Circuit and Systems

# Assignment - 2

Date ______
Page ______

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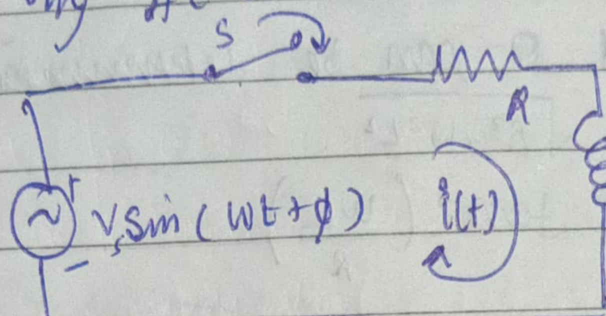
Ans1. Transient response of series RL circuit having AC excitation




applying KVL,

$$L\frac{di}{dt}(t) + Ri(t) = V_m \sin(\omega t + \phi)$$

$$\frac{di(t)}{dt} + \frac{R}{L} i(t) = \frac{V_m}{L} \sin(\omega t + \phi) \quad -(1)$$

This is non-homogeneous eq$^n$. The current $i(t)$ consists of sum of complementary function $i(t)$ and particular integral of $p(t)$ i.e.

$$i(t) = i_c(t) + i_p(t)$$

$$i_c(t) = K e^{-R/L\,t}$$

And particular integral of eq$^n$

$$i_p(t) = e^{-R/L\,t} \int \frac{V_m}{L} \sin(\omega t + \phi)\, e^{R/L\,t}\, dt$$

$$i_p(t) = \frac{V_m e^{-R/L\,t}}{2jL} \left\{ \frac{e^{j(\omega t + \phi) + R/L\,t}}{j\omega + R/L} - \frac{e^{-j(\omega t + \phi) + R/L\,t}}{-j\omega + R/L} \right.$$

$$= \frac{V_m}{2jL} \left[ \frac{e^{j(\omega t + \phi)}}{j\omega + R/L} - \frac{e^{-j(\omega t + \phi)}}{-j\omega + R/L} \right]$$

$$= \frac{V_m}{2jL} \left[ \frac{e^{j(\omega t + \phi)}(-j\omega + R/L) - e^{-j(\omega t + \phi)}(j\omega + R/L)}{(j\omega + R/L)(-j\omega + R/L)} \right.$$

$$= \frac{V_m}{L} \left[ \frac{R/L \sin(\omega t + \phi) - \omega \cos(\omega t + \phi)}{\omega^2 + R^2/L^2} \right]$$

Date ______
Page ______

$$= \frac{V_m}{R^2+W^2L^2} \left( R \sin(wt+\phi) - wL \cos(wt+\phi) \right)$$

This can be reduced to single sinusoidal in the form, $i_p(t) = \frac{V_m}{R^2+W^2L^2} \left( C \sin(wt+\phi+\theta) \right)$

where C and $\theta$ can be determined as

$$C = \sqrt{R^2+W^2L^2}$$

$$\theta = \tan^{-1}\left(\frac{WL}{R}\right)$$

subs. C and $\theta$

$$i_p(t) = \frac{V_m}{\sqrt{R^2+W^2L^2}} \cdot \sin\left(wt+\phi-\tan^{-1}\frac{WL}{R}\right)$$

$$\therefore \ i(t) = \frac{V_m}{\sqrt{R^2+W^2L^2}} \sin\left(wt+\phi+\tan^{-1}\frac{WL}{R} + ke^{-R/L\,t}\right)$$

Since inductor behaves as an open circuit At switching.

$$i(0^+) = 0$$

$$0 = \frac{V_m}{\sqrt{R^2+W^2L^2}} \sin\left(\phi - \tan^{-1}\frac{WL}{R}\right) + k$$

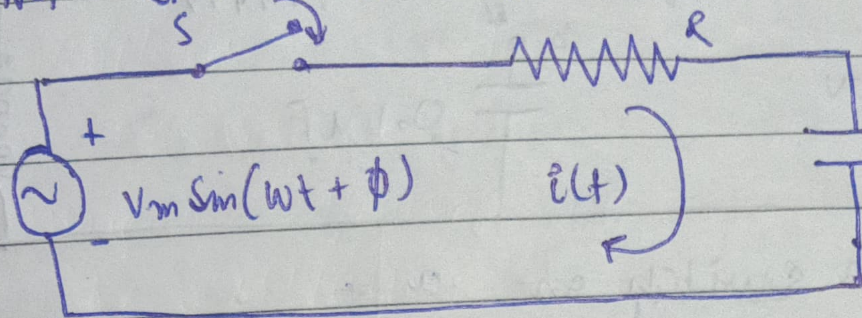
$$k = \frac{V_m}{\sqrt{R^2+W^2L^2}} \sin\left(\phi - \tan^{-1}\frac{WL}{R}\right)$$

