TUTORIAL 1

1. Solve the partial differential equation

$$(D^2 - 4DD' + 4D'^2)z = e^{2x+y}$$

2. Solve the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x\,\partial y} + 2\frac{\partial^2 z}{\partial y^2} = x + y$$

3. Solve the partial differential equation

$$(D^2 - DD' - 2D'^2 + 2D + 2D')z = \sin(2x + y)$$

4. Solve the partial differential equation

$$(D - 3D' - 2)^2 z = 2x e^{2x} \tan(y + 3x)$$

5. Using the method of separation of variables, solve

$$3\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \text{ , } u(x,0) = 4e^{-x}$$

6. Find the solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$ under the conditions $u = 0$ when $x = 0$ and $x = \pi$, $\frac{\partial u}{\partial t} = 0$ when $t = 0$ and $u(x,0) = x$; $0 < x < \pi$.

7. Find the solution of the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions $u(0,y) = u(l,y) = u(x,0) = 0$ and $u(x,a) = \sin(n\pi x / l)$.

8. An insulated rod of length l has its ends *A* and *B* maintained at 0^o**C** and 100^o**C** respectively until steady state conditions prevail. If *B* is suddenly reduced to 0^o**C** and maintained at 0^o**C**. Show that the temperature at a distance *x* from *A* at time *t* is given by

$$u(x,t) = \frac{200}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin\frac{n\pi x}{l}\, e^{-\frac{n^2\pi^2c^2t}{l^2}}$$

ANSWERS

Here ϕ_1, ϕ_2 are arbitrary functions.

1. $z = \phi_1(y+2x) + x\phi_2(y+2x) + \frac{x^2}{2}e^{2x+y}$
2. $z = \phi_1(y-x) + \phi_2(y-2x) + \frac{x^2 y}{2} - \frac{x^3}{3}$
3. $z = e^{-2x}\phi_1(y+2x) + \phi_2(y-x) - \frac{1}{6}\cos(2x+y)$
4. $z = e^{2x}\phi_1(y+3x) + xe^{2x}\phi_2(y+3x) + \frac{1}{3}x^3 e^{2x}\tan(y+3x)$
5. $u(x,y) = 4e^{(3y-2x)/2}$
6. $u(x,t) = 2\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\cos(nat)\sin(nx)$
7. $u(x,y) = \frac{\sinh(n\pi y / l)}{\sinh(n\pi a / l)}\sin\left(\frac{n\pi x}{l}\right)$

TUTORIAL 2

1. The odds against ***A*** solving a certain problem are **5** to **7** and the odds in favour of ***B*** solving the same problem are **3** to **4**. What is the probability that if both of them try, the problem would be solved.

2. An insurance company insured **2000** scooter drivers**, 4000** car drivers and **6000** truck drivers. The probability of an accident involving a scooter driver, a car driver and a truck driver is **0.01, 0.03** and **0.15** respectively. One of the insured person meets with an accident. What is the probability that he is a scooter driver?

3. A random variable ***X*** has the probability distribution given by

x :	**−3**	**6**	**9**
P(X= x):	**1/6**	**1/2**	**1/3**

Find **E**(X) and **E**(X^2). Hence evaluate **E**$(2X+1)^2$.

4. The first four moments about the working mean **28.5** of a distribution are **0.294, 7.144, 42.409** and **454.98**. Calculate mean and all the four moments about the mean.

5. Find the moment generating function about any arbitrary point '***a***' of the distribution

x :	**−1**	**1**
P(X= x) :	**1/2**	**1/2**

6. In a Binomial distribution consisting of **5** independent trials, probabilities of **1** and **2** successes are **0.4096** and **0.2048** respectively. Find the parameter ***p*** of the distribution.

7. A manufacturer of pins knows that on an average **5%** of his product is defective. He sells pins in boxes of **100** and guarantees that not more than **4** pins will be defective. What is the probability that the box will fail to meet the guaranteed quality? $(e^{-5} = 0.0067)$.

