3.2 Characters in Mathematics Mode

All the characters on the keyboard have their standard meaning in mathematics mode, with the exception of the characters

`# $ % & ~ _ ^ \ { } '`

Letters are set in italic type. In mathematics mode the character `'` has a special meaning: typing `$u' + v''$` produces $u' + v''$ When in mathematics mode the spaces you type between letters and other symbols do not affect the spacing of the final result, since LaTeX determines the spacing of characters in formulae by its own internal rules. Thus `$u v + w = x$` and `$uv+w=x$` both produce $uv + w = x$ You can also type carriage returns where necessary in your input file (e.g., if you are typing in a complicated formula with many Greek characters and funny symbols) and this will have no effect on the final result if you are in mathematics mode.

To obtain the characters

`#    $    %    &    _    {    }`

in mathematics mode, one should type

`\#    \$    \%    \&    \_    \{    \} .`

To obtain in mathematics mode, one may type `\backslash`.

3.3 Superscripts and Subscripts

Subscripts and superscripts are obtained using the special characters `_` and `^` respectively. Thus the identity

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 - c^2 dt^2$$

is obtained by typing

```
\[ ds^2 = dx_1^2 + dx_2^2 + dx_3^2 - c^2 dt^2 \]
```

It can also be obtained by typing

```
\[ ds^2 = dx^2_1 + dx^2_2 + dx^2_3 - c^2 dt^2 \]
```

since, when a superscript is to appear above a subscript, it is immaterial whether the superscript or subscript is the first to be specified.

Where more than one character occurs in a superscript or subscript, the characters involved should be enclosed in curly brackets. For example, the polynomial $x^{17} - 1$ is obtained by typing `$x^{17} - 1$`.

One may not type expressions such as `$s^n^j$` since this is ambiguous and could be interpreted either as s^{nj} or as s^{n^j} The first of these alternatives is obtained by typing `$s^{n j}$`, the second by typing `$s^{n^j}$`. A similar remark applies to subscripts. Note that one can obtain in this way double superscripts (where a superscript is placed on a superscript) and double subscripts.

It is sometimes necessary to obtain expressions in which the horizontal ordering of the subscripts is significant. One can use an 'empty group' `{}` to separate superscripts and subscripts that must follow one another. For example, the identity

$$R_i{}^j{}_{kl} = g^{jm} R_{imkl} = - g^{jm} R_{mikl} = - R^j{}_{ikl}$$

can be obtained by typing

```
\[ R_i{}^j{}_{kl} = g^{jm} R_{imkl}
   = - g^{jm} R_{mikl} = - R^j{}_{ikl} \]
```

3.4 Greek Letters

Greek letters are produced in mathematics mode by preceding the name of the letter by a backslash `\`. Thus to obtain the formula $A = \pi r^2$ one types `A = \pi r^2`.

Here are the control sequences for the standard forms of the lowercase Greek letters:-

α	`\alpha`	ι	`\iota`	ρ	`\rho`
β	`\beta`	κ	`\kappa`	σ	`\sigma`
γ	`\gamma`	λ	`\lambda`	τ	`\tau`
δ	`\delta`	μ	`\mu`	υ	`\upsilon`
ϵ	`\epsilon`	ν	`\nu`	ϕ	`\phi`
ζ	`\zeta`	ξ	`\xi`	χ	`\chi`
η	`\eta`	o	`o`	ψ	`\psi`
θ	`\theta`	π	`\pi`	ω	`\omega`

There is no special command for omicron: just use `o`.

Some Greek letters occur in variant forms. The variant forms are obtained by preceding the name of the Greek letter by 'var'. The following table lists the usual form of these letters and the variant forms:-

ϵ	`\epsilon`	ε	`\varepsilon`
θ	`\theta`	ϑ	`\vartheta`
π	`\pi`	ϖ	`\varpi`
ρ	`\rho`	ϱ	`\varrho`
σ	`\sigma`	ς	`\varsigma`
ϕ	`\phi`	φ	`\varphi`

Upper case Greek letters are obtained by making the first character of the name upper case. Here are the control sequence for the uppercase letters:—

Γ	`\Gamma`	Ξ	`\Xi`	Φ	`\Phi`
Δ	`\Delta`	Π	`\Pi`	Ψ	`\Psi`
Θ	`\Theta`	Σ	`\Sigma`	Ω	`\Omega`
Λ	`\Lambda`	Υ	`\Upsilon`		

3.5 Mathematical Symbols

There are numerous mathematical symbols that can be used in mathematics mode. These are obtained by typing an appropriate control sequence.

