Pulse Code Modulation PCM

Pulse Modulation

Pulse Analog Modulation | Pulse Digital Modulation

In Pulse ~~Amplitude~~ Analog Modulation, only time is expressed in digital form and any one of pulse parameters (amplitude, duration or position) is varied in continuous manner in accordance with message signal.
Eg. PAM, PWM, PPM.

In these Modulation Methods, Info is transmitted in analog form at discrete times.

Pulse Digital Modulation is a method which converts analog signal to its corresponding digital form.

Here time and Pulse Parameter usually Amplitude occurs in discrete form and digital coded form respectively.

PCM is a Pulse Digital Modulation method in which the message signal is first sampled and then amplitude of each sample is rounded off to finite set of quantization levels, so both time and amplitude are in discrete form.
This process is known as Discretization in time and Amplitude.

PCM System

It consists of Transmitter, Transmission Path and Receiver.

1) Transmitter – Essential operation in Transmitter of PCM system are sampling, Quantization and Encoding.

Both Quantization and Encoding operations are performed in a same block known as Analog To Digital Convertor (ADC).

Combined Use of Quantizing and Coding distinguishes Pulse Code Modulation from Analog Modulation techniques.

2) Receiver :- Operations in Receiver are Regeneration of Impaired Signals, Decoding and Demodulation of Quantized Samples These Operations are Performed by Digital To Analog Convertor (DAC

3) Transmission Path :- Along the Transmission route, Regenerative Repeaters are used to Reconstruct the Transmitted sequence of coded Pulses, and to remove signal distortion and Noise.

