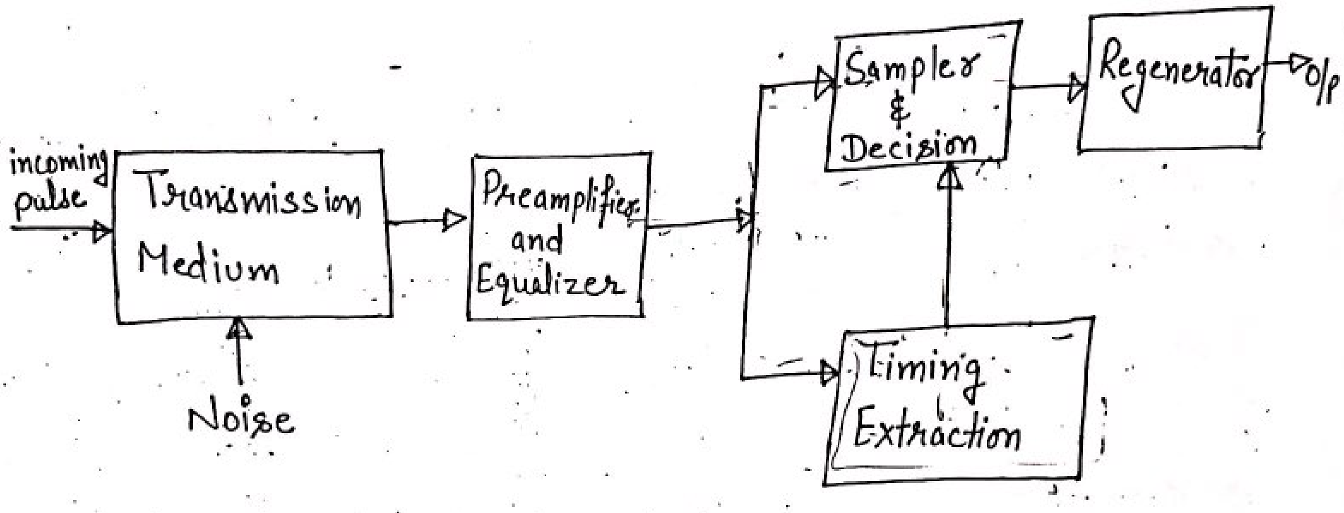
Digital Receiver :-

Digital Receiver performs basically three operations :-

1.) Reshaping the incoming pulses → through Equalizer

2.) Extracting the timing information → for sampling the incoming pulses at optimum instants.

3.) Making symbol detection decision → through sampled pulse.



Transmission Medium :-

As the medium is always unideal, therefore, noise is added to signal & the resulting signal is attenuated & distorted.

Preamplifier and Equalizer :-

Preamplifier and Equalizer receives an attenuated and distorted signal. It amplifies (boost up) the signal.

Preamplifier → compensate attenu...

Equalizer → compensate distortion.

Equalizer :-

Signal distortion occurs due to inter-symbol interference. Which results in pulse distortion.

Equalizer has the inverse characterstics to that of transmission medium.

It can restore the frequency components of pulse and removes distortion.

Preamplifier boost up the signal. As the signal also contains noise, then signal will become more noisy. This undesirable phenomenon is also called as Noise Amplification.

Therefore in this whole process, if distortion reduction is increased then noise amplification will also increase and hence a compromise is to be done between distortion reduction and noise amplification. and hence equalizer is designed accordingly.

The receiving signal need not to be received completely, as the signal at sampling instants is enough to be received, with the help of what, we can decide weather the signal received is '0' or '1' because in digital comm. signals are received in the form of '0' or '1' and hence it makes the work of equalizer easier.

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Equalizer

- Zero-forcing Equalizer
- Minimum Mean Square Error (MSME) Equalizer
- Adaptive Equalizer

Timing Extraction :-

There are 3 methods of Timing Extraction :-

1.) Derivation from a primary or secondary standard :-

In this method, a third device operates the transmitter and receiver.
Its main drawback is its high cost. but at the same time, it has a very high speed.

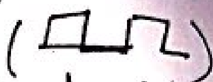
2.) Transmitting a separate synchronizing signal :-

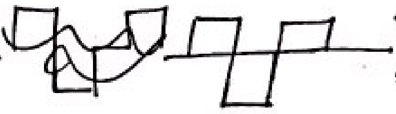
In this method, a pilot carrier is transmitted with the transmitting signal.
It is used only where the channel capacity in comparison to the transmitting data is more so that the carrier signal can also be transmitted through the same channel.
It uses more power but the cost is less.

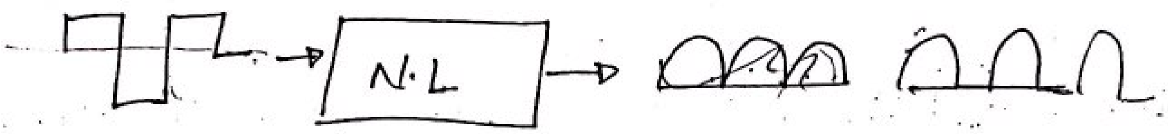
3.) Self Synchronization :-

In this method, timing information is extracted from the transmitted signal itself.
It consumes less power, low cost and better than the other two.

timing extraction.

In self-synchronization, for unipolar signals () (also called on-off signals) which contains discrete components, ~~timi~~ clock pulses are generated using these descrete components.

for bipolar signals () no discrete components are present. Hence a Null-Linear device is used which converts the bipolar signals into d.c signal with negative voltage removed. and then clock pulses are generated.



In timing extraction, timing jitter occurs. Timing jitter means variation of pulse position.

To reduce timing jitter, buffering is done using highly stable phase locked loop.

Generally at every 200 miles, timing jitter occurs.

Sampler and Decision :-

For sampling, time interval of the signal is known through timing extraction. Now sampling is done at the exact mid of the signal in the given time interval.

Let the signal be with max. amplitude Ap as,

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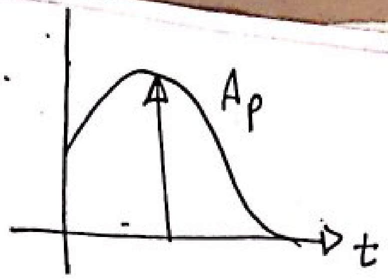
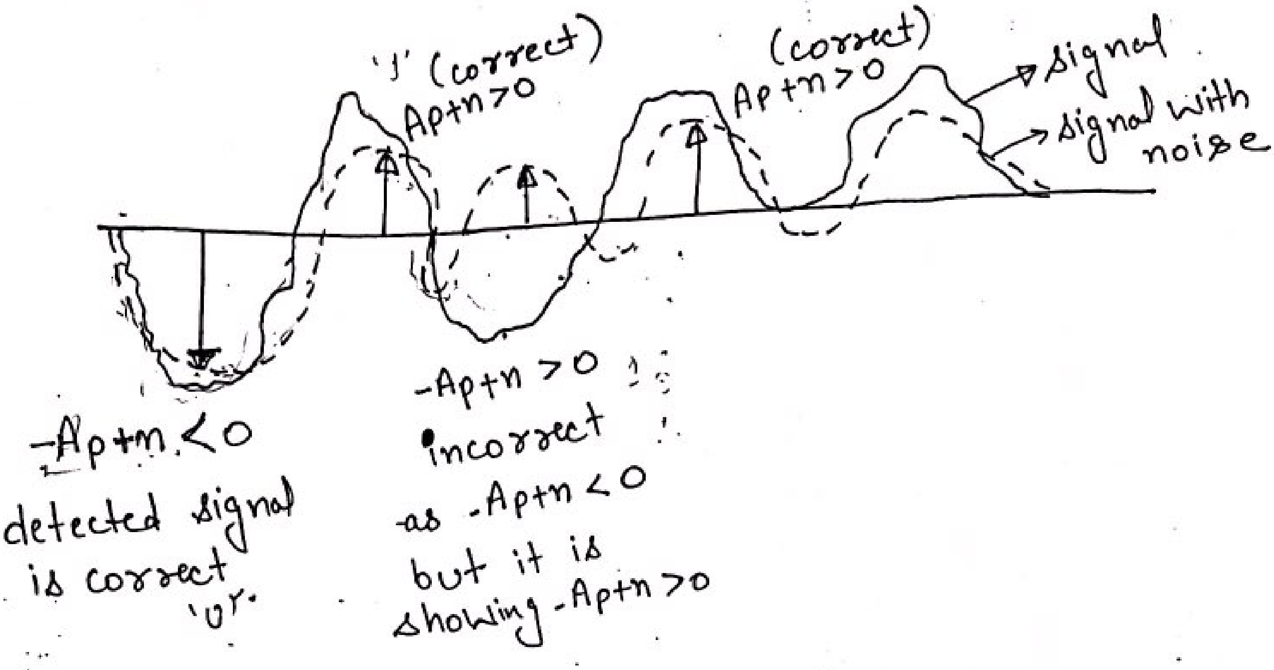
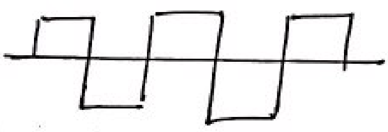




As the detected signal also contains noise, the signal with noise is compared with original and at the sampled instant and decision is made weather '0' is received or '1' is received.

Decision making is based on threshold level ($\pm A_p + n$).

<u>Regenerator</u> :- Regenerator regenerates the ~~recei~~ pulses according to the received data.

10101 → 

Optimum Filter (i.e. Optimum Receiver) :-

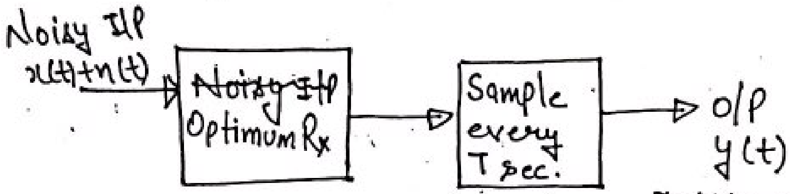
Optimum filters receive/recover the pulses with best possible S/N ratio and free from ISI.

Let the received signal be a binary waveform.

for binary '1' $\quad x_1(t) = +A$

for binary '0' $\quad x_2(t) = -A$

$\therefore$ I/P signal $x(t)$ will be either $x_1(t)$ or $x_2(t)$



$x_{01}(t) + n_0(t)$

$x_{02}(t) + n_0(t)$

in absence of noise,

$$y(t) = x_{01}(t) \quad \text{for } x(t) = x_1(t)$$

$$\& \quad y(t) = x_{02}(t) \quad \text{for } x(t) = x_2(t)$$

Here decisions are taken clearly.

If noise is present, then we select $x_1(t)$ if $y(t)$ is closer to $x_{01}(t)$ than $x_{02}(t)$ and we select $x_2(t)$ if $y(t)$ is closer to $x_{02}(t)$.

$\therefore$ Decision boundary will be in the middle of $x_{01}(t)$ and $x_{02}(t)$.

$$\Rightarrow \frac{x_{01}(t) + x_{02}(t)}{2}$$

Calculation of (P_e) for Optimum Filters :-

Let $x_2(t)$ was transmitted but $x_{01}(t) > x_{02}(t)$

if noise $n_0(T)$ is positive & larger compared to voltage difference $x_{01}(t) - x_{02}(T)$

$\therefore$ error will be generated if

$$n_0(T) \geq \frac{x_{01}(t) - x_{02}(T)}{2}$$

PDF of $n_0(T)$ is given as,

$$f_X[n_0(T)] = \frac{1}{\sigma\sqrt{2\pi}} e^{-[n_0(T)]^2/2\sigma^2}$$

To evaluate P_e, integrate

$$P_e = P\left[n_0(T) > \frac{x_{01}(t) - x_{02}(T)}{2}\right]$$

$$\Rightarrow \int_{\frac{x_{01}(T)-x_{02}(T)}{2}}^{\infty} f_X[n_0(T)]\, d[n_0(T)] = \int_{\frac{x_{01}(T)-x_{02}(T)}{2}}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-[n_0(T)]^2/2\sigma^2}\, d[n_0(T)]$$

Let $\dfrac{[n_0(T)]^2}{2\sigma^2} = y^2$

$$\Rightarrow n_0(T) = \sigma\sqrt{2}\, y \quad \Rightarrow \quad y = \frac{n_0(t)}{\sigma\sqrt{2}}$$

$$d[n_0(T)] = \sigma\sqrt{2}\, dy$$

When $n_0(t) = \infty$; $y = \infty$

When $n_0(t) = \dfrac{x_{01}(T) - x_{02}(T)}{2}$, $y = \dfrac{x_{01}(T) - x_{02}(T)}{2\sqrt{2}\,\sigma}$

Now,

$$P_e = \int_{\frac{x_{01}(T)-x_{02}(T)}{2\sqrt{2}\sigma}}^{\infty} \frac{1}{\cancel{\sigma}\sqrt{2\pi}} e^{-y^2}\, \cancel{\sigma}\sqrt{2}\, dy$$

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$$\Rightarrow \frac{1}{\sqrt{\pi}} \int_{\frac{x_{01}(T)-x_{02}(T)}{2\sqrt{2}\,\sigma}}^{\infty} e^{-y^2} dy$$

To solve this integration, let us use the following standard result,

$$\frac{2}{\sqrt{\pi}} \int_{u}^{\infty} e^{-y^2} dy = \text{erfc}(u)$$

$\therefore$ after rearranging the eq^n, we have

$$P_e = \frac{1}{2}\left[\frac{2}{\sqrt{\pi}}\right] \int_{\frac{x_{01}(T)-x_{02}(T)}{2\sqrt{2}\,\sigma}}^{\infty} e^{-y^2} dy = \frac{1}{2}\,\text{erfc}\left[\frac{x_{01}(T)-x_{02}(T)}{2\sqrt{2}\,\sigma}\right]$$

Transfer fu^n for Optimum Filters :-

Let $x_0(T) = x_{01}(T) - x_{02}(T)$

S/N ratio for optimum filters is given as

$$\left[\frac{S}{N}\right]_0 = \frac{x_0^2(T)}{\sigma^2}$$

$\therefore$ We have to maximize the ratio $\frac{x_0^2(T)}{\sigma^2}$. For this, transfer fu^n is to be found

$x_0^2(T)$ is normalized signal power.

