(ii) If data for function $f(x)$ at equidistant points is

x	0	x_1	x_2	$\cdots$	x_{k-1}	x_k
$f(x)$	y_0	y_1	y_2	$\cdots$	y_{k-1}	y_k

and we are to find half-range sine-series, then for odd extension of $f(x)$, we must have $y_0 = 0$, otherwise odd extension cannot be possible. Further, if $y_k = 0$ then $x_k = l$ $(\because f(-l) = f(l)$ and $f(-l) = -f(l) \Rightarrow f(l) = 0)$ and the entry y_k should not be taken but if $y_k \neq 0$ then $x_k \neq l$ and $y_{k+1} = 0$ at $x_{k+1} = l$ and hence entry y_k should be taken.

mple 3.51: Obtain the Fourier sine series for $f(x)$ containing three non-zero terms where is given in the following table:

(ii)

x	0	1	2	3	4	5
$f(x)$	0	10	15	8	5	3

$l = 6$

Solution: Here, $l = 6$

$\therefore$ The Fourier sine series for $f(x)$ is given by

$$y = b_1 \sin\frac{\pi x}{6} + b_2 \sin\frac{2\pi x}{6} + b_3 \sin\frac{3\pi x}{6} + \cdots$$

x	$f(x)$	$\frac{\pi x}{6} = \theta$	$\sin\theta$	$\sin 2\theta$	$\sin 3\theta$
0	0	0	0	0	0
1	10	$\frac{\pi}{6}$	$1/2$	$\sqrt{3}/2$	1
2	15	$\frac{\pi}{3}$	$\sqrt{3}/2$	$\sqrt{3}/2$	0
3	8	$\frac{\pi}{2}$	1	0	-1
4	5	$\frac{2\pi}{3}$	$\sqrt{3}/2$	$-\sqrt{3}/2$	0
5	3	$\frac{5\pi}{6}$	$1/2$	$-\sqrt{3}/2$	1

$\therefore$

$$b_1 = \frac{2}{6}\sum f(x)\sin\theta = \frac{1}{3}\left[\left(5+8+\frac{3}{2}\right)+(15+5)\frac{\sqrt{3}}{2}\right]$$

$$= \frac{1}{3}\left[14.5+10\sqrt{3}\right] = 10.607$$

$$b_2 = \frac{2}{6}\sum f(x)\sin 2\theta = \frac{1}{3}\left[\frac{\sqrt{3}}{2}(10+15-5-3)\right] = 4.907$$

$$b_3 = \frac{2}{6}\sum f(x)\sin 3\theta = \frac{1}{3}[10-8+3] = \frac{5}{3} = 1.667$$

$\therefore$ Fourier sine series for $f(x)$ is

$$f(x) = 10.607\sin\frac{\pi x}{6} + 4.907\sin\frac{2\pi x}{6} + 1.667\sin\frac{3\pi x}{6} + \cdots$$

(iii) If data for function $f(x)$ at equidistant points is

x	0	x_1	x_2	$\cdots$	x_{k-1}	x_k
$f(x)$	y_0	y_1	y_2	$\cdots$	y_{k-1}	y_k

and we are to find half-range cosine series. Then for even extension of $f(x)$, y_0 may or may not be zero. Further, for even extension $f(-l) = f(l)$ and hence $f(l)$ may or may not be zero. Hence y_k may or may not be zero, therefore, we have $x_k = l$. Thus, for even extension by trapezoidal rule

Example 3.50: Obtain the first three coefficients in the Fourier cosine series for y, where y is given in the following table:

x	0	1	2	3	4	5
y	4	8	15	7	6	2

$\therefore x \; l = l$

(111)

Solution: Here, $l = 5$

The Fourier cosine series for y is given by

$$y = \frac{a_0}{2} + a_1 \cos\frac{\pi x}{5} + a_2 \cos\frac{2\pi x}{5} + \cdots$$

x	y	$\frac{\pi x}{5}$	$\cos\frac{\pi x}{5}$	$\cos\frac{2\pi x}{5}$
0	4	0	1	1
1	8	$\frac{\pi}{5}$	0.8090	0.3090
2	15	$\frac{2\pi}{5}$	0.3090	−0.8090
3	7	$\frac{3\pi}{5}$	−0.3090	−0.8090
4	6	$\frac{4\pi}{5}$	−0.8090	0.3090
5	2	π	−1	1

$$\frac{a_0}{2} = \frac{1}{2}\cdot\frac{2}{5}\left[\frac{4+2}{2} + (8+15+7+6)\right] = \frac{1}{5}(39) = 7.8$$

$$a_1 = \frac{2}{5}\left[\frac{4+(-2)}{2} + \{(8-6)0.8090 + (15-7)0.3090\}\right] = \frac{2}{5}(5.0900) = 2.036$$

$$a_2 = \frac{2}{5}\left[\frac{4+2}{2} + \{(8+6)0.3090 - (15+7)0.8090\}\right] = \frac{2}{5}(-10.4720) = -4.1888$$

$\therefore$ The first three coefficients in the Fourier cosine series are

$$\frac{a_0}{2} = 7.8,\ a_1 = 2.036 \text{ and } a_2 = -4.1888.$$

Example 3.51: ... series for $f(x)$ containing three non-zero terms ...