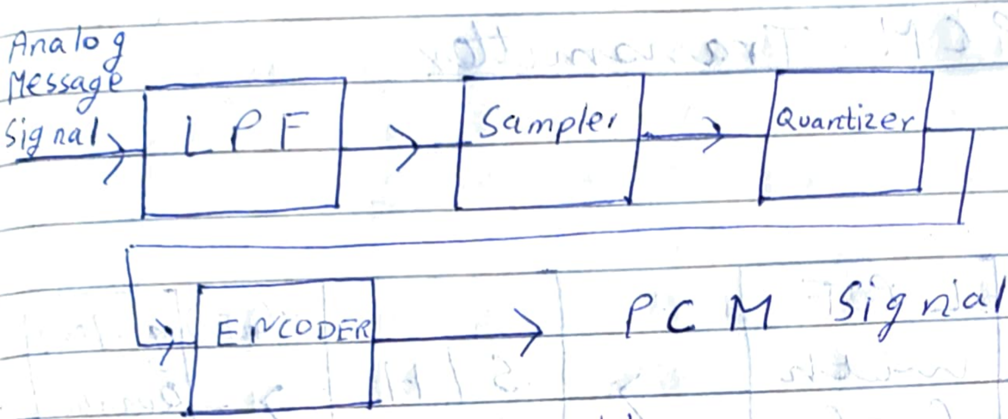


Fig 1 Transmitter

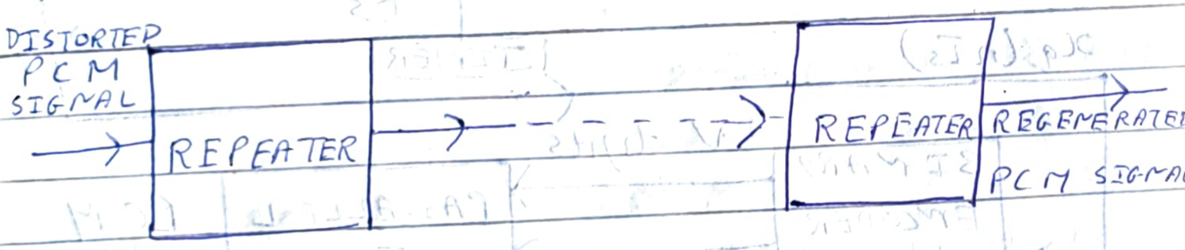


Fig 2 Transmission Path

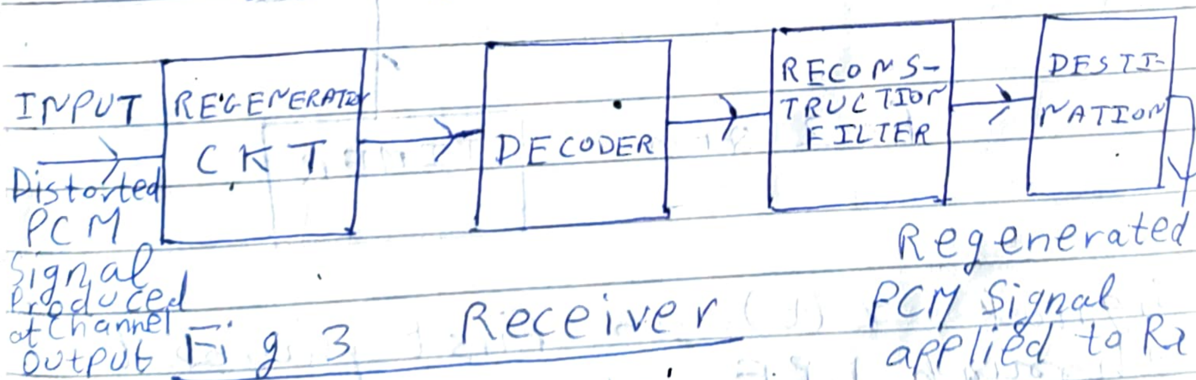
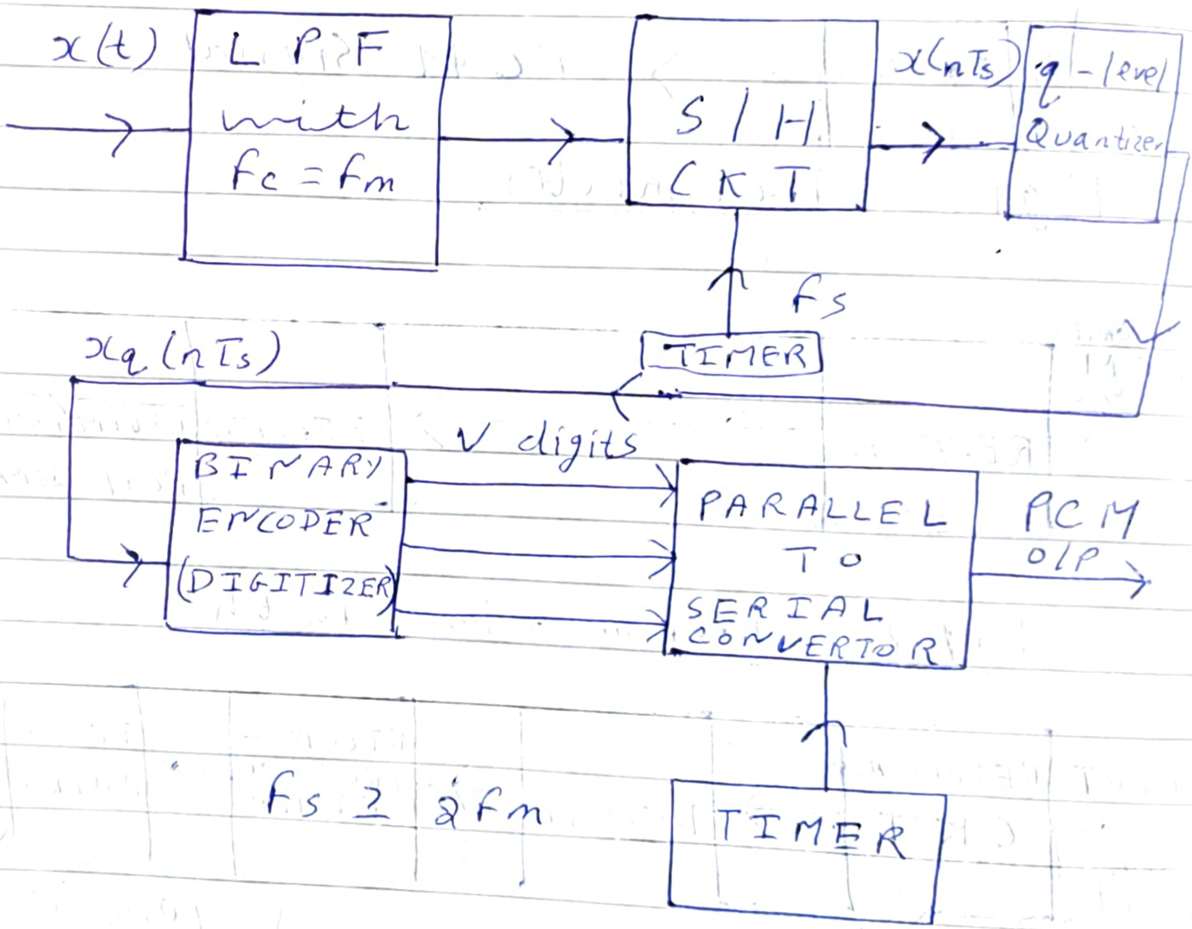


