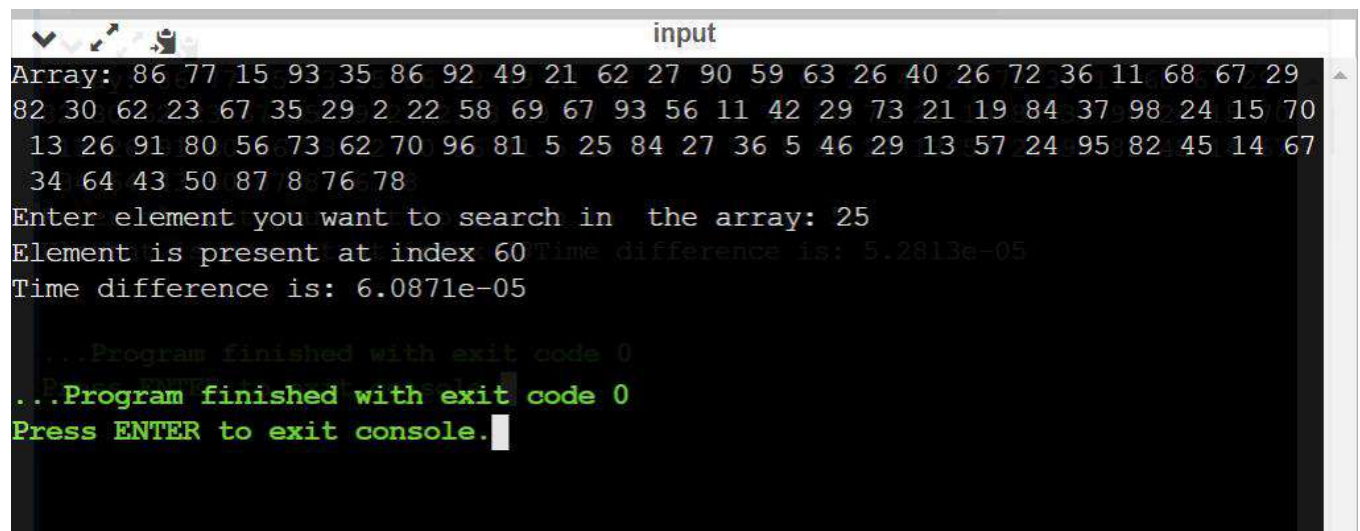
    (result == -1)
        ? cout << "Element is not present in array"
        : cout << "Element is present at index " << result;

    double time_taken = chrono::duration_cast<chrono::nanoseconds>(end -
start).count();
    time_taken *= 1e-9;

    cout << "Time difference is: " << time_taken << setprecision(6) << endl;

    return 0;
}
```

Output:



```
input
Array: 86 77 15 93 35 86 92 49 21 62 27 90 59 63 26 40 26 72 36 11 68 67 29
82 30 62 23 67 35 29 2 22 58 69 67 93 56 11 42 29 73 21 19 84 37 98 24 15 70
13 26 91 80 56 73 62 70 96 81 5 25 84 27 36 5 46 29 13 57 24 95 82 45 14 67
34 64 43 50 87 8 76 78
Enter element you want to search in the array: 25
Element is present at index 60
Time difference is: 5.2813e-05
Time difference is: 6.0871e-05
...Program finished with exit code 0
...Program finished with exit code 0
Press ENTER to exit console.
```

```
input
Array: 86 77 15 93 35 86 92 49 21 62 27 90 59 63 26 40 26 72 36 11 68 67 29
82 30 62 23 67 35 29 2 22 58 69 67 93 56 11 42 29 73 21 19 84 37 98 24 15 70
13 26 91 80 56 73 62 70 96 81 5 25 84 27 36 5 46 29 13 57 24 95 82 45 14 67
34 64 43 50 87 8 76 78
Enter element you want to search in the array: 0
Element is not present in array
Time difference is: 4.7726e-05

...Program finished with exit code 0
Press ENTER to exit console.
```

```
input
Array: 86 77 15 93 35 86 92 49 21 62 27 90 59 63 26 40 26 72 36 11 68 67 29
82 30 62 23 67 35 29 2 22 58 69 67 93 56 11 42 29 73 21 19 84 37 98 24 15 70
13 26 91 80 56 73 62 70 96 81 5 25 84 27 36 5 46 29 13 57 24 95 82 45 14 67
34 64 43 50 87 8 76 78
Enter element you want to search in the array: 12
Element is not present in array
Time difference is: 6.6163e-05

...Program finished with exit code 0
Press ENTER to exit console.
```

Batch Analysis:

Source Code:

```
#include <bits/stdc++.h>
using namespace std;

int linearSearch(int arr[], int n, int x)
{
    for (int i = 0; i < n; i++)
        if (arr[i] == x)
            return i;

    return -1;
}

void printArray(double arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << ", ";

    cout << endl;
}

void printArray(int arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << ", ";

    cout << endl;
}

void printArray(float arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << ", ";

    cout << endl;
}

double search(int n)
{
    int arr[n];
```

```
    for (int i = 0; i < n; i++)
    {
        arr[i] = rand() % 100;
    }
    sort(arr, arr + n);

    int x = rand() % 100;

    auto start = chrono::high_resolution_clock::now();
    ios_base::sync_with_stdio(false);

    linearSearch(arr, n, x);

    auto end = chrono::high_resolution_clock::now();

    double time_taken = chrono::duration_cast<chrono::nanoseconds>(end -
start).count();
    time_taken *= 1e-9;

    return time_taken;
}

int main()
{
    double times[10];
    int ns[10];

    for (int x = 0; x < 10; x++)
    {
        int n = rand() % 100;
        ns[x] = n;
        times[x] = search(n);
    }

    cout << "value of n's: " << endl;
    printArray(ns, 10);

