In digital modulations, instead of transmitting one bit at a time, we transmit two or more bits simultaneously. This is known as M-ary transmission. This type of transmission results in reduced channel bandwidth. However, sometimes, we use two quadrature carriers for modulation. This process is known as **Quadrature modulation.**

Thus, we see that there are a number of modulation schemes available to the designer of a digital communication system required for data transmission over a bandpass channel.

Every scheme offers system trade-offs of its own. However, the final choice made by the designer is determined by the way in which the available primary communication resources such as transmitted power and channel bandwidth are best exploited. In particular, the choice is made in favour of a scheme which possesses as many of the following design characteristics as possible:

(*i*) Maximum data rate,
(*ii*) Minimum probability of symbol error,
(*iii*) Minimum transmitted power,
(*iv*) Maximum channel bandwidth,
(*v*) Maximum resistance to interfering signals,
(*vi*) Minimum circuit complexity.

14.3. Types of Digital Modulation Techniques

(U.P. Tech., Sem. Examination, 2003-2004)

Basically, digital modulation techniques may be classified into coherent or non-coherent techniques, depending on whether the receiver is equipped with a phase-recovery circuit or not. The phase-recovery circuit ensures that the oscillator supplying the locally generated carrier wave receiver is synchronized* to the oscillator supplying the carrier wave used to originally modulate the incoming data stream in the transmitter.

(*i*) Coherent Digital Modulation Techniques

Coherent digital modulation techniques are those techniques which employ coherent detection. In coherent detection, the local carrier generated at the receiver is phase locked with the carrier at the transmitter. Thus, the detection is done by correlating received noisy signal and locally generated carrier. The coherent detection is a synchronous detection.

(*ii*) Non-coherent Digital Modulation Techniques

Non-coherent digital modulation techniques are those techniques in which the detection process does not need receiver carrier to be phase locked with transmitter carrier.

The advantage of such type of system is that the system becomes simple. But the drawback of such a system is that the error probability increases.

In fact, the different digital modulation techniques are used for various specific application areas.

14.4. Coherent Binary Modulation Techniques

As mentioned earlier, the binary (*i.e.*, digital) modulation has three basic forms amplitude-shift keying (ASK), phase-shift keying (PSK) and frequency-shift keying (FSK). In this section, let us discuss different coherent binary modulation techniques.

14.5. Coherent Binary Amplitude Shift Keying or On-Off Keying

Amplitude shift keying (ASK) or ON-OFF keying (OOK) is the simplest digital modulation technique. In this method, there is only one unit energy carrier and it is switched on or off depending upon the input binary sequence. The ASK waveform may be represented as,

$$s(t) = \sqrt{2P_s}\cos(2\pi f_c t) \quad \text{(To transmit `1')} \qquad \ldots(14.1)$$

* In both frequency and phase.

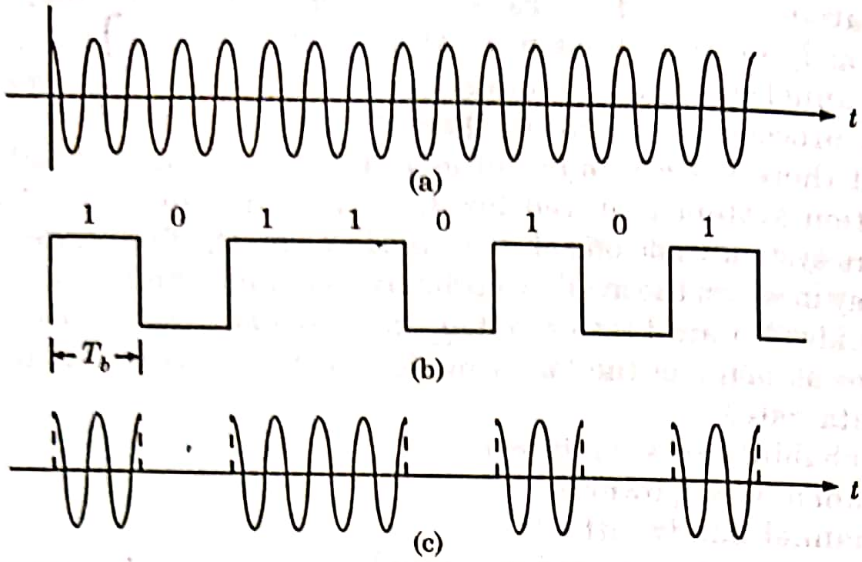


Fig. 14.2. *Amplitude-shift keying waveforms, (a) Unmodulated carrier, (b) Unipolar bit sequence, (c) ASK waveform.*

To transmit symbol '0', the signal $s(t) = 0$ *i.e.*, no signal is transmitted. Signal $s(t)$ contains some complete cycles of carrier frequency 'f_c'.

Hence, the ASK waveform looks like an ON-OFF of the signal. Therefore, it is also known as the **ON-OFF keying (OOK)**. Figure 14.2 shows the ASK waveform.

14.5.1. Signal Space Diagram of ASK

The ASK waveform of equation (14.1) for symbol '1' can be represented as,

$$s(t) = \sqrt{P_s T_b} \cdot \sqrt{2/T_b} \cos(2\pi f_c t)$$

or

$$s(t) = \sqrt{P_s T_b}\,\phi_1(t) \quad \text{...(14.2)}$$

This means that there is only one carrier function $\phi_1(t)$. The signal space diagram will have two points on $\phi_1(t)$. One will be at zero and other will be at $\sqrt{P_s T_b}$. Figure 14.3 shows this aspect.

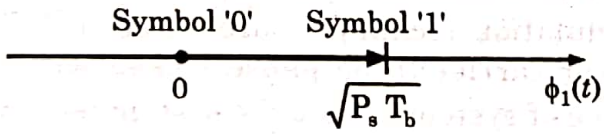


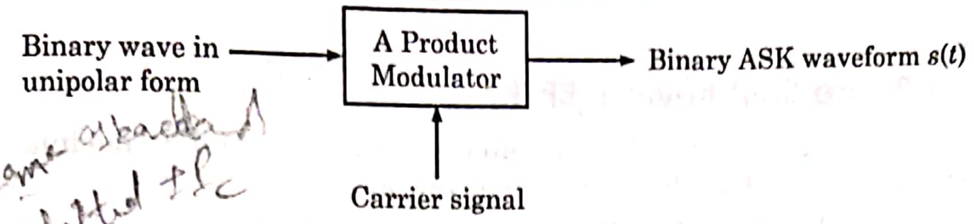
Fig. 14.3. *Signal space diagram of ASK.*

Thus, the distance between the two signal points is,

$$d = \sqrt{P_s T_b} = \sqrt{E_b} \quad \text{...(14.3)}$$

14.5.2. Generation of ASK Signal

ASK signal may be generated by simply applying the incoming binary data (represented in unipolar form) and the sinusoidal carrier to the two inputs of a product modulator (*i.e.*, balanced modulator). The resulting output will be the ASK waveform. This is shown in figure 14.4. Modulation causes a shift of the baseband signal spectrum. The ASK signal, which is basically the product of the binary sequence and the carrier signal, has a power spectral density (PSD) same as that of the baseband on-off signal but shifted in the frequency domain by $\pm f_c$. This is shown in figure 14.5. It may be noted that two impulses occur at $\pm f_c$. The spectrum of the ASK signal shows that it has an infinite bandwidth. However for practical purpose, the bandwidth is often defined as the bandwidth of an ideal bandpass filter centered at f_c whose output contains about 95% of the total average power content of the ASK signal. It may be proved that according to this criterion the bandwidth of the ASK signal is approximately $3/T_b$ Hz. The bandwidth of the ASK signal can however, be reduced by using smoothed versions of the pulse waveform instead of rectangular pulse waveforms.



PSD → Same as baseband
freq → shifted $\pm f_c$
& BW = $2/T_b$ Hz

Fig. 14.4. *Generation of binary ASK waveform.*

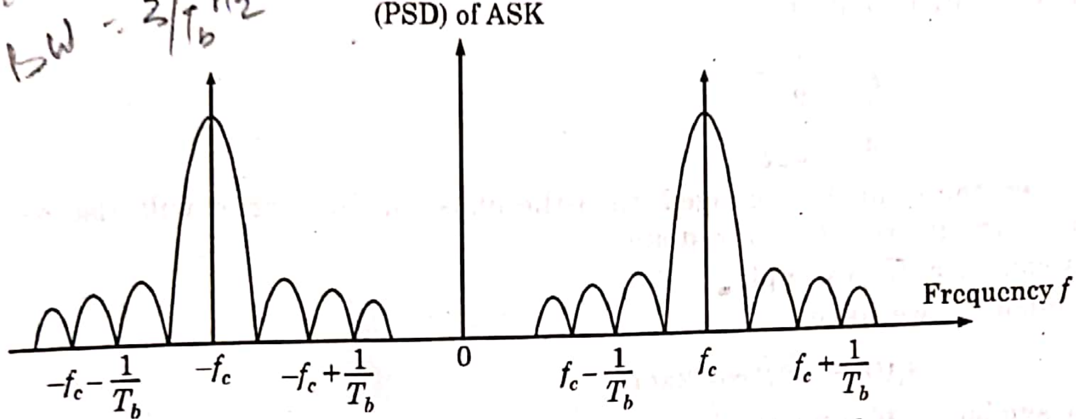


Fig. 14.5. *Power spectral density of ASK signal.*

14.5.3. Coherent Demodulation of Binary ASK

The demodulation of binary ASK waveform can be achieved with the help of *coherent detector* as shown in figure 14.6. It consists of a product modulator which is followed by an integrator and a decision-making device. The incoming ASK signal is applied to one input of the product modulator. The other input of the product modulator is supplied with a sinusoidal carrier which is generated with the help of a local oscillator. The output of the product modulator goes to input of the integrator. The integrator operates on the output of the multiplier for successive bit intervals and essentially performs a low-pass filtering action. The output of the integrator goes to the input of a decision-making device.

Now, the decision-making device compares the output of the integrator with a preset threshold. It makes a decision in favour of symbol 1 when the threshold is exceeded and in favour of symbol 0 otherwise. The *coherent detection* makes the use of linear operation. In this method we have assumed that the local carrier is in perfect synchronisation with the carriers used in the transmitter. This means that the frequency and phase of the locally generated carrier is same as those of the carriers used in the transmitter.

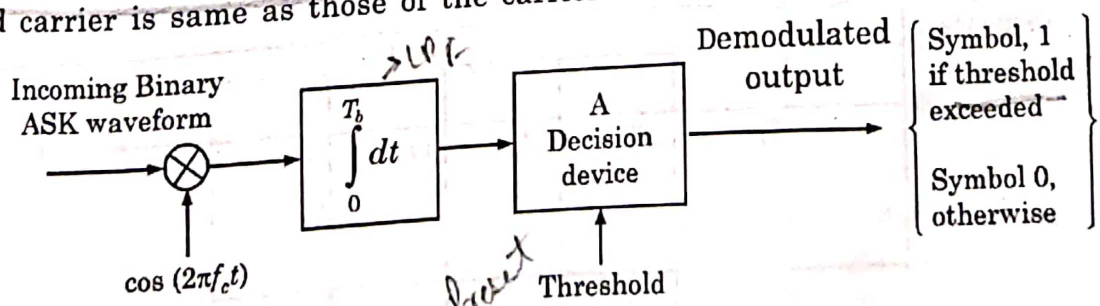


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Fig. 14.6. *Coherent detection of binary ASK signals.*

The following two forms of synchronisation are required for the operation of coherent (or synchronous detector):

(i) *Phase synchronisation which ensures that carrier wave generated locally in the receiver is locked in phase with respect to one that is employed in the transmitter.*

(ii) Timing synchronisation which enable proper timing of the decision making operation in the receiver with respect to switching instants (switching between 1 and 0) in the original binary data.

14.6. Binary Phase Shift Keying (BPSK)

In a binary phase shift keying (BPSK), the binary symbols '1' and '0' modulate the phase of the carrier. Let us assume that the carrier is given as,

$$s(t) = A \cos (2\pi f_c t) \quad ...(14.4)$$

Here 'A' represents peak value of sinusoidal carrier. For the standard 1 Ω load resistor, the power dissipated would be,

$$P = \frac{1}{2}A^2$$

or

$$A = \sqrt{2P} \quad ...(14.5)$$

Now, when the symbol is changed, then the phase of the carrier will also be changed by an amount of 180 degrees (*i.e.*, π radians).

Let us consider, for example,

For symbol '1', we have

$$s_1(t) = \sqrt{2P}\cos(2\pi f_c t) \quad ...(14.6)$$

If next symbol is '0', then we have

For symbol '0', we have

$$s_2(t) = \sqrt{2P}\cos(2\pi f_c t + \pi) \quad ...(14.7)$$

Now, because cos (θ + π) = – cos θ, therefore, the last equation can be written as,

$$s_2(t) = -\sqrt{2P}\cos(2\pi f_c t) \quad ...(14.8)$$

With the above equation, we can define BPSK signal combinely as,

$$s(t) = b(t)\sqrt{2P}\cos(2\pi f_c t) \quad ...(14.9)$$

where $b(t)$ = + 1 when binary '1' is to be transmitted.

– 1 when binary '0' is to be transmitted

Figure 14.7 illustrates binary signal and its equivalent signal $b(t)$.

Note: *It may be observed from figure 14.7(b) that the signal b(t) is a* ***NRZ bipolar signal.*** *In fact, this signal directly modulates the carrier signal cos (2π f_c t).*

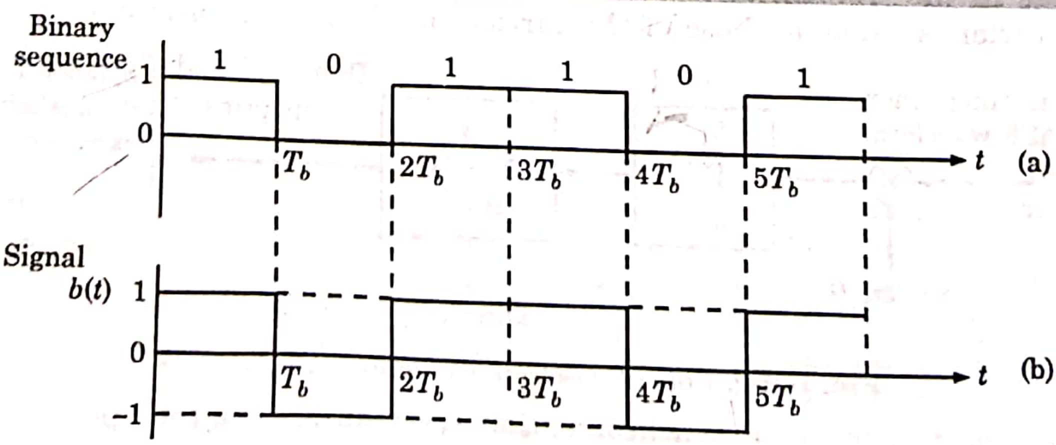


Fig. 14.7. *(a) Binary sequence, (b) The corresponding bipolar signal b(t).*

14.6.1. Generation of BPSK Signal

The BPSK signal may be generated by applying carrier signal to a balanced modulator. Here, the bipolar signal $b(t)$ is applied as a modulating signal to the balanced modulator.

Figure 14.8 shows the block diagram of a BPSK signal generator.

A NRZ level encoder converts the binary data sequence into bipolar NRZ signal.

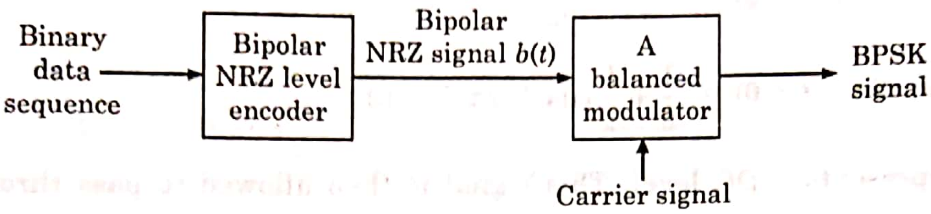


Fig. 14.8. *Generation of BPSK.*

14.6.2. Reception (*i.e.* Detection) of BPSK Signal

Figure 14.9 shows the block diagram of the scheme to recover baseband signal from BPSK signal. The transmitted BPSK signal is given as

$$s(t) = b(t)\sqrt{2P}\cos(2\pi f_c t)$$

This signal undergoes the phase change depending upon the time delay from transmitter end to receiver end. This phase change is, usually, a fixed phase shift in the transmitted signal.

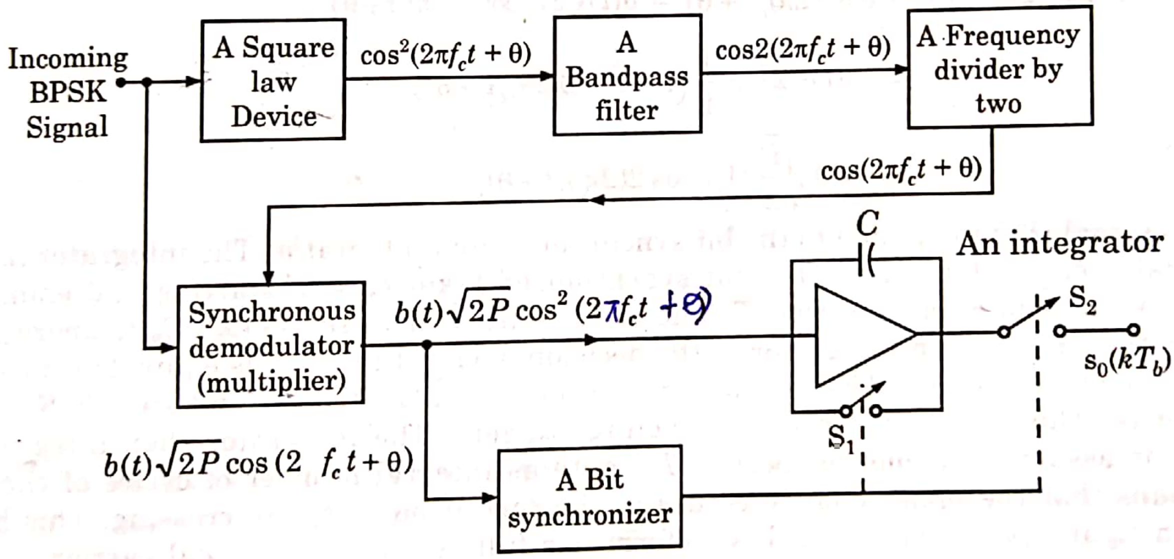


Fig. 14.9. *Reception of baseband signal in BPSK signal.*

Let us consider that this phase shift is θ. Because of this, the signal at the input of the receiver can be written as

$$s(t) = b(t)\sqrt{2P}\cos(2\pi f_c t + \theta) \qquad \text{...(14.10)}$$

Now, from this received signal, a carrier is separated because this is coherent detection. As shown in the figure 14.9, the received signal is allowed to pass through a square law device. At the output of the square law device, we get a signal which is given as

$$\cos^2(2\pi f_c t + \theta)$$

Here, it may be noted that we have neglected the amplitude, since we are only interested in the carrier of the signal.

Also, in the k^{th} bit interval, we can write output signal as under:

$$s_0(kT_b) = b(kT_b)\sqrt{\frac{P}{2}}\int_{(k-1)T_b}^{kT_b}[1+\cos(2\pi f_c t+\theta)]\,dt$$

This equation gives the output of an interval for k^{th} bit. Hence, integration is performed from $(k-1)T_b$ to kT_b. Here, T_b is the one bit period. We can write the above equation as under :

$$s_0(kT_b) = b(kT_b)\sqrt{\frac{P}{2}}\left[\int_{(k-1)T_b}^{kT_b} 1dt + \int_{(k-1)T_b}^{kT_b}\cos(2\pi f_c t+\theta)dt\right]$$

where $\int_{(k-1)T_b}^{kT_b}\cos 2(2\pi f_c t+\theta)\,dt = 0$, since average value of sinusoidal waveform is zero if integration is done over full cycles. Hence, we can write above equation as,

$$s_0(kT_b) = b(kT_b)\sqrt{\frac{P}{2}}\int_{(k-1)T_b}^{kT_b} 1dt$$

or
$$s_0(kT_b) = b(kT_b)\sqrt{\frac{P}{2}}\,[t]_{(k-1)T_b}^{kT_b}$$

or
$$s_0(kT_b) = b(kT_b)\sqrt{\frac{P}{2}}\{kT_b-(k-1)T_b\}$$

or
$$s_0(kT_b) = b(kT_b)\sqrt{\frac{P}{2}}\,T_b \qquad ...(14.12)$$

The last eqaution shows that the output of the receiver depends on input.

Thus, $s_0(kT_b)\,\alpha\, b(kT_b)$

Depending upon the value of $b(kT_b)$, the output $s_0(kT_b)$ is generated in receiver.

This signal is then applied to a decision device which decides whether transmitted symbol was zero or one.

