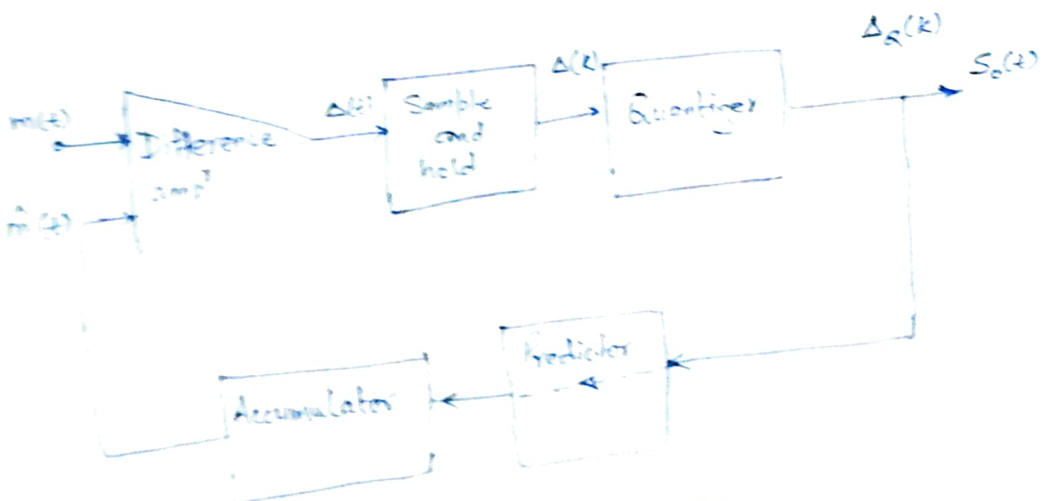
DIFFERENTIAL PULSE CODE MODULATION (DPCM):

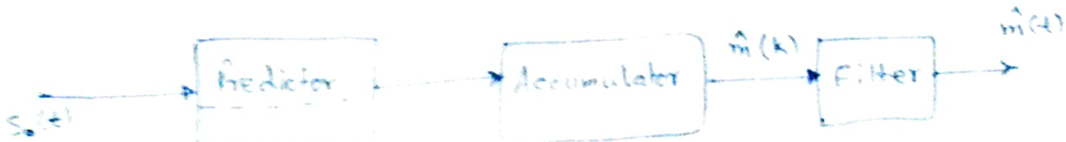
- In DPCM we transmit the difference between the sample value $m(k)$ at sampling time k and the sample value $m(k-1)$ ~~at~~ sampled at time instant $k-1$, at each sampling time.

If such changes are transmitted, then simply by adding up (accumulation) these changes we shall generate at the receiver a waveform $\hat{m}(t)$ identical to $m(t)$.

- The merit of DPCM is that the differences $m(k) - m(k-1)$ will be smaller than the sample value themselves. Hence fewer levels will be required to quantize and hence number of bits as well as required Bandwidth is reduced for same SQNR in comparison to PCM.

