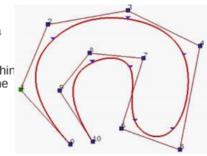
Maharaja Agrasen Institute of Technology
ETCS 211
Computer Graphics & Multimedia
UNIT 2

B-SPLINE CURVE

What is B-Spline curve?

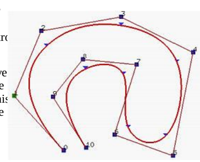
B-spline allows the local control over the **curve** surface because each vertex affects the shape of a **curve** only over a range of parameter values where its associated basis function is nonzero. The **curve** exhibits the variation diminishin property. The **curve** generally follows the shape of defining polygon.



What is B-Spline curve?

The main problem with Bezier curves is their lack of local control. Simply increasing the number of control points adds little local contro to the curve.

They combine all the points to create the curve The obvious solution is to combine only those points nearest to the current parameter. For this we define our points to lie in parametric space equal intervals.



What is B-Spline curve?

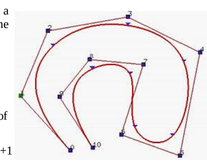
Given $n + 1$ control points $\mathbf{P}_0, \mathbf{P}_1, ..., \mathbf{P}_n$ and a knot vector $U = \{ u_0, u_1, ..., u_m \}$, the B-spline curve of degree p defined by these control points and knot vector U is

$$\mathbf{C}(u) = \sum_{i=0}^{n} N_{i,p}(u)\mathbf{P}_i$$

wh spline basis functions of degree p.

set of n+1 control points, a knot vector of m+1 knots, and a degree p.

n, m and p must satisfy $m = n + p + 1$.



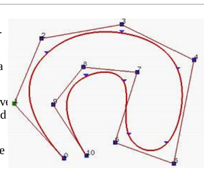
What is B-Spline curve?

if a knot vector of $m + 1$ knots and $n + 1$ control points are given, the degree of the B-spline curve is $p = m - n - 1$.

The point on the curve that corresponds to a knot u_i, $\mathbf{C}(u_i)$, is referred to as a *knot point*.

Hence, the knot points divide a B-spline curve into curve segments, each of which is defined on a knot span.

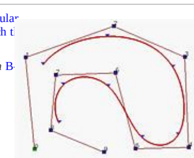
To change the shape of a B-spline curve, one can modify one or more of these control parameters: the positions of control points, the positions of knots, and the degree of the curve.



What is B-Spline curve?

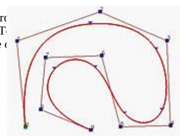
If the knot vector does not have any particular structure, the generated curve will not touch t first and last legs of the control polyline.

This type of B-spline curves is called *open* B-spline curves.



What is B-Spline curve?

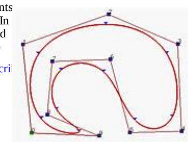
curve is clamped so that it is tangent to the first and the last legs at the first and last contro points, respectively, as a Bézier curve does. T do so, the first knot and the last knot must be multiplicity p+1. This will generate the so-called *clamped* B-spline curves.



What is B-Spline curve?

By repeating some knots and control points the generated curve can be a *closed* one. In case, the start and the end of the generated curve join together forming a closed loop

We use open, clamped and closed to descri three types of B-spline curves.

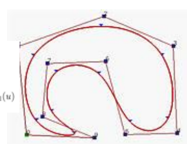


What is B-Spline curve?

B-spline basis functions as follows:

$$N_{i,0}(u) = \begin{cases} 1 & \text{if } u_i \le u < u_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

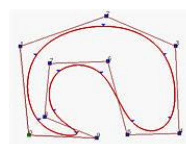
$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u)$$



Properties of B-Spline curve

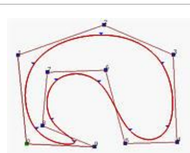
B-spline basis functions as follows:

1. **$N_{i,p}(u)$ is a degree p polynomial in u**
2. **Nonnegativity -- For all i, p and u, $N_{i,p}(u)$ is non-negative**
3. **Local Support -- $N_{i,p}(u)$ is a non-zero polynomial on $[u_i,u_{i+p+1})$**
   This has been discussed on previous page.
4. **On any span $[u_i, u_{i+1})$, at most p+1 degree p basis functions are non-zero, namely: $N_{i-p,p}(u)$, $N_{i-p+1,p}(u)$, $N_{i-p+2,p}(u)$, ..., and $N_{i,p}(u)$**



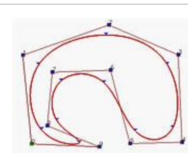
Properties of B-Spline curve

5. **Partition of Unity -- The sum of all non-zero degree p basis functions on span $[u_i, u_{i+1})$ is 1**:
   The previous property shows that $N_{i-p,p}(u)$, $N_{i-p+1,p}(u)$, $N_{i-p+2,p}(u)$, ..., and $N_{i,p}(u)$ are non-zero on $[u_i, u_{i+1})$. This one states that the sum of these p+1 basis functions is 1.
6. **If the number of knots is m+1, the degree of the basis functions is p, and the number of degree p basis functions is n+1, then $m = n + p + 1$** :
   This is not difficult to see. Let $N_{n,p}(u)$ be the last degree p basis function. It is non-zero on $[u_n, u_{n+p+1})$. Since it is the last basis function, u_{n+p+1} must be the last knot u_m. Therefore, we have $u_{n+p+1} = u_m$ and $n + p + 1 = m$. In summary, given m and p, let $n = m - p - 1$ and the degree p basis functions are $N_{0,p}(u)$, $N_{1,p}(u)$, $N_{2,p}(u)$, ..., and $N_{n,p}(u)$.



Properties of B-Spline curve

5. **Basis function $N_{i,p}(u)$ is a composite curve of degree p polynomials with joining points at knots in $[u_i, u_{i+p+1})$**
   The example shown on the previous page illustrates this property well. For example, $N_{0,2}(u)$, which is non-zero on [0,3), is constructed from three parabolas defined on [0,1), [1,2) and [2,3). They are connected together at the knots 2 and 3.
6. **At a knot of multiplicity k, basis function $N_{i,p}(u)$ is C^{p-k} continuous.**
   Therefore, increasing multiplicity decreases the level of continuity, and increasing degree increases continuity. The above mentioned degree two basis function $N_{0,2}(u)$ is C^1 continuous at knots 2 and 3, since they are simple knots (k = 1).



Cubic B-Spline curve

Uniform **cubic B-spline curves** are based on the assumption that a nice **curve** corresponds to using **cubic** functions for each segment and constraining the points that joint the segments to meet three continuity requirements:

1. **Positional Continuity** (0 order): i.e. the end point of segment *i* is the same as the starting point of segment *i* + 1.
2. **Tangential Continuity** (1st order): i.e. no abrupt change in slope occurs at the transition between segment *i* and segment *i* + 1.
3. **Curvature Continuity** (2nd order): i.e. no polarity changes in slope at the transition between segment *i* and segment *i* + 1.