Fig 3 Receiver

PCM O/P is in Coded Digital Form.
It is in the form of digital pulses of constant amplitude, width and Position.

PCM Transmitter



Signal $x(t)$ is first passed through LPF of cutoff freq. f_m Hz. Now signal is bandlimited to f_m Hz.

S/H ckt then samples this signal at Rate $f_s \geq 2f_m$ to Avoid Aliasing.

O/P of S/H Ckt is $x(nTs)$.

A q - level Quantizer compares input $x(nTs)$ with its fixed digital levels, It assigns any one of digital level to $x(nTs)$ which results in min. distortion or error. This error is called Quantization Error.
Thus O/P of Quantizer is $x_q(nTs)$

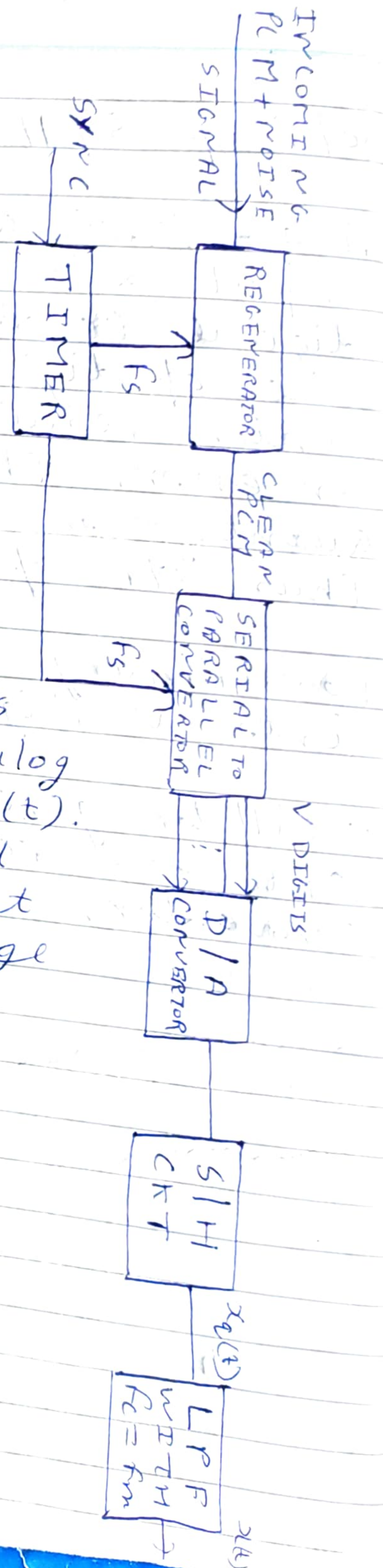
Now Binary Encoder converts it into v digits binary word.

Oscillator generates clocks for S/H Ckt and Parallel to serial convertor.

PCM Receiver

First Regenerator reconstructs the signal and remove the noise. This signal is then converted into Parallel Digital Words for ~~ead~~ each Sample.

