

DATE :

Assignment - 4

Name = Syeda Reeha Ansar
Roll no. - 14114802719
Group - 4C7

Ques:- A company manufactures two types of cloths, using three different colors of wool. One yard length of type A cloth required 4 oz of red wool, 5 oz of green wool and 3 oz of yellow wool. One yard length of type B cloth requires 5 oz of red wool, 2 oz of green wool and 8 oz of yellow wool. The wool available for manufacture is 1000 oz. of red wool, 1000 oz of green wool and 1200 oz of yellow wool. The manufacture can make a profit of Rs 5 on one yard of type A cloth and Rs. 3 on one yard of type B cloth. formulate this problem as a linear programming problem to find the best combination of the quantities of type A and Type B. cloth using which gives maximum profit.

Sol^h Let the manufacturer decide to produce

x_1 yards of type A cloth
& x_2 yards of type B cloth

then the total income in rupees, from
these units of cloth is given by

$$Z = 5x_1 + 3x_2$$

to produce these units of two types
of cloth, he requires

$$\text{red wool} = 4x_1 + 5x_2 \text{ oz}$$

$$\text{green wool} = 5x_1 + 2x_2 \text{ oz}$$

$$\text{yellow wool} = 3x_1 + 8x_2 \text{ oz}$$

Since the manufacturer does not
have more than 1000 oz of red
wool, 1000 oz of green wool, and
1200 oz of yellow wool.

$$\therefore 4x_1 + 5x_2 \leq 1000$$

$$5x_1 + 2x_2 \leq 1000$$

$$3x_1 + 8x_2 \leq 1200$$

$$x_1, x_2 \geq 0$$

DATE:

2. Using graphical method. Solve the following L.P.P.

$$\text{Min } Z = x_1 + x_2$$

$$\text{s.t. } x_1 + 2x_2 \leq 10$$

$$x_1 + x_2 \geq 1$$

$$x_1 \leq 4$$

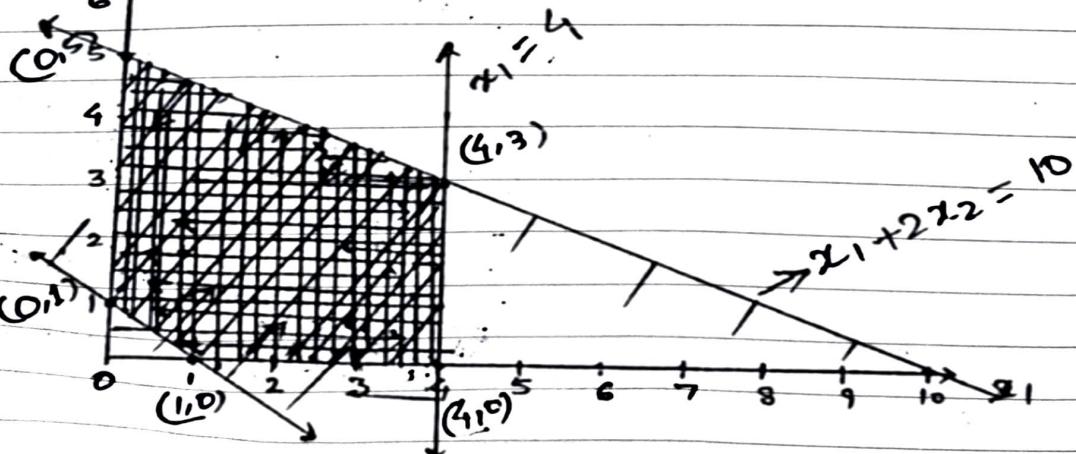
$$x_1, x_2 \geq 0$$

$$\text{Soln. } x_1 + 2x_2 = 10$$

x_1	0	4
x_2	5	3

$$x_1 + x_2 = 1$$

x_2	x_1	0	1
x_2	1	0	



4C7

Syedahieha Ansar

14114802719

DATE:

corner points	$Z = x_1 + x_2$
A(0,5)	5
B(0,1)	1
C(1,0)	1
D(4,0)	4
E(4,3)	7

The minimum value of Z occurs
at B(0,1) & C(1,0) = 1 Ans

3. Que:- Using graphical method, find the maximum value of

$$Z = 2x + 3y$$

$$\text{s.t. } x + y \leq 10$$

$$y \geq 3$$

$$x \geq 1$$

$$0 \leq x \leq 10$$

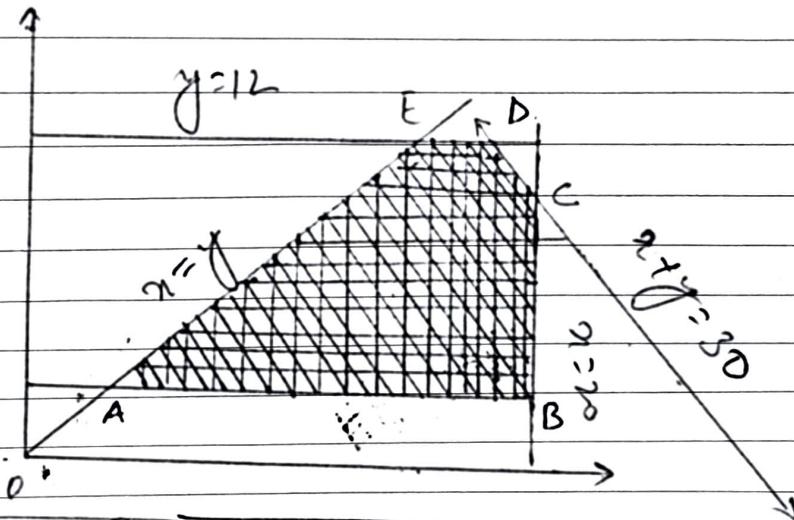
$$0 \leq y \leq 12$$

DATE:

SII". Any point (x,y) satisfying the conditions
 $x \geq 0, y \geq 0$ lies in the 1st quadrant
 Only. Also since $x \leq 30, y \leq 3, y \leq 12$
 and $x \leq 20$, the desired point (x,y)
 lies within the convex region

A B C D E . Its vertices are

A(3,3), B(20,3), C(20,10), D(18,12) and E(12,12)



Corners Points	$Z = 2x + 3y$
A(3,3)	15
B(20,3)	49
C(20,10)	70
D(18,12)	72
E(12,12)	60

DATE:

Since the maximum value of Z is
72 which occurs at point D.

The solution to the L.P.P. is

$x = 18, y = 12$ and maximum value
 $= 72$

4. Que:- Convert the following L.P.P.
to the standard form:

$$\text{Max } Z = 2x_1 + 3x_2 + 5x_3$$

$$\text{s.t. } 6x_1 - 3x_2 \leq 5$$

$$3x_1 + 2x_2 + 4x_3 \leq 10$$

$$4x_1 + 3x_3 \leq 2$$

$$x_1, x_2 \geq 0$$

$$\text{Soln. } x_3 = (x_3' - x_3'')$$

$$\text{Max } Z = 2x_1 + 3x_2 + 5x_3' - 5x_3'' + 0S_1 + 0S_2 + 0S_3$$

$$\text{s.t. } 6x_1 - 3x_2 + S_1 = 5$$

$$3x_1 + 2x_2 + 4(x_3' - x_3'') - S_2 = 10$$

$$\Rightarrow 3x_1 + 2x_2 + 4x_3' - 4x_3'' - S_2 = 10$$

$$\& 4x_1 + 3(x_3' - x_3'') + S_3 = 2$$

$$\Rightarrow 4x_1 + 3x_3' - 3x_3'' + S_3 = 2$$

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DATE:

5. Que:- Solve the following L.P.P. to
the standard form by simplex method

$$\text{Max } Z = 5x_1 + 3x_2$$

$$\text{s.t. } 3x_1 + 5x_2 \leq 15$$

$$5x_1 + 2x_2 \leq 10$$

$$x_1, x_2 \geq 0$$

$$\text{Sol"} \quad \text{Max } Z = 5x_1 + 3x_2$$

$$3x_1 + 5x_2 + S_1 = 15$$

$$5x_1 + 2x_2 + S_2 = 10$$

	C_j	5	3	0	0		
C_B	X_B	b	x_1	x_2	S_1	S_2	min ratio
0	S_1	15	3	5	1	0	$15/3$
0	S_2	10	<u>5</u>	<u>2</u>	0	1	$10/5$ ← outgoing variable

key element

$$Z_j \quad 0 \quad 0 \quad 0 \quad 0$$

$$C_j \quad 5 \quad 3 \quad 0 \quad 0$$

$$Z_j - C_j \quad -5 \quad -3 \quad 0 \quad 0$$

↑ entering variable

4C7

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14114802719

DATE:

C_B	x_B	b	x_1	x_2	S_1	S_2	min ratio
0	S_1	9	0	$\frac{17}{5}$	1	$\frac{-3}{5}$	$\frac{45}{19}$
5	x_1	2	1	$\frac{2}{5}$	0	$\frac{1}{5}$	$\frac{10}{2}$

z_j	5	2	0	1
-------	---	---	---	---

C_j	5	3	0	0
$z_j - C_j$	0	-1	0	1

C_B	x_B	b	x_1	x_2	S_1	S_2
3	x_2	$\frac{45}{19}$	0	1	$\frac{5}{19}$	$\frac{-3}{19}$
5	x_1	$\frac{20}{19}$	1	0	$\frac{-2}{19}$	$\frac{5}{19}$

$$z_j - j C_j \quad 0 \quad 0 \quad \frac{5}{19} \quad \frac{16}{19}$$

$$\text{all } z_j - C_j \geq 0$$

$$x_1 = \frac{20}{19}$$

$$x_2 = \frac{45}{19}$$

$$\max Z = \frac{5 \times 20}{19} + 3 \times \frac{45}{19}$$

$$= \frac{100}{19} + \frac{135}{19}$$

$$\boxed{\max Z = \frac{235}{19}}$$

DATE:

6. Que:- Solve the following L.P. by Simplex method

$$\text{Min } Z = x_1 - 3x_2 + 3x_3$$

$$s.t. \quad 3x_1 - x_2 + 2x_3 \leq 7$$

$$2x_1 + 4x_2 \geq -12$$

$$-4x_1 + 3x_2 + 8x_3 \leq 10$$

$$x_1, x_2, x_3 \geq 0$$

$$\text{Sol}: \quad \text{Max } Z' = -x_1 + 3x_2 - 3x_3$$

$$\text{now, } -2x_1 - 4x_2 \leq 12 \\ \text{in Standard form}$$

$$\text{now, Max } Z' = -x_1 + 3x_2 - 3x_3 + OS_1 + OS_2 + OS_3$$

$$\text{s.t } 3x_1 - x_2 + 2x_3 + S_1 + OS_2 + OS_3 = 7$$

$$-2x_1 - 4x_2 + 0x_3 + OS_1 + S_2 + OS_3 = 12$$

$$-4x_1 + 3x_2 + 8x_3 + OS_1 + OS_2 + S_3 = 10$$

$$x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$$

find initial basic feasible solution is

$$x_1 = x_2 = x_3 = 0 \text{ (non basic)}; S_1 = 7, S_2 = 12$$

$$S_3 = 10 \text{ (basic)}$$

\therefore Initial basic feasible solution is given by the table below:

4C7

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14114802719

DATE:

C_B	Basis	C_j	-1	3	-3	0	0	0	b	θ
0	S_1	x_1	x_2	x_3	S_1	S_2	S_3			
0	S_2		-2	-4	0	0	1	0	12	$12/(-4)$
0	S_3		-4	(3)	8	0	0	1	10	$10/3 \leftarrow$
$Z_j = \sum_{B} B_{ij} C_j$			0	0	0	0	0	0	0	
$C_j = C_j - Z_j$			-1	3	-3	0	0	0		
						↑				

As C_j is positive under 2nd column, the initial basic feasible solution is not optimal and we proceed further.

here,

x_2 is the incoming variable, S_3 is the outgoing variable and (3) is the key element

C_B	Basis	C_j	-1	3	-3	0	0	0	b	θ
0	S_1	x_1	x_2	x_3	S_1	S_2	S_3			
0	S_1	(5/3)	0	$14/3$	1	0	$1/3$	$31/3$	$31/5$	
0	S_2	($-2^{2/3}$)	0	$32/3$	0	1	$1/3$	$76/3$	$-38/11$	
3	x_2	$-4/3$	1	8.3	0	0	$1/3$	$10/3$	$-5/2$	

$$Z_j = -4 \quad 3 \quad 8 \quad 0 \quad 0 \quad 1 \quad 10$$

$$C_j \quad 3 \quad 0 \quad -11 \quad 0 \quad 0 \quad -1$$

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DATE:

As C_j is positive under 1st column

the solution is not optimal and we proceed further x_1 is the incoming variable, s_1 is the outgoing variable and (G_3) is the key element.

