Control Systems Project

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Problem Statement

Consider the following tasks for the inverted pendulum system:

- Check the stability of the system using all known methods.
- Simulate the unstable system and show that its response is unstable.
- Compute the controllability and observability of the system. If the system is controllable, place the controller poles at $(-4, -3, -8, -5)$ and observer poles faster than the controller poles.
- Simulate the stable system and design a PID controller.
- Compute the steady-state errors before and after designing controllers.

State-space Representation

The state-space representation of the system is given by:

$$\dot{x} = Ax + Bu, \quad y = Cx + Du$$

Where:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.818 & 2.6727 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.4545 & 31.1818 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1.8182 \\ 0 \\ 4.5455 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, \quad D = 0$$

Stability Analysis

The eigenvalues of the system matrix A are:

$$\lambda = \{0, 5.5670, -5.6067, -0.7783\}$$

The poles of the transfer function are:

$$\text{Poles} = \{0, 5.5670, -5.6067, -0.7783\}$$

Since one eigenvalue and one pole have positive real parts, the system is unstable in open-loop configuration.

Step Response of the System (Unstable)

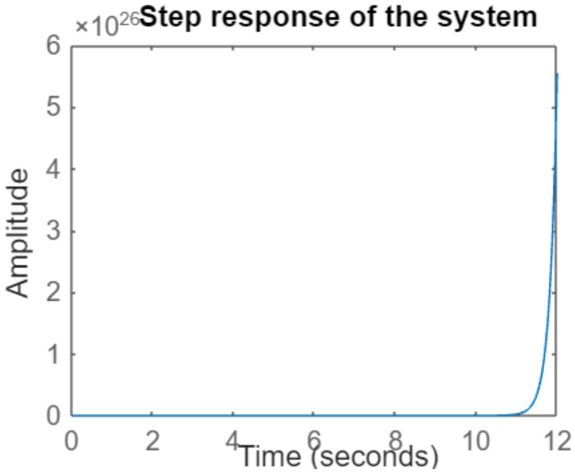

Figure: Step Response of the System

Controllability and Observability

The controllability and observability matrices have full rank, confirming that the system is controllable and observable.

```
P=[B A*B A*A*B A*A*A*B];
Q=[ C;
    C*A;
    C*A*A;
    C*A*A*A
    ];
disp('The rank of matrix P is')
rank(P)
disp('The rank of matrix Q is')
rank(Q)
disp('Order of system is')
size(A,1)
%both are 4 = order of matrix A
%System is controllable
```

Figure: Code for Controllability and Observability

```
The rank of matrix P is
ans = 4
The rank of matrix Q is
ans = 4
Order of system is
ans = 4
```

Figure: Output of Controllability and Observability

State-Feedback and Observer Design

State-Feedback Controller (SFC):

$$K = \begin{bmatrix} -10.7754 & -7.8698 & 37.9423 & 7.3679 \end{bmatrix}$$

Observer Feedback Controller (OFC):

$$L = \begin{bmatrix} 0.0212 \\ 0.1849 \\ 0.4650 \\ 2.5916 \end{bmatrix}$$

Closed-loop system matrices:

$$A_{clp} = A - BK, \quad A_{clp2} = A - LC$$

Controller Design Simulation

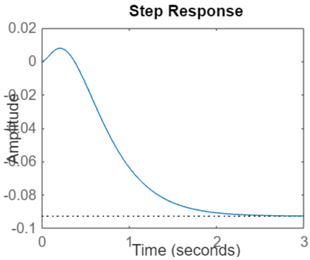

Figure: Response for State Feedback Controller

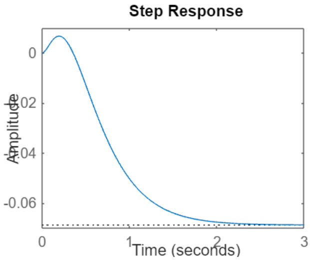

Figure: Response for Observer Feedback Controller

PID Controller Design

The PID controller parameters are:

$$K_p = -0.0552, \quad K_i = -0.000684, \quad K_d = -1.11$$

The closed-loop system transfer function is:

$$\mathsf{sys_{now}} = \frac{-2.024s^4 - 0.1003s^3 + 49.6s^2 + 2.458s + 0.03046}{s^5 - 1.206s^4 - 31.28s^3 + 25.3s^2 + 2.458s + 0.03046}$$

Step Response with PID Controller

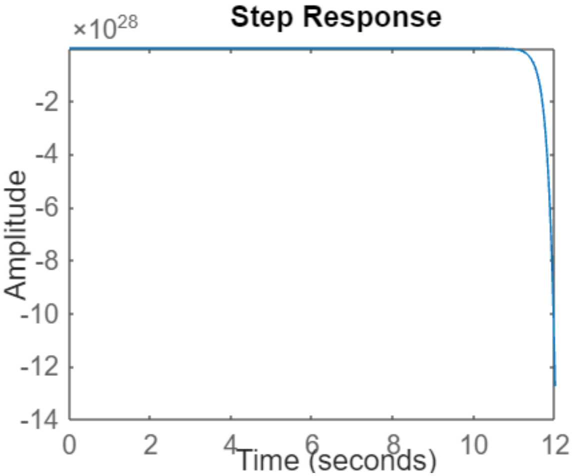

Figure: Response for PID Controller

Steady-State Errors and Results

DC gains before and after controllers for Step :

$$\text{DC gain (Open-loop)} = \infty,$$
$$\text{DC gain (SFC)} = -0.0928,$$
$$\text{DC gain (OFC)} = -0.0687,$$
$$\text{DC gain (PID)} = 1$$

Steady-state errors for Step:

$$\text{Error (Open-loop)} = 0,$$
$$\text{Error (SFC)} = 1.1023,$$
$$\text{Error (OFC)} = 1.0738,$$
$$\text{Error (PID)} = 0.5$$

Simulation Results in Simulink

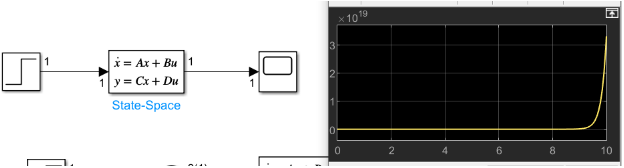

Figure: Simulink System Response

Simulink Controllers

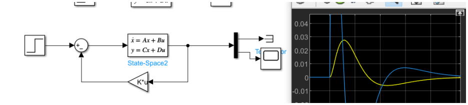

Figure: Simulink State Feedback Controller

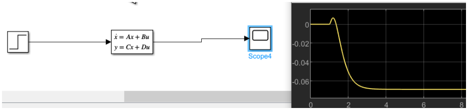

Figure: Simulink Observer Feedback Controller