$$i(t) = \frac{V_m}{L} \left( \sin(wt+\phi+\theta) - \sin(\phi+\theta) e^{-R/L\,t} \right)$$

$$\theta = -\tan^{-1}\frac{WL}{R} \qquad Z = \sqrt{R^2+W^2L^2}$$

# TRANSIENT RESPONSE OF SERIES RC CIRCUIT

## HAVING SINUSOIDAL EXCITATION :




Applying KVL,

$$Ri(t) + \frac{1}{C}\int_0^t i(t)\,dt + v_C(0^+) = V_m \sin(\omega t + \phi)$$

On differentiating,
$$\frac{di(t)}{dt} + \frac{1}{RC}\, i(t) = \frac{V_m \omega}{R}\cos(\omega t + \phi)$$

General solⁿ: $i(t) = i_c(t) + i_p(t)$

$$= Ke^{-t/RC} + \frac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \sin\left(\omega t + \phi + \tan^{-1}\frac{1}{\omega CR}\right)$$

Since capacitor behaves as short circuit at switching $i(0^+) = \frac{V_m \sin\phi}{R}$

$$\frac{V_m \sin\phi}{R} = K + \frac{V_m}{\sqrt{R^2 + (1/\omega C)^2}} \sin\left[\phi + \tan^{-1}\frac{1}{\omega CR}\right]$$

$$K = \frac{V_m \sin\phi}{R} - \frac{V_m}{\sqrt{R^2 + (1/\omega C)^2}} \sin\left(\phi + \tan^{-1}\frac{1}{\omega CR}\right)$$

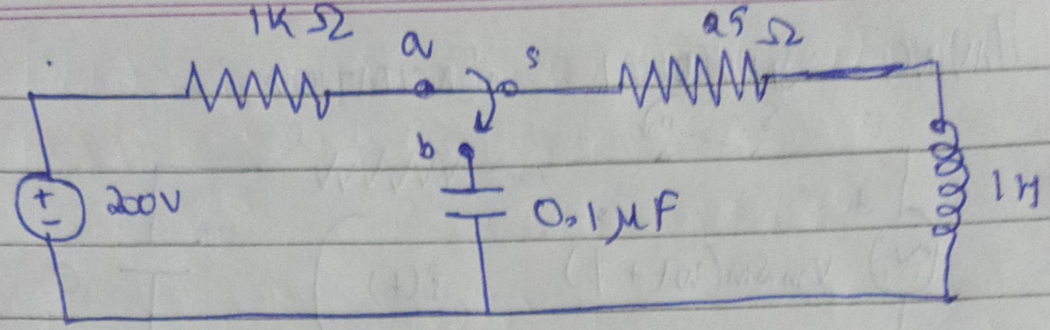
$$i(t) = \left[\frac{V_m \sin\phi}{R} - \frac{V_m}{\sqrt{R^2 + (1/\omega C)^2}} \sin\left(\phi + \tan^{-1}\frac{1}{\omega CR}\right)\right] e^{-t/RC}$$

$$i(t) = \left[\frac{V_m \sin\phi}{R} - \frac{V_m \sin(\phi + \theta)}{Z}\right] e^{-t/RC} + \frac{V_m}{Z}\sin(\omega t + \phi + \theta)$$

$$\theta = \tan^{-1}\left(\frac{1}{\omega CR}\right) \quad \& \quad Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

Date
Page

Ans 2.




with switch on a.

$$i(0^-) = \frac{200}{3000} = \frac{2}{30} = 0.06 \text{ A}$$

when switch on b

$$i(0^+) = i(0^-) = 0.06 \text{ A}$$

Applying, KVL,

$$\frac{di(t)}{dt} + 2000\, i(t) + \frac{1}{0.1\times10^{-6}} \int_0^t i(t)\,dt = 0 \quad -①$$

Since initial capacitor is uncharged and at switching instant capacitor behave as short

i.e. $\frac{1}{0.1\times10^{-6}} \int_0^t i(t)\,dt = 0$

Then from eqn ① $\frac{di(t)}{dt} + 2000\, i(t) = 0$

$$\frac{di(t)}{dt} = -2000\, i(t)$$

$$\frac{di(0^+)}{dt} = -2000\, i(0^+) = -2000 \times \frac{0.06}{100} = -120 \text{ A/sec}$$

$$\frac{d^2 i(t)}{dt^2} + 2000 \frac{di(t)}{dt} + \frac{1}{0.1\times10^{-6}}\, i(t) = 0$$

$$\frac{d^2 i(0^+)}{dt^2} = -2000 \times (-120) - \frac{1}{0.1\times10^{-6}} (-0.06)$$