$\sigma^2 = \overline{n_0^2(T)} = E[n_0^2(T)] \rightarrow$ normalized noise power

$$\left[\frac{S}{N}\right]_0 = \frac{x_0^2(T)}{\overline{n_0^2(T)}} \text{ or } \frac{x_0^2(T)}{E[n_0^2(T)]} \text{ or } \frac{x_0^2(T)}{\sigma^2}$$

$$x_0(T) = \int_{-\infty}^{\infty} H(f)\, X(f)\, e^{j2\pi f T}\, df$$

where $X(f) = \text{IFT}[x(t)]$

& $H(f) =$ Transfer fu^n

∴ I/P & O/P power spectral density of noise are related as,

$$S_{no}(f) = |H(f)|^2 S_{ni}(f)$$

Normalized noise power can be obtained by integrating PSD.

$$\sigma^2 = \int_{-\infty}^{\infty} S_{no}(f)\,df = \int_{-\infty}^{\infty} |H(f)|^2 S_{ni}(f)\,df$$

$$\therefore \left[\frac{S}{N}\right]_0 = \frac{\left|\int_{-\infty}^{\infty} H(f)X(f)e^{j2\pi fT}\,df\right|^2}{\int_{-\infty}^{\infty} |H(f)|^2 S_{ni}(f)\,df}$$

Acc. to Schwarz's inequality,

$$\left|\int_{-\infty}^{\infty} \theta_1(x)\,\theta_2(x)\,dx\right|^2 \le \int_{-\infty}^{\infty} |\theta_1(x)|^2\,dx \int_{-\infty}^{\infty} |\theta_2(x)|^2\,dx$$

Let $\theta_1(f) = \sqrt{S_{ni}(f)}\,H(f)$ and $\theta_2(f) = \frac{1}{\sqrt{S_{ni}(f)}}X(f)e^{j2\pi fT}$

then eq^n becomes,

$$\left[\frac{S}{N}\right]_0 = \frac{\left|\int_{-\infty}^{\infty} \theta_1(f)\cdot\theta_2(f)\,df\right|^2}{\int_{-\infty}^{\infty} |\theta_1(f)|^2\,df}$$

Applying Schwarz's inequality to the numerator of above eq^n,

$$\left[\frac{S}{N}\right]_0 \le \frac{\int_{-\infty}^{\infty} \cancel{|\theta_1(f)|^2\,df} \int_{-\infty}^{\infty} |\theta_2(f)|^2\,df}{\cancel{\int_{-\infty}^{\infty} |\theta_1(f)|^2\,df}}$$

$$\Rightarrow \left[\frac{S}{N}\right]_0 \le \int_{-\infty}^{\infty} |\theta_2(f)|^2\,df \le \int_{-\infty}^{\infty} \frac{1}{S_{ni}(f)} |X(f)e^{j2\pi fT}|^2\,df$$

$$|X(f)e^{j2\pi fT}|^2 = |X(f)|^2 \quad [\text{since}, |e^{j2\pi fT}| = 1]$$

$\therefore$

$$\left[\frac{S}{N}\right]_0 \le \int_{-\infty}^{\infty} \frac{|X(f)|^2}{S_{ni}(f)} df$$

$\frac{S}{N}$ ratio will be maximum when we consider equality.

$$\therefore \left[\frac{S}{N}\right]_{omax} = \int_{-\infty}^{\infty} \frac{|X(f)|^2}{S_{ni}(f)} df$$

$$\boxed{H(f) = k \cdot \frac{X^*(f)}{S_{ni}(f)} e^{-j2\pi fT}}$$

Matched Filter

A matched filter is a linear filter designed to provide max. $\frac{S}{N}$ ratio at its O/P for a given transmitted symbol waveform. For optimum filters, noise considered is generalized Gaussian noise. When this noise is white gaussian noise, then the optimum filter is known as matched filter.

Power spectral density of white gaussian noise :-

$$S_{ni}(f) = \frac{N_0}{2}$$

Transfer fun of optimum filter,

$$H(f) = k \cdot \frac{X^*(f)}{S_{ni}(f)} e^{-j2\pi ft}$$

Putting $S_{ni}(f) = \frac{N_0}{2}$, the transfer fun of optimum filter becomes transfer fun of Matched filter.

$$H(f) = 2k \cdot \frac{X^*(f)}{N_0} e^{-j2\pi ft}$$

From the property of F.T, $X^*(f) = X(-f)$

then, $H(f) = \frac{2k}{N_0} X(-f) e^{-j2\pi f t}$

$h(t)$ Impulse response of a matched filter can be evaluated by taking IFT of above eqⁿ i.e. $h(t) = IFT[H(f)]$

$$h(t) = IFT[H(f)] = IFT\left[\frac{2k}{N_0} e^{-j2\pi f t} X(-f)\right]$$

$$F.T[x(-t)] = X(-f)$$

$$F.T[x(T-t)] = X(-f) e^{-j2\pi f t}$$

$\therefore$ $h(t) = \frac{2k}{N_0}[x(T-t)]$

As, $x(t) = x_1(t) - x_2(t)$

$\therefore$ $h(t) = \frac{2k}{N_0}[x_1(T-t) - x_2(T-t)]$

Calculation of Probability of error (P_e) :-

For optimum filters,

$$P_e = \frac{1}{2} \operatorname{erfc}\left[\frac{x_{01}(T) - x_{02}(T)}{2\sqrt{2}\,\sigma}\right] \quad - (1)$$

In this eqⁿ, we have

$$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2_{max} = \int_{-\infty}^{\infty} \frac{|X(f)|^2}{S_{ni}(f)} df$$

& $S_{ni}(f) = \frac{N_0}{2}$

then, $$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2_{max} = \frac{2}{N_0}\int_{-\infty}^{\infty} |X(f)|^2 df \quad - (2)$$

Parseval's power theorem states that,

$$\int_{-\infty}^{\infty} |X(f)|^2 df = \int_{-\infty}^{\infty} x^2(t)dt = \int_0^T x^2(t)dt$$

We have taken limits from 0 to T because x(t) exists from 0 to T only. We know that $x(t) = x_1(t) - x_2(t)$

Hence, above equation becomes,

$$\int_{-\infty}^{\infty} |X(f)|^2 df = \int_0^T [x_1(t) - x_2(t)]^2 dt$$

$$\Rightarrow \int_0^T [x_1^2(t) + x_2^2(t) - 2x_1(t)x_2(t)]dt$$

$$\Rightarrow \int_0^T x_1^2(t)dt + \int_0^T x_2^2(t)dt - 2\int_0^T x_1(t)x_2(t)dt$$

$\int_0^T x_1^2(t)dt = E_1$ i.e. energy of $x_1(t)$

$\int_0^T x_2^2(t)dt = E_2$ i.e. energy of $x_2(t)$

$\int_0^T x_1(t)x_2(t) = E_{12}$ represents energy due to autocorrelation b/w $x_1(t)$ and $x_2(t)$

Now if we choose $x_1(t) = -x_2(t)$ then these energies will be equal, i.e., $E_1 = E_2 = -E_{12} = E$

$$\therefore \int_{-\infty}^{\infty} |X(f)|^2 df = [E + E - 2(-E)] = 4E$$

Substituting this value of $\int_{-\infty}^{\infty} |X(f)|^2 df$ in eq^n 2

$$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2_{max} = \frac{2}{N_0} \cdot 4E = \frac{8E}{N_0}$$

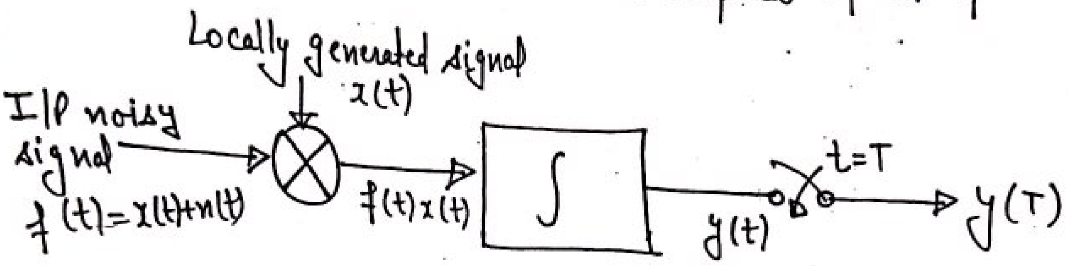
$$\Rightarrow \left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]_{max} = 2\sqrt{2}\sqrt{\frac{E}{N_0}}$$

Substituting the value in eq^n 1 $\Rightarrow$ $\boxed{P_e = \frac{1}{2} \text{erfc} \sqrt{\frac{E}{N_0}}}$

∴ error probability depends upon the signal energy. It does not depend on the shape of signal.

Correlator (Coherent Reception) :-

The mathematical operation of a correlator is correlation; a signal is correlated with a replica of itself.



[Block diagram of a correlator]

$f(t)$ represents i/p noisy signal

$$f(t) = x(t) + n(t)$$

The signal $f(t)$ is multiplied to the locally generated replica of i/p signal $x(t)$. Then, result of multiplication $f(t)\cdot x(t)$ is ~~generated~~ integrated. The o/p of the integrator is sampled at $t=T$. Then based on this sampled value, decision is made. This is how the correlator works. It is known as correlator because it correlates the received signal $f(t)$ with a stored replica of the known signal $x(t)$.

After integration, o/p ~~of~~ $y(t)$ will be

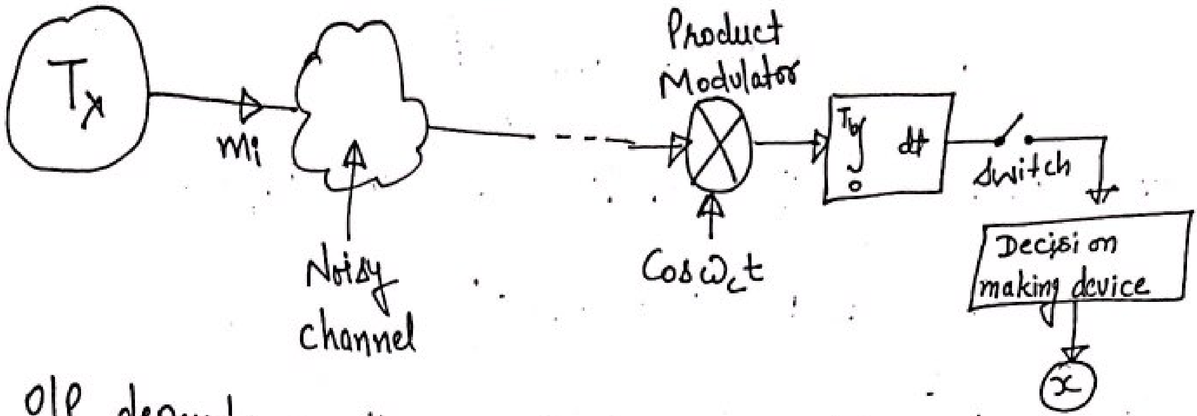
$$y(t) = \int_0^T f(t)\,x(t)\,dt$$

At $t = T$

$$y(T) = \int_0^T f(t)\,x(t)\,dt \,. \quad [\text{o/p of Correlator}]$$

2. Maximum Likelihood Receiver :-

Reception of Digital Rx :-



O/P depends on the amount of noise.

After multiplying to some carrier, it is passed through LPF which is an integrator.

$T_b \rightarrow$ bit interval

L.P.F converts the digital variation into analog variation.

Switch will be on when signal comes.

By this mechanism, we nullify the effect of noise to a great extent.

Decision making device will make decision weather the signal transmitted is in favour of '0' or '1'.

$x \longrightarrow$ '0' or '1'

Max. Likelihood Rx is a nature of probability means there is some sense of possibility.

This probability is given by some probability functions or we can say likelihood functions (denoted by $f_x(x/m_i)$).

$f_x(x/m_i)$ = prob. of receiving some symbol x when m_i is transmitted.

This kind of probability has to be maximized.

$$f_x(x/m_i) = L[m_i]$$

$L[m_i] \rightarrow$ Likelihood fun for m_i.

Because of noisy channel, probability of error (P_e) is,

$$P_e = P_e[m_i/x] = P[m_i \text{ not sent}/x]$$

$$P_e = 1 - P[m_i \text{ sent}/x]$$

Max. Likelihood R_x will work over certain optimum decision rules which says that,

1.) estimated message $(\hat{m}) = m_i$

if $P[m_i/x] \geq P[m_k \text{ sent}/x]$

means if you want to search m_i out of x, then prob. of sending m_i should be greater than prob. of sending any other k signals because this T_x can transmit any symbol $[m_1, m_2, m_3 \text{--} m_i \text{----} m_k]$

2.) estimated message $\hat{m} = m_i$

if we can maximize

$$P_k\left[\frac{f_x(x/m_i)}{f_x(x)}\right] \text{ is maximum for } k = i$$

means $f_x(x/m_k)$ is valid for all the possible values but it is maximum when we are concentrating over particular values of transmitting signal. There are certain things which we keep in mind while applying this rule ...

$f_X(x)$ is Independent of transmitting symbol m_k.
k can be any value. It can also be i.

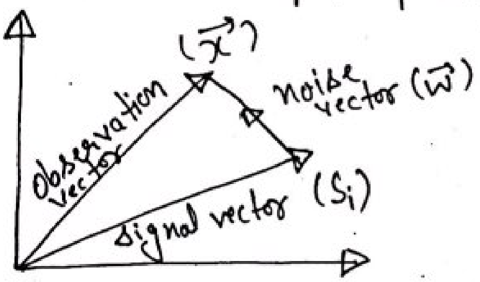
* P_k (Priori probability) = P_i (When all the source symbols are transmitted with equal probabilities).

* $f_X(x/m_k)$ is in one to one relation with $\log(m_k)$.

__Detection Principle__ :-

When we receive some receive some vector, then because of noisy channel, we get a noisy cloud and out of that noisy cloud, we will detect our main signal.