14.6.3. The Spectrum of BPSK Signals

Type of we know that the waveform $b(t)$ is a NRZ binary waveform. In this waveform, there are rectangular pulses of amplitude $\pm V_b$. If we assume that each pulse is $\pm\frac{T_b}{2}$ around its centre, then it becomes easy to find Furrier transform of such pulse. The Fourier transform of this type of pulse is given as,

$$X(f) = V_b\,T_b\frac{\sin(\pi f T_b)}{(\pi f T_b)}\,* \qquad ...(14.13)$$

For a large number of such positive and negative pulses, the power spectral density $S(f)$ is expressed as

$$S(f) = \frac{\overline{|X(f)|^2}}{T_s} \qquad ...(14.14)$$

Here, $\overline{X(f)}$ denotes average value of $X(f)$ due to all the pulses in $b(t)$. And T_s is symbol duration. Substituting value of $X(f)$ from equation (13.13) in equation (14.14), we get

* Making use of standard relation.

$$S(f) = \frac{V_b^2 T_b^2}{T_s}\left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2$$

For BPSK, because only one bit is transmitted at a time, therefore, symbol and bit durations are same *i.e.*, $T_b = T_s$. Then the last equation becomes,

$$S(f) = V_b^2\, T_b\left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2 \qquad ...(14.15)$$

This equation gives the power spectral density (psd) of baseband signal $b(t)$. The BPSK signal is generated by modulating a carrier by the baseband signal $b(t)$. Due to modulation of the carrier of frequency f_c, the spectral components are translated from f to $f_c + f$ and $f_c - f$. The magnitude of these components is divided by half.

Therefore, from equation (14.15) we can write the power spectral density of BPSK signal as under:

$$S_{BPSK}(f) = V_b^2\, T_b\left\{\frac{1}{2}\left[\frac{\sin \pi(f_c - f)\,T_b}{\pi(f_c - f)T_b}\right] + \frac{1}{2}\left[\frac{\sin \pi(f_c + f)T_b}{\pi(f_c + f)\,T_b}\right]^2\right\}$$

It may be noted that this equation consists of two half magnitude spectral components of same frequency 'f' above and below f_c. Let us assume that the value of $\pm V_b = \pm\sqrt{P}$. This means that the NRZ signal is having amplitudes of $+\sqrt{P}$ and $-\sqrt{P}$. Then the last equation becomes,

$$S_{BPSK}(f) = \frac{PT_b}{2}\left\{\left[\frac{\sin \pi(f - f_c)T_b}{\pi(f - f_c)T_b}\right]^2 + \frac{1}{2}\left[\frac{\sin \pi(f_c + f)T_b}{\pi(f_c + f)T_b}\right]^2\right\} \qquad ...(14.16)$$

This equation gives power spectral density (psd) of BPSK signal for modulating signal $b(t)$ having amplitudes equal to $\pm\sqrt{P}$.

Further, we know that the modulated signal is given as

$$s(t) = \pm\sqrt{2P}\cos(2\pi f_c t) \qquad [\because A = \sqrt{2P}]$$

If $b(t) = \pm\sqrt{P}$, then the carrier becomes,

$$\phi(t) = \sqrt{2}\cos(2\pi f_c t) \qquad ...(14.17)$$

Equation (14.15) describes power spectral density (psd) of the NRZ waveform. For one rectangular pulse, the shape of $S(f)$ will be a sinc pulse as shown in figure 14.11.

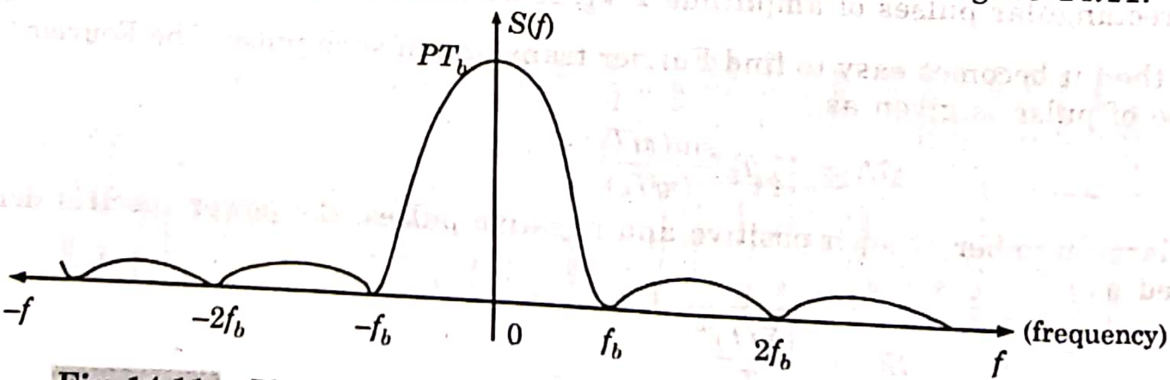


Fig. 14.11. *Plot of power spectral density (psd) of NRZ baseband signal.*

It may be observed from this figure that the main lobe ranges from $-f_b$ to $+f_b$.* Because we have taken $\pm V_b = \pm\sqrt{P}$ in equation (14.15), therefore, the peak value of the main lobe is PT_b. Now let us consider the power spectral density (psd) of BPSK signal expressed by equation (14.16).

Figure 14.12 shows the plot of this equation. This figure, thus, clearly shows that there are two lobes, one at f_c and other at $-f_c$. The same spectrum of figure 14.11 has been placed at $+f_c$ and $-f_c$. However, the amplitudes of main lobes are $\frac{PT_b}{2}$ in figure 14.12.

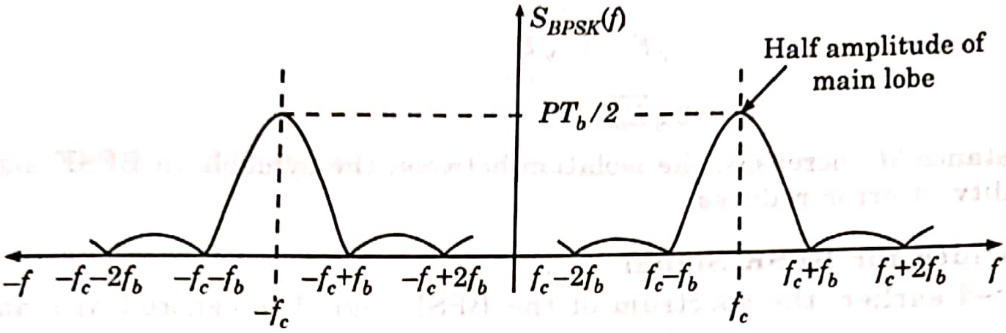


Fig. 14.12. *Plot of power spectral density of BPSK signal.*

Hence, they are reduced to half. The spectrum of $S(f)$ as well as $S_{BPSK}(f)$ extends overall the frequencies.

14.6.4. A Geometrical Representation for BPSK Signals

We know that BPSK signal carries the information about two symbols. These symbols are symbol '1' and symbol '0'. We can represent BPSK signal geometrically to show those two symbols. From equation (14.9), we know that BPSK signal is expressed as,

$$s(t) = b(t)\sqrt{2P}\cos(2\pi f_c t) \qquad ...(14.18)$$

Let us rearrange the last equation as,

$$s(t) = b(t)\sqrt{PT_b}\cdot\sqrt{\frac{2}{T_b}}\cos(2\pi f_c t) \qquad ...(14.19)$$

Now, let $\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t)$ represents an orthonormal carrier signal. Equation (14.17) lso gives equation for carrier. It is slightly different than $\phi_1(t)$ defined here. Then, we may rite equation (14.19) as,

$$s(t) = b(t)\sqrt{PT_b}\,\phi_1(t) \qquad ...(14.20)$$

The bit energy E_b is defined in terms of power 'P' and bit duration T_b as,

$$E_b = PT_b \qquad ...(14.21)$$

Therefore, equation (14.20) becomes,

$$s(t) = \pm\sqrt{E_b}\,\phi_1(t) \qquad ...(14.22)$$

Here, $b(t)$ is simply ± 1.

Thus, on the single axis of $\phi_1(t)$, there will be two points. One point will be located at $\overline{E_b}$ and other point will be located at $-\sqrt{E_b}$. This has been shown in figure 14.13.

* Here, $f_b = \frac{1}{T_b}$.

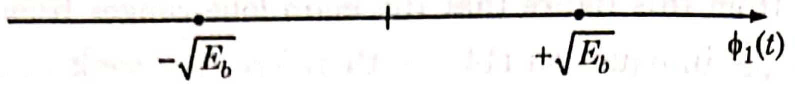


Fig. 14.13. *Geometrical representation of BPSK signal.*

At the receiver end, the point at $+\sqrt{E_b}$ on $\phi_1(t)$ represents symbol '1' and point at $-\sqrt{E_b}$ represents symbol '0'. The separation between these two points represents the isolation in symbols '1' and '0' in BPSK signal. This separation is generally called distance 'd'. From figure 14.13, it is obvious that the distance between the two points is,

$$d = +\sqrt{E_b} - \left(-\sqrt{E_b}\right)$$

or

$$d = 2\sqrt{E_b} \qquad ...(14.23)$$

As this distance 'd' increases, the isolation between the symbols in BPSK signal is more. Thus, probability of error reduces.

14.6.5. Bandwidth for BPSK Signal

As discussed earlier, the spectrum of the BPSK signal is centred around the carrier frequency f_c.

If $f_b = \frac{1}{T_b}$, then for BPSK, the maximum frequency in the baseband signal will be f_b as shown in figure 13.12. In this figure, the main lobe is centred around carrier frequency f_c and extends from $f_c - f_b$ to $f_c + f_b$.

Therefore Bandwidth of BPSK signal will be,

BW = Highest frequency – Lowest frequency in the main lobe

$$BW = f_c + f_b - (f_c - f_b)$$

or

$$BW = 2f_b \qquad ...(14.24)$$

Hence, the minimum bandwidth of BPSK signal is equal to twice of the highest frequency contained in baseband signal.

14.6.6. Error Probability of BPSK Signal Employing Coherent Reception or Matched Filter

The BPSK receiver shown in figure 14.9 is equivalent to the coherent receiver. It is nothing but coherent receiver. Let us say that the signal $x(t)$ represents $s_1(t)$ and $s_2(t)$. Then, the BPSK receiver of figure 14.9 can be approximated to coherent as shown in figure 14.14.

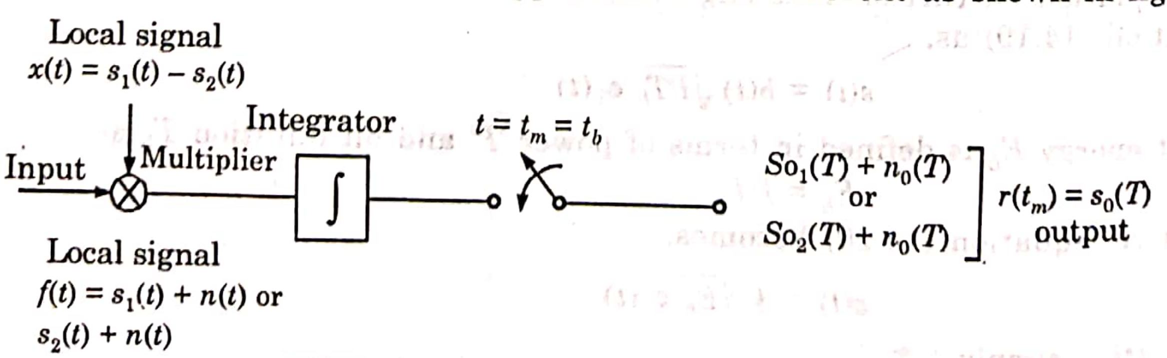


Fig. 14.14. *Coherent reception system*

Because, the correlator of figure 14.14 is equivalent to matched filter, we can apply the analysis of matched filter detection. We know that signal to noise ratio ρ_{max} at the output of receiver is given by

$$\rho_{max} = \frac{2}{N_0} \int_{-\infty}^{\infty} x^2(t)\, dt \quad \text{...(14.25)}$$

Here, $x(t) = s_1(t) - s_2(t)$ *i.e.*, input difference signal, then we have

$$\rho_{max} = \frac{2}{N_0} \int_{-\infty}^{\infty} [s_1(t) - s_2(t)]^2 dt$$

or

$$\rho_{max} = \frac{2}{N_0} \left[\int_{-\infty}^{\infty} s_1^2(t)\, dt + \int_{-\infty}^{\infty} s_2^2(t) dt - 2 \int_{-\infty}^{\infty} s_1(t) s_2(t) dt \right]$$

Since $s_1(t) = -\, s_2(t)$, we have

$$\int_{-\infty}^{\infty} s_1(t)\, s_2(t)\, dt = -\, E_b$$

and $\int_{-\infty}^{\infty} s_1^2(t)\, dt = \int_{-\infty}^{\infty} s_2^2(t)\, dt = E_b$

Hence, $\rho_{max} = \frac{2}{N_0} [E_b + E_b + 2E_b]$ by putting values

or

$$\rho_{max} = \frac{8E_b}{N_0}$$

For the correlator, we can write

$$\left[\frac{s_{01}(t) - s_{02}(t)}{\sigma} \right]^2 = \frac{8E_b}{N_0}$$

or

$$\frac{s_{01}(t) - s_{02}(t)}{\sigma} = 2\sqrt{2} \sqrt{\frac{E_b}{N_0}} \quad \text{...(14.26)}$$

The probability of error is given as,

$$P(e) = \frac{1}{2}\, erfc \left[\frac{s_{01}(T) - s_{02}(T)}{2\sqrt{2}\, \sigma} \right]$$

From equation (14.26), we can write above equation as under:

$$P(e) = \frac{1}{2}\, erfc \left[\frac{1}{2\sqrt{2}} \cdot 2\sqrt{2} \sqrt{\frac{E_b}{N_0}} \right]$$

or

$$\boxed{P(e) = \frac{1}{2}\, erfc \sqrt{\frac{E_b}{N_0}}} \quad \text{...(14.27)}$$

This is the required expression for error probability of BPSK reception using coherent/ matched filter detection.

14.6.7. Drawbacks of BPSK

Figure 14.9 shows the block diagram of BPSK receiver. To regenerate the carrier in the receiver, we start by squaring $b(t)\sqrt{2P}\cos(2\pi f_c t + \theta)$. If the received signal is $-b(t)\sqrt{2P}\cos(2\pi f_c t + \theta)$, then the squared signal remains same as before. Hence, the recovered carrier is unchanged even if the input signal has changed its sign. Therefore, it is not possible to determine whether the received signal is equal to $b(t)$ or $-\,b(t)$. Infact, this results in ambiguity in the output signal.

This problem can be removed if we use differential phase shift keying (DPSK). However, differential phase shift keying (DPSK) also has some other problems. DPSK will be discussed in detail later on in this chapter. Other problems of BPSK are ISI and Interchannel interference. However, these problems can be reduced to some extent by making use of filters.

14.7. Coherent Binary Frequency Shift Keying (BFSK)

In binary frequency shift keying (BFSK), the frequency of the carrier is shifted according to the binary symbol. However, the phase of the carrier is unaffected. This means that we have two different frequency signals according to binary symbols. Let there be a frequency shift by Ω. Then we can write following equations.

If $\quad b(t) = \text{'1'}, \quad$ then $s_H(t) = \sqrt{2P_s}\cos(2\pi f_c + \Omega)\,t$ ...(14.28)

If $\quad b(t) = \text{'0'}, \quad$ then $s_L(t) = \sqrt{2P_s}\cos(2\pi f_c - \Omega)\,t$ ...(14.29)

Hence, there is increase or decrease in frequency by Ω. Let us use the following conversion table to combine above two FSK equations:

Table 14.1. Conversion table for BPSK representation

$b(t)$ Input	$d(t)$	$P_H(t)$	$P_L(t)$
1	+ 1V	+ 1V	0V
0	– 1V	0V	+ 1V

The equations (14.28) and (14.29) combinely may be written as

$$s(t) = \sqrt{2P_s}\cos\left[(2\pi f_c + d(t)\Omega)\,t\right] \qquad \text{...(14.30)}$$

Hence, if symbol '1' is to be transmitted, the carrier frequency will be $f_c + \left(\frac{\Omega}{2\pi}\right)$. If symbol '0' is to be transmitted, then the carrier frequency will be $f_c - \left(\frac{\Omega}{2\pi}\right)$.

Therefore, we have

Thus, $\quad f_H = f_c + \frac{\Omega}{2\pi} \quad$ for symbol '1' ...(14.31)

$\quad f_L = f_c - \frac{\Omega}{2\pi} \quad$ for symbol '0' ...(14.32)

14.7.1. Generation of BFSK

It may be observed from Table 14.1 that $P_H(t)$ is same as $b(t)$ and also $P_L(t)$ is inverted version of $b(t)$. The block diagram for BFSK generation is shown in figure 14.15.

We know that input sequence $b(t)$ is same as $P_H(t)$. An inverter is added after $b(t)$ to get $P_L(t)$. The level shifter $P_H(t)$ and $P_L(t)$ are unipolar signals. The level shifter converts the '+1' level to $\sqrt{P_s T_b}$. Zero level is unaffected. Thus, the output of the level shifters will be either $\sqrt{P_s T_b}$ (if '+1') or zero (if input is zero). Further, there are product modulators after level shifter. The two carrier signals $\phi_1(t)$ and $\phi_2(t)$ are used. $\phi_1(t)$ and $\phi_2(t)$ are orthogonal to each other. In one bit period of input signal (*i.e.*, T_b), $\phi_1(t)$ or $\phi_2(t)$ have integral number of cycles. Thus, the modulated signal is having continuous phase. Figure 14.16 shows such type of BFSK signal. The adder then adds the two signals.

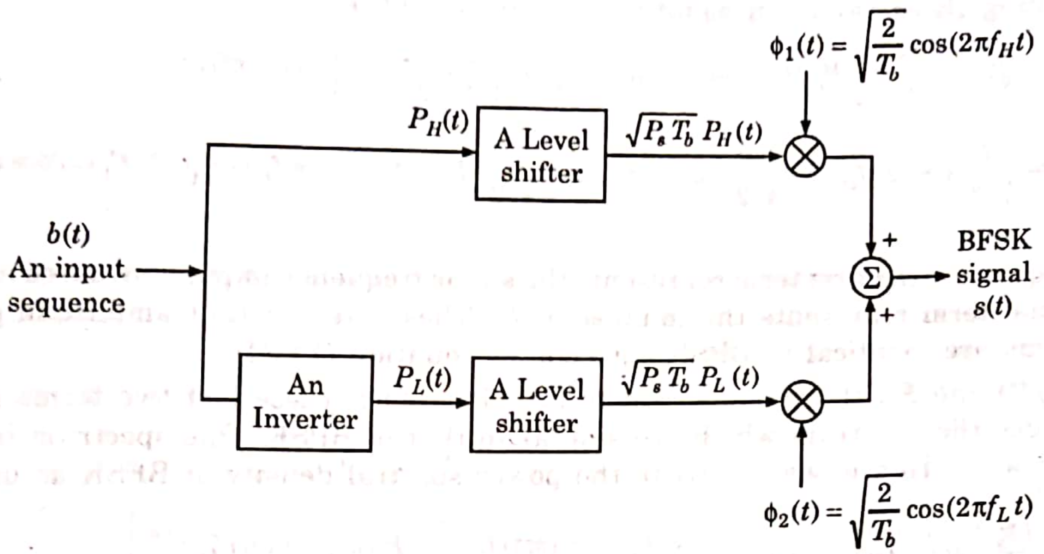


Fig. 14.15. *Block diagram for BFSK generation.*

Note: *Here it may be noted that outputs from both the multipliers are not possible at a time. This is because $P_H(t)$ and $P_L(t)$ are complementary to each other. Therefore, if $P_H(t) = 1$, then output will be only due to upper modulator and lower modulator output will be zero [since $P_L(t) = 0$)].*

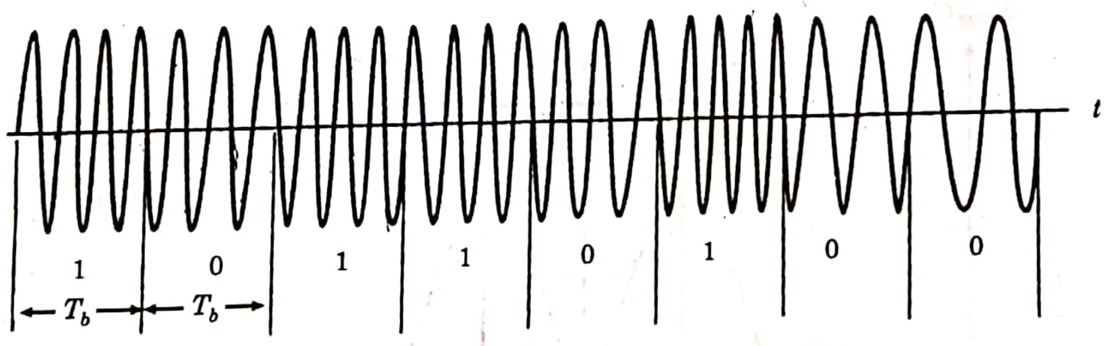


Fig. 14.16. *The BFSK signal.*

14.7.2. The Spectrum of BFSK Signal

In figure 14.15, the BFSK signal $s(t)$ may be written as,

$$s(t) = \sqrt{2P_s}\, P_H(t)\cos(2\pi f_H t) + \sqrt{2P_s}\, P_L(t)\cos(2\pi f_L t) \quad \text{...(14.33)}$$

This is the expression for BFSK signal. Let us compare this equation with BPSK equation which is written below:

$$S_{BPSK}(t) = b(t)\sqrt{2P}\cos(2\pi f_c t) \quad \text{...(14.34)}$$

It may be noted that this equation is identical to BFSK equation. In BPSK equation, $b(t)$ is a bipolar signal where as in BFSK, the similar coefficients $P_H(t)$ or $P_L(t)$ are unipolar. Hence, let us convert these coefficients in bipolar form as under:

$$P_H(t) = \frac{1}{2} + \frac{1}{2}P'_H(t) \quad \text{...(14.35)}$$

and

$$P_L(t) = \frac{1}{2} + \frac{1}{2}P'_L(t) \quad \text{...(14.36)}$$

where $P'_H(t)$ and $P'_L(t)$ will be bipolar (*i.e.*, + 1 or – 1).