    cout << "time for each n: " << endl;
    printArray(times, 10);
}
```

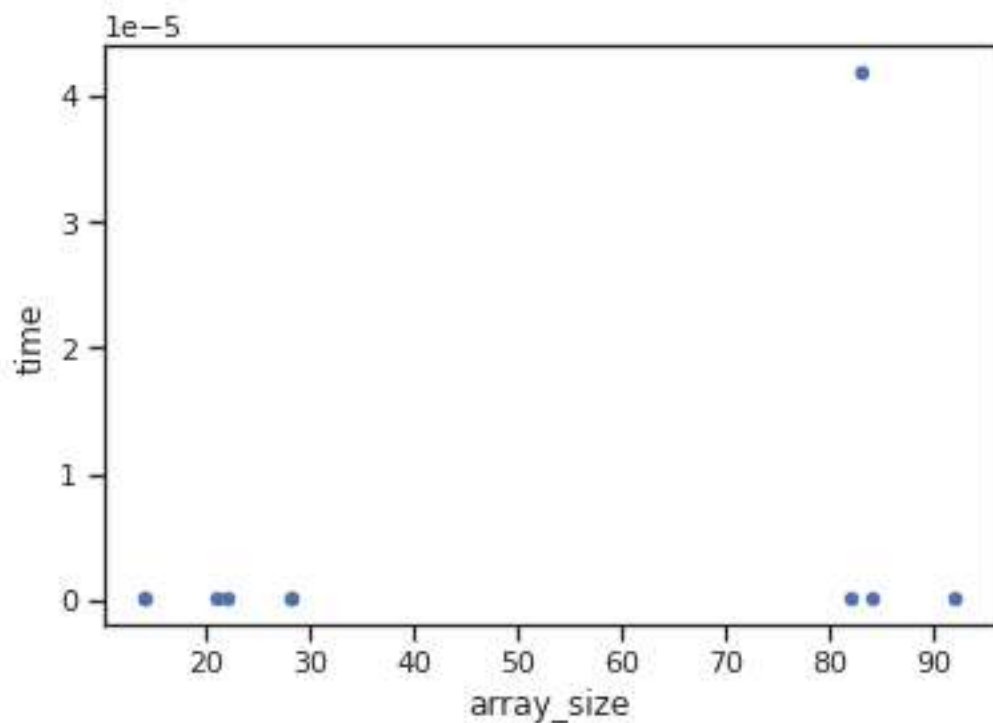

Output:

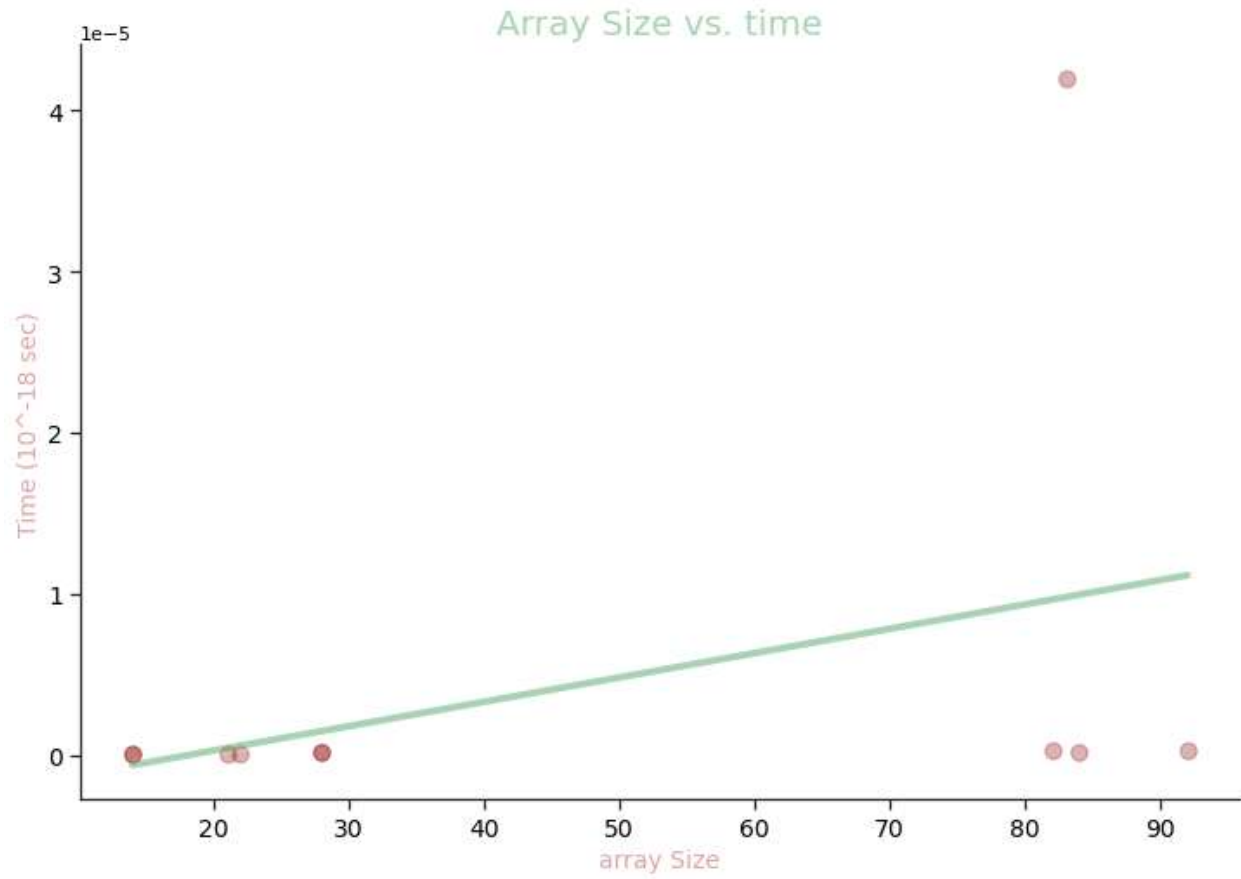
```
input
value of n's:
83, 84, 14, 21, 28, 22, 82, 92, 14, 28,
time for each n:
3.1241e-05, 1.35e-07, 1.01e-07, 9.3e-08, 1.43e-07, 1.14e-07, 2.44e-07, 2.25e-07, 5.8e-08, 1.33e-07,

...Program finished with exit code 0
Press ENTER to exit console.
```

```
arraySize = [83, 84, 14, 21, 28, 22, 82, 92, 14, 28]
```

```
time = [4.1967e-05, 1.51e-07, 1.03e-07, 1.02e-07, 1.65e-07, 1.46e-07, 2.93e-07, 2.79e-07, 1.04e-07, 1.68e-07]
```





Linear Search

Binary search:

Source Code:

```
#include <bits/stdc++.h>
using namespace std;

int binarySearch(int arr[], int l, int r, int x)
{
    if (r >= l)
    {
        int mid = l + (r - l) / 2;

        if (arr[mid] == x)
            return mid;

        else if (arr[mid] > x)
            return binarySearch(arr, l, mid - 1, x);

        return binarySearch(arr, mid + 1, r, x);
    }
    return -1;
}

int binarySearchIter(int arr[], int l, int r, int x)
{
    while (l <= r)
    {
        int m = l + (r - l) / 2;

        if (arr[m] == x)
            return m;

        if (arr[m] < x)
            l = m + 1;

        else
            r = m - 1;
    }
    return -1;
}
```

```
void printArray(int arr[], int size)
{
    int i;

    for (i = 0; i < size; i++)
        cout << arr[i] << " ";

    cout << endl;
}

int main()
{
    // int arr[] = { 2, 3, 4, 10, 40 };
    // int n = sizeof(arr) / sizeof(arr[0]);

    int n = rand() % 100;
    int arr[n];

    for (int i = 0; i < n; i++)
    {
        arr[i] = rand() % 100;
    }

    sort(arr, arr + n);

    cout << "Array: ";
    printArray(arr, n);

    int x;

    cout << "Enter element you want to search in the array: ";
    cin >> x;

    cout << "\n Recursive Binary Search" << endl;

    auto start = chrono::high_resolution_clock::now();
    // unsync the I/O of C and C++.
    ios_base::sync_with_stdio(false);

    int result = binarySearch(arr, 0, n, x);

    auto end = chrono::high_resolution_clock::now();

    (result == -1)
        ? cout << "Element is not present in array" << endl
```

```
        : cout << "Element is present at index " << result << endl;

    double time_taken = chrono::duration_cast<chrono::nanoseconds>(end -
start).count();
    time_taken *= 1e-9;

    cout << "Time difference is: " << time_taken << setprecision(6) << endl;

    cout << "\n Iterative Binary Search" << endl;

    start = chrono::high_resolution_clock::now();

    int resultIter = binarySearchIter(arr, 0, n, x);

    end = chrono::high_resolution_clock::now();

    (resultIter == -1)
        ? cout << "Element is not present in array" << endl
        : cout << "Element is present at index " << resultIter << endl;

    time_taken = chrono::duration_cast<chrono::nanoseconds>(end - start).count();
    time_taken *= 1e-9;

    cout << "Time difference is: " << time_taken << setprecision(6) << endl;

    return 0;
}
```

Output:

```
input
Array: 2 5 5 8 11 11 13 13 14 15 15 19 21 21 22 23 24 24 25 26 26 26 27 27 2
9 29 29 29 30 34 35 35 36 36 37 40 42 43 45 46 49 50 56 56 57 58 59 62 62 62
63 64 67 67 67 67 68 69 70 70 72 73 73 76 77 78 80 81 82 82 84 84 86 86 87
90 91 92 93 93 95 96 98
Enter element you want to search in the array: 43

Recursive Binary Search
Element is present at index 37
Time difference is: 6.3252e-05

Iterative Binary Search
Element is present at index 37
Time difference is: 2.71e-07

...Program finished with exit code 0
Press ENTER to exit console.
```

```
input
Array: 2 5 5 8 11 11 13 13 14 15 15 19 21 21 22 23 24 24 25 26 26 26 27 27 2
9 29 29 29 30 34 35 35 36 36 37 40 42 43 45 46 49 50 56 56 57 58 59 62 62 62
63 64 67 67 67 67 68 69 70 70 72 73 73 76 77 78 80 81 82 82 84 84 86 86 87
90 91 92 93 93 95 96 98
Enter element you want to search in the array: 0

Recursive Binary Search
Element is not present in array
Time difference is: 5.0812e-05

Iterative Binary Search
Element is not present in array
Time difference is: 1.85e-07

...Program finished with exit code 0
Press ENTER to exit console.
```

Batch Analysis:

Source Code:

```
#include <bits/stdc++.h>
using namespace std;

int binarySearch(int arr[], int l, int r, int x)
{
    if (r >= l)
    {
        int mid = l + (r - l) / 2;

        if (arr[mid] == x)
            return mid;

        else if (arr[mid] > x)
            return binarySearch(arr, l, mid - 1, x);

        return binarySearch(arr, mid + 1, r, x);
    }
    return -1;
}

int binarySearchIter(int arr[], int l, int r, int x)
{
    while (l <= r)
    {
        int m = l + (r - l) / 2;
        if (arr[m] == x)
```

```
        return m;

    if (arr[m] < x)
        l = m + 1;

    else
        r = m - 1;
    }
    return -1;
}

void printArray(double arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << ", ";

    cout << endl;
}

void printArray(int arr[], int size)
{
    for (int i = 0; i < size; i++)
        cout << arr[i] << ", ";

    cout << endl;
}

void printArray(float arr[], int size)
{
    for (int i = 0; i < size; i++)
```



```
        cout << arr[i] << ", ";

    cout << endl;
}

double search(int n)
{
    int arr[n];
    for (int i = 0; i < n; i++)
    {
        arr[i] = rand() % 100;
    }
    sort(arr, arr + n);

    int x = rand() % 100;

    auto start = chrono::high_resolution_clock::now();
    ios_base::sync_with_stdio(false);

    binarySearch(arr, 0, n, x);

    auto end = chrono::high_resolution_clock::now();

    double time_taken = chrono::duration_cast<chrono::nanoseconds>(end -
start).count();
    time_taken *= 1e-9;

    return time_taken;
}

int main()
```

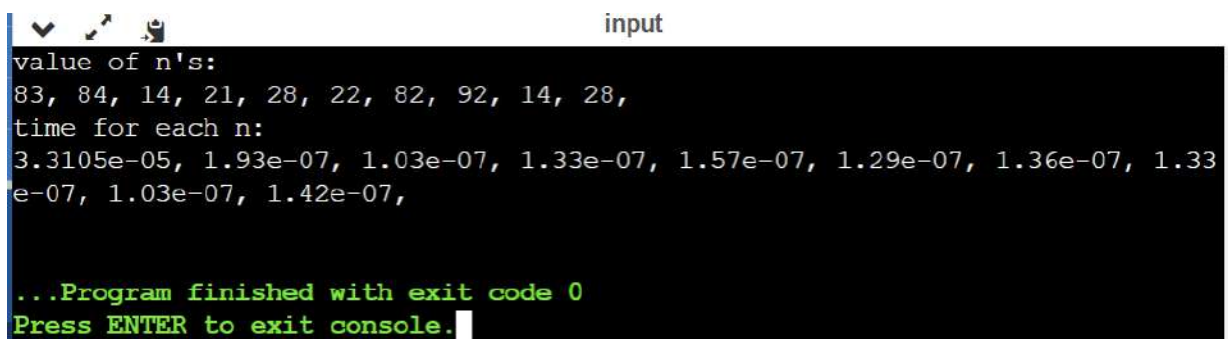
```
{
    double times[10];
    int ns[10];