This cloud is a cluster of many symbols, out of which main symbol is detected (complex process).



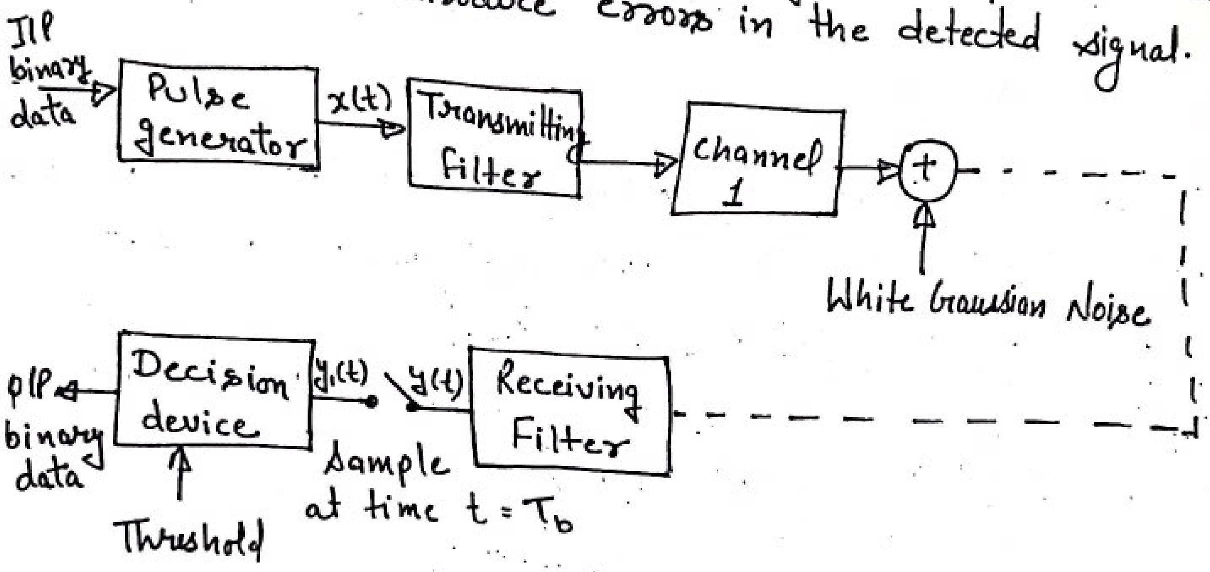
Apply trianglar law of vector addition,

$$\boxed{\vec{x} = \vec{S_i} + \vec{w}}$$

After this logic of maximization is applied, decision making device will work on max. likelihood logic.

ISI (Intersymbol Interference) :-

In a communication system, when the data is being transmitt in the form of pulses, the o/p produced at the receiver due to other bits or symbols interferes with the o/p produced by the desired bit. This is known as intersymbol interference (ISI).
The ISI will introduce errors in the detected signal.



(Baseband binary data transmission system)

The I/P signal consists of a binary data sequence (b_K) with a bit duration of T_b seconds.
This sequence is applied to a pulse generator to produce a discrete PAM signal given as

$$x(t) = \sum_{k=-\infty}^{\infty} a_K \, v(t - kT_b)$$

where $v(t)$ denotes the basic pulse normalized such that $v(0) = 1$.
Pulse Amplitude Modulator converts the I/P sequence into polar form as under :

if $b_K = 1$, $a_K = 1$

$b_K = 0$, $a_K = -1$

The PAM signal $x(t)$ is then passed through a transmitting filter. O/P of the filter is then transmitted over transmission channel. Let the impulse response of this channel be $h(t)$.

2. A random noise is then added to the signal when travels over the channel. Thus, the signal received at the receiving end is contaminated with noise.

3. The channel O/P is passed through a receiving filter. This filter O/P is sampled synchronously with the Tx.

4. The sequence of samples obtained at the O/P of receiving filter is used to reconstruct the original data sequence with the help of decision making device.

5. Each sample is compared to a threshold level then it is decided ~~th~~ weather symbol '1' or '0' is received.

Receiving filter O/P can be written as,

$$y(t) = \mu \sum_{k=-\infty}^{\infty} a_k \, p(t - kT_b) + n(t)$$

where μ is a scaling factor and noise $n(t)$ is the noise. $p(t-kT_b)$ represents the combined impulse response of the receiving filter.

$y(t)$ is sampled at time $t_i = iT_b$,

$$y(t_i) = \mu \sum_{k=-\infty}^{\infty} a_k \, p(iT_b - kT_b) + n(t_i)$$

or

$$\boxed{y(t_i) = \mu a_i + \mu \sum_{k=-\infty}^{\infty} a_k \, p(iT_b - kT_b) + n(t_i)}$$

This is the Rx O/P $y(t)$ at instant $t = t_i$

The eq^n has two terms

1. 1st term μa_i is produced by the i^{th} transmitted bit. Only this term should be present.
2. IInd term represents the residual effect of all the transmitted bits. This residual effect is known as ISI.

Factors responsible for ISI :-

The ISI arises due to the imperfections in the overall frequency response of the system. When a short pulse of duration T_b seconds is transmitted, the pulse appearing at the o/p of the system will be dispersed over an Interval which is longer than T_b seconds. Due to this dispersion, symbols each of duration T_b will interfere with each other. This will result in the ISI.

Effects of ISI :-

1. In the absence of ISI and noise, the transmitted bit can be decoded correctly at the receiver.
2. The presence of ISI will introduce errors in the decision at the R_x o/p.
3. Hence, the R_x can make an error in deciding weather it has received a logic '1' or a logic '0'.

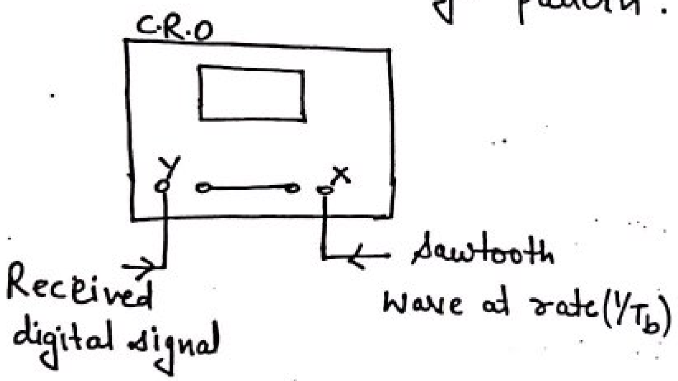
Remedy to reduce ISI :-

1. The function which produces a zero ISI is a sinc function. Hence, instead of a rectangular pulse, if we transmit a sinc pulse then ISI can be reduced to zero.

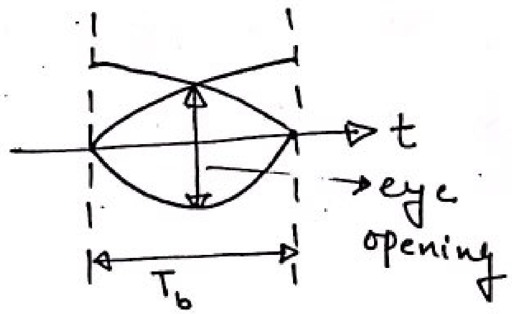
2.

Eye Diagram :-

Eye pattern is a pattern displayed on the screen of a CRO (Cathode Ray Oscilloscope). The shape of this pattern resembles with the shape of human eye. Hence, it is known as eye pattern. Eye pattern is a practical way to study the ISI and its effects on a PCM or data communication system. Eye pattern is obtained on the CRO interference by applying the received signal to vertical deflection plates (Y-plates) of the CRO and a sawtooth wave at the transmission symbol rate ($1/T_b$) to the horizontal deflection plates (X-plates). The resulting oscilloscope display is called as the eye pattern.



(Oscilloscope connections)



(Eye pattern seen on CRO screen)

The interior region of the eye pattern is known as the eye opening. Eye pattern provides a great deal of information about the performance of the system.

Information obtained from eye pattern :-

1. The width of the eye opening defines the time interval over ~~the~~ which the received wave can be sampled without an error due to ISI. The best time for sampling is when the eye is open widest.

2. The sensitivity of the system to the timing error is determined by the rate of closure of the eye as the sampling rate is varied. The height of eye opening at a specified sampling time defines the margin over noise.

3. When the effect of ISI is severe, the eye is completely closed and it is impossible to avoid errors due to the combined presence of ISI and noise in the system.

Orthogonal Representation of a Signal :-

Orthogonal Representation of signal is a useful technique to represent any arbitrary signal in terms of orthogonal basis functions. It also leads to vector representation of signal, especially in digital communication, that simplifies estimation problems.

Let us consider a set of functions $g_1(x), g_2(x) ---- g_n(x)$, defined over the interval $x_1 \leq x \leq x_2$ and which are related to one another in the very special way that any two different ones of the set satisfy the condition

$$\boxed{\int_{x_1}^{x_2} g_i(x)\, g_j(x)\, dx = 0}$$

That is when we multiply two different functions and then integrate over the interval from x_1 to x_2, the result is zero.

A set of functions which has this property is described as being orthogonal over the interval x_1 to x_2.

Similarly, in terms of vectors, the scalar product of two vectors V_i and V_j is a scalar quantity V_{ij} defined as,

$$V_{ij} = |V_i||V_j| \cos(V_i, V_j) = V_{ji}$$

$|V_i|$ and $|V_j|$ are the magnitudes of the respective vectors and $\cos(V_i, V_j)$ is the cosine of angle b/w the two vectors. If it should turn out that $V_{ij} = 0$ then (ignoring the trivial cases in which $V_i = 0$ or $V_j = 0$) $\cos(V_i, V_j)$ must be zero & it means that the vectors are perpendicular to one another.

Optimum [illegible]

Hence vectors whose scalar product is zero are physically orthogonal to one another and, functions whose integrated product is zero are also orthogonal to one another.

Now consider that we have some arbitrary function $f(x)$ and that we are interested in $f(x)$ only in range from x_1 to x_2 i.e. & further suppose we write $f(x)$ as a linear sum of the functions $g_n(x)$ i.e.

$$f(x) = C_1 g_1(x) + C_2 g_2(x) + \cdots + C_n g_n(x) + \cdots$$

$C \rightarrow$ numerical coefficients

The orthogonality of g's make it very easy to calculate C_n.
To calculate C_n, multiply both side of eq^n by $g_n(x)$ & integrate over the interval of orthogonality.

$$\int_{x_1}^{x_2} f(x) g_n(x)\, dx = C_1 \int_{x_1}^{x_2} g_1(x) g_n(x)\, dx + C_2 \int_{x_1}^{x_2} g_2(x) g_n(x)\, dx + \cdots + C_n \int_{x_1}^{x_2} g_n^2(x)\, dx$$

Because of orthogonality, all terms become zero with a single exception,

$$\int_{x_1}^{x_2} f(x) g_n(x)\, dx = C_n \int_{x_1}^{x_2} g_n^2(x)\, dx$$

$$\Rightarrow C_n = \frac{\int_{x_1}^{x_2} f(x) g_n(x)\, dx}{\int_{x_1}^{x_2} g_n^2(x)\, dx}$$

The mechanism by which we use the orthogonality of fu^n to drain away all terms except the term that involves the

Coefficient we are evaluating is often called the 'orthogonality sieve'.

Next, suppose that each $g_n(x)$ is selected so that the denominator has the value,

$$\int_{x_1}^{x_2} g_n^2(x)\,dx = 1$$

then

$$C_n = \int_{x_1}^{x_2} f(x) g_n(x)\,dx$$

When the orthogonal functions $g_n(x)$ are selected like this, they are described as being normalized.
A set of functions which is both orthogonal and normalized is called an orthonormal set.

The Gram-Schmidt Procedure :-

The orthogonal set itself has only a finite no. of fu^ns and the Gram-Schmidt procedure ~~which we not~~ ~~describe~~ allows ~~us~~ to construct this orthogonal set.

Let there are time functions which are to be expanded : $S_1(t), S_2(t)$ ---- $S_N(t)$ and the orthonormal functions $u_1(t), u_2(t), ---- u_N(t)$.

$$S_1(t) = S_{11} u_1(t) + S_{12} u_2(t) + ---- S_{1N} u_N(t) \quad (1)$$

$$S_2(t) = S_{21} u_1(t) + S_{22} u_2(t) + ---- S_{2N} u_N(t) \quad (2)$$

$$\vdots$$

$$S_N(t) = S_{N1} u_1(t) + S_{N2} u_2(t) + ----- S_{NN} u_N(t)$$

$S_{ij} \rightarrow$ coefficient

or, $$S_i(t) = \sum_{j=1}^{N} S_{ij}\, u_j(t) \qquad i = 1, 2 \ldots N$$

The orthogonality of the functions $u(t)$ over the interval T is expressed by,

$$\int_T u_j(t)\, u_k(t)\, dt = \begin{cases} 1 & \text{if } j = k \\ 0 & \text{if } j \neq k \end{cases}$$

Step 1

in eq^n (1) set all the coefficients equal to zero except S_{11},

$$S_1(t) = S_{11}\, u_1(t)$$

Squaring & integrating,

Since $u_1(t)$ is to be normalized fo^n we find that,

$$S_{11} = \left[\int_T S_1^2(t)\, dt\right]^{1/2}$$

and $u_1(t) = S_1(t)/S_{11}$ can now be determined.