Substituting these values in equation (14.33), we obtain

$$s(t) = \sqrt{2P_s}\left[\frac{1}{2}+\frac{1}{2}P_H'(t)\right]\cos(2\pi f_H t)+\sqrt{2P_s}\left[\frac{1}{2}+\frac{1}{2}P_L'(t)\right]\cos(2\pi f_L t)$$

or $$s(t) = \sqrt{\frac{P_s}{2}}\cos(2\pi f_H t)+\sqrt{\frac{P_s}{2}}\cos(2\pi f_L t)+\sqrt{\frac{P_s}{2}}P_H'(t)\cos(2\pi f_H t)+\sqrt{\frac{P_s}{2}}P_L'(t)\cos(2\pi f_L t)$$

...(14.37)

In this equation, the first term represents the single frequency impulse situated at frequency f_H. The second term represents the impulse at f_L. These are constant amplitude pulses. The last two terms are identical to BPSK equation of equation (14.34).

Here $P_H'(t)$ and $P'_L(t)$ are equivalent to $b(t)$. Therefore, these last two terms in equation (14.37) produce the spectrum which are similar to that of BPSK. One spectrum is located at f_H and other at f_L. Hence, we can write the power spectral density of BFSK as under:

$$S(f) = \sqrt{\frac{P_s}{2}}\left[\delta(f-f_H)+\delta(f+f_L)+\frac{P_sT_b}{2}\left\{\frac{\sin(\pi f_H T_b)}{\pi f_H T_b}\right\}^2+\frac{P_sT_b}{2}\left\{\frac{\sin(\pi f_L T_b)}{\pi f_L T_b}\right\}^2\right]$$ ...(14.38)

Figure 14.17 illustrates the plot of power spectral density of BFSK signal expressed by equation (14.38).

Also, f_H and f_L are selected such that,

$$f_H - f_L = 2f_b$$ ...(14.39)

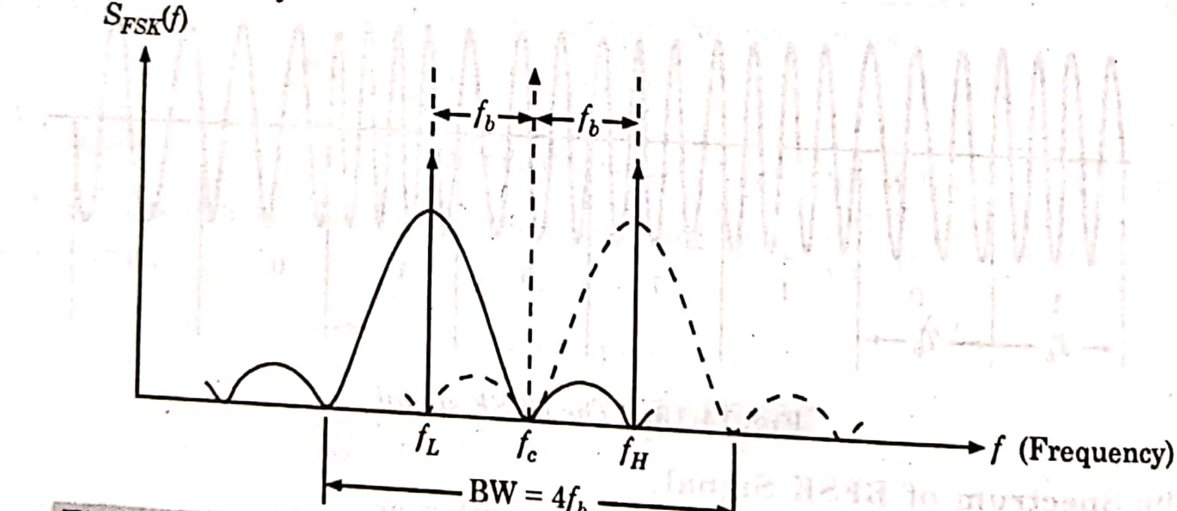


Fig. 14.17. *Illustration of Power spectral density (psd) of a BFSK signal.*

With such types of selection, it is obvious from the spectrums in the above figure that the two frequencies f_H and f_L may be identified properly. The interference between the spectrums is not much with the above assumption.

14.7.3. Bandwidth of BFSK Signal

From figure 14.17, it is obvious that the width of one lobe is $2f_b$. The two main lobes due to f_H and f_L are placed such that the total width due to both main lobes is $4f_b$.

Therefore, we have

Bandwidth of BFSK = $2f_b + 2f_b$

or $$BW = 4f_b$$ ...(14.40)

Now, if we compare this bandwidth with that of BPSK, we note that,

$$BW\,(BFSK) = 2 \times BW(BPSK)$$ ...(14.41)

14.7.4. Detection of BFSK

Figure 14.18 shows the block diagram of a scheme for demodulation of BFSK wave using coherent detection technique. The detector consists of two correlators that are individually tuned to two different carrier frequencies to represent symbols '1' and '0'. A correlator consists of a multiplier followed by an integrator. Then, the received binary FSK signal is applied to the multipliers of both the correlators. To the other input of the multipliers, carriers with frequency f_{c1} and f_{c2} are applied as shown in figure 14.18. The multiplied output of each multiplier is subsequently passed through integrators generating output l_1 and l_2 in the two paths. The output of the two integrators are then fed to the decision making device. The decision making device is essentially a comparator which compares the output l_1 (in the upper path) and output l_2 (in the lower path). If the output l_1 produced in the upper path (associated with frequency f_{c1}) is greater than the output l_2 produced in the lower path (associated with frequency f_{c2}), the detector makes a decision in favour of symbol 1. If the output l_1 is less than l_2, then the decision making device decides in favour of symbol 0 (say). This type of digital communication receivers are also called *correlation receivers*. As discussed earlier, the detector based upon coherent detection requires phase and timing synchronisation.

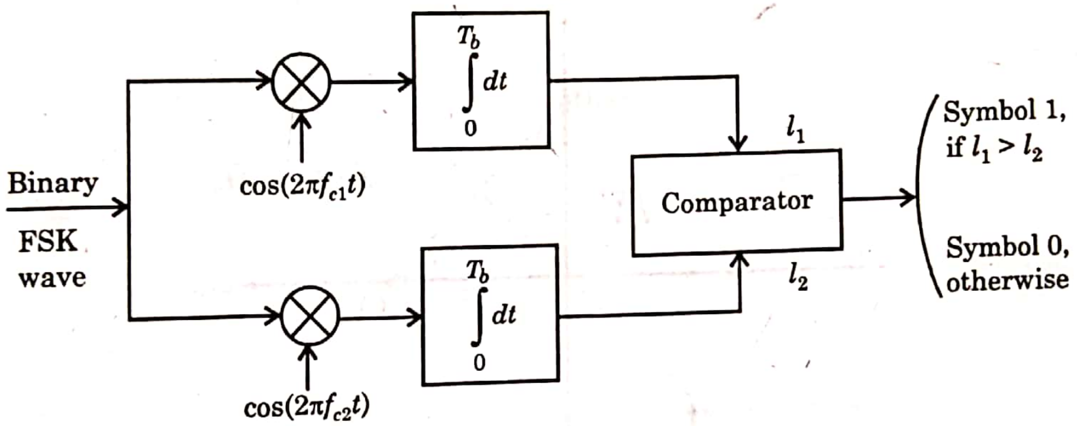


Fig. 14.18. *Block diagram of BFSK receiver (detection of BFSK).*

14.7.5. Geometrical Representation of Orthogonal BFSK

As a matter of fact, orthogonal carriers are used for M-ary PSK and QASK. The different signal points are represented geometrically in $\phi_1\ \phi_2$-plane. For geometrical representation of BFSK signals, such orthogonal carriers are required. From figure 14.15, we know that two carriers $\phi_1(t)$ and $\phi_2(t)$ of two different frequencies f_H and f_L are used for modulation. To make $\phi_1(t)$ and $\phi_2(t)$ orthogonal, the frequencies f_H and f_L must be some integer multiple of band frequency 'f_b'.

Thus,
$$f_H = m\ f_b \quad ...(14.42)$$
and
$$f_L = n\ f_b \quad ...(14.43)$$

Here; $f_b = \frac{1}{T_b}$, then the carriers would be

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi m\ f_b\ t) \quad ...(14.44)$$

and
$$\phi_2(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi\ m f_b\ t) \quad ...(14.45)$$

The carriers $\phi_1(t)$ and $\phi_2(t)$ are orthogonal over the period T_b. We can write equation (14.28) and equation (14.29) as,

$$s_H(t) = \sqrt{P_s T_b} \sqrt{\frac{2}{T_b}} \cos(2\pi\ f_H t)$$

and
$$s_L(t) = \sqrt{P_s T_b}\sqrt{\frac{2}{T_b}}\cos(2\pi f_L t)$$

Here
$$f_H = f_c + \frac{\Omega}{2\pi}$$

and
$$f_L = f_c - \frac{\Omega}{2\pi}$$

Using the relations in equations (14.42) to (14.45), we can write above equations as,
$$s_H(t) = \sqrt{P_s T_b}\cdot\phi_1(t) \quad \text{...(14.46)}$$

and
$$s_L(t) = \sqrt{P_s T_b}\cdot\phi_2(t) \quad \text{...(14.47)}$$

Thus, based on the above two equations, we can draw the signal space diagram as shown in figure 14.19.

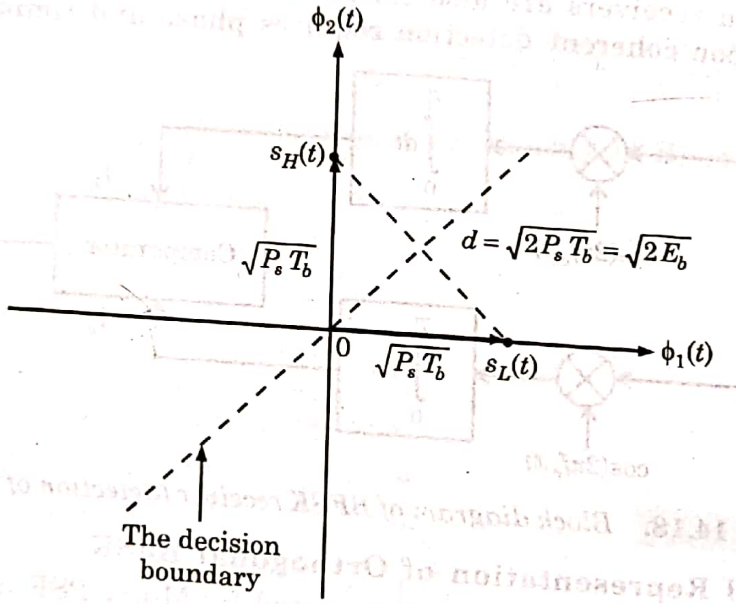


Fig. 14.19. *Illustration of signal space representation of orthogonal BFSK signal.*

14.7.5.1. Distance Between Signal Points

Note that there are two signal points in the signal space. The distance between these two points may be evaluated as under:
$$d^2 = \left(\sqrt{P_s T_b}\right)^2 + \left(\sqrt{P_s T_b}\right)^2$$

or
$$d^2 = 2P_s T_b \quad \text{or} \quad d = \sqrt{2P_s T_b} \quad \text{...(14.48)}$$

Since $P_s T_b = E_b$, we can write above relation (*i.e.*, equation (14.48)) as under:
$$d = \sqrt{2E_b} \quad \text{...(14.49)}$$

As compared to the distance of BPSK, we may observe that this distance is smaller than BPSK.

14.7.6. Geometrical Representation of Non-Orthogonal BFSK Signals

As a matter of fact, whenever the carriers $\phi_1(t)$ and $\phi_2(t)$ are non-orthogonal, then the signal point $S_H(t)$ or $S_L(t)$ would not lie exactly on the axes $\phi_1(t)$ and $\phi_2(t)$. Such a representation has been shown in figure 14.20.

The distance 'd' for non-orthogonal signal shown in figure 14.20 may be given approximately as,

$$d^2 = 2E_b\left[1 - \frac{\sin 2\pi(f_H - f_L)T_b}{2\pi(f_H - f_L)T_b}\right] \quad ...(14.50)$$

14.7.7. Probability of Error, P_e of BFSK Signals

The generated equation derived using union bound approximation gives probability of error as under:

$$P(e) = \frac{1}{2}\sum_{\substack{k=1 \\ k\neq i}}^{M} erfc\left(\frac{d_{ik}}{2\sqrt{N_0}}\right) \quad \text{for all } i \quad ...(14.50a)$$

Here, we have only two points in BFSK. Let $S_H(t)$ be $S_1(t)$ and $S_L(t)$ be $S_2(t)$. Then, distance between these two points can be written as,

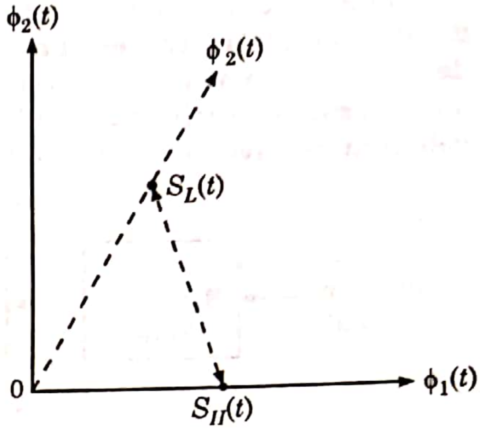


Fig. 14.20. *Geometrical representation of non-orthogonal BFSK signals.*

$$d = d_{12} = \sqrt{2E_b}$$

Since there are only two points (*i.e.*, $M = 2$) take $i = 2$ and $k = 1$, then equation (14.50*a*) becomes,

$$P(e) = \frac{1}{2} erfc\left(\frac{d_{12}}{2\sqrt{N_0}}\right)$$

or

$$P(e) = \frac{1}{2} erfc\left(\frac{\sqrt{2E_b}}{2\sqrt{N_0}}\right)$$

or

$$\boxed{P(e) = \frac{1}{2} erfc\sqrt{\frac{E_b}{2N_0}}}$$

This equation shows that error probability of BFSK is higher compared to that of BPSK.

14.7.8. Advantages and Disadvantages of BFSK Signals

Even though the generation of BFSK is easier, it has many disadvantages compared to BPSK signal. Firstly, its bandwidth is greater than $4f_b$, which is almost double the bandwidth of BPSK. Also, the distance between the signal points is less in case of BFSK. Therefore, the error rate of BFSK is more compared to BPSK.

14.8. Non-Coherent Binary Modulation Techniques

As discussed earlier, coherent detection exploits knowledge of the carrier wave's phase reference, and thus providing the optimum error performance attainable with a digital modulation format of interest. However, when it is impractical to have knowledge of the carrier phase at the receiver, we make use of **non-coherent detection**. Thus, in this section, we shall study non-coherent binary modulation techniques *i.e.*, we shall study non-coherent detection of ASK and FSK. In the case of phase-shift keying (PSK), we cannot have "non-coherent PSK" since non-coherent means doing without phase information. However, there is a 'pseudo PSK' technique known as differential phase-shift keying (DPSK) which can be viewed as the non-coherent form of PSK.

14.9. Non-Coherent Binary Amplitude Shift Keying (ASK)

In the binary ASK case, the transmitted signal is defined as

$$s(t) = \sqrt{2P_s}\cos(2\pi f_c t)$$

Binary ASK signal can also be demodulated non-coherently using envelope detector. This greatly simplifies the design consideration required in synchronous detection. Non-coherent detection schemes do not require a phase-coherent local oscillator. This method involves some form of rectification and low pass filtering at the receiver. The block diagram of a non-coherent receiver for ASK signal has been shown in figure 14.21.

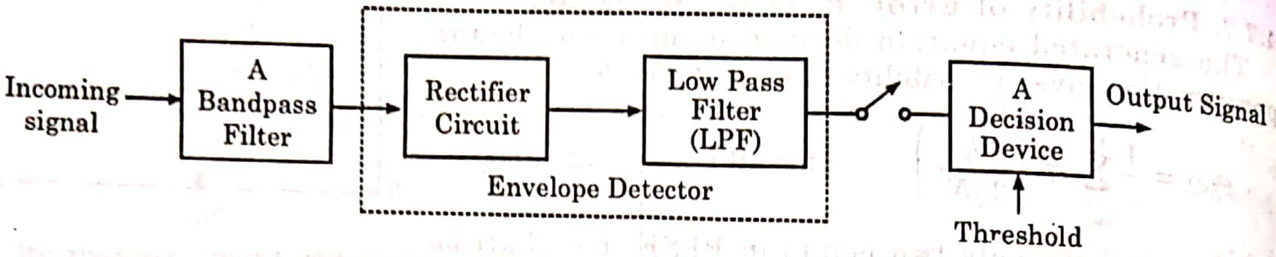


Fig. 14.21. *Non-coherent ASK detector.*

14.10. Non-Coherent Detection of FSK

Binary FSK waves may be demodulated non-coherently using envelope detector. The received FSK signal is applied to a bank of two bandpass filters, one tuned to frequency f_{c1} and the other tuned to f_{c2}. Each filter is followed by an envelope detector. The resulting outputs of the two envelope detectors are sampled and then compared with each other. The arrangement for non-coherent detection of FSK signal has been shown in figure 14.22.

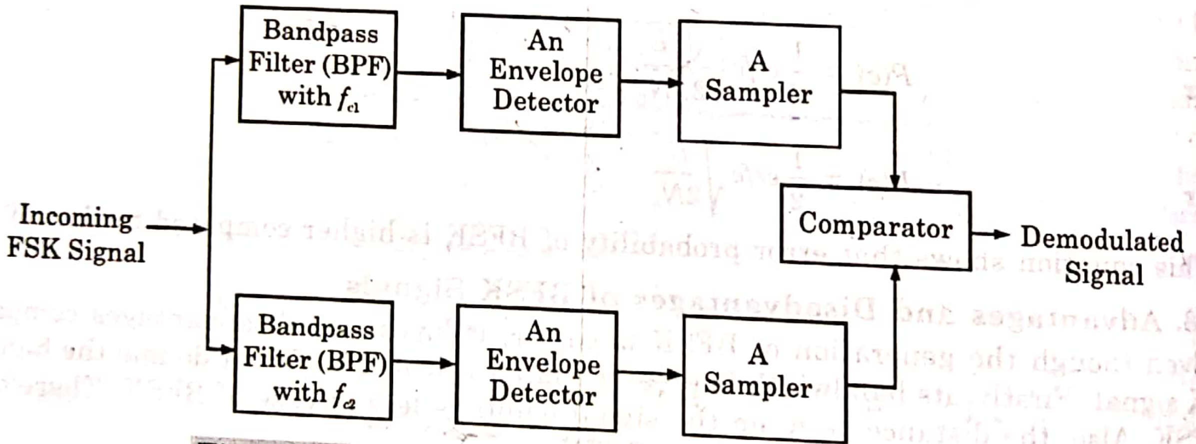


Fig. 14.22. *Non-coherent detection of FSK binary signals.*

A decision is made in favour of symbol '1' if the envelope detector output derived from the filter tuned to frequency f_{c1} is larger than that derived from the second filter. Otherwise, a decision is made in favour of the symbol 0.

14.11. Differential Phase Shift Keying (DPSK)

(U.P. Tech., Semester, Examination, 2003-2004)

We can view differential phase-shift keying as the non-coherent version of the PSK. Differential phase shift keying (DPSK) is differentially coherent modulation method. DPSK does not need a synchronous (coherent) carrier at the demodulator. The input sequence of binary bits is modified such that the next bit depends upon the previous bit. Therefore, in the receiver, the previous received bits are used to detect the present bit.

14.11.1. Generation of DPSK

Thus, in order to eliminate the need for phase synchronisation of coherent receiver with PSK, a differential encoding system can be used with PSK. The digital information content of the binary data is encoded in terms of signal transitions. As an example, the symbol 0 may be used to represent transition in a given binary sequence (with respect to the previous encoded bit) and symbol '1' to indicate no transition. This new signaling technique which combines differential encoding with phase-shift keying (PSK) is known as *differential phase-shift keying (DPSK).*

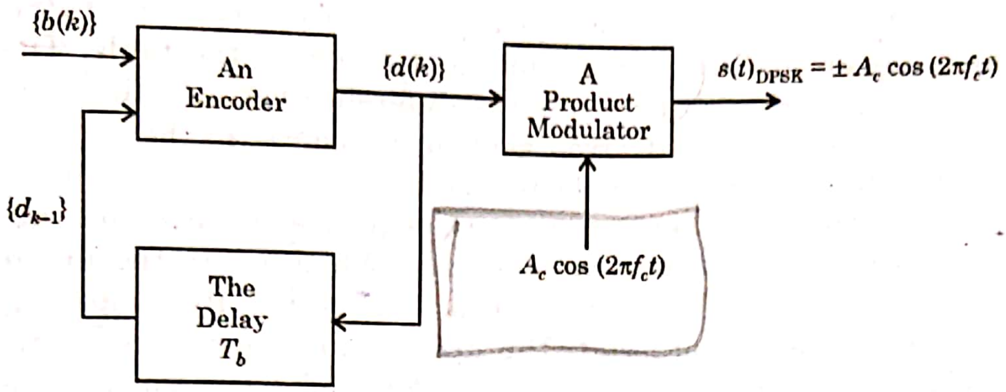


Fig. 14.23. *Illustration of the scheme to generate DPSK signals.*

A schematic arrangement for generating DPSK signal has been shown in figure 14.23. The data stream $b(t)$ is applied to the input of the encoder. The output of the encoder is applied to one input of the product modulator. To the other input of this product modulator, a sinusoidal carrier of fixed amplitude and frequency is applied. The relationship between the binary sequence and its differentially encoded version is illustrated in Table 14.2 for a assumed data sequence 0 0 1 0 0 1 0 0 1 1. In this illustration it has been assumed that the encoding has been done in such a way that *transition* in the given bianry sequence with respect to the previous encoded bit is represented by a symbol 0 and *no transition* by symbol '1'. It may be noted that an extra bit (symbol 1) has been arbitrarily added as an initial bit. This is essential to determine the encoded sequence. The phase of the generated DPSK signal has been shown in the third row of Table 14.2.