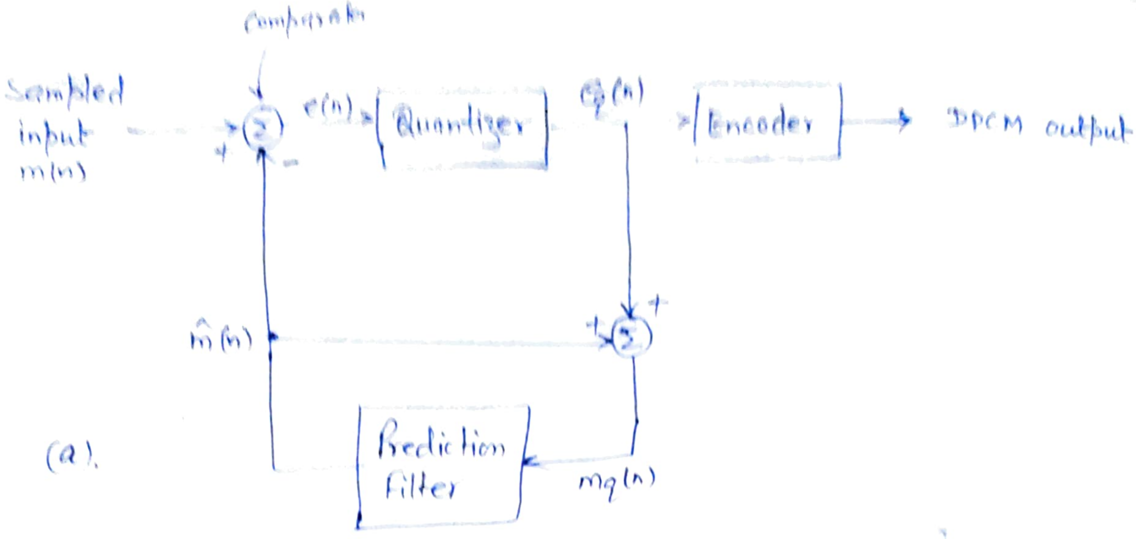


(a) Transmitter

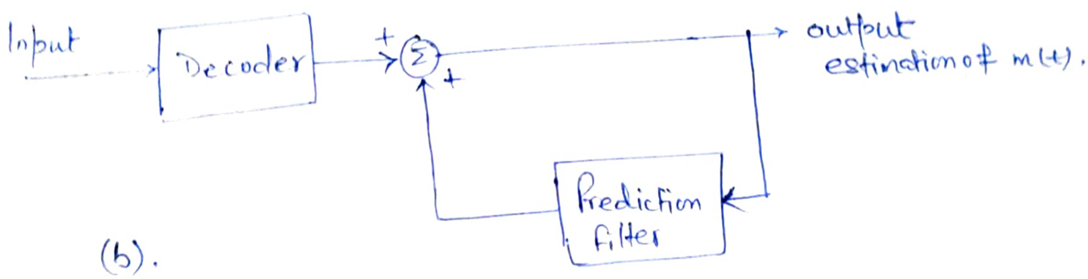


(b) Receiver

Figure. The DPCM system (Taub, Schilling)



(a).



(b).

Figure DPCM system (a) Transmitter (b) Receiver. (Symon Haykin)

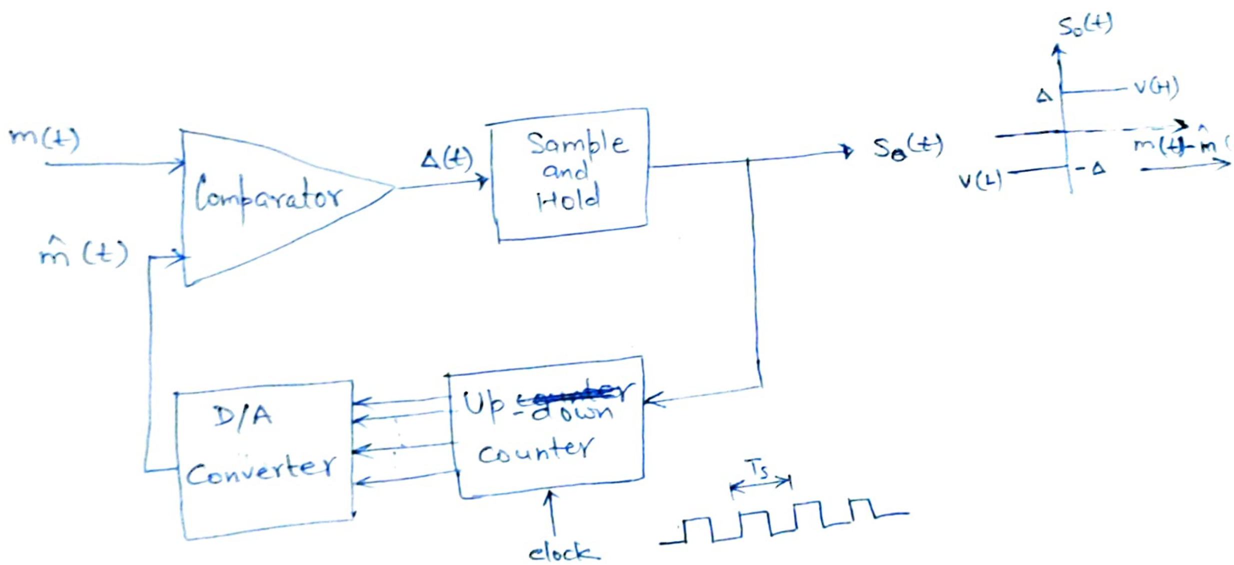
DELTA MODULATION :

- Delta Modulation (DM) is a DPCM scheme in which the difference signal $\Delta(t) = m(k) - m(k-1)$ in encoded into just a single bit.

