Control Systems Project Report

Syed Fahd Ali
Section C, DCSE
Enrollment Number: 21PWCSE1992

January 8, 2024

1. Problem

Consider the following system with state-space representation for an inverted pendulum:

1. Check the stability of the system using all known methods.
2. Simulate the unstable system and show that its response is unstable.
3. Compute the controllability and observability of the system. If the system is controllable, place the controller poles at $(-4, -3, -8, -5)$ and observer poles at locations faster than the controller poles.
4. Simulate the stable system .Design a PID controller also.
5. Compute the steady state errors before and after designing controlles.

2. Solution

2.1 State-space Representation

The state-space representation of the given system is:

$$\dot{x} = Ax + Bu, \quad y = Cx + Du$$

Where the matrices are:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.818 & 2.6727 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.4545 & 31.1818 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1.8182 \\ 0 \\ 4.5455 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, \quad D = 0$$

2.2 Stability Analysis

The eigenvalues of the system matrix A are:

$$\lambda = \{0, 5.5670, -5.6067, -0.7783\}$$

The poles of the transfer function are:

$$\text{Poles} = \{0, 5.5670, -5.6067, -0.7783\}$$

As one eigenvalue and one pole have positive real parts, the system is unstable in open-loop configuration. The Step Response of the system shows that system is unstable in open-loop configuration.

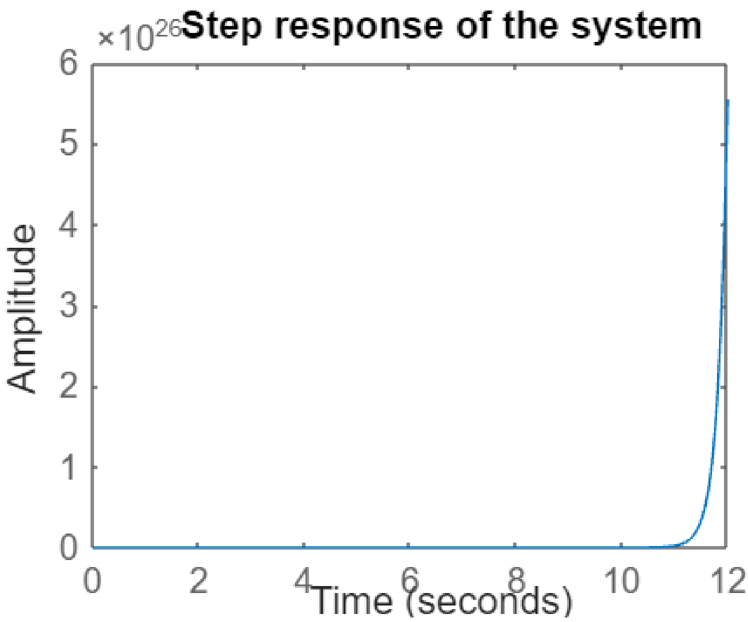

Figure 1: Step Response of the System

2.3 Controllability and Observability

The ranks of the controllability and observability matrices are both full (Rank = 4), confirming that the system is controllable and observable.

2.4 Controller and Observer Design

- **State-Feedback Controller (SFC):** The controller gain matrix is:

$$K = \begin{bmatrix} -10.7754 & -7.8698 & 37.9423 & 7.3679 \end{bmatrix}$$

- **Observer Feedback Controller (OFC):** The observer gain matrix is:

$$L = \begin{bmatrix} 0.0212 \\ 0.1849 \\ 0.4650 \\ 2.5916 \end{bmatrix}$$

- Closed-loop system matrices:

$$A_{clp} = A - BK, \quad A_{clp2} = A - LC$$

```
P=[B A*B A*A*B A*A*A*B];
Q=[ C;
    C*A;
    C*A*A;
    C*A*A*A
    ];
disp('The rank of matrix P is')
rank(P)
disp('The rank of matrix Q is')
rank(Q)
disp('Order of system is')
size(A,1)
%both are 4 = order of matrix A
%System is controllable
```

