Differentiate w.r.t. 'x'

Question # 1

$x^4 + 2x^3 + x^2$

Solution Let $y = x^4 + 2x^3 + x^2$

Differentiating w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^4 + 2x^3 + x^2\right)$$
$$= \frac{d}{dx}x^4 + 2\frac{d}{dx}x^3 + \frac{d}{dx}x^2$$
$$= 4x^{4-1} + 2\left(3x^{3-1}\right) + 2x^{2-1}$$
$$= 4x^3 + 6x^2 + 2x$$

Question # 2

$x^{-3} + 2x^{-\frac{3}{2}} + 3$

Solution Let $y = x^{-3} + 2x^{-\frac{3}{2}} + 3$

Diff. w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^{-3} + 2x^{-\frac{3}{2}} + 3\right)$$
$$= \frac{d}{dx}x^{-3} + 2\frac{d}{dx}x^{-\frac{3}{2}} + \frac{d}{dx}(3)$$
$$= -3x^{-3-1} + 2\left(-\frac{3}{2}x^{-\frac{3}{2}-1}\right) + 0$$
$$\Rightarrow \frac{dy}{dx} = -3x^{-4} - 3x^{-\frac{5}{2}}$$
$$\text{or} \quad \frac{dy}{dx} = -3\left(\frac{1}{x^4} + \frac{1}{x^{5/2}}\right)$$

Question # 3

$\dfrac{a+x}{a-x}$

Solution Let $y = \dfrac{a+x}{a-x}$

Now $$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{a+x}{a-x}\right) = \frac{(a-x)\frac{d}{dx}(a+x) - (a+x)\frac{d}{dx}(a-x)}{(a-x)^2}$$
$$= \frac{(a-x)(0+1) - (a+x)(0-1)}{(a-x)^2}$$

$$= \frac{(a-x)(1)-(a+x)(-1)}{(a-x)^2}$$

$$= \frac{a-x+a+x}{(a-x)^2} \quad = \frac{2a}{(a-x)^2} \qquad \textit{Answer}$$

Question # 4

$\dfrac{2x-3}{2x+1}$

Solution Let $y = \dfrac{2x-3}{2x+1}$

Now $\dfrac{dy}{dx} = \dfrac{d}{dx}\left(\dfrac{2x-3}{2x+1}\right)$

$$= \frac{(2x+1)\frac{d}{dx}(2x-3)-(2x-3)\frac{d}{dx}(2x+1)}{(2x+1)^2}$$

$$= \frac{(2x+1)(2-0)-(2x-3)(2+0)}{(2x+1)^2}$$

$$= \frac{(2x+1)(2)-(2x-3)(2)}{(2x+1)^2}$$

$$= \frac{2(2x+1-2x+3)}{(2x+1)^2}$$

$$= \frac{2(4)}{(2x+1)^2} \quad = \frac{8}{(2x+1)^2} \qquad \textit{Answer}$$

Question # 5

$(x-5)(3-x)$

Solution Let $y = (x-5)(3-x)$

$$= 3x - x^2 - 15 + 5x$$

$$= -x^2 + 8x - 15$$

Now

$$\frac{dy}{dx} = \frac{dy}{dx}\left(-x^2+8x-15\right)$$

$$= \frac{dy}{dx}\left(-x^2\right) + 8\frac{d}{dx}(x) - \frac{d}{dx}(15)$$

$$= -2x^{2-1} + 8(1) - 0 \quad = -2x + 8 \qquad \textit{Answer}$$

Question # 6

$\left(\sqrt{x} - \dfrac{1}{\sqrt{x}}\right)^2$

Solution Let $y=\left(\sqrt{x}-\dfrac{1}{\sqrt{x}}\right)^2$

$$=\left(\sqrt{x}\right)^2+\left(\frac{1}{\sqrt{x}}\right)^2-2\left(\sqrt{x}\right)\left(\frac{1}{\sqrt{x}}\right)$$

$$=x+\frac{1}{x}-2 \quad =x+x^{-1}-2$$

Now diff. w.r.t x

$$\frac{dy}{dx}=\frac{d}{dx}\left(x+x^{-1}-2\right)=\frac{d}{dx}(x)+\frac{d}{dx}(x^{-1})-\frac{d}{dx}(2)$$

