Electrostatics

• Charges at rest by the help of insulators.

• Atom can be charged by addition or removal of $e^{\ominus}$

Core of atom: Nucleus + Completely filled shell.

⇒ Electric Charge & Coulomb's law:

• From 600 BC to 1600 AD ancient Greeks discovered electric and magnetic minerals.

• Gilbert first time studied interactions.

* Empirical laws:- laws which are based on observations.

• Coulomb's law is according to Newton's third law.

* Position Vector:- locales the position of an object with respect to origin

* Unit Vector:- locales the position with respect to origin and its magnitude is 1

⇒ Vector & Scaler diffrence.

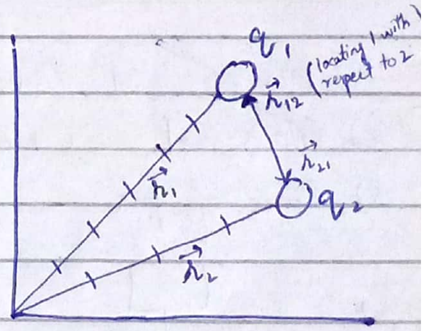
• Scaler contains

1) Magnitude 2) Unit

• While Vector contains

1) Magnitude 2) Direction

3) Unit 4) Should obey law of Vector → v.Imp.



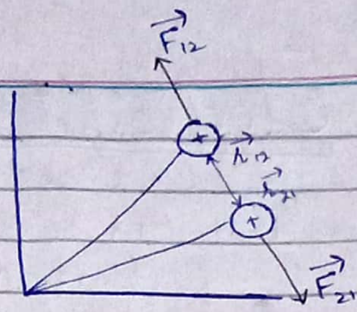
⇒ Current & rotation have directions but are not vectors.

⇒ Relative position vectors are equal in magnitude and opposite in direction

$$\vec{r}_{12} = -\vec{r}_{21}$$

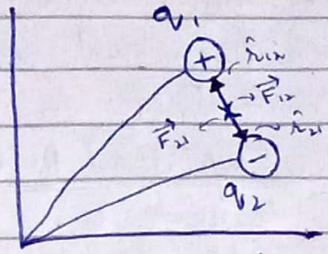
⇒ Electric Charge:-

- Mutual force
- "line of action of two forces is always same"



$\boxed{\vec{F}_{12} = (\quad) \hat{r}_{12}}$ → For Repulsion

$\boxed{\vec{F}_{12} = (\quad) \hat{r}_{21}}$ → For Attraction



Conclusion:-

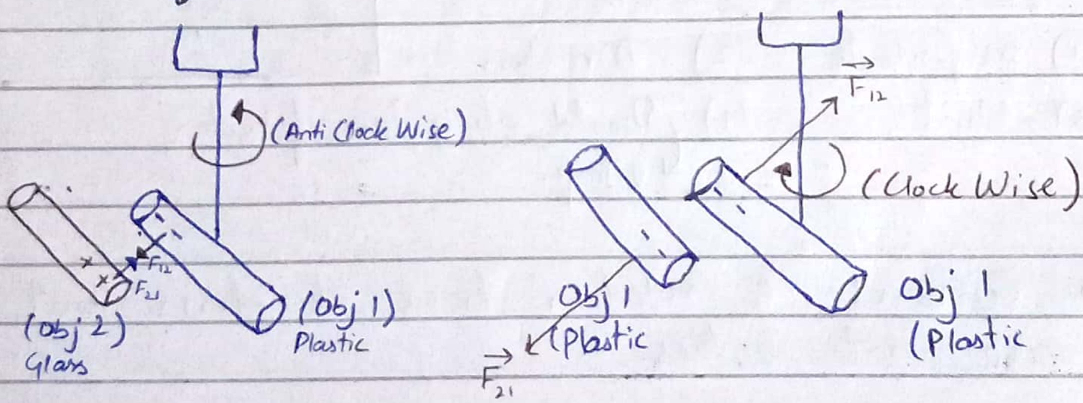
- like charges repels & Unlike charges attracts
- Electrostatic force of attraction obeys Newton's third law of motion.

Experiment:-

1) Plastic rod (-ve)
2) Glass rod (+ve)

} → When rubbed with fur.

