LECTURE-35 REPRESENTATION & MINIMIZATION OF BOOLEAN FUNCTIONS

SIMPLIFICATION USING ALGEBRAIC FUNCTIONS

IN THIS APPROACH, ONE BOOLEAN EXPRESSION IS MINIMIZED INTO AN EQUIVALENT EXPRESSION BY APPLYING BOOLEAN IDENTITIES.

PROBLEM 1

MINIMIZE THE FOLLOWING BOOLEAN EXPRESSION USING BOOLEAN IDENTITIES –

F(A,B,C)=A'B+BC'+BC+AB'C'

SOLUTION

GIVEN, F(A,B,C)=A'B+BC'+BC+AB'C'

OR, F(A,B,C)=A'B+(BC'+BC')+BC+AB'C'

OR,F(A,B,C)=A'B+(BC'+BC)+(BC'+AB'C')

[BY IDEMPOTENT LAW, BC' = BC' + BC']

$$OR, F(A,B,C)=A'B+B(C'+C)+C'(B+AB')$$

[BY DISTRIBUTIVE LAWS]

$$OR, F(A,B,C)=A'B+B.1+C'(B+A)$$

$$[(C' + C) = 1 \text{ AND ABSORPTION LAW } (B + AB') = (B + A)]$$

$$OR, F(A,B,C)=A'B+B+C'(B+A)$$

$$\begin{bmatrix} B.1 = B \end{bmatrix}$$

$$OR, F(A,B,C)=B(A'+1)+C'(B+A)$$

OR,
$$F(A,B,C)=B.1+C'(B+A)$$

$$[(A'+1)=1]$$

$$OR, F(A,B,C)=B+C'(B+A)$$

$$[AS, B.1 = B]$$

$$OR, F(A,B,C)=B+BC'+AC'$$

$$OR, F(A,B,C)=B(1+C')+AC'$$

OR,
$$F(A,B,C)=B.1+AC'$$

$$[AS, (1 + C') = 1]$$

$$OR, F(A,B,C)=B+AC'$$

[AS, B.1 = B]

SO, F(A,B,C)=B+AC'

IS THE MINIMIZED FORM.

PROBLEM 2

MINIMIZE THE FOLLOWING BOOLEAN EXPRESSION USING BOOLEAN IDENTITIES –

F(A,B,C)=(A+B)(A+C)

SOLUTION:

Solution

Given, F(A,B,C) = (A+B)(A+C)

Or,
$$F(A, B, C) = A.A + A.C + B.A + B.C$$

Or, $F(A, B, C) = A + A \cdot C + B \cdot A + B \cdot C$

Or,
$$F(A, B, C) = A(1 + C) + B \cdot A + B \cdot C$$

Or, $F(A, B, C) = A + B \cdot A + B \cdot C$

Or,
$$F(A, B, C) = (A + 1) \cdot A + B \cdot C$$

Or, F(A, B, C) = 1.A + B.C

Or,
$$F(A, B, C) = A + B. C$$

So, F(A, B, C) = A + BC is the minimized form.

[Applying distributive Rule]

[Applying Idempotent Law]

[Applying distributive Law]

[Applying dominance Law]

[Applying distributive Law]

[Applying dominance Law]

[Applying dominance Law]

KARNAUGH MAPS

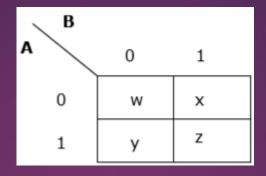
THE KARNAUGH MAP (K–MAP), INTRODUCED BY MAURICE KARNAUGHIN IN 1953, IS A GRID-LIKE REPRESENTATION OF A TRUTH TABLE WHICH IS USED TO SIMPLIFY BOOLEAN ALGEBRA EXPRESSIONS. A KARNAUGH MAP HAS ZERO AND ONE ENTRIES AT DIFFERENT POSITIONS. IT PROVIDES GROUPING TOGETHER BOOLEAN EXPRESSIONS WITH COMMON FACTORS AND ELIMINATES UNWANTED VARIABLES FROM THE EXPRESSION. IN A K-MAP, CROSSING A VERTICAL OR HORIZONTAL CELL BOUNDARY IS ALWAYS A CHANGE OF ONLY ONE VARIABLE.

EXAMPLE 1

AN ARBITRARY TRUTH TABLE IS TAKEN BELOW -

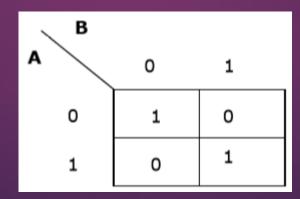
- A B A OPERATION B
- o o W
- 0 1 X
- 1 0 Y
- $1 \quad 1 \quad Z$

NOW WE WILL MAKE A K-MAP FOR THE ABOVE TRUTH TABLE –



EXAMPLE 2

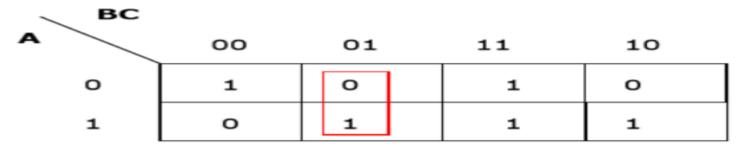
NOW WE WILL MAKE A K-MAP FOR THE EXPRESSION – AB+ A'B'



