CDF Numericals

Que.) In an experiment, a trial consists of four successive tosses of a coin. If we define an RV x as the number of heads appearing in a trial, determine $P_X(x)$ and $F_X(x)$.

Sol. Sample space

$S = \{HHHH, HHHT, HHTH, HTHH, HHTT, HTHT, HTTH, TTHH, THTH, THHT, HTTT, TTTH, TTHT, THTT, THHH, TTTT\}$

$P(0 \text{ Heads}) = P(X=0) = \frac{1}{16}$

$P(1 \text{ Head}) = P(X=1) = \frac{4}{16} = \frac{1}{4}$

$P(2 \text{ Heads}) = P(X=2) = \frac{6}{16} = \frac{3}{8}$

$P(3 \text{ Heads}) = P(X=3) = \frac{4}{16} = \frac{1}{4}$

$P(4 \text{ Heads}) = P(X=4) = \frac{1}{16}$

CDF :- $F_X(x_0) = P(X \leq x_0) = P(X < x_0) + P(X = x_0)$

$F_X(x_0) = \frac{1}{16}$

$F_X(x_1) = P(X \leq x_1) = P(X \leq x_0) + P(X = x_1)$

$= F_X(x_0) + P(X = x_1)$

$= \frac{1}{16} + \frac{4}{16} = \frac{5}{16}$

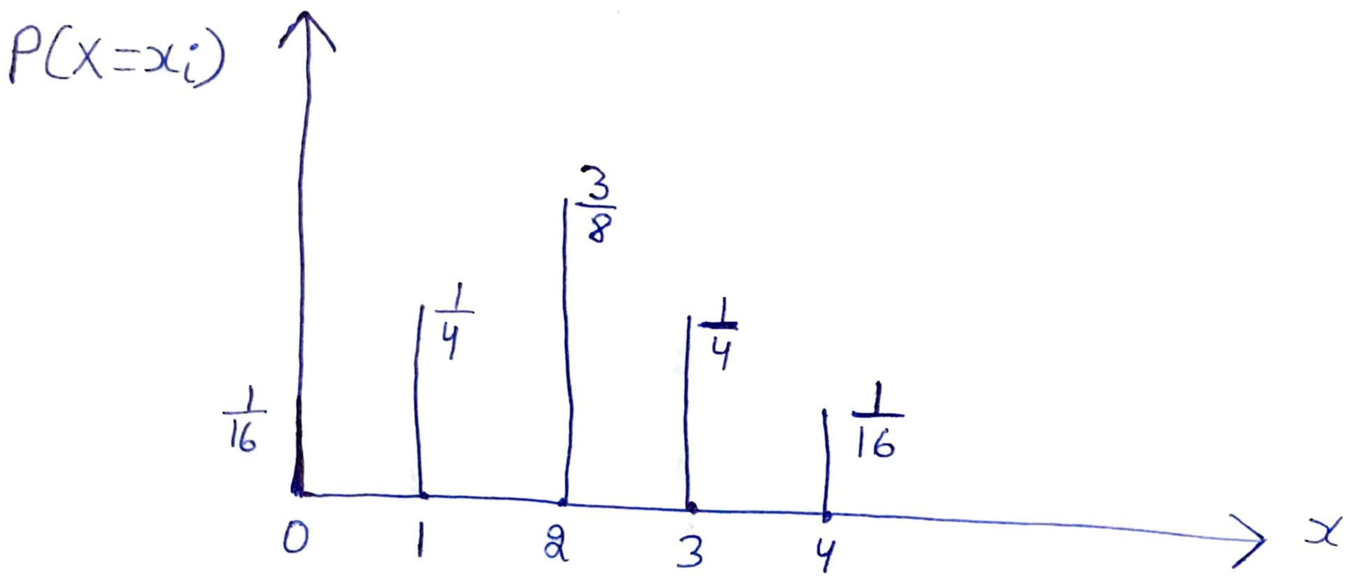
$F_X(x_2) = P(X \leq x_2) = P(X \leq x_1) + P(X = x_2)$

