LECTURE-34 RINGS, BOOLEAN FUNCTION & EXPRESSION

Ring – Let addition (+) and Multiplication (.) be two binary operations defined on a non empty set R. Then R is said to form a ring w.r.t addition (+) and multiplication (.) if the following conditions are satisfied:

1. (R, +) is an abelian group (i.e commutative group)
2. (R, .) is a semigroup
3. For any three elements a, b, c $\in$ R the left distributive law a.(b+c) =a.b + a.c and the right distributive property (b + c).a =b.a + c.a holds.

Therefore a non- empty set R is a ring w.r.t to binary operations + and . if the following conditions are satisfied.

1. For all a, b ϵ R, a+bϵ R,
2. For all a, b, c ϵ R a+(b+c)=(a+b)+c,
3. There exists an element in R, denoted by 0 such that a+0=a for all a ϵ R
4. For every a ϵ R there exists an y ϵ R such that a+y=0. y is usually denoted by -a
5. a+b=b+a for all a, b ϵ R.
6. a.b ϵ R for all a.b ϵ R.
7. a.(b.c)=(a.b).c for all a, b ϵ R
8. For any three elements a, b, c ϵ R a.(b+c) =a.b + a.c and (b + c).a =b.a + c.a. And the ring is denoted by (R, +, .).

R is said to be a commutative ring if the multiplication is commutative.

BOOLEAN ALGEBRA :

BOOLEAN ALGEBRA IS ALGEBRA OF LOGIC. IT DEALS WITH VARIABLES THAT CAN HAVE TWO DISCRETE VALUES, 0 (FALSE) AND 1 (TRUE); AND OPERATIONS THAT HAVE LOGICAL SIGNIFICANCE. THE EARLIEST METHOD OF MANIPULATING SYMBOLIC LOGIC WAS INVENTED BY GEORGE BOOLE AND SUBSEQUENTLY CAME TO BE KNOWN AS BOOLEAN ALGEBRA.

BOOLEAN ALGEBRA HAS NOW BECOME AN INDISPENSABLE TOOL IN COMPUTER SCIENCE FOR ITS WIDE APPLICABILITY IN SWITCHING THEORY, BUILDING BASIC ELECTRONIC CIRCUITS AND DESIGN OF DIGITAL COMPUTERS.

BOOLEAN FUNCTIONS :

Boolean Functions

A Boolean function is a special kind of mathematical function $f : X^n \to X$ of degree n, where $X = \{0, 1\}$ is a Boolean domain and n is a non-negative integer. It describes the way how to derive Boolean output from Boolean inputs.

Example – Let, $F(A, B) = A'B'$. This is a function of degree 2 from the set of ordered pairs of Boolean variables to the set $\{0, 1\}$ where $F(0, 0) = 1, F(0, 1) = 0, F(1, 0) = 0$ and $F(1, 1) = 0$

BOOLEAN EXPRESSIONS & IDENTITIES :

Boolean Expressions

A Boolean expression always produces a Boolean value. A Boolean expression is composed of a combination of the Boolean constants (True or False), Boolean variables and logical connectives. Each Boolean expression represents a Boolean function.

Example – $AB'C$ is a Boolean expression.

Boolean Identities

Double Complement Law

$$\sim(\sim A) = A$$

Complement Law

$A + \sim A = 1$ (OR Form)

$A . \sim A = 0$ (AND Form)

Idempotent Law

$A + A = A$ (OR Form)

$A . A = A$ (AND Form)

