Circuit and Systems

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Assignment - 4

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Ans 1. Necessary and sufficient conditions for any rational function to be +ve real func^n :

1) Both A(s) and B(s) polynomials in $F(s) = \frac{A(s)}{B(s)}$ are Hurwitz so poles and zeroes of PR func^n cannot have +ve real points i.e. they can't be in right half of S-plane

2) Highest and lowest powers of A(s) and B(s) differ by unity

3) If P(s) is a PR func^n, then reciprocal of F(s) is also a PR func^n.

4) The sum of PR func^n is also a PR func^n but diff of two PR func^n is not necessary a PR func^n

Ans 2. Properties of LC, RC and RL network func^n

1. $Z_{LC}(s)$ and hence $Y_{LC}(s)$ are the ratio of even to odd to even polynomials. (This property is also called as "Foster Reactance Theorem"

2. Since both $M_i(s)$ and $N_j(s)$ are Hurwitz they have only imaginary roots and it follows that the poles and zeroes of $Z_{LC}(s)$ on $Y_{LC}(s)$ are on the imaginary axis (including origin)

3. The poles and zeroes interlace (or alternate) on $j\omega$ axis
4. The highest power of Numerator and denominator must differ by unity.
5. The lowest power of Numerator and denominator must also differ by unity.
6. There must be zero or a pole at origin and infinity.

Properties of R-L admittance func^n or R-C impedance

1. The poles and zeroes lie on -ve real axis (including origin) of the complex s-plane
2. The poles and zero interlace (or alternate) along the -ve real axis
3. The residues of poles of $Z_{R-C}(s)$ or $Y_{R-L}(s)$ must be real and +ve.
   The residues of $Y_{R-C}(s)$ or $Z_{R-L}(s)$ are real and -ve
4. The singularity nearest to (or at) origin must be a pole i.e. func^n Z_{R-C} or Y_{R-L} $\to \infty$ with $s \to 0$
5. The singularity nearest to (or at) the origin must be a pole i.e. func^n Z_{R-C} or $Y_{R-L} \to 0$ and $s \to \infty$

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Properties of R-L impedance or R-C admittance

1. The poles and zeroes lie on -ve real axis (including origin) of the complex s-plane
2. The poles and zeros interlace along -ve real axis
3. a) the residue of poles of $\frac{Z_{R-L}(s)}{s}$ or $Y_{R-C}(s)$ are real or -ve
   b) Residue at poles $Y_{R-L}(s)$ or $Z_{R-C}(s)$ must be real and +ve.
4. The singularity nearest to (or at) the minus ($-\infty$) must be a ~~pole~~ zero i.e. func^n $Z_{R-L}(s)$ or $Y_{R-C}(s) \to 0$ with $s \to 0$
5. The singularity nearest to (or at) the origin must be ($-\infty$) must be a pole i.e. func^n $Z_{R-L}(s)$ or $Y_{R-C}(s) \to \infty$ with $s \to \infty$.

Ans3. a) $Z(s) = \dfrac{s^3 + 5s^2 + 9s + 3}{s^3 + 4s^2 + 7s + 9}$

$N(s) = s^3 + 7s \qquad M(s) = 4s^2 + 9$

$$4s^2 + 9 \,\big)\, s^3 + 7s \,\big(\, s/4$$
$$(-)\; s^3 + 9s/4$$
$$19s/4 \,\big)\, 4s^2 + 9 \,\big)\, 16s/19$$
$$(-)\; 4s^2$$
$$9 \,\big)\, 19/4\, s \,\big(\, 19/36\, s$$
$$-19s/4$$
$$\times$$

$F(s) = M_1 M_2 - N_1 N_2$

$s = j\omega \geq 0$

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$M_1(s) = 5s^2+3$ $\quad N_1(s) = s^3+9s$

