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Assignment - 4

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1. Que:- A company manufactures two types of cloths, using three different colors of wool. One yard length of type A cloth requires 4 oz of red wool, 5 oz of green wool and 3 oz of yellow wool. One yard length of type B cloth requires 5 oz of red wool, 2 oz of green wool and 8 oz of yellow wool. The wool available for manufacture is 1000 oz. of red wool, 1000 oz of green wool and 1200 oz of yellow wool. The manufacture can make a profit of Rs 5 one one yard of type A cloth and Rs. 3 one one yard of type B cloth formulate this problem as a linear programming problem to find the best combination of the quantities of type A and Type B. cloth ~~using~~ which gives maximum profit.

Sol^n Let the manufactures decide to produce

x_1 yards of type A cloth

& x_2 yards of type B cloth

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then the total income in rupees, from these units of cloth is given by

$$z = 5x_1 + 3x_2$$

to produce these units of two types of cloth, he requires

red wool $= 4x_1 + 5x_2$ oz

green wool $= 5x_1 + 2x_2$ oz

yellow wool $= 3x_1 + 8x_2$ oz

Since the manufactures he does not have more than 1000 oz of red wool, 1000 oz of green wool, and 1200 oz of yellow wool.

$\therefore$ $4x_1 + 5x_2 \leq 1000$

$5x_1 + 2x_2 \leq 1000$

$3x_1 + 8x_2 \leq 1200$

$x_1, x_2 \geq 0$

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2. Using graphical method. Solve the following L.P.P.

$$\text{Min } z = x_1 + x_2$$

s.t. $x_1 + 2x_2 \leq 10$

$x_1 + x_2 \geq 1$

$x_1 \leq 4$

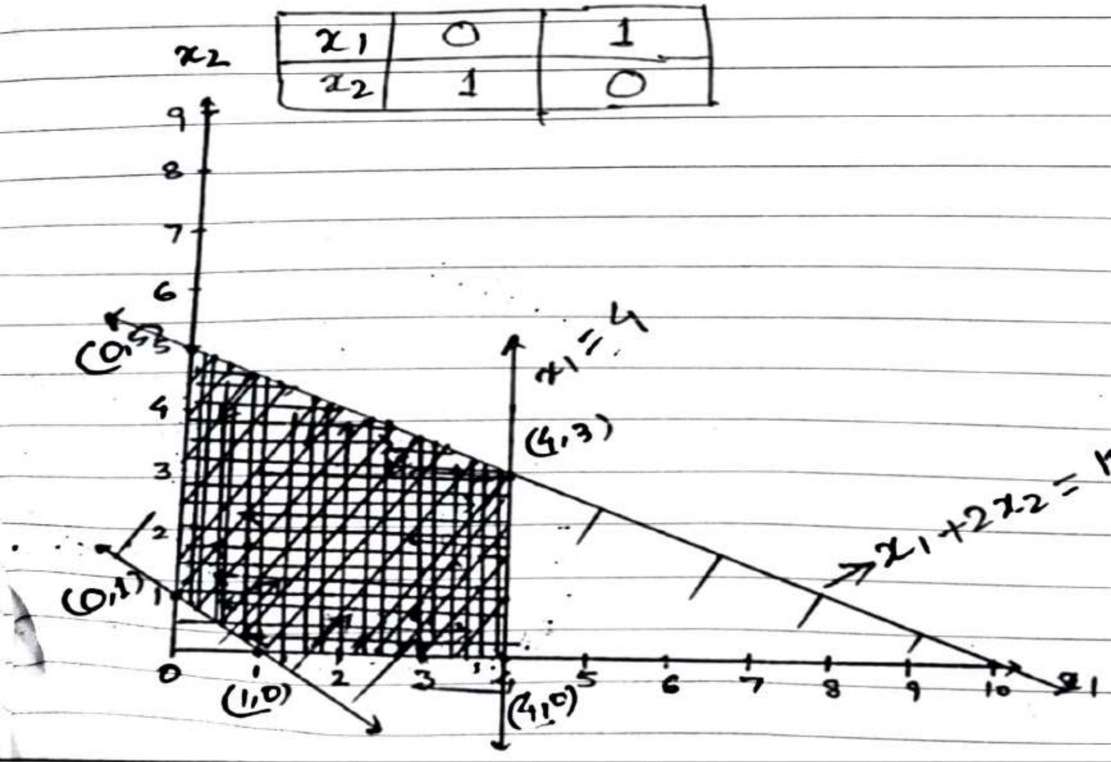
$x_1, x_2 \geq 0$

Sol^n. $x_1 + 2x_2 = 10$

x_1	0	4
x_2	5	3

$x_1 + x_2 = 1$

x_1	0	1
x_2	1	0



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Corner Points	$z = x_1 + x_2$
A(0,5)	5
B(0,1)	1
C(1,0)	1
D(4,0)	4
E(4,3)	7