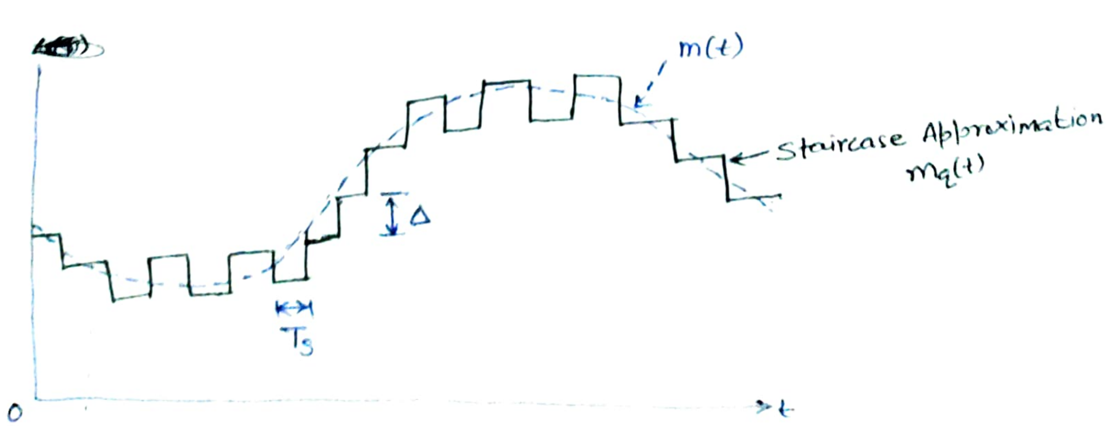
The single bit corresponding to two quantization level $(\pm\Delta)$

- The comparator has one fixed output $V(H)$ (corresponding to $+\Delta$) when $m(t) > \hat{m}(t)$ and $V(L)$ (corresponding to $-\Delta$) when $m(t) < \hat{m}(t)$

- The maximum quantization error $= \Delta$.