$M_2(s) = 4s^2+9$ $\quad N_2(s) = s^3+7s$

$F(s) = (5s^2+3)(4s^2+9) - (s^3+9s)(s^3+7s)$

$= -s^6 + 4s^4 - 6s^2 + 27$ $\quad s = j\omega$

$= -(j\omega)^6 + 4(j\omega)^4 - 6(j\omega)^2\ 27$

$= \omega^6 + 4\omega^4 + 6\omega^2 + 27 \geq 0$

the given func^n is PR

Ans 3. b) $f(s) = \dfrac{s^2+s+1}{s^2+s+4}$ $\quad M_1M_2 - N_1N_2 \geq 0$

$M_1 = s^2+1$ $\quad N_1 = s$

$M_2 = s^2+4$ $\quad N_2 = s$

$(s^2+1)(s^2+4) - s^2 \geq 0$

$s^4 + 5s^2 + 4 - s^2 \geq 0$

$s^4 + 4s^2 + 4 \ |_{j\omega} \geq 0$

$(j\omega)^4 + 4(j\omega)^2 + 4 \geq 0$

$\omega^4 + 4\omega^2 + 4 \geq 0$

For any value ω it will remain +ve

So it is a PR func^n

c) $\dfrac{s^2+10s+4}{s+2} = F(s)$

$M_1M_2 - N_1N_2 > 0$

$M_1 = s^2+4$ $\quad N_1 = 10s$

$M_2 = 2$ $\quad N_2 = s$

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$$(s^2+4)\,2 - 10s^2 \geq 0$$

$$2s^2 - 8 - 10s^2 \geq 0$$

$$-8s^2 + 8 \,\Big|_{jw} \geq 0$$

$$-8(jw)^2 + 8 \geq 0$$

$$8w^2 + 8 \geq 0$$

for every value of w it will remain +ve so it is a PR funcⁿ

Ans 4. Foster -1 Form

$$Z(s) = \frac{s(s^2+4)}{2(s^2+1)(s^2+9)} \qquad [1/s = C]$$

$$= \frac{s^3 + 4s}{2s^4 + 20s^2 + 18}$$

using Partial fraction expression.

$$\frac{s(s^2+4)}{2(s^2+1)(s^2+9)} = \frac{1}{2}\left[\frac{As+B}{s^2+1} + \frac{(Cs+D)}{(s^2+9)}\right]$$

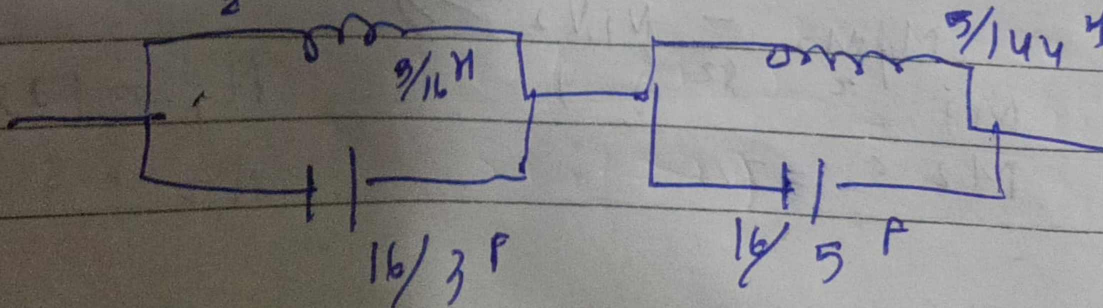
$A = 3/8$, $B = 0$, $D = 0$, $C = 5/8$

$$\therefore \;\; = \left[\frac{3s/16}{s^2+1} + \frac{5s/16}{s^2+9}\right]$$

$L_0 = 0$, $C_0 = 0$ $\quad$ $C_1 = 16/3$, $\quad$ $C_2 = 16/5$

$d_1 = 3/16$ $\quad$ $d_2 = 5/144$

$$a = \frac{1}{L_2\, 16/5}, \qquad L_2 = \frac{5}{144}$$



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Form-II (LC network)

Step I $\rightarrow$ $Y(s) = \frac{1}{Z(s)} = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)}$

$= \frac{2s^4+20s^2+18}{s^3+4s}$ $LC \Rightarrow N/D$

$\frac{s^4}{s^3} = s$

Step II $\rightarrow$ $N > D$

$$\begin{array}{r|l} s^3+4s \,\big)\; 2s^4 + 20s^2 + 18 & 2s \\ 2s^4 + 8s^2 & \\ - \quad - & \\ \hline 12s^2 + 18 & \end{array}$$