The minimum value of z occurs at B(0,1) & C(1,0) = 1 Ans

3. Que:- Using graphical method, find the maximum value of

$$z = 2x + 3y$$

$$\text{s.t.} \quad x + y \leq 30$$

$$y \geq 3$$

$$x \geq y$$

$$0 \leq x \leq 20$$

$$0 \leq y \leq 12$$

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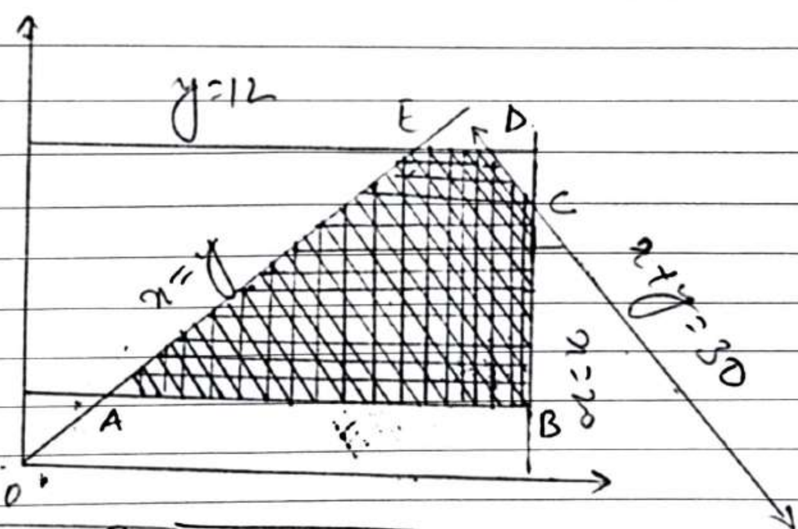
Sol^n. Any point (x, y) satisfying the conditions $x \geq 0$, $y \geq 0$ lies in the 1st quadrant only. Also since $x+y \leq 30$, $y \geq 3$, $y \leq 12$ and $x \leq 20$, the desired point (x, y) lies within the convex region ABCDE. Its vertices are

A(3,3), B(20,3), C(20,10), D(18,12) and E(12,12)



Corner points	$Z = 2x+3y$
A(3,3)	15
B(20,3)	49
C(20,10)	70
D(18,12)	72
E(12,12)	60

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Since the maximum value of z is 72 which occurs at point D. The solution to the L.P.P. is $x = 18, y = 12$ and maximum value $= 72$

4. Que:- Convert the following L.P.P. to the standard form :

$$\text{Max } Z = 2x_1 + 3x_2 + 5x_3$$

S.t. $6x_1 - 3x_2 \leq 5$

$$3x_1 + 2x_2 + 4x_3 \geq 10$$

$$4x_1 + 3x_3 \leq 2$$

$$x_1, x_2 \geq 0$$

Soln: $x_3 = (x_3' - x_3'')$

$$\text{Max } Z = 2x_1 + 3x_2 + 5x_3' - 5x_3'' + 0S_1 + 0S_2 + 0S_3$$

S.t. $6x_1 - 3x_2 + S_1 = 5$

$$3x_1 + 2x_2 + 4(x_3' - x_3'') - S_2 = 10$$

$$\Rightarrow 3x_1 + 2x_2 + 4x_3' - 4x_3'' - S_2 = 10$$

& $4x_1 + 3(x_3' - x_3'') + S_3 = 2$

$$\Rightarrow 4x_1 + 3x_3' - 3x_3'' + S_3 = 2$$

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5. Que:- Solve the following L.P.P. to the standard form by simplex method

$$\text{Max } Z = 5x_1 + 3x_2$$

s.t. $3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$

$x_1, x_2 \geq 0$

Sol^n max $z = 5x_1 + 3x_2$

$3x_1 + 5x_2 + S_1 = 15$

$5x_1 + 2x_2 + S_2 = 10$

	C_j		5	3	0	0	
C_B	X_B	b	x_1	x_2	S_1	S_2	min ratio
0	S_1	15	3	5	1	0	15/3
0	S_2	10	$\boxed{5}$ key element	2	0	1	10/5 ← outgoing variable
Z_j			0	0	0	0	
C_j			5	3	0	0	
$Z_j - C_j$			-5 ↑ entering variable	-3	0	0	