Step 2

in eq^n (2) set all coefficient to zero except first two, S_{21} and S_{22};

$$S_2(t) = S_{21}\, u_1(t) + S_{22}\, u_2(t)$$

Multiplying both sides by $u_1(t)$ & integrating over the interval T

$$\int_T S_2(t)\, u_1(t) = \int_T S_{21}\, u_1(t)\, u_1(t) + \int_T S_{22}\, u_2(t)\, u_1(t)$$

$\cancel{S_{21} =}$

$$\therefore \int_T S_2(t)\, u_1(t) = S_{21} + 0.$$

$$S_{21} = \int S_2(t)\, u_1(t)$$

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$S_{22}u_2(t) = S_2(t) - S_{21}u_1(t)$

Squaring and integrating,

$$S_{22}^2 \int_T u_2^2(t)\,dt = \int_T [S_2(t) - S_{21}u_1(t)]^2 dt$$

$$S_{22}^2 = \left[\int_T S_2(t) - S_{21}u_1(t)\right]^2 dt$$

$$S_{22} = \left[\left[\int_T S_2(t) - S_{21}u_1(t)dt\right]^2\right]^{1/2}$$

As S_{21} & S_{22} are known, $u_2(t)$ can be given as,

$$u_2(t) = \frac{1}{S_{22}}\left[S_2(t) - S_{21}u_1(t)\right]$$

Putting value of $u_1(t)$

$$= \frac{1}{S_{22}}\left[S_2(t) - \frac{S_{21}S_1(t)}{S_{11}}\right]$$

Step 3 :-

Continuing the pattern,

$$S_3(t) = S_{31}u_1(t) + S_{32}u_2(t) + S_{33}u_3(t)$$

$$S_{31} = \int_T S_3(t)u_1(t)\,dt$$

$$S_{32} = \int_T S_3(t)u_2(t)\,dt$$

$$S_{33} = \left[\left[\int_T S_3(t) - S_{31}u_1(t) - S_{32}u_2(t)\right]^2 dt\right]^{1/2}$$

&

$$u_3(t) = \frac{S_3(t) - S_{31}u_1(t) - S_{32}u_2(t)}{S_{33}}$$

Step 4

We will continue the procedure until we have used all N equations & shall finally have N orthonormal fun^ns $u_1(t), u_2(t) --- u_N(t)$.

We shall have also evaluated the coefficients S_{ij} needed to express the functions $S_1(t), S_2(t) --- S_N(t)$ in terms of the $u(t)$s.

herent Binary Amplitude Shift keying or N-OFF Keying

(I) Definition

Amplitude shift Keying (ASK) or ON-OFF Keying (OOK) is the simplest digital modulation technique. There is only one unit energy carrier & it is switched on or off depending upon the I/P binary sequence.

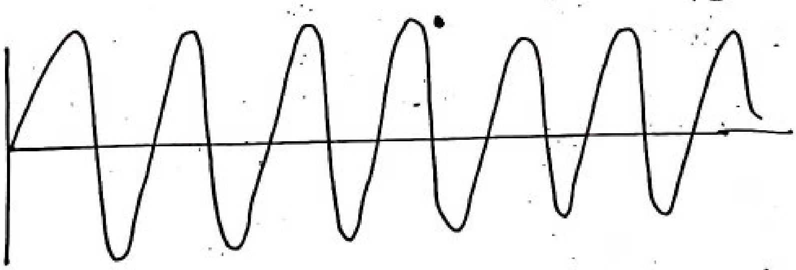
(II) Expression and Waveforms :-

The ASK Waveform may be represented as,

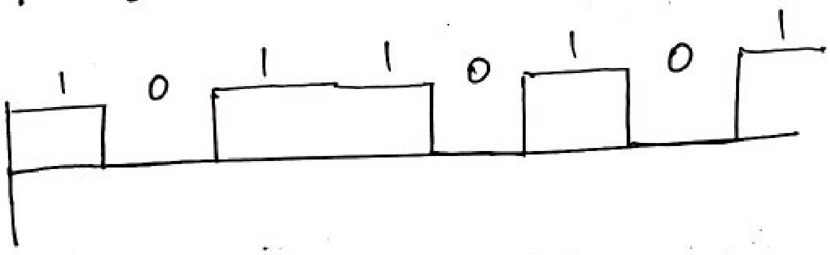
$$S(t) = \sqrt{2P_s}\, Cos(2\pi f_c t) \quad [\text{To transmit '1'}]$$

To transmit symbol '0', the signal $S(t) = 0$ i.e. no signal is transmitted.

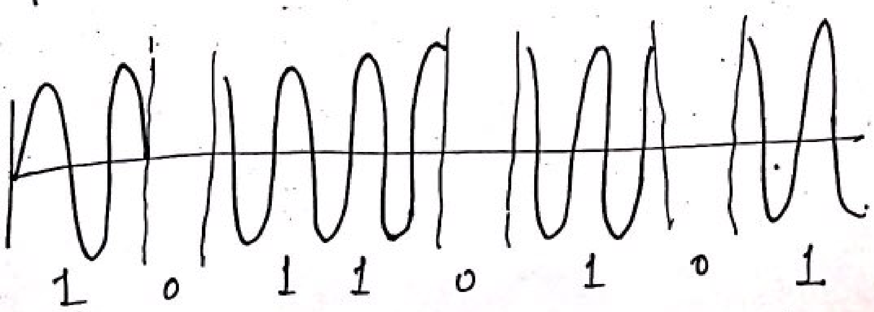
Hence ASK Waveform looks like an ON-OFF of the signal.



Unmodulated carrier



Bit sequence

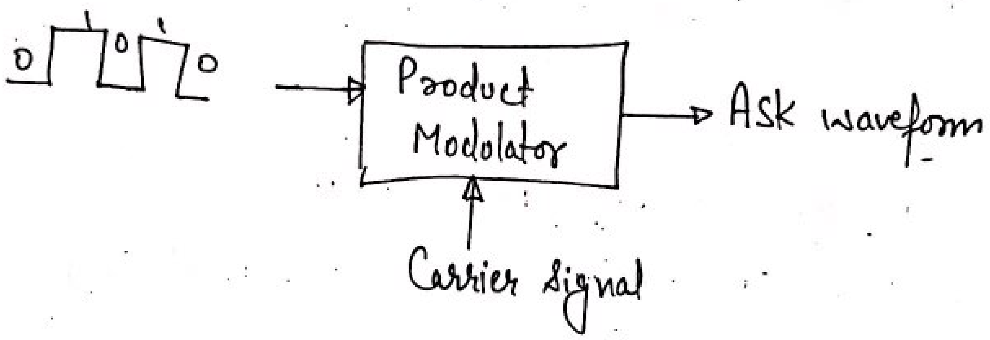


ASK Waveform

Generation of ASK signal

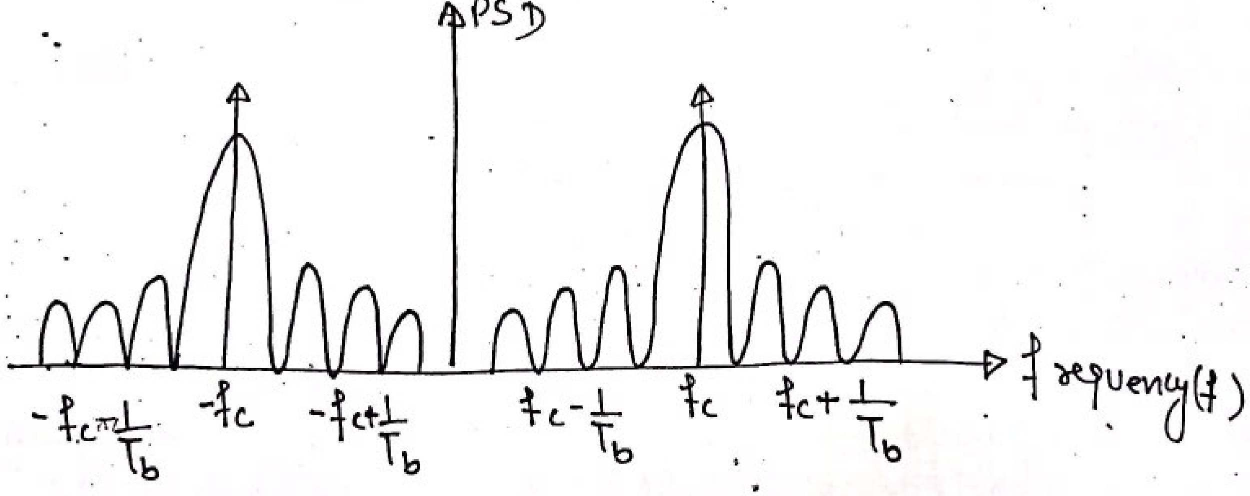
(1) Working Operation

ASK signal may be generated by simply applying the incoming binary data (in unipolar form) & the sinusoidal carrier to the two I/Ps of a product modulator. The resulting O/P will be the ASK waveform.



(2) Power Spectral Density

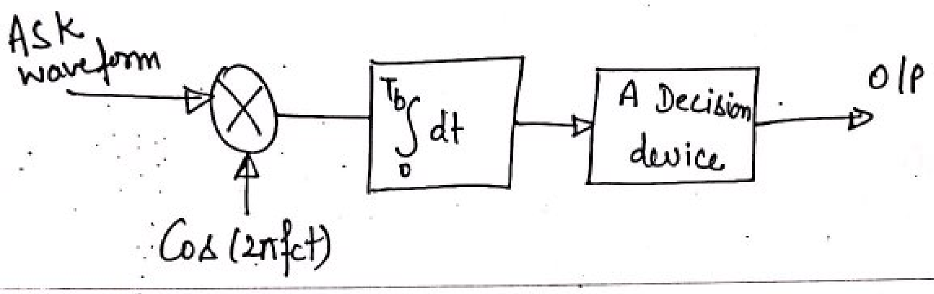
The ASK signal, which is basically the product of the binary sequence & carrier signal has PSD same as that of the baseband on-off signal but shifted in the frequence domain by $\pm f_c$. Two impulses occur at $\pm f_c$.



Bandwidth :- $(3/T_b)$

Spectrum shows, it has an infinite B.W., but for practical purpose, the B.W is centred at f_c whose o/p contains about 95% of total avg. power content of ASK signal. Acc. to this criterion, the B.W of ASK signal is approx. $\frac{3}{T_b}$. B.W can be reduced by using smoothed versions of pulse waveform instead of rectangular pulse waveforms.

BASK reception



Incoming ASK signal is applied to one i/p of the product modulator. The ~~o/p~~ other i/p of modulator is supplied with a sinusoidal carrier which is generated with the help of local oscillator. The o/p of product modulator goes to the i/p of integrator. The integrator operates on the o/p of the multiplier for successive bit intervals & performs a low pass filtering action. o/p of integrator goes to the i/p of Decision making device. This device compares the o/p of integrator with a preset threshold.

If threshold is exceeded → it makes a decision '1'
" " " lowered → " " " " '0'

In this, we have assumed that the local carrier is perfect synchronisation with carriers used in transmitter.

Synchronization Requirement :-

1.) Phase synchronization which ensures that carrier wave generated locally in the R_x is locked in phase w.r.t one that is employed in the T_x.

II) Timing synchronization which enable proper timing of the decision-making operation in the R_x w.r.t switching instants in the original binary data.

Salient feature of BASK :-

1.) Its simplicity
2.) It is easy to generate and detect.

Drawback :-

1.) It is very sensitive to noise.
2.) It is used at very low bit rates, upto 100 bits per sec.

Binary Phase Shift Keying (BPSK)

Definition :-

BPSK is the most efficient of the three modulations, ASK, FSK & PSK. It is used for high bit rates. In BPSK, phase of the sinusoidal carrier is changed according to the data bit to be transmitted. Also, a bipolar NRZ signal is used to represent the digital data.

Expression for BPSK :-

In BPSK, binary symbols '1' and '0' modulate the phase of the carrier. Let the carrier is given as,

$$s(t) = A\cos[2\pi f_c t]$$

$A \rightarrow$ peak value of sinusoidal carrier,

$$P = \frac{1}{2}A^2 \quad \text{(power dissipated by carrier signal)}$$

$$A = \sqrt{2P}$$

When the symbol is changed, then the phase of the carrier will also be changed by an amount of 180° (i.e. π radians).

For symbol '1', we have

$$S_1(t) = \sqrt{2P}\cos(2\pi f_c t)$$

If next symbol is '0' then

$$S_2(t) = \sqrt{2P}\cos(2\pi f_c t + \pi)$$

because $\cos(\theta+\pi) = -\cos\theta$, then

$$\Rightarrow S_2(t) = -\sqrt{2P}\cos(2\pi f_c t)$$

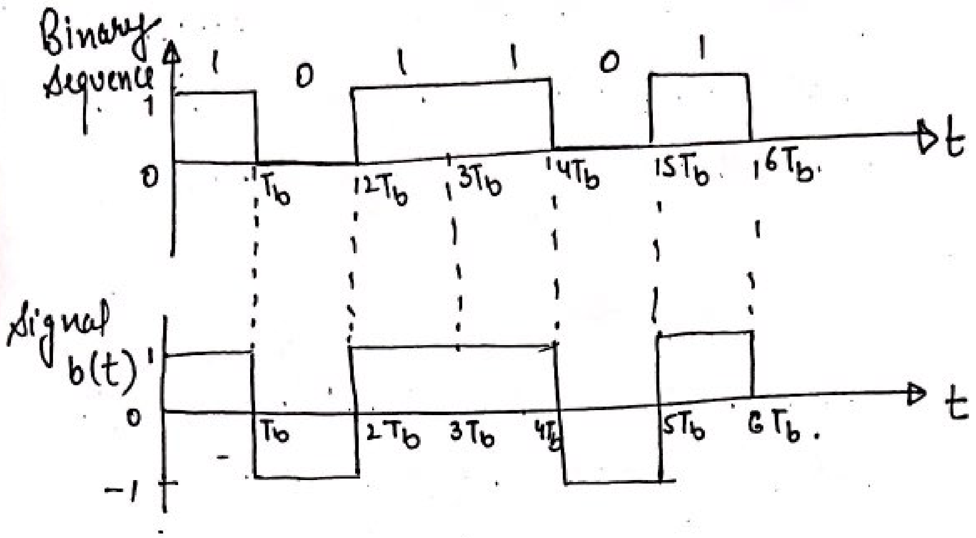
$\therefore$ We can write BPSK signal combinely as,

$$S(t) = b(t)\sqrt{2P}\cos(2\pi f_c t)$$

$b(t) = +1$ when binary '1' is to be transmitted

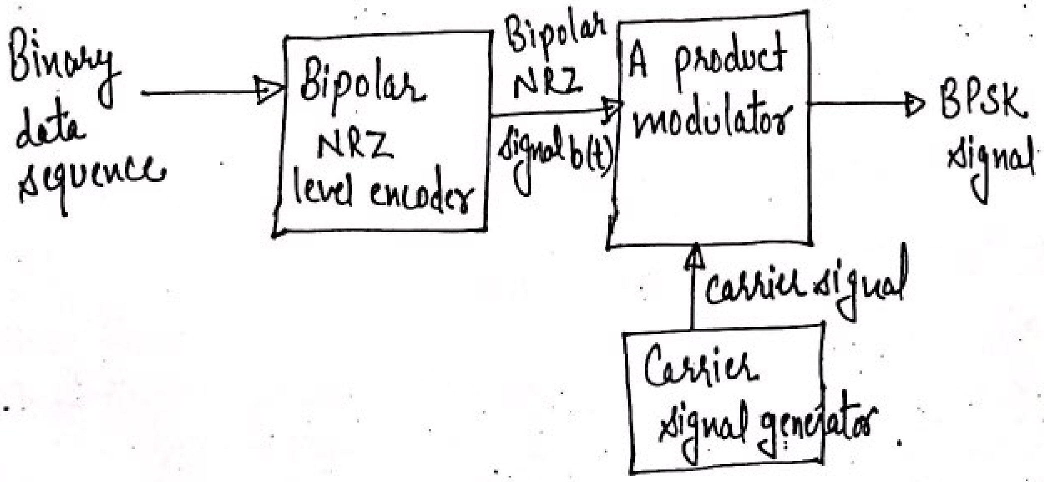
-1 " " '0' " " " "