Identity Law

$$A + 0 = A \quad \text{(OR Form)}$$

$$A.1 = A \quad \text{(AND Form)}$$

Dominance Law

$$A + 1 = 1 \quad \text{(OR Form)}$$

$$A.0 = 0 \quad \text{(AND Form)}$$

Commutative Law

$$A + B = B + A \quad \text{(OR Form)}$$

$$A.B = B.A \quad \text{(AND Form)}$$

Associative Law

$$A + (B + C) = (A + B) + C \quad \text{(OR Form)}$$

$$A.(B.C) = (A.B).C \quad \text{(AND Form)}$$

Absorption Law

$$A.(A+B) = A$$

$$A+(A.B) = A$$

Simplification Law

$$A.(\sim A+B) = A.B$$

$$A+(\sim A.B) = A+B$$

Distributive Law

$$A+(B.C) = (A+B).(A+C)$$

$$A.(B+C) = (A.B)+(A.C)$$

De-Morgan's Law

$$\sim(A.B) = \sim A + \sim B$$

$$\sim(A+B) = \sim A. \sim B$$

CANONICAL FORMS

FOR A BOOLEAN EXPRESSION THERE ARE TWO KINDS OF CANONICAL FORMS –

1. THE SUM OF MINTERMS (SOM) FORM
2. THE PRODUCT OF MAXTERMS (POM) FORM

THE SUM OF MINTERMS (SOM) OR SUM OF PRODUCTS (SOP) FORM

A MINTERM IS A PRODUCT OF ALL VARIABLES TAKEN EITHER IN THEIR DIRECT OR COMPLEMENTED FORM. ANY BOOLEAN FUNCTION CAN BE EXPRESSED AS A SUM OF ITS 1-MINTERMS AND THE INVERSE OF THE FUNCTION CAN BE EXPRESSED AS A SUM OF ITS 0-MINTERMS. HENCE,

F (LIST OF VARIABLES) = ∑ (LIST OF 1-MINTERM INDICES)

AND

F' (LIST OF VARIABLES) = ∑ (LIST OF 0-MINTERM INDICES)

A	B	C	Term	Minterm
0	0	0	x’y’z’	m_0
0	0	1	x’y’z	m_1
0	1	0	x’yz’	m_2
0	1	1	x’yz	m_3
1	0	0	xy’z’	m_4
1	0	1	xy’z	m_5
1	1	0	xyz’	m_6
1	1	1	xyz	m_7

Example :

Let, $F(x,y,z)=x'y'z'+ xy'z+ xyz'+ xyz$

Or, $F(x,y,z)=m0+m5+m6+m7$

Hence,

$F(x,y,z)=\sum(0,5,6,7)$

Now we will find the complement of $F(x,y,z)$

$F'(x,y,z)=x'yz+x'y'z+x'yz'+xy'z'$

Or, $F'(x,y,z)=m3+m1+m2+m4$

Hence,

$F'(x,y,z)=\sum(3,1,2,4)=\sum(1,2,3,4)$

THE PRODUCT OF MAXTERMS (POM) OR PRODUCT OF SUMS (POS) FORM

The Product of Maxterms (POM) or Product of Sums (POS) form

A maxterm is addition of all variables taken either in their direct or complemented form. Any Boolean function can be expressed as a product of its 0-maxterms and the inverse of the function can be expressed as a product of its 1-maxterms. Hence,

F(list of variables) = π (list of 0-maxterm indices).

and

F'(list of variables) = π (list of 1-maxterm indices).

A	B	C	Term	Maxterm
0	0	0	x + y + z	M_0
0	0	1	x + y + z’	M_1
0	1	0	x + y’ + z	M_2
0	1	1	x + y’ + z’	M_3
1	0	0	x’ + y + z	M_4
1	0	1	x’ + y + z’	M_5
1	1	0	x’ + y’ + z	M_6
1	1	1	x’ + y’ + z’	M_7

EXAMPLE :

LET $F(X,Y,Z)=(X+Y+Z).(X+Y+Z').(X+Y'+Z).(X'+Y+Z)$

OR, $F(X,Y,Z)=M0.M1.M2.M4$

HENCE,

$F(X,Y,Z)=\Pi(0,1,2,4)$

$F''(X,Y,Z)=(X+Y'+Z').(X'+Y+Z').(X'+Y'+Z).(X'+Y'+Z')$

OR, $F(X,Y,Z)=M3.M5.M6.M7$

HENCE,

$F'(X,Y,Z)=\Pi(3,5,6,7)$