    for (int x = 0; x < 10; x++)
    {
        int n = rand() % 100;
        ns[x] = n;
        times[x] = search(n);
    }

    cout << "value of n's: " << endl;
    printArray(ns, 10);

    cout << "time for each n: " << endl;
    printArray(times, 10);
}
```

Output:

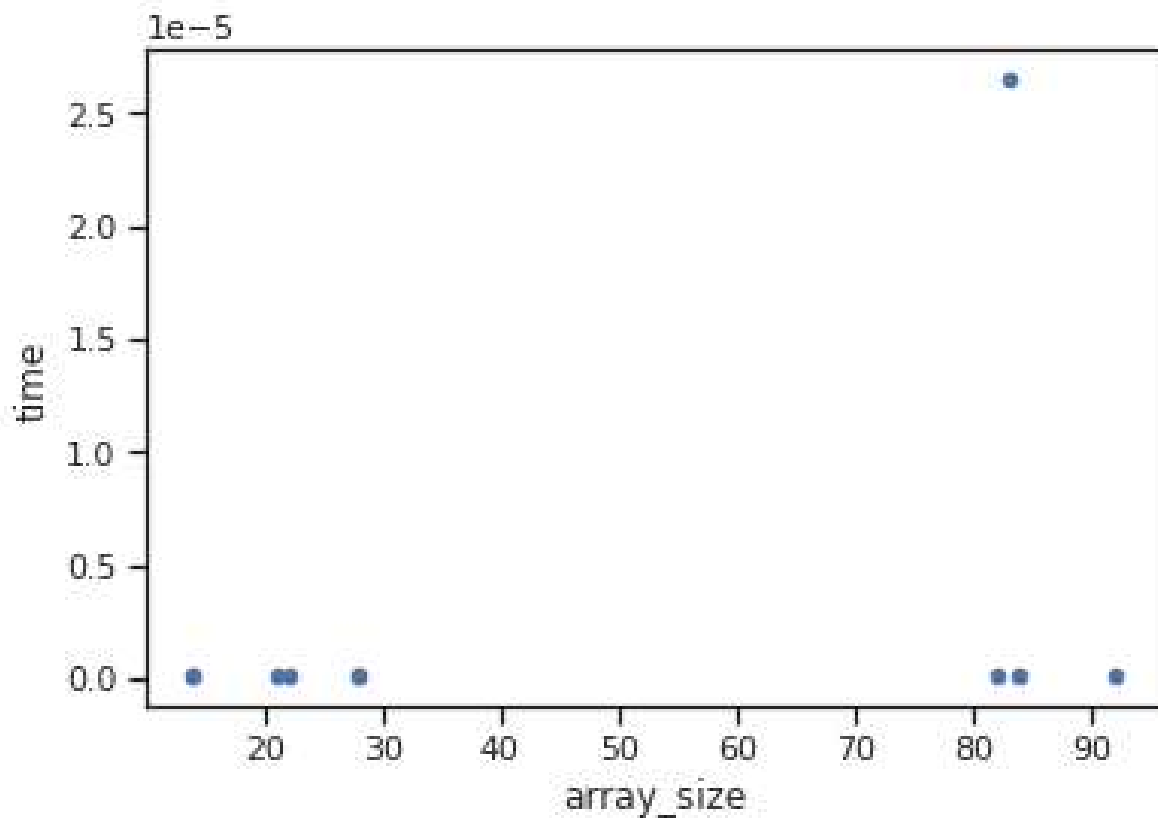


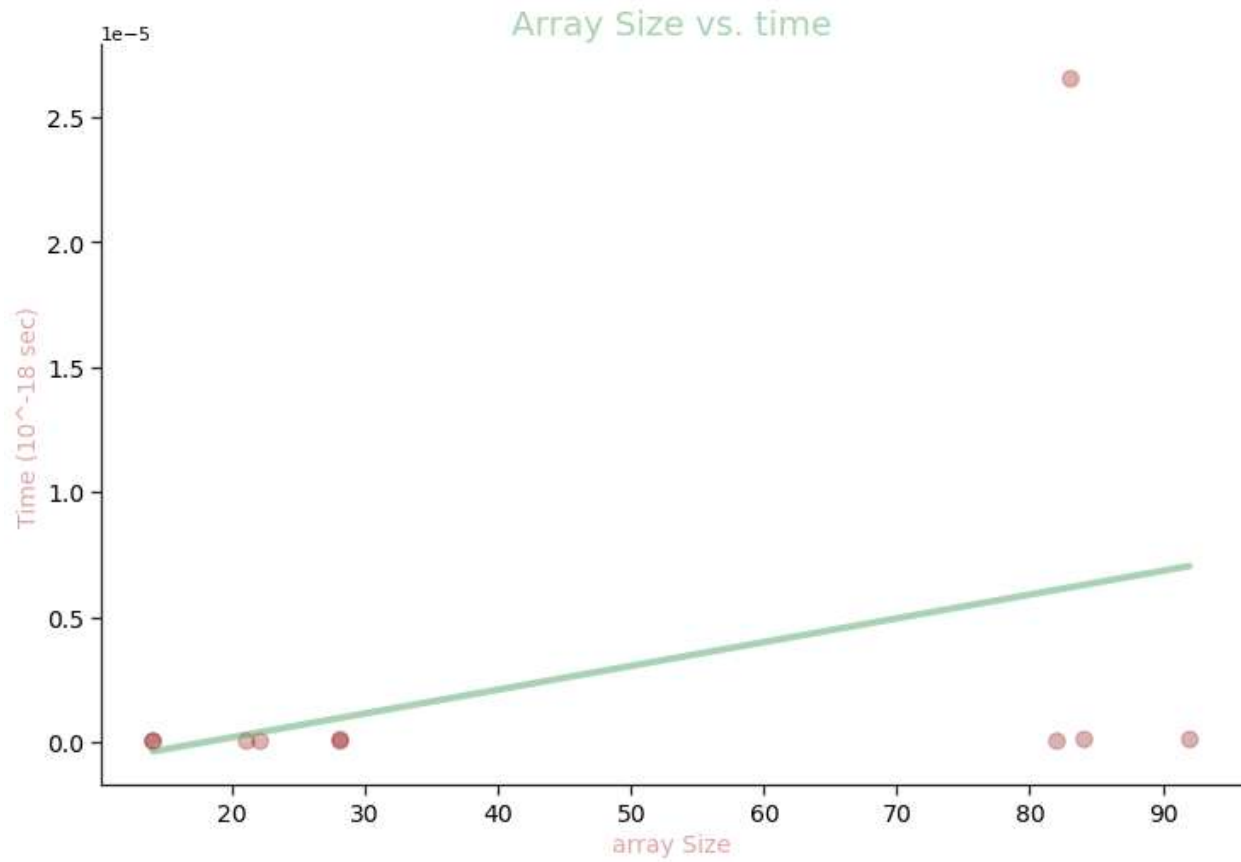
```
input
value of n's:
83, 84, 14, 21, 28, 22, 82, 92, 14, 28,
time for each n:
3.3105e-05, 1.93e-07, 1.03e-07, 1.33e-07, 1.57e-07, 1.29e-07, 1.36e-07, 1.33
e-07, 1.03e-07, 1.42e-07,

