Foundation of Computer Science

Lecture 6

Rules of Inference with Quantifiers
Resolution Principle

Rules of Inference with Quantifiers

1. Universal Instantiation

premises: $\forall \mathbf{x}\ P(\mathbf{x})$

conclusion: P(c), for any c

Example: Our domain consists of all dog & Fido is a dog

"All dogs are cuddly"
"Therefore , Fido is cuddly"

Rules of Inference with Quantifiers

1. Universal Generalization

premises:	P(c), for any c
conclusion:	$\forall \mathbf{x}\ P(\mathbf{x})$

Rules of Inference with Quantifiers

3. **Existential Instantiation**

premises: $\exists \mathbf{x}\ P(\mathbf{x})$

conclusion: P(c), for some element c

Example:

“ There is someone who got an “A” in the course.

“Lets call her ‘x’ and say ‘x’ got an ‘A’

Rules of Inference with Quantifiers

4. **Existential Generalization**

premises: P(c) for some element c

conclusion: $\exists\, \mathbf{x}\ P(\mathbf{x})$

Example:

"Renu got 'A' in the class

Therefore, someone got 'A' in the class

Applying Rules of Inferences

- Example 1: It is known that
  1. A student in this class has not read the book.
  2. Everyone in this class passed the first exam.

- Can you conclude that “Someone who passed the first exam has not read the book”?

Solution

- To simplify the discussion, let

  $C(\mathbf{x})$:= $\mathbf{x}$ isa student in the class

  $B(\mathbf{x})$:= $\mathbf{x}$ has read thebook

  $P(\mathbf{x})$:= $\mathbf{x}$ passed the firstexam

- We will give a valid argument with premises:
  $\exists \mathbf{x} (C(\mathbf{x}) \land \neg B(\mathbf{x}))$,

  $\forall \mathbf{x} (C(\mathbf{x}) \to P(\mathbf{x}))$

  conclusion: $\exists \mathbf{x} (P(\mathbf{x}) \land \neg B(\mathbf{x}))$

Solution

Step	Reason
1. $\exists \mathbf{x} (C(\mathbf{x}) \wedge \neg B(\mathbf{x}))$	Premise
2. $C(a) \wedge \neg B(a)$	Existential Instantiation
3. $C(a)$	Simplification by (2)
4. $\forall \mathbf{x} (C(\mathbf{x}) \rightarrow P(\mathbf{x}))$	Premise
5. $C(a) \rightarrow P(a)$	Universal Instantiation Modus
6. $P(a)$	Ponens by (3) and (5)
7. $\neg B(a)$	Simplification by (2)
8. $P(a) \wedge \neg B(a)$	Conjunction by (6) and (7)
9. $\exists \mathbf{x} (P(\mathbf{x}) \wedge \neg B(\mathbf{x}))$	Existential Generalization

Solve the question

Using the rules of inference, construct a valid argument to show that

"John Smith has two legs" is a consequence of the premises:

- "Every man has two legs."
- "John Smith is a man."

Resolution Principal

- Another way to prove the validity of arguments is using resolution principle
- The rule of inference called resolution is based on the tautology:

  $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$
- If we express the hypotheses and the conclusion as clauses (possible by CNF, a conjunction of clauses), we can use resolution as the only inference rule to build proofs!
- This is used in programming languages like Prolog.
- It can be used in automated theorem proving systems.

Proofs that use exclusively resolution as inference rule

- Step 1 Convert hypotheses and conclusion into clauses:

Original hypothesis	equivalent CNF	Hypothesis as list of clauses
$(p \wedge q) \vee r$	$(p \vee r) \wedge (q \vee r)$	$(p \vee r)$, $(q \vee r)$
$r \rightarrow s$	$(\neg r \vee s)$	$(\neg r \vee s)$

Conclusion	equivalent CNF	Conclusion as list of clauses
$p \vee s$	$(p \vee s)$	$(p \vee s)$

- Step 2 Write a proof based o resolution

Step	Reason
1. $p \vee r$	hypothesis
2. $\neg r \vee s$	hypothesis
3. $p \vee s$	resolution of 1 and 2

Show that the hypotheses:

- $\neg s \wedge c$ translates to clauses: $\neg s$, c
- $w \rightarrow s$ translates to clause: $(\neg w \vee s)$
- $\neg w \rightarrow t$ translates to clause: $(w \vee t)$
- $t \rightarrow h$ translates to clause: $(\neg t \vee h)$

lead to the conclusion:

- h (it is already a trivial clause)

Note that the fact that p and $\neg p \vee q$ implies q (called disjunctive syllogism) is a special case of resolution, since $p \vee F$ and $\neg p \vee q$ give us $F \vee q$ which is equivalent to q.

Resolution-based proof:

Step	Reason
1. $\neg s$	hypothesis
2. $\neg w \vee s$	hypothesis
3. $\neg w$	resolution of 1 and 2
4. $w \vee t$	hypothesis
5. t	resolution of 3 and 4
6. $\neg t \vee h$	hypothesis
7. h	resolution of 5 and 6

Solve the question using resolution

Show the following argument is valid

If today is Tuesday, I have a test in mathematics or Economics. If my Economics Professor is sick, I will not have test in economics. Today is Tuesday and my Economics Professor is sick . Therefore I have a test in Mathematics

Thank You