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		c_j	5	3	0	0	
C_B	X_B	b	x_1	x_2	S_1	S_2	min ratio
0	S_1	9	0	19/5	1	-3/5	45/19
5	x_1	2	1	2/5	0	1/5	10/2
z_j			5	2	0	1	
c_j			5	3	0	0	
$z_j - c_j$			0	-1	0	1	
				↑			

C_B	X_B	b	x_1	x_2	S_1	S_2
3	x_2	45/19	0	1	5/19	-3/19
5	x_1	20/19	1	0	-2/19	5/19
$z_j - c_j$			0	0	5/19	16/19

all $z_j - c_j \geq 0$

$x_1 = 20/19$

$x_2 = 45/19$

$$\max z = \frac{5 \times 20}{19} + \frac{3 \times 45}{19}$$

$$= \frac{100}{19} + \frac{135}{19}$$

$$\boxed{\max z = \frac{235}{19}}$$

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6. Que:- Solve the following L.P.P. by Simplex method

$$\text{Min } Z = x_1 - 3x_2 + 3x_3$$

$$\text{s.t. } 3x_1 - x_2 + 2x_3 \leq 7$$

$$2x_1 + 4x_2 \geq -12$$

$$-4x_1 + 3x_2 + 8x_3 \leq 10$$

$$x_1, x_2, x_3 \geq 0$$

Sol^n. $\text{Max } Z' = -x_1 + 3x_2 - 3x_3$

now, $-2x_1 - 4x_2 \leq 12$

In Standard form

now, $\text{Max } Z' = -x_1 + 3x_2 - 3x_3 + 0S_1 + 0S_2 + 0S_3$

S.t $3x_1 - x_2 + 2x_3 + S_1 + 0S_2 + 0S_3 = 7$

$-2x_1 - 4x_2 + 0x_3 + 0S_1 + S_2 + 0S_3 = 12$

$-4x_1 + 3x_2 + 8x_3 + 0S_1 + 0S_2 + S_3 = 10$

$x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

find initial basic feasible solution is $x_1 = x_2 = x_3 = 0$ (non basic) ; $S_1 = 7, S_2 = 12$

$S_3 = 10$ (basic)

$\therefore$ Initial basic feasible solution is given by the table below:

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C_B	c_j / Basis	x_1	x_2	x_3	S_1	S_2	S_3	b	θ
		-1	3	-3	0	0	0		
0	S_1	3	-1	2	1	0	0	7	7/(-1)
0	S_2	-2	-4	0	0	1	0	12	12/(-4)
0	S_3	-4	(3)	8	0	0	1	10	10/3 ←
$z_j = \sum c_B a_{ij}$		0	0	0	0	0	0	0	
$C_j = c_j - z_j$		-1	3 ↑	-3	0	0	0		

As C_j is Positive under 2nd column, the initial basic feasible solution is not optimal and we Proceed further

here,

x_2 is the incoming variable, S_3 is the outgoing variable and (3) is the key element

C_B	c_j / Basis	x_1	x_2	x_3	S_1	S_2	S_3	b	θ
		-1	3	-3	0	0	0		
0	S_1	(5/3)	0	14/3	1	0	1/3	31/3	31/5
0	S_2	(-22/3)	0	32/3	0	1	4/3	76/3	-38/11
3	x_2	-4/3	1	8.3	0	0	1/3	10/3	-5/2
	z_j	-4	3	8	0	0	1	10	
	C_j	3 ↑	0	-11	0	0	-1		

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As C_j is positive under 1st column the solution is not optimal and we proceed further x_1 is the incoming variable, S_1 is the outgoing variable and $(5/3)$ is the key element.