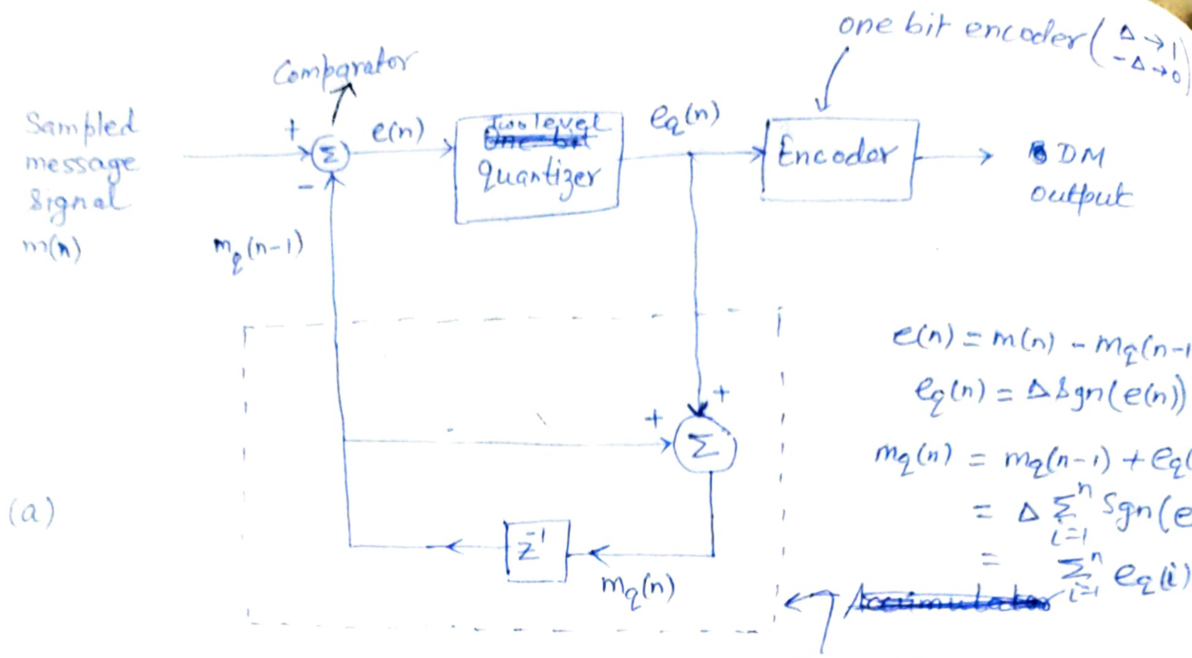
Linear Delta Modulator Block diagram



Binary sequence at modulator output

0 0 1 0 1 0 1 1 1 1 0 1 0 1 0 0 0

Illustration of delta Modulation



$$e(n) = m(n) - m_q(n-1)$$
$$e_q(n) = \Delta \operatorname{sgn}(e(n))$$
$$m_q(n) = m_q(n-1) + e_q(n)$$
$$= \Delta \sum_{i=1}^{n} \operatorname{sgn}(e[i])$$
$$= \sum_{i=1}^{n} e_q(i)$$

~~Accumulator~~

Accumulator

$$m_q(n) = m(n) + q(n)$$
$$e(n) = m(n) - m(n-1) - q(n-1)$$

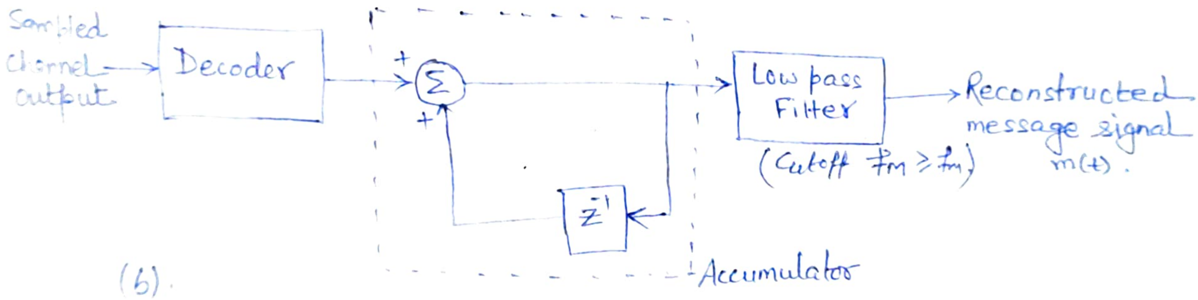
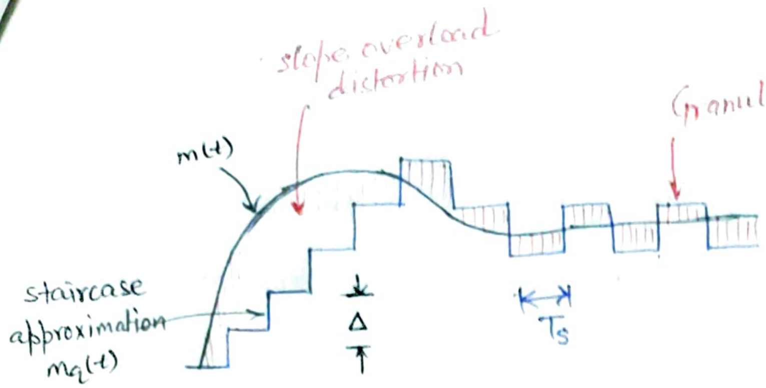


Figure DM system (a) Transmitter (b) Receiver



$T_s\left(=\frac{1}{f_s}\right) \rightarrow$ sampling interval

Fig: Illustration of the two different forms of quantization error in DM.

