Method of false position or Regula-falsi method (7)

This method is used for finding the real root of an eq $f(x)=0$ & closely resembles the bisection method.

Choose x_0 & x_1 s.t $f(x_0)\cdot f(x_1) < 0$

Eq^n of the chord joining $A[x_0, f(x_0)]$ & $B[x_1, f(x_1)]$ is

$$y - f(x_0) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0) \quad \text{--- (1)}$$

The method consists of replacing the curve AB by means of the chord AB and taking the pt. of intersection of the chord with the x-axis as an approximation to the root. So the abscissa of the pt. where the chord cuts the x axis $(y=0)$ is given by

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0)$$

which is the approximation to the root.

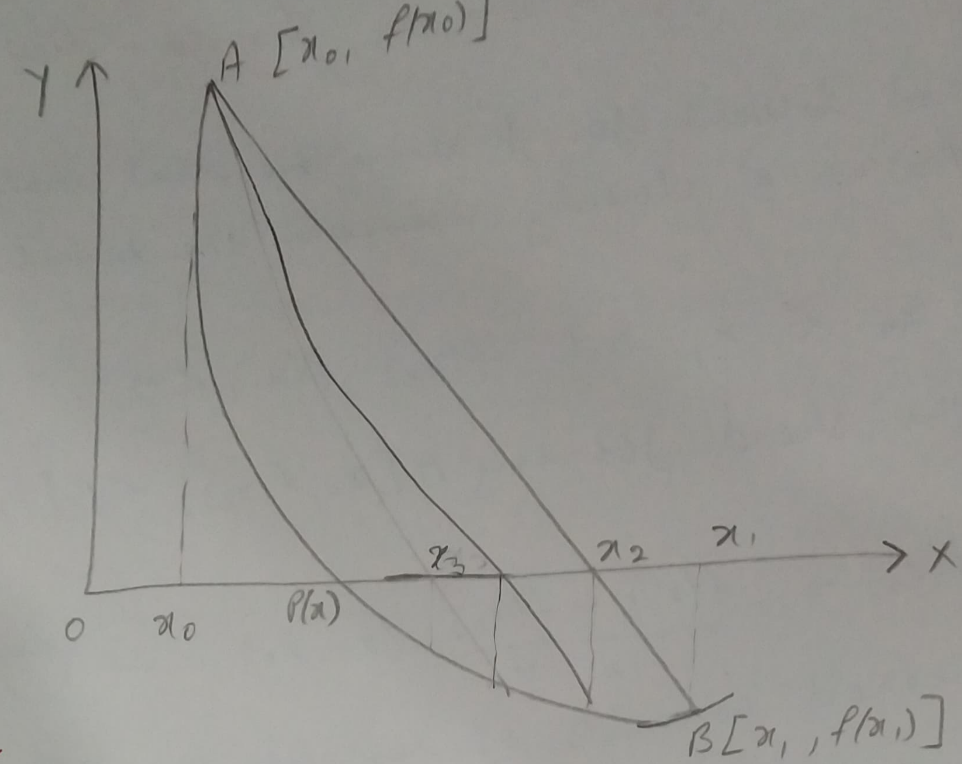
Repeat the procedure until the root is found to the desired accuracy.

Remark:- 1) Rate of convergence (ROC) is much faster than that of bisection method.

2) Linear Rate of convergence

Q. Apply bisection method to ... root

(8)



Ans.

Find a real root of the eq. $x^3 - 2x - 5 = 0$ by the method of false position correct to 3 decimal places.

x_5 Ans. Let $f(x) = x^3 - 2x - 5$

$f(2) = -1$ & $f(3) = 16$

x_6 $\therefore$ root lies b/w 2 & 3.

$\therefore$ taking $x_0 = 2$, $x_1 = 3$, $f(x_0) = -1$, $f(x_1) = 16$, in

$x_7 =$ the method of false position, we get

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} \cdot f(x_0) = 2 + \frac{1}{17} = 2.0588$$

$f(x_2) = f(2.0588) = -0.3908$

$\therefore$ the root lies b/w 2.0588 and 3.

taking $x_0 = 2.0588$, $x_1 = 3$, $f(x_0) = -0.3908$, $f(x_1) = 16$

$$x_3 = 2.0588 - \frac{0.9412}{16.3908}(-0.3908) = 2.0813$$

Repeating this process, the successive approximations are.

$x_4 = 2.0862$, $x_5 = 2.0915$, $x_6 = 2.0934$,

$x_7 = 2.0941$, $x_8 = 2.0943$

Hence, the root is 2.094 correct to 3 decimal places.