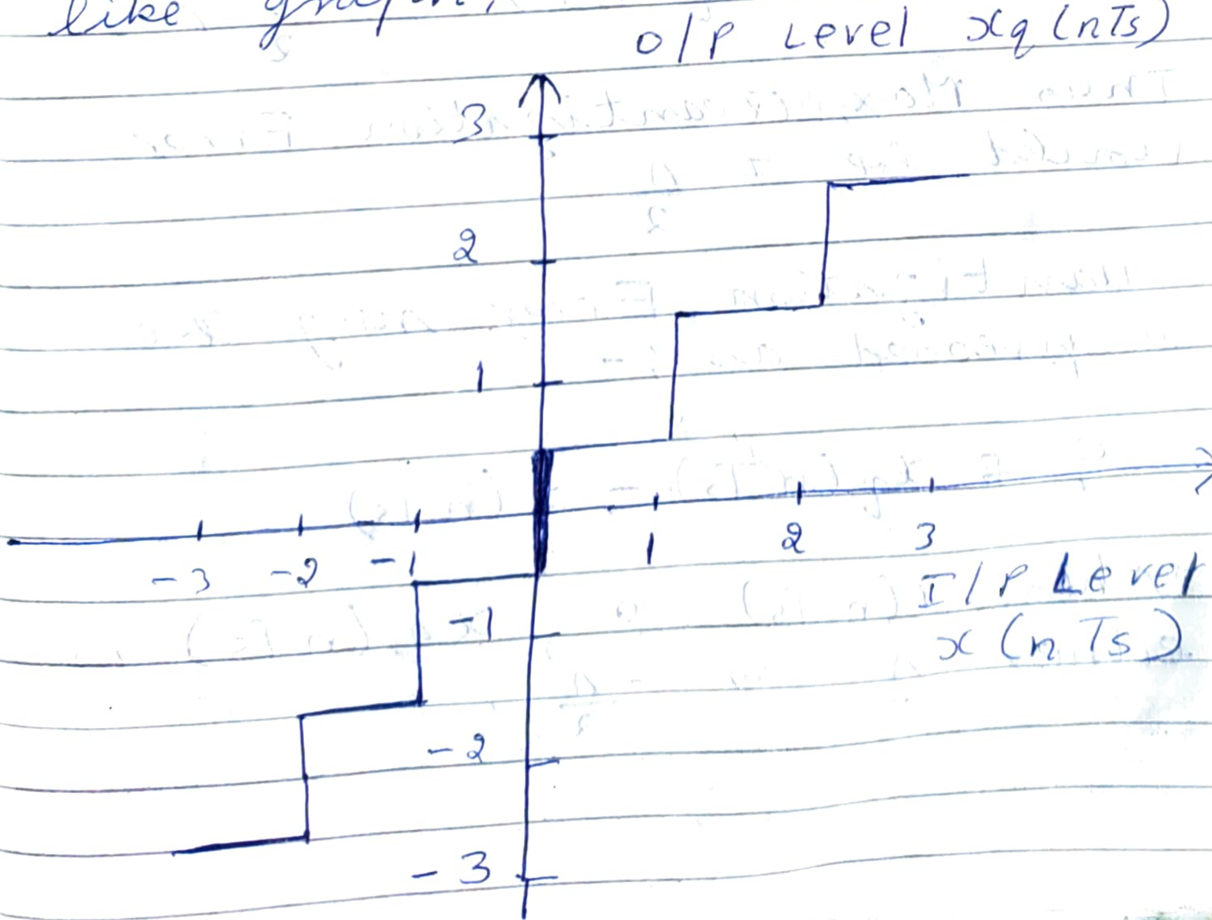
Now S/H Ckt Converts Digital Word To Analog signal denoted by $x_q(t)$. Then it is passed through LPF to get the original message signal $x(t)$.



Principle of Quantizer

A q - level Quantizer compares discrete time I/P $x(nT_s)$ with its fixed digital which results in min. distortion or error. This error is called Quantization Error.

Consider Transfer Characteristics of Uniform Quantizer of Midrise type.
In Midrise, origin lies in the middle of rising part of staircase like graph.