...Program finished with exit code 0
Press ENTER to exit console.
```

```
arraySize = [83, 84, 14, 21, 28, 22, 82, 92, 14, 28]
```

```
time = [2.6536e-05, 1.6e-07, 7.9e-08, 9.3e-08, 1.31e-07, 8.7e-08, 1e-07,  
1.15e-07, 8.9e-08, 1.06e-07]
```





Binary Search

Viva Questions

1. The sequential search, also known as _____?

Ans.

Linear Search

One of the most straightforward and elementary searches is the sequential search, also known as **a linear search**.

2. What is the primary requirement for implementation of binary search in an array?

Ans.

The one pre-requisite of binary search is that an array should be in sorted order, whereas the linear search works on both sorted and unsorted array. The binary search algorithm is based on the divide and conquer technique, which means that it will divide the array recursively.

3. Is binary search applicable on array and linked list both?

Ans.

Yes, Binary search is possible on the linked list if the list is ordered and you know the count of elements in list. But While sorting the list, you can access a single element at a time through a pointer to that node i.e. either a previous node or next node.

4. What is the principle of working of Binary search?

Ans.

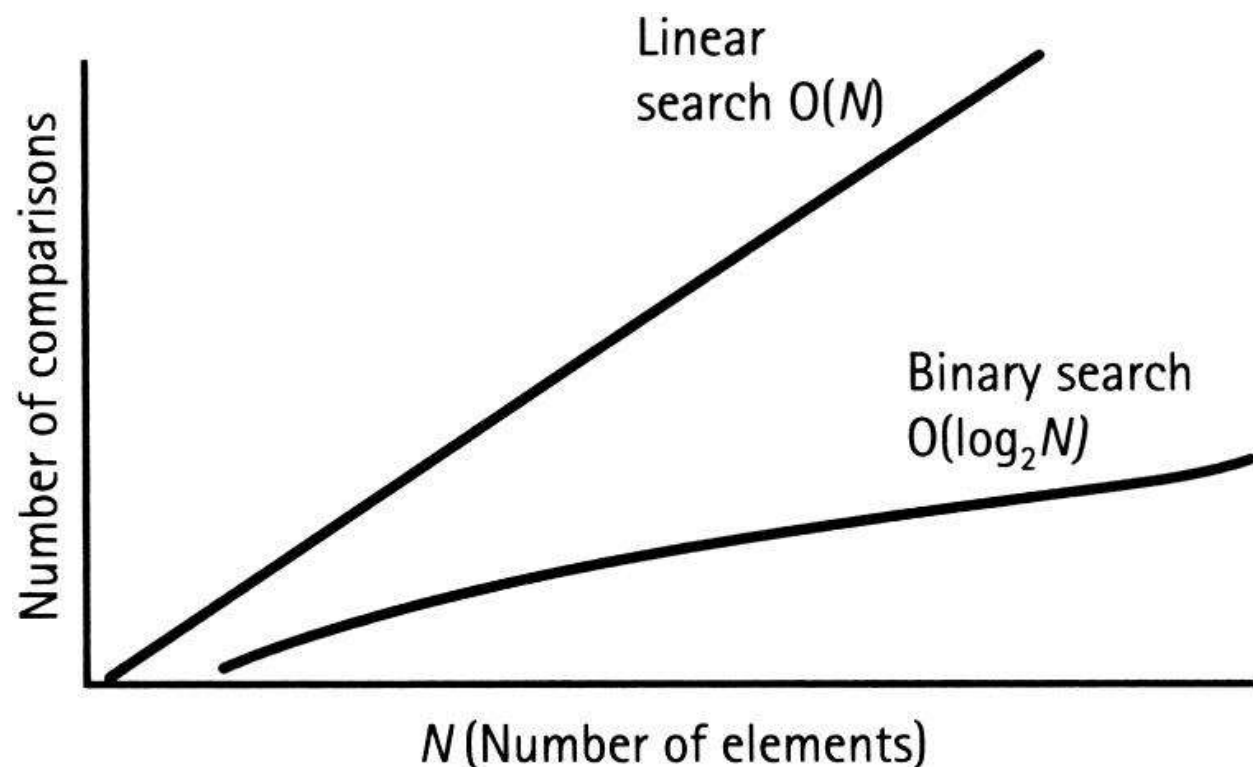
Divide and Conquer

Binary search is a fast search algorithm with run-time complexity of $O(\log n)$. This search algorithm works on the principle of **divide and conquer**. For this algorithm to work properly, the data collection should be in the sorted form.

5. Which searching algorithm is efficient one?

Ans.

Binary search is a more efficient search algorithm which relies on the elements in the list being sorted. We apply the same search process to progressively smaller sub-lists of the original list, starting with the whole list and approximately halving the search area every time.



6. Under what circumstances, binary search cannot be applied to a list of elements?

Ans.

In case the list of elements is not sorted, there's no way to use binary search because the median value of the list can be anywhere and when the list is split into two parts, the element that you were searching for could be cut off.

The main problem that binary search takes **$O(n)$ time in Linked List** due to fact that in linked list we are not able to do indexing which led traversing of each element in Linked list take $O(n)$ time. In this paper a method is implemented through which binary search can be done with time complexity of $O(\log_2 n)$.



EXPERIMENT - 3

Algorithms Design and Analysis Lab

Aim

To implement Matrix Multiplication and analyse its time complexity.

Syeda Reeha Quasar

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4C7

EXPERIMENT – 3

Aim:

To implement Matrix Multiplication and analyse its time complexity.

Theory:

Given a sequence of matrices $A_1, A_2 \dots A_n$, insert parentheses so that the product of the matrices, in order, is unambiguous and needs the minimal number of multiplications

Matrix multiplication is associative: $A_1 (A_2 A_3) = (A_1 A_2) A_3$

A product is unambiguous if no factor is multiplied on both the left and the right and all factors are either a single matrix or an unambiguous product.

Multiplying an $i \times j$ and a $j \times k$ matrix requires ijk multiplications. Each element of the product requires j multiplications, and there are ik elements

$$\begin{array}{ccc}
 & \vec{b}_1 & \vec{b}_2 \\
 & \downarrow & \downarrow \\