- Delta modulation is subject to two types of error:
  (1) Slope overload distortion (2). Granular noise

- Slope overload error (distortion): occurs when the step size, Δ is not sufficiently large, to ~~eath~~ follow the message signal properly. To avoid this, we required,

$$\boxed{\frac{\Delta}{T_s} \geq \max\left|\frac{dm(t)}{dt}\right|}$$

slope of message signal, m(t)

slope of staircase approximation

- To avoid the slopoverload distortion, we need to either increase the step size, Δ or increase the sampling frequency, $f_s = \frac{1}{T_s}$

- For sinusoidal input, $m(t) = A_m \sin(2\pi f_m t)$.

$$\left|\frac{d\,m(t)}{dt}\right|_{max} = A_m 2\pi f_m \cos(2\pi f_m t)\Big|_{max} = A\cdot 2\pi f_m$$

Hence, $\frac{\Delta}{T_s} \geq A\, 2\pi f_m \Rightarrow \boxed{\Delta \geq \frac{2\pi A_m f_m}{f_s}}$

- In contrast to slope-overload distortion, granular noise occurs where message signal, $m(t)$ remains at constant level, ~~...~~ (ie. dc signal). ~~then~~ The staircase approximation level changes alternatively.

- The noise (Granular) corresponds to $e(n) = m(n) - m_q(n-1)$, is similar to quantization noise, in PCM, ie. $\sigma_Q^2 = \frac{\Delta^2}{12}$.
  noise power

- Note that unlike slope overload noise, Granular noise increases with increase in step size, Δ.

- Hence the best ~~m~~ way to decrease ~~th~~ both the noise we should increase the sampling frequency, f_s, while keeping Δ as small as possible.

- SNR in DM system:
  - We know that to avoid slope overload distortion

$$A_m < \frac{\Delta}{2\pi}\left(\frac{f_s}{f_m}\right)$$

$A_m \rightarrow$ Amplitude of message signal (sinusoidal).

- Therefore max output signal power, $P_{max} = \left(\frac{A_m}{\sqrt{2}}\right)^2 = \frac{A_m^2}{2} = \frac{\Delta^2 f_s^2}{8\pi^2 f_m^2}$

Now the pdf of quantization Noise, can be assumed to be uniformly distributed between $-\Delta$ to $+\Delta$; ~~where~~ Δ This means the max quantization error in DM (Granular Noise), $\epsilon_{max} = \pm\Delta$

Hence quantization Noise Power, $\sigma_q^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx$

$$= \int_{-\Delta}^{\Delta} \frac{1}{2\Delta} x^2\,dx = \frac{\Delta^2}{3}$$

- Normalized noise power at Rx LPF output, $N_q = \frac{\Delta^2}{3} \times \frac{f_M}{f_s}$

$f_M \rightarrow$ LPF cutoff freq.
$f_s \rightarrow$ sampling freq.
(assuming that Noise Power is distributed uniformly over freq band upto f_s.)

Hence $$\boxed{(SNR)_{DM} = \frac{P_{max}}{N_q} = \frac{3}{8\pi^2 f_M f_m^2 T_s^3}}$$

ADAPTIVE DELTA MODULATION :

- To overcome the quantization errors due to slope overload and granular noise, the step size Δ is made adaptive to variations in the input signal $x(t)$. Particularly in the steep segment of the signal the step size is increases. Similarly if the input is slow varying the step size is reduced. to minimize the granular noise.

- A particular simple rule to change the step size is given by

$$\boxed{\Delta_n = \Delta_{n-1}\, K^{\epsilon_n \times \epsilon_{n-1}}}$$

$$\epsilon_n \times \epsilon_{n-1} = \begin{cases} 1 \times 1 = 1 \rightarrow K \\ 1 \times -1 = -1 \rightarrow \frac{1}{K} \\ -1 \times 1 = -1 \rightarrow \frac{1}{K} \\ -1 \times -1 = 1 \rightarrow K \end{cases}$$

where ϵ_n is the output of the quantizer before being scaled by step size. K is some constant larger than 1.