	C_j	-1	3	-3	0	0	0		
C_B	Basis	x_1	x_2	x_3	S_1	S_2	S_3	b	θ
-1	x_1	1	0	14/5	3/5	0	1/5	31/5	
0	S_2	0	0	156/5	22/5	1	14/5	354/5	
3	x_2	0	1	32/5	4/5	0	3/5	58/5	
	Z_j	-1	3	82/5	9/5	0	8/5	143/5	
	C_j	0	0	-97/5	-9/5	0	-8/5		

Now, since each $C_j \leq 0$. Therefore it gives the optimal solution

$x_1 = 31/5$

$x_2 = 58/5$

$x_3 = 0$ (non basic)

and $Z'_{max} = 143/5$

Hence $Z_{min} = -143/5$

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7. Que:- Write the dual of the following Problem :

$$\text{Max } Z = 4x_1 + 9x_2 + 2x_3$$

$$\text{s.t. } 2x_1 + 3x_2 + 2x_3 \leq 7$$

$$3x_1 - 2x_2 + 4x_3 = 5$$

$$x_1, x_2, x_3 \geq 0$$

Solⁿ. Let y_1 and y_2 by the dual variables associated with the first and 2nd constraints. Then the dual problem is

$$\text{Minimize } W = 7y_1 + 5y_2,$$

$$\text{Subject to } 2y_1 + 3y_2 \leq 4,$$

$$3y_1 - 2y_2 \leq 9,$$

$$2y_1 + 4y_2 \leq 2,$$

$$y_1 \geq 0,$$

y_2 is Unrestricted in Sign.

8. Que:- Using dual Simplex method Solve the following L.P.P.

$$\text{Max } Z = -3x_1 - 2x_2$$

$$\text{s.t} = x_1 + x_2 \geq 1$$

$$x_1 + x_2 \leq 7$$

$$x_1 + 2x_2 \geq 10$$

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$x_2 \leq 3$

$x_1, x_2 \geq 0$

Solⁿ. Step 1. (i) Convert the 1st and 3rd constraints into ($\leq$) type.

$-x_1 - x_2 \leq -1, \; -x_1 - 2x_2 \leq -10$

Step (ii) Express the Problem in standard form.

Max $Z = -3x_1 - 2x_2 + 0S_1 + 0S_2 + 0S_3 + 0S_4$

Subject to, $-x_1 - x_2 + S_1 = -1,$

$x_1 + x_2 + S_2 = 7$

$-x_1 + 2x_2 + S_3 = -10$

$x_2 + S_4 = 3, \; x_1, x_2, S_1, S_2, S_3, S_4 \geq 0$

Step 2: Find the initial basic solⁿ

$x_1 = x_2 = 0, \; S_1 = -1, S_2 = 7, S_3 = -10, S_4 = 3$

and $Z = 0$.

$\therefore$	C_j	-3	-2	0	0	0	0	
C_B	Basis	x_1	x_2	S_1	S_2	S_3	S_4	b
0	S_1	-1	-1	1	0	0	0	-1
0	S_2	1	1	0	1	0	0	7
0	S_3	-1	(-2)	0	0	1	0	-10 ←
0	S_4	0	1	0	0	0	1	3
$z_j = \sum c_B a_{ij}$		0	0	0	0	0	0	
$c_j = c_j - z_j$		-3	-2 ↑	0	0	0	0	

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Step 4: Mark the outgoing variable

$\because$ b_3 is negative and numerically largest, the 3rd row is the key row and S_3 is the outgoing variable.

Step 5: Calculate the ratios of elements

Step 6:

(i)

C_B	Basis	x_1	x_2	S_1	S_2	S_3	S_4	b
	c_j	-3	-2	0	0	0	0	
0	S_1	$-1/2$	0	1	-0	$-1/2$	0	4
0	S_2	$1/2$	0	0	$1/2$	$1/2$	0	2
-2	x_2	$1/2$	1	0	-0	$1/2$	0	5
0	S_4	$(-1/2)$	0	0	0	$1/2$	1	$-2 \leftarrow$
	$Z_j = \Sigma C_B a_{ij}$	-1	-2	0	0	1	0	-10
	$C_j = c_j - Z_j$	-2 $\uparrow$	0	0	0	-1	0	

