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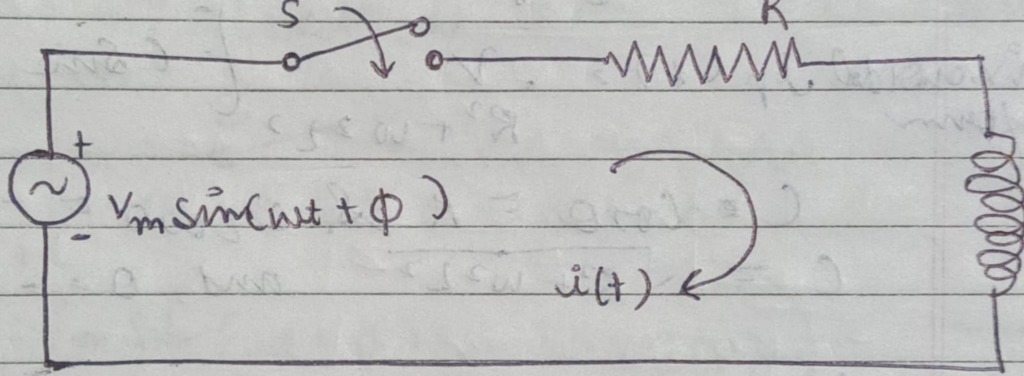
CIRCUIT SYSTEM

→ Transient Response of R-L circuit having sinusoidal

Q Consider a series R-L circuit excited by a sinusoidal voltage source as shown in fig. below. The switch 'S' is closed at time t = 0. Find the response (current) i(t)



A/S Applying KVL,

$$L\frac{di(t)}{dt} + R\, i(t) = V_m \sin(\omega t + \phi) \quad -①$$

$$i(t) = i_c(t) + i_p(t)$$

The complementary funcⁿ of eqⁿ ① is

$$i_c(t) = K e^{R/L\, t}$$

finding integral of eqⁿ ①

$$i_p(t) = e^{R/L\, t} \int \frac{V_m}{L} \sin(\omega t + \phi) \cdot e^{R/L\, t}\, dt$$

$$= \frac{V_m\, e^{-R/L\, t}}{2jL} \int \{ e^{j(\omega t+\phi)} - e^{-j(\omega t+\phi)} \} e^{R/L\, t}\, dt$$

$$= \frac{V_m\, e^{R/L\, t}}{2jL} \left[\frac{e^{j(\omega t+\phi) + R/L\, t}}{j\omega + R/L} - \frac{e^{-j(\omega t+\phi)\, j\omega t\, R/L}}{-j\omega + R/L} \right.$$

$$\frac{V_m \sin\phi}{R} = K + \frac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \sin\left(\phi + \tan^{-1}\frac{1}{\omega CR}\right)$$

$$i(t) = \left[\frac{V_m \sin\phi}{R} - \frac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \sin\left(\phi + \tan^{-1}\frac{1}{\omega CR}\right)\right] \cdot e^{-t/RC}$$

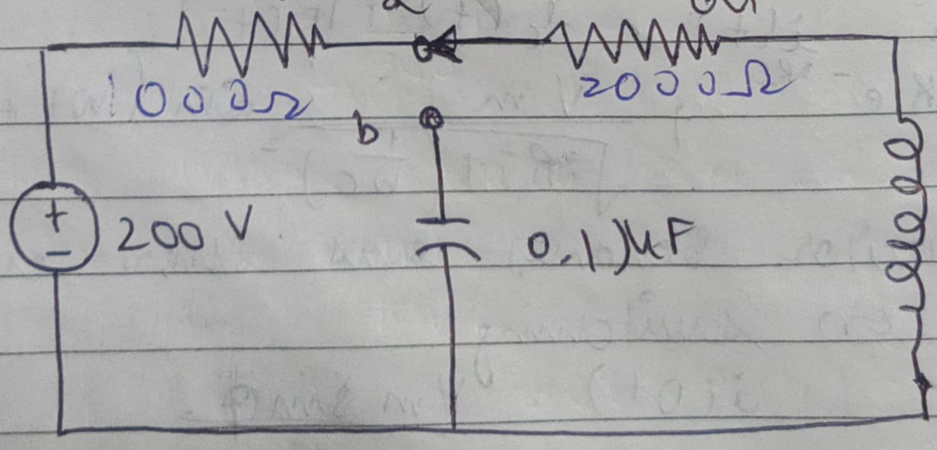
$$+ \frac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \sin\left(\omega t + \phi + \tan^{-1}\frac{1}{\omega CR}\right)$$

$$i(t) = \left[\frac{V_m \sin\phi}{R} - \frac{V_m}{Z} \sin(\phi + \theta)\right] e^{-t/RC} + \frac{V_m}{Z} \sin(\omega t + \phi + \theta)$$

where $\theta = \tan^{-1}\frac{1}{\omega CR}$

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \text{ : impedance}$$

Ans 3. In the given circuit shown the switch S is changed from position a to b at time $t=0$. Find out an expression for current $i(t)$, $\frac{di(t)}{dt}$ and $\frac{d^2 i(t)}{dt^2}$ at $t=0^+$