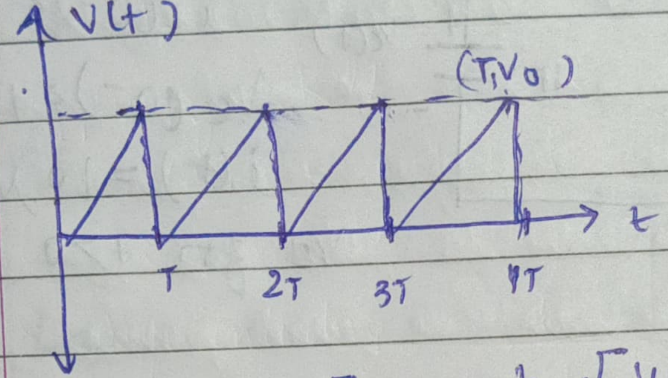
Date ______
Page ______

$$\frac{d^2 i(0^+)}{dt^2} = +240000 - 10^7 \times \frac{6}{100}$$

$= 240000 - 600000 = 36 \times 10^4$ A/sec

$= 36 \times 10^4$ A/ sec$^2$

Ans 5.




This is a periodic func^n with period T

Laplace transform first cycle

$$\mathcal{L}F(t) = \mathcal{L}\left[\frac{V_0}{T} t \cdot [u(t) - u(t-T)]\right]$$

$$= \frac{V_0}{T} \cdot \frac{1}{s^2} - \frac{\mathcal{L}V_0}{T} t\, u(t-T) = \frac{V_0}{Ts^2} - \mathcal{L}\frac{V_0}{T}(t-T+T)u(t-T)$$

$$= \frac{V_0}{Ts^2} - \mathcal{L}\frac{V_0}{T}(t-T)u(t-T) - \mathcal{L}\frac{V_0}{T} u(t-T)$$

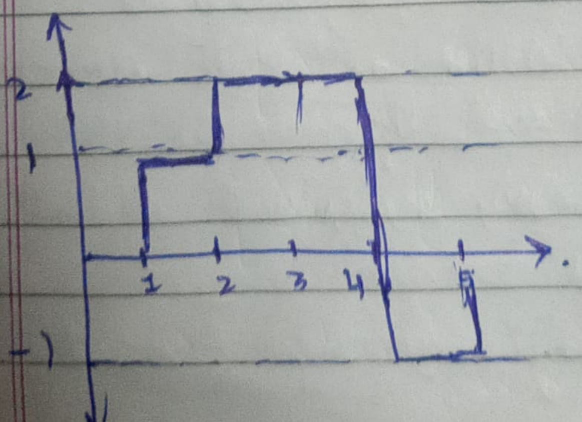
$$= \frac{V_0}{Ts^2} - \frac{V_0}{T} e^{-Ts}\frac{1}{s^2} - \frac{V_0}{T} \cdot \frac{e^{-Ts}}{s}$$

Laplace transform of periodic func^n with period

$$T = \frac{1}{1-e^{-Ts}} = \frac{1}{1-e^{-Ts}}\left(\frac{V_0}{Ts^2} - \frac{V_0}{T}\cdot\frac{e^{-Ts}}{s^2} - \frac{V_0}{T}\frac{e^{-Ts}}{s}\right)$$

Ans

Ans 6.



