## 1. Conditional Independence

- **Definition**: Two variables A and B are conditionally independent given C if, once C is known, knowing A gives no additional information about B.
- **Explanation**: This simplifies computations in probabilistic models, especially Bayesian networks. For example, if you know the weather (C), then whether you carry an umbrella (A) doesn't affect whether the ground is wet (B).

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## 2. Bayesian Networks

- **Definition**: A graphical model representing conditional dependencies between random variables using a Directed Acyclic Graph (DAG).
- **Explanation**: They allow efficient computation of joint and conditional probabilities, and are useful in diagnostics, decision-making, and AI.

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## 3. Markov Chains

- **Definition**: A stochastic process where the next state depends only on the current state.
- **Explanation**: Used in modeling memoryless systems like board games, genetics, and text generation.

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## 4. Stationarity in Stochastic Processes

- **Definition**: A process whose statistical properties (mean, variance, autocorrelation) do not change over time.
- **Explanation**: Stationary processes are easier to analyze and forecast since their behavior is consistent over time.

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## 5. Ergodicity

- **Definition**: A process is ergodic if time averages equal ensemble (population) averages.
- **Explanation**: Important in practice—if a process is ergodic, one long observation gives insight into the overall behavior.

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## 6. Poisson Process

- **Definition**: A process where events happen independently and at a constant average rate.

- **Explanation**: Used to model arrivals like phone calls, decay events, or customer visits.

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### 7. Renewal Theory

- **Definition**: Studies times between successive renewals (events) in a process.
- **Explanation**: Generalizes Poisson processes and is key in reliability theory, queues, and maintenance modeling.

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### 8. Queueing Theory

- **Definition**: The mathematical study of waiting lines.
- **Explanation**: Used to optimize service efficiency in systems like banks, hospitals, and networks.

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### 9. Absorbing Markov Chains

- **Definition**: A Markov chain with one or more states that, once entered, cannot be left.
- **Explanation**: Useful in modeling systems with terminal states (e.g., extinction, game over, system failure).

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### 10. Hidden Markov Models (HMMs)

- **Definition**: A model where the system is a Markov process with hidden states and observable outputs.
- **Explanation**: Common in speech recognition, bioinformatics, and time series analysis.

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### 11. Gibbs Sampling

- **Definition**: A Markov Chain Monte Carlo method that samples one variable at a time from its conditional distribution.
- **Explanation**: Useful when direct sampling from a multivariate distribution is hard but conditionals are easy.

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### 12. Metropolis-Hastings Algorithm

- **Definition**: A general MCMC algorithm that uses a proposal distribution and accepts/rejects samples based on a ratio.

- **Explanation**: Can sample from complicated distributions and is very flexible.

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## 13. Importance Sampling

- **Definition**: Estimate expectations by sampling from an alternative distribution and reweighting.
- **Explanation**: Efficiently estimates rare event probabilities or integrals.

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## 14. Rejection Sampling

- **Definition**: Generate samples from a target distribution by accepting samples from a proposal with certain probability.
- **Explanation**: Simple conceptually but can be inefficient if proposal and target are very different.

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## 15. Monte Carlo Integration

- **Definition**: Numerical integration using random sampling.
- **Explanation**: Especially powerful in high dimensions where standard quadrature methods fail.

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## 16. Variance Reduction in Monte Carlo Methods

- **Definition**: Techniques to reduce estimator variance without increasing the number of samples.
- **Explanation**: Improves efficiency of Monte Carlo simulations.

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## 17. Antithetic Variates

- **Definition**: Use negatively correlated samples to cancel out variation.
- **Explanation**: Improves estimator accuracy with the same number of samples.

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## 18. Control Variates

- **Definition**: Use another variable with known expected value and correlation to reduce variance.
- **Explanation**: Adjusts the estimator based on deviation from expected behavior.

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### 19. Stratified Sampling in Monte Carlo

- **Definition**: Divide sample space into strata and sample from each.
- **Explanation**: Ensures better coverage and lower variance than plain random sampling.

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### 20. Quasi-Monte Carlo

- **Definition**: Use low-discrepancy sequences instead of random numbers.
- **Explanation**: More evenly spread samples improve convergence in deterministic ways.

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### 21. Copula in Probability

- **Definition**: A function that couples multivariate distributions to their marginals.
- **Explanation**: Useful for modeling dependencies, especially in finance and risk.

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### 22. Tail Dependence

- **Definition**: Measures the likelihood of joint extreme events.
- **Explanation**: Critical in risk management (e.g., market crashes, insurance).

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### 23. Extreme Value Theory

- **Definition**: Studies the statistics of the extremes (maxima/minima).
- **Explanation**: Models rare events in finance, climate, and insurance.

