**Maharaja Agrasen Institute of Technology**
**ETCS 211**
**Computer Graphics & Multimedia**

EXPERIMENT 6

2D AND 3D TRANSFORMATIONS



# AIM

Write C Programs for the implementation of 2D and 3D transformations.



# THEORY

Transformation is converting a graphic image into another graphic image by applying algorithms. We can perform the following actions to transform the images-
•Change the position of an image.
•Increase or decrease the size of an image.
•Change the angle of the image.
By applying the above actions a new image will be formed and this process is called Transformation.

**Object Transformation requires**

•**Geometric Transformation:** moving the picture and keeping the background fixed.
•**Coordinate Transformation:** moving the background and the picture is fixed.

**Types of transformation :** **2D and 3D transformation**



# THEORY

**The Two-Dimensional Transformation includes-**

1. **2D Translation:** it is used to move the object from one position to another position on the screen.
2. **2D Rotation:** it is used to rotate the object from origin to a particular angle.
3. **2D Scaling:** it is a process or technique used to resize the object in two-dimensional plane.
4. **2D Reflection:** it is a process in which we can rotate the object at the angle of 180°.
5. **2D Shearing:** it is a process that is used to perform slanting on the object.



# THEORY

**2D Translation:** it is used to move the object from one position to another position on the screen.

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

Translation Matrix

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Translation Matrix
(Homogeneous Coordinates Representation)

$X_{new} = X_{old} + T_x$
$Y_{new} = Y_{old} + T_y$



# THEORY

**2D Rotation:** it is used to rotate the object from origin to a particular angle

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Rotation Matrix

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Rotation Matrix
(Homogeneous Coordinates Representation)

•$X_{new} = X_{old} \times \cos\theta - Y_{old} \times \sin\theta$
•$Y_{new} = X_{old} \times \sin\theta + Y_{old} \times \cos\theta$

# THEORY

**2D Scaling:** it is a process or technique used to resize the object in two-dimensional plane

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Scaling Matrix

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Scaling Matrix
(Homogeneous Coordinates Representation)

- $X_{new} = X_{old} \times S_x$
- $Y_{new} = Y_{old} \times S_y$



# THEORY

**2D Scaling:** it is a process or technique used to resize the object in two-dimensional plane

- Scaling may be used to increase or reduce the size of object.
- Scaling subjects the coordinate points of the original object to change.
- Scaling factor determines whether the object size is to be increased or reduced.

If scaling factor > 1, then the object size is increased.

If scaling factor < 1, then the object size is reduced.



# THEORY

**2D Reflection:** it is a process in which we can rotate the object at the angle of 180°.

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Reflection Matrix
(Reflection Along X Axis)

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Reflection Matrix
(Reflection Along X Axis)
(Homogeneous Coordinates Representation)

- $X_{new} = X_{old}$
- $Y_{new} = -Y_{old}$



# THEORY

**2D Shearing:** it is a process that is used to perform slanting on the object.

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & Sh_x \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Shearing Matrix
(In X axis)

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ Sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Shearing Matrix
(In X axis)
(Homogeneous Coordinates Representation)

- $X_{new} = X_{old} + Sh_x \times Y_{old}$
- $Y_{new} = Y_{old}$



# EXAMPLE

1. Given a circle C with radius 10 and center coordinates (1, 4). Apply the translation with distance 5 towards X axis and 1 towards Y axis. Obtain the new coordinates of C without changing its radius.

Old center coordinates of C = $(X_{old}, Y_{old})$ = (1, 4)

Translation vector = $(T_x, T_y)$ = (5, 1)

Applying the translation equations, we have-

- $X_{new} = X_{old} + T_x = 1 + 5 = 6$
- $Y_{new} = Y_{old} + T_y = 4 + 1 = 5$

New center coordinates of C = (6, 5).

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$



# EXAMPLE

2. Given a line segment with starting point as (0, 0) and ending point as (4, 4). Apply 30 degree rotation anticlockwise direction on the line segment and find out the new coordinates of the line.

Old ending coordinates of the line = $(X_{old}, Y_{old})$ = (4, 4)

Rotation angle = θ = 30º

new ending coordinates of the line after rotation = $(X_{new}, Y_{new})$.

$X_{new}$

$= X_{old} \times \cos\theta - Y_{old} \times \sin\theta$

= 4 x cos30º – 4 x sin30º

= 4 x (√3 / 2) – 4 x (1 / 2)

= 2√3 – 2

= 2(√3 – 1)

= 2(1.73 – 1)

= 1.46

$Y_{new}$

$= X_{old} \times \sin\theta + Y_{old} \times \cos\theta$

= 4 x sin30º + 4 x cos30º

= 4 x (1 / 2) + 4 x (√3 / 2)

= 2 + 2√3

= 2(1 + √3)

= 2(1 + 1.73)

= 5.46

New ending coordinates of the line after rotation = (1.46, 5.46).