3) Piece of fur



- ★ line of action of force also same here.
- charge on electron $e = 1.602 \times 10^{-19}$ C
- ★ Electron is not divisible.
- ★ Coulumb is a very large unit. i.e

$\boxed{6 \times 10^{18} e = 1C}$

→ **Quantization of Charge:** Electric charge on an object is always an integer multiple of elementary charge.

- elementary Charge = $1e = 1.6 \times 10^{-19}$ C.

$q = Ne \qquad q/e = N \qquad$ where $N = 0, \pm 1, \pm 2, \ldots, n$)

$\boxed{N = 0, 1, 2, 3,\ N \neq 1.1, 1.2 \ldots}$ [illegible] electron [illegible]

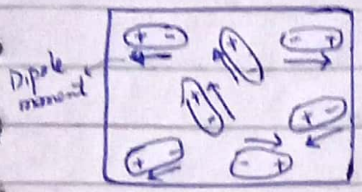
→ **Types of Materials:**

Insulators & conductors.

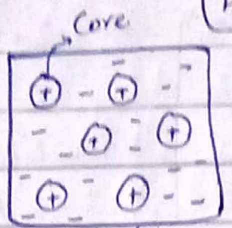
- There are three ways to charge a material.

1) Charging by friction.
2) Charging by contact
3) Charging by induction

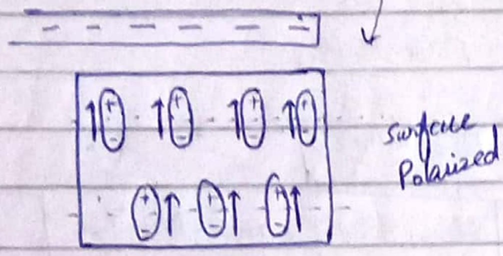
* conductor cannot be charged as charge cannot stay on them due to their charge carrying ability. However insulators like Plastic, rubber etc can easily be charged.



Insulators
(Alignment)



Conductors
(Relative movement



- In Contact → Charge gets balanced
- Induction → charge overall neutral (dipole)

Question:-

if
$A^+ \xleftrightarrow{att} B^-$
$B^- \xleftrightarrow{repel} C^-$
$A^+ \xleftrightarrow{?} C^-$ → Attract.

$A^+ \xleftrightarrow{attract} B^-$
$B^- \xleftrightarrow{attract} C^+$
$A^+ \xleftrightarrow{?} C^+$ → repel

$A^+ \xleftrightarrow{repel} B^+$
$B^+ \xleftrightarrow{repel} C^+$
$A^+ \xleftrightarrow{?} C^+$ → repel

Question:-

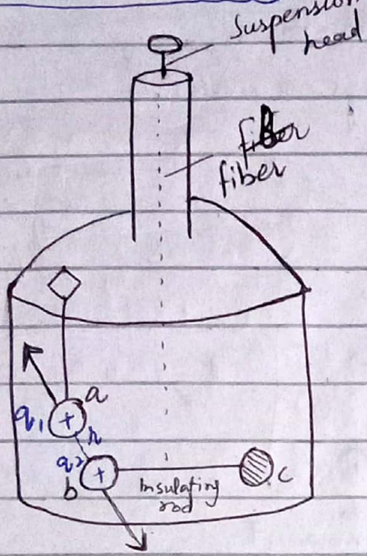
When you rub copper coin briskly with your hand it does not gets charged. why?

Frictional energy provided by hand to coin is not enough to charge copper.

⇒ Coulumb's law

In 1785, the first successful quantitative experiments were performed to study the electrostatic force by C.A Coulumb.

⇒ Torsion Balance



* Spheres a and b are charged
* C is uncharged.
* insulating rod is used so that charge does not travels to C.

Observations

- $F \propto q_1 q_2$
- $F \propto \frac{1}{r^2}$

$$F \propto \frac{q_1 q_2}{r^2}$$

This is the magnitude of coulumb's forces.

$$\boxed{F = \frac{K q_1 q_2}{r^2}}$$

$$\boxed{K = 8.99 \times 10^9 \; Nm^2/C^2}$$

$$K = \frac{1}{4\pi \varepsilon_0}$$

$\varepsilon_0 = 8.85 \times 10^{-12} \; C^2/Nm^2$

ε_0 = Permitivity of free space.