Let I/P To Quantizer $x(nT_s)$ varies from -3Δ to $+3\Delta$.
Δ is the Step Size.

Now Fixed Digital levels are available at $\pm\frac{\Delta}{2}$, $\pm\frac{3\Delta}{2}$ and $\pm\frac{5\Delta}{2}$

Now when $x(nT_s) = 3\Delta$
then $x_q(nT_s) = \frac{5\Delta}{2}$

If $x(nT_s) = -3\Delta$
$x_q(nT_s) = -\frac{5}{2}\Delta$

Thus Max. Quantization Error would be $\pm\frac{\Delta}{2}$

Quantization Error may be expressed as :-

$$\varepsilon = x_q(nT_s) - x(nT_s)$$

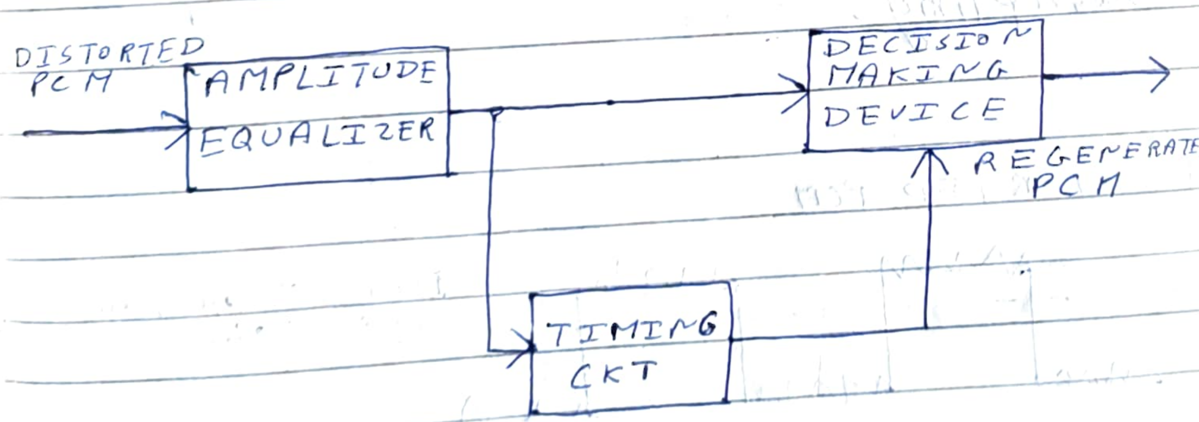
When $x(nT_s) = 0$, $x_q(nT_s)$ is either $\frac{\Delta}{2}$ or $-\frac{\Delta}{2}$.

Quantization Error will be :-

$$\varepsilon = x_q(nT_s) - x(nT_s)$$

$$= \frac{\Delta}{2} - 0 = \frac{\Delta}{2}$$

Block Diagram of Repeater



Amplitude Equalizer shapes the distorted PCM and removes the Amplitude and Phase distortions.

The Timing CKT provides the Periodic Pulse Train which is derived from

B.W. of PCM System

Assume that Quantizer uses 'v' binary digits to represent each level.

No of Levels will be :-

$$q = 2^v$$

For eg If $v = 4$
$q = 2^4 = 16$ levels

Each sample is converted Into 'v' binary bits.

No of Bits per sample = v

No of Samples per sec = f_s

No of Bits per sec = No of Bits per sample × No of samples per sec

$$r = v f_s$$

$r \rightarrow$ Signalling Rate of PCM

where $f_s \geq 2 f_m$

$$BW \geq \frac{1}{2} r$$

$$\geq \frac{1}{2} v f_s$$

$$\geq \frac{1}{2} v \, 2 f_m$$

$$\boxed{BW \geq v f_m}$$

Quantization Noise / Error in PCM System

For Linear or Uniform Quantization

Quantization Error is given as:-

$$\varepsilon = x_q(nT_s) - x(nT_s)$$

Let us assume I/P $x(nT_s)$ to a Linear or Uniform Quantizer has Continuous Amplitude in the Range $-x_{max}$ to $+x_{max}$.