## EXAMPLE

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} \cos 30 & -\sin 30 \\ \sin 30 & \cos 30 \end{bmatrix} \times \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 4 \times \cos 30 - 4 \times \sin 30 \\ 4 \times \sin 30 + 4 \times \cos 30 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 4 \times \cos 30 - 4 \times \sin 30 \\ 4 \times \sin 30 + 4 \times \cos 30 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1.46 \\ 5.46 \end{bmatrix}$$



## EXAMPLE

3. Given a square object with coordinate points A(0, 3), B(3, 3), C(3, 0), D(0, 0). Apply the scaling parameter 2 towards X axis and 3 towards Y axis and obtain the new coordinates of the object.

Old corner coordinates of the square = A (0, 3), B(3, 3), C(3, 0), D(0, 0)
Scaling factor along X axis = 2
Scaling factor along Y axis = 3



## EXAMPLE

3. Given a square object with coordinate points A(0, 3), B(3, 3), C(3, 0), D(0, 0). Apply the scaling parameter 2 towards X axis and 3 towards Y axis and obtain the new coordinates of the object.

**For Coordinates A(0, 3)**

- $X_{new} = X_{old} \times S_x = 0 \times 2 = 0$
- $Y_{new} = Y_{old} \times S_y = 3 \times 3 = 9$

**For Coordinates B(3, 3)**

- $X_{new} = X_{old} \times S_x = 3 \times 2 = 6$
- $Y_{new} = Y_{old} \times S_y = 3 \times 3 = 9$

**For Coordinates C(3, 0)**

- $X_{new} = X_{old} \times S_x = 3 \times 2 = 6$
- $Y_{new} = Y_{old} \times S_y = 0 \times 3 = 0$

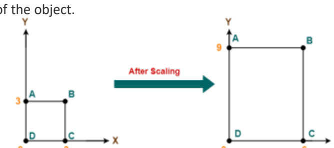
**For Coordinates D(0, 0)**

- $X_{new} = X_{old} \times S_x = 0 \times 2 = 0$
- $Y_{new} = Y_{old} \times S_y = 0 \times 3 = 0$



## EXAMPLE

3. Given a square object with coordinate points A(0, 3), B(3, 3), C(3, 0), D(0, 0). Apply the scaling parameter 2 towards X axis and 3 towards Y axis and obtain the new coordinates of the object.






## EXAMPLE

4. Given a triangle with coordinate points A(3, 4), B(6, 4), C(5, 6). Apply the reflection on the X axis and obtain the new coordinates of the object.

**For Coordinates A(3, 4)**

- $X_{new} = X_{old} = 3$
- $Y_{new} = -Y_{old} = -4$

**For Coordinates B(6, 4)**

- $X_{new} = X_{old} = 6$
- $Y_{new} = -Y_{old} = -4$

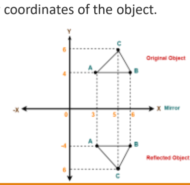
**For Coordinates C(5, 6)**

- $X_{new} = X_{old} = 5$
- $Y_{new} = -Y_{old} = -6$



## EXAMPLE

4. Given a triangle with coordinate points A(3, 4), B(6, 4), C(5, 6). Apply the reflection on the X axis and obtain the new coordinates of the object.



## EXAMPLE

5. Given a triangle with points (1, 1), (0, 0) and (1, 0). Apply shear parameter 2 on X axis and 2 on Y axis and find out the new coordinates of the object.

- Old corner coordinates of the triangle = A (1, 1), B(0, 0), C(1, 0)
- Shearing parameter towards X direction ($Sh_x$) = 2
- Shearing parameter towards Y direction ($Sh_y$) = 2

**Shearing in X Axis-**

**For Coordinates A(1, 1)**

$X_{new} = X_{old} + Sh_x \times Y_{old} = 1 + 2 \times 1 = 3$

$Y_{new} = Y_{old} = 1$

**For Coordinates B(0, 0)**

$X_{new} = X_{old} + Sh_x \times Y_{old} = 0 + 2 \times 0 = 0$

$Y_{new} = Y_{old} = 0$



## EXAMPLE

5. Given a triangle with points (1, 1), (0, 0) and (1, 0). Apply shear parameter 2 on X axis and 2 on Y axis and find out the new coordinates of the object.