Table 14.2. **Differentially encoded sequences with phase.**

Binary data $\{b(k)\}$		0	0	1	0	0	1	0	0	1	1
Differentially encoded data $\{d(k)\}$	1*	0	1	1	0	1	1	0	1	1	1
Phase of DPSK	0	π	0	0	π	0	0	π	0	0	0
Shifted differentially encoded data $\{d_{k-1}\}$		1	0	1	1	0	1	1	0	1	1
Phase of shifted DPSK		0	π	0	0	π	0	0	π	0	0
Phase comparison output		–	–	+	–	–	+	–	–	+	+
Detected binary sequence		0	0	1	0	0	1	0	0	1	1

* Arbitrary starting reference bit.

14.11.2. Detection of DPSK

For detection of the differentially encoded PSK (*i.e.*, DPSK), we can use the receiver arrangement as shown in figure 14.24. The received DPSK signal is applied to one input of the multiplier. To the other of the multiplier, a delayed version of the received DPSK signal by the time interval T_b is applied. The delayed version of the received DPSK signal (in the absence of channel noise) has been shown in the 4th row of the table. The output of the difference is proportional to cos (ϕ), here ϕ is the difference between the carrier phase angle of the received DPSK signal and its delayed version, measured in the same bit interval. The phase angle of the DPSK signal and its delayed version have been shown in 3rd and 5th rows respectively. The phase difference between the two sequences for each bit interval is used to determine the sign of the phase comparator output. When $\phi = 0$, the integrator output is positive whereas when $\phi = \pi$, the integrator output is negative. By comparing the integrator output with a decision level of zero volt, the decision device can reconstruct the binary sequence by assigning a symbol '0' for negative output and a symbol '1' for positive output. The reconstructed binary data is shown in the last row of the table. It is thus seen that in the absence of noise, the receiver can reconstruct the transmitted bianry data exactly. *DPSK may be viewed as a non-coherent version of PSK. It may also be noted that the reconstruction is invariant with the choice of the initial bit in the encoded data. This has been illustrated in the example 14.1 given below.*

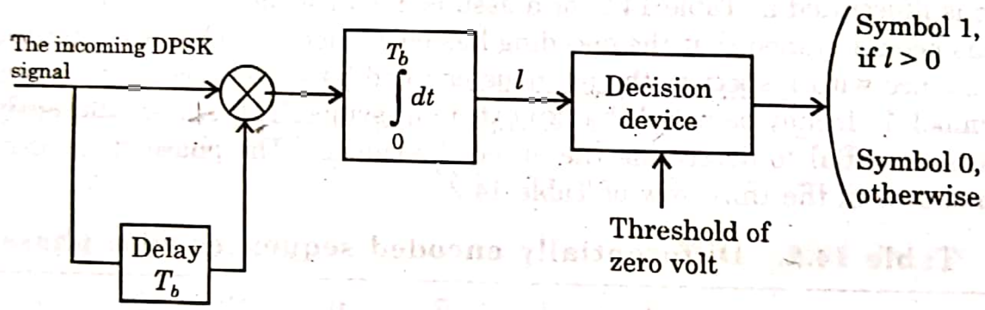


Fig. 14.24. *Receiver for the detection of DPSK signals.*

Example 14.1. A binary data strem 0 0 1 0 0 1 0 0 1 1 needs to be transmitted using DPSK technique. Prove that the reconstruction of the DPSK signal by the technique discussed in the previous article is independent of the choice of the extra bit.

Solution: In the last article, we have observed that DPSK signal can be detected accurately (in the absence of channel noise) without having a local oscillator for generation of synchronous carrier. The initial bit in the differentially encoded data was assumed to be '1'. In this example, we use the initial bit to be symbol '0' and verify that the reconstruction is invariant with the choice of the initial bit. The results obtained for this case are given in Table 14.3. It can be easily verified that the extra chosen bit 0 changes the phase of the DPSK sequence but the detected sequence remains invariant.

Scanned by CamScanner

Table 14.3. Differentially encoded sequences with phase.

Binary data $\{b(k)\}$		0	0	1	0	0	1	0	0	1	1
Differentially encoded data $\{d(k)\}$	0*	1	0	0	1	0	0	1	0	0	0
Phase of DPSK	π	0	π	π	0	π	π	0	π	π	π
Shifted differentially encoded data $\{d_{k-1}\}$		0	1	0	0	1	0	0	1	0	0
Phase of shifted DPSK		π	0	π	π	0	π	π	0	π	π
Phase comparison output		−	−	+	−	−	+	−	−	+	+
Detected binary sequence		0	0	1	0	0	1	0	0	1	1

* Starting reference bit.

14.11.3. Evaluation of Bandwidth of DPSK Signal

As discussed earlier that one previous bit is used to decide the phase shift of next bit. Thus, change in $b(t)$ occurs only if input bit is at level '1'. No change happens if input bit is at level '0'. Because, one previous bit is always used to define the phase shift in next bit, therefore, the symbol can be said to have two bits. Hence, one symbol duration (T) is equivalent to two bits duration ($2T_b$) *i.e.*, ...(14.51)

Symbol duration $T_s = 2T_b$

Bandwidth is expressed as

$$BW = \frac{2}{T_s} = \frac{1}{T_b} = f_b \qquad \text{...(14.52)}$$

Hence, the minimum bandwidth in DPSK is equal to f_b, *i.e.*, maximum baseband signal frequency.

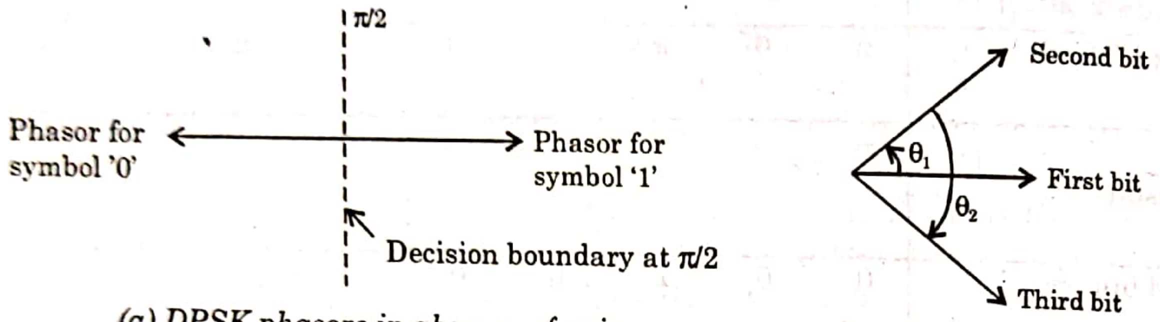
14.11.4. Error Probability of DPSK

Figure 14.25(a) shows the phasor diagram of DPSK signal when no noise is present. This means that in the absence of noise and transmission delay, the phase shift of the DPSK signal is either '0' or 'π'. Therefore, a decision boundary is drawn at $\frac{\pi}{2}$ as shown in figure 14.25(a). Therefore, we consider that the transmitted symbol is '1', if the phase difference between two consecutive bits differs by less than $\frac{\pi}{2}$. If the pahse difference between two consecutive bits differs by more than $\frac{\pi}{2}$, then decision is taken in favour of zero.

Figure 14.25(b) shows three consecutive bits. The first bit signal contains no noise, hence its phasor is along the horizontal line. Therefore, the symbol transmitted in first bit is assumed to be '1'. Because of noise, there is a phase difference of 'θ_1' between first and second bit. Since $\theta_1 < \frac{\pi}{2}$, second bit is also taken as symbol '1'. The phase difference between second and third bit is θ_2. From figure, it is clear that $\theta_2 > \frac{\pi}{2}$, hence third bit is taken as symbol zero. Since the phase differences are not exact between two successive bits, some error is introduced in the decision. Therefore, DPSK system is called 'sub-optimum' in nature. If the synchronization is used to make phase diferences exact, then it becomes PSK system.

Because of the sub-optimum nature of DPSK, the error probability is higher than that of BPSK. The average probability of error of non-coherent receiver is given as,

$$P(e) = \frac{1}{2} e^{-E/2N_0}$$



(a) DPSK phasors in absence of noise   *(b) Three consecutive bits*

Fig. 14.25.

Here, E is the energy per symbol and $\frac{N_0}{2}$ is spectral density of white Gaussian noise. We know that symbol duration $T = 2T_b$. Hence, energy of the symbol will be

$$E = 2E_b$$

Hence, expression for $P(e)$ becomes,

$$P(e) = \frac{1}{2} e^{-E_b/N_0}$$

This is called average probability of error or bit error rate (BER) of DPSK system.

14.11.5. Advantages and Disadvantages of DPSK

We have observed in above discussion that DPSK has some advantages over BPSK, however, at the same time, it has some drawbacks.

(i) Advantages

(i) DPSK does not need carrier at the receiver end. This means that the complicated circuitry for generation of local carrier is not required.

(ii) The bandwidth requirement of DPSK is reduced as compared to that of BPSK.

(ii) Disadvantages

(i) The probability or error (*i.e.*, bit error rate) of DPSK is higher than that of BPSK.

(ii) Because DPSK uses two successive bits for its reception, error in the first bit creates error in the second bit. Therefore, error propagation in DPSK is more. On the other hand, in BPSK single bit can go in error since detection of each bit is independent.

(iii) Noise interference in DPSK is more.

Note: *In DPSK, previous bit is used to detect next bit. Hence, if error is present in previous bit, detection of next bit can also be wrong. Hence, error is created in next bit also. Therefore, there is tendency of appearing errors in pairs in DPSK.*

14.12. Quadrature Phase Shift Keying (QPSK)

As a matter of fact, in communication systems, we have two main resources. These are the transmission power and the channel bandwidth. The channel bandwidth depends upon the bit rate or signaling rate f_b. In digital bandpass transmission, we use a carrier for transmission. This carrier is transmitted over a channel. If two or more bits are combined in some symbols, then the signaling rate will be reduced. Thus, the frequency of the carrier needed is also reduced. This reduces the transmission channel bandwidth. Hence, because of grouping of bits in symbols, the transmission channel bandwidth can be reduced. In quadrature phase shift keying (QPSK), two successive bits in the data sequence are grouped together. This reduces the bits rate or signaling rate (*i.e.*, f_b) and thus reduces the bandwidth of the channel.

In case of BPSK, we know that when symbol changes the level, the phase of the carrier is changed by 180°. Because, there were only two symbols in BPSK, the phase shift occurs in two levels only. However, in QPSK, two successive bits are combined. Infact, this combination of two bits forms four distinct symbols. When the symbol is changed to next symbol, then the phase of the carrier is changed by 45° ($\pi/4$ radians). Table 14.4 shows these symbols and their phase shifts.

Table 14.4. Symbol and corresponding phase shifts in QPSK

S. No.	*Input successive bits*		*Symbol*	*Phase shift in carrier*
$i = 1$	1(1 V)	0 (−1 V)	S_1	$\pi/4$
$i = 2$	0(−1 V)	0(−1 V)	S_2	$3\pi/4$
$i = 3$	0(−1 V)	1(1 V)	S_3	$5\pi/4$
$i = 4$	1(1 V)	1(1 V)	S_4	$7\pi/4$

Hence as shown in Table 14.4, there are four symbols and the phase is shifted by $\pi/4$ for each symbol.

14.13. Generation of QPSK

Figure 14.26 shows the block diagram of QPSK transmitter. Here, the input binary sequence is first converted to a bipolar NRZ type of signal. This signal is denoted by $b(t)$. It represents binary '1' by + 1 V and binary '0' by − 1 V. This signal has been shown in figure 14.27(*a*). The demultiplexer divides $b(t)$ into two separate bit streams of the odd numbered and even numbered bits. Here, $b_e(t)$ represents even numbered sequence and $b_0(t)$ represents odd numbered sequence. The symbol duration of both of these odd and even numbered sequences is $2T_b$. Hence, each symbol consists of two bits. Figure 14.27(*b*) and (*c*) illustrate the waveform of $b_e(t)$ and $b_0(t)$.

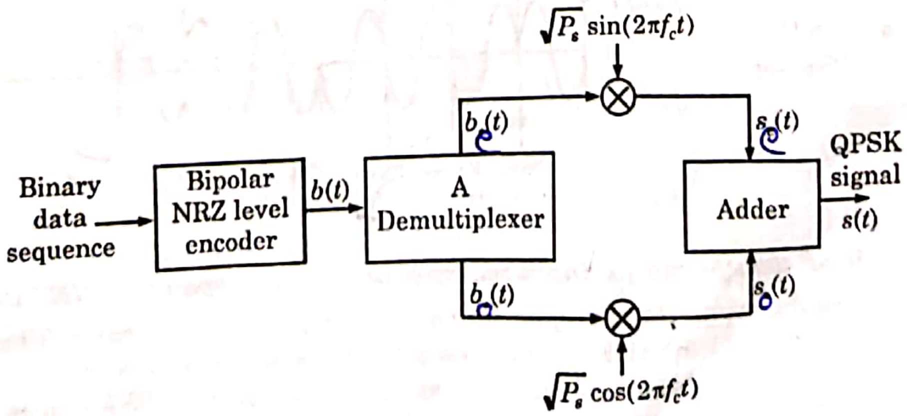


Fig. 14.26. *Generation of QPSK.*

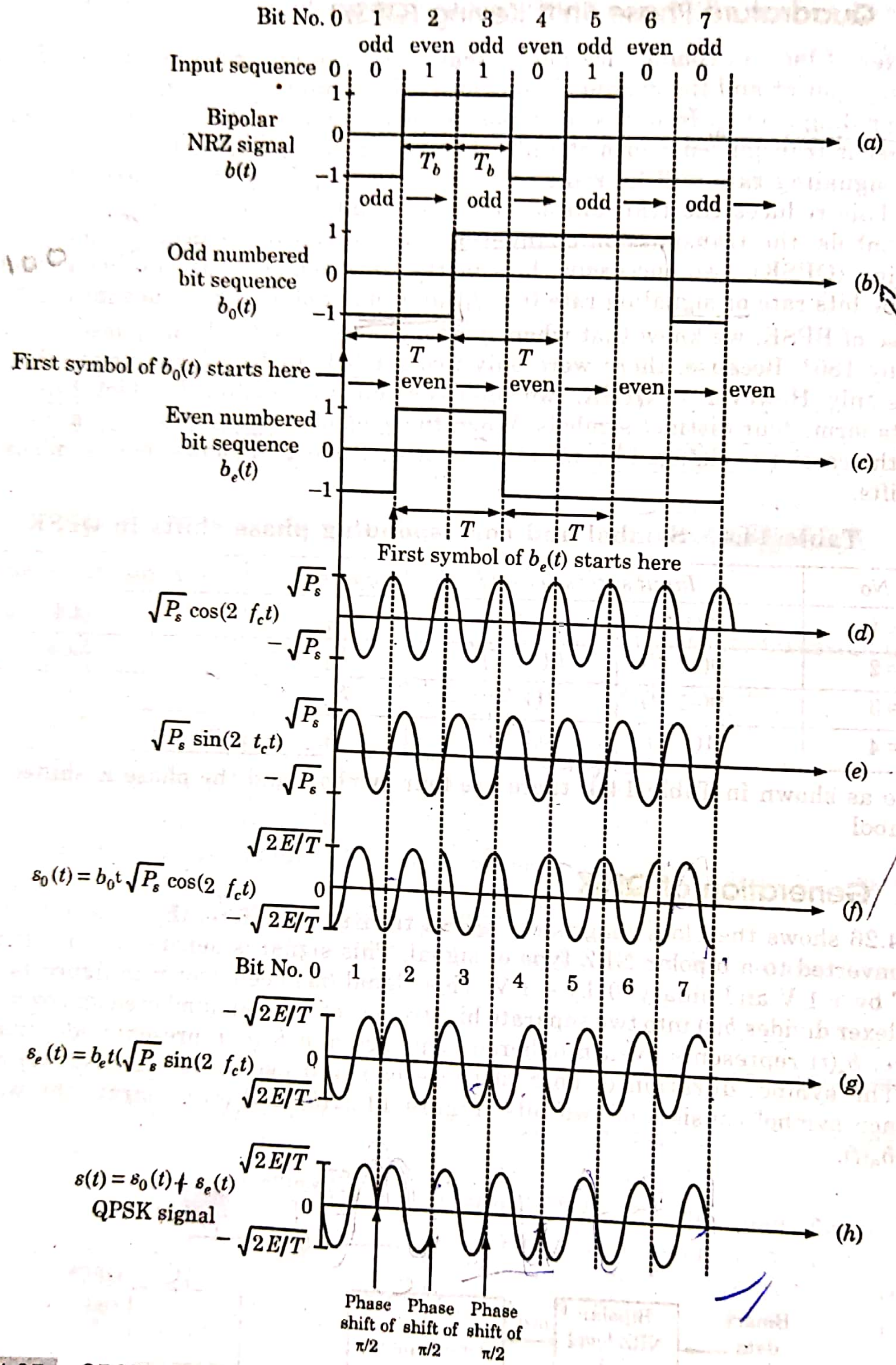


Fig. 14.27. *QPSK waveforms, (a) Input sequence and its corresponding NRZ waveform, (b) Odd numbered bit sequence and its corresponding waveform (c) Even numbered bit sequence and its NRZ waveform (d) Basis function $\phi_1(t)$ (e) Basis function $\phi_2(t)$ (f) Binary PSK waveform for odd numbered channel (g) Binary PSK waveform for even numbered channel (h) Final QPSK waveform.*

It may be observed that the first even bit occurs after the first odd bit. Hence, even numbered bit sequence $b_e(t)$ starts with the delay of one bit period due to first odd bit. Thus, first symbol of $b_e(t)$ is delayed by one bit period 'T_b' with respect to first symbol of $b_0(t)$. This delay of T_b is known as offset. This shows that the change in levels of $b_e(t)$ and $b_0(t)$ cannot occur at the same time due to offset or staggering.

Also, the bit steam $b_e(t)$ modulates carrier $\sqrt{P_s}\cos(2\pi f_c t)$ and $b_0(t)$ modulates $\sqrt{P_s}\sin(2\pi f_c t)$. These modulators are the balanced modulators. The two carriers $\sqrt{P_s}\cos(2\pi f_c t)$ and $\sqrt{P_s}\sin(2\pi f_c t)$ have been shown in figure 14.27(*d*) and (*e*). There carriers are also known as **quadrature carriers.**

The two modulated sigals can be written as,

$$s_e(t) = b_e(t)\sqrt{P_s}\sin(2\pi f_c t) \quad ...(14.53)$$

and

$$s_0(t) = b_0(t)\sqrt{P_s}\cos(2\pi f_c t) \quad ...(14.54)$$

Hence, $s_e(t)$ and $s_0(t)$ are basically BPSK signals. The only difference is that $T = 2T_b$ here. The value of $b_e(t)$ and $b_0(t)$ would be $+1V$ or $-1V$. Figure 14.27 (*f*) and (*g*) shows the waveforms of $s_e(t)$ and $s_0(t)$. The adder in figure 14.26 adds these two signals $b_e(t)$ and $b_0(t)$.

The output of the adder is QPSK signal and it is given by,

$$s(t) = s_0(t) + s_e(t)$$

or

$$s(t) = b_0(t)\sqrt{P_s}\cos(2\pi f_c t) + b_e(t)\sqrt{P_s}\sin(2\pi f_c t) \quad ...(14.55)$$

Figure 14.27(*h*) shows the QPSK signal represented by equation (14.55). In QPSK signal in figure 14.27(*h*), if there is any phase change, it occurs at minimum duration of T_b. This is because the two signals $s_e(t)$ and $s_0(t)$ have an offset of 'T_b'. Due to this offset, the phase shift in QPSK signal is $\frac{\pi}{2}$.

