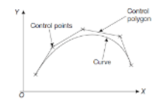
Maharaja Agrasen Institute of Technology
ETCS 211
Computer Graphics & Multimedia
UNIT 2

BEZIER CURVE

What is curve and surfaces?

A complete curve is split into curve segments, each defined by a cubical polynomial.

There are many ways to represent curves and surfaces



Curves can be broadly classified into three categories − **explicit, implicit,** and **parametric curves**.

What is curve and surfaces?

Implicit curve

a)Two dimensional curve(s) g(x,y)=0

b)Much more robust

All lines ax+by+c=0

Circles x2+y2-r2=0

c)Three dimensions g(x,y,z)=0 defines a surface

d)we could intersect two surfaces to get a curve

What is curve and surfaces?

Explicit curve

a)Most familiar form of curve in 2D y=f(x)

b)Cannot represent all curves

Vertical lines

Circles

c)Extension to 3D : y=f(x), z=g(x)

d) gives a curve of the form y = f(x,z) defines a surface

What is curve?

Objects are not flat all the time and we need to draw curves many times to draw an object.

A curve is an infinitely large set of points. Each point has two neighbors except endpoints. Curves can be broadly classified into three categories − **explicit, implicit,** and **parametric curves**.

What is curve and surfaces?

Parametric curve

a)Separate equation for each spatial variable

x=x(u) , y=y(u) , z=z(u)

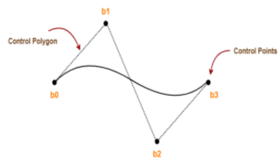
p(u)=[x(u), y(u), z(u)]T

b)•For u_{max} >= u >= u_{min} we trace out a curve in two or three dimensions

What is Bezier curve?

A **Bezier curve** is a parametric **curve** used in computer graphics and related fields. The **curve**, which is related to the Bernstein polynomial, is named after Pierre **Bzier**

- The bezier curve is defined by a set of control points b_0, b_1, b_2 and b_3.
- Points b_0 and b_3 are ends of the curve.
- Points b_1 and b_2 determine the shape of the curve.



Bezier Curve Example

Properties of Bezier curve

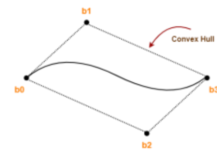
Few important properties of a bezier curve are-

Property-01:

Bezier curve is always contained within a polygon called as convex hull of its control points.

Property-02:

- Bezier curve generally follows the shape of its defining polygon.
- The first and last points of the curve are coincident with the first and last points of the defining polygon.



Bezier Curve With Convex Hull

Properties of Bezier curve

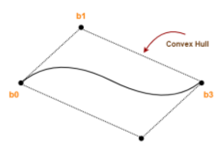
Few important properties of a bezier curve are-

Property-03:

The degree of the polynomial defining the curve segment is one less than the total number of control points.

Property-04:

The order of the polynomial defining the curve segment is equal to the total number of control points.



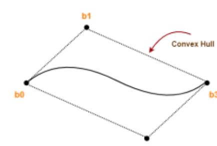
Bezier Curve With Convex Hull

Properties of Bezier curve

Few important properties of a bezier curve are-

Property-05:

- Bezier curve exhibits the variation diminishing property.
- It means the curve do not oscillate about any straight line more often than the defining polygon.



Bezier Curve With Convex Hull

Equation of Bezier curve

Here,

- t is any parameter where 0 <= t <= 1
- P(t) = Any point lying on the bezier curve
- B_i = i^{th} control point of the bezier curve
- n = degree of the curve
- $J_{n,i}(t)$ = Blending function = $C(n,i)t^i(1-t)^{n-i}$ where $C(n,i) = n! / i!(n-i)!$

$$\boxed{P(t) = \sum_{i=0}^{n} B_i J_{n,i}(t)}$$

Bezier Curve Equation

Equation of Bezier curve

Cubic bezier curve is a bezier curve with degree 3.

The total number of control points in a cubic bezier curve is 4.

substituting n = 3 for a cubic bezier curve, we get-

$$P(t) = \sum_{i=0}^{3} B_i J_{3,i}(t)$$

Expanding the equation, we get-

$P(t) = B_0J_{3,0}(t) + B_1J_{3,1}(t) + B_2J_{3,2}(t) + B_3J_{3,3}(t)$ ....(1)

Equation of Bezier curve

$$J_{3,0}(t) = \frac{3!}{0!\,(3-0)!}\, t^0 (1-t)^{3-0}$$
$$J_{3,0}(t) = (1-t)^3 \quad \text{.............}(2)$$

$$J_{3,1}(t) = \frac{3!}{1!\,(3-1)!}\, t^1 (1-t)^{3-1}$$
$$J_{3,1}(t) = 3t(1-t)^2 \quad \text{.............}(3)$$

$$J_{3,2}(t) = \frac{3!}{2!\,(3-2)!}\, t^2 (1-t)^{3-2}$$
$$J_{3,2}(t) = 3t^2(1-t)^2 \quad \text{.............}(4)$$

$$J_{3,3}(t) = \frac{3!}{3!\,(3-3)!}\, t^3 (1-t)^{3-3}$$
$$J_{3,3}(t) = t^3 \quad \text{.............}(5)$$

Using (2), (3), (4) and (5) in (1), we get-

$P(t) = B_0(1-t)^3 + B_1 3t(1-t)^2 + B_2 3t^2(1-t) + B_3 t^3$

This is the required parametric equation for a cubic bezier curve.

