

TUTORIAL - 6

Ques 1. Explain homogeneous coordinate system.

Ans. To perform sequence of transformations such as translation followed by rotation and scaling. To shorten this process we have to use 3×3 transformation matrix instead of 2×2 . To do so we use extra dummy coordinate w .

→ We can represent points by 3 members instead of 2

$$(x, y) \rightarrow (x_w, y_w, w)$$

$$\left(\frac{x}{w}, \frac{y}{w} \right) \leftarrow \begin{matrix} \downarrow & \downarrow \\ x * w & y * w \end{matrix} (x_w, y_w, w)$$

Translation

$$x' = x + tx$$

$$y' = y + ty$$

$$\begin{bmatrix} tx \\ ty \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotation

$$x' = x \cos \theta - y \sin \theta$$

$$y' = x \sin \theta + y \cos \theta$$

anti-clockwise

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ h \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & h \end{bmatrix} \begin{bmatrix} x \\ y \\ h \end{bmatrix}$$

Scaling

$$x' = x \cdot S_x$$

$$y' = y \cdot S_y$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ h \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

x shearing \rightarrow y shearing \leftarrow

$$\begin{bmatrix} 1 & S_{xy} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ S_{yx} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Reflection

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

origin \rightarrow y axis \rightarrow $x = y$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Ques 2. Translate the polygon with coordinates $A(2, 5)$, $B(7, 10)$ and $C(10, 2)$ by 3 units in x-direction and 4 units in y-direction.

Ans

$A(2, 5)$, $B(7, 10)$, $C(10, 2)$

$$t_x = 3, \quad t_y = 4$$

$$A' = (2+3, 5+4) = (5, 9)$$

$$B' = (7+3, 10+4) = (10, 14)$$

$$C' = (10+3, 2+4) = (13, 6)$$

New coordinates are $A(5, 9)$

$B(10, 14)$

$C(13, 6)$

$$\boxed{\begin{matrix} x' = x + t_x \\ y' = y + t_y \end{matrix}}$$

Ques 3. Scale the polygon with coordinates $A(2, 5)$, $B(7, 10)$ and $C(10, 2)$ by two units in x-direction and two units in y-direction.

Ans

$A(2, 5)$, $B(7, 10)$, $C(10, 2)$

$$dx = 2$$

$$dy = 2$$

for scaling

$$x' = x \cdot dx$$

$$y' = y \cdot dy$$

$$A' = (2 \cdot 2, 5 \cdot 2) = (4, 10)$$

$$B' = (7 \cdot 2, 10 \cdot 2) = (14, 20)$$

$$C' = (10 \cdot 2, 2 \cdot 2) = (20, 4)$$

New coordinates are $A(4, 10)$,
 $B(14, 20)$ and $C(20, 4)$.

Ans 4. Find transformation matrix for
reflections about line $y = x$
Ans $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ matrix req. for same

$A(2, 5)$, $B(7, 10)$, $C(10, 2) \rightarrow \Delta$
matrix for given Δ

$$\begin{bmatrix} 2 & 7 & 10 \\ 5 & 10 & 2 \end{bmatrix}$$

to get reflected matrix about line
 $y = x$:

$$\Rightarrow \begin{bmatrix} 2 & 7 & 10 \\ 5 & 10 & 2 \end{bmatrix} \xrightarrow{\text{reflected matrix about line } y=x} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 7 & 10 \\ 5 & 10 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5 & 10 & 2 \\ 2 & 7 & 10 \end{bmatrix}$$

\therefore Reflected coordinates are
 $A(5, 2)$
 $B(10, 7)$
 $C(2, 10)$

Ans 5. Define the following terms in computer graphics:

a) parallel Projection

A parallel projection is a projection of 3D object in 3D space onto fixed plane called projection / image plane, where rays / line of sight / projection lines are parallel to each other. It can be divided into two sub categories depending upon DOP (direction of projection) as orthographic or oblique projections.

b) View plane

The plane in which the object is projected in a parallel or perspective projection is called the view plane.

c) Perspective projection

It is a method of projection in which a 3D object can be represented by projecting points upon a picture plane using straight lines converging at a fixed point representing the eye of the viewer.

Q1

Shearing :

It is the process of modifying the shape of an object in 2D plane to the change in shape of the object. In this sliding of the layers of object occurs. The shear can be in one direction or in 2 directions.