 \begin{array}{l} \vec{a}_1 \rightarrow \\ \vec{a}_2 \rightarrow \end{array} & \begin{bmatrix} 1 & 7 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & 3 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} \vec{a}_1 \cdot \vec{b}_1 & \vec{a}_1 \cdot \vec{b}_2 \\ \vec{a}_2 \cdot \vec{b}_1 & \vec{a}_2 \cdot \vec{b}_2 \end{bmatrix} \\
 A & B & C
 \end{array}$$

Number of Parenthesizations

Given the matrices A_1, A_2, A_3, A_4 . Assume the dimensions of $A_1 = d_0 \times d_1$, etc. Below are the five possible parenthesizations of these arrays, along with the number of multiplications:

1. $(A_1A_2)(A_3A_4):d_0d_1d_2+d_2d_3d_4+d_0d_2d_4$
2. $((A_1A_2)A_3)A_4:d_0d_1d_2+d_0d_2d_3+d_0d_3d_4$
3. $(A_1(A_2A_3))A_4:d_1d_2d_3+d_0d_1d_3+d_0d_3d_4$
4. $A_1((A_2A_3)A_4):d_1d_2d_3+d_1d_3d_4+d_0d_1d_4$
5. $A_1(A_2(A_3A_4)):d_2d_3d_4+d_1d_2d_4+d_0d_1d_4$

The number of parenthesizations is at least $T(n) \geq T(n-1) + T(n-1)$. Since the number with the first element removed is $T(n-1)$, which is also the number with the last removed. Thus the number of parenthesizations is $\Omega(2^n)$. The number is actually $T(n) = n-1 \sum_{k=1}^{n-1} T(k)T(n-k)$ which is related to the *Catalan numbers*. This is because the original product can be split into 2 sub products in k places. Each split is to be parenthesized optimally. This recurrence is related to the *Catalan numbers*.

Characterizing the Optimal Parenthesization

An optimal parenthesization of $A_1 \dots A_n$ must break the product into two expressions, each of which is parenthesized or is a single array. Assume the break occurs at position k . In the optimal solution, the solution to the product $A_1 \dots A_k$ must be optimal otherwise, we could improve $A_1 \dots A_n$ by improving $A_1 \dots A_k$ but the solution to $A_1 \dots A_n$ is known to be optimal. This is a contradiction. Thus the solution to $A_1 \dots A_n$ is known to be optimal.

Principle of Optimality

This problem exhibits the Principle of Optimality. The optimal solution to product $A_1 \dots A_n$ contains the optimal solution to two sub products. Thus we can use Dynamic Programming. Consider a recursive solution

Then improve its performance with memoization or by rewriting bottom up

Matrix Dimensions

Consider matrix product $A_1 \times \dots \times A_n$. Let the dimensions of matrix A_i be $d_{i-1} \times d_i$. Thus the dimensions of matrix product $A_i \times \dots \times A_j$ are $d_{i-1} \times d_j$.

Recursive Solution

Let $M[i,j]$ represent the number of multiplications required for matrix product $A_i \times \dots \times A_j$. For $1 \leq i \leq j \leq n$: $M[i,i] = 0$ since no product is required

The optimal solution of $A_i \times A_j$ must break at some point, k , with $i \leq k < j$

Thus, $M[i,j] = M[i,k] + M[k+1,j] + d_{i-1}d_kd_j$

Thus, $M[i,j] = \begin{cases} 0 & \text{if } i=j \\ \min_{i \leq k < j} \{M[i,k] + M[k+1,j] + d_{i-1}d_kd_j\} & \text{if } i < j \end{cases}$

Pseudo code:

// Matrix A_i has dimension $p[i-1] \times p[i]$ for $i = 1..n$

MATRIX-CHAIN-ORDER (p)

$n \leftarrow \text{length}[p] - 1$

for $i \leftarrow 1$ to n

do $m[i,i] \leftarrow 0$

for $l \leftarrow 2$ to n

do for $i \leftarrow 1$ to $n-l+1$

do $j \leftarrow i+l-1$

$m[i,j] \leftarrow \infty$

for $k \leftarrow i$ to $j-1$

do $q \leftarrow m[i,k] + m[k+1,j] + p_{i-1}p_kp_j$

if $q < m[i,j]$

then $m[i,j] \leftarrow q$

$s[i,j] \leftarrow k$

return m and s

PRINT-OPTIMAL-PARENS (s, i, j)

