

LECTURE-35 REPRESENTATION & MINIMIZATION OF BOOLEAN FUNCTIONS

SIMPLIFICATION USING ALGEBRAIC FUNCTIONS

IN THIS APPROACH, ONE BOOLEAN EXPRESSION IS MINIMIZED INTO AN EQUIVALENT EXPRESSION BY APPLYING BOOLEAN IDENTITIES.

PROBLEM 1

MINIMIZE THE FOLLOWING BOOLEAN EXPRESSION USING BOOLEAN IDENTITIES –

$$F(A,B,C)=A'B+BC'+BC+AB'C'$$

SOLUTION

$$\text{GIVEN, } F(A,B,C)=A'B+BC'+BC+AB'C'$$

$$\text{OR, } F(A,B,C)=A'B+(BC'+BC')+BC+AB'C'$$

$$[\text{BY IDEMPOTENT LAW, } BC' = BC' + BC']$$

$$\text{OR, } F(A,B,C)=A'B+(BC'+BC)+(BC'+AB'C')$$

$$\text{OR, } F(A,B,C)=A'B+B(C'+C)+C'(B+AB')$$

[BY DISTRIBUTIVE LAWS]

$$\text{OR, } F(A,B,C)=A'B+B.1+C'(B+A)$$

[$(C' + C) = 1$ AND ABSORPTION LAW $(B + AB') = (B + A)$]

$$\text{OR, } F(A,B,C)=A'B+B+C'(B+A)$$

[$B.1 = B$]

$$\text{OR, } F(A,B,C)=B(A'+1)+C'(B+A)$$

$$\text{OR, } F(A,B,C)=B.1+C'(B+A)$$

[$(A' + 1) = 1$]

$$\text{OR, } F(A,B,C)=B+C'(B+A)$$

[AS, $B.1 = B$]

$$\text{OR, } F(A,B,C)=B+BC'+AC'$$

$$\text{OR, } F(A,B,C)=B(1+C')+AC'$$

$$\text{OR, } F(A,B,C)=B.1+AC'$$

[AS, $(1 + C') = 1$]

OR, $F(A,B,C)=B+AC'$

[AS, B.1 = B]

SO, $F(A,B,C)=B+ AC'$

IS THE MINIMIZED FORM.

PROBLEM 2

MINIMIZE THE FOLLOWING BOOLEAN EXPRESSION USING BOOLEAN IDENTITIES –

$$F(A,B,C)=(A+B)(A+C)$$

SOLUTION :

Solution

$$\text{Given, } F(A, B, C) = (A + B)(A + C)$$

$$\text{Or, } F(A, B, C) = A.A + A.C + B.A + B.C \quad [\text{Applying distributive Rule}]$$

$$\text{Or, } F(A, B, C) = A + A.C + B.A + B.C \quad [\text{Applying Idempotent Law}]$$

$$\text{Or, } F(A, B, C) = A(1 + C) + B.A + B.C \quad [\text{Applying distributive Law}]$$

$$\text{Or, } F(A, B, C) = A + B.A + B.C \quad [\text{Applying dominance Law}]$$

$$\text{Or, } F(A, B, C) = (A + 1).A + B.C \quad [\text{Applying distributive Law}]$$

$$\text{Or, } F(A, B, C) = 1.A + B.C \quad [\text{Applying dominance Law}]$$

$$\text{Or, } F(A, B, C) = A + B.C \quad [\text{Applying dominance Law}]$$

$$\text{So, } F(A, B, C) = A + BC \text{ is the minimized form.}$$

KARNAUGH MAPS

THE KARNAUGH MAP (K-MAP), INTRODUCED BY MAURICE KARNAUGHIN IN 1953, IS A GRID-LIKE REPRESENTATION OF A TRUTH TABLE WHICH IS USED TO SIMPLIFY BOOLEAN ALGEBRA EXPRESSIONS. A KARNAUGH MAP HAS ZERO AND ONE ENTRIES AT DIFFERENT POSITIONS. IT PROVIDES GROUPING TOGETHER BOOLEAN EXPRESSIONS WITH COMMON FACTORS AND ELIMINATES UNWANTED VARIABLES FROM THE EXPRESSION. IN A K-MAP, CROSSING A VERTICAL OR HORIZONTAL CELL BOUNDARY IS ALWAYS A CHANGE OF ONLY ONE VARIABLE.

