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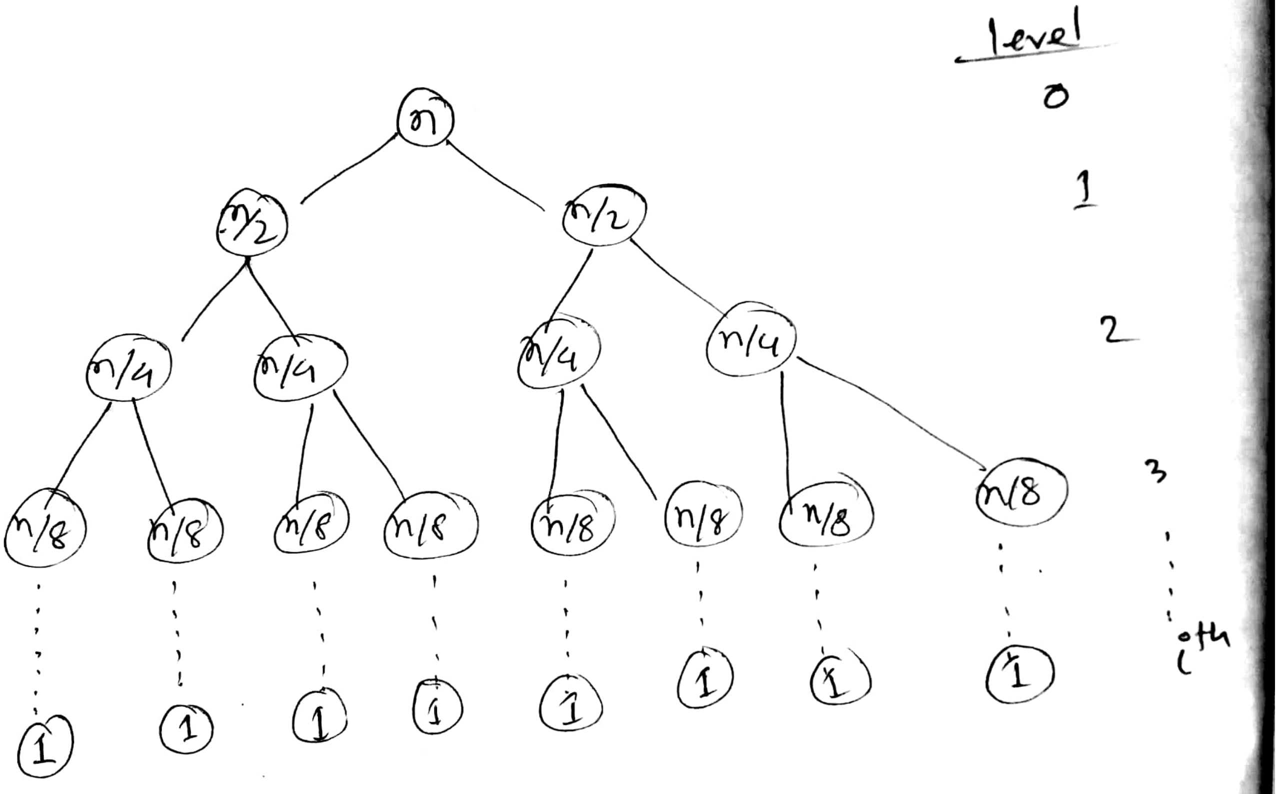
Time complexity of Merge Sort

Merge sort is a recursive and divide-and-conquere algorithm. Following is the recursive tree :



Level	total cost	#node	size of each node
0	cn	$1=2^0$	$n=\frac{n}{2^0}$
1	$c(n/2)+c(n/2)=cn$	$2=2^1$	$\frac{n}{2}=\frac{n}{2^1}$
2	$c(n/4)+c(n/4)+c(n/4)+c(n/4)$	$4=2^2$	$\frac{n}{4}=\frac{n}{2^2}$
3	......	$8=2^3$	
⋮	⋮	⋮	⋮
i	cn	$n=2^i$	$n/2^i$

From Base condition,

$$n/2^i = 1$$

$$\Rightarrow n = 2^i$$

$$\Rightarrow i = \log_2 n$$

$\therefore$ Total level $= (\log n + 1)$

$\therefore$ Total cost $= cn(\log_2 n + 1)$

$= cn\log_2 n + cn$

$\therefore$ worst case complexity $= O(n\log_2 n)$