Dirichlet's Condition

Any function f(x) can be developed as a Fourier series

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

where a_0, a_n, b_n are constants.

1) f(x) is periodic, single-valued and finite.

2) f(x) has a finite number of discontinuities in any one period.

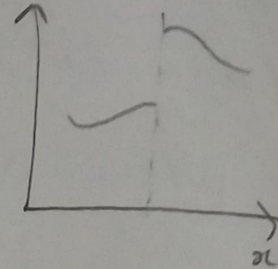
3) f(x) has at the most a finite number of maxima and minima.

In fact :- f(x) as a fourier series depends upon the evaluation of integrals

$\frac{1}{\pi}\int f(x)\cos nx\,dx$, $\frac{1}{\pi}\int f(x)\sin nx\,dx$ within limits $(0, 2\pi)$, $(-\pi, \pi)$ or $(\alpha, \alpha+2\pi)$

Functions having points of Discontinuity

If in the interval $(\alpha, \alpha + 2\pi)$, $f(x)$ is defined by.



$$f(x) = \phi(x), \quad \alpha < x < c$$
$$= \psi(x), \quad c < x < \alpha + 2\pi, \text{ i.e}$$

c is the point of discontinuity, then

$$a_0 = \frac{1}{\pi}\left[\int_{\alpha}^{c} \phi(x)\,dx + \int_{c}^{\alpha+2\pi} \psi(x)\,dx\right]$$

$$a_n = \frac{1}{\pi}\left[\int_{\alpha}^{c} \phi(x)\cos nx\,dx + \int_{c}^{\alpha+2\pi} \psi(x)\cos nx\,dx\right]$$

$$b_n = \frac{1}{\pi}\left[\int_{\alpha}^{c} \phi(x)\sin nx\,dx + \int_{c}^{\alpha+2\pi} \psi(x)\sin nx\,dx\right]$$

At the point of discontinuity, there are finite jump. Both the limit on the left (i.e $f(c-0)$) and the limit on the right [i.e $f(c+0)$] exist and are different. At such a point, fourier series gives the value of $f(x)$ as the ~~arith~~ arithmetic mean of these 2 ~~...~~ limits.

i.e, at $x = c$, $f(x) = \frac{1}{2}\left[f(c-0) + f(c+0)\right]$