Binary Sequence and its Equivalent Signal b(t) :-

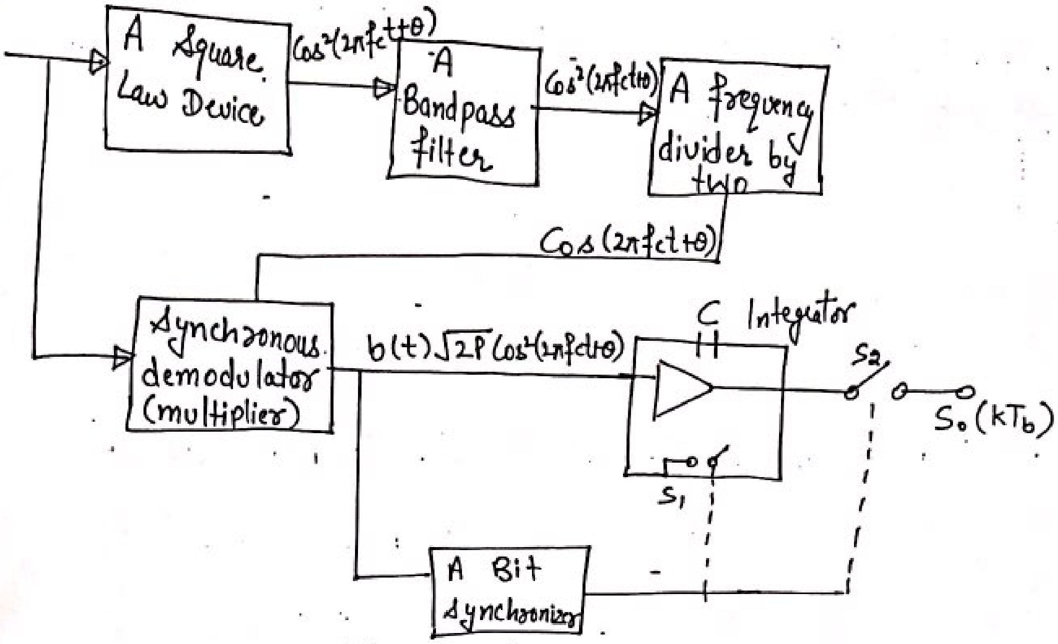


Generation of BPSK signal :-

BPSK signal may be generated by applying carrier signal to a balanced modulator. The binary data signal is converted into a NRZ bipolar signal by an NRZ encoder. Here, the bipolar signal b(t) is applied as a modulating signal b(t) to the modulator.



Reception of BPSK signal :-



The transmitted BPSK signal undergoes the phase change depending upon the time delay from transmitter end to receiver end. This phase change is a fixed phase shift in the transmitted signal.

Let this phase shift is θ.

$\therefore$ I/P of rx is, $s(t) = b(t)\sqrt{2P}\,Cos(2\pi f_c t+\theta)$

Now, from this received signal, a carrier is separated because this is coherent detection. The received signal is allowed to pass through a sq. law device. O/P of this device will be,

$$Cos^2(2\pi f_c t+\theta)$$

As $Cos^2\theta = \dfrac{1+Cos\,2\theta}{2}$

then, $Cos^2[2\pi f_c t+\theta] = \dfrac{1+Cos\,2(2\pi f_c t+\theta)}{2} = \dfrac{1}{2} + \dfrac{1}{2}Cos\,2(2\pi f_c t+\theta)$

Here, $\frac{1}{2}$ represents a DC level. This signal is then allowed to pass through a bandpass filter whose passband is centred around $2f_c$. B.P.F removes the d.c level of $\frac{1}{2}$, & the O/P will be,

$$\Rightarrow \cos 2(2\pi f_c t+\theta)$$

This signal is having freq. equal to $2f_c$. Now it is passed through freq. divider. O/P of freq. divider will be

$$\cos(2\pi f_c t+\theta)$$

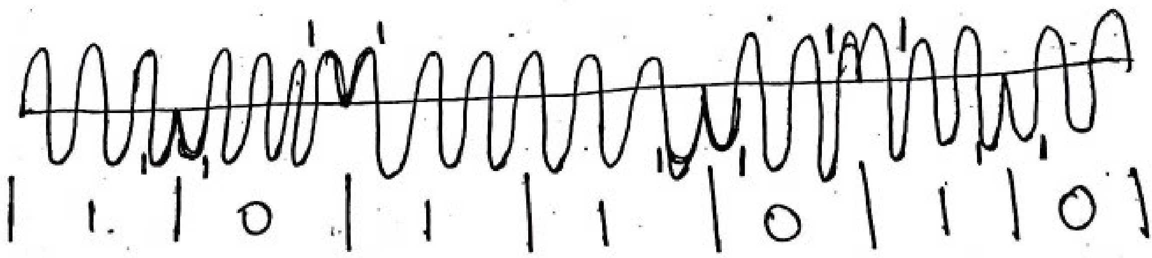
The synchronous demodulator multiplies the I/P signal & the recovered carrier. Hence O/P of multiplier will be,

$$b(t)\sqrt{2P}\cos(2\pi f_c t+\theta)\times\cos(2\pi f_c t+\theta) = b(t)\sqrt{2P}\cos^2(2\pi f_c t+\theta)$$

$$= b(t)\sqrt{2P}\times\frac{1}{2}\left[1+\cos 2[2\pi f_c t+\theta]\right]$$

$$= b(t)\sqrt{\frac{P}{2}}\left[1+\cos 2(2\pi f_c t+\theta)\right]$$

This signal is then applied to the bit synchronizer and integrator. Integrator integrates the signal over one bit period. The bit synchronizer takes care of starting and ending times of a bit. At the end of bit duration T_b, the bit synchronizer closes switch S_2 temporarily. This connects the o/p of an integrator to the decision device. The synchronizer then opens switch S_2 and switch S_1 is closed temporarily. This resets the integrator voltage to zero. The integrator then integrates the next bit.



the k^{th} bit interval, we can write o/p signal as under,

$$S_o(kT_b) = b(kT_b)\sqrt{\frac{P}{2}}\int_{(k-1)T_b}^{kT_b}\left[1+\cos 2(2\pi f_c t+\theta)\right]dt$$

$$\Rightarrow b(kT_b)\sqrt{\frac{P}{2}}\left[\int_{(k-1)T_b}^{kT_b} 1\,dt + \int_{(k-1)T_b}^{kT_b}\cos 2(2\pi f_c t+\theta)\,dt\right]$$

Avg value of sinusoidal waveform is zero if integration is done over full cycles.

$$\therefore \; S_o(kT_b) = b(kT_b)\sqrt{\frac{P}{2}}\int_{(k-1)T_b}^{kT_b} 1\; dt$$

$$\Rightarrow b(kT_b)\sqrt{\frac{P}{2}}\left[kT_b-(k-1)T_b\right] = b(kT_b)\sqrt{\frac{P}{2}}\,T_b$$

$\therefore$ O/P of the receiver depends on input,

$$S_o(kT_b) \propto b(kT_b)$$

Spectrum of BPSK Signals

As the waveform $b(t)$ is a NRZ binary waveform. In this waveform, there are rectangular pulses of amplitude $\pm V_b$. The Fourier Transform of this type of pulse is given as,

$$X(f) = V_b T_b \frac{\sin(\pi f T_b)}{(\pi f T_b)}$$

The power spectral density $S(f)$ is expressed as;

$$S(f) = \frac{|X(f)|^2}{T_s}$$

$T_s \rightarrow$ symbol duration.

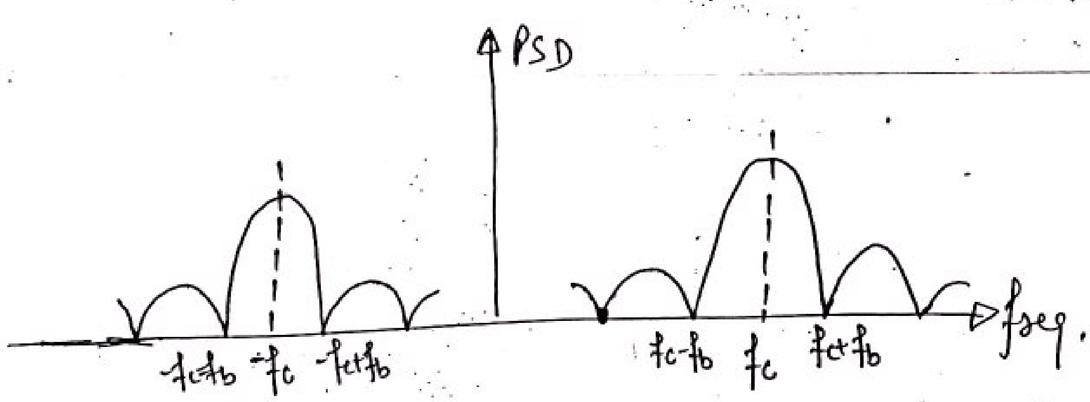
$$\Rightarrow S(f) = \frac{V_b^2 T_b^2}{T_s}\left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2$$

In BPSK only one bit is transmitted at a time, therefore symbol and bit durations are same, $T_b = T_s$

$$S(f) = V_b^2 T_b\left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2$$

This equation gives the P.S.D of signal $b(t)$. The BPSK signal is generated by modulating a carrier with signal $b(t)$. Due to modulation of the carrier of freq. 'f_c'. the spectral components are translated from f to f_c+f and f_c-f. The magnitude of these carriers is divided by half.

$$S_{BPSK}(f) = V_b^2 T_b\left\{\frac{1}{2}\left[\frac{\sin\pi(f_c-f)T_b}{\pi(f_c-f)T_b}\right] + \frac{1}{2}\left[\frac{\sin\pi(f_c+f)T_b}{\pi(f_c+f)T_b}\right]\right\}^2$$



Bandwidth :- $\quad B.W = f_c + f_b - (f_c - f_b)$

$$= \boxed{2f_b}$$

Salient features of BPSK :-

→ BPSK has a B.W lower than that of BFSK.
→ BPSK has best performance of all three modulation techniques.
→ good noise immunity.

Drawbacks :- To regenerate the carrier in the R_x, we start by squaring $b(t)\sqrt{2P}\cos(2\pi f_c t+\theta)$. If received signal is $-b(t)\sqrt{2P}\cos(2\pi f_c t+\theta)$, then the squared signal remains same as before.

→ It can be removed by using DPSK.

Binary Frequency Shift Keying (BFSK)

In BFSK, freq. of carrier is shifted according to binary symbol. This means that we have two frequency signals according to binary symbols. Let there be a frequency shift by Ω.

If $b(t)=$ '1' then $S_H(t)=\sqrt{2P_s}\cos(2\pi f_c+\Omega)t$

If $b(t)=$ '0' then $S_L(t)=\sqrt{2P_s}\cos(2\pi f_c-\Omega)t$

The eq^ns may be written as,

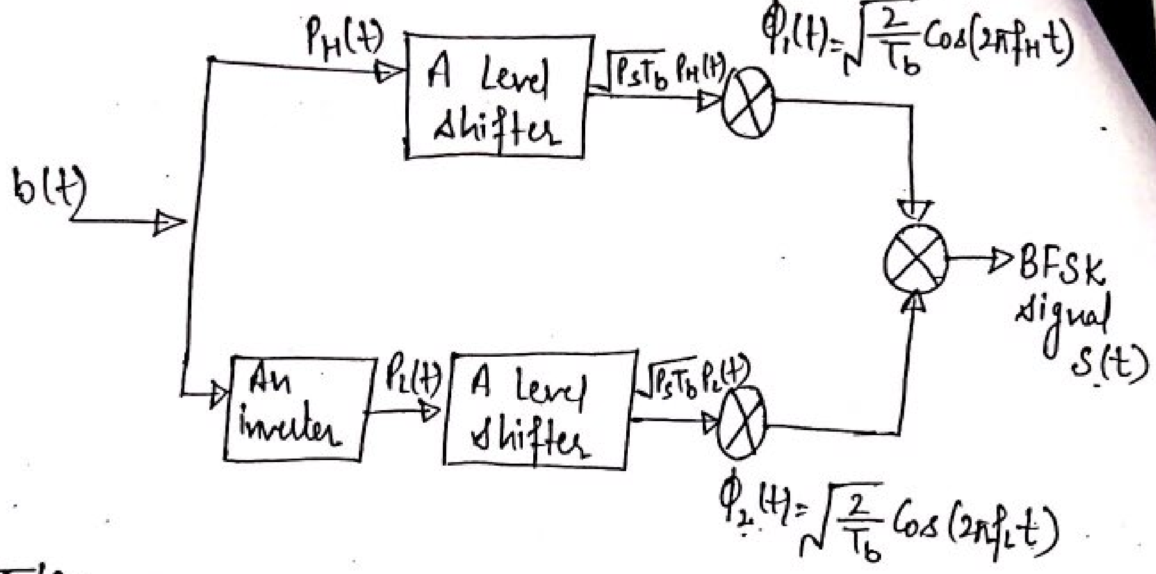
$$s(t)=\sqrt{2P_s}\cos\left[2\pi f_c+d(t)\Omega\, t\right]$$

If symbol '1' is to be transmitted, carrier freq. will be $f_c+\frac{\Omega}{2\pi}$ & if '0' is to be transmitted carrier freq. will be $f_c-\frac{\Omega}{2\pi}$.

