Foundation of Computer Science

Lecture 5

Rules of Inference

Outline

- Mathematical Argument
- Rules of Inference

Argument

- In mathematics, an argument is a sequence of propositions (called premises) followed by a proposition (called conclusion)

A valid argument is one that, if all its premises are true, then the conclusion is true.

Valid Argument Form

- By definition, if a valid argument form consists
  - premises: $p_1, p_2, \ldots, p_k$
  - conclusion: q

  then $(p_1 \wedge p_2 \wedge \ldots \wedge p_k) \rightarrow q$ is a tautology
- Ex: $((p \rightarrow q) \wedge p) \rightarrow q$ is a tautology
- Some simple valid argument forms, called rules of inference, are derived and can be used to construct complicated argument form

Valid Argument Form

Example :

"If it rains, I drive to school."

"It rains."

"I drive to school."

$p \rightarrow q$ p

$\therefore q$

- This is called a valid argument form
- The argument is valid since $((p \rightarrow q) \wedge p) \rightarrow q$ is a tautology.

Rules of Inference

1. Modus Ponens (method of affirming)

premises: $p, \quad p \to q$

conclusion: q

2. Modus Tollens (method of denying)

premises: $\neg q, \quad p \to q$

conclusion: $\neg p$

Rules of Inference

3. Hypothetical Syllogism

   premises: $p \rightarrow q, \quad q \rightarrow r$

   conclusion: $p \rightarrow r$

4. Disjunctive Syllogism

   premises: $\neg p, \quad p \vee q$

   conclusion: q

Rules of Inference

5. Addition premises: p

   conclusion: $p \vee q$

6. Simplification premises: $p \wedge q$

   conclusion: p

Rules of Inference

7. Conjunction premises: p, q

   conclusion: $p \wedge q$

8. Resolution premises: $p \vee q$, $\neg p \vee r$ conclusion:

   $q \vee r$

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$[p \wedge (p \to q)] \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$[\neg q \wedge (p \to q)] \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	$[(p \vee q) \wedge \neg p] \to q$	Disjunctive syllogism
p $\therefore p \vee q$	$p \to (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \to p$	Simplification
p q $\therefore p \wedge q$	$[(p) \wedge (q)] \to (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$[(p \vee q) \wedge (\neg p \vee r)] \to (q \vee r)$	Resolution

Rules of Inference - Rules for Propositional Logic

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Applying Rules of Inferences

- Example 1: It is known that
  1. It is not sunny this afternoon, and it is colder than yesterday.
  2. We will go swimming only if it is sunny.
  3. If we do not go swimming, we will play basketball.
  4. If we play basketball, we will go home early.
- Can you conclude "we will go home early"?

Solution

- To simplify the discussion, let

  p := It is sunny this afternoon

  q := It is colder than Yesterday r :=

  We will go swimming

  s := We will play basketball

  t := We will go home early

- We will give a valid argument with

  premises: $\neg p \wedge r$, $r \to p$, $\neg r \to s$, $s \to t$

  conclusion: t

Solution

Step	Reason
1. $\neg p \wedge r$	Premise
2. $\neg p$	Simplification using (1)
3. $r \rightarrow p$	Premise
4. $\neg r$	Modus Tollens using (2) and (3)
5. $\neg r \rightarrow s$	Premise
6. s	Modus Ponens using (4) and (5)
7. $s \rightarrow t$	Premise
8. t	Modus Ponens using (6) and (7)

Applying Rules of Inferences

- Example 2: It is known that
  1. If you send me an email, then I will finish my program.
  2. If you do not send me an email, then I will go to sleep early.
  3. If I go to sleep early, I will wake up refreshed.
- Can you conclude "If I do not finish my program, then I will wake up refreshed"?

Solution

- To simplify the discussion,
- let p := You send me an email
- q := I finish my program
- r := I go to sleep early
- s := I wake up refreshed

- We will give a valid argument with

premises: $p \rightarrow q$, $\neg p \rightarrow r$, $r \rightarrow s$

conclusion: $\neg q \rightarrow s$

Solution

Step	**Reason**
1. $p \to q$	Premise
2. $\neg q \to \neg p$	Contrapositive of (1)
3. $\neg p \to r$	Premise
4. $\neg q \to r$	Hypothetical Syllogism by (2) and (3)
5. $r \to s$	Premise
6. $\neg q \to s$	Hypothetical syllogism by (4) and (5)

Show that the hypotheses:
It is not sunny this afternoon and it is colder than yesterday. $\neg s \land c$
We will go swimming only if it is sunny. $w \rightarrow s$
If we do not go swimming, then we will take a canoe trip. $\neg w \rightarrow t$
If we take a canoe trip, then we will be home by sunset. $t \rightarrow h$

lead to the conclusion:
We will be home by the sunset. h

Step	Reason
1. $\neg s \land c$	hypothesis
2. $\neg s$	simplification
3. $w \rightarrow s$	hypothesis
4. $\neg w$	modus tollens of 2 and 3
5. $\neg w \rightarrow t$	hypothesis
6. t	modus ponens of 4 and 5
7. $t \rightarrow h$	hypothesis
8. h	modus ponens of 6 and 7

Thank You