Since all C_j values are ≤ 0 and $b_4 = -2$

(ii) mark the outgoing variable

(iii) Calculate ratios of elements in C_j row

(iv) Drop S_4 and introduce x_1

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C_B	Basis	x_1	x_2	S_1	S_2	S_3	S_4	b
	C_j	-3	-2	0	0	0	0	
0	S_1	0	0	1	0	-1	-1	6
0	S_2	0	0	0	1	1	1	0
-2	x_2	0	1	0	0	0	1	3
-3	x_1	1	0	0	0	-10	-2	4
Z_j		-3	-2	0	0	3	4	-18
$\bar{C}_j$		0	0	0	0	-3	-4	

Since all $\bar{c}_j$ values are ≤ 0 and all b's are ≥ 0. Therefore this solution is optimal. and feasible.

Thus the optimal solution is

$$\boxed{x_1 = 4,\ x_2 = 3 \text{ and } Z_{max} = -18}$$

9. Que:- An Engineer wants to assign 3 jobs J_1, J_2, J_3 to three machines M_1, M_2, M_3 in such a way that each job is assigned to some machine and no machine works on more than one job. find the optimal solution using Hungarian method if the cost matrix is as follows

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	M_1	M_2	M_3
J_1	15	10	9
J_2	9	15	10
J_3	10	12	8

⇓

Sol^n

	M_1	M_2	M_3
J_1	6	1	0
J_2	0	6	1
J_3	2	4	0

⇓

	M_1	M_2	M_3
J_1	6	$\boxed{0}$	$\cancel{0}$
J_2	$\boxed{0}$	5	1
J_3	2	3	$\boxed{0}$

		Cost
J_1 –	M_2	10
J_2 –	M_1	9
J_3 –	M_3	8
		27

$\therefore$ $\boxed{\text{Min cost} = 27}$

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10. Que:- Solve the following transportation problem by VAM Method

		Destination				
	from | TO	A	B	C	D	Availability
Source	I	21	16	25	13	11
	II	17	18	24	23	13
	III	33	27	18	41	19
	Requirement	6	10	12	15	43

Table 1.

Solⁿ.

21	16	25	11 13	11 (3)
17	18	14	23	13 (3)
32	27	18	41	19(9)
6	10	12	15	
(4)	(2)	(4)	(10)	

Table 2

17	18	14	4 23	13 (3)
32	27	18	41	19 (9)
6	10	12	4 +	
(15)	(9)	(4)	(18)	

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Table 3

$\boxed{6}$ 17	18	14	9(3)
32	27	18	19(9)
6	10	12	
(15)	(9)	(4)	

Table 4

$\boxed{3}$ 18	14	3(4)
27	18	19(9)
10	12	
(9)	(4)	

Table 5

$\boxed{7}$ 27	$\boxed{12}$ 18	19
7	12 +	

Table 6.

21	16	25	$\boxed{11}$ 13
$\boxed{6}$ 17	$\boxed{3}$ 18	14	$\boxed{4}$ 23
32	$\boxed{7}$ 27	$\boxed{12}$ 18	41

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Table 7

$u_i \backslash v_j$	17	18	9	23
-10	(-) 21	(-) 16	(-) 25	13
0	17	18	14	23
9	32	27	18	41

now, Apply optimality check

no. of allocations = $m+n-1$ i.e 6

We can apply MODI method

(i) We have $u_2+v_1=17$

$u_2+v_2=18$

$u_3+v_2=27$

$u_3+v_3=18$

$u_1+u_4=13$

$u_2+u_4=23$

let $u_2=0$, then $V_1=17$, $V_2=18$,

$u_3=9$, $u_4=23$, $u_1=-10$

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(ii) Net evaluations $w_{ij} = (u_i + v_j) - c_{ij}$

for all empty cell are

$w_{11} = -14,\ w_{12} = -8,\ w_{13} = -26,$

$w_{23} = -5,\ w_{31} = -6,\ w_{34} = -9$

(iii) Since all net evaluations are negative. The current solution is optimal. Hence the optimal allocation is given by

$x_{14} = 11,\ x_{21} = 6,\ x_{22} = 3,\ x_{24} = 4$

$,\ x_{32} = 7$ and $x_{33} = 12$

$\therefore$ The optimal (minimum) transportation cost is

$= 11 \times 13 + 6 \times 17 + 3 \times 18 + 4 \times 23 + 7 \times 27 + 12 \times 18$

$= 796$ Ans