Equation of Bezier curve

$$J_{3,0}(t) = \frac{3!}{0!\,(3-0)!}\, t^0 (1-t)^{3-0}$$
$$J_{3,0}(t) = (1-t)^3 \quad \text{.............}(2)$$

$$J_{3,1}(t) = \frac{3!}{1!\,(3-1)!}\, t^1 (1-t)^{3-1}$$
$$J_{3,1}(t) = 3t(1-t)^2 \quad \text{.............}(3)$$

$$J_{3,2}(t) = \frac{3!}{2!\,(3-2)!}\, t^2 (1-t)^{3-2}$$
$$J_{3,2}(t) = 3t^2(1-t)^2 \quad \text{.............}(4)$$

$$J_{3,3}(t) = \frac{3!}{3!\,(3-3)!}\, t^3 (1-t)^{3-3}$$
$$J_{3,3}(t) = t^3 \quad \text{.............}(5)$$

Using (2), (3), (4) and (5) in (1), we get-

$P(t) = B_0(1-t)^3 + B_1 3t(1-t)^2 + B_2 3t^2(1-t) + B_3 t^3$

This is the required parametric equation for a cubic bezier curve.

Example of Bezier curve :

Given a bezier curve with 4 control points-
B_0[1 0] , B_1[3 3] , B_2[6 3] , B_3[8 1]
Determine any 5 points lying on the curve. Also, draw a rough sketch of the curve.

Solution:

We have-

- The given curve is defined by 4 control points.
- So, the given curve is a cubic bezier curve.

The parametric equation for a cubic bezier curve is-

$P(t) = B_0(1-t)^3 + B_1 3t(1-t)^2 + B_2 3t^2(1-t) + B_3 t^3$

Substituting the control points B_0, B_1, B_2 and B_3, we get-

$P(t) = [1\ 0](1-t)^3 + [3\ 3]3t(1-t)^2 + [6\ 3]3t^2(1-t) + [8\ 1]t^3$ ……..(1)

Now,

To get 5 points lying on the curve, assume any 5 values of t lying in the range

$0 <= t <= 1$.

Let 5 values of t are 0, 0.2, 0.5, 0.7, 1

Example of Bezier curve :

Given a bezier curve with 4 control points-
B_0[1 0] , B_1[3 3] , B_2[6 3] , B_3[8 1]
Determine any 5 points lying on the curve. Also, draw a rough sketch of the curve.

Solution:

For t = 0:

Substituting t=0 in (1), we get-

$P(0) = [1\ 0](1-0)^3 + [3\ 3]3(0)(1-t)^2 + [6\ 3]3(0)^2(1-0) + [8\ 1](0)^3$

P(0) = [1 0] + 0 + 0 + 0

P(0) = [1 0]

For t = 0.2:

Substituting t=0.2 in (1), we get-

$P(0.2) = [1\ 0](1-0.2)^3 + [3\ 3]3(0.2)(1-0.2)^2 + [6\ 3]3(0.2)^2(1-0.2) + [8\ 1](0.2)^3$

$P(0.2) = [1\ 0](0.8)^3 + [3\ 3]3(0.2)(0.8)^2 + [6\ 3]3(0.2)^2(0.8) + [8\ 1](0.2)^3$

P(0.2) = [1 0] x 0.512 + [3 3] x 3 x 0.2 x 0.64 + [6 3] x 3 x 0.04 x 0.8 + [8 1] x 0.008

P(0.2) = [1 0] x 0.512 + [3 3] x 0.384 + [6 3] x 0.096 + [8 1] x 0.008

P(0.2) = [0.512 0] + [1.152 1.152] + [0.576 0.288] + [0.064 0.008]

P(0.2) = [2.304 1.448]

Example of Bezier curve :

Given a bezier curve with 4 control points-
B_0[1 0] , B_1[3 3] , B_2[6 3] , B_3[8 1]
Determine any 5 points lying on the curve. Also, draw a rough sketch of the curve.

Solution:

For t = 1:

Substituting t=1 in (1), we get-

$P(1) = [1\ 0](1-1)^3 + [3\ 3]3(1)(1-1)^2 + [6\ 3]3(1)^2(1-1) + [8\ 1](1)^3$

P(1) = [1 0] x 0 + [3 3] x 3 x 1 x 0 + [6 3] x 3 x 1 x 0 + [8 1] x 1

P(1) = 0 + 0 + 0 + [8 1]

P(1) = [8 1]

Following is the required rough sketch of the curve-