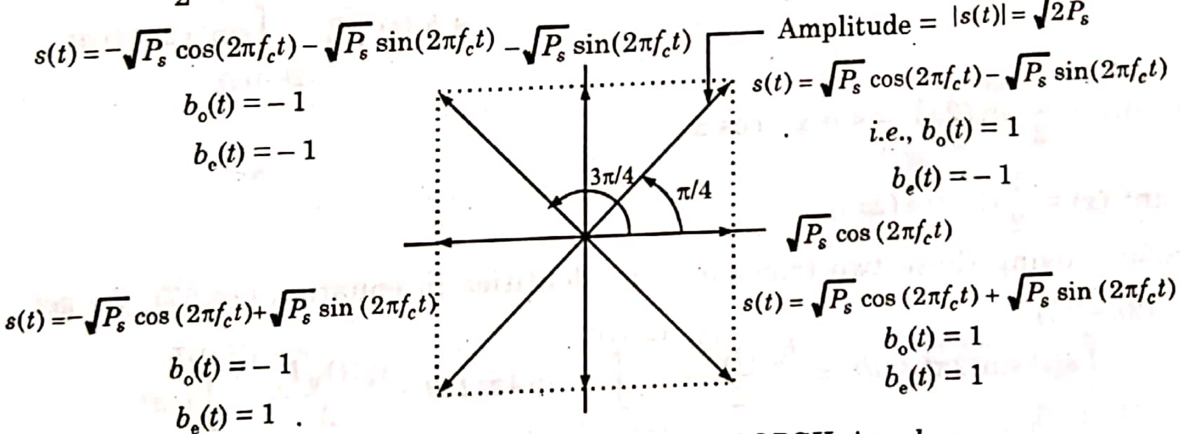


Fig. 14.28. *The Phasor diagram of QPSK signal.*

14.13.1. Reception of QPSK (*i.e.* Detection of QPSK)

Figure 14.29 shows the QPSK receiver. This is synchronous reception. Hence, the coherent carrier is to be recovered from the received signal $s(t)$. The received signal $s(t)$ is first raised to its 4th power, *i.e.*, $s^4(t)$. After that, it is allowed to pass through a bandpass filter (BPF) which is centred around $4f_c$. The output of the bandpass filter is a coherent carrier of frequency $4f_c$. This is divided by 4 and it provides two coherent quadrature carriers, *i.e.*, $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$. These coherent carriers are applied to two synchronous demodulators. These synchronous demodulators consist of multiplier and an integrator.

The incoming signal is applied to both the multipliers. Here, the integrator integrates the product signal over two bit interval (*i.e.*, $T_s = 2T_b$). At the end of this period, the output of integrator is sampled. The outputs of the two integrators are sampled at the offset of one bit period, T_b. Hence, the output of the multiplexer is the signal $b(t)$. This means that the odd and even sequences are combined by multiplexer.

Now, let us consider the product signal at the output of upper multiplier, *i.e.*,

$$s(t)\sin(2\pi f_c t) = b_0(t)\sqrt{P_s}\cos(2\pi f_c t)\sin(2\pi f_c t) + b_e(t)\sqrt{P_s}\sin^2(2\pi f_c t) \quad ...(14.56)$$

This signal is integrated by the upper integrator in figure 14.29.

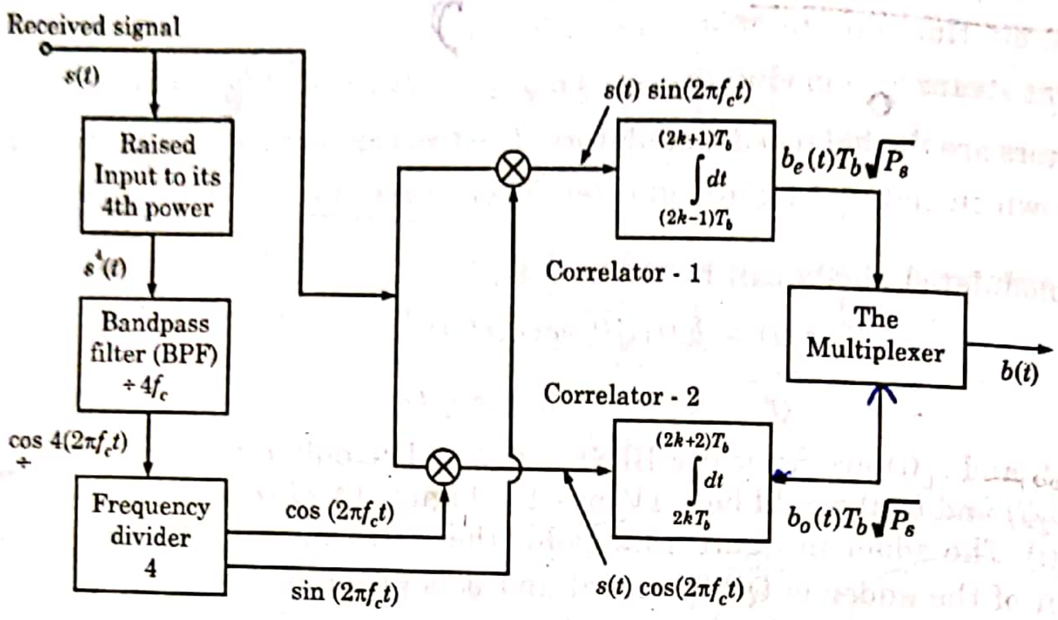


Fig. 14.29. *Reception of QPSK.*

Therefore, we have

$$\int_{(2k-1)T_b}^{(2k+1)T_b} s(t)\sin(2\pi f_c t)\,dt = b_0(t)\sqrt{P_s}\int_{(2k-1)T_b}^{(2k+1)T_b}\cos(2\pi f_c t)\sin(2\pi f_c t)\,dt$$

$$+ b_e(t)\sqrt{P_s}\int_{(2k-1)T_b}^{(2k+1)T_b}\sin^2(2\pi f_c t)\,dt \quad ...(14.57)$$

Now, since $\frac{1}{2}\sin(2x) = \sin x \cdot \cos x$

and $\sin^2(x) = \frac{1}{2}[1-\cos(2x)]$

Therefore, using these two trigonometric identities in equation (14.57), we get

$$\int_{(2k-1)T_b}^{(2k+1)T_b} s(t)\sin(2\pi f_c t)\,dt = \frac{b_o(t)\sqrt{P_s}}{2}\int_{(2k-1)T_b}^{(2k+1)T_b}\sin 4\pi f_c t\,dt + \frac{b_e(t)\sqrt{P_s}}{2}\int_{(2k-1)T_b}^{(2k+1)T_b} 1\cdot dt$$

$$-\frac{b_e(t)\sqrt{P_s}}{2}\int_{(2k-1)T_b}^{(2k+1)T_b}\cos 4\pi f_c t\,dt$$

In this equation, the first and third integration terms involve integration of sinusoidal carriers over two bit period. They have full (integral number of) cycles over two bit periods and thus integration will be zero, *i.e.*,

$$\int_{(2k-1)T_b}^{(2k+1)T_b} s(t)\sin(2\pi f_c t)\,dt = \frac{b_e(t)\sqrt{P_s}}{2}[t]_{(2k-1)T_b}^{(2k+1)T_b}$$

$$= \frac{b_e(t)\sqrt{P_s}}{2}\times 2T_b = b_e(t)\sqrt{P_s}\,T_b \quad ...(14.58)$$

Hence, the upper integrator responds to even sequence only. Similarly, we can obtain the output of lower integrator as $b_0(t)\sqrt{P_s T_b}$.

Note: *Even though bit synchronizer has not been shown in figure 14.29, it is assumed to be present with the integrator to locate starting and ending times of integration. The multiplexer is also operated by bit synchronizer. The amplitudes of voltage marked in figure 14.29 are arbitrary. They can change depending upon the gains of the integrator.*

14.13.2. Concept of Carrier Synchronization in QPSK

Both the carriers are to be synchronized properly in coherent detection in QPSK. Figure 14.30 shows the PLL system for carrier synchronization in QPSK.

The fourth power of the input signal consists of discrete frequency component at $4f_c$.

We know that,

$$\cos 4(2\pi f_c t) = \cos(8\pi f_c t + 2\pi N)$$

where 'N' is the number of cycles over the bit period. It is always an integer value. When the frequency division by four takes place, the RHS of this equation becomes $\cos\left(2\pi f_c t + \frac{N\pi}{2}\right)$.

This indicates that the output has a fixed phase error of $\frac{N\pi}{2}$. Differential encoding can be used to nullify the phase error events. The PLL remains locked with the phase of '$4f_c$' and then output of PLL is divided by 4. This provides a coherent carrier. A 90° phase shift is added to this carrier to produce a quadrature carrier.

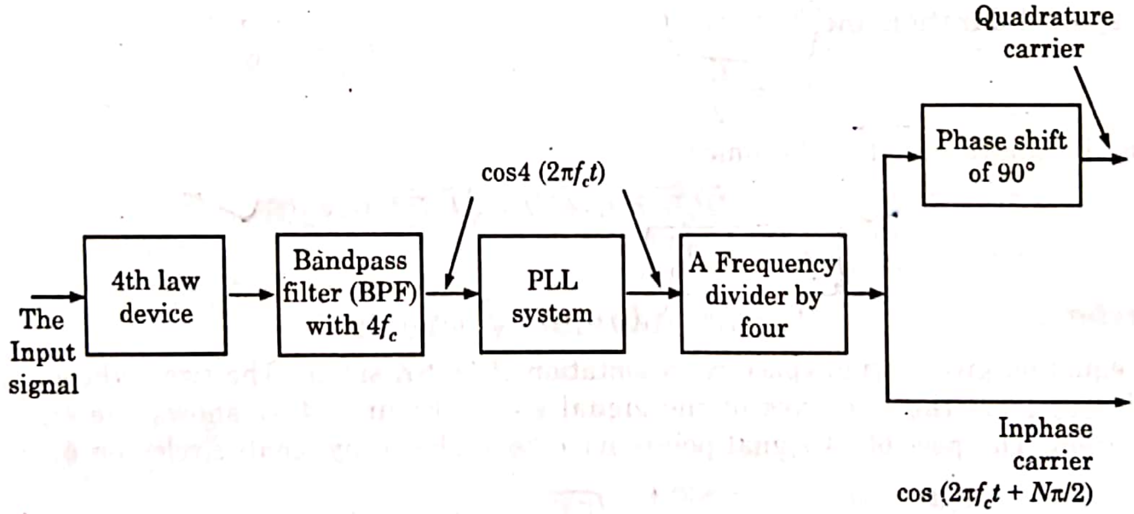


Fig. 14.30. *PLL system used for carrier synchronization in QPSK.*

14.13.3. Signal Space Representation in QPSK Signals

Figure 14.31 shows the phasor diagram of QPSK signal. Depending upon the combination of two successive bits, the phase shift occurs in carrier. This means that the QPSK signal in equation (14.55) can be written as,

$$s(t) = \sqrt{2P_s}\cos\left[2\pi f_c t + (2m+1)\frac{\pi}{4}\right] \qquad m = 0, 1, 2, 3 \qquad \text{...(14.60)}$$

Here, this equation takes four values and they represent the phasors of figure 14.31. This equation can be expanded as under:

$$s(t) = \sqrt{2P_s}\cos(2\pi f_c t)\cos\left[(2m+1)\frac{\pi}{4}\right] - \sqrt{2P_s}\sin(2\pi f_c t)\sin\left[(2m+1)\frac{\pi}{4}\right]$$

Let us rearrange the above equation as under:

$$s(t) = \left\{\sqrt{P_s T_s}\cos\left[(2m+1)\frac{\pi}{4}\right]\right\}\sqrt{\frac{2}{T_s}}\cos(2\pi f_c t)$$

$$-\left\{\sqrt{P_s T_s}\sin\left[(2m+1)\frac{\pi}{4}\right]\right\}\sqrt{\frac{2}{T_s}}\sin(2\pi f_c t) \quad \text{...(14.61)}$$

Again, let $\phi_1(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi f_c t)$ ...(14.62)

and $\phi_2(t) = \sqrt{\frac{2}{T_s}}\sin(2\pi f_c t)$ ...(14.63)

These two signals are known as orthogonal signals and they are used as carriers in QPSK modulator.

Let $b_0(t) = \sqrt{2}\cos\left[(2m+1)\frac{\pi}{4}\right]$ ...(14.64)

and $b_e(t) = -\sqrt{2}\sin\left[(2m+1)\frac{\pi}{4}\right]$ ...(14.65)

With the use of equations (14.61) to (14.64) we can write equation (14.60) as under:

$$s(t) = \sqrt{P_s T_s}\cdot\frac{1}{\sqrt{2}}b_0(t)\,\phi_1(t) + \sqrt{P_s T_s}\cdot\frac{1}{\sqrt{2}}b_e(t)\,\phi_2(t)$$

or

$$s(t) = \sqrt{P_s\cdot\frac{T_s}{2}}\,b_0(t)\,\phi_1(t) + \sqrt{P_s\frac{T_s}{2}}\,b_e(t)\,\phi_2(t)$$

T_s = symbol duration and $\boxed{T_s = 2T_b}$

or $T_b = \frac{T_s}{2}$ ...(14.66)

Then the above equation becomes,

$$s(t) = \sqrt{P_s T_b}\,b_0(t)\,\phi_1(t) + \sqrt{P_s T_b}\,b_e(t)\,\phi_2(t) \quad \text{...(14.67)}$$

Since bit energy $\boxed{E_b = P_s T_b}$

Therefore, $s(t) = \sqrt{E_b}\,b_0(t)\,\phi_1(t) + \sqrt{E_b}\,b_e(t)\,\phi_2(t)$ ...(14.68)

This equation gives signal space representation of QPSK signal. The two orthogonal signals $\phi_1(t)$ and $\phi_2(t)$ form the two axes of the signal space. Figure 14.31 shows the signal space representation. The possible 4 signal points have been shown by small circles on ϕ_1 ϕ_2-plane.

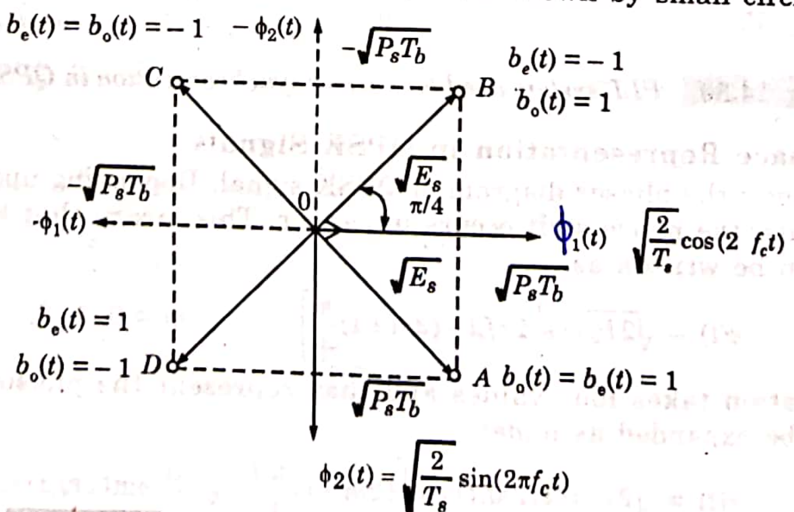


Fig. 14.31. *The signal space representation for QPSK signals.*

From each signal point, we obtain two bits. For example, from point 'A', we obtain two bits as (1, 1) and from 'B' we obtain bits as (– 1, 1). The distance of any signal point from origin 'O', given as,

$$\text{Distance } 'OB' = \sqrt{P_s T_b + P_s T_b} = \sqrt{2 P_s T_b}$$

$$= \sqrt{P_s T_s} \qquad [\because 2T_b = T_s] \qquad ...(14.69)$$

or

$$'OB' = \sqrt{E_s} \qquad \boxed{[\because P_s\, T_s = E_s]} \qquad ...(14.70)$$

Hence, the length of each signal point from origin is $\sqrt{E_s}$. We know that $b_e(t)$ and $b_0(t)$ represent two successive bits. There is an offset of 'T_b' between $b_e(t)$ and $b_o(t)$. Therefore, $b_e(t)$ and $b_o(t)$ both cannot change their levels simultaneously. Hence, either $b_e(t)$ or $b_o(t)$ can change at a time.

Let us say that $b_e(t) = b_o(t) = 1$ is representing signal point 'A' in figure 14.31. In the next bit interval, if $b_o(t) = -1$, then signal point will be 'D'. Otherwise, if $b_e(t)$ changes its level [*i.e.*, $b_e(t) = -1$], then next signal point will be 'B'. Hence, from signal point 'A', then next signal points will be either 'D' or 'B'.

14.13.3.1. *Distance Between Signal Points*

As a matter of fact, the ability to determine a bit without error is measured by the distance between two nearest possible signal points in the signal space. Such points differed in signal bit. For example, signal points 'A' and 'B' are two nearest points since they differ by a signal bit $b_e(t)$. As 'A' and 'B' become closer to each other, the possibility of error increases. Therefore, this distance must be as large as possible. This distance is denoted by 'd'. In figure 14.31, the distance between signal points 'A' and 'B' can be given by,

$$d^2 = \left(\sqrt{E_s}\right)^2 + \left(\sqrt{E_s}\right)^2 = \sqrt{2E_s} \qquad ...(14.71)$$

$E_S = P_S T_S$

$E_b = P_S T_b$

or

$$d = 2\sqrt{P_s T_b} = 2\sqrt{E_b} \qquad ...(14.72)$$

Note: *If we compare this distance with the distance of BPSK signals, then this shows that the distance for QPSK is the same as that for BPSK. Because, this distance represents noise immunity of the system, it shows that noise immunities of BPSK and QPSK are same.*

14.13.4. Spectrum of QPSK Signal

The input sequence $b(t)$ is of bit duration T_b. Also, it is a NRZ bipolar waveform. Recall, the power spectral density of such waveform can be given as,

$$S(f) = V_b^2\, T_b \left[\frac{\sin(\pi f\, T_b)}{\pi f\, T_b}\right]^2$$

Also, $V_b = \sqrt{P_s}$, then this equation becomes,

$$S(f) = P_s T_b \left[\frac{\sin(\pi f\, T_b)}{\pi f\, T_b}\right]^2 \qquad ...(14.73)$$

This equation gives power spectral density (psd) of signal $b(t)$. This signal is divided into $b_e(t)$ and $b_0(t)$ each of bit period $2T_b$. If we consider that symbols 1 and 0 are equally likely, then we can write power spectral densities (psds) of $b_e(t)$ and $b_0(t)$ as,

$$S_e(f) = P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2 \qquad ...(14.74)$$

and $$S_0(f) = P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2 \quad ...(14.75)$$

In these two equations, we have just replaced T_b by T_s and T_s is the period of bit in $b_e(t)$ and $b_0(t)$. Because, inphase and quadrature components $[b_e(t)$ and $b_0(t)]$ are statistically independent, the baseband power spectral density of QPSK signal equals the sum of the individual power spectral densities of $b_e(t)$ and $b_0(t)$ *i.e.*,

$$S_B(f) = S_e(f) + S_0(f)$$

or $$S_B(f) = 2P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2 \quad ...(14.76)$$

This equation gives baseband power spectral density of QPSK signal. Upon modulation of carrier of frequency f_c, the spectral density given by above equation is shifted at $\pm f_c$. Thus plots of power spectral density of QPSK will be similar to that BPSK.

14.13.5. Bandwidth of QPSK Signal

We have observed that the bandwidth of BPSK signal is equal of $2f_b$. Here, $T_b = \frac{1}{f_b}$ is the one bit period. In QPSK, the two waveforms $b_e(t)$ and $b_0(t)$ form the baseband signals. One bit period for both of these signals is equal to $2T_b$. Therefore, bandwidth of QPSK signal will be

$$BW = 2 \times \frac{1}{2T_b} = f_b \quad ...(14.77)$$

Hence, the bandwidth of QPSK signal is half of the bandwidth of BPSK signal. Earlier, we have observed that noise immunity of QPSK and BPSK is same. This shows that inspite of the reduction in bandwidth in QPSK, the noise immunity remains same as compared to BPSK. *BW* of QPSK can also be obtained by plotting equation (14.71) as shown in figure 14.32 below.

$$BW = \text{Highest frequency} - \text{Lowest frequency in main lobe}$$

$$BW = \frac{1}{T_s} - \left(-\frac{1}{T_s}\right) \text{ since carrier frequency } f_c \text{ cancels our}$$

$$BW = \frac{2}{T_s}$$

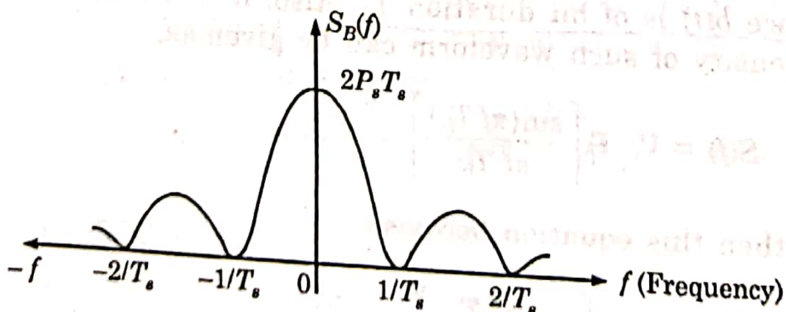


Fig. 14.32. *Plot of power spectral density (psd) of QPSK signal.*

We know that $$T_s = 2T_b$$

or $$BW = \frac{2}{2T_b} = \frac{1}{T_b}$$

or $$BW = f_b \quad ...(14.78)$$

14.13.6. Probability of Error of QPSK System

Observe figure 14.31 carefully. Between signal phasors 'OA' and 'OB', the axis $\phi_1(t)$ can be called decision boundary. Let us say signal vector 'OB' is present, but because of imperfect phase synchronization, it is detected as 'OA'. For this to happen, the phase shift must be at least $\frac{\pi}{4}$. The same thing can happen in case of other phasors also.