```

if i=j
then print "Ai"
    else print " ( "
        PRINT-OPTIMAL-PARENS (s, i, s[i,j])
        PRINT-OPTIMAL-PARENS (s, s[i,j]+1, j)
    print " ) "

```

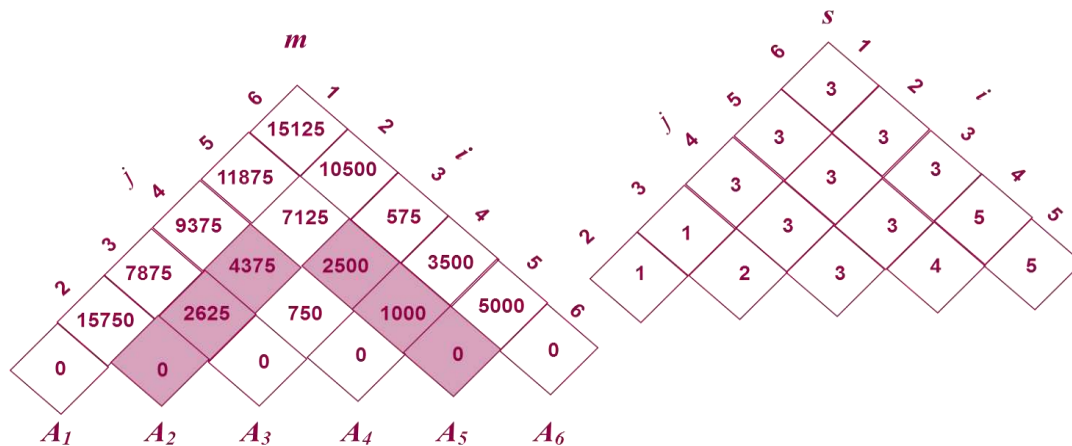
Conditions:

1. The main condition of matrix multiplication is that the number of columns of the 1st matrix must equal to the number of rows of the 2nd one.
2. As a result of multiplication you will get a new matrix that has the same quantity of rows as the 1st one has and the same quantity of columns as the 2nd one.
3. For example if you multiply a matrix of 'n' x 'k' by 'k' x 'm' size you'll get a new one of 'n' x 'm' dimension.

Sample Example:

matrix	dimension
A_1	30 x 35
A_2	35 x 15
A_3	15 x 5
A_4	5 x 10
A_5	10 x 20
A_6	20 x 25

$$\begin{aligned}
 m[2,5] &= \min \begin{cases} m[2,2] + m[3,5] + p_1 p_2 p_5 = 0 + 2500 + 35 \cdot 15 \cdot 20 = 13000 \\ m[2,3] + m[4,5] + p_1 p_3 p_5 = 2625 + 100 + 35 \cdot 5 \cdot 20 = 7125 \\ m[2,4] + m[5,5] + p_1 p_4 p_5 = 4375 + 0 + 35 \cdot 10 \cdot 20 = 11375 \end{cases} \\
 &= (7125)
 \end{aligned}$$



An optimal solution can be constructed from the computed information stored in the table $s[1 \dots n, 1 \dots n]$. The earlier matrix multiplication can be computed recursively.

$$p1 = a(f - h)$$

$$p3 = (c + d)e$$

$$p5 = (a + d)(e + h)$$

$$p7 = (a - c)(e + f)$$

$$p2 = (a + b)h$$

$$p4 = d(g - e)$$

$$p6 = (b - d)(g + h)$$

The $A \times B$ can be calculated using above seven multiplications.

Following are values of four sub-matrices of result C

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} p5 + p4 - p2 + p6 & p1 + p2 \\ p3 + p4 & p1 + p5 - p3 - p7 \end{bmatrix}$$

A B C

A, B and C are square matrices of size $N \times N$

a, b, c and d are submatrices of A, of size $N/2 \times N/2$

e, f, g and h are submatrices of B, of size $N/2 \times N/2$

$p1, p2, p3, p4, p5, p6$ and $p7$ are submatrices of size $N/2 \times N/2$

Result and Analysis

Clearly, the space complexity of this procedure is $O(n^2)$. Since the tables m and s require $O(n^2)$ space. As far as the time complexity is concern, a simple inspection of the for-loop(s) structures gives us a running time of the procedure. Since, the three for-loops are nested three deep, and each one of them iterates at most n times (that is to say indices L , i , and j takes on at most $n - 1$ values). Therefore, the running time of this procedure is $O(n^3)$.

Addition and Subtraction of two matrices takes $O(N^2)$ time. So time complexity can be written as

$$T(N) = 7T(N/2) + O(N^2)$$

From Master's Theorem, time complexity of this method is

$O(N^{\log_2 7})$ which is approximately $O(N^{2.8074})$

Matrix Multiplication:

Source Code:

```
#include <bits/stdc++.h>
#include <chrono>

using namespace std;

void printMatrix(int arr[10][10], int r, int c)
{
    for (int i = 0; i < r; ++i)
        for (int j = 0; j < c; ++j)
        {
            cout << " " << arr[i][j];
            if (j == c - 1)
                cout << endl;
        }
}

int main()
{
    int a[10][10], b[10][10], multipliedMatrix[10][10], r1, c1, r2, c2, i, j, k;

    cout << "Enter rows and columns for first matrix: ";
    cin >> r1 >> c1;
    cout << "Enter rows and columns for second matrix: ";
    cin >> r2 >> c2;

    while (c1 != r2) // matrix multiplication validation check
    {
        cout << "Error! column of first matrix not equal to row of second.";

        cout << "Enter rows and columns for first matrix: ";
        cin >> r1 >> c1;

        cout << "Enter rows and columns for second matrix: ";
        cin >> r2 >> c2;
    }

    cout << endl
        << "\n Enter elements of matrix 1:" << endl;
```

```
for (i = 0; i < r1; ++i)
    for (j = 0; j < c1; ++j)
    {
        cin >> a[i][j];
    }

cout << endl
    << "\n Enter elements of matrix 2:" << endl;
for (i = 0; i < r2; ++i)
    for (j = 0; j < c2; ++j)
    {
        cin >> b[i][j];
    }

// Initializing elements of matrix multipliedMatrix to 0.
for (i = 0; i < r1; ++i)
    for (j = 0; j < c2; ++j)
    {
        multipliedMatrix[i][j] = 0;
    }

auto start = chrono::high_resolution_clock::now();
// unsync the I/O of C and C++.
ios_base::sync_with_stdio(false);


for (i = 0; i < r1; ++i) // multiplication
    for (j = 0; j < c2; ++j)
        for (k = 0; k < c1; ++k)
        {
            multipliedMatrix[i][j] += a[i][k] * b[k][j];
        }

auto end = chrono::high_resolution_clock::now();

cout << "multiply of the matrix=\n";
printMatrix(multipliedMatrix, r1, c2);

double time_taken = chrono::duration_cast<chrono::nanoseconds>(end -
start).count();
time_taken *= 1e-9;
cout << "Time difference is: " << time_taken << setprecision(6) << endl;

return 0;
}
```


Output:

```
Enter rows and columns for first matrix: 2 2
Enter rows and columns for second matrix: 3 3
Error! column of first matrix not equal to row of second.
Enter rows and columns for first matrix: 2 2
Enter rows and columns for second matrix: 2 2

Enter elements of matrix 1:
3 4
2 1

Enter elements of matrix 2:
1 5
3 7
multiply of the matrix=
15 43
5 17
Time difference is: 4.7856e-05

...Program finished with exit code 0
Press ENTER to exit console.
```

```
multiply of the matrix=
15 43
5 17
Time difference is: 4.7856e-05

...Program finished with exit code 0
Press ENTER to exit console.
```

Strassen algorithm:

Source Code:

```
#include <bits/stdc++.h>
#include <chrono>

using namespace std;

void printMatrix(int arr[2][2], int r, int c)
{
    for (int i = 0; i < r; ++i)
        for (int j = 0; j < c; ++j)
        {
            cout << " " << arr[i][j];
            if (j == c - 1)
                cout << endl;
        }
}

int main()
{
    int a[2][2], b[2][2], mult[2][2];
    int m1, m2, m3, m4, m5, m6, m7, i, j;

    cout << "Matrix Multiplication Strassen's method: \n";
    cout << "Enter the elements of 2x2 Matrix 1:\n";
    for (i = 0; i < 2; i++)
    {
        for (j = 0; j < 2; j++)
        {
            cin >> a[i][j];
        }
    }

    cout << "Enter the elements of 2x2 Matrix 2:\n";
    for (i = 0; i < 2; i++)
    {
        for (j = 0; j < 2; j++)
        {
            cin >> b[i][j];
        }
    }
}
```