Miscellaneous Symbols:

$\aleph$	`\aleph`	$\prime$	`\prime`	$\forall$	`\forall`
$\hbar$	`\hbar`	$\emptyset$	`\emptyset`	$\exists$	`\exists`
$\imath$	`\imath`	∇	`\nabla`	$\neg$	`\neg`
$\jmath$	`\jmath`	$\surd$	`\surd`	$\flat$	`\flat`
ℓ	`\ell`	$\top$	`\top`	$\natural$	`\natural`
$\wp$	`\wp`	$\bot$	`\bot`	$\sharp$	`\sharp`
$\Re$	`\Re`	$\\|$	`\\|`	$\clubsuit$	`\clubsuit`
$\Im$	`\Im`	$\angle$	`\angle`	$\diamondsuit$	`\diamondsuit`
∂	`\partial`	$\triangle$	`\triangle`	$\heartsuit$	`\heartsuit`
∞	`\infty`	$\backslash$	`\backslash`	$\spadesuit$	`\spadesuit`

“Large” Operators:

$\sum$	`\sum`	$\bigcap$	`\bigcap`	$\bigodot$	`\bigodot`
$\prod$	`\prod`	$\bigcup$	`\bigcup`	$\bigotimes$	`\bigotimes`
$\coprod$	`\coprod`	$\bigsqcup$	`\bigsqcup`	$\bigoplus$	`\bigoplus`
$\int$	`\int`	$\bigvee$	`\bigvee`	$\biguplus$	`\biguplus`
$\oint$	`\oint`	$\bigwedge$	`\bigwedge`		

Binary Operations:

$\pm$	\pm	$\cap$	\cap	$\vee$	\vee
$\mp$	\mp	$\cup$	\cup	$\wedge$	\wedge
$\setminus$	\setminus	$\uplus$	\uplus	$\oplus$	\oplus
$\cdot$	\cdot	$\sqcap$	\sqcap	$\ominus$	\ominus
$\times$	\times	$\sqcup$	\sqcup	$\otimes$	\otimes
$\ast$	\ast	$\triangleleft$	\triangleleft	$\oslash$	\oslash
$\star$	\star	$\triangleright$	\triangleright	$\odot$	\odot
$\diamond$	\diamond	$\wr$	\wr	$\dagger$	\dagger
$\circ$	\circ	$\bigcirc$	\bigcirc	$\ddagger$	\ddagger
$\bullet$	\bullet	$\bigtriangleup$	\bigtriangleup	$\amalg$	\amalg
$\div$	\div	$\bigtriangledown$	\bigtriangledown		

Relations:

$\leq$	\leq	$\geq$	\geq	$\equiv$	\equiv
$\prec$	\prec	$\succ$	\succ	$\sim$	\sim
$\preceq$	\preceq	$\succeq$	\succeq	$\simeq$	\simeq
$\ll$	\ll	$\gg$	\gg	$\asymp$	\asymp
$\subset$	\subset	$\supset$	\supset	$\approx$	\approx
$\subseteq$	\subseteq	$\supseteq$	\supseteq	$\cong$	\cong
$\sqsubseteq$	\sqsubseteq	$\sqsupseteq$	\sqsupseteq	$\bowtie$	\bowtie
$\in$	\in	$\ni$	\ni	$\propto$	\propto
$\vdash$	\vdash	$\dashv$	\dashv	$\models$	\models
$\smile$	\smile	$\mid$	\mid	$\doteq$	\doteq
$\frown$	\frown	$\parallel$	\parallel	$\perp$	\perp

Negated Relations:

$\not<$	\not<	$\not>$	\not>	$\not=$	\not=
$\not\leq$	\not\leq	$\not\geq$	\not\geq	$\not\equiv$	\not\equiv
$\not\prec$	\not\prec	$\not\succ$	\not\succ	$\not\sim$	\not\sim
$\not\preceq$	\not\preceq	$\not\succeq$	\not\succeq	$\not\simeq$	\not\simeq
$\not\subset$	\not\subset	$\not\supset$	\not\supset	$\not\approx$	\not\approx
$\not\subseteq$	\not\subseteq	$\not\supseteq$	\not\supseteq	$\not\cong$	\not\cong
$\not\sqsubseteq$	\not\sqsubseteq	$\not\sqsupseteq$	\not\sqsupseteq	$\not\asymp$	\not\asymp

Arrows:

$\leftarrow$	`\leftarrow`	$\rightarrow$	`\rightarrow`
$\longleftarrow$	`\longleftarrow`	$\longrightarrow$	`\longrightarrow`
$\Leftarrow$	`\Leftarrow`	$\Rightarrow$	`\Rightarrow`
$\Longleftarrow$	`\Longleftarrow`	$\Longrightarrow$	`\Longrightarrow`
$\leftrightarrow$	`\leftrightarrow`	$\Leftrightarrow$	`\Leftrightarrow`
$\longleftrightarrow$	`\longleftrightarrow`	$\Longleftrightarrow$	`\Longleftrightarrow`
$\hookleftarrow$	`\hookleftarrow`	$\hookrightarrow$	`\hookrightarrow`
$\leftharpoonup$	`\leftharpoonup`	$\rightharpoonup$	`\rightharpoonup`
$\leftharpoondown$	`\leftharpoondown`	$\rightharpoondown$	`\rightharpoondown`
$\uparrow$	`\uparrow`	$\downarrow$	`\downarrow`
$\Uparrow$	`\Uparrow`	$\Downarrow$	`\Downarrow`
$\updownarrow$	`\updownarrow`	$\Updownarrow$	`\Updownarrow`
$\nearrow$	`\nearrow`	$\nwarrow$	`\nwarrow`
$\searrow$	`\searrow`	$\swarrow$	`\swarrow`
$\mapsto$	`\mapsto`	$\longmapsto$	`\longmapsto`
$\rightleftharpoons$	`\rightleftharpoons`		

Openings:

$[$	`\lbrack`	$\lfloor$	`\lfloor`	$\lceil$	`\lceil`
$\{$	`\lbrace`	$\langle$	`\langle`		

Closings:

$]$	`\rbrack`	$\rfloor$	`\rfloor`	$\rceil$	`\rceil`
$\}$	`\rbrace`	$\rangle$	`\rangle`		

Alternative Names:

$\neq$	`\ne` or `\neq`	(same as `\not=`)
$\leq$	`\le`	(same as `\leq`)
$\geq$	`\ge`	(same as `\geq`)
$\{$	`\{`	(same as `\lbrace`)
$\}$	`\}`	(same as `\lbrace`)
$\to$	`\to`	(same as `\rightarrow`)
$\gets$	`\gets`	(same as `\leftarrow`)
$\owns$	`\owns`	(same as `\ni`)
$\land$	`\land`	(same as `\wedge`)
$\lor$	`\lor`	(same as `\vee`)
$\lnot$	`\lnot`	(same as `\neg`)
$\vert$	`\vert`	(same as `|`)
$\Vert$	`\Vert`	(same as `\\|`)
$\iff$	`\iff`	(same as `\Longleftrightarrow`, but with extra space at each end)
$\colon$	`\colon`	(same as `:`, but with less space around it and less likelihood of a line break after it)

3.6 Changing Fonts in Mathematics Mode

(The following applies to LaTeX2ε, a recent version of LaTeX. It does not apply to older versions of LaTeX.)

The 'math italic' font is automatically used in mathematics mode unless you explicitly change the font. The rules for changing the font in mathematics mode are rather different to those applying when typesetting ordinary text. In mathematics mode any change only applies to the single character or symbol that follows (or to any text enclosed within curly brackets immediately following the control sequence). Also, to change a character to the roman or boldface font, the control sequences `\mathrm` and `\mathbf` must be used (rather than `\textrm` and `\textbf`).

The following example illustrates the use of boldface in mathematical formulae. To obtain

> Let $\mathbf{u}$,$\mathbf{v}$ and $\mathbf{w}$ be three vectors in $\mathbf{R}^3$. The volume V of the parallelepiped with corners at the points $\mathbf{0}$, $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$, $\mathbf{u}+\mathbf{v}$, $\mathbf{u}+\mathbf{w}$, $\mathbf{v}+\mathbf{w}$ and $\mathbf{u}+\mathbf{v}+\mathbf{w}$ is given by the formula
>
> $$V = (\mathbf{u}\times\mathbf{v})\cdot\mathbf{w}.$$

one could type

```
Let $\mathbf{u}$,$\mathbf{v}$ and $\mathbf{w}$ be three
```

```
vectors in ${\mathbf R}^3$. The volume~$V$ of the
parallelepiped with corners at the points
$\mathbf{0}$, $\mathbf{u}$, $\mathbf{v}$,
$\mathbf{w}$, $\mathbf{u}+\mathbf{v}$,
$\mathbf{u}+\mathbf{w}$, $\mathbf{v}+\mathbf{w}$
and $\mathbf{u}+\mathbf{v}+\mathbf{w}$
is given by the formula
\[ V = (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}.\]
```