$$=1+(-1\cdot x^{-1-1})-0=1-x^{-2}$$

$$=1-\frac{1}{x^2}=\frac{x^2-1}{x^2} \quad \textit{Answer}$$

Question # 7

$$\frac{\left(1+\sqrt{x}\right)\left(x-x^{3/2}\right)}{\sqrt{x}}$$

Solution Consider $y=\dfrac{\left(1+\sqrt{x}\right)\left(x-x^{3/2}\right)}{\sqrt{x}}$

$$=\frac{\left(1+\sqrt{x}\right)\ x\left(1-x^{\frac{1}{2}}\right)}{\sqrt{x}}$$

$$=\frac{x\left(1+\sqrt{x}\right)\left(1-\sqrt{x}\right)}{\sqrt{x}} \qquad \text{Since } x^{\frac{3}{2}}=x^{1+\frac{1}{2}}$$

$$=\frac{\left(\sqrt{x}\right)^2\left(1-\left(\sqrt{x}\right)^2\right)}{\sqrt{x}}$$

$$=\sqrt{x}(1-x)=x^{\frac{1}{2}}(1-x)=x^{\frac{1}{2}}-x^{\frac{3}{2}}$$

Now

$$\frac{dy}{dx}=\frac{d}{dx}\left(x^{\frac{1}{2}}-x^{\frac{3}{2}}\right)$$

$$=\frac{1}{2}x^{\frac{1}{2}-1}-\frac{3}{2}x^{\frac{3}{2}-1}$$

$$=\frac{1}{2}x^{-\frac{1}{2}}-\frac{3}{2}x^{\frac{1}{2}}$$

$$=\frac{1}{2}\left(\frac{1}{\sqrt{x}}-3\sqrt{x}\right) \quad \textit{Answer}$$

Question # 8

$$\frac{\left(x^2+1\right)^2}{x^2-1}$$

Solution Let $y = \dfrac{\left(x^2+1\right)^2}{x^2-1}$

Differentiating w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\left(x^2+1\right)^2}{x^2-1}\right)$$

$$= \frac{\left(x^2-1\right)\,\frac{d}{dx}\left(x^2+1\right)^2-\left(x^2+1\right)^2\frac{d}{dx}\left(x^2-1\right)}{\left(x^2-1\right)^2}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\left(x^2-1\right)\,2\left(x^2+1\right)^{2-1}\frac{d}{dx}\left(x^2+1\right)-\left(x^2+1\right)^2(2x)}{\left(x^2-1\right)^2}$$

$$= \frac{\left(x^2-1\right)\,2\left(x^2+1\right)(2x)-\left(x^2+1\right)^2(2x)}{\left(x^2-1\right)^2}$$

$$= \frac{2x\left(x^2+1\right)\left[2\left(x^2-1\right)-\left(x^2+1\right)\right]}{\left(x^2-1\right)^2}$$

$$= \frac{2x\left(x^2+1\right)\left[2x^2-2-x^2-1\right]}{\left(x^2-1\right)^2}$$

$$= \frac{2x\left(x^2+1\right)\left(x^2-3\right)}{\left(x^2-1\right)^2} \quad \textit{Answer}$$

Question # 9

$$\frac{x^2+1}{x^2-3}$$

Solution Let $y = \dfrac{x^2+1}{x^2-3}$

Differentiating w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2+1}{x^2-3}\right)$$

$$= \frac{\left(x^2-3\right)\,\frac{d}{dx}\left(x^2+1\right)-\left(x^2+1\right)\frac{d}{dx}\left(x^2-3\right)}{\left(x^2-3\right)^2}$$

$$= \frac{(x^2-3)\ (2x)-(x^2+1)(2x)}{(x^2-3)^2} = \frac{2x(x^2-3-x^2-1)}{(x^2-3)^2}$$

$$= \frac{2x(-4)}{(x^2-3)^2} = \frac{-8x}{(x^2-3)^2} \qquad \textit{Answer}$$

Question # 10

$$\frac{\sqrt{1+x}}{\sqrt{1-x}}$$

Solution Let $y = \dfrac{\sqrt{1+x}}{\sqrt{1-x}} = \left(\dfrac{1+x}{1-x}\right)^{1/2}$

