Foundation of Computer Science

Lecture 6

Rules of Inference with Quantifiers Resolution Principle

1. Universal Instantiation

premises: $\forall \mathbf{x} P(\mathbf{x})$

conclusion: P(c), for any c

Example: Our domain consists of all dog & Fido is a dog

"All dogs are cuddly"
"Therefore, Fido is cuddly"

1. Universal Generalization

premises: P(c), for any c

conclusion: $\forall \mathbf{x} P(\mathbf{x})$

3. Existential Instantiation

premises: $\exists \mathbf{x} P(\mathbf{x})$

conclusion: P(c), for some element c

Example:

"There is someone who got an "A" in the course. "Lets call her 'x' and say 'x' got an 'A'

4. Existential Generalization

premises: P(c) for some element c

conclusion: $\exists \mathbf{x} P(\mathbf{x})$

Example:

"Renu got 'A' in the class

Therefore, someone got 'A' in the class

Applying Rules of Inferences

- Example 1: It is known that
 - 1. A student in this class has not read the book.
 - 2. Everyone in this class passed the first exam.

 Canyou conclude that "Someone who passed the first exam has not read the book"?

Solution

• To simplify the discussion, let

```
C(\mathbf{x}) := \mathbf{x} is a student in the class
```

 $B(\mathbf{x}) := \mathbf{x}$ has read the book

 $P(\mathbf{x}) := \mathbf{x}$ passed the first exam

• We will give a valid argument with premises:

$$\exists \mathbf{x} (C(\mathbf{x}) \land \neg B(\mathbf{x})),$$

$$\forall \mathbf{x} (C(\mathbf{x}) \rightarrow P(\mathbf{x}))$$

conclusion: $\exists \mathbf{x} (P(\mathbf{x}) \land \neg B(\mathbf{x}))$

Solution

Step

- 1. $\exists \mathbf{x} (C(\mathbf{x}) \land \neg B(\mathbf{x}))$
- 2. $C(a) \land \neg B(a)$
- 3. C(a)
- 4. $\forall \mathbf{x} (C(\mathbf{x}) \rightarrow P(\mathbf{x}))$
- 5. $C(a) \rightarrow P(a)$
- 6. P(a)
- 7. $\neg B(a)$
- 8. $P(a) \wedge \neg B(a)$
- 9. $\exists x (P(x) \land \neg B(x))$

Reason

Premise

Existential Instantiation

Simplification by (2)

Premise

Universal Instantiation Modus

Ponens by (3) and (5)

Simplification by (2)

Conjunction by (6) and (7)

Existential Generalization

Solve the question

Using the rules of inference, construct a valid argument to show that

- "John Smith has two legs" is a consequence of the premises:
- "Every man has two legs."
- "John Smith is a man."

Resolution Principal

- Another way to prove the validity of arguments is using resolution principle
- The rule of inference called resolution is based on the tautology:

$$((p Vq) \land (\neg p V r)) \rightarrow (qVr)$$

- If we express the hypotheses and the conclusion as clauses (possible by CNF, a conjunction of clauses), we can use resolution as the only inference rule to build proofs!
- This is used in programming languages like Prolog.
- It can be used in automated theorem proving systems.

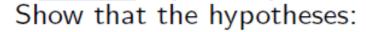
Proofs that use exclusively resolution as inference rule

Step 1 Convert hypotheses and conclusion into clauses:

Original hypothesis	equivalent CNF	Hypothesis as list of clauses
$(p \wedge q) \vee r$	$(p \lor r) \land (q \lor r)$	(- /
r o s	$(\neg r \lor s)$	$(\neg r \lor s)$
Conclusion	equivalent CNF	Conclusion as list of clauses
$p \vee s$	$(p \lor s)$	$(p \lor s)$

Step 2 Write a proof based o resolution

Step	Reason
1. $p \vee r$	hypothesis
2. $\neg r \lor s$	hypothesis
3. $p \vee s$	resolution of 1 and 2



- $\neg s \land c$ translates to clauses: $\neg s, c$
- $w \to s$ translates to clause: $(\neg w \lor s)$
- $\bullet \neg w \rightarrow t$ translates to clause: $(w \lor t)$
- $t \rightarrow h$ translates to clause: $(\neg t \lor h)$

lead to the conclusion:

h (it is already a trivial clause)

Note that the fact that p and $\neg p \lor q$ implies q (called disjunctive syllogism) is a special case of resolution, since $p \lor F$ and $\neg p \lor q$ give us $F \lor q$ which is equivalent to q.

Resolution-based	proof:

Step	Reason
1. ¬s	hypothesis
2. $\neg w \lor s$	hypothesis
3. ¬ <i>w</i>	resolution of 1 and 2
4. $w \lor t$	hypothesis
5. <i>t</i>	resolution of 3 and 4
6. $\neg t \lor h$	hypothesis
7. <i>h</i>	resolution of 5 and 6_

Solve the question using resolution

Show the following argument is valid

If today is Tuesday, I have a test in mathematics or Economics. If my Economics Professor is sick, I will not have test in economics. Today is Tuesday and my Economics Professor is sick. Therefore I have a test in Mathematics

Thank You