Newton - Raphson method or Newton's Iteration method or method of Tangents

Let x_0 be an approximate root of the equation $f(x) = 0$. If $x_1 = x_0 + h$ be the exact root, then $f(x_1) = 0$

$\therefore$ expanding $f(x_0 + h)$ by Taylors series

$$f(x_0) + h f'(x_0) + \frac{h^2}{2!} f''(x_0) + \cdots = 0$$

Since h is small, neglecting h^2 & higher powers of h, we get.

$$f(x_0) + h f'(x_0) = 0 \quad \text{or} \quad h = -\frac{f(x_0)}{f'(x_0)} \quad \text{--- (1)}$$

$\therefore$ a closer approximation to the root is given by

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

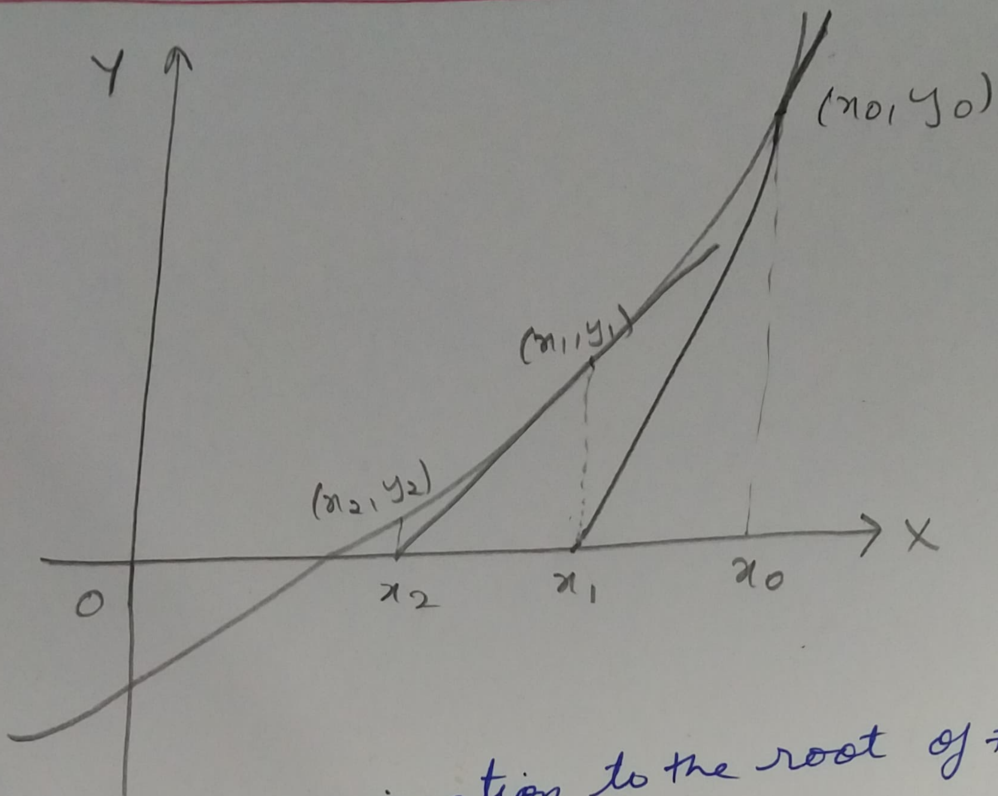
IIly, starting with x_1 a still better approx. x_2 is given by.

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

In general, $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Note Newton's method has a second order of quadratic cgs.

Geometrical Interpretation



Let x_0 is an approximation to the root of $f(x) = 0$.
we find the eq. of tangent at (x_0, y_0) to the graph of the curve $y = f(x)$ where $y_0 = f(x_0)$. Let this tangent meets x-axis at x_1 then x_1 will be the next approximation and we find (x_1, y_1) on the graph & draw tangent at (x_1, y_1) to the curve $y = f(x)$.
Its intersection with x-axis will be x_2.
Proceeding in this way, when approximation x_n is found then intersection of tangent at (x_n, y_n) with x-axis will give the next approx. x_{n+1}.

Qus: Find the positive root of $x^4 - x = 10$ correct to 3 decimal places, using N-R method.

Ans: Let $f(x) = x^4 - x - 10$

s.t $f(1) = -10$ (-ve), $f(2) = 16 - 2 - 10 = 4$ (+ve)

$\therefore$ a root lies b/w 1 & 2.

Let us take $x_0 = 2$

$f'(x) = 4x^3 - 1$

N-R formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad \text{— (1)}$$

Put $n = 0$,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{f(2)}{f'(2)} = 1.871$$

Put $n = 1$,

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \frac{(1.871)^4 - (1.871) - 10}{4(1.871)^3 - 1}$$

$$= 1.856$$

Put $n = 2$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.856$$

Here $x_2 = x_3$. Hence the desired root is 1.856 correct to 3 decimal places.