Now $\dfrac{dy}{dx} = \dfrac{d}{dx}\left(\dfrac{1+x}{1-x}\right)^{1/2}$

$$= \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}-1} \frac{d}{dx}\left(\frac{1+x}{1-x}\right)$$

$$= \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}}\left(\frac{(1-x)\frac{d}{dx}(1+x)-(1+x)\frac{d}{dx}(1-x)}{(1-x)^2}\right)$$

$$= \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{\frac{1}{2}}\left(\frac{(1-x)(1)-(1+x)(-1)}{(1-x)^2}\right)$$

$$= \frac{1}{2}\frac{(1-x)^{\frac{1}{2}}}{(1+x)^{\frac{1}{2}}}\left(\frac{1-x+1+x}{(1-x)^2}\right) = \frac{(1-x)^{\frac{1}{2}}}{2(1+x)^{\frac{1}{2}}}\left(\frac{2}{(1-x)^2}\right)$$

$$= \frac{1}{(1+x)^{\frac{1}{2}}(1-x)^{2-\frac{1}{2}}} = \frac{1}{\sqrt{1+x}\ (1-x)^{\frac{3}{2}}} \qquad \textit{Answer}$$

Question # 11

$$\frac{2x-1}{\sqrt{x^2+1}}$$

Solution Let $y = \dfrac{2x-1}{\sqrt{x^2+1}}$

Differentiating w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2x-1}{(x^2+1)^{1/2}}\right)$$

$$= \frac{(x^2+1)^{1/2}\ \frac{d}{dx}(2x-1)-(2x-1)\frac{d}{dx}(x^2+1)^{1/2}}{\left((x^2+1)^{1/2}\right)^2}$$

$$= \frac{\left(x^2+1\right)^{1/2}\,(2)-(2x-1)\,\frac{1}{2}\left(x^2+1\right)^{-1/2}\frac{d}{dx}(x^2+1)}{\left(x^2+1\right)}$$

$$= \frac{2\left(x^2+1\right)^{1/2}-(2x-1)\frac{1}{2\left(x^2+1\right)^{1/2}}(2x)}{\left(x^2+1\right)}$$

$$= \frac{1}{\left(x^2+1\right)}\left(2\left(x^2+1\right)^{1/2}-\frac{2x^2-x}{\left(x^2+1\right)^{1/2}}\right)$$

$$= \frac{1}{\left(x^2+1\right)}\left(\frac{2x^2+2-2x^2+x}{\left(x^2+1\right)^{1/2}}\right)$$

$$= \frac{x+2}{\left(x^2+1\right)\sqrt{x^2+1}} \quad \text{or} \quad \frac{x+2}{\left(x^2+1\right)^{3/2}} \quad \textit{Answer}$$

Question # 12

$$\frac{\sqrt{a-x}}{\sqrt{a+x}}$$

Solution

Do yourself as Question # 10

Question # 13

$$\frac{\sqrt{x^2+1}}{\sqrt{x^2-1}}$$

Solution Let $y = \dfrac{\sqrt{x^2+1}}{\sqrt{x^2-1}}$

$$= \left(\frac{x^2+1}{x^2-1}\right)^{\frac{1}{2}}$$

Differentiating w.r.t x.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2+1}{x^2-1}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(\frac{x^2+1}{x^2-1}\right)^{-\frac{1}{2}}\frac{d}{dx}\left(\frac{x^2+1}{x^2-1}\right)$$

$$= \frac{1}{2}\left(\frac{x^2-1}{x^2+1}\right)^{\frac{1}{2}}\left(\frac{(x^2-1)(2x)-(x^2+1)(2x)}{\left(x^2-1\right)^2}\right)$$

$$= \frac{1}{2}\frac{\sqrt{x^2-1}}{\sqrt{x^2+1}}\left(\frac{2x^3-2x-2x^3-2x}{\left(x^2-1\right)^2}\right)$$

$$= \frac{1}{2}\frac{\sqrt{x^2-1}}{\sqrt{x^2+1}}\left(\frac{-4x}{\left(x^2-1\right)^2}\right)$$

$$= \frac{-2\sqrt{x^2-1}}{\left(x^2-1\right)^2\sqrt{x^2+1}} = \frac{-2}{\left(x^2-1\right)^{2-\frac{1}{2}}\sqrt{x^2+1}}$$

$$= \frac{-2}{\left(x^2-1\right)^{\frac{3}{2}}\sqrt{x^2+1}} \quad \textit{Answer}$$

