LECTURE-15 LATTICES

Lattices – A Poset in which every pair of elements has both, a least upper bound and a greatest lower bound is called a lattice.

There are two binary operations defined for lattices –

1. **Join –** The join of two elements is their least upper bound. It is denoted by $\lor$, not to be confused with disjunction.
2. **Meet –** The meet of two elements is their greatest lower bound. It is denoted by $\land$, not to be confused with conjunction.

Sub Lattice – A sublattice of lattice L is a subset $S \subseteq L$ such that if $a, b \in S$, $a \land b \in S$ and $a \lor b \in S$.

Identities for join and meet –

- $x \land x = x$ $\quad and\, x \lor x = x$
- $x \land y = y \land x$ $\quad and\, x \lor y = y \lor x$
- $(x \land y) \land z = x \land (y \land z)$ $\quad and\, (x \lor y) \lor z = x \lor ($
- $x \land (y \lor x) = x$ $\quad and\, x \lor (y \land x) = x$

DISTRIBUTIVE LAWS MAY OR MAY NOT HOLD TRUE FOR A LATTICE :

1. $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$
2. $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$

Note – A lattice is called a distributive lattice if the distributive laws hold for it.

But Semidistributive laws hold true for all lattices :

1. $x \wedge (y \vee z) \geq (x \wedge y) \vee (x \wedge z)$.
2. $x \vee (y \wedge z) \leq (x \vee y)$.

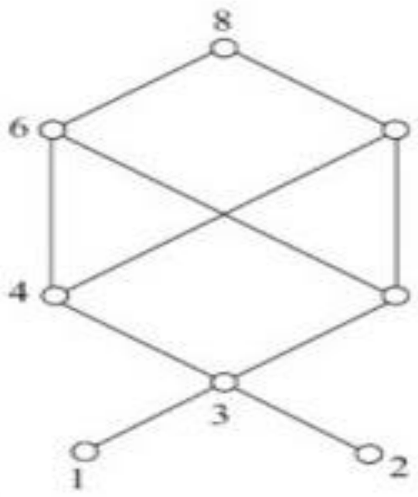
Two important properties of Distributive Lattices –

1. In any distributive lattice $a \wedge y = a \wedge x$ and $a \vee y = a \vee x$ together imply that $x = y$.
2. If $a \wedge x = O$ and $a \vee x = I$, where O and I are the least and greatest element of lattice, then a and x are said to be a complementary pair. O and I are a trivially complementary pair.

QUESTIONS:

1.

Consider the partially-ordered set $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$ under the relation whose Hasse diagrams is shown below:



Consider the subsets $S_1 = \{1, 2\}$, $S_2 = \{3, 4, 5\}$ of A. Find

(i) All the lower and upper bounds of S_1 and S_2.

(ii) glb S_1, lub S_1, glb S_2, lub S_2.

QUESTIONS:

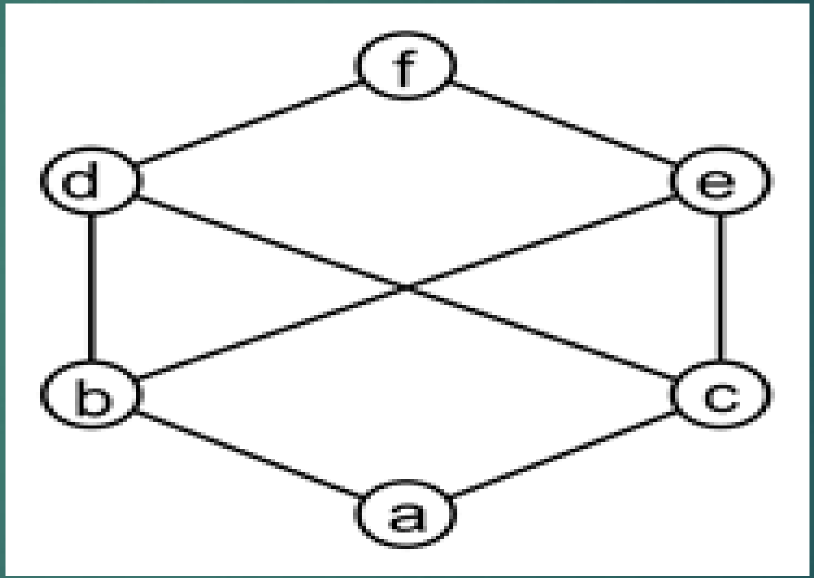
2. THE GRAPH GIVEN BELOW IS AN EXAMPLE OF __________