There is also a 'calligraphic' font available in mathematics mode. This is obtained using the control sequence `\cal`. *This font can only be used for uppercase letters.* These calligraphic letters have the form

$$\mathcal{ABCDEFGHIJKLMNOPQRSTUVWXYZ}.$$

3.7 Standard Functions (sin, cos etc.)

The names of certain standard functions and abbreviations are obtained by typing a backlash \ before the name. For example, one obtains

$$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$

by typing

```
\[ \cos(\theta + \phi) = \cos \theta \cos \phi
      - \sin \theta \sin \phi \]
```

The following standard functions are represented by control sequences defined in LaTeX:

`\arccos`	`\cos`	`\csc`	`\exp`	`\ker`	`\limsup`	`\min`	`\sinh`
`\arcsin`	`\cosh`	`\deg`	`\gcd`	`\lg`	`\ln`	`\Pr`	`\sup`
`\arctan`	`\cot`	`\det`	`\hom`	`\lim`	`\log`	`\sec`	`\tan`
`\arg`	`\coth`	`\dim`	`\inf`	`\liminf`	`\max`	`\sin`	`\tanh`

Names of functions and other abbreviations not in this list can be obtained by converting to the roman font. Thus one obtains $\mathrm{cosec} A$ by typing `$\mathrm{cosec} A$`. Note that if one were to type simply `$cosec A$` one would obtain $cosec A$, because LaTeX has treated `cosec A` as the product of six quantities c, o, s, e, c and A and typeset the formula accordingly.

3.8 Text Embedded in Displayed Equations

Text can be embedded in displayed equations (in LaTeX) by using `\mbox{`*embedded text*`}`. For example, one obtains

$$M^\perp = \{ f \in V' : f(m) = 0 \mbox{ for all } m \in M \}.$$

by typing

```
\[ M^\bot = \{ f \in V’ : f(m) = 0 \mbox{ for all } m \in M \}.\]
```

Note the blank spaces before and after the words ‘for all’ in the above example. Had we typed

```
\[ M^\bot = \{ f \in V’ : f(m) = 0 \mbox{for all} m \in M \}.\]
```

we would have obtained

$$M^\perp = \{ f \in V' : f(m) = 0\mbox{for all}m \in M \}.$$

(In Plain TeX one should use `\hbox` in place of `\mbox`.)

3.9 Fractions and Roots

Fractions of the form

$$\frac{\textit{numerator}}{\textit{denominator}}$$

are obtained in LaTeX using the construction

`\frac{`*numerator*`}{`*denominator*`}`.

For example, to obtain

> The function f is given by
> $$f(x) = 2x + \frac{x-7}{x^2+4}$$
> for all real numbers x.

one would type

```
The function $f$ is given by
\[ f(x) = 2x + \frac{x - 7}{x^2 + 4}\]
for all real numbers $x$.
```

To obtain square roots one uses the control sequence

\sqrt{*expression*}.

For example, to obtain

> The roots of a quadratic polynomial $ax^2 + bx + c$ with $a \neq 0$ are given by the formula
> $$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

one would type

```
The roots of a quadratic polynomial $a x^2 + bx + c$ with
$a \neq 0$ are given by the formula
\[ \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
```

In LaTeX, an nth root is produced using

\sqrt[n]{*expression*}.

For example, to obtain

> The roots of a cubic polynomial of the form $x^3 - 3px - 2q$ are given by the formula
> $$\sqrt[3]{q + \sqrt{q^2 - p^3}} + \sqrt[3]{q - \sqrt{q^2 - p^3}}$$
> where the values of the two cube roots must are chosen so as to ensure that their product is equal to p.

in LaTeX, one would type

```
The roots of a cubic polynomial of the form $x^3 - 3px - 2q$
are given by the formula
\[ \sqrt[3]{q + \sqrt{ q^2 - p^3 }}
  + \sqrt[3]{q - \sqrt{ q^2 - p^3 }} \]
where the values of the two cube roots must are chosen
so as to ensure that their product is equal to $p$.
```

3.10 Ellipsis (i.e., 'three dots')

Ellipsis (three dots) is produced in mathematics mode using the control sequences `\ldots` (for dots aligned with tbe baseline of text), and `\cdots` (for dots aligned with the centreline of mathematical formulae). Thus the formula

$$f(x_1, x_2, \ldots, x_n) = x_1^2 + x_2^2 + \cdots + x_n^2$$

is obtained by typing

```
\[ f(x_1, x_2,\ldots, x_n) = x_1^2 + x_2^2 + \cdots + x_n^2 \]
```