Question # 14

$$\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$$

Solution Assume $y = \dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$

$$= \frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\cdot\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}} \qquad \textit{Rationalizing}$$

$$= \frac{\left(\sqrt{1+x}-\sqrt{1-x}\right)^2}{\left(\sqrt{1+x}\right)^2-\left(\sqrt{1-x}\right)^2}$$

$$= \frac{\left(\sqrt{1+x}\right)^2+\left(\sqrt{1-x}\right)^2-2\left(\sqrt{1+x}\right)\left(\sqrt{1-x}\right)}{1+x-1+x}$$

$$= \frac{1+x+1-x-2\sqrt{(1+x)(1-x)}}{2x}$$

$$= \frac{2-2\sqrt{1-x^2}}{2x} = \frac{2\left(1-\left(1-x^2\right)^{\frac{1}{2}}\right)}{2x}$$

$$= \frac{1-\left(1-x^2\right)^{\frac{1}{2}}}{x}$$

Now differentiation w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1-\left(1-x^2\right)^{\frac{1}{2}}}{x}\right)$$

$$= \frac{x\,\frac{d}{dx}\left(1-\left(1-x^2\right)^{\frac{1}{2}}\right)-\left(1-\left(1-x^2\right)^{\frac{1}{2}}\right)\frac{d}{dx}x}{x^2}$$

$$= \frac{1}{x^2}\cdot\left[x\left(0-\frac{1}{2}\left(1-x^2\right)^{\frac{1}{2}-1}\frac{d}{dx}\left(1-x^2\right)\right)-\left(1-\left(1-x^2\right)^{\frac{1}{2}}\right)(1)\right]$$

$$= \frac{1}{x^2}\cdot\left[x\left(-\frac{1}{2}\left(1-x^2\right)^{-\frac{1}{2}}(-2x)\right)-1+\left(1-x^2\right)^{\frac{1}{2}}\right]$$

$$= \frac{1}{x^2}\cdot\left[\frac{x^2}{\left(1-x^2\right)^{\frac{1}{2}}}-1+\left(1-x^2\right)^{\frac{1}{2}}\right] = \frac{1}{x^2}\cdot\left[\frac{x^2-\left(1-x^2\right)^{\frac{1}{2}}+1-x^2}{\left(1-x^2\right)^{\frac{1}{2}}}\right]$$

$$= \frac{1}{x^2}\cdot\left[\frac{1-\left(1-x^2\right)^{\frac{1}{2}}}{\left(1-x^2\right)^{\frac{1}{2}}}\right] = \frac{1-\sqrt{1-x^2}}{x^2\sqrt{1-x^2}} \quad \textit{Answer}$$

Question # 15

$$\frac{x\sqrt{a+x}}{\sqrt{a-x}}$$

Solution Let $y = \dfrac{x\sqrt{a+x}}{\sqrt{a-x}} = x\left(\dfrac{a+x}{a-x}\right)^{1/2}$

Diff. w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}x\left(\frac{a+x}{a-x}\right)^{1/2}$$

$$= x\frac{d}{dx}\left(\frac{a+x}{a-x}\right)^{1/2} + \left(\frac{a+x}{a-x}\right)^{1/2}\frac{d}{dx}x \quad \ldots\ldots\ldots\ldots \text{(i)}$$

Now $\dfrac{d}{dx}\left(\dfrac{a+x}{a-x}\right)^{1/2} = \dfrac{1}{2}\left(\dfrac{a+x}{a-x}\right)^{1/2-1}\dfrac{d}{dx}\left(\dfrac{a+x}{a-x}\right)$

$$= \frac{1}{2}\left(\frac{a+x}{a-x}\right)^{-1/2}\left(\frac{(a-x)\frac{d}{dx}(a+x)-(a+x)\frac{d}{dx}(a-x)}{(a-x)^2}\right)$$

$$= \frac{1}{2}\left(\frac{a-x}{a+x}\right)^{1/2}\left(\frac{(a-x)(1)-(a+x)(-1)}{(a-x)^2}\right)$$

$$= \frac{1}{2}\frac{(a-x)^{\frac{1}{2}}}{(a+x)^{\frac{1}{2}}}\left(\frac{a-x+a+x}{(a-x)^2}\right) = \frac{1}{2}\frac{1}{(a+x)^{\frac{1}{2}}(a-x)^{-\frac{1}{2}}}\cdot\left(\frac{2a}{(a-x)^2}\right)$$

$$= \frac{a}{(a+x)^{\frac{1}{2}}(a-x)^{2-\frac{1}{2}}} = \frac{a}{(a+x)^{\frac{1}{2}}(a-x)^{\frac{3}{2}}}$$