8. In a normal distribution, **31%** of the items are under **45** and **8%** are over **64**. Find the mean and S.D. of the distribution.

(Given that $P(0 \le Z \le 0.5) = 0.19$, $P(0 \le Z \le 1.4) = 0.42$)

ANSWERS

1. $\frac{16}{21}$ 2. $\frac{1}{52}$ 3. $\frac{11}{2}, \frac{93}{2}, 209$

4. $\text{Mean} = 28.794$, $\mu_1 = 0$, $\mu_2 = 7.058$, $\mu_3 = 36.151$, $\mu_4 = 408.738$

5. $M_a(t) = e^{-at} \cosh t$ 6. 1/5 7. 0.562 8. $\text{Mean} = 50$, $\text{S.D.} = 10$

TUTORIAL 3

1. By the method of least squares, fit a straight line $y = mx + c$ from the following data:

x :	50	70	100	120
y:	12	15	21	25

2. Two random variables have the regression lines with equations $x = 19.13 - 0.87y$ and $y = 11.64 - 0.50x$. Find the mean values of x and y. Also, find the correlation coefficient between x and y.

3. Prove that $r = \dfrac{\sigma_x^2 + \sigma_y^2 - \sigma_{x-y}^2}{2\,\sigma_x \sigma_y}$, where σ_x & σ_y are the S.D.'s of x and y-series respectively and r is the correlation coefficient.

4. A sample of **900** members is found to have a mean of **3.4** cm. Can it be reasonably regarded as a truly random sample from a large population with mean **3.25**cm and standard deviation **1.61** cm?

5. A random sample of **10** boys had the following I.Q.:

70, 120, 110, 101, 88, 83, 95, 98, 107, 100.

Do these data support the assumption of a population mean I.Q. of **100** (at 5% level of significance)?

6. In experiments on pea breeding, the following frequencies of seeds were obtained:

Round and yellow	Wrinkled and yellow	Round and green	Wrinkled and green	Total
315	101	108	32	556

Theory predicts that the frequencies should be in proportion **9:3:3:1.** Examine the correspondence between theory and experiment.

7. A random sample of **900** measurements from a large population gave a mean value of **64**. If this sample has been drawn from a normal population with standard deviation of **20**. Find the **95%** confidence limits for the mean in the population.

8. Two independent samples of sizes 7 and 6 have the following values:

Sample A:	28	30	32	33	33	29	34
Sample B:	29	30	30	24	27	29	

Examine whether the samples have been drawn from normal populations having the same variance.

ANSWERS

1. $y = 2.2759 + 0.1879x$ 2. $\bar{x} = 15.79$, $\bar{y} = 3.74$, $r = -0.66$ 4. No 5. Yes
6. There is a very high degree of agreement between theory and experiments. 7. 62.693 and 65.307 8. Yes

TUTORIAL 4

1. A toy company manufactures two types of toys, type **A** and type **B**. Each toy of type **B** takes twice as long to produce as one of type **A**, and the company would have time to make a maximum of **2000** toys per day. The supply of plastic is sufficient to produce **1500** toys per day(both **A** and **B** combined). Type **B** toy requires a dress of which there are only **600** per day available. If the company makes a profit of Rs.**3.00** and Rs.**5.00** per toy, respectively on type **A** and **B**, then how many of each types of toy should be produced per day in order to maximize the total profit. Formulate this problem.

2. Using graphical method, solve the following L.P.P.

$$\text{Max } Z = 3x_1 + 5x_2 \quad \text{s.t.} \quad x_1 + 2x_2 \le 200 \ , \ x_1 + x_2 \le 150 \ , \ x_1 \le 60 \ , \ x_1, \ x_2 \ge 0$$

3. Convert the following L.P.P. to the standard form: $\text{Min } Z = 3x_1 + 4x_2$

$$s.t. \ 2x_1 - x_2 - 3x_3 = -4 \ , \ 3x_1 + 5x_2 + x_4 = 10 \ , \ x_1 - 4x_2 = 12 \ , \ x_1 \ , \ x_2 \ , \ x_3 \ , \ x_4 \ge 0$$