There are two correlators in the QPSK receiver. One correlator is used to detect even bits and other detects odd bits. Thus, any correlator can make a mistake if phase shift of $\frac{\pi}{4}$ occurs in the corresponding carrier. Therefore, the probability that correlator 1 or correlator 2 will make a mistake is given as,

$$P'_1(e) = P'_2(e) = \frac{1}{2}\, erfc \sqrt{\frac{E\cos^2\theta}{N_0}}$$

This equation has been written because each correlator is independent PSK receiver. 'E_b' is replaced by 'E' in above equation, since energy of one bit in $b_e(t)$ or $b_0(t)$ is E (also written as E_s). Putting phase shift $\theta = \frac{\pi}{4}$ in above equation we obtain,

$$P'_1(e) = P'_2(e) = \frac{1}{2}\, erfc \sqrt{\frac{E\cos^2\frac{\pi}{4}}{N_0}}$$

or $$P'_1(e) = \frac{1}{2}\, erfc \sqrt{\frac{E}{2N_0}}$$

We can verify the above relation for BPSK as under:

We know that $E = E_s = P_s\, T_s$

and $T_s = 2\, T_b$

Therefore, $E = P_s\, 2T_b = 2P_s\, T_b$

$E_b = P_s\, T_b$

$E = 2E_b$

By using the above relation, we obtain error probability of BPSK *i.e.*,

$$P'_1(e) = P'_2(e) = \frac{1}{2}\, erfc \sqrt{\frac{2E_b}{2N_0}}$$

or $$P'_1(e) = \frac{1}{2}\, erfc \sqrt{\frac{E_b}{N_0}}$$

The above equation gives bit error probability of QPSK. Thus, bit error probability of QPSK and BPSK is same.

The probability $P(e)$ that the QPSK receiver will correctly detect the transmitted signal is equal to product of probabilities that both correlator 1 and correlator 2 will receive their bits correctly. The probability of correct reception of correlators 1 and 2 is,

$$P_1(e) = 1 - P'_1(e) \quad \text{and} \quad P_2(e) = 1 - P'_2(e)$$

$$\therefore \quad P(e) = P'_1(e) \times P'_2(e)$$
$$= [1 - P'_1(e)] \times [1 - P'_2(e)]$$
$$= 1 - P'_1(e) - P'_2(e) + P'_1(e) \times P'_2(e)$$

$\therefore \quad P'_1(e) = P'_2(e)$ we can write above equation as,

$$P(e) = 1 - 2P'(e) + P'^2(e)$$

Here, $\quad P'_1(e) = P'_2(e) = P'^2(e).$

The term $P'^2(e)$ will be very very small and can be neglected. Hence,

$$P(e) = 1 - 2\,P'(e)$$

$\therefore$ Probability of error of QPSK system is,

$$P(e) = 1 - P'(e)$$

or $$P(e) = 1 - 1 + 2\,P'(e)$$

or $$P(e) = 2\,P'(e)$$

Putting value of $P'(e) = \frac{1}{2}\,erfc\sqrt{\frac{E}{2N_0}}$, we get

$$\boxed{P(e) = erfc\sqrt{\frac{E}{2N_0}} = erfc\sqrt{\frac{E_b}{N_0}}} \quad (\text{since } E = 2E_b)$$

The individual probabilities $P'_1(e)$ and $P'_2(e)$ correlators are some times called as bit error probabilities or Bit Error Rate (BER). Thus bit error rate of QPSK is given as,

$$BER = \frac{1}{2}\,erfc\sqrt{\frac{E_b}{N_0}}$$

14.13.7. Advantages of QPSK

QPSK has some certain advantages as compared to BPSK and DPSK as under:

(i) For the same bit error rate, the bandwidth required by QPSK is reduced to half as compared to BPSK.

(ii) Because of reduced bandwidth, the information transmission rate of QPSK is higher.

14.14. Minimum Shift Keying (MSK)

(U.P. Tech., Sem., Examination 2003-2004)

We have discussed QPSK technique in last article. The bandwidth requirement of QPSK is high. Filters or other methods can overcome these problems, but they have other side effects. For example, filters alter the amplitude of the waveform.

MSK overcomes these problems. In MSK, the output waveform is continuous in phase hence there are no abrupt changes in amplitude. The sidelobes of MSK are very small hence bandpass filtering is not required to avoid interchannel interference. Figure 14.33 shows the waveform of MSK. The binary bit sequence is shown at the top. Figure 14.33(*a*) shows the corresponding NRZ waveform $b(t)$. From $b(t)$, two waveforms are generated for odd and even bits. $b_0(t)$ represents odd bits and $b_e(t)$ represents even bits. Figure 14.33(*b*) and (*c*) shows the waveform of $b_o(t)$ and $b_e(t)$. As shown in those waveforms b_1, b_3, b_5 etc. are represented by odd waveform *i.e.*, $b_o(t)$.

The duration of each bit in $b_o(t)$ or $b_e(t)$ is $2T_b$, whereas it is T_b in $b(t)$ *i.e.*,

$$T_s = 2T_b \qquad ...(14.79)$$

The waveforms $b_o(t)$ and $b_e(t)$ have an offset of T_b. This offset is essential in MSK. Two waveforms sin $2\pi(t/4T_b)$ and cos $2\pi(t/4T_b)$ are generated as shown in figure 14.33(*d*). The waveform of sin $2\pi(t4T_b)$ passes through zero at the end of symbol time in $b_o(t)$. Hence, one symbol duration of $b_o(t)$ consists of complete half cycle of cos $2\pi(t/4T_b)$. This means that similarly, one symbol duration of $b_e(t)$ contains half cycle of sin $2\pi(t/T_b)$. Thus there is a phase shift of 'T_b' in sine and cosine waveforms. $b_e(t)$ is multiplied by sin $2\pi(t/4T_b)$ and $b_o(t)$ is multiplied by cos $2\pi(t/4T_b)$. These product waveforms are shown in figure 14.33 (*e*) and (*f*). The transmitted MSK signal is represented as under:

$$s(t) = \sqrt{2P_s}[b_e(t)\sin(2\pi t/4T_b)]\cos(2\pi f_c t) + \sqrt{2P_s}[b_0(t)\cos(2\pi t/4T_b)]\sin(2\pi f_c t) \quad ...(14.80)$$

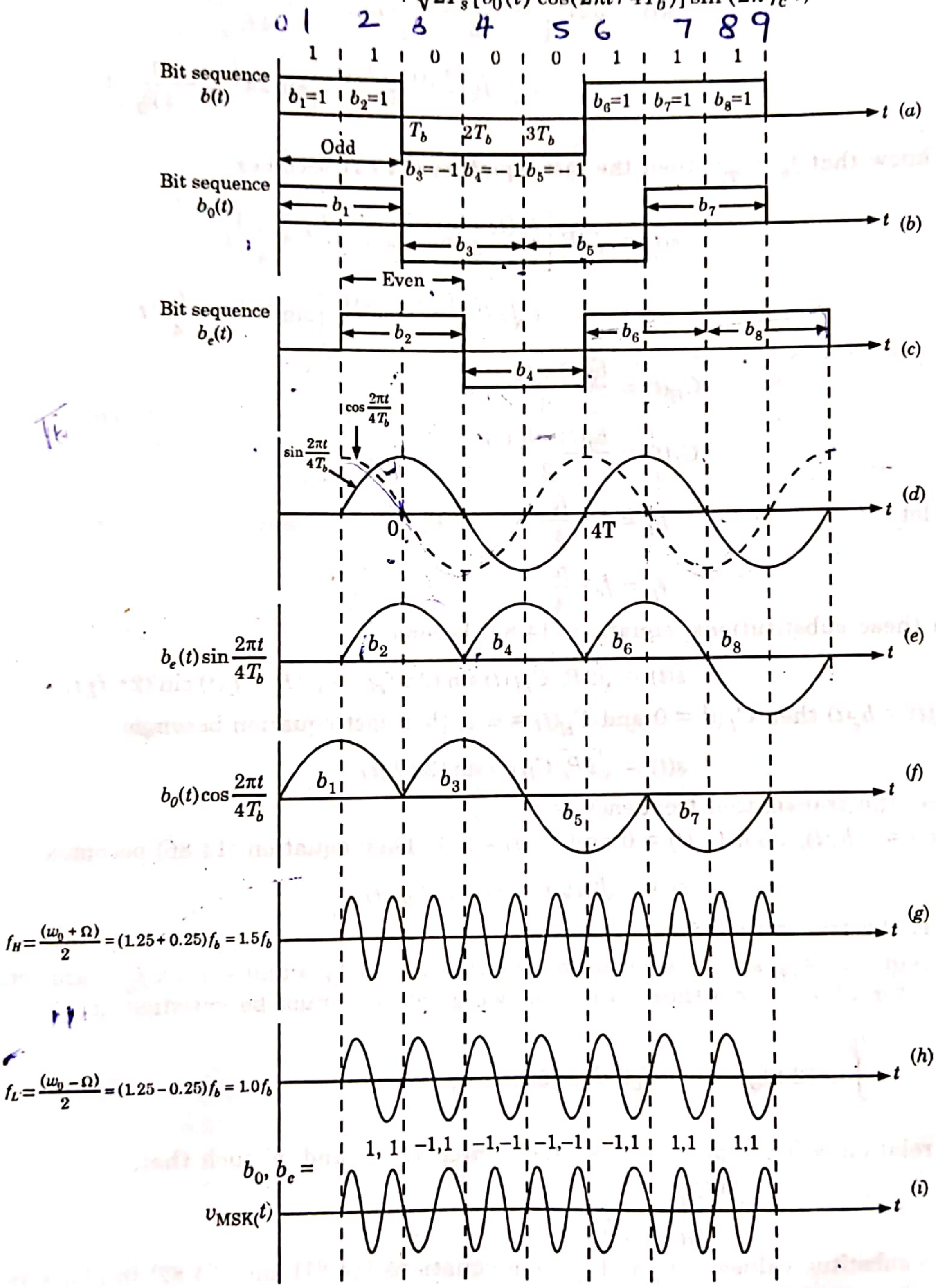


Fig. 14.33. *(a) Bipolar NRZ waveform representing bit sequence (b) Odd bit sequence waveforms $b_0(t)$ (c) Even bit sequence waveform $b_e(t)$ (d) Waveforms of frequency $f_b/4$ used for smoothing of $b_e(t)$ and $b_o(t)$ (e) Modulating waveform of even sequence (f) Modulating waveform of odd sequence (g) Waveform of frequency f_H (h) Waveform of frequency f_L (i) MSK waveform.*

This means that the product signal $b_e(t)\sin(2\pi t/4T_b)$ and $b_0(t)\cos(2\pi t/4T_b)$ modulate the quadrature carriers of frequency f_c. We can write last equation as,

$$s(t) = \sqrt{2P_s}\left[\frac{b_o(t)+b_e(t)}{2}\right]\sin 2\pi\left(f_c+\frac{1}{4T_b}\right)t$$

$$+\sqrt{2P_s}\left[\frac{b_o(t)-b_e(t)}{2}\right]\sin 2\pi\left(f_c-\frac{1}{4T_b}\right)t \quad ...(14.81)$$

We know that $f_b = \frac{1}{T_b}$, then the last equation (14.81) becomes,

$$s(t) = \sqrt{2P_s}\left[\frac{b_0(t)+b_e(t)}{2}\right]\sin 2\pi\left(f_c+\frac{f_b}{4}\right)t$$

$$+\sqrt{2P_s}\left[\frac{b_0(t)-b_e(t)}{2}\right]\sin 2\pi\left(f_c-\frac{f_b}{4}\right)t \quad ...(14.82)$$

Let $$C_H(t) = \frac{b_0(t)+b_e(t)}{2}$$

and $$C_L(t) = \frac{b_0(t)-b_e(t)}{2}$$

and let $$f_H = f_c+\frac{f_b}{4} \quad ...(14.83)$$

and $$f_L = f_c-\frac{f_b}{4} \quad ...(14.84)$$

with these substitutions, equation (14.82) becomes,

$$s(t) = \sqrt{2P_s}\, C_H(t)\sin(2\pi f_H t)+\sqrt{2P_s}\, C_L(t)\sin(2\pi f_L t) \quad ...(14.85)$$

If $b_0(t) = b_e(t)$ then $C_L(t) = 0$ and $C_H(t) = \pm 1$, then last equation becomes,

$$s(t) = \sqrt{2P_s}\, C_H(t)\sin(2\pi f_H t) \quad ...(14.86)$$

Hence, the transmitted frequency is f_H.

If $b_0(t) = -b_e(t)$, then $C_H(t) = 0$ and $C_L(t) = \pm 1$. Then equation (14.86) becomes,

$$s(t) = \sqrt{2P_s}\, C_H(t)\sin(2\pi f_L t) \quad ...(14.87)$$

Hence, the transmitted frequency is f_L.

The frequencies f_H and f_L are chosen such that $\cos(2\pi f_H t)$ and $\sin(2\pi f_L t)$ are orthogonal over the interval T_b. For orthogonality following relation must be satisfied *i.e.*,

$$\int_0^{T_b}\sin(2\pi f_H t)\sin(2\pi f_L t)\,dt = 0 \quad ...(14.88)$$

This relation will be satisfied if we have integers 'm' and 'n' such that,

$$2\pi(f_H-f_L)T_b = n\pi \quad ...(14.89)$$

and $$2\pi(f_H+f_L)T_b = m\pi \quad ...(14.90)$$

Let us substitue values of f_H and f_L from equations (14.81) and (14.82) in above relations. From equation (14.87), we get

$$2\pi\left(f_c+\frac{f_b}{4}-f_c+\frac{f_b}{4}\right)T_b = n\pi$$

or $$f_b\, T_b. = n$$

or $$f_b \times \frac{1}{f_b} = n \quad \Rightarrow \quad n = 1 \qquad ...(14.91)$$

Similarly from equation (14.90), we get

$$2\pi\left(f_c + \frac{f_b}{4} + f_c - \frac{f_b}{4}\right)T_b = m\,\pi$$

or $$4f_c\,T_b = m$$

or $$4f_c \times \frac{1}{f_b} = m \quad \Rightarrow \quad f_c = \frac{m}{4} f_b \qquad ...(14.92)$$

with $n = 1$ in equation (14.89), we get

$$2\pi(f_H - f_L)\,T_b = 1 \times \pi$$

or $$(f_H - f_L) = \frac{1}{2T_b} = \frac{f_b}{2} \qquad ...(14.93)$$

Here $n = 1$ means the difference between f_H and f_L is minimum and at the same time, (MSK) they are orthogonal. Therefore, this technique is called **minimum shift keying (MSK)**. This minimum difference is given by equation (14.93) above. From equation (14.92), we know that $f_c = \frac{m}{4} f_b$. This shows that carrier frequency 'f_c' is integer multiple of $\frac{f_b}{4}$. Substituting, this value of f_c in equation (14.83), we get

$$f_H = f_c + \frac{f_b}{4} = m\frac{f_b}{4} + \frac{f_b}{4}$$

or $$f_H = (m+1)\frac{f_b}{4} \qquad ...(14.94)$$

Similarly, substituting $f_b = m\frac{f_b}{4}$ in equation (14.84), we get

$$f_L = (m-1)\frac{f_b}{4} \qquad ...(14.95)$$

Figure 14.32(g) and (h) shows the waveforms of sin($2\pi\, f_H\, t$) and sin ($2\pi f_L\, t$). For these waveforms $m = 5$. Using equations (14.94) and (14.90), f_H and f_L are calculated with $m = 5$. Figure 14.33(i) shows the final MSK waveform. From equation (14.86), we know that if $b_0(t) = b_e(t)$, then transmitted waveform is of frequency f_H. And if $b_0(t) = -\,b_e(t)$ then the transmitted waveform is given by equation (14.82), which has frequency of f_L. This shows that MSK is basically FSK with reduced bandwidth and continuous phase.

14.14.1. Signal Space Representation of MSK and Distance between the Signal Points (*i.e.*, Geometrical Representation of MSK)

Let us rearrange equation (14.80) as follows

$$s(t) = C_H(t)\sqrt{P_s T_s}\cdot\sqrt{\frac{2}{T_s}}\sin(2\pi f_H t) + C_L(t)\sqrt{P_s T_s}\cdot\sqrt{\frac{2}{T_s}}\sin(2\pi f_L t) \qquad ...(14.96)$$

Here let, $$\phi_H(t) = \sqrt{2/T_s}\,\sin(2\pi f_H t) \qquad ...(14.97)$$

$$\phi_L(t) = \sqrt{2/T_s}\,\sin(2\pi f_L t) \qquad ...(14.98)$$

The carriers $\phi_H(t)$ and $\phi_L(t)$ are in quadrature. They are in quadrature because their frequencies are in quadrature. In QPSK the carriers are in quadrature because of phase shift. Depending on the values of $C_H(t)$ and $C_L(t)$, there will be four signal points in $\phi_H\ \phi_L$

plane. This has benn illustrated in figure 14.34. The distance of each signal point from the origin is $\sqrt{P_s T_s}$.

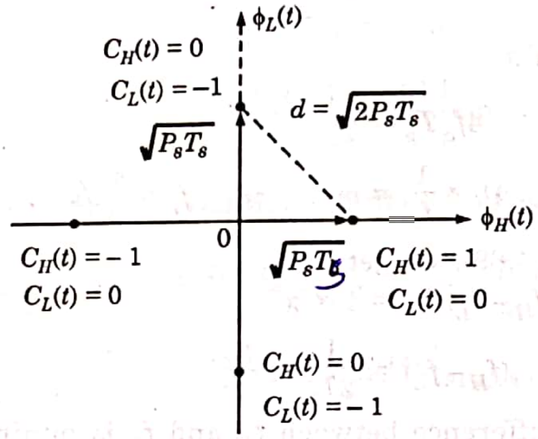


Fig. 14.34. *Geometrical (Signal Space) representation of MSK signals.*

Distance Between Signal Points

Since the points are symmetric, the distance between any two nearest points is same, *i.e.*,

$$d^2 = \left(\sqrt{P_s T_s}\right)^2 + \left(\sqrt{P_s T_s}\right)^2$$

or $$d = \sqrt{2P_s T_s} \qquad \text{...(14.99)}$$

or $$d = \sqrt{2E_s} \quad (\text{since } P_s\, T_s = E_s) \qquad \text{...(14.100)}$$

or $$d = \sqrt{4E_b} \quad (\text{since } E_s = 2E_b) = 2\sqrt{E_b}\,) \qquad \text{...(14.101)}$$

These relations give distance between signal points in MSK. This distance is same as in QPSK.

14.14.2. Power Spectral Density (psd) and Bandwidth of MSK

Let us consider the baseband signal of equation (14.85). The waveform which modulates $\sin(2\pi\, f_c t)$ is,

$$p(t) = \sqrt{2P_s}\,[b_0(t)\cos(2\pi t / 4T_b)] \qquad \text{...(14.102)}$$

$$= \sqrt{2P_s}\; b_0(t)\cos(\pi f_b t / 2) \qquad \text{...(14.103)}$$

The power spectral density (psd) of above waveform is expressed as,

$$S_p(f) = \frac{32E_b}{\pi^2}\left[\frac{\cos(2\pi f T_b)}{1-(4fT_b)^2}\right]^2 \qquad \text{...(14.105)}$$

when this signal modulates the carrier 'f_c' then the total power spectral density (psd) of baseband signal is divided by '4' and is placed at $\pm f_c$, *i.e.*,

$$S(f) = \frac{8E_b}{\pi^2}\left\{\frac{\cos 2\pi(f-f_c)T_b}{1-[4(f-f_c)T_b]}\right\}^2 + \frac{8E_b}{\pi^2}\left\{\frac{\cos 2\pi(f+f_c)T_b}{1-[4(f+f_c)T_b]^2}\right\}^2 \qquad \text{...(14.106)}$$

The above equation gives power spectral density (psd) of MSK signal. Figure 14.34 shows the normalized spectral densities of MSK and QPSK. Normalization means maximum amplitudes of signals are scaled with respect to '1'.

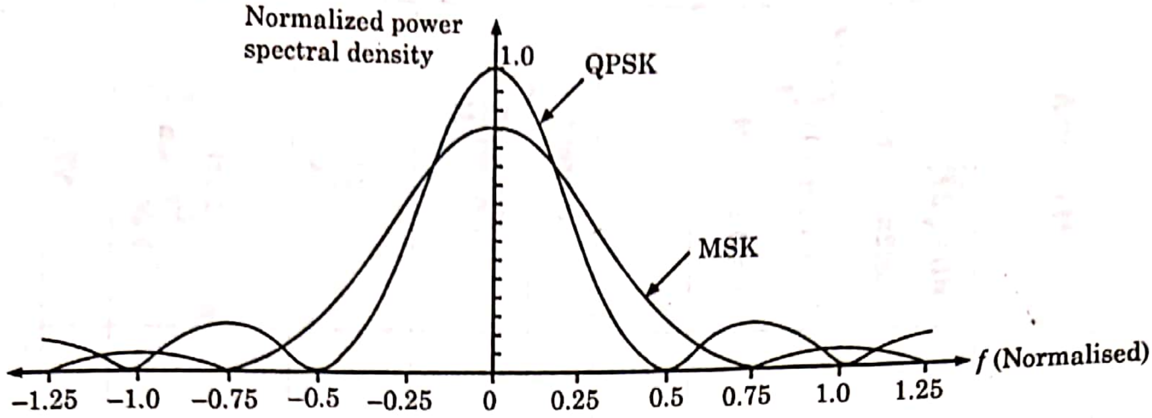


Fig. 14.35. *Power spectral densities (psd) of MSK and QPSK.*

The above plots show that the main lobe in MSK is wider than QPSK. The sides lobes in MSK are very small compared to QPSK.