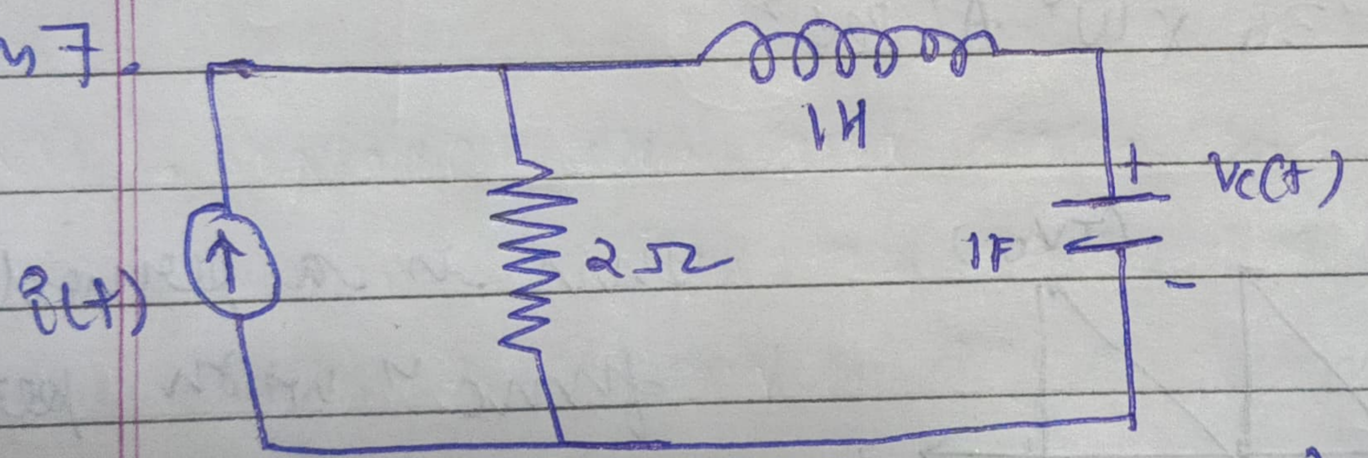

$$F(t) = 1[u(t-1) - u(t-2)]$$
$$+ 2[u(t-2) - u(t-4)]$$
$$- 1[u(t-4) - u(t-5)]$$

Date
Page

$$F_s(t) = u(t-1) + u(t-2) - 3u(t-4) + u(t-5)$$

$$F(s) = \frac{e^{-s}}{s} + \frac{e^{-2s}}{s} - \frac{3e^{-4s}}{s} + \frac{e^{-5s}}{s}$$

Ans 7.




$i_L(0^-) = 5A$

$V_c(0^-) = 10\,V$

$i(t) = 10\,u(t)$

$V_c$ for $t>0$ ?

$i(0^+)$ :

KVL,

$$288\,u(t) = 24 i_1 + 2\frac{di_1}{dt} + 80(i_1 - i_2) \quad \text{— (1)}$$

$$i_2 = i_1 \cdot \frac{80}{20+80}$$ (By current division rule)

$$i_2 = 0.8\, i_1$$

$$288\, v(t) = 24 i_1 + 2\frac{di_1}{dt} + 80(i_1 - 0.8 i_1)$$

$$= 40 i_1 + 2\frac{di_1}{dt}$$

t) inductor open circuit

$$\frac{di_1}{dt} + 20 i_1 = 144(t)$$

$$i_1(t) = \frac{144}{20} + k e^{-20t}$$

$t = 0^+$, inductor is open circuit

$$\therefore\ i_1(t) = \frac{144}{20}(1 - e^{-20t}) = 7.2(1 - e^{-20t})$$

$$i\ \cancel{(t)} = i_2(t) = 0.8(t) = 5.76(1 - e^{-20t})$$

$i_1(0^+) = 0$

$i_2(0^+) = 0$ $\qquad v_1(0^+) = 288\,V$

$i_1(\infty) = 7.2\,A$ $\qquad i_2 = 5.76\,A$

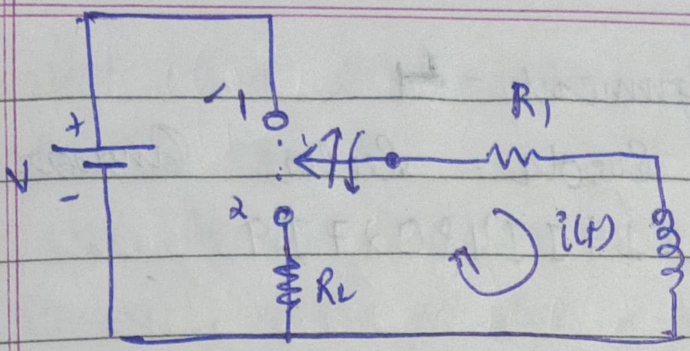
$$v_1(\infty) = L\frac{di_1(\infty)}{dt} = 2 \times 0 = 0$$

(short circuit

$V_C = 20 - 10e^{-t} - 5e^{-7} \, V$

Date
Page

Ans 8.




$i(t) = ?$

At position ① 1, $i(0^-) = \frac{V}{R_1}$

At position 2, $i(0^+) = i(0^-) = \frac{V}{R_1}$

$L\frac{di}{dt}(t) + R_1 i(t) + R_2 i(t) = 0$

$\frac{di(t)}{dt} + \left(\frac{R_1 + R_2}{L}\right) i(t) = 0$ $\qquad i(t) = Ke^{-(R_1+R_2/L)t}$

$i(t) = Ke^{-(R_1+R_2/L)t}$ $\qquad i(0^+) = \frac{V}{R_1}$

$\frac{V}{R_1} = K$ $\qquad i(t) = \frac{V}{R_1} e^{-(R_1+R_2/L)t}$

Ans