SIMPLIFICATION USING K-MAP

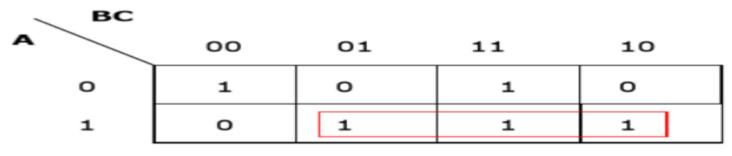
K-MAP USES SOME RULES FOR THE SIMPLIFICATION OF BOOLEAN EXPRESSIONS BY COMBINING TOGETHER ADJACENT CELLS INTO SINGLE TERM. THE RULES ARE DESCRIBED BELOW –





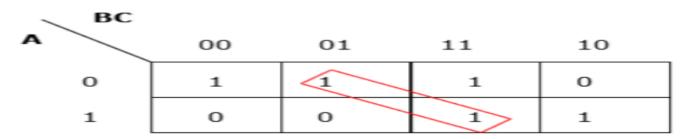
Wrong grouping

Rule 2 - Groups must contain 2n cells (n starting from 1).

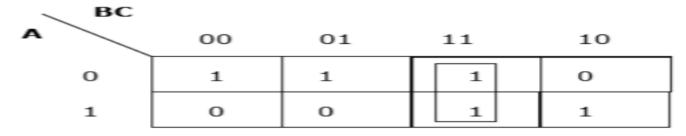


Wrong grouping

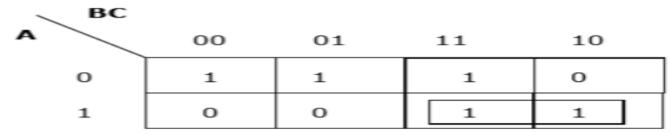
Rule 3 - Grouping must be horizontal or vertical, but must not be diagonal.



Wrong diagonal grouping

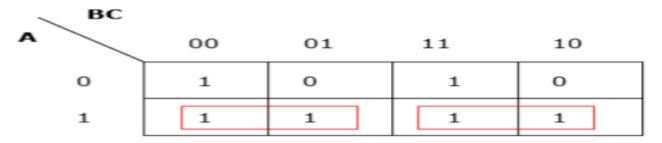


Proper vertical grouping

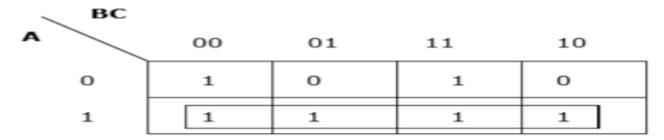


Proper horizontal grouping

Rule 4 - Groups must be covered as largely as possible.

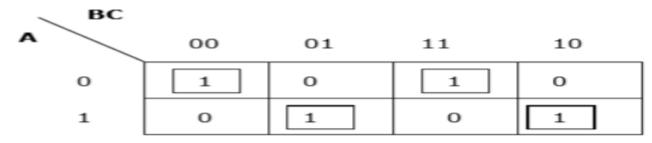


Insufficient grouping



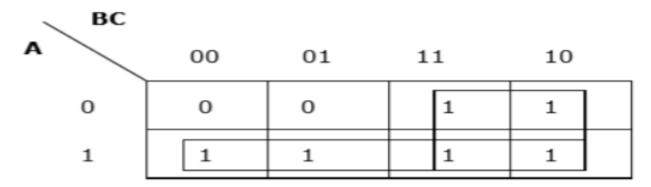
Proper grouping

Rule 5 - If 1 of any cell cannot be grouped with any other cell, it will act as a group itself.



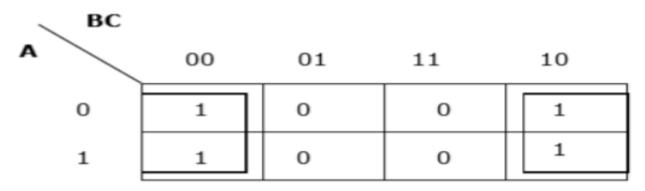
Proper grouping

Rule 6 - Groups may overlap but there should be as few groups as possible.



Proper grouping

Rule 7 - The leftmost cell/cells can be grouped with the rightmost cell/cells and the topmost cell/cells can be grouped with the bottommost cell/cells.



Proper grouping

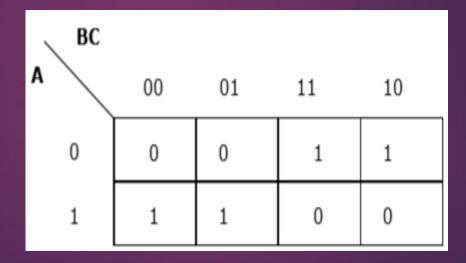
PROBLEM:

MINIMIZE THE FOLLOWING BOOLEAN EXPRESSION USING K-MAP –

F(A,B,C)=A'BC+A'BC'+AB'C'+AB'C

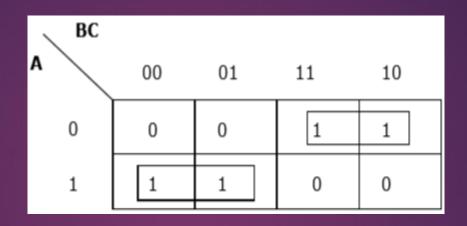
SOLUTION:

EACH TERM IS PUT INTO K-MAP AND WE GET THE FOLLOWING -



K-MAP FOR F (A, B, C)

NOW WE WILL GROUP THE CELLS OF 1 ACCORDING TO THE RULES STATED ABOVE –



K-MAP FOR F (A, B, C)

WE HAVE GOT TWO GROUPS WHICH ARE TERMED AS A'B AND AB'. HENCE, F(A,B,C)=A'B+AB'=A⊕B. IT IS THE MINIMIZED FORM.