EXAMPLE 1

AN ARBITRARY TRUTH TABLE IS TAKEN BELOW –

A	B	A OPERATION B
0	0	W
0	1	X
1	0	Y
1	1	Z

NOW WE WILL MAKE A K-MAP FOR THE ABOVE TRUTH TABLE –

		B	
		0	1
A	0	w	x
	1	y	z

EXAMPLE 2

NOW WE WILL MAKE A K-MAP FOR THE EXPRESSION – $AB + A'B'$

		B	
		0	1
A	0	1	0
	1	0	1

SIMPLIFICATION USING K-MAP

K-MAP USES SOME RULES FOR THE SIMPLIFICATION OF BOOLEAN EXPRESSIONS BY COMBINING TOGETHER ADJACENT CELLS INTO SINGLE TERM. THE RULES ARE DESCRIBED BELOW –

Rule 1 – Any cell containing a zero cannot be grouped.

A \ BC				
	00	01	11	10
0	1	0	1	0
1	0	1	1	1

Wrong grouping

Rule 2 – Groups must contain 2^n cells (n starting from 1).

A \ BC				
	00	01	11	10
0	1	0	1	0
1	0	1	1	1

Wrong grouping

Rule 3 – Grouping must be horizontal or vertical, but must not be diagonal.

		BC			
		00	01	11	10
A	0	1	1	1	0
	1	0	0	1	1

Wrong diagonal grouping

		BC			
		00	01	11	10
A	0	1	1	1	0
	1	0	0	1	1

Proper vertical grouping

		BC			
		00	01	11	10
A	0	1	1	1	0
	1	0	0	1	1

Proper horizontal grouping

Rule 4 – Groups must be covered as largely as possible.

		BC			
		00	01	11	10
A	0	1	0	1	0
	1	1	1	1	1

Insufficient grouping

		BC			
		00	01	11	10
A	0	1	0	1	0
	1	1	1	1	1

Proper grouping

Rule 5 – If 1 of any cell cannot be grouped with any other cell, it will act as a group itself.

		BC			
		00	01	11	10
A	0	1	0	1	0
	1	0	1	0	1

Proper grouping

Rule 6 – Groups may overlap but there should be as few groups as possible.

		BC			
A		00	01	11	10
	0	0	0	1	1
	1	1	1	1	1

Proper grouping

Rule 7 – The leftmost cell/cells can be grouped with the rightmost cell/cells and the topmost cell/cells can be grouped with the bottommost cell/cells.

		BC			
A		00	01	11	10
	0	1	0	0	1
	1	1	0	0	1

Proper grouping

PROBLEM :

MINIMIZE THE FOLLOWING BOOLEAN EXPRESSION USING K-MAP –

$$F(A,B,C)=A'BC+A'BC'+AB'C'+AB'C$$

SOLUTION :

EACH TERM IS PUT INTO K-MAP AND WE GET THE FOLLOWING –

		BC			
		00	01	11	10
A	0	0	0	1	1
	1	1	1	0	0

K-MAP FOR F (A, B, C)

NOW WE WILL GROUP THE CELLS OF 1 ACCORDING TO THE RULES STATED ABOVE –

		BC			
		00	01	11	10
A	0	0	0	1	1
	1	1	1	0	0

K-MAP FOR F (A, B, C)

WE HAVE GOT TWO GROUPS WHICH ARE TERMED AS $A'B$ AND AB' . HENCE,
 $F(A,B,C)=A'B+AB'=A\oplus B$. IT IS THE MINIMIZED FORM.