	C_j	-1	3	-3	0	0	0	
C_B	Basis	x_1	x_2	x_3	s_1	s_2	s_3	b
-1	x_1	1	0	$\frac{1}{5}$	$\frac{3}{5}$	0	$\frac{1}{5}$	$\frac{3}{5}$
0	s_2	0	0	$\frac{16}{5}$	$\frac{2}{5}$	1	$\frac{1}{5}$	$\frac{32}{5}$
3	x_2	0	1	$\frac{3}{5}$	$\frac{4}{5}$	0	$\frac{3}{5}$	$\frac{58}{5}$
	Z_j	-1	3	$\frac{8}{5}$	$\frac{9}{5}$	0	$\frac{8}{5}$	$\frac{143}{5}$
	c_j	0	0	$-\frac{9}{5}$	$\frac{9}{5}$	0	$-\frac{8}{5}$	

Now, since each $C_j \leq 0$. Therefore it gives the optimal solution

$$x_1 = \frac{3}{5}$$

$$x_2 = \frac{58}{5}$$

$$x_3 = 0 \text{ (non basic)}$$

$$\text{and } Z^{\max} = \frac{143}{5}$$

$$\text{Hence } Z_{\min} = -\frac{143}{5}$$

DATE:

7. Que:- Write the dual of the following problem :

$$\text{Max } Z = 4x_1 + 9x_2 + 2x_3$$

$$\text{s.t. } 2x_1 + 3x_2 + 2x_3 \leq 7$$

$$3x_1 - 2x_2 + 4x_3 = 5$$

$$x_1, x_2, x_3 \geq 0$$

Soln, let y_1 and y_2 by the dual variables associated with the first and 2nd constraints. Then the dual problem is

$$\text{Minimize } W = 7y_1 + 5y_2,$$

$$\text{subject to } 2y_1 + 3y_2 \leq 4,$$

$$3y_1 - 2y_2 \leq 9,$$

$$2y_1 + 4y_2 \leq 2,$$

$$y_1 \geq 0,$$

y_2 is Unrestricted in sign.

8. Que:- Using dual simplex method solve the following L.P.P.

$$\text{Max } Z = -3x_1 - 2x_2$$

$$\text{s.t. } x_1 + x_2 \geq 1$$

$$x_1 + x_2 \leq 7$$

$$x_1 + 2x_2 \geq 10$$

4C7

Syedahiba Ansar

14114802719

DATE:

$$x_2 \leq 9$$

$$x_1, x_2 \geq 0$$

Soln. Step 1. (i) Convert the 1st and 3rd constraints into (\leq) type.

$$-x_1 - x_2 \leq -1, -x_1 - 2x_2 \leq -10$$

Step ① Express the Problem in standard form.

$$\text{Max } Z = -3x_1 - 2x_2 + 0S_1 + 0S_2 + 0S_3 + 0S_4$$

$$\text{Subject to, } -x_1 - x_2 + S_1 = -1,$$

$$x_1 + x_2 + S_2 = 7$$

$$-x_1 + 2x_2 + S_3 = -10$$

$$x_2 + S_4 = 3, x_1, x_2, S_1, S_2, S_3, S_4 \geq 0$$

Step 2: Find the initial basic soln

$$x_1 = x_2 = 0, S_1 = -1, S_2 = 7, S_3 = -10, S_4 = 3$$

and $Z = 0$.

C_j	-3	-2	0	0	0	0	b
C_B	x_1	x_2	S_1	S_2	S_3	S_4	
0	S_1	-1	-1	1	0	0	-1
0	S_2	1	1	0	1	0	7
0	S_3	-1	(-2)	0	0	1	0
0	S_4	0	1	0	0	0	1
$Z_j = \sum c_B a_{ij}$	0	0	0	0	0	0	3
$C_j = a_j - z_j$	-3	-2	0	0	0	0	

DATE:

Step 4: Mark the outgoing variable

$\therefore b_3$ is negative and numerically largest. the 3rd row is the key row and s_3 is the outgoing variable.