4. Solve the following L.P.P. by simplex method

$$\begin{aligned} &\text{Max } Z = 3x_1 + 2x_2 + 5x_3 \\ s.t. \quad & x_1 + 2x_2 + x_3 \le 430 \\ & 3x_1 + 2x_3 \le 460 \\ & x_1 + 4x_2 \le 420 \\ & x_1 \ , \ x_2 \ , x_3 \ge 0 \end{aligned}$$

5. Write the dual of the following problem: $\text{Max } Z = 3x_1 + 2x_2$

$$s.t. \ \ x_1 - x_2 \le 1 \ , \ x_1 + x_2 \ge 3 \ , \ x_1 \ge 0 \ , \ x_2 \ \textbf{is unrestricted in sign}.$$

6. Using dual simplex method solve the following L.P.P.

$$\text{Max } Z = -2x_1 - x_3$$

$$s.t. \quad x_1 + x_2 - x_3 \ge 5 \ , \ x_1 - 2x_2 + 4x_3 \ge 8 \ \text{and} \ x_1 \ , \ x_2 \ , x_3 \ge 0.$$

7. A car hire company has one car at each of five depots ***a, b, c, d*** and ***e***. A customer requires a car in each town, namely ***A, B, C, D*** and ***E***. Distances (in kms) between depots(origins) and towns (destinations) are given in the following distance matrix:

	a	***b***	***c***	***d***	***e***
A	160	130	175	190	200
B	135	120	130	160	175
C	140	110	155	170	185
D	50	50	80	80	110
E	55	35	70	80	105

How should cars be assigned to customers so as to minimize the distance travelled?

8. Find the initial basic feasible solution of the following transportation problem by **VAM** method:

From\To	W_1	W_2	W_3	W_4	Supply
F_1	11	20	7	8	50
F_2	21	16	10	12	40
F_3	8	12	18	9	70
Demand	30	25	35	40	

ANSWERS

1. Suppose the company produce x_1 toys of type **A** and x_2 toys of type **B** per day. Then

$$\text{Max } Z = 5x_1 + 3x_2 \ \text{ s.t. } \ x_1 + 2x_2 \le 2000 \ , \ x_1 + x_2 \le 1500 \ , \ x_2 \le 600 \ , \ x_1, x_2 \ge 0$$

2. $x_1 = 100 \ , \ x_2 = 0 \ , \ Max\, Z = 550$

3. $$\text{Max } Z'(= -Z) = -3x_1 - 4x_2' + 4x_2''$$

$$s.t. \ -2x_1 + x_2' - x_2'' + 3x_3 = 4$$

$$3x_1 + 5x_2' - 5x_2'' + x_4 = 10$$

$$x_1 - 4x_2' - 4x_2'' = 12$$

$$x_1 \ , \ x_2' \ , \ x_2'' \ , \ x_3 \ , \ x_4 \ge 0$$

4. $x_1 = 0 \ , \ x_2 = 100 \ , \ x_3 = 230$ and $\text{Max } Z = 1350$

5. $$\text{Min } Z_d = y_1 - 3y_2$$

$$s.t. \quad y_1 - y_2 \ge 3$$

$$-y_1 - y_2 = 2$$

$$y_1 \ , \ y_2 \ge 0$$

6. $x_1 = 0 \ , \ x_2 = 14 \ , \ x_3 = 9$ and $\text{Max } Z = -9$

7. $A \to e \ , \ B \to c \ , \ C \to b \ , \ D \to a \ , \ E \to d \ , \ \text{Min cost} = 570 \text{ kms}$

8. From F_1 transport **25** units to W_3 and **25** units to W_4;

   From F_2 transport **10** units to W_3;

   From F_3 transport **30** units to W_1, **25** units to W_2 and **15** units to W_4 respectively;

   Optimal transport cost = Rs. **1150**