Using in eq. (i)

$$\frac{dy}{dx} = x\cdot\frac{a}{(a+x)^{\frac{1}{2}}(a-x)^{\frac{3}{2}}} + \left(\frac{a+x}{a-x}\right)^{\frac{1}{2}}(1)$$

$$= \frac{ax}{(a+x)^{\frac{1}{2}}(a-x)^{\frac{3}{2}}} + \frac{(a+x)^{\frac{1}{2}}}{(a-x)^{\frac{1}{2}}}$$

$$= \frac{ax+(a+x)(a-x)}{(a+x)^{\frac{1}{2}}(a-x)^{\frac{3}{2}}} = \frac{ax+a^2-x^2}{\sqrt{a+x}\,(a-x)^{\frac{3}{2}}} \quad \textit{Answer}$$

Question # 16

If $y=\sqrt{x}-\frac{1}{\sqrt{x}}$, show that $2x\frac{dy}{dx}+y= 2\sqrt{x}$

Solution Since $y = \sqrt{x}-\frac{1}{\sqrt{x}}$

$$= x^{\frac{1}{2}} - x^{-\frac{1}{2}}$$

Diff. w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^{\frac{1}{2}} - x^{-\frac{1}{2}}\right)$$

$$= \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$$

Multiplying by $2x$

$$2x\frac{dy}{dx} = 2x\left(\frac{1}{2}x^{-\frac{1}{2}}\right)+2x\left(\frac{1}{2}x^{-\frac{3}{2}}\right) \Rightarrow 2x\frac{dy}{dx} = x^{-\frac{1}{2}+1}+x^{-\frac{3}{2}+1}$$

$$2x\frac{dy}{dx} = x^{\frac{1}{2}}+x^{-\frac{1}{2}}$$

Adding y on both sides

$$2x\frac{dy}{dx}+y= x^{\frac{1}{2}}+x^{-\frac{1}{2}}+y$$

$$\Rightarrow 2x\frac{dy}{dx}+y= x^{\frac{1}{2}}+x^{-\frac{1}{2}}+x^{\frac{1}{2}}-x^{-\frac{1}{2}} \qquad \because\ y=x^{\frac{1}{2}}-x^{-\frac{1}{2}}$$

$$\Rightarrow 2x\frac{dy}{dx}+y= 2x^{\frac{1}{2}} \quad \Rightarrow 2x\frac{dy}{dx}+y= 2\sqrt{x} \qquad \textit{Proved}$$

Question # 17

If $y = x^4+2x^2+2$, prove that $\frac{dy}{dx} = 4x\sqrt{y-1}$

Solution Since $y = x^4 + 2x^2 + 2$

Now $\dfrac{dy}{dx} = \dfrac{d}{dx}\left(x^4 + 2x^2 + 2\right)$

$$\Rightarrow \frac{dy}{dx} = 4x^{4-1} + 2\left(2x^{2-1}\right) + 0$$

$$= 4x^3 + 4x$$

$$\Rightarrow \frac{dy}{dx} = 4x\left(x^2 + 1\right) \ \ldots\ldots\ldots \text{(i)}$$

Now $y = x^4 + 2x^2 + 2$

$$\Rightarrow y - 1 = x^4 + 2x^2 + 2 - 1$$

$$= x^4 + 2x^2 + 1 = \left(x^2 + 1\right)^2$$

$$\Rightarrow \sqrt{y-1} = \left(x^2 + 1\right) \quad \text{i.e.} \quad \left(x^2 + 1\right) = \sqrt{y-1}$$

Using it in eq. (i), we have

$$\Rightarrow \frac{dy}{dx} = 4x\sqrt{y-1} \qquad \text{as required.}$$