$= F_X(x_1) + P(X = x_2)$

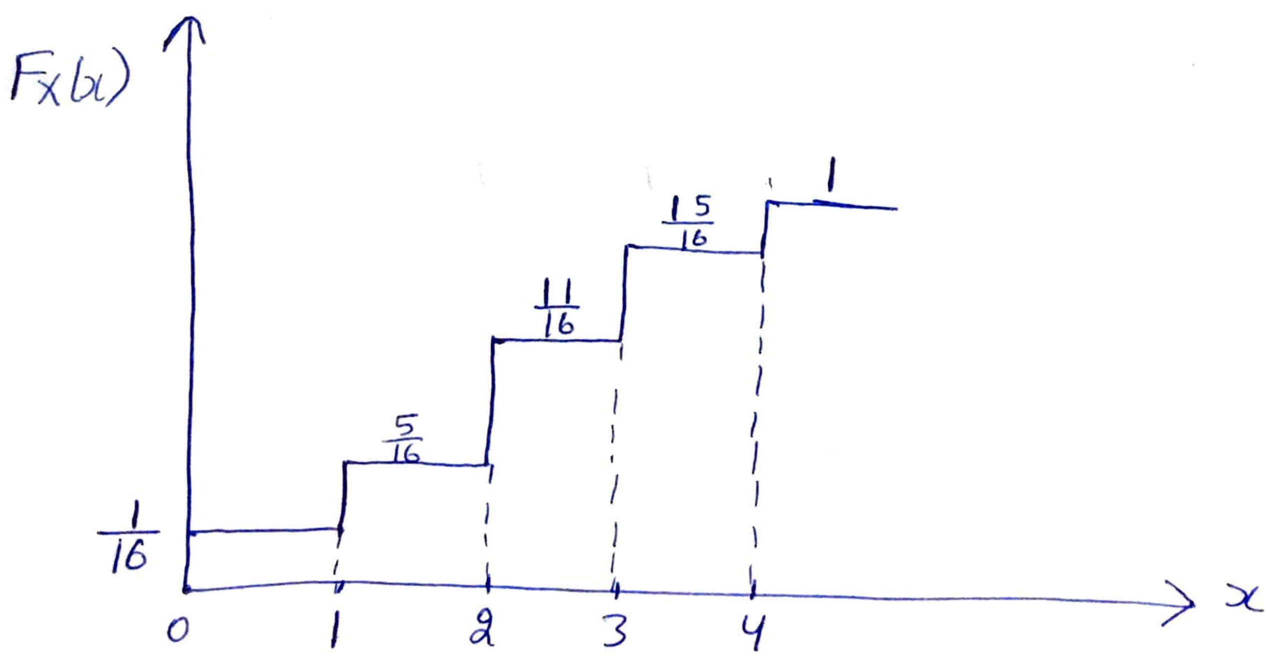
$= \frac{5}{16} + \frac{6}{16} = \frac{11}{16}$

$$F_X(x_3) = P(X \le x_3) = P(X \le x_2) + P(X = x_3)$$
$$= F_X(x_2) + P(X = x_3)$$
$$= \frac{11}{16} + \frac{4}{16} = \frac{15}{16}$$

$$F_X(x_4) = P(X \le x_4) = P(X \le x_3) + P(X = x_4)$$
$$= \frac{15}{16} + \frac{1}{16} = 1$$



CDF



Que.) A three digit message is transmitted over a noisy channel having probability of error $P(E) = \frac{2}{5}$ per digit. Find out the corresponding CDF.

$$P(E) = \frac{2}{5}, \quad P(C) = \frac{3}{5}$$

$$S = \{CCC, CCE, CEC, ECC, CEE, EEC, ECE, EEE\}$$

Let X be a RV which denote no of errors in the received message.

$$X = \{0, 1, 2, 3\}$$

For $X < x_0$ $\quad F_X(x) = 0$

For $X = x_0$ $\quad F_X(x_0) = P(X \leq x_0)$

$$= P(X < x_0) + P(X = x_0)$$

$$= 0 + \frac{27}{125} = \frac{27}{125}$$

For $X = x_1$ $\quad F_X(x_1) = P(X \leq x_1)$

$$= P(X \leq x_0) + P(X = x_1)$$

$$= F_X(x_0) + P(X = x_1)$$

$$= \frac{27}{125} + \frac{54}{125}$$

$$= \frac{81}{125}$$

For $X = x_2$

$$
\begin{aligned}
F_X(x_2) &= P(X \le x_2) \\
&= P(X \le x_1) + P(X < x_2) \\
&= F_X(x_1) + P(X < x_2) \\
&= \frac{81}{125} + \frac{36}{125} = \frac{117}{125}
\end{aligned}
$$

For $X = x_3$

$$
\begin{aligned}
F_X(x_3) &= P(X \le x_3) \\
&= P(X \le x_2) + P(X = x_3) \\
&= F_X(x_2) + P(X = x_3) \\
&= \frac{117}{125} + \frac{8}{125} = 1
\end{aligned}
$$