Thus, $f_H = f_c+\frac{\Omega}{2\pi}$ for '1'

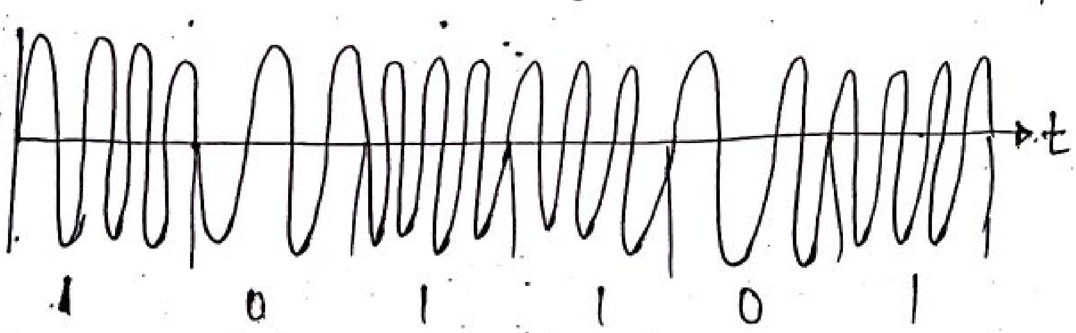
$f_L = f_c-\frac{\Omega}{2\pi}$ for '0'

Generation of BFSK :-



I/P sequence b(t) is same as $P_H(t)$. An inverter is added after b(t) to get $P_L(t)$. The level shifter P. converts the '+1' level to $\sqrt{P_sT_b}$, or zero. Thus, when a binary '0' is to be transmitted, $P_L(t) = 1$ and $P_H(t) = 0$ for binary '1', $P_H(t) = 1$ and $P_L(t) = 0$.

Hence transmitted signal will have frequency of either f_H or f_L. In product modulators, two carrier signals $\phi_1(t)$ & $\phi_2(t)$ are used. $\phi_1(t)$ and $\phi_2(t)$ are orthogonal to each other. The adder then adds the two signals.



...ctrum of BFSK Signal :-

BFSK signal $S(t)$ may be written as,

$$S(t) = \sqrt{2P_s}\, P_H(t) \cos(2\pi f_H t) + \sqrt{2P_s}\, P_L(t) \cos(2\pi f_L t)$$

Let us compare this equation with BPSK equation,

$$S_{BPSK}(t) = b(t)\sqrt{2P}\cos(2\pi f_c t)$$

In BPSK $b(t)$ is a bipolar signal whereas in BFSK, the similar coefficients $P_H(t)$ & $P_L(t)$ are unipolar.
Let us convert these coefficients in bipolar form,

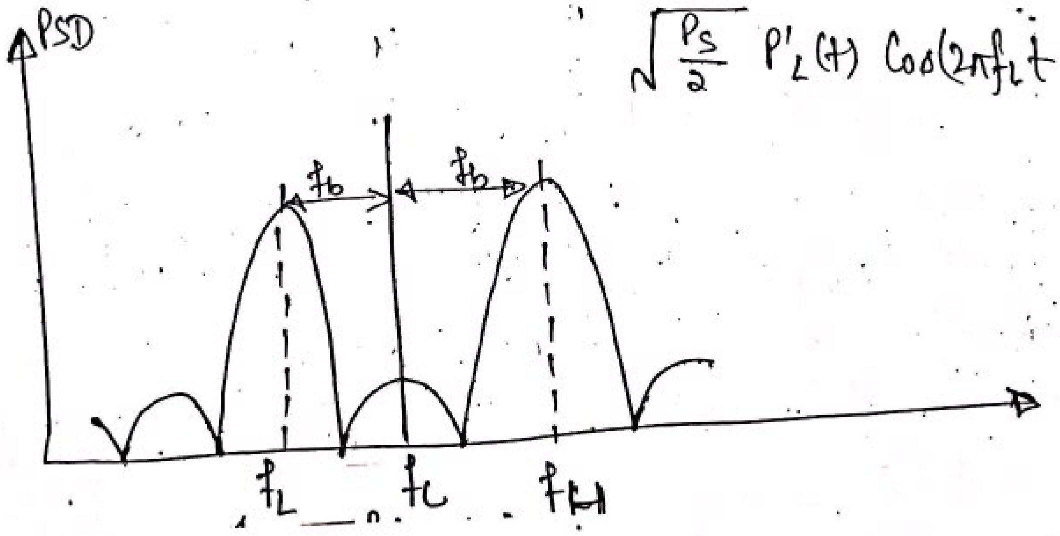
$$P_H(t) = \frac{1}{2} + \frac{1}{2}P'_H(t)$$

$$P_L(t) = \frac{1}{2} + \frac{1}{2}P'_L(t)$$

Where $P'_H(t)$ & $P'_L(t)$ are bipolar (ie +1 or -1)

$$\therefore\ S(t) = \sqrt{2P_s}\left[\frac{1}{2} + \frac{1}{2}P'_H(t)\right]\cos(2\pi f_H t) + \sqrt{2P_s}\left[\frac{1}{2} + \frac{1}{2}P'_L(t)\right]\cos(2\pi f_L t)$$

$$\Rightarrow \sqrt{\frac{P_s}{2}}\cos(2\pi f_H t) + \sqrt{\frac{P}{2}}\cos(2\pi f_L t) + \sqrt{\frac{P_s}{2}}P'_H(t)\cos(2\pi f_H t) + \sqrt{\frac{P_s}{2}}P'_L(t)\cos(2\pi f_L t)$$

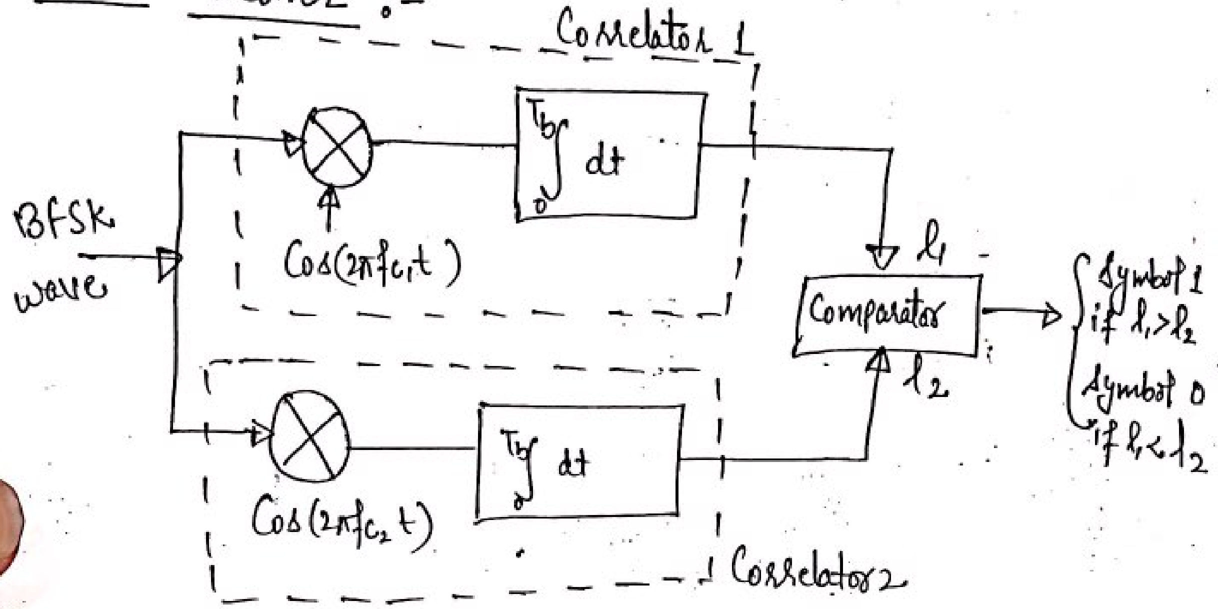


Bandwidth :-

$$B.W = 2f_b + 2f_b$$
$$= 4f_b.$$

$$BW(BFSK) = 2 \times BW(BPSK).$$

BFSK Receiver :-



The Receiver consists of two correlators that are individually tuned to two different carrier frequencies to represent symbols '1' and '0'. A correlator consists of a multiplier followed by an integrator.

Then, the received binary FSK signal is applied to the multipliers of both the correlators. To the other I/P of the multipliers, carriers with frequency f_{c_1} and f_{c_2} are applied. The multiplied o/p of each multiplier is subsequently passed through integrators generating output l_1 and l_2 in the two paths. The o/p of the two integrators are then fed to the cision making device which is a comparator. Comparator compares the o/p l_1 and l_2.

if $l_1 > l_2$, makes a decision in favour of '1'

if $l_1 < l_2$, " " " " " " '0'.

Salient features of BFSK :-

→ BFSK is relatively easy to implement

→ Better noise Immunity.

Drawback :-

→ High bandwidth

Quadrature Phase shift Keying (QPSK) :-

The modulation schemes discussed so far are all two level modulations, because they can represent only two states of the digital data (0 or 1). Hence bit rate and baud rate are same for these systems. We can keep the baudrate same and increase the bit rate by using multilevel modulation techniques. In this type of systems, the data groups are divided into groups of two or more bits and each group of bits is represented by a specific value of amp., freq. and phase of the carrier. QPSK is an example of such multilevel phase of modulation.

Phase Shift in QPSK :-

In QPSK, two successive bits in a bit stream are combined together to form a message and each message is represented by a distinct value of phase shift of carrier.

0 0	0 1	1 0	1 1
S1 symbol	S2 symbol	S3 symbol	S4 symbol.

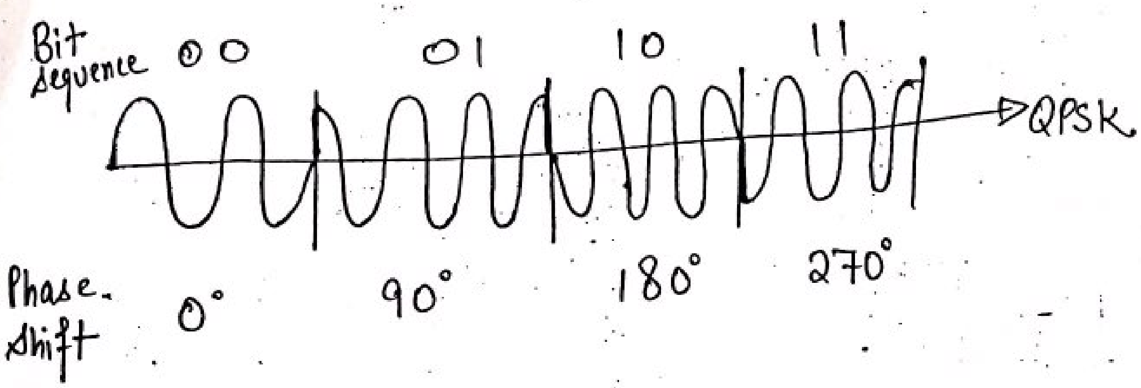
(Grouping of bits in QPSK.)

Symbol	Phase
0 0	0
0 1	90
1 0	180
1 1	270

(Phase shift in QPSK)

→ Each symbol contains two bits, Hence symbol duration $\boxed{T_s = 2T_b}$.

QPSK Waveforms :-



Alternative Representation of QPSK

Symbol	Bits	Phase shift in Carrier
S_1	0 0	$\pi/4$ radian
S_2	0 1	$3\pi/4$ rad.
S_3	1 0	$5\pi/4$ rad.
S_4	1 1	$7\pi/4$ rad.

Mathematical Representation of QPSK :-

$$V_{QPSK}(t) = \sqrt{2P_s}\,\cos\left[\omega_c t + (2m+1)\frac{\pi}{4}\right]$$

$$m = 0, 1, 2, 3$$

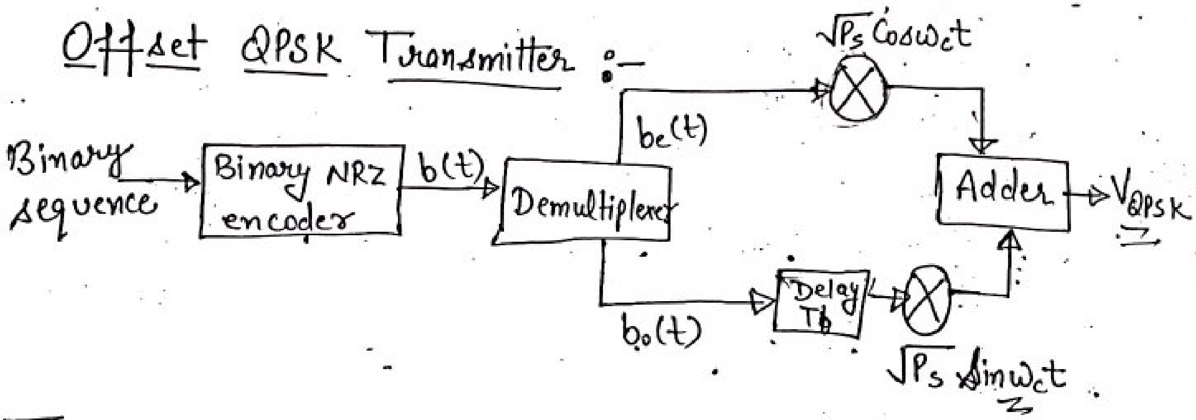
By substituting the values of m from 0 to 3, we get the four messages,

$$V_{QPSK} = S_1 = \sqrt{2P_s}\,\cos\left[\omega_c t + \frac{\pi}{4}\right] \quad \text{for } m=0$$

$$S_2 = \sqrt{2P_s}\,\cos\left[\omega_c t + \frac{3\pi}{4}\right] \quad \text{for } m=1$$

Similarly for $m=2$ & $m=3$.