Total Amp. Range $= x_{max} - (-x_{max})$
$= 2 x_{max}$

When Total Amp. Range is Divided into q levels of Quantizer, then step size will be :-

$$\Delta = \frac{2 x_{max}}{q}$$

When I/P $x(t)$ is Normalised to Min and Max Values equal to 1, then :-

$x_{max} = 1$, $-x_{max} = -1$

Step size $\Delta = \frac{2}{q}$

If Δ is Very Small then Quantization Error ϵ will be an Uniformly Distributed Random Variable.

Max Quantization Error :-

$$\epsilon_{max} = \left| \frac{\Delta}{2} \right|$$

$$-\frac{\Delta}{2} \le \mathcal{E}_{max} \le \frac{\Delta}{2}$$

Hence, over the Interval $\left(-\frac{\Delta}{2}, \frac{\Delta}{2}\right)$, $\mathcal{E}$ is assumed to be Uniformly Distributed Random Variable.

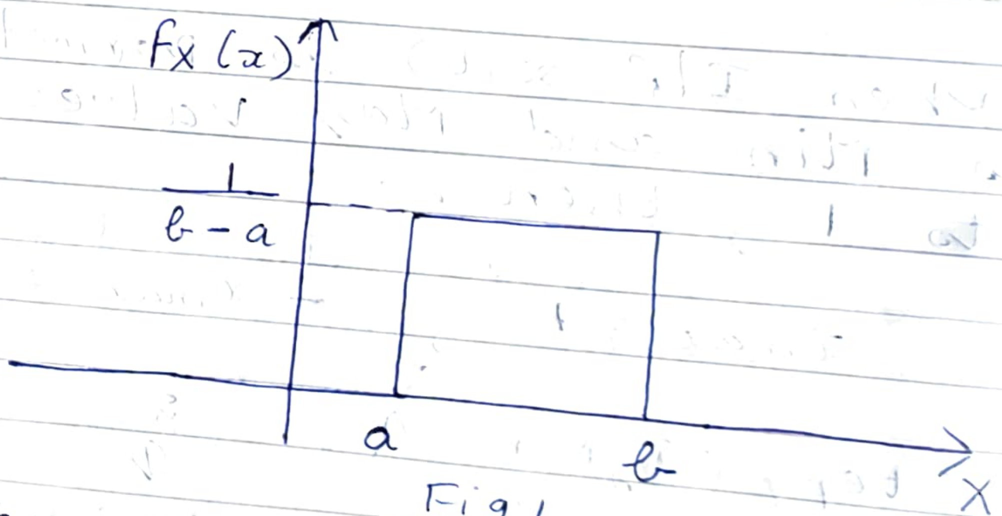


Fig 1

In Fig 1, X is Uniformly Distributed Random Variable over an interval (a, b).

PDF of X is given as:-

$$f_X(x) = \begin{cases} 0 & \text{for } x \le a \\ \frac{1}{b-a} & \text{for } a < x \le b \\ 0 & \text{for } x > b \end{cases} \quad \text{—(1)}$$

