Harmonic Analysis

If function is given in discrete form, then we use this method to calculate fourier coefficients.

$$y = f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{2}{p} \sum_{i=1}^{p} y_i$$

$$a_n = \frac{2}{p} \sum_{i=1}^{p} y_i \cos n x_i$$

$$b_n = \frac{2}{p} \sum_{i=1}^{p} y_i \sin n x_i$$

Qus 1 Find the first three harmonics for the fn $f(\theta)$ given in the following table.

θ°	0	60	120	180	240	300	360
$f(\theta)$	0.8	0.6	0.4	0.7	0.9	1.1	0.8

Ans.

θ°	$f(\theta)$	$\cos\theta$	$\sin\theta$	$\cos 2\theta$	$\sin 2\theta$	$\cos 3\theta$	$\sin 3\theta$
0	0.8	1	0	1	0	1	0
60	0.6	0.5	$\sqrt{3}/2$	-0.5	$\sqrt{3}/2$	-1	0
120	0.4	-0.5	$\sqrt{3}/2$	-0.5	$-\sqrt{3}/2$	1	0
180	0.7	-1	0	1	0	-1	0
240	0.9	-0.5	$-\sqrt{3}/2$	-0.5	$\sqrt{3}/2$	1	0
300	1.1	0.5	$-\sqrt{3}/2$	-0.5	$-\sqrt{3}/2$	-1	0

$$a_1 = \frac{2}{6}\sum f(\theta)\cos\theta = \frac{1}{3}\left[0.8 - 0.7 + 0.5(0.6 - 0.4 - 0.9 + 1.1)\right] = 0.1$$ 26

$$b_1 = \frac{2}{6}\sum f(\theta)\sin\theta = \frac{1}{3}\left[\frac{\sqrt{3}}{2}(0.6 + 0.4 - 0.9 - 1.1)\right] = -0.3$$

$$a_2 = \frac{2}{6}\sum f(\theta)\cos 2\theta = \frac{1}{3}\left[0.8 + 0.7 - 0.5(0.6 + 0.4 + 0.9 + 1.1)\right] = 0$$

$$b_2 = \frac{2}{6}\sum f(\theta)\sin 2\theta = \frac{1}{3}\cdot\frac{\sqrt{3}}{2}(0.6 - 0.4 + 0.9 - 1.1) = 0$$

$$a_3 = \frac{2}{6}\sum f(\theta)\cos 3\theta = \frac{1}{3}\left[0.8 - 0.6 + 0.4 - 0.7 + 0.9 - 1.1\right] = -0.1$$

$$b_3 = \frac{2}{6}\sum f(\theta)\sin 3\theta = 0$$

$\therefore$ first 3 harmonics are

first harmonic $= a_1\cos\theta + b_1\sin\theta = 0.1\cos\theta - 0.3\ldots$

Second " $= a_2\cos 2\theta + b_2\sin 2\theta = 0$

third " $= a_3\cos 3\theta + b_3\sin 3\theta = -0.1\cos 3\theta$

Q2: Obtain the constant term and the coeff. of the first two sine and first two cosine terms in the Fourier expansion of $y(x)$ tabulated below

x	0	1	2	3	4	5
y	9	18	24	28	26	20

Also find the amplitude of the first harmonic.

Take the interval as $\theta = \frac{360}{6} = 60°$

Ans.

x	y	$\frac{\pi x}{3} = \theta$	$\cos\theta$	$\sin\theta$	$\cos 2\theta$	$\sin 2\theta$
0	9	0 0°	1	0	1	0
1	18	$\pi/3$ 60°	0.5	$\sqrt{3}/2$	-0.5	$\frac{\sqrt{3}}{2}$
2	24	$\frac{2\pi}{3}$ 120°	-0.5	$\sqrt{3}/2$	-0.5	$-\frac{\sqrt{3}}{2}$
3	28	π 180°	-1	0	1	0
4	26	$\frac{4\pi}{3}$ 240°	-0.5	$-\frac{\sqrt{3}}{2}$	-0.5	$\frac{\sqrt{3}}{2}$
5	20	$\frac{5\pi}{3}$ 300°	0.5	$-\frac{\sqrt{3}}{2}$	-0.5	$-\frac{\sqrt{3}}{2}$

Constant term $= \frac{1}{2} a_0 = \frac{2}{12} \Sigma y = \frac{1}{6}(125) = 20.83$

$$a_1 = \frac{2}{6} \underbrace{\Sigma y \cos \frac{\pi x}{3}}_{\Sigma y \cos\theta} = \frac{1}{3}\left[9 - 28 + 0.5(18 - 24 - 26 + 20)\right] = -8.33$$

$$b_1 = \frac{2}{6} \underbrace{\Sigma y \sin \frac{\pi x}{3}}_{y \sin\theta} = \frac{1}{3} \frac{\sqrt{3}}{2}\left[18 + 24 - 26 - 20\right] = -1.15$$

$$a_2 = \frac{2}{6} \underbrace{\Sigma y \cos \frac{2\pi x}{3}}_{y \cos 2\theta} = \frac{1}{3}\left[9 + 28 - 0.5(18 + 24 + 26 + 20)\right] = -2.$$

$$b_2 = \frac{2}{6} \underbrace{\Sigma y \sin \frac{2\pi x}{3}}_{\sin 2\theta} = \frac{1}{3} \frac{\sqrt{3}}{2}\left[18 - 24 + 26 - 20\right] = 0$$

$\therefore$ Constant term $= \frac{a_0}{2} = 20.83$

Coeff. of $\sin \frac{\pi x}{3} = b_1 = -1.15$

" " $\sin \frac{2\pi x}{3} = b_2 = 0$

" " $\cos \frac{\pi x}{3} = a_1 = -8.33$

" " $\cos \frac{2\pi x}{3} = a_2 = -2.33$

Amplitude of first harmonic $= \sqrt{a_1^2 + b_1^2}$

$= \sqrt{(-8.33)^2 + (-1.15)^2}$

$= 8.41$