Similarly the formula

$$\frac{1 - x^{n+1}}{1 - x} = 1 + x + x^2 + \cdots + x^n$$

is produced using `\cdots`, by typing

```
\[ \frac{1 - x^{n+1}}{1 - x} = 1 + x + x^2 + \cdots + x^n \]
```

3.11 Accents in Mathematics Mode

There are various control sequences for producing underlining, overlining and various accents in mathematics mode. The following table lists these control sequences, applying them to the letter a:

$\underline{a}$	`\underline{a}`
$\overline{a}$	`\overline{a}`
$\hat{a}$	`\hat{a}`
$\check{a}$	`\check{a}`
$\tilde{a}$	`\tilde{a}`
$\acute{a}$	`\acute{a}`
$\grave{a}$	`\grave{a}`
$\dot{a}$	`\dot{a}`
$\ddot{a}$	`\ddot{a}`
$\breve{a}$	`\breve{a}`
$\bar{a}$	`\bar{a}`
$\vec{a}$	`\vec{a}`

It should be borne in mind that when a character is underlined in a mathematical manuscript then it is normally typeset in bold face without any underlining. Underlining is used very rarely in print.

The control sequences such as `\'` and `\"`, used to produce accents in ordinary text, may not be used in mathematics mode.

3.12 Brackets and Norms

The frequently used left delimiters include (, [and {, which are obtained by typing `(`, `[` and `\{` respectively. The corresponding right delimiters are of course obtained by typing `)`, `]` and `\}`. In addition $|$ and $\|$ are used as both left and right delimiters, and are obtained by typing `|` and `\|` respectively. For example, we obtain

Let X be a Banach space and let $f \colon B \to \mathbf{R}$ be a bounded linear functional on X. The *norm* of f, denoted by $\|f\|$, is defined by

$$\|f\| = \inf\{ K \in [0,+\infty) : |f(x)| \leq K \|x\| \mbox{ for all } x \in X \}.$$

by typing

```
Let $X$ be a Banach space and let $f \colon B \to \textbf{R}$
be a bounded linear functional on $X$. The \textit{norm} of
$f$, denoted by $\|f\|$, is defined by
\[ \|f\| = \inf \{ K \in [0,+\infty) :
           |f(x)| \leq K \|x\| \mbox{ for all } x \in X \}.\]
```

Larger delimiters are sometimes required which have the appropriate height to match the size of the subformula which they enclose. Consider, for instance, the problem of typesetting the following formula:

$$f(x,y,z) = 3y^2 z \left(3 + \frac{7x+5}{1 + y^2} \right).$$

The way to type the large parentheses is to type `\left(` for the left parenthesis and `\right)` for the right parenthesis, and let LaTeX do the rest of the work for you. Thus the above formula was obtained by typing

```
\[ f(x,y,z) = 3y^2 z \left( 3 + \frac{7x+5}{1 + y^2} \right).\]
```

If you type a delimiter which is preceded by `\left` then LaTeX will search for a corresponding delimiter preceded by `\right` and calculate the size of the delimiters required to enclose the intervening subformula. One is allowed to balance a `\left(` with a `\right]` (say) if one desires: there is no reason why the enclosing delimiters have to have the same shape. One may also nest pairs of delimiters within one another: by typing

```
\[ \left| 4 x^3 + \left( x + \frac{42}{1+x^4} \right) \right|.\]
```

we obtain

$$\left| 4 x^3 + \left(x + \frac{42}{1+x^4} \right) \right|.$$

By typing `\left.` and `\right.` one obtains *null delimiters* which are completely invisible. Consider, for example, the problem of typesetting

$$\left. \frac{du}{dx} \right|_{x=0}.$$

We wish to make the vertical bar big enough to match the derivative preceding it. To do this, we suppose that the derivative is enclosed by delimiters, where the left delimiter is invisible and the right delimiter is the vertical line. The invisible delimiter is produced using `\left.` and thus the whole formula is produced by typing

$$\left.\frac{du}{dx}\right|_{x=0}.$$

3.13 Multiline Formulae in LaTeX

Consider the problem of typesetting the formula

$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1.\end{aligned}$$

It is necessary to ensure that the = signs are aligned with one another. In LaTeX, such a formula is typeset using the `eqnarray*` environment. The above example was obtained by typing the lines

```
\begin{eqnarray*}
\cos 2\theta & = & \cos^2 \theta - \sin^2 \theta \\
             & = & 2 \cos^2 \theta - 1.
\end{eqnarray*}
```

Note the use of the special character `&` as an *alignment tab*. When the formula is typeset, the part of the second line of the formula beginning with an occurrence of `&` will be placed immediately beneath that part of the first line of the formula which begins with the corresponding occurrence of `&`. Also `\\` is used to separate the lines of the formula.