$Y = Q + R/D = \frac{2s + 12s^2 + 18}{s^3+4s}$

Step III using partial fraction : $Y(s)$

$\frac{12s^2+18}{s(s^2+4)} = \frac{A}{s} + \frac{Bs+C}{s^2+4}$

$A = 9/2$, $C = 0$ and $B = 15/2$

$Y(s) = 2s + \frac{9/2}{s} + \frac{15/2\, s}{s^2+4}$

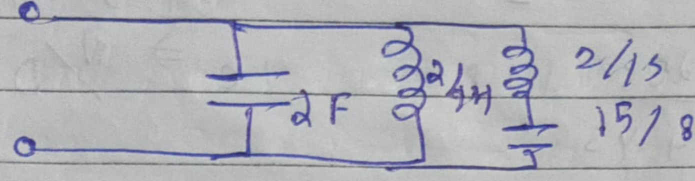
Step IV $\frac{1}{L_0 s} + C_0 s + \frac{s/L_1}{\left(\frac{1}{L_1C_1}+s^2\right)} + \frac{s/L_2}{\left(\frac{1}{L_2C_2}+s^2\right)} + \text{---}$

$Y(s) = 2s + \frac{9/2}{s} + \frac{15/2}{s^2+4}$

$Y(s) = 2s + \frac{1}{(2/9)s} + \frac{s/(2/15)}{s^2+4}$ //

$L_1C_1 = \frac{1}{4}$ or $\frac{1}{L_1C_1} = 4$

$\frac{1}{C_1} = 4L_1$ $\qquad \frac{1}{C_1} = \frac{8}{15}$ $\qquad C_1 = \frac{15}{8}$



Ans 5. $Z(s) = \dfrac{2s^5 + 12s^3 + 16s}{s^4 + 4s^2 + 3}$

Cauer - I form $\qquad$ LC

$\frac{s^5}{s^4} = s$, so L will come first

$s^4 + 4s^2 + 3 \,\big)\, 2s^5 + 12s^3 + 16s \,\big(\, 2s$

$2s^5 + 8s^3 + 6s$

$(-) \quad (-) \quad (-)$

$4s^3 + 10s \,\big)\, s^4 + 4s^2 + 3 \,\big(\, s/4$

$s^4 + 5/2\, s^2$

$- \quad -$

$3/2\, s^2 + 3 \,\big)\, 4s^3 + 10s \,\big(\, 8/3\, s$

$4s^3 + 8s$

$- \quad -$

$2s \,\big)\, 3/2\, s^2 + 3 \,\big(\, 3/4\, s$

$3/2\, s^2$

$- \quad -$

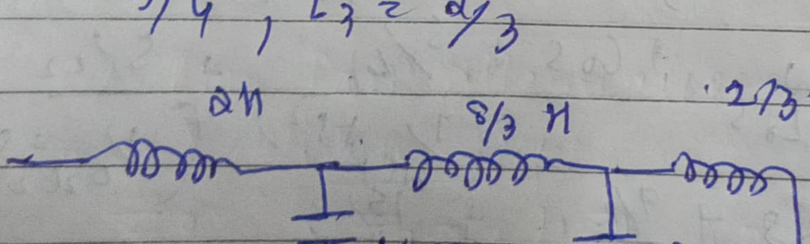
$3 \,\big)\, 2s \,\big(\, 2/3\, s$

$2s$

$\times$

$L_1 = 2, \; C_1 = 1/4, \; L_2 = 8/3$

$C_2 = 3/4, \; L_3 = 2/3$



Ans 6. $Z(s) = \dfrac{(s^2+1)(s^2+3)}{s(s^2+2)} = \dfrac{s^4 + 3s^2 + s^2 + 3}{s^3 + 2s}$

$$= \frac{s^4 + 4s^2 + 3}{s^3 + 2s}$$

Cauer - II form

LC, $\frac{s^4}{s^3} = s \rightarrow$ L will be first element

$$2s + s^3 \,\overline{)\, 3 + 4s^2 + s^4 \,}\; \frac{3}{2s}$$

$3 + \frac{3}{2}s^2$

$\frac{5}{2}s^2 + s^4 \,\overline{)\, 2s + s^3 \,}\; \frac{4}{5s}$

$2s + \frac{4}{5}s^3$

$\frac{1}{5}s^3 \,\overline{)\, \frac{5}{2}s^2 + s^4 \,}\; \frac{25}{2s}$

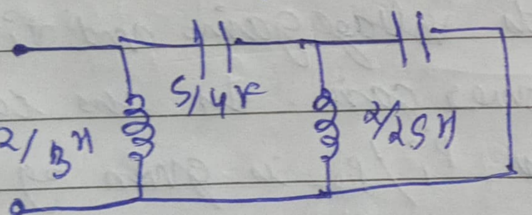
$\frac{5}{2}s^2 + 0$

$s^4 \,\overline{)\, \frac{1}{5}s^3 \,}\; \frac{1}{5s}$

$\frac{1}{5}s^3$

$\times$

$L_1 = 2/3$, $C_2 = 5/4$,
$L_2 = 2/25$, $C_3 = 5$



Ans 7. Parameters of a filter

→ character impedance Z_c / Z_0

The character impedance of a filter must be chosen such that the filter may fit into a given line or b/w two types of equipments