Offset QPSK Transmitter :-



The input binary sequence is first converted into a bipolar NRZ signal, $b(t)$.

$$b(t) = +1 \quad \text{for logic '1'}$$
$$= -1 \quad \text{for logic '0'}$$

Demultiplexer will divide $b(t)$ into two separate bit streams named $b_o(t)$ and $b_e(t)$. $b_e(t)$ contains only even numbered bits i.e. 2, 4, 6 ---- & $b_o(t)$ " " odd " bits i.e 1, 3, 5 ----.

Filters :-

Optimum Filters .

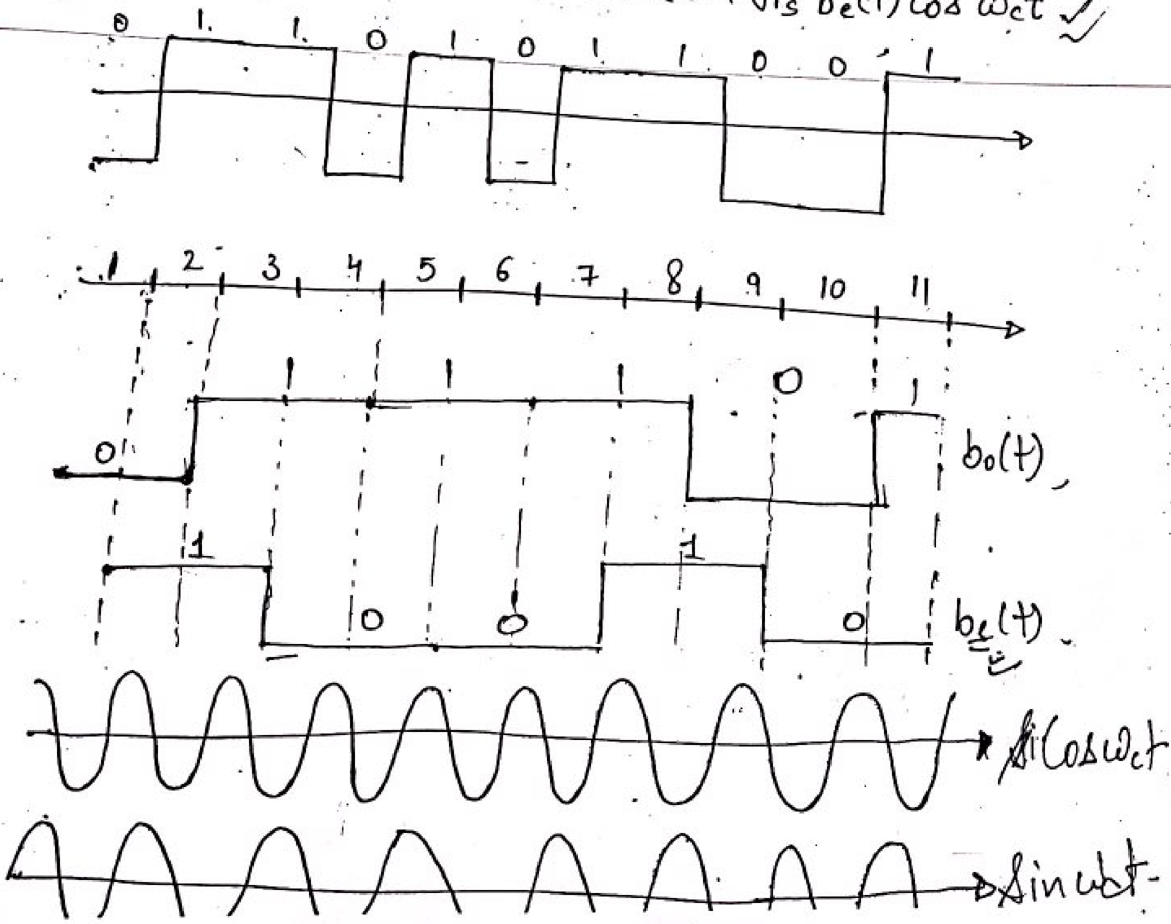
Each bit in the even or odd bit stream will be held for a period of $2T_b$.

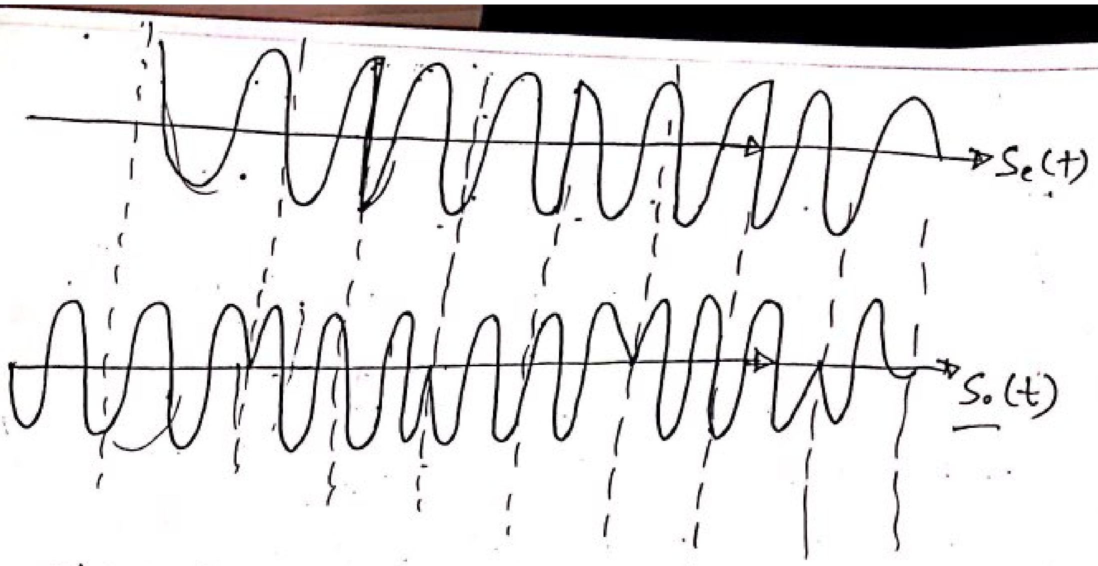
The first odd bit will occur before the first even bit. Hence even bit stream $b_e(t)$ will start with a delay of one bit period (T_b).

This delay is called as offset.

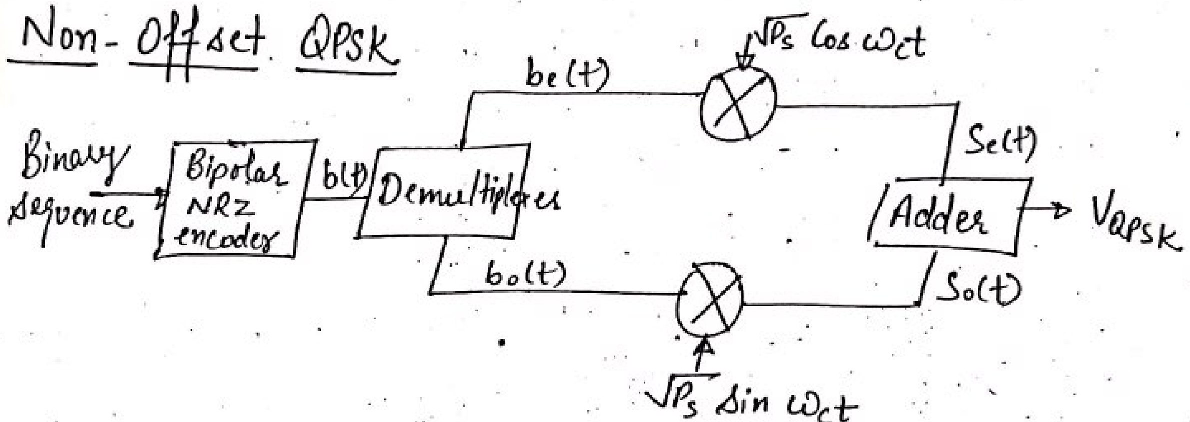
The bit streams $b_e(t)$ is superimposed on a carrier $\sqrt{P_s}\cos\omega_c t$ and $b_o(t)$ is superimposed on a carrier $\sqrt{P_s}\sin\omega_c t$ by the use of multipliers to generate two signals S_1 and S_2. These two signals are basically BPSK signals. These signals are then added together to generate QPSK output signal ($V_{QPSK}(t)$).

$$V_{QPSK}(t) = \sqrt{P_s}\, b_o(t)\sin\omega_c t + \sqrt{P_s}\, b_e(t)\cos\omega_c t$$



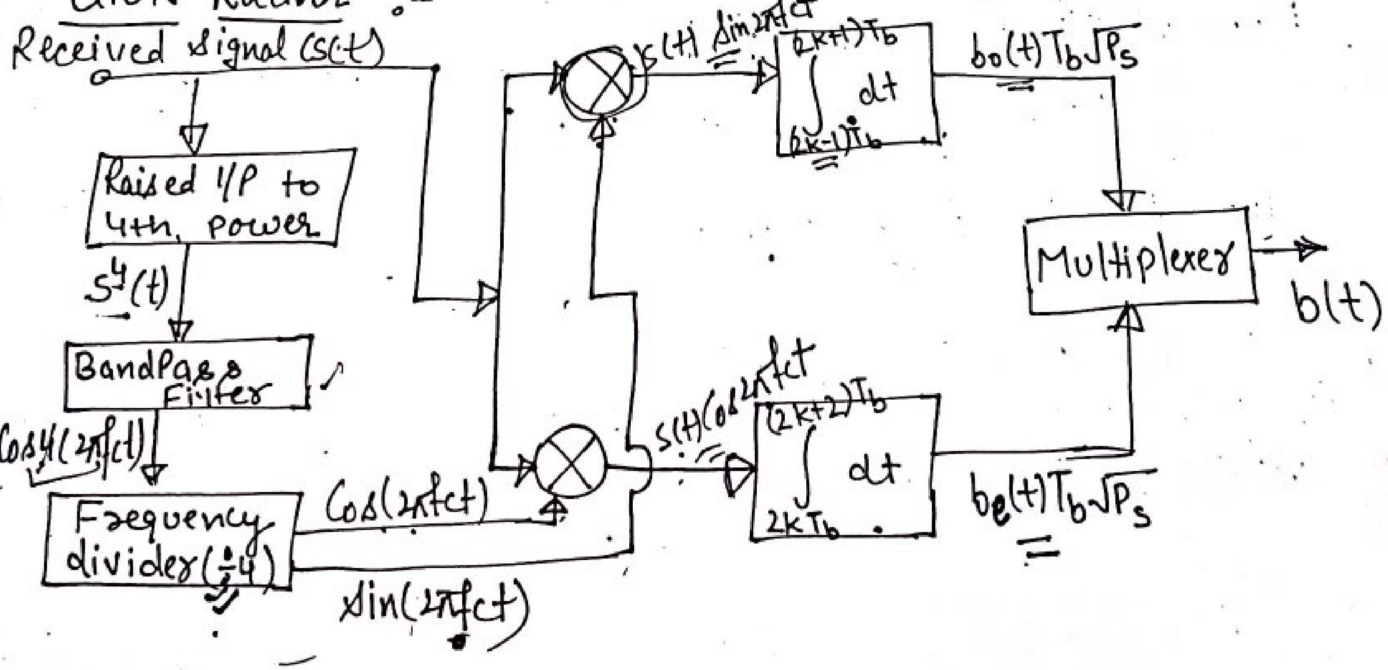


Non-Offset QPSK



The transmitter of the non-offset QPSK is exactly same as that of the offset QPSK except for one change. There will be no time delay in non-offset QPSK.

QPSK Receiver :-



~~$b[kT_b][kT_b - kT_b + T_b]$~~

~~$b(kT_b)[T_b]$. depending on value of $b(kT_b)$~~

The received QPSK signal $s(t)$ is raised to fourth power i.e. $s^4(t)$. This signal is then filtered by using a bandpass filter with a center frequency of $4\omega_c t$. The o/p of bandpass filter is $\cos 4\omega_c t$. A freq. divider which divides the freq. at the o/p by 4 generates two carrier signals $\sin\omega_c t$ and $\cos\omega_c t$. The incoming signal $s(t)$ is applied to two synchronous demodulators consisting of a multiplier followed by an integrator. Each integrator integrates over a two-bit interval of duration $T_s = 2T_b$.

The i/p to the integrator (upper) is given by,

$$s(t)\sin\omega_c t = b_o(t)\sqrt{P_s}\,\sin^2\omega_c t + b_e(t)\sqrt{P_s}\,\sin\omega_c t\cos\omega_c t$$

This integrator will integrate its i/p signal over a symbol period of $T_s = 2T_b$. Upper integrator o/p is given by,

$$= \int_{(2k-1)T_b}^{(2k+1)T_b} s(t)\sin\omega_c t\, dt$$

$$\Rightarrow b_o(t)\sqrt{P_s}\int_{(2k-1)T_b}^{(2k+1)T_b} \sin^2\omega_c t\, dt + b_e(t)\sqrt{P_s}\int_{(2k-1)T_b}^{(2k+1)T_b} \sin\omega_c t\cos\omega_c t\, dt$$

As $\sin^2\omega_c t = \frac{1}{2}[1 - \cos 2\omega_c t]$

& $\sin\omega_c t\cos\omega_c t = \frac{1}{2}\sin 2\omega_c t$

By substituting these values, we get

Integrator O/P $= \frac{1}{2} b_o(t)\sqrt{P_s}\int_{(2k-1)T_b}^{(2k+1)T_b}[1-\cos 2\omega_c t]dt + \frac{1}{2} b_e(t)\sqrt{P_s}\int_{(2k-1)T_b}^{(2k+1)T_b} \sin 2\omega_c t \, dt$

$$\Rightarrow \underbrace{\frac{1}{2} b_o(t)\sqrt{P_s}\int_{(2k-1)T_b}^{(2k+1)T_b} 1 \, dt}_{\mathrm{I}} - \underbrace{\frac{1}{2} b_o(t)\sqrt{P_s}\int_{(2k-1)T_b}^{(2k+1)T_b} \cos 2\omega_c t \, dt}_{\mathrm{II}} + \underbrace{\frac{1}{2} b_e(t)\sqrt{P_s}\int_{(2k-1)T_b}^{(2k+1)T_b} \sin 2\omega_c t \, dt}_{\mathrm{III}}$$

The value of 2nd & IIIrd term is ~~been~~ zero because integration of a sinusoidal signal over a period corresponding to its integral no. of cycles is zero.