Although we have placed corresponding occurrences of `&` beneath one another in the above example, it is not necessary to do this in the input file. It was done in the above example merely to improve the appearance (and readability) of the input file.

The more complicated example

If $h \leq \frac{1}{2}|\zeta - z|$ then

$$|\zeta - z - h| \geq \frac{1}{2}|\zeta - z|$$

and hence

$$\left|\frac{1}{\zeta - z - h} - \frac{1}{\zeta - z}\right| = \left|\frac{(\zeta - z) - (\zeta - z - h)}{(\zeta - z - h)(\zeta - z)}\right|$$

$$\begin{aligned} &= \left|\frac{h}{(\zeta - z - h)(\zeta - z)}\right| \\ &\leq \frac{2|h|}{|\zeta - z|^2}. \end{aligned}$$

was obtained by typing

```
If $h \leq \frac{1}{2} |\zeta - z|$ then
\[ |\zeta - z - h| \geq \frac{1}{2} |\zeta - z|\]
and hence
\begin{eqnarray*}
\left| \frac{1}{\zeta - z - h} - \frac{1}{\zeta - z} \right|
& = & \left|
\frac{(\zeta - z) - (\zeta - z - h)}{(\zeta - z - h)(\zeta - z)}
\right| \\  & = &
\left| \frac{h}{(\zeta - z - h)(\zeta - z)} \right| \\
  & \leq & \frac{2 |h|}{|\zeta - z|^2}.
\end{eqnarray*}
```

The asterisk in `eqnarray*` is put there to suppress the automatic equation numbering produced by LaTeX. If you wish for an automatically numbered multiline formula, you should use `\begin{eqnarray}` and `\end{eqnarray}`.

3.14 Matrices and other arrays in LaTeX

Matrices and other arrays are produced in LaTeX using the **array** environment. For example, suppose that we wish to typeset the following passage:

> The *characteristic polynomial* $\chi(\lambda)$ of the 3×3 matrix
>
> $$\left(\begin{array}{ccc} a & b & c \\ d & e & f \\ g & h & i \end{array} \right)$$
>
> is given by the formula
>
> $$\chi(\lambda) = \left| \begin{array}{ccc} \lambda - a & -b & -c \\ -d & \lambda - e & -f \\ -g & -h & \lambda - i \end{array} \right|.$$

This passage is produced by the following input:

```
The \emph{characteristic polynomial} $\chi(\lambda)$ of the
$3 \times 3$~matrix
\[ \left( \begin{array}{ccc}
a & b & c \\
d & e & f \\
g & h & i \end{array} \right)\]
is given by the formula
\[ \chi(\lambda) = \left| \begin{array}{ccc}
\lambda - a & -b & -c \\
-d & \lambda - e & -f \\
-g & -h & \lambda - i \end{array} \right|.\]
```

First of all, note the use of `\left` and `\right` to produce the large delimiters around the arrays. As we have already seen, if we use

`\left)` ... `\right)`

then the size of the parentheses is chosen to match the subformula that they enclose. Next note the use of the alignment tab character `&` to separate the entries of the matrix and the use of `\\` to separate the rows of the matrix, exactly as in the construction of multiline formulae described above. We begin the array with `\begin{array}` and end it with `\end{array}`. The only thing left to explain, therefore, is the mysterious `{ccc}` which occurs immediately after `\begin{array}`. Now each of the `c`'s in `{ccc}` represents a column of the matrix and indicates that the entries of the column should be *centred.* If the `c` were replaced by `l` then the corresponding column would be typeset with all the entries flush *left*, and `r` would produce a column with all entries flush *right.* Thus

```
\[ \begin{array}{lcr}
\mbox{First number} & x & 8 \\
\mbox{Second number} & y & 15 \\
\mbox{Sum} & x + y & 23 \\
\mbox{Difference} & x - y & -7 \\
\mbox{Product} & xy & 120 \end{array}\]
```

produces

$$\begin{array}{lcr}
\mbox{First number} & x & 8 \\
\mbox{Second number} & y & 15 \\
\mbox{Sum} & x + y & 23 \\
\mbox{Difference} & x - y & -7 \\
\mbox{Product} & xy & 120 \end{array}$$

We can use the array environment to produce formulae such as

$$|x| = \left\{ \begin{array}{ll} x & \mbox{if $x \geq 0$};\\ -x & \mbox{if $x < 0$}.\end{array} \right.$$