Figure 2: Code for Controllability and Observability

```
The rank of matrix P is
ans = 4
The rank of matrix Q is
ans = 4
Order of system is
ans = 4
```

Figure 3: Output of Controllability and Observability

```
egnvalues_sfc=[-4 -3 -8 -5];
egnvalues_ofc=[-4 -3 -9 -6];
%for state feedback controller
K=place(A,B,egnvalues_sfc)
%for observer feedback controller
L=place(A',C',egnvalues_ofc)'
```

Figure 4: Code for K and L matrices

```
A_clp = A-B*K
B_clp= B
C_clp = C
D_clp = D
A_clp2 = A-L*C
%response of state feeback controller
step(A_clp,B_clp,C_clp,D_clp)

%response of state feeback controller
step(A_clp2,B_clp,C_clp,D_clp)
```

Figure 5: Code for state feedback and observer feedback controller

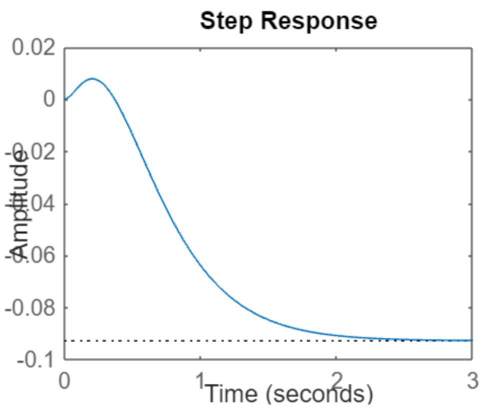

Figure 6: Response for State Feedback Controller

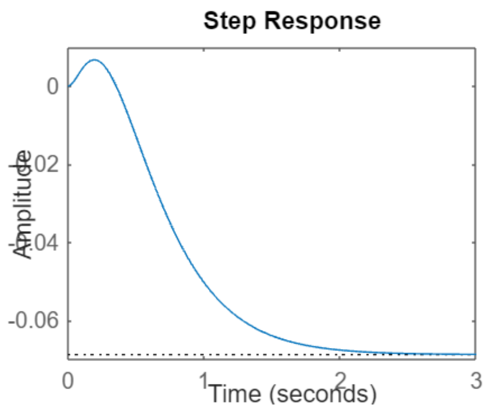

Figure 7: Response for Observer Feedback Controller

2.5 PID Controller Design

The PID controller parameters are:

$$K_p = -0.0552, \quad K_i = -0.000684, \quad K_d = -1.11$$

The closed-loop system transfer function is:

$$\mathrm{sys}_{\mathrm{now}} = \frac{-2.024s^4 - 0.1003s^3 + 49.6s^2 + 2.458s + 0.03046}{s^5 - 1.206s^4 - 31.28s^3 + 25.3s^2 + 2.458s + 0.03046}$$

The step response of the system with PID controller is:

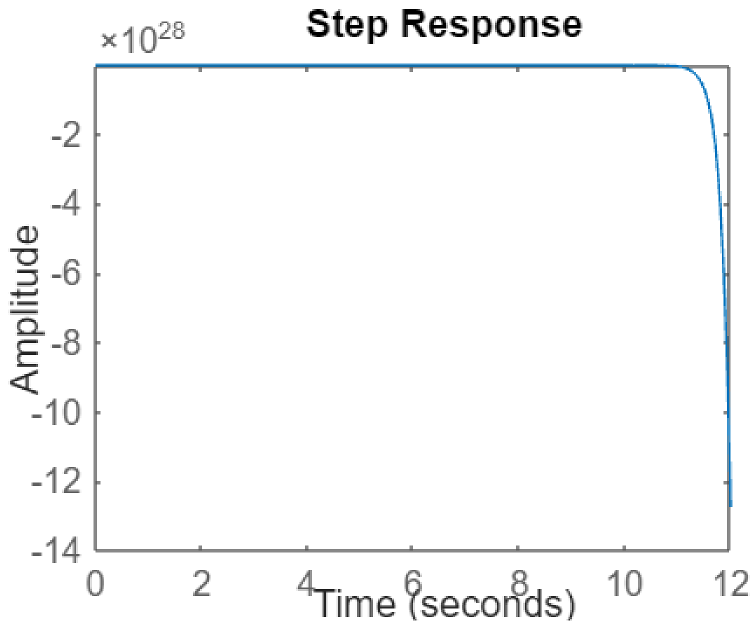

Figure 8: Response for PID Controller

2.6 Simulation Results

Step response of the system confirms performance improvement with controllers.DC gains before and after controllers:

DC gain (Open-loop) $= \infty$, DC gain (SFC) $= -0.0928$, DC gain (OFC) $= -0.0687$, DC gain (PI

- Steady-state errors:

Error (Open-loop) $= 0$, Error (SFC) $= 1.1023$, Error (OFC) $= 1.0738$, Error (PID) $= 0.5$

```
dc_gain_before_controller = dcgain(A,B,C,D)
dc_gain_sfc = dcgain(A_clp,B_clp,C_clp,D_clp)
dc_gain_obs = dcgain(A_clp2,B_clp,C_clp,D_clp)
dc_gain_pid = dcgain(sys_now)
```

Figure 9: DC gain code

```
dc_gain_before_controller = Inf
dc_gain_sfc = -0.0928
dc_gain_obs = -0.0687
dc_gain_pid = 1
```

Figure 10: DC gain Output

```
error_before_controller = 1/(1+dc_gain_before_controller)
error_sfc= 1/(1+dc_gain_sfc)
error_obs= 1/(1+dc_gain_obs)
error_pid= 1/(1+dc_gain_pid)
```

Figure 11: Error code

```
error_before_controller = 0
error_sfc = 1.1023
error_obs = 1.0738
error_pid = 0.5000
```

Figure 12: Error Output

3. Results and Discussion

This section summarizes the results obtained from the simulations and discusses the implications of the findings. The project successfully demonstrates the design and analysis of controllers for an unstable system. The MATLAB simulations validate the improvements in system dynamics and performance, confirming the effectiveness of the designed controllers.