A) NON-LATTICE POSET

B) SEMILATTICE

C) PARTIAL LATTICE

D) BOUNDED LATTICE

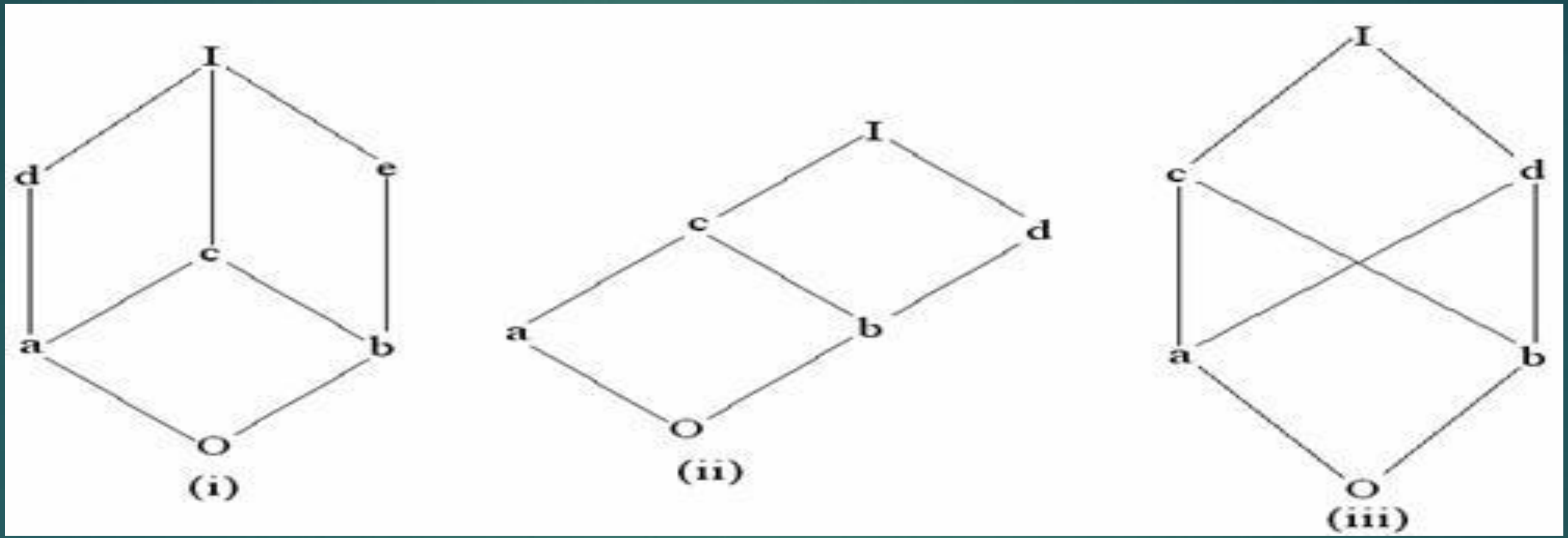


ANSWER: A

EXPLANATION: THE GRAPH IS AN EXAMPLE OF NON-LATTICE POSET WHERE B AND C HAVE COMMON UPPER BOUNDS D, E AND F BUT NONE OF THEM IS THE LEAST UPPER BOUND.

QUESTIONS:

3. HOW TO RECOGNIZE WHICH LATTICES ARE DISTRIBUTIVE OR NOT ONLY BY LOOKING ON THEIR DIAGRAMS? IS IT EVEN POSSIBLE?



YES, IT'S POSSIBLE. A LATTICE IS DISTRIBUTIVE IF AND ONLY IF IT DOES NOT CONTAIN THE DIAMOND LATTICE OR THE PENTAGON LATTICE AS A SUBLATTICE

QUESTIONS:

4.

Which of the Hasse diagrams represent lattices?