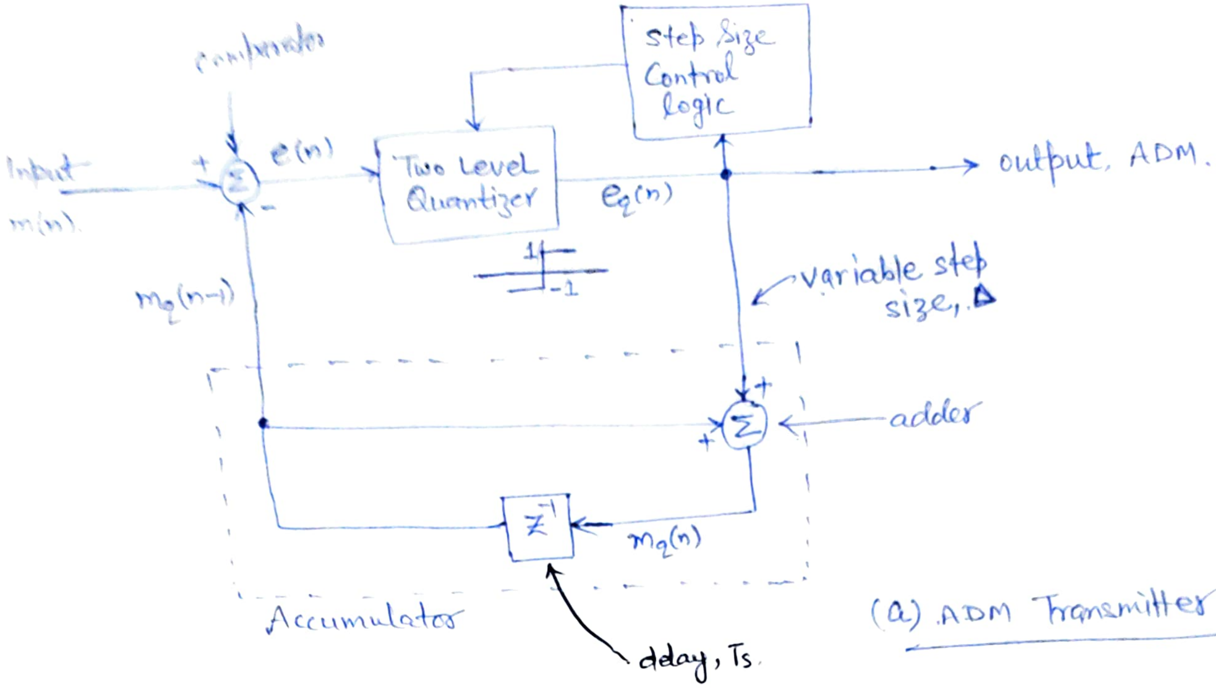
It has been verified that in the 20-60 kbits/sec range, with the choice of $K = 1.5$, the performance of Adaptive delta modulation system is 5-10 dB better than the performance of delta modulation for speech signal.

Adv

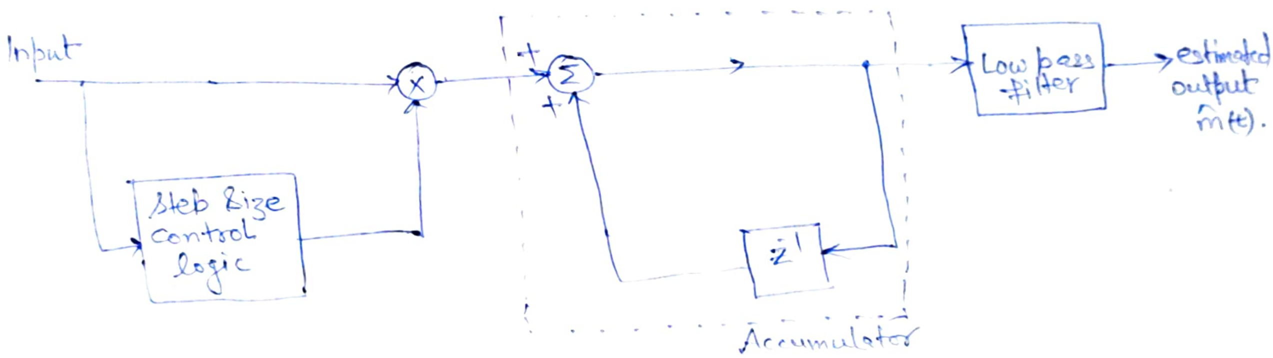
- Dynamic Range of ADM is wider than Conventional DM.
- Utilization of BW is better than delta modulation.

disadv

- Complexity is slightly higher than DM.



(a) ADM Transmitter



(b). ADM Receiver.