Note that both columns of this array are set flush left. Thus we use `{ll}` immediately after `\begin{array}`. The large curly bracket is produced using `\left\{`. However this requires a corresponding `\right` delimiter to match it. We therefore use the *null delimiter* `\right.` discussed earlier. This delimiter is invisible. We can therefore obtain the above formula by typing

```
\[ |x| = \left\{ \begin{array}{ll}
         x & \mbox{if $x \geq 0$};\\
        -x & \mbox{if $x < 0$}.\end{array} \right. \]
```

3.15 Derivatives, Limits, Sums and Integrals

The expressions

$$\frac{du}{dt} \mbox{ and } \frac{d^2 u}{dx^2}$$

are obtained in LaTeX by typing `\frac{du}{dt}` and `\frac{d^2 u}{dx^2}` respectively. The mathematical symbol ∂ is produced using `\partial`. Thus the Heat Equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

is obtained in LaTeX by typing

```
\[\frac{\partial u}{\partial t}
   = \frac{\partial^2 u}{\partial x^2}
      + \frac{\partial^2 u}{\partial y^2}
      + \frac{\partial^2 u}{\partial z^2} \]
```

To obtain mathematical expressions such as

$$\lim_{x \to +\infty}, \inf_{x > s} \mbox{ and } \sup_K$$

in displayed equations we type `\lim_{x \to +\infty}`, `\inf_{x > s}` and `\sup_K` respectively. Thus to obtain

$$\lim_{x \to 0} \frac{3x^2 +7}{x^2 +1} = 3.$$

(in LaTeX) we type

```
\[ \lim_{x \to 0} \frac{3x^2 +7x^3}{x^2 +5x^4} = 3.\]
```

To obtain a summation sign such as

$$\sum_{i=1}^{2n}$$

we type `sum_{i=1}^{2n}`. Thus

$$\sum_{k=1}^n k^2 = \frac{1}{2} n(n+1).$$

is obtained by typing

```
\[ \sum_{k=1}^n k^2 = \frac{1}{2} n (n+1).\]
```

We now discuss how to obtain *integrals* in mathematical documents. A typical integral is the following:

$$\int_a^b f(x)\,dx.$$

This is typeset using

```
\[ \int_a^b f(x)\,dx.\]
```

The integral sign $\int$ is typeset using the control sequence `\int`, and the *limits of integration* (in this case a and b are treated as a subscript and a superscript on the integral sign.

Most integrals occurring in mathematical documents begin with an integral sign and contain one or more instances of d followed by another (Latin or Greek) letter, as in dx, dy and dt. To obtain the correct appearance one should put extra space before the d, using `\,`. Thus

$$\int_0^{+\infty} x^n e^{-x} \,dx = n!.$$

$$\int \cos\theta \,d\theta = \sin\theta.$$

$$\int_{x^2+y^2\leq R^2} f(x,y)\,dx\,dy = \int_{\theta=0}^{2\pi}\int_{r=0}^{R} f(r\cos\theta, r\sin\theta) r\,dr\,d\theta.$$

and

$$\int_0^R \frac{2x\,dx}{1+x^2} = \log(1+R^2).$$

are obtained by typing

```
\[ \int_0^{+\infty} x^n e^{-x} \,dx = n!.\]

\[ \int \cos \theta \,d\theta = \sin \theta.\]
```

```
\[ \int_{x^2 + y^2 \leq R^2} f(x,y)\,dx\,dy
   = \int_{\theta=0}^{2\pi} \int_{r=0}^R
      f(r\cos\theta,r\sin\theta) r\,dr\,d\theta.\]
```

and

```
\[ \int_0^R \frac{2x\,dx}{1+x^2} = \log(1+R^2).\]
```

respectively.

In some multiple integrals (i.e., integrals containing more than one integral sign) one finds that LaTeX puts too much space between the integral signs. The way to improve the appearance of of the integral is to use the control sequence `\!` to remove a thin strip of unwanted space. Thus, for example, the multiple integral

$$\int_0^1 \! \int_0^1 x^2 y^2\,dx\,dy.$$

is obtained by typing

```
\[ \int_0^1 \! \int_0^1 x^2 y^2\,dx\,dy.\]
```

Had we typed

```
\[ \int_0^1 \int_0^1 x^2 y^2\,dx\,dy.\]
```

we would have obtained

$$\int_0^1 \int_0^1 x^2 y^2\,dx\,dy.$$

A particularly noteworthy example comes when we are typesetting a multiple integral such as

$$\int \!\!\! \int_D f(x,y)\,dx\,dy.$$

Here we use `\!` three times to obtain suitable spacing between the integral signs. We typeset this integral using

```
\[ \int \!\!\! \int_D f(x,y)\,dx\,dy.\]
```