**Shearing in X Axis-**

**For Coordinates C(1, 0)**

- $X_{new} = X_{old} + Sh_x \times Y_{old} = 1 + 2 \times 0 = 1$
- $Y_{new} = Y_{old} = 0$

New coordinates of the triangle after shearing in X axis = A (3, 1), B(0, 0), C(1, 0).



## QUESTIONS ON 2D TRANSFORMATION

1. Translate the square ABCD whose co-ordinate are A(0,0), b(3,0), C(3,3), D(0,3) by 2 units in both direction and then scale it by 1.5 units in x direction and 0.5 units in y direction.
2. Write the equation and matrix for shearing along the y-axis.
3. Magnify the triangle with vertices A(0,0), B(1,1) and C(5,2) to twice its size while keeping C(5,2) fixed.
4. What are the uses of doing transformation?



## THEORY

Transformation is converting a graphic image into another graphic image by applying algorithms. We can perform the following actions to transform the images-
- Change the position of an image.
- Increase or decrease the size of an image.
- Change the angle of the image.

By applying the above actions a new image will be formed and this process is called Transformation.

**Object Transformation requires**

- **Geometric Transformation:** moving the picture and keeping the background fixed.
- **Coordinate Transformation:** moving the background and the picture is fixed.

**Types of transformation :** **2D and 3D transformation**



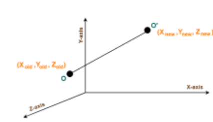
## THEORY

- 3D Transformations take place in a three dimensional plane.
- 3D Transformations are important and complex than 2D Transformations.
- Transformations are helpful in changing the position, size, orientation, shape etc of the object.



## THEORY

- Initial coordinates of the object O = ($X_{old}$, $Y_{old}$, $Z_{old}$)
- New coordinates of the object O after translation = ($X_{new}$, $Y_{new}$, $Z_{old}$)
- Translation vector or Shift vector = ($T_x$, $T_y$, $T_z$)
- $T_x$ defines the distance the $X_{old}$ coordinate has to be moved.
- $T_y$ defines the distance the $Y_{old}$ coordinate has to be moved.
- $T_z$ defines the distance the $Z_{old}$ coordinate has to be moved.



3D Translation in Computer Graphics

## THEORY

$X_{new} = X_{old} + T_x$ (This denotes translation towards X axis)

$Y_{new} = Y_{old} + T_y$ (This denotes translation towards Y axis)

$Z_{new} = Z_{old} + T_z$ (This denotes translation towards Z axis)

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Translation Matrix



## THEORY

- Initial coordinates of the object O = $(X_{old}, Y_{old}, Z_{old})$
- Initial angle of the object O with respect to origin = Φ
- Rotation angle = θ
- New coordinates of the object O after rotation = $(X_{new}, Y_{ne}$
- X-axis Rotation
- Y-axis Rotation
- Z-axis Rotation



## THEORY

**For X-Axis Rotation-**

$X_{new} = X_{old}$

$Y_{new} = Y_{old} \times \cos\theta - Z_{old} \times \sin\theta$

$Z_{new} = Y_{old} \times \sin\theta + Z_{old} \times \cos\theta$

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Rotation Matrix
(For X-Axis Rotation)



## THEORY

**For Y-Axis Rotation-**

- $X_{new} = Z_{old} \times \sin\theta + X_{old} \times \cos\theta$
- $Y_{new} = Y_{old}$
- $Z_{new} = Y_{old} \times \cos\theta - X_{old} \times \sin\theta$

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Rotation Matrix
(For Y-Axis Rotation)



## THEORY

**For z-Axis Rotation-**

- $X_{new} = X_{old} \times \cos\theta - Y_{old} \times \sin\theta$
- $Y_{new} = X_{old} \times \sin\theta + Y_{old} \times \cos\theta$
- $Z_{new} = Z_{old}$

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Rotation Matrix
(For Z-Axis Rotation)



## THEORY

Initial coordinates of the object O = $(X_{old}, Y_{old}, Z_{old})$

Scaling factor for X-axis = $S_x$

Scaling factor for Y-axis = $S_y$

Scaling factor for Z-axis = $S_z$

New coordinates of the object O after scaling = $(X_{new}, Y_{new}, Z_{new})$

$X_{new} = X_{old} \times S_x$

$Y_{new} = Y_{old} \times S_y$

$Z_{new} = Z_{old} \times S_z$

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ Z_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ Z_{old} \\ 1 \end{bmatrix}$$

3D Scaling Matrix