Bandwidth Calculation of MSK

From figure 14.35, we observe that the width of main lobe in MSK is ± 0.75 *i.e.*,

$$fT_b = \pm 0.75$$

or

$$f = \pm 0.75 f_b$$

Hence, bandwidth will be equal to width of the main lobe *i.e.*,

$$BW = 0.75 f_b - (-0.75 f_b) = 1.5 f_b \quad \text{...(14.107)}$$

Thus, the BW of MSK is higher than that of QPSK.

14.14.3. Generation of MSK

Figure 14.36 shows the block diagram of MSK transmitter. The two sinusoidal signals $\sin(2\pi f_c t)$ and $\cos(2\pi t/4T_b)$ are mixed (*i.e.*, multiplied). The bandpass filters then pass only sum and difference components $f_c + \frac{f_b}{4}$ and $f_c - \frac{f_b}{4}$. The outputs of bandpass filters (BPFs) are

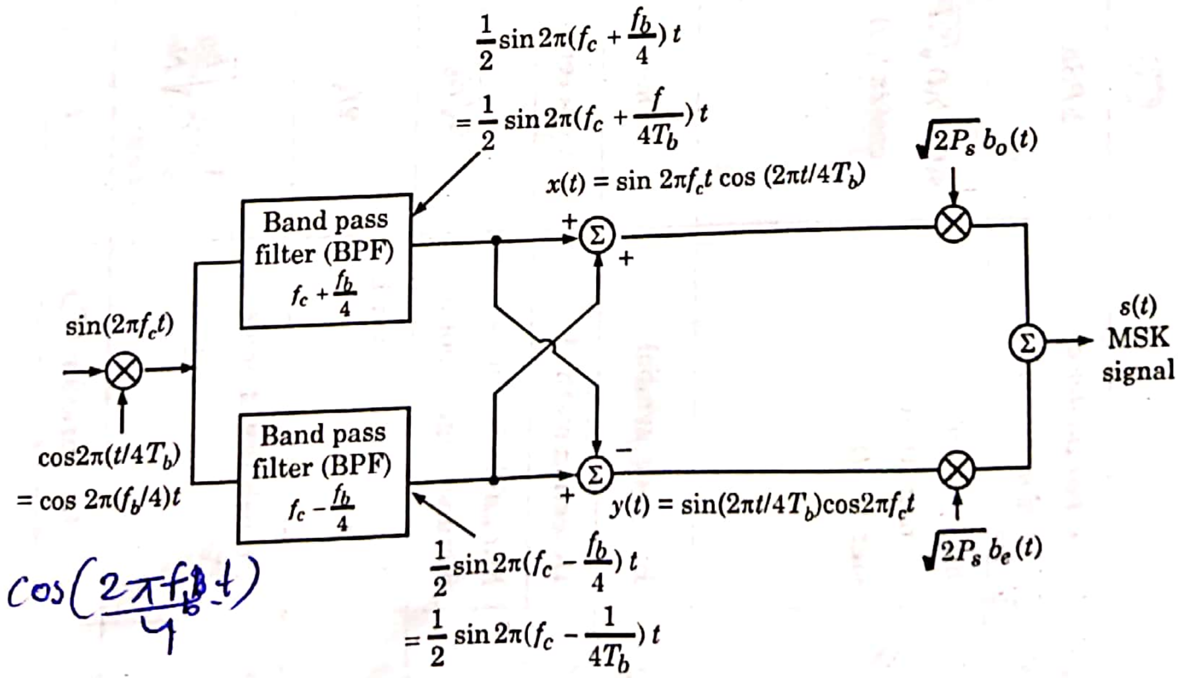


Fig. 14.36. *MSK transmitter block diagram.*

then added and subtracted such that two signals $x(t)$ and $y(t)$ are generated. Signal $x(t)$ is multiplied by $\sqrt{2P_s}\, b_0(t)$ and $y(t)$ is multiplied by $\sqrt{2P_s}\, b_e(t)$. The outputs of the multiplies are then added to give final MSK signal. Thus the block diagram of figure 14.35 is the step to step implementation of equation (14.80).

14.14.4. Reception of MSK (*i.e.* Detection of MSK)

Figure 14.37 shows the block diagram of MSK receiver. MSK uses synchronous detection. The signals $x(t)$ and $y(t)$ are multiplied with the received MSK signal. Here $x(t)$ and $y(t)$ have same values as shown in transmitter block diagram of figure 14.37. The outputs of the multipliers are $b_e(t)$ and $b_0(t)$. The integrators integrate over the period of $2T_b$. For the upper correlator, the sampling switch samples output of integrator at $t = (2k + 1)T_b$. Then the decision device decides whether $b_0(t)$ is + 1 or – 1. Similarly, lower correlator output is $b_e(t)$. The outputs of two decision devices are staggered by T_b. The switch S_3 operates at $t = kT_b$ and simply multiplexes the two correlator outputs.

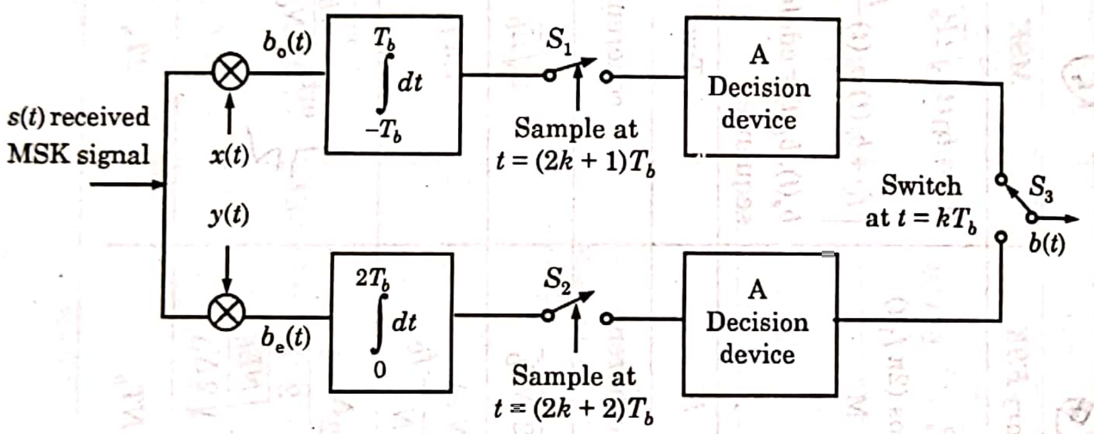


Fig. 14.37. *MSK receiver block diagram.*

14.14.5. Advantages and Disadvantages of MSK as Compared to QPSK

From the discussion of MSK, we can now compare the advantages of MSK over QPSK.

Advantages:

1. The MSK baseband waveforms are smoother compared to QPSK.
2. MSK signal have continuous phase in all the cases, whereas QPSK has abrupt phase shift of $\frac{\pi}{2}$ or π.
3. MSK waveform does not have amplitude variations, whereas QPSK signals have abrupt amplitude variations.
4. The main lobe of MSK is wider than that of QPSK. Main lobe of MSK contains around 99% of signal energy whereas QPSK main lobe contains around 90% signal energy.
5. Side lobes of MSK are smaller compared to that of QPSK. Hence, interchannel interference because of side lobes is significantly large in QPSK.
6. To avoid interchannel interference due to sidelobes, QPSK needs bandpass filtering, whereas it is not required in MSK.
7. Bandpass filtering changes the amplitude waveform of QPSK because of abrupt changes in phase. This problem does not exist in MSK.

The distance between signal points is same in QPSK as will as in MSK. Hence, the probability of error is also same. However, there are some drawback of MSK.

(ii) Drawbacks

1. The bandwidth requirement of MSK is 1.5 f_b, whereas it is f_b in QPSK. Actually, this cannot be said serious drawback of MSK. Because power to bandwidth ratio of MSK is more. In fact, 99% of signal power can be transmitted within the bandwidth of 1.2 f_b in MSK. While QPSK needs around 8 f_b to transmit the same power.
2. The generation and detection of MSK is slightly complex. Because of incorrect synchronization, phase jitter can be present in MSK. This degrades the performance of MSK.

14.15. Comparison of Digital Modulation Techniques

Table 14.5 shows the comparison of various digital modulation techniques. They are compared on the basis of various parameters like bits transmitted per symbol, detection method, Euclidean distance, bandwidth, error probability, symbol duration etc. Various other important parameters like bandwidth efficiency, spectrum of transmitted signal etc., are not compared. QPSK, ASK have amplitude variations hence noise interference is more in these techniques. Normally, PSK and FSK methods have less noise interference. M-ary techniques are more complex compared to binary techniques.

SUMMARY

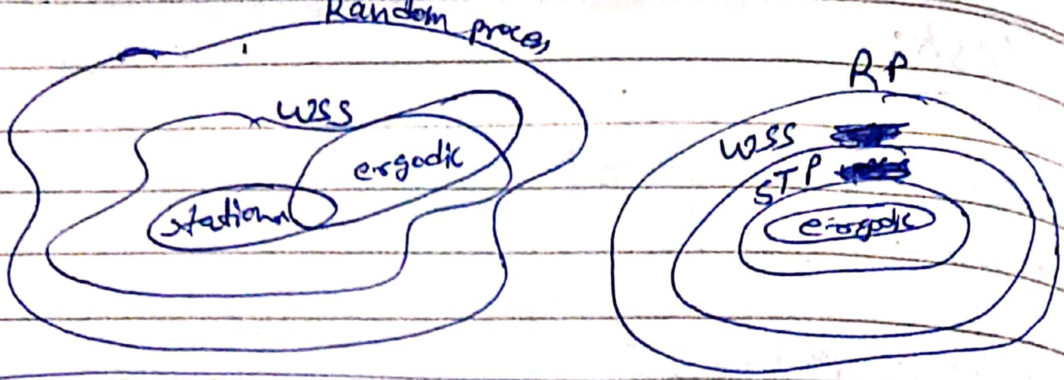
1. Modulation is defined as the process by which some characteristics of a carrier is varied in accordance with a modulating signal.
2. In digital communications, the modulating signal consists of binary data or an M-ary encoded version of it.
3. The channel may be a telephone channel, microwave radio link, satellite channel or an optical fiber. In digital communication, the modulation process involves switching or keying the amplitude, frequency or phase of the carrier in accordance with the input data.
4. There are three basic modulation techniques for the transmission of digital data. They are known as amplitude-shift keying (ASK), frequency shift keying (FSK) and phase-shift keying (PSK) which can be viewed as special cases of amplitude modulation frequency modulation and phase modulation respectively.
5. When we have to transmit a digital signal over a long distance, we need continuous-wave (CW) modulation. For this purpose, the transmission medium can be in form of radio, cable or other type of channel. Also, a carrier signal having some frequency f_c is used for modulation. Then the modulating digital signal modulates some prameter like frequency, phase or amplitude of the carrier.
6. There is some deviation in carrier frequency f_c. This deviation is known as the bandwidth of the channel. This means that the channel has to transmit some range or band of frequencies. Such type of transmission is known as bandpass transmission and the communication channel is known as bandpass channel.
7. When it is required to transmit digital signals on a bandpass channel, the amplitude, frequency or phase of the sinusoidal carrier is varied in accordance with the incoming digital data. Since the digital data is in discrete steps, the modulation of the bandpass sinusoidal carrier is also done in discrete steps. Due to this reason, this type of modulation (*i.e.*, Digital modulation) is also known as switching or signaling.
8. Because of constant amplitude of FSK and PSK, the effect of non-linearities, noise interference is minimum on signal detection. However, these effects are more pronounced on ASK. Therefore, FSK and PSK are preferred over ASK.
9. In digital modulations, instead of transmitting one bit at a time, we transmit two or more bits simulteneously. This is known as M-ary transmission. This type of transmission results in reduced

Table 14.5.

Sr. No.	*Parameter of Comparision*	3 *BPSK*	4 *DPSK*	5 *QPSK*	8 *M-ary PSK*
1	Equation of the transmitted signal $s(t)$	$s(t) = b(t)\sqrt{2P_s}$ $\cos(2\pi f_C t)$	$s(t) = b(t)\sqrt{2P_s}$ $\cos(2\pi f_C t)$ ✓ $b(t)$ differentially coded	$s(t) = \sqrt{2P_s}\cos$ $[2\pi f_C t + (2m+1)\frac{\pi}{4}]$ $m = 0, 1, 2, 3, \ldots$	$s(t) = \sqrt{2P_s}$ $\cos(2\pi f_C t + \phi_m)$ $\phi_m = (2m+1)\frac{p}{M}$ p → π $m = 0, 1, 2, \ldots M-1$
2	Bits per symbol	One	one	Two	N
3	Detection method	Coherent	Non coherent	coherent	coherent
4	Minimum Euclidean distance signal points	$2\sqrt{E_b}$	–	$2\sqrt{E_b}$	$2\sqrt{E_s}\sin\frac{\pi}{M}$
5	Minimum Bandwidth (BW)	$2f_b$	f_b	f_b	$\frac{2f_b}{N}$
6	Probability of error $P(e)$	$\frac{1}{2}\,erfc\sqrt{\frac{E_b}{No}}$	$\frac{1}{2}e^{-E_b/No}$	$\frac{1}{2}\,\text{erfc}\sqrt{\frac{E_b}{N}}$	$\leq \text{erfc}\left(\sqrt{\frac{E_s}{No}}\sin\frac{\pi}{M}\right)$
7	Symbol duration (T_s)	T_b	$2T_b$	$2T_b$	NT_b

(Contd.)

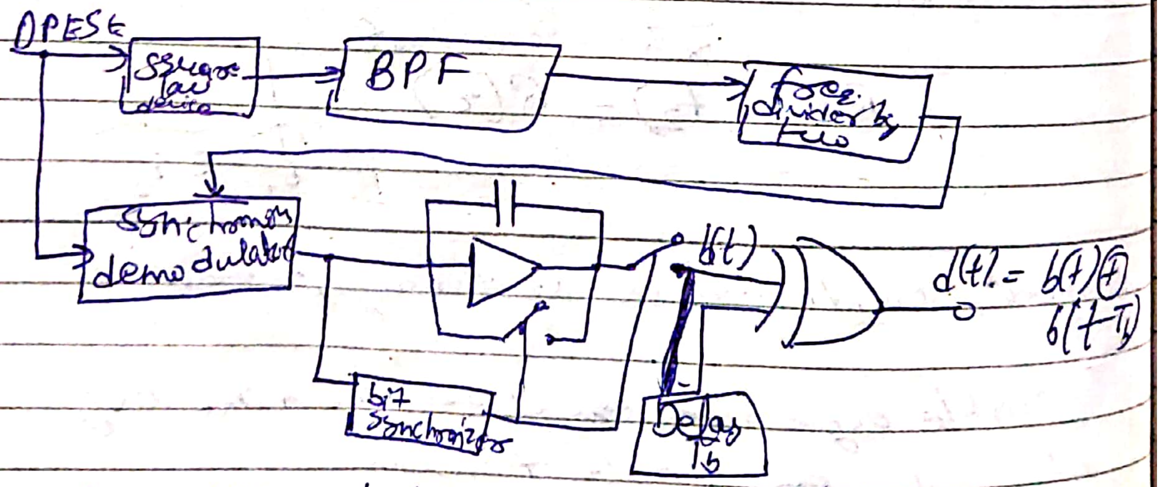
* 4	3	7	6	1
QASK	*BFSK*	*M-ary FSK*	*MSK*	*ASK*
$s(t) = k_1\sqrt{0.2P_s}$ $\cos 2\pi f_C t) + k_2\sqrt{0.2P_s}$ $\sin(2\pi f_C t)$ $k_1, k_2 = \pm 1$ or ± 3 for $M = 16$	$s(t) = \sqrt{2P_s}\cos$ $[(2\pi f_C + d(t)\,\Omega)t]$	$s(t) = \sqrt{2P_s}$ $\cos(2\pi f_1 t)$ $i = 1, 2, \ldots M$	$s(t) = b_0(t)\sqrt{2P_s}\sin 2\pi$ $\left[f_C + b_e(t)\,b_o(t)\frac{f_b}{4}\right]t$ $b_e(t), b_0(t)$ = odd/even sequence	$s(t) = \sqrt{2P_s}$ $\cos(2\pi f_C t)$ for symbol '1' = 0 for symbol '0'
N	one	N	Two	one
coherent	non coherent	non coherent	coherent	coherent
$\sqrt{0.4E_s}$ for $M = 16$	$\sqrt{2E_b}$	$\sqrt{2NE_b}$	$2\sqrt{E_b}$	$\sqrt{E_b}$
$\frac{2f_b}{N}$	$4f_b$	$\frac{2^{N+1}}{N}f_b$	$1.5f_b$	–
$\leq 2\,\text{erfc}\sqrt{\frac{0.4E_b}{N_o}}$ for $M = 16$	$\frac{1}{2}\text{erfc}\sqrt{\frac{E_b}{2N_o}}$	$\leq \frac{2^N - 1}{2}\text{erfc}\sqrt{\frac{NE_b}{2N_0}}$	$\frac{1}{2}$ $\text{erfc}\sqrt{\frac{E_b}{N_o}}$	$\frac{1}{2}\text{erfc}\sqrt{\frac{E_b}{4N_o}}$
NT_b	T_b	NT_b	$2T_b$	T_b



strict sense stationary

DEPSK : The transmitter of DEPSK is identical to the DPSK transmitter, but the receiver is completely different. The signal b(t) is recovered from the received signal, using the synchronous demod tech. This is same as the BPSK.

once of the signal b(t) is recovered, it's applied to one i/p of an exor gate. The signal b(t) is also applied to a time delay circuit and the delayed signal $b(t-T_b)$ is applied to the other i/p of the exor gate.



if $b(t) = b(t-T_b)$ then o/p of Ex-or will be 0

i.e. $d(t) = 0$, if $b(t) = b(t-T_b)$

and if $b(t) = \overline{b(t-T_b)}$, o/p will be '1'
if $b(t) = b(t-T_b)$.
$\therefore d(t) = 1$

Ad:
. Less P_e comp to DPSK.
. In DPSK demod, the delay generating device T_b has to operate at the carrier freq but in DEPSK demod, the delay device operates at the baseband freq (f_b). This reduces the hardware cost of the delay device.

DIS:
. complex demod is req.
. error occurs in pairs. Thus one error in $b(t)$ will give rise to two bit errors.

$\boxed{\text{* Maximum likelihood receiver}}$

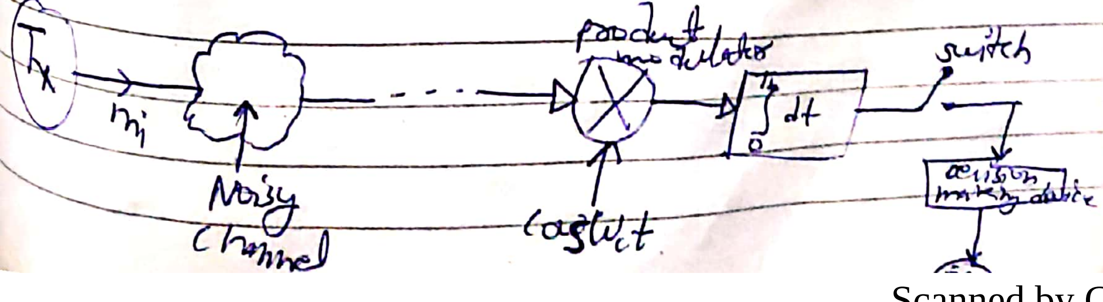
likelihood f^n, $f_{\vec{x}}(\vec{x}|m_i) = \prod_{i=1}^{N} f_{X_1}(x_1|m_i)$

Decision rule to estimate, $f_{\vec{x}}(\vec{x}|m_i)$ is maximum
$\hat{m} = m_i$ for $k = i$

$\downarrow$ using log fn

$\ln f_{\vec{x}}(\vec{x}|m_k)$ is max for $k = i$

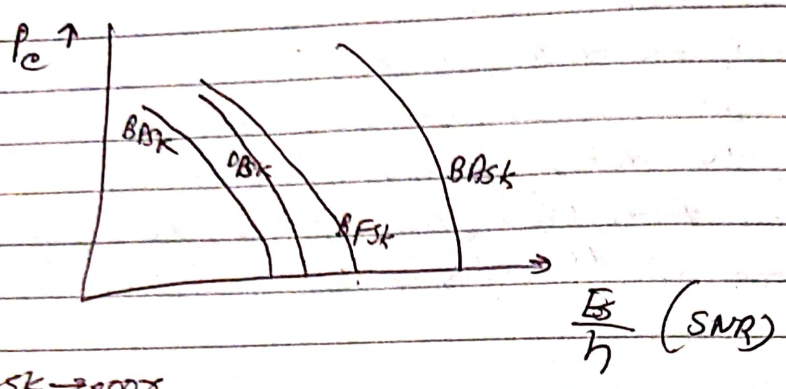
Max likelihood decision = Max likelihood receiver



imp

★ P_e of DPSK → complex → SNR

$$\boxed{P_e = \frac{1}{2} e^{-E_s/\eta}}$$ → Gaussian noise constant

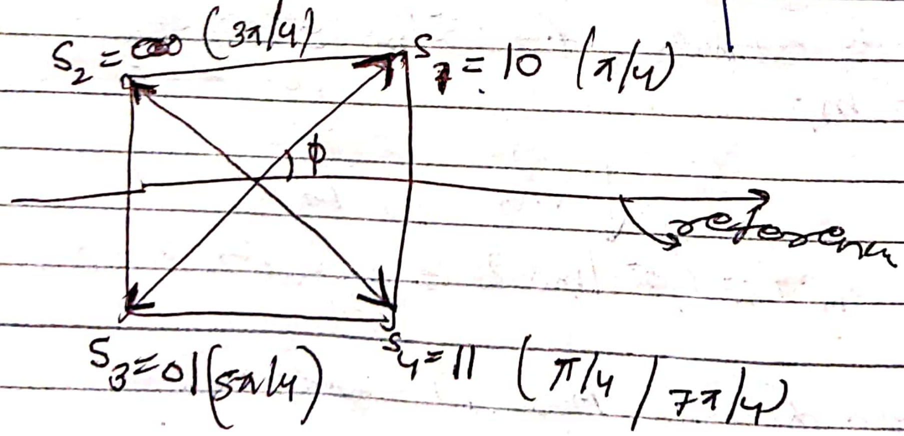


ASK → poor

⇒ QPSK constellation : ✓ imp

	Bits	Phase
S_2	00	$3\pi/4$
S_3	01	$5\pi/4$
S_1	10	$\pi/4$
S_4	11	$7\pi/4$

S_1	1	0	$\pi/4$
S_2	0	0	$3\pi/4$
S_3	0	1	$5\pi/4$
S_4	1	1	$7\pi/4$



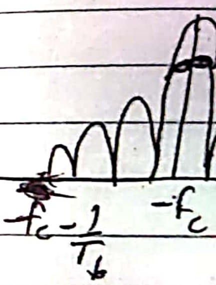
. imp

★ ASK

• Signal spa...
fo...