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At position a :
Steady state value of current $i(0^-)$:-

$$i(0^-) = \frac{200}{1000+2000} = \frac{2}{30} = \frac{1}{15}\ A$$

At position b :

$$i(0^+) = i(0^-) = 0.0666\ A \text{ or } \frac{1}{15}\ A$$

Applying KVL :

$$2000\ i(t) + 1 \cdot \frac{di(t)}{dt} = \frac{1}{0.1 \times 10^{-6}} \int_0^t i(t)\, dt = 0$$

$$\Rightarrow \frac{di(t)}{dt} + 2000\, i(t) = 0$$ (∵ ~~capacitor initially is~~ uncharged ; At $t = 0^+$ capacitor s.c.)

$$\Rightarrow \frac{di(0^+)}{dt} = -2000\, i(0^+) = \frac{-2000}{15}\ A/sec$$

$$\frac{di(0^+)}{dt} = -133.33\ A/sec$$

diff wrt : t

$$\frac{d^2 i(0^+)}{dt^2} + 2000 \frac{di(t)}{dt} + 10^7 i(t) = 0$$

$$\Rightarrow \frac{d^2 i(0^+)}{dt^2} = -2000(-133.33) - 10^7 (0.66)$$

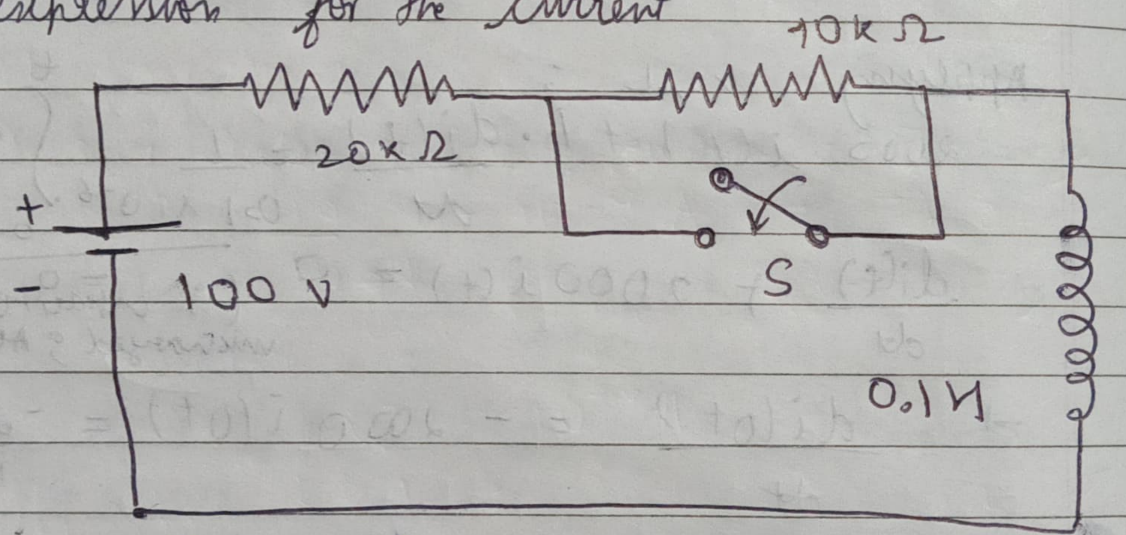
$$\frac{d^2 i(0^+)}{dt^2} = -\left\{2000\left(-\frac{2000}{15}\right) + \frac{10^7}{15}\right\}$$

$$= -\frac{10^6}{15}\{-4+10\} = -\frac{10^6}{15}\{-4+10\}$$

$= \frac{-6}{15}$

$\Rightarrow \frac{d^2 i(0^+)}{dt^2} = -4 \times 10^5 \, A/sec^2$

Ques 4. A dc voltage of 100V is applied in the adjoining circuit and the switch S is open. The switch S is closed at t = 0. Find the complete expression for the current.