→ Pass band

Band in which ideal filter have no ~~attenuate~~ pass all frequencies without reduc^n in mag.

are reffered to as pass band

→ Stop band:
Band, in which ideal filter have no alternate freq. are referred to as stop frequencies

→ Unit of attenuation
The attenuation of a wave filter can be expressed in decibels (dB) or Nepers or beta
Let V_i, I_i and P_i be the input currents and input power respectively of a filter. Similarly V_o, I_o and P_o represent output voltage, current and power.
The V_0/V_i represents voltage gain and similarly P_o/P_i represents power gain. In the case of filter $P_o < P_i$ so, that P_i/P_o is given by attenuation

Ans. a) Low Pass attenuation
for T network filter,

$$Z_{0T} = \sqrt{Z_1 Z_2\left(1+\frac{Z_1}{4Z_2}\right)} = \sqrt{\frac{L}{C}\left(1-\frac{\omega^2 LC}{4}\right)}$$

$$Z_{0T} = \sqrt{\frac{L}{C}}\sqrt{1-\left(\frac{f}{f_0}\right)^2} = R_0\sqrt{1-\left(\frac{f}{f_c}\right)^2}$$

In the pass band, $f < f_c$, Z_{0T} is real

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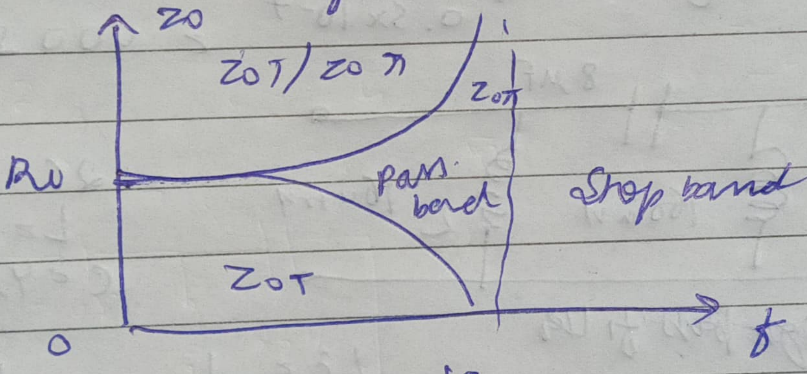
In stop band $f > f_c$, Z_{0T} is imaginary
and if $f = f_c$, $Z_{0T} = 0$

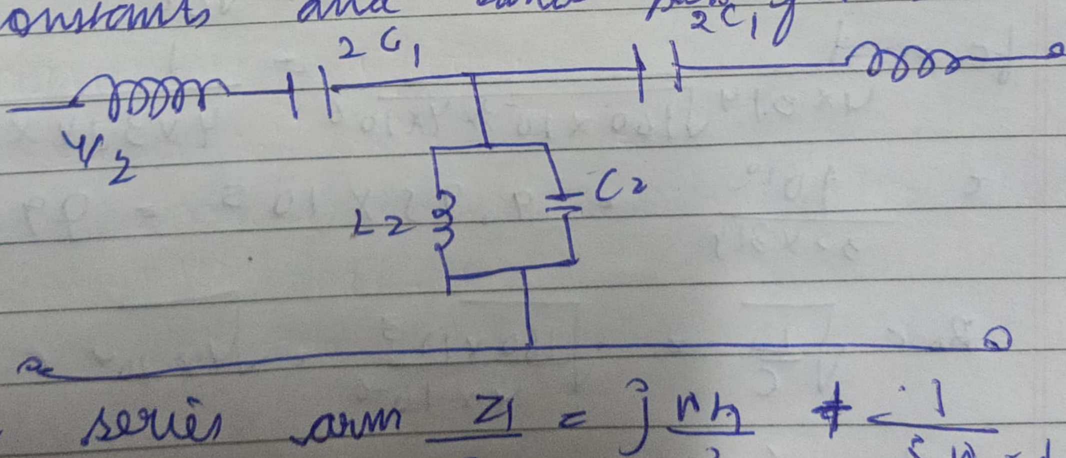
for π network filter $\Rightarrow$

$$Z_{0\pi} = \sqrt{\frac{Z_1 Z_2}{1 + Z_1/4Z_2}} = \sqrt{\frac{L/C}{1 - \omega^2 LC/4}} = \sqrt{\frac{L/C}{1 - (f/f_c)^2}}$$