Dist...

• PSD



• Bandw...

★ BPS...

$S(t) =$ ...
$b(t)$ ...

imp

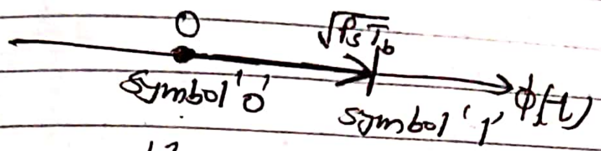
* ASK :

$$s(t) = \sqrt{2P_s}\cos(2\pi f_c t) \quad \text{(to transmit '1')}$$

- Signal space diagram :

for symbol 1 :

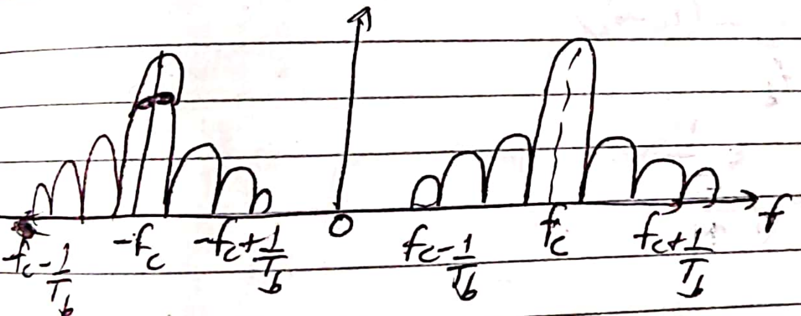
$$s(t) = \sqrt{P_s T_b}\,\sqrt{2/T_b}\,\cos(2\pi f_c t)$$
$$= \sqrt{P_s T_b}\,\phi_1(t)$$



Distance b/w signal points :

$$\boxed{d = \sqrt{P_s T_b} = \sqrt{E_b}}$$

- PSD



- Bandwidth $= \dfrac{3}{T_b}$ Hz

* BPSK :

$$s(t) = b(t)\sqrt{2P}\cos(2\pi f_c t)$$

$$b(t) = +1 \quad \text{for '1'}$$
$$= -1 \quad \text{for '0'}$$

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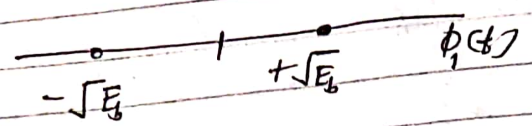
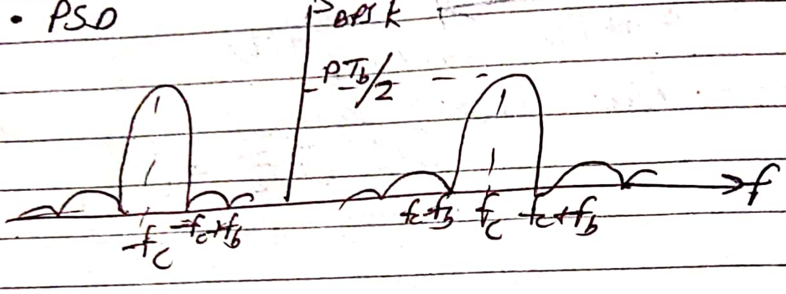


$$\boxed{d = 2\sqrt{E_b}}$$

$\boxed{d \text{ inc prob of error reduces}}$

• PSD



$$\bullet\ BW = f_c+f_b-(f_c-f_b)$$
$$= 2f_b$$

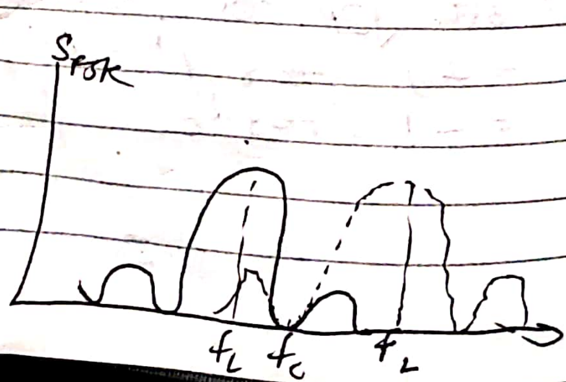
$$\bullet\ P_e = \frac{1}{2}\,\text{erfc}\sqrt{\frac{E_b}{N}}$$

* $\boxed{BFSK}$

$$s(t) = \sqrt{2P_s}\,\cos\left[\left(2\pi f_c + d(t)\Omega\right) t\right]$$

$d(t) = +1$ for '1'
$d(t) = -1$ for '0'

• PSD



• BW = $4f_b$

• signal-spa...

$\phi_1(t) =$

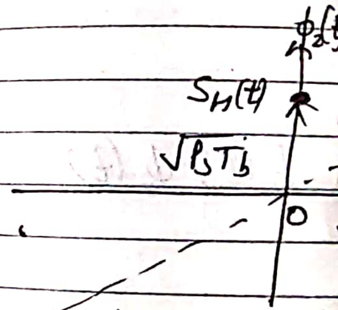
$\phi_2(t) =$

$S_H(t) = \sqrt{P_sT_b}$

$S_L(t) \quad \sqrt{P_sT_b}$

$\therefore S_H(t) =$

$S_L(t) =$



decision bounda...

• $P_e = \frac{1}{2}\,\text{erfc}$

• $BW = 4f_b$

• signal-space ~~representation~~:

$$\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi m f_3 t)$$

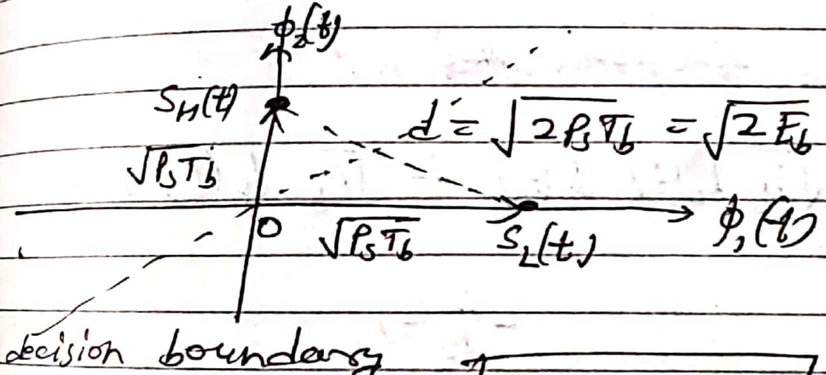
$$\phi_2(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi m f_5 t)$$

$$s_H(t) = \sqrt{P_s T_b}\sqrt{\frac{2}{T_b}}\cos(2\pi f_H t)$$

$$s_L(t) = \sqrt{P_s T_b}\sqrt{\frac{2}{T_b}}\cos(2\pi f_L t)$$

$$\therefore\ s_H(t) = \sqrt{P_s T_b}\,\phi_1(t)$$

$$s_L(t) = \sqrt{P_s T_b}\,\phi_2(t)$$



$$\boxed{d = \sqrt{2 E_b}}$$

• $P_e = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{E_b}{2N_0}}$

DPSK

- $s(t) = \pm A_c \cos(2\pi f_c t)$
- $BW = \frac{2}{T_s} = \frac{2}{2T_b} = \frac{1}{T_b} = f_b$

  as $T = 2T_b$
- $P_e = \frac{1}{2} e^{-E_b/N_0}$

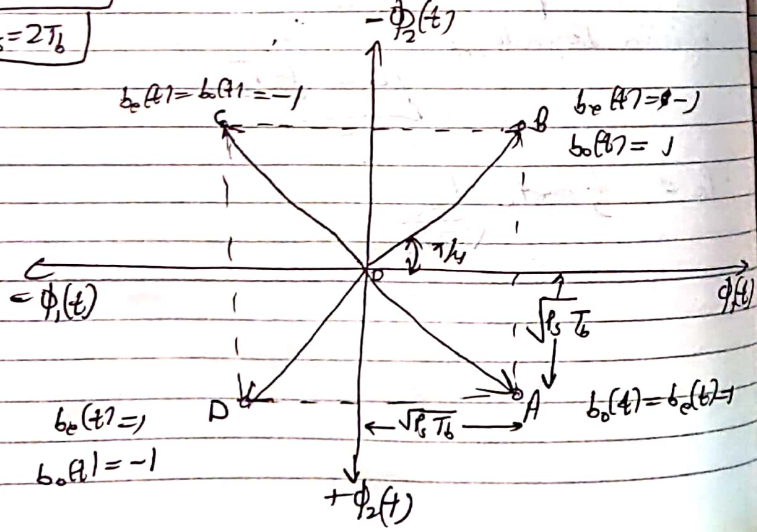
QPSK

- $s(t) = b_o(t)\sqrt{P_s}\cos(2\pi f_c t) + b_e(t)\sqrt{P_s}\sin(2\pi f_c t)$
- signal space representation:

$$s(t) = \sqrt{E_b}\, b_o(t)\, \phi_1(t) + \sqrt{E_b}\, b_e(t)\, \phi_2(t)$$

$\boxed{E_b = P_s T_b}$

$\boxed{T_s = 2T_b}$



$OB = \sqrt{P_sT_b + P_sT_b} = \sqrt{2P_sT_b}$

$= \sqrt{P_sT_s}$

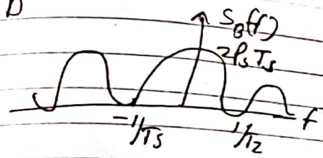
$OB = \sqrt{E_s}$

and d = distance b/w A and B $= 2\sqrt{P_sT_b}$

$\boxed{d = 2\sqrt{E_b}}$

same as BPSK

- PSD



- $BW = \frac{2}{T_s} = \frac{2}{2T_b} = \frac{1}{T_b} = f_b$
- $P_e = \frac{1}{2}\,\text{erfc}\sqrt{\frac{E_b}{N_0}}$

MSK

$s(t) = \sqrt{2P_s}\,[b_e(t)\sin(2\pi t/4T_b)]\cos(2\pi f_c t)$

$+ \sqrt{2P_s}\,[b_o(t)\cos(2\pi t/4T_b)]\sin(2\pi f_c t)$

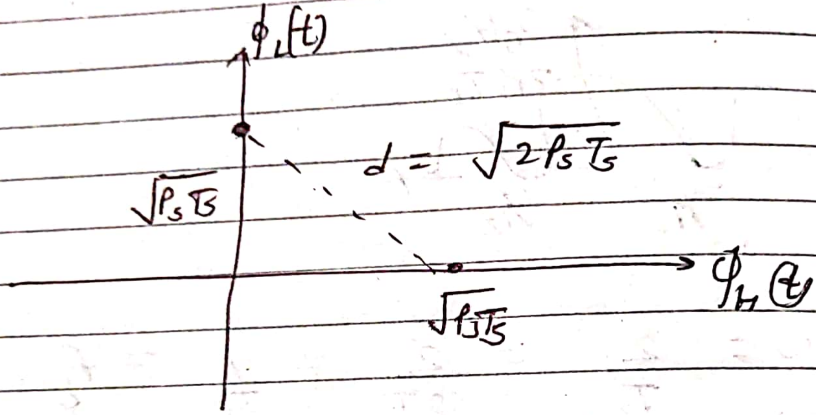
- $BW = 1.5 f_b$

- Signal space representation:

$$s(t) = c_H(t)\sqrt{P_sT_s}\sqrt{\frac{2}{T_s}}\sin(2\pi f_H t) + c_L(t)\sqrt{P_sT_s}\sqrt{\frac{2}{T_s}}\sin(2\pi f_L t)$$

$$\phi_H(t) = \sqrt{2/T_s}\,\sin(2\pi f_H t)$$

$$\phi_L(t) = \sqrt{2/T_s}\,\sin(2\pi f_L t)$$

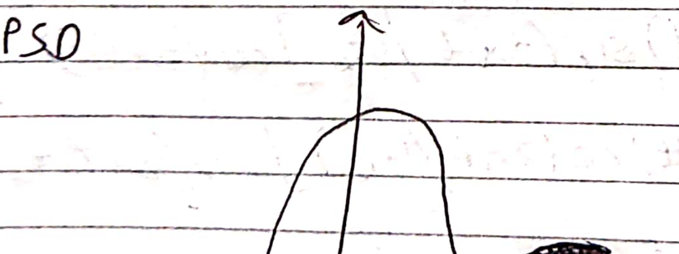


$$d = \sqrt{2P_sT_s} = \sqrt{2E_s}$$

$\boxed{E_s = 2E_b}$

$\boxed{d = \sqrt{4E_b} = 2\sqrt{E_b}}$
same as QPSK

- PSD



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$\cos(n\omega t+\phi)$

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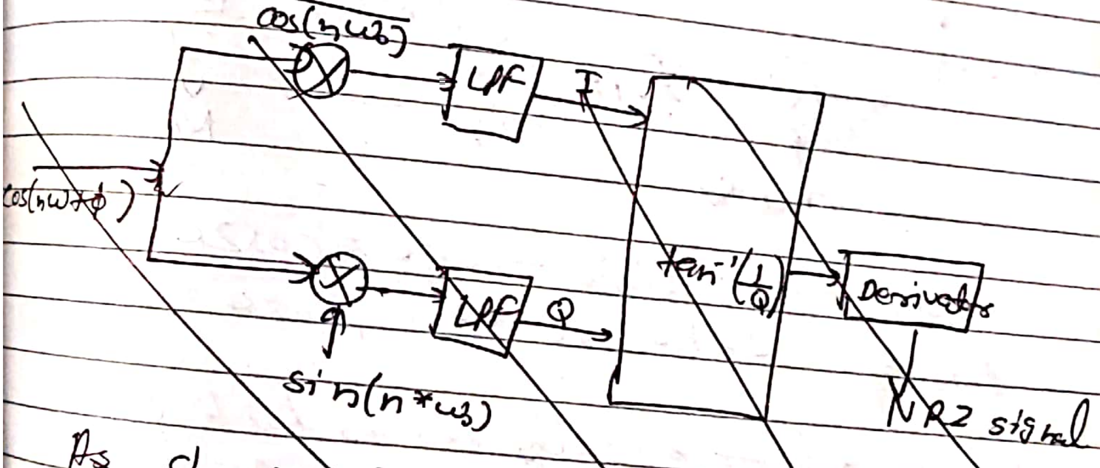
* GMSK Modulation

Gaussian MSK is used in GSM / CDPD (cellular Digital Packet data) technologies. There are 2 main methods to generate GMSK. The first one is FSK modulation & the second one is QPSK modulation. The first one is based on GMSK VCO architecture & is not ideal for coherent demodulator.

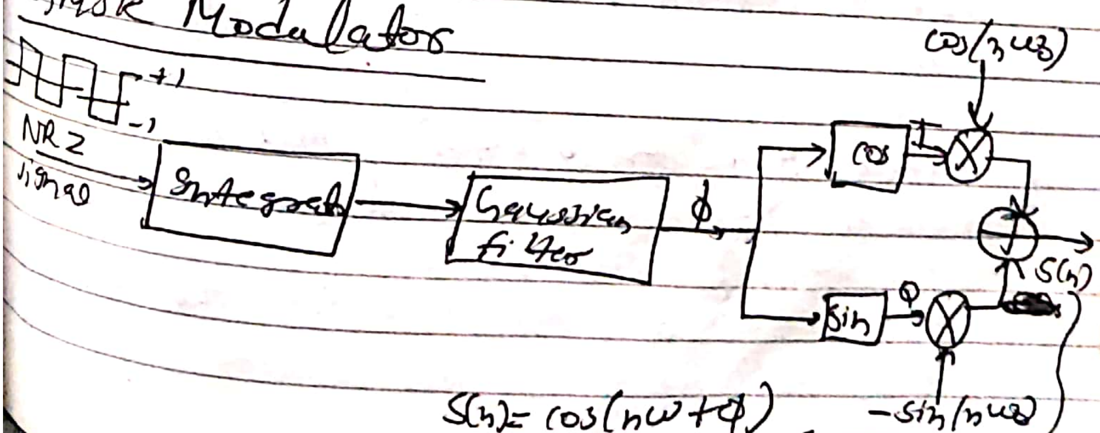
We generate GMSK using 2nd method.

GMSK Modulator



~~As shows Gaussian filter is applied to NRZ signal after it is passed through integrator block. This gives φ. By~~

GMSK Modulator

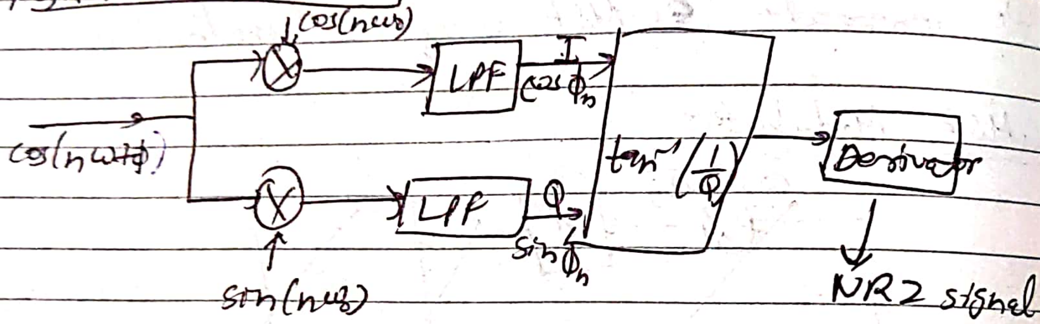


$S(n) = \cos(n\omega + \phi)$

Gaussian filter is applied to NRZ signal after it is passed through integrator block. This gives ϕ. By applying cos & SIN function to this ϕ give out I and Q components which bits with cos f^n & sin f^n respectively using mixing f^n. Both the chains are summed up & will give out $s(n)$.

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

GMSK Demodulator



$$\cos 2\theta = \cos^2\theta - \sin^2\theta \rightarrow \cos^2\theta = \cos 2\theta + \sin^2\theta$$
$$\sin 2\theta = 2\sin\theta\cos\theta \rightarrow \sin\theta\cos\theta = \sin 2\theta/2$$

GMSK modulator basically derives back ϕ using arctan f^n, which is applied to derivator block to obtain NRZ signal back. Before doing this mixing & Low pass filtering is done to obtain I & Q components from two chains.

Advantage: Modulated carrier in MSK contains no phase discontinuities & freq changes occur at zero crossing of carrier.

- GMSK spectral efficiency is better than MSK
- GMSK ~~both~~ has less demodulator complexity.

optimum filter:

$$H(f) = k \cdot \frac{X^*(f)}{S_{ni}(f)} e^{-j2\pi fT}$$

$$S_{ni}(f) = \frac{N_0}{2}$$

psd

Matched, $h(t) = \frac{2k}{N_0} \cdot x(T-t)$

optimum filter

$$P_e = \frac{1}{2} \operatorname{erfc}\left[\frac{x_1(T) - x_2(T)}{2\sqrt{2}\sigma}\right]$$

Matched $P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E}{N_0}}$

$$\left(\frac{S}{N}\right)_{omax} = \int_{-\infty}^{\infty} \frac{|X(f)|^2}{S_{ni}(f)} df$$

optimum filter

$$\left(\frac{x_1(T) - x_2(T)}{\sigma}\right)^2_{max} = \int_{-\infty}^{\infty} \frac{|X(f)|^2}{S_{no}(f)} df$$

$$\left(\frac{S}{N_0}\right)_{max} = \frac{x(t) \text{ energy}}{\text{psd of white noise}}$$

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