Ans When switch S was open :
steady state current $i(0^-) = \frac{100}{20000 + 10000} A$
(∵ Inductor → S. C at t = ∞)

$i(0^-) = \frac{1}{300} A$

Now, when switch S is closed at t = 0
10 Ω Resistance → short circuited

Applying KVL :

$0.1 \frac{di(t)}{dt} + 20000 \, i(t) = 100$

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$$\frac{di(t)}{dt} + \left(\frac{2\cancel{\times}10^{5}}{\cancel{b}}\right) i(t) = \frac{10^3}{2}$$ Type II (1st order non homogeneous) D.E.

$\Rightarrow$ General solution — ①

$$i(t) = \frac{10^3}{2\times10^5} + Ke^{-(2\times10^5)t}$$

Initially conditions $\Rightarrow$ $i(0^+) = i(0^-) = \frac{1}{300}$ A

$$= \frac{1}{300} = \frac{1}{200} + Ke^{-(2\times10^5)(0)}$$

$$\Rightarrow K = \frac{1}{300} - \frac{1}{200} \Rightarrow K = \frac{-1}{600}$$

$$\therefore ① \; i(t) = \frac{1}{200} - \frac{1}{600} e^{-(2\times10^5)t}$$

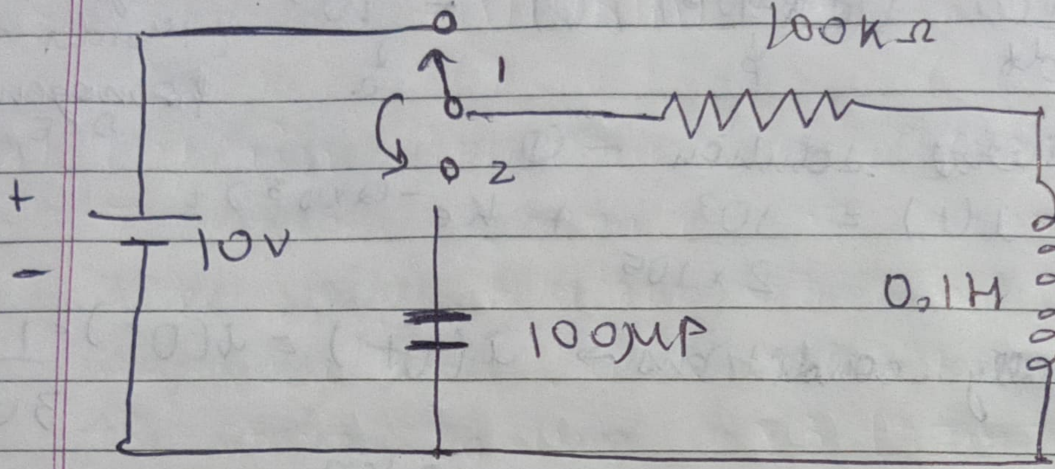
$$A \Rightarrow i(t) = \frac{10}{2}\left(1 - \frac{1}{3} e^{-200t}\right) \text{ mA}$$ (t → msec)

$$i(t) = 5\left(1 - \frac{1}{3} e^{-200t}\right),$$ $i(t) \to$ mA, $t \to$ msec

$i'(t) =$ 66.6 mA − 133.2 A/sec

$$\frac{d^2 i(t)}{dt^2} = -400 \times 10^3 \text{ A/sec}^2$$

Ques 3. In the circuit, the switch is moved from position 1 to 2 at $t = 0$. Determine i, $\frac{di}{dt}$ and $\frac{d^2 i}{dt^2}$ at $t = 0^+$.



Sol

At position 1 :

steady state value of current $i(0^-)$:

$$i(0^-) = \frac{10}{100k} = 10^{-4}\,A \Rightarrow i(0^-) = 0.1\,mA$$

At position 2 :

$$i(0+) = i(0^-) = 10^{-4}\,A$$

Applying KVL :

$$(100 \times 10^3)\, i(t) + 0.1 \frac{di(t)}{dt} + \frac{1}{100 \times 10^{-6}} \int_0^t i(t)\,dt = 0$$

0

$$\frac{d(i(t))}{dt} + 10^6\, i(t) = 0$$

$$\frac{di(0+)}{dt} = -10^6(10^{-4}) = -100\,A/sec$$

$$\frac{d^2 i(t)}{dt^2} + 10^6 \frac{di(t)}{dt} + 10^5\, i(t) = 0$$

$$\therefore \frac{d^2 i(0+)}{dt^2} = -10^6(100) - 10^3(10^{-4})$$

$$\Rightarrow \frac{d^2 i(0^+)}{dt^2} = -(10^8 + 10)\ A/sec^2$$

$$\frac{d^2(i(0^+))}{dt^2} \approx -10^8\ A/sec^2$$