LECTURE -30 PERMUTATION GROUP

- **Definition of Permutation**

A permutation is one to one mapping of non empty set P , say onto itself

- Example :

Let $S = \{1,2,3\}$

Then function $f : S \to S$

$f(1) = 2$

$f(2) = 3$

$f(3) = 1$ Then permutation

$P_1 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ $P_2 = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 1 & 3 \end{pmatrix}$

$P3 = \begin{pmatrix} 2 & 1 & 3 \\ 3 & 2 & 1 \end{pmatrix}$ $P4 = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 1 & 2 \end{pmatrix}$

$P5 = \begin{pmatrix} 3 & 1 & 2 \\ 1 & 2 & 3 \end{pmatrix}$ $P6 = \begin{pmatrix} 3 & 2 & 1 \\ 1 & 3 & 2 \end{pmatrix}$

There are n! of pattern of expressing Permutation .

So if Set has 3 elements then pattern of expressing permutation is 3! = 6

- **Equality of Permutations :**

➢ Let f and g be two permutations defined on a non empty set P. Then $f = g$ if and only if $f(x) = g(x)\ \forall x \in P$

❖ Example

1) Let $S = \{1,2,3,4\}$ and let permutation f and g are equal or not..

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix} \qquad g = \begin{pmatrix} 4 & 1 & 3 & 2 \\ 4 & 3 & 2 & 1 \end{pmatrix}$$

2) Let $S = \{1,2,3,4\}$ and let permutation f and g are equal or not..

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix} \qquad g = \begin{pmatrix} 2 & 4 & 3 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

- **Permutation Identity**

➢ An Identity permutation on S , denoted by I , is defined as $I(a) = a$ $\forall a \in S$

➢ For example :

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

Note : In identity permutation the image of element is element itself

- **Composition of Permutation (Product of Permutation)**
- Let f and g be two arbitrary permutations of like degree , given by,

$$f = \begin{pmatrix} a1 & a2 & a3 & a4 & \ldots\ldots\ldots\ldots & an \\ b1 & b2 & b3 & b4 & \ldots\ldots\ldots\ldots & bn \end{pmatrix}$$

$$g = \begin{pmatrix} b1 & b2 & b3 & b4 & \ldots\ldots\ldots\ldots & bn \\ c1 & c2 & c3 & c4 & \ldots\ldots\ldots\ldots & cn \end{pmatrix}$$

on non empty set A. Then the composition (or Product) of f and g is defined as

$$f \circ g = \begin{pmatrix} a1 & a2 & a3 & a4 & \ldots\ldots\ldots\ldots & an \\ b1 & b2 & b3 & b4 & \ldots\ldots\ldots\ldots & bn \end{pmatrix} \circ \begin{pmatrix} b1 & b2 & b3 & b4 & \ldots\ldots\ldots\ldots & bn \\ c1 & c2 & c3 & c4 & \ldots\ldots\ldots\ldots & cn \end{pmatrix}$$

$$= \begin{pmatrix} a1 & a2 & a3 & a4 & \ldots\ldots & an \\ c1 & c2 & c3 & c4 & \ldots\ldots & cn \end{pmatrix}$$

- Example

Let $P_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$ $P_2 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ $P_3 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$

Check $P_1 \circ (P_2 \circ P_3) = (P_1 \circ P_2) \circ P_3$

- **Inverse Permutation**

Every permutation f on set $P = \{a_1, a_2, a_3, \ldots, a_n\}$ Possesses a unique inverse permutation , denoted by f^{-1} thus if

$$f = \begin{pmatrix} a1 & a2 & \ldots\ldots\ldots & an \\ b1 & b2 & \ldots\ldots\ldots & bn \end{pmatrix}$$

Then

$$f^{-1} = \begin{pmatrix} b1 & b2 & \ldots\ldots\ldots & bn \\ a1 & a2 & \ldots\ldots\ldots & an \end{pmatrix}$$

Cyclic Permutation

- Let $t_1, t_2, \ldots, t_r$ be r distinct elements of the set $P = \{t_1, t_2, \ldots, t_n\}$. Then the permutation $p : P \rightarrow P$ is defined by

$p(t_1) = t_2$, $p(t_2) = t_3, \ldots, p(t_r - 1) = t_r$, $p(t_r) = t_1$ is called a ***cyclic permutation*** of length r.

- **Example** : The permutation

$$P = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 6 & 5 & 3 \end{pmatrix}$$

is written as (1,2) , (3,4,6) , (5).. The cycle (1,2) has length 2 , The cycle length 3,The cycle 1.