$\therefore$ Integrator O/P $= \frac{1}{2} b_o(t)\sqrt{P_s}\int_{(2k-1)T_b}^{(2k+1)T_b} 1 \, dt$

$$\Rightarrow \frac{1}{2} b_o(t)\sqrt{P_s}\left[(2k+1)T_b - (2k-1)T_b\right]$$

$$\Rightarrow \frac{1}{\cancel{2}} b_o(t)\sqrt{P_s}\left[\cancel{2}T_b\right] \Rightarrow b_o(t)\sqrt{P_s}\, T_b.$$

Similarly, o/p of lower integrator is given by $b_e(t)\sqrt{P_s}\,T_b$.

Spectrum of QPSK :-

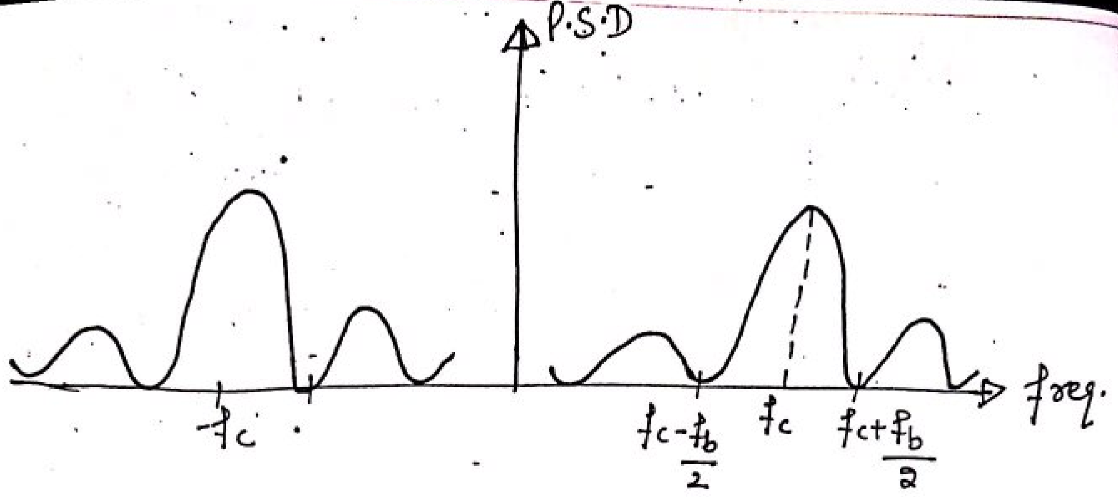
P.S.D of an NRZ bipolar signal is given as,

$$S(f) = P_s T_b\left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2$$

In QPSK this signal $b(t)$ is divided into even and odd bit streams i.e. $b_e(t)$ and $b_o(t)$. Their P.S.D are given by,

$$S_e(f) = P_s T_s\left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2$$

$$S_o(f) = P_s T_s\left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2 \qquad [T_s = 2T_b]$$



Bandwidth :-

B.W of QPSK system is one half of the B.W of BPSK.

$$\therefore \quad B.W = \frac{2f_b}{2} = \cancel{B.W}\ f_b.$$

[or]

$$\left(f_c + \frac{f_b}{2}\right) - \left(f_c - \frac{f_b}{2}\right) = f_b.$$

Thus, the advantage of multilevel modulation is reduction in required B.W.

Advantages:-
- Very good noise immunity.
- Low error probability
- High bit rate data transmission

Drawback:- The generation and detection of QPSK is quite complex.

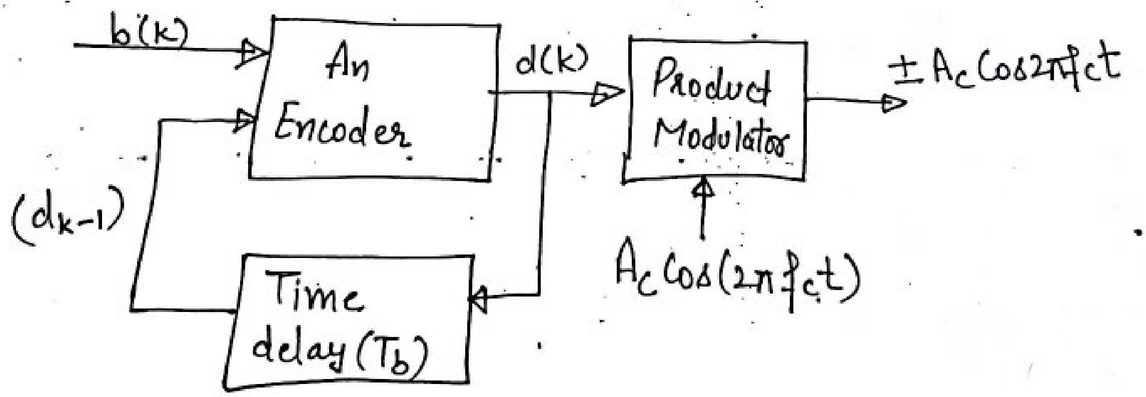
Differential Phase Shift Keying (DPSK) :-

DPSK is the non-coherent version of the PSK. DPSK does not need a synchronous carrier at the demodulator. The input sequence of binary bits is modified such that the next bit depends upon the previous bit. Therefore, in the Rx, the previous received bits are used to detect the present bit.

Encoding refers to the procedure of encoding of data differentially. The presence of binary '1' or '0' is manifested by the symbol's similarity or difference when compared with the preceding symbol.

Generation of DPSK :-

The digital information content of the binary data is encoded in terms of signal transitions. Symbol '0' is used to represent transition in a given binary sequence and symbol '1' to indicate no transition.



The data stream b(k) is applied to the I/P of the encoder. O/P of the encoder is applied to one I/P of the product modulator. To the other I/P of this product modulator, a sinusoidal carrier is applied

The relationship b/w the binary sequence and its differential encoded version is illustrated in below table for assumed data sequence 0 0 1 0 0 1 0 0 1 1.

Encoding has been done in such a way that transition in the given binary sequence with respect to previous encoded bit is represented by a symbol '0' and no transition by symbol '1'.

An extra bit ('1') has been added as an initial bit.

The phase of generated DPSK signal is shown in third row.

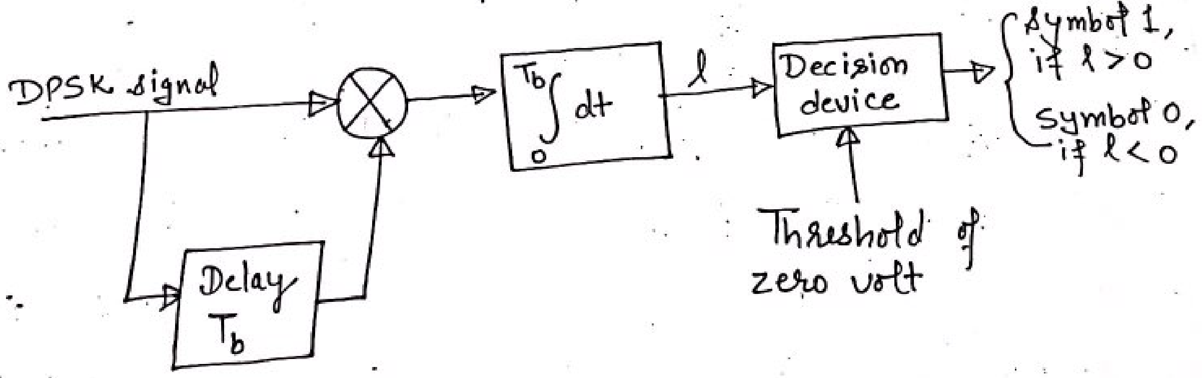
Binary data b(k)		0	0	1	0	0	1	0	0	1	1
Encoded data {d(k)}	1*	0	1	1	0	1	1	0	1	1	1
Phase of DPSK	0	π	0	0	π	0	0	π	0	0	0
shifted encoded data $\{d_{k-1}\}$		1	0	1	1	0	1	1	0	1	1
Phase of shifted DPSK		0	π	0	0	π	0	0	π	0	0
Phase comparison O/P		−	−	+	−	−	+	−	−	+	+
Detected binary sequence		0	0	1	0	0	1	0	0	1	1

Detection of DPSK :-

For detection of the differentially encoded PSK, the received DPSK signal is applied to one i/p of the multiplier. To the other i/p of the multiplier, a delayed version of the received DPSK signal by the time interval T_b is applied. The delayed version of the received DPSK signal is shown in 4th row of the table. The o/p of the difference is proportional to $\cos(\phi)$, 'ϕ' is the difference b/w carrier phase angle of the received DPSK and its delayed version which is shown in 6th row of the table.

When $\phi = 0$, integrator o/p is positive; whereas when $\phi = \pi$, " " " negative.

By comparing the integrator o/p with a decision level of zero volt, the decision device can reconstruct the binary sequence by assigning symbol '0' for negative o/p and symbol '1' for positive output. The reconstructed binary data is shown in last row of the table.



Bandwidth of DPSK signal :-

In DPSK, one previous bit is always used to define the phase shift in next bit, therefore, the symbol can be said to have two bits. Hence, one symbol duration (T) is equivalent to two bits duration ($2T_b$)

i.e. $T = 2T_b$

$$B.W = \frac{2}{T} = \frac{2}{2T_b} = \frac{1}{T_b} = \boxed{f_b}$$

Salient Features of DPSK :-

→ DPSK does not need carrier at the receiver end. This means that the complicated circuitry for generation of local carrier is not required.

→ B.W requirement of DPSK is reduced as compared to that of BPSK.

Drawbacks :-

→ The probability of error of DPSK is higher than that of BPSK.

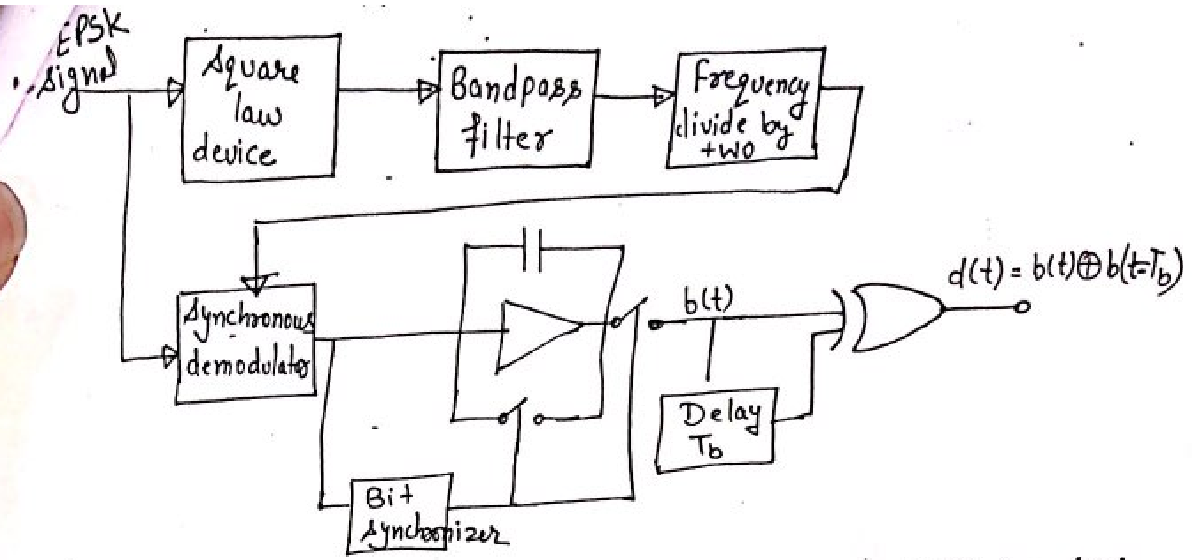
→ Error in the first bit creates error in the second bit. Therefore, error propagation in DPSK is more.

→ Noise interference in DPSK is more.

Differentially Encoded PSK (DEPSK) :-

The transmitter of DEPSK system is identical to the DPSK transmitter, but the receiver is completly different. The signal b(t) is recovered from the received signal, using the synchronous demodulation technique. This is same as the BPSK.

Once the signal b(t) is recovered, it is applied to one I/P of an Ex-OR gate. The signal b(t) is also applied to a time delay circuit and the delayed signal $b(t-T_b)$ is applied to the other I/P of the Ex-OR gate.



if $b(t) = b(t-T_b)$, then o/p of Ex-OR gate will be '0'.

$\therefore \quad d(t) = 0 \quad$ if $b(t) = b(t-T_b)$

and

if $b(t) = \overline{b(t-T_b)}$, the o/p of Ex-OR gate will be '1'

$\therefore \quad d(t) = 1 \quad$ if $b(t) = \overline{b(t-T_b)}$

Advantages of DEPSK :-

- The DEPSK system has less probability of error as compared to DPSK.
- In DPSK demodulator, the delay generating device (T_b) has to operate at the carrier frequency but in DEPSK demodulator, the delay device operates at the baseband frequency (f_b). This reduces the hardware cost of the delay device.

Drawbacks :-

- Complex demodulator is required.
- Errors occur in pairs. Thus one error in $b(t)$ will give rise to two errors in $d(t)$.

Optimum Filters :-

Optimum Filters receive/recove