Step 5: Calculate the ratios of elements

Step 6:

C_B	C_j	-3	-2	0	0	0	0	b
		x_1	x_2	s_1	s_2	s_3	s_4	
0	s_1	$-1/2$	0	1	$-1/2$	$-1/2$	0	4
0	s_2	$1/2$	0	0	$1/2$	$1/2$	0	2
-2	x_2	$1/2$	1	0	$-1/2$	$-1/2$	0	5
0	s_4	$(-1/2)$	0	0	$1/2$	$1/2$	1	-2

$$Z_j = \sum C_B a_{Bj} - 1 - 2 + 0 + 0 + 1 + 0 - 10$$

$$C_j = C_j - Z_j - 2 \uparrow 0 + 0 + 0 - 1 0$$

Since all C_j values are ≤ 0 and $b_4 = -2$

(i) mark the outgoing variable

(ii) calculate ratios of elements in G_R

(iii) drop s_4 and introduce x_1

DATE:

C_j	-3	-2	0	0	0	0	
C_B Basis	x_1	x_2	S_1	S_2	S_3	S_4	b
0 S_1	0	0	1	0	-1	-1	6
0 S_2	0	0	0	1	1	1	0
-2 x_2	0	1	0	0	0	1	3
-3 x_1	1	0	0	0	-10	-2	9
Z_j	-3	-2	0	0	3	4	-18
C_j	0	0	0	0	-3	-4	

Since all c_j values are ≤ 0 and all b 's are > 0 . Therefore this solution is optimal and feasible.

Thus the optimal solution is

$$x_1 = 9, x_2 = 3 \text{ and } Z_{\max} = -18$$

9. Que: An Engineer Wants to assign 3 jobs J_1, J_2, J_3 to three machines M_1, M_2, M_3 in such a way that each job is assigned to some machine and no machine works on more than one job. Find the optimal solution using Hungarian method if the cost matrix is as follows

4C7

Syedahieha Ansar

14114802719

DATE:

	M ₁	M ₂	M ₃
J ₁	15	10	9
J ₂	9	15	10
J ₃	10	12	8

↓

SPLⁿ

	M ₁	M ₂	M ₃
J ₁	6	1	0
J ₂	0	6	1
J ₃	2	4	0

↓

	M ₁	M ₂	M ₃
J ₁	6	0	✓
J ₂	0	5	1
J ₃	2	3	0

Cost

J₁ - M₂

10

J₂ - M₁

9

J₃ - M₃

8

27

 $\therefore \boxed{\text{Min Cost} = 27}$

4C7

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14114802719

DATE:

10. Que:- Solve the following transportation problem by VAM Method

		Destination				
From / To		A	B	C	D	Availability
Source	I	21	16	25	13	11
	II	17	18	24	23	13
	III	33	27	18	41	19
Requirement		6	10	12	15	43

Table 1.

Solⁿ,

21	16	25	13	11 (3)
17	18	14	23	13 (3)
32	27	18	41	19 (9)
6	10	12	15	
(4)	(2)	(4)	(10)	

Table 2

17	18	14	23	13 (3)
32	27	18	41	19 (9)
6	10	12	14	
(15)	(9)	(4)	(18)	

4C7

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14114802719

DATE:

Table 3

6	17	18	14	9(3)
32	27	18	19(9)	

6 10 12

(15) (9) (4)

Table 4

3	18	14	3(4)
27		18	19(9)

10 12

(9) (6)

Table 5

7	12	18	19
27			

7 12 +

Table 6.

21	16	25	13	11
6	17	31	14	4
32	27	12	18	23

4C7

Syedahieha Ansar

14114802719

DATE:

Table 7

$U_i \backslash V_j$	17	18	9	23
-10	21	16	25	13
0	17	18	14	23
9	32	27	18	41

now, Apply optimality check

no. of allocations = m+n-1 i.e 6

we can apply MODI method

(i) we have $U_2 + V_1 = 17$

$U_2 + V_2 = 18$

$U_3 + V_2 = 27$

$U_3 + V_3 = 18$

$U_1 + U_4 = 13$

$U_2 + U_4 = 23$

let $U_2 = 0$, then $V_1 = 17$, $V_2 = 18$,

$U_3 = 9$, $U_4 = 23$, $U_1 = -10$

DATE:

(i) Net evaluations $w_{ij} = (u_i + v_j) - c_{ij}$

for all empty cell are

$$w_{11} = -14, w_{12} = -8, w_{13} = -26,$$

$$w_{23} = -5, w_{31} = -6, w_{34} = -9$$

(ii) Since all net evaluations are negative. The current solution is optimal. Hence the optimal allocation is given by

$$x_{14} = 11, x_{21} = 6, x_{22} = 3, x_{24} = 4$$

$$x_{32} = 7 \text{ and } x_{33} = 12$$

\therefore The optimal (minimum) transportation cost is

$$= 11 \times 13 + 6 \times 17 + 3 \times 18 + 4 \times 23 + 7 \times 27 + 12 \times 18$$

$$= 796 \text{ Ans}$$