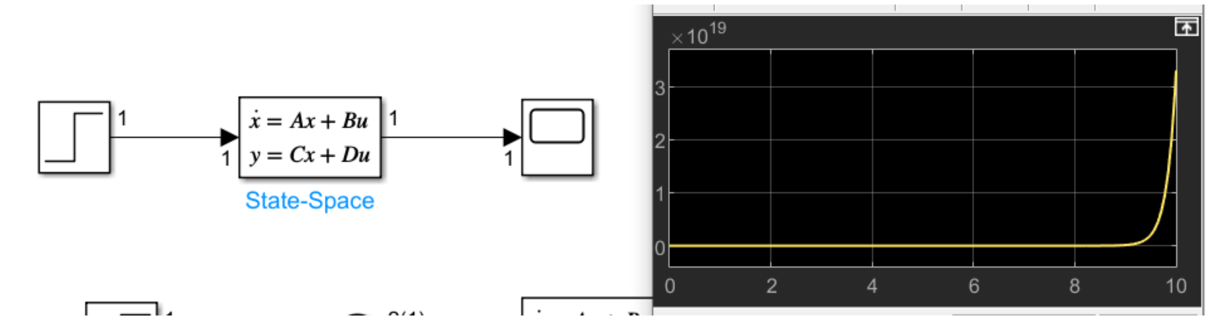

Figure 13: Simulink System Response

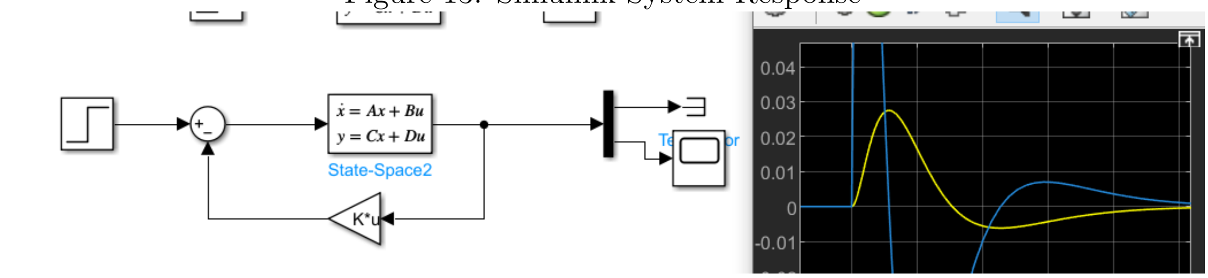

Figure 14: Simulink State Feedback Controller

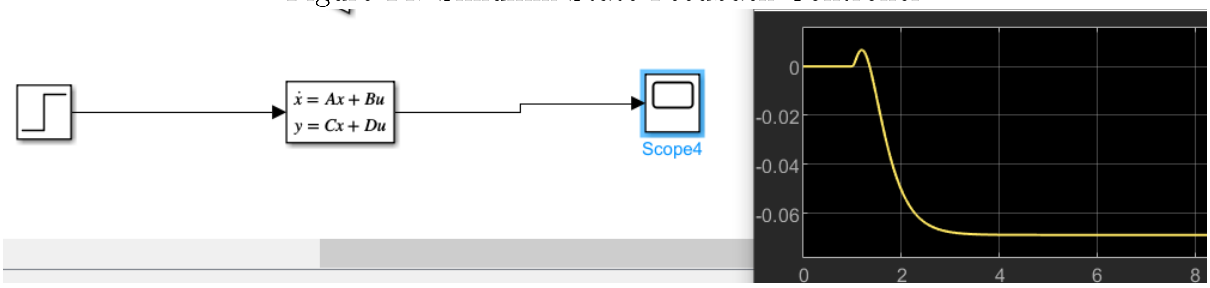

Figure 15: Simulink Observer Feedback Controller

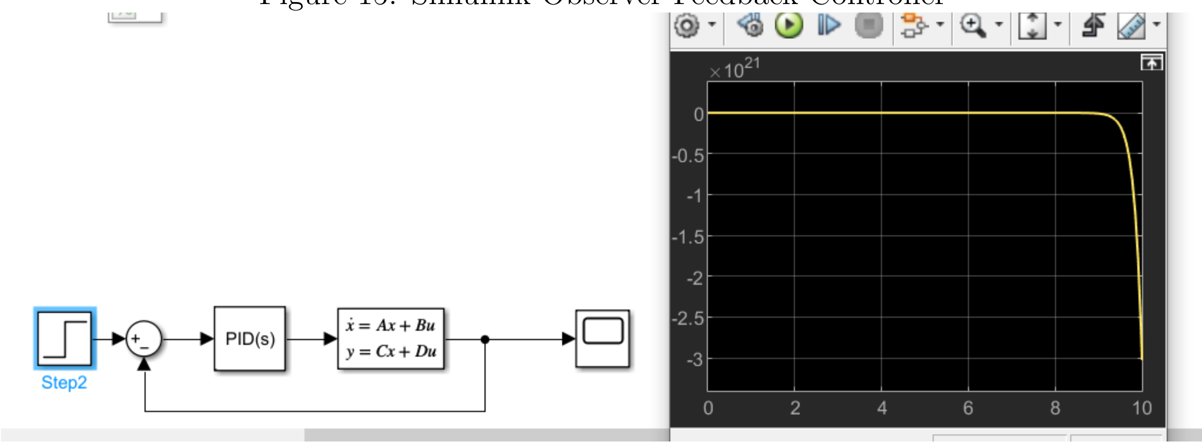

Figure 16: Simulink PID Feedback Controller