In pass-band $f < f_c$, $Z_{0\pi}$ is real
In stop-band if $f > f_c$ $Z_{0\pi}$ is imaginary
and if $f_c = f \Rightarrow Z_{\pi} = 0$
if scan verified by $Z_{0T}\, Z_{0\pi} = R_0^2$



b) Constants and band pass filter

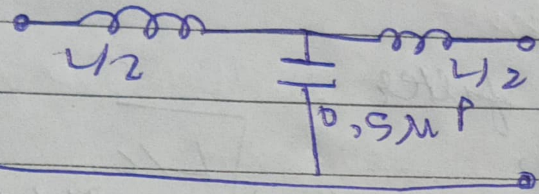


for series arm $\frac{Z_1}{2} = j\frac{\omega L_1}{2} + \frac{1}{j\omega 2C_1}$

$\rightarrow Z_1 = j\omega L_1 + \frac{1}{j\omega C_1} = j\left(\frac{\omega^2 L_1 C_1 - 1}{\omega C_1}\right)$

$\therefore Z_1 Z_2 = \frac{L_2}{C_1}\left(\frac{1-\omega^2 L_1 C_1}{1-\omega^2 L_2 C_2}\right)$

Ans 9. a)



Low pass filter

$\omega_C = \sqrt{4/LC}$

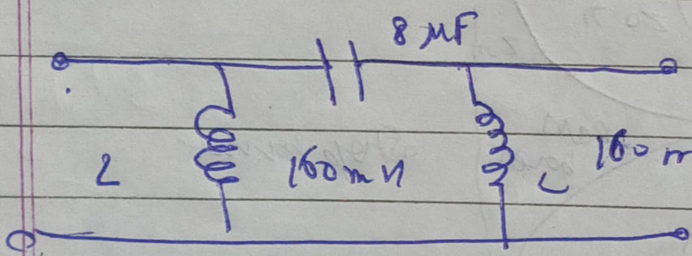
$= \sqrt{\frac{4}{80/2 \times 2 \times 10^{-3} \times 0.5 \times 10^{-7}}} = \sqrt{\frac{4}{400 \times 10^{-10}}}$

$\sqrt{10^8} = 10^4 = 10\,kHz$

$R_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{80 \times 10^{-3}}{0.5 \times 10^{-7}}} = \sqrt{6 \times 10^4} = 4 \times 10^2$

$= 400\,\Omega$

b)



$2C = 8\,\mu F$

$L = 160\,mH$

$C = 4\,\mu F$

High pass filter

$f_c = \frac{1}{4\pi\sqrt{LC}}$

$f_c = \frac{1}{4 \times 3.14 \sqrt{160 \times 10^{-3} \times 4 \times 10^{-6}}} = \frac{1}{4 \times 3.14 \times 8 \sqrt{10^{-8}}}$

$= \frac{10^{10}}{32 \times 3.14} = 9.95 \times 10^5 = 995\,kHz$

$R_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{160 \times 10^{-3}}{4 \times 10^{-6}}} = \sqrt{4 \times 10^4} = 20\,\Omega$

Ans 10. $R_0 = 600\,\Omega$ $f_{c1} = 2\,kHz$

$f_{c2} = 5\,kHz$

$$L_1 = \frac{R_0}{\pi(f_{c2}-f_{c1})} = \frac{600}{3.14(5-2)\times 10^3} = 63.69\ mH$$

$$C_2 = \frac{f_{c2}-f_{c1}}{4\pi R_0 f_{c1} f_{c2}} = \frac{(5-3)\times 10^3}{4\times 3.14\times 600\times 2\times 5\times 10^{-6}} = 0.02\ \mu F$$

$$L_2 = \frac{(f_{c2}-f_{c1})R_0}{4\pi f_{c1} f_{c2}} = \frac{3\times 10^3\times 600}{4\times 3.14\times 5\times 2\times 10^6} = 14.3\ mH$$

$$C_2 = \frac{1}{\pi R_0(f_{c2}-f_{c1})} = \frac{1}{3.14\times 600(3)\times 10^3} = 0.17\ \mu F$$

Hence $\frac{L_1}{2} = 31.84\ mH$

$2C_1 = 0.04\ \mu F$