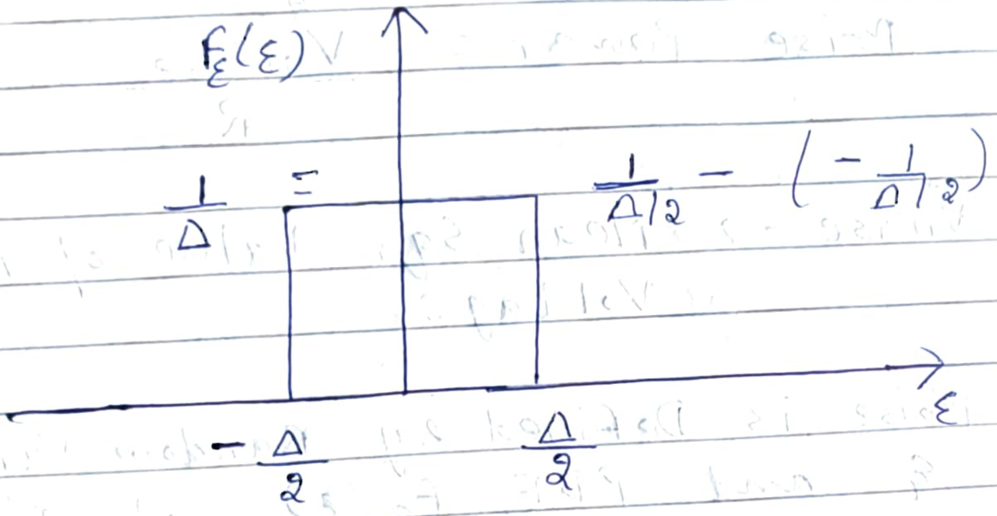


Fig 2

PDF for Quantization Error may be defined as :-

$$f_{\varepsilon}(\varepsilon) = \begin{cases} 0 & \text{for } \varepsilon \le -\frac{\Delta}{2} \\ \frac{1}{\Delta} & \text{for } -\frac{\Delta}{2} \le \varepsilon \le \frac{\Delta}{2} \\ 0 & \text{for } \varepsilon > \frac{\Delta}{2} \end{cases}$$

Signal To Noise Ratio of the Quantizer is defined as :-

$$\frac{S}{N} = \frac{\text{Signal Power (Norm.)}}{\text{Noise Power (Norm.)}}$$

$$\text{Noise Power} = \frac{V^2_{noise}}{R}$$

$V^2_{noise} \rightarrow$ Mean Sqr Value of Noise Voltage

Noise is Defined by Random Variable ξ and PDF $f_\varepsilon(\varepsilon)$, its Mean sqr value is given as:-

$$E[\xi^2] = \overline{\xi^2} = V^2_{noise}$$

$$E[\xi^2] = \int_{-\infty}^{\infty} \xi^2 \, f_\varepsilon(\varepsilon) \, d\xi$$

$$E[\xi^2] = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \xi^2 \times \frac{1}{\Delta} \, d\xi$$

$$= \frac{1}{\Delta} \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \varepsilon^2 \, d\xi$$

$$= \frac{1}{\Delta} \left[\frac{\xi^3}{3} \right]_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}}$$

$$E[\varepsilon^2] = \frac{1}{3\Delta}\left[\frac{\Delta^3}{8} - \left(-\frac{\Delta^3}{8}\right)\right]$$

$$= \frac{1}{3\Delta} \frac{\cancel{2}\,\Delta^3}{\cancel{8}\,4}$$

$$E[\varepsilon^2] = \frac{\Delta^2}{12}$$

$$V^2_{noise} = \frac{\Delta^2}{12}$$

If Load Resistance $R = 1\,\Omega$, then Noise Power is Normalised

$$\text{Noise Power (Norm.)} = \frac{V^2_{noise}}{1}$$

$$= \frac{\Delta^2}{12}$$

So Normalised Noise Power or Quantization Noise Power OR

Quantization Error (in Terms of Power)

$$= \frac{\Delta^2}{12}$$

Eg 1 24 Voice Signals are sampled Uniformly and then have to be TDM. Highest Freq. component of each voice signal is equal to 3.4 KHz. Now

i) If Signals are Pulse Amplitude Modulated Using Nyquist Rate Sampling, what would be the Min Channel B.W. Required.

ii) If signals are PCM Modulated with an 8 bit encoder, what would be the sampling Rate. Bit Rate of system is 1.5×10^6 bits/sec

For N channels TDM, Min Transmission B.W. would be :-

$$B.W. = N f_m$$

$$B.W. = 24 \times 3.4$$

$$B.W. = 81.6 \text{ KHz}$$

ii) Signalling Rate $= 1.5 \times 10^6$ bits/sec

Since there are 24 Channels, Bit Rate for Each Channel :-

$$r = \frac{1.5 \times 10^6}{8} = 62,500 \text{ bits/sec}$$

$$r = v f_s$$

$$f_s = \frac{r}{v}$$

$$= \frac{62500}{8}$$

$$f_s = 7812.5 \text{ Hz}$$