Had we typed

```
\[ \int \int_D f(x,y)\,dx\,dy.\]
```

we would have obtained

$$\int \int_D f(x,y)\,dx\,dy.$$

The following (reasonably complicated) passage exhibits a number of the features which we have been discussing:

In non-relativistic wave mechanics, the wave function $\psi(\mathbf{r},t)$ of a particle satisfies the *Schrödinger Wave Equation*

$$i\hbar\frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi + V \psi.$$

It is customary to normalize the wave equation by demanding that

$$\int \!\!\! \int \!\!\! \int_{\textbf{R}^3} \left| \psi(\mathbf{r},0) \right|^2\,dx\,dy\,dz = 1.$$

A simple calculation using the Schrödinger wave equation shows that

$$\frac{d}{dt} \int \!\!\! \int \!\!\! \int_{\textbf{R}^3} \left| \psi(\mathbf{r},t) \right|^2\,dx\,dy\,dz = 0,$$

and hence

$$\int \!\!\! \int \!\!\! \int_{\textbf{R}^3} \left| \psi(\mathbf{r},t) \right|^2\,dx\,dy\,dz = 1$$

for all times t. If we normalize the wave function in this way then, for any (measurable) subset V of $\mathbf{R}^3$ and time t,

$$\int \!\!\! \int \!\!\! \int_{V} \left| \psi(\mathbf{r},t) \right|^2\,dx\,dy\,dz$$

represents the probability that the particle is to be found within the region V at time t.

One would typeset this in LaTeX by typing

```
In non-relativistic wave mechanics, the wave function
$\psi(\mathbf{r},t)$ of a particle satisfies the
\textit{Schr\"{o}dinger Wave Equation}
\[ i\hbar\frac{\partial \psi}{\partial t}
  = \frac{-\hbar^2}{2m} \left(
    \frac{\partial^2}{\partial x^2}
    + \frac{\partial^2}{\partial y^2}
    + \frac{\partial^2}{\partial z^2}
  \right) \psi + V \psi.\]
It is customary to normalize the wave equation by
demanding that
\[ \int \!\!\! \int \!\!\! \int_{\textbf{R}^3}
      \left| \psi(\mathbf{r},0) \right|^2\,dx\,dy\,dz = 1.\]
A simple calculation using the Schr\"{o}dinger wave
equation shows that
```

```
\[ \frac{d}{dt} \int \!\!\! \int \!\!\! \int_{\textbf{R}^3}
      \left| \psi(\mathbf{r},t) \right|^2\,dx\,dy\,dz = 0,\]
and hence
\[ \int \!\!\! \int \!\!\! \int_{\textbf{R}^3}
      \left| \psi(\mathbf{r},t) \right|^2\,dx\,dy\,dz = 1\]
for all times~$t$. If we normalize the wave function in this
way then, for any (measurable) subset~$V$ of $\textbf{R}^3$
and time~$t$,
\[ \int \!\!\! \int \!\!\! \int_V
      \left| \psi(\mathbf{r},t) \right|^2\,dx\,dy\,dz\]
represents the probability that the particle is to be found
within the region~$V$ at time~$t$.
```

4 Further Features of LaTeX

4.1 Producing White Space in LaTeX

To produce (horizontal) blank space within a paragraph, use `\hspace`, followed by the length of the blank space enclosed within curly brackets. The length of the skip should be expressed in a unit recognized by LaTeX. These recognized units are given in the following table:

`pt`	point	(1 in = 72.27 pt)
`pc`	pica	(1 pc = 12 pt)
`in`	inch	(1 in = 25.4 mm)
`bp`	big point	(1 in = 72 bp)
`cm`	centimetre	(1 cm = 10 mm)
`mm`	millimetre	
`dd`	didot point	(1157 dd = 1238 pt)
`cc`	cicero	(1 cc = 12 dd)
`sp`	scaled point	(65536 sp = 1 pt)

Thus to produce a horizontal blank space of 20 mm in the middle of a paragraph one would type `\hspace{20 mm}`.

If LaTeX decides to break between lines at a point in the document where an `\hspace` is specified, then no white space is produced. To ensure that white space is produced even at points in the document where line breaking takes place, one should replace `\hspace` by `\hspace*`

To produce (vertical) blank space between paragraphs, use `\vspace`, followed by the length of the blank space enclosed within curly brackets. Thus to obtain