Derivative of Exponential functions:

$$\frac{d}{dx} e^x = e^x$$

Example: $y = e^{x^2+1}$ find $\frac{dy}{dx}$.

Let $u = x^2 + 1$,

$$\Rightarrow y = e^u$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{d}{du} e^u \cdot \frac{d}{dx} (x^2+1)$$

$$= e^{x^2+1} \cdot 2x$$

$$\frac{d}{dx} a^x = a^x (\ln a).$$

Example: $y = a^{\sqrt{x}}$, find $\frac{dy}{dx}$.

Let $u = \sqrt{x}$

$$\Rightarrow y = a^u$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{d}{du} a^u \cdot \frac{d}{dx} \sqrt{x}$$

$$= a^{\sqrt{x}} \ln a \cdot \frac{1}{2\sqrt{x}}.$$

Example : $y = a^x$, find $\dfrac{dy}{dx}$.

$$y = a^x$$
$$= e^{x \ln a}$$

$e^{\ln a^x} = a^x$.

$$\dfrac{dy}{dx} = e^{x \ln a} \cdot \dfrac{d}{dx}(x \ln a).$$

$$= e^{x \ln a} \cdot \ln a.$$

$$= a^x \cdot \ln a.$$

Derivative of the logarithmic function:

$$\frac{d}{dx}(\ln x) = \frac{1}{x}.$$

$$\frac{d}{dx}\left[\log_a x\right] = \frac{1}{x} \cdot \frac{1}{\ln a}.$$

Example: Differentiate $\ln(x^2+2x)$ w.r.t. x.

Let, $\quad y = \ln(x^2+2x)$

$$\frac{dy}{dx} = \frac{1}{(x^2+2x)} \cdot \frac{d}{dx}(x^2+2x)$$

$$= \frac{2x+2}{(x^2+2x)}.$$

Example: $y = \log_{10}(ax^2+bx+c)$

Let $\quad u = (ax^2+bx+c)$

$\Rightarrow \quad y = \log_{10} u$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{d}{du}(\log_{10} u) \cdot \frac{d}{dx}(ax^2+bx+c)$$

$$= \frac{1}{(ax^2+bx+c)\ln 10} \cdot (2ax+b).$$

EXERCISE 2.6

1. Find $f'(x)$ if

   (i) $f(x) = e^{\sqrt{x}-1}$

   (ii) $f(x) = x^3 e^{\frac{1}{x}} \ (x \neq 0)$

   (iii) $f(x) = e^x (I + \ln x)$

   (iv) $f(x) = \dfrac{e^x}{e^{-x}+1}$

   (v) $\ln(e^x + e^{-x})$

   (vi) $fx = \dfrac{e^{ax} - e^{-ax}}{e^{ax} + e^{-ax}}$

   (vii) $f(x) = \sqrt{\ln(e^{2x} + e^{-2x})}$

   (viii) $f(x) = \ln\left(\sqrt{e^{2x} + e^{-2x}}\right)$

2. Find $\dfrac{dy}{dx}$ if

   (i) $y = x^2 \ln\sqrt{x}$

   (ii) $y = x\sqrt{\ln x}$

   (iii) $y = \dfrac{x}{\ln x}$

   (iv) $y = x^2 \ln\dfrac{1}{x}$

   (v) $y = \ln\sqrt{\dfrac{x^2 - 1}{x^2 + 1}}$

   (vi) $y = \ln\left(x + \sqrt{x^2 + 1}\right)$

   (vii) $y = \ln(9 - x^2)$

   (viii) $y = e^{-2x} \sin 2x$

   (ix) $y = e^{-x}(x^3 + 2x^2 + 1)$

   (x) $y = x e^{\sin x}$

   (xi) $y = 5e^{3x-4}$

   (xii) $y = (x+1)^x$

   (xiii) $y = (\ln x)^{\ln x}$

   (xiv) $y = \dfrac{\sqrt{x^2 - 1}(x+1)}{(x^3 + 1)^{3/2}}$

3. Find $\dfrac{dy}{dx}$ if

   (i) $y = \cosh 2x$

   (ii) $y = \sinh 3x$

   (iii) $y = \tanh^{-1}(\sin x) \quad \dfrac{\pi}{2} \ x \ \dfrac{\pi}{2}$

   (iv) $y = \sinh^{-1}(x^3)$

   (v) $y = \ln(\tanh x)$

   (vi) $y = \sinh^{-1}\left(\dfrac{x}{2}\right)$