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### 24. Pareto Distribution

- **Definition**: A power-law distribution modeling "the rich get richer."
- **Explanation**: Common in economics and natural phenomena with heavy tails.

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### 25. Heavy-Tailed Distribution

- **Definition**: Distributions with large probability of extreme outcomes.
- **Explanation**: Characterized by slower decay, important in risk and network theory.

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### 26. Stable Distribution

- **Definition**: Distributions invariant under linear combinations.
- **Explanation**: Includes Gaussian and Cauchy; may have infinite variance.

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### 27. Lévy Flight

- **Definition**: A random walk with step lengths from a heavy-tailed distribution.
- **Explanation**: Models animal foraging, financial movement, and optimization.

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### 28. Stochastic Dominance

- **Definition**: One distribution is always "better" than another under risk-averse preferences.
- **Explanation**: Used in economics and decision theory to compare investments.

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### 29. Risk-Neutral Probability

- **Definition**: A probability measure where all assets grow at the risk-free rate.
- **Explanation**: Essential in pricing derivatives via expected discounted payoff.

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### 30. Martingale

- **Definition**: A process with no net gain over time; future expectation equals the present value.
- **Explanation**: Models fair games and underpins many stochastic finance results.

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### 31. Brownian Motion

- **Definition**: Continuous-time stochastic process with Gaussian, independent increments.
- **Explanation**: Foundation of stochastic calculus and financial models.

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### 32. Geometric Brownian Motion

- **Definition**: Brownian motion applied to logarithmic returns.
- **Explanation**: Standard model for stock prices due to non-negative values.

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### 33. Itô's Lemma

- **Definition**: A formula to compute the differential of a function of a stochastic process.
- **Explanation**: Key tool in deriving option pricing and solving SDEs.

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### 34. Black-Scholes Model

- **Definition**: A model to price European options using GBM and no-arbitrage.
- **Explanation**: First closed-form solution for options, cornerstone of financial engineering.

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### 35. Stochastic Differential Equations

- **Definition**: Differential equations with a stochastic (random) component.
- **Explanation**: Models continuous-time systems with uncertainty, e.g., temperature, finance.

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### 36. Fokker-Planck Equation

- **Definition**: Governs the evolution of probability densities in stochastic processes.
- **Explanation**: Complements SDEs by describing how probabilities flow.

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### 37. Chapman-Kolmogorov Equation

- **Definition**: Describes consistency of transition probabilities across time.
- **Explanation**: Foundation of Markov process theory.

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### 38. First Passage Time

- **Definition**: Time taken for a process to reach a threshold for the first time.
- **Explanation**: Used in reliability, neuroscience, and finance.

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### 39. Hitting Probability

- **Definition**: Probability a process hits a particular state before another.
- **Explanation**: Important in absorption and exit problems.

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### 40. Absorption Probability

- **Definition**: The chance of eventually reaching an absorbing state.
- **Explanation**: Used in modeling disease spread, finance defaults, etc.

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### 41. Branching Process

- **Definition**: Each individual produces offspring independently.
- **Explanation**: Models population dynamics and extinction scenarios.

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### 42. Galton-Watson Process

- **Definition**: A specific branching process with i.i.d. offspring distributions.
- **Explanation**: Introduced to study extinction in family names.

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### 43. Birth-Death Process

- **Definition**: A Markov process with arrivals (births) and departures (deaths).
- **Explanation**: Used in queues, populations, and chemical reactions.

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### 44. Poisson-Binomial Distribution

- **Definition**: Sum of independent but not identically distributed Bernoulli trials.
- **Explanation**: Generalizes binomial to varied success probabilities.

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### 45. Dirichlet Distribution

- **Definition**: A multivariate distribution over probabilities.
- **Explanation**: Used as a prior in Bayesian models involving proportions.

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### 46. Multinomial Distribution

- **Definition**: Counts of outcomes over multiple categories.
- **Explanation**: Generalizes the binomial to more than two outcomes.

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## 47. Hypergeometric Distribution

- **Definition**: Probability of successes in draws without replacement.
- **Explanation**: Models sampling from finite populations (e.g., lottery, cards).

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## 48. Negative Binomial Distribution

- **Definition**: Number of trials to achieve a fixed number of successes.
- **Explanation**: Models overdispersed count data and failures before success.

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## 49. Beta-Binomial Distribution

- **Definition**: A binomial with a beta-distributed success probability.
- **Explanation**: Captures uncertainty and overdispersion in probabilities.

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## 50. Zero-Inflated Models

- **Definition**: Count models with excess zeros beyond what standard models expect.
- **Explanation**: Useful for insurance claims, rare event modeling, and manufacturing.