```
}

auto start = chrono::high_resolution_clock::now();
// unsync the I/O of C and C++.
ios_base::sync_with_stdio(false);

m1 = (a[0][0] + a[1][1]) * (b[0][0] + b[1][1]);
m2 = (a[1][0] + a[1][1]) * b[0][0];
m3 = a[0][0] * (b[0][1] - b[1][1]);
m4 = a[1][1] * (b[1][0] - b[0][0]);
m5 = (a[0][0] + a[0][1]) * b[1][1];
m6 = (a[1][0] - a[0][0]) * (b[0][0] + b[0][1]);
m7 = (a[0][1] - a[1][1]) * (b[1][0] + b[1][1]);

mult[0][0] = m1 + m4 - m5 + m7;
mult[0][1] = m3 + m5;
mult[1][0] = m2 + m4;
mult[1][1] = m1 - m2 + m3 + m6;


auto end = chrono::high_resolution_clock::now();

cout << "\nProduct of matrices is: \n";
printMatrix(mult, 2, 2);

double time_taken = chrono::duration_cast<chrono::nanoseconds>(end -
start).count();
time_taken *= 1e-9;

cout << "Time difference is: " << time_taken << setprecision(6) << endl;

return 0;
}
```

Output:


```
Matrix Multiplication Strassen's method:
Enter the elements of 2x2 Matrix 1:
3 4
2 1
Enter the elements of 2x2 Matrix 2:
1 5
3 7

Product of matrices is:
15 43
5 17
Time difference is: 7.2175e-05

...Program finished with exit code 0
Press ENTER to exit console.
```

```
Product of matrices is:
15 43
5 17
Time difference is: 7.2175e-05

...Program finished with exit code 0
Press ENTER to exit console.
```



```
Matrix Multiplication Strassen's method:
Enter the elements of 2x2 Matrix 1:
2 4
3 6
Enter the elements of 2x2 Matrix 2:
9 3
2 3

Product of matrices is:
 26 18
 39 27
Time difference is: 6.3373e-05

...Program finished with exit code 0
Press ENTER to exit console.
```

Viva Questions

1. What is the recurrence relation for MCM problem?

Ans.

Matrix Chain Multiplication + Dynamic Programming + Recurrence Relation

Given a sequence of matrices, find the most efficient way to multiply these matrices together. The problem is not actually to perform the multiplications, but merely to decide in which order to perform the multiplications.

We have many options to multiply a chain of matrices because matrix multiplication is associative. In other words, no matter how we parenthesize the product, the result will be the same. For example, if we had four matrices A, B, C, and D, we would have:

$$(ABC)D = (AB)(CD) = A(BCD) = \dots$$

However, the order in which we parenthesize the product affects the number of simple arithmetic operations needed to compute the product, or the efficiency. For example, suppose A is a 10×30 matrix, B is a 30×5 matrix, and C is a 5×60 matrix. Then,

$$(AB)C = (10 \times 30 \times 5) + (10 \times 5 \times 60) = 1500 + 3000 = 4500 \text{ operations}$$

$$A(BC) = (30 \times 5 \times 60) + (10 \times 30 \times 60) = 9000 + 18000 = 27000 \text{ operations.}$$

Clearly the first parenthesization requires less number of operations.

Given an array $p[]$ which represents the chain of matrices such that the i th matrix A_i is of dimension $p[i-1] \times p[i]$. We need to write a function `MatrixChainOrder()` that should return the minimum number of multiplications needed to multiply the chain.

2. Let A_1, A_2, A_3, A_4, A_5 be five matrices of dimensions $2 \times 3, 3 \times 5, 5 \times 2, 2 \times 4, 4 \times 3$ respectively. Find the minimum number of scalar multiplications required to find the product $A_1 A_2 A_3 A_4 A_5$ using the basic matrix multiplication method?

Ans.

We have many ways to do matrix chain multiplication because matrix multiplication is associative. In other words, no matter how we parenthesize the product, the result of the matrix chain multiplication obtained will remain the same. Here we have four matrices A_1, A_2, A_3 , and A_4 , we would have:

$$((A_1 A_2) A_3) A_4 = ((A_1 (A_2 A_3)) A_4) = (A_1 A_2) (A_3 A_4) = A_1 ((A_2 A_3) A_4) = A_1 (A_2 (A_3 A_4)).$$

However, the order in which we parenthesize the product affects the number of simple arithmetic operations needed to compute the product, or the efficiency. Here, A_1 is a 10×5 matrix, A_2 is a 5×20 matrix, and A_3 is a 20×10 matrix, and A_4 is 10×5 .

If we multiply two matrices A and B of order $l \times m$ and $m \times n$ respectively, then the number of scalar multiplications in the multiplication of A and B will be $l \times m \times n$.

Then,

The number of scalar multiplications required in the following sequence of matrices will be :

$$A_1 ((A_2 A_3) A_4) = (5 \times 20 \times 10) + (5 \times 10 \times 5) + (10 \times 5 \times 5) = 1000 + 250 + 250 = 1500.$$

All other parenthesized options will require number of multiplications more than 1500.

3. Consider the two matrices P and Q which are 10×20 and 20×30 matrices respectively. What is the number of multiplications required to multiply the two matrices?

Ans.

The number of multiplications required is $10 \times 20 \times 30$.

4. Consider the matrices P, Q and R which are 10 x 20, 20 x 30 and 30 x 40 matrices respectively. What is the minimum number of multiplications required to multiply the three matrices?

Ans.

The minimum number of multiplications are 18000. This is the case when the matrices are parenthesized as $(P*Q)*R$.

5. Can this problem be solved by greedy approach. Justify?

Ans.

Unfortunately, **there is no good "greedy choice"** for Matrix Chain Multiplication, meaning that for any choice that's easy to compute, there is always some input sequence of matrices for which your greedy algorithm will not find the optimum parenthesization.

At each step only least value is selected among all elements of in the array $p[x, y]$, so that the multiplication cost is kept minimum at each step. This greedy approach ensures that the solution is optimal with least cost involved and the output is a fully parenthesized product of matrices.