↳ shows the electric behaviour of light

μ_0 → shows the magnetic behaviour of light

$$\boxed{c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}} \rightarrow \text{speed of light}$$

* Coulumb's law in vector form

$$\vec{F}_{12} = \frac{k q_1 q_2}{r_{12}^2} (\hat{r}_{12})$$

for like charge
$\vec{F}_{12}$ same dir. $\hat{r}_{12}$.

$$\vec{F}_{21} = \frac{k q_1 q_2}{r_{12}^2} (\hat{r}_{21})$$

for like charges

$$\boxed{\hat{r}_{21} = -\hat{r}_{12}} \rightarrow \text{Always true}$$

$$\vec{F}_{12} = \frac{k q_1 q_2}{r_{12}^2} (-\hat{r}_{21})$$

$$\vec{F}_{12} = -\frac{k q_1 q_2}{r_{12}^2} (\hat{r}_{21})$$

$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

$$\vec{F}_{12} = \frac{k q_1 q_2}{r_{12}^3} \vec{r}_{12}$$

$F_{12} \propto \frac{1}{r^3}$ ✗

$$F_{12} = \frac{k q_1 q_2}{r_{12}^{\cancel{3}\,2}} \cancel{r_{12}} \hat{r}_{12}$$

$\boxed{F_{12} \propto \frac{1}{r^2}}$ ✓

Comparision b/w Gravitational & Electrostatic force.

Gravitational (Newton's law)	Electrostatic (Coulumb's law)
$F_g = \frac{G m_1 m_2}{r^2}$	$F = \frac{k q_1 q_2}{r^2}$
• $F \propto \frac{1}{r^2}$	• $F \propto \frac{1}{r^2}$
• Very Weak force	• Strong force
• long range.	• Short range.
• Can't be sheilded	• Can be shielded
• Only Attractive	• Attractive as well as repulsive
• Conservative force	• Conservative force.

$F_g = 3.6 \times 10^{-47}$ N

$\rightarrow F_e = 8.2 \times 10^{-8}$ N

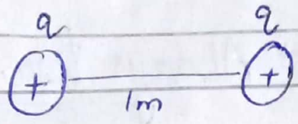
F_g is 10^{39} times weaker.

pg 574

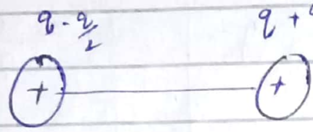
sample problem 25.2, 25.3, 25.4

$F = 1.69 \times 10^{16}$ N also 2×10^{12} Tons

- We cannot disturb the electrical neutrality of the objects very much.



$\boxed{F_o = kq^2}$ force b/w two identical & like charges



$F' = \frac{q}{2} \times \frac{3q}{2}$

$F' = \frac{q \times 3q}{2}$

$F = \frac{4q}{2}$

$F = 2q$

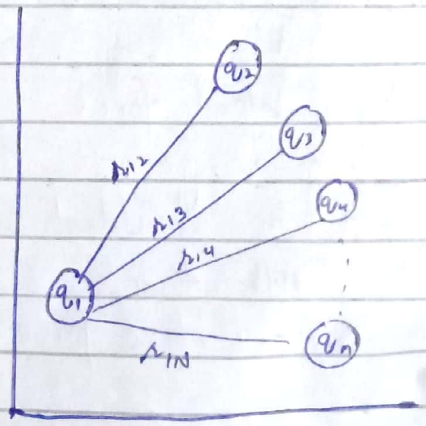
$\frac{q}{2} \times \frac{3q}{2}$

$F = \frac{3q}{4}$

$\boxed{F = \frac{3}{4} F}$

$\Rightarrow$ Superposition of forces $\rightarrow$ V.Imp

Consider N number of charges.



Statment

"Electrostatic force acting on one charge due to another charge is independent of weather or not other charges are present. so we calculate the forces separately and then take their vector sum to find out the resultant force"

$$\vec{F_{1}} = \vec{F_{12}} + \vec{F_{13}} + \vec{F_{14}} + \cdots\cdots \vec{F_{1N}}$$

$$\boxed{\vec{F_1} = \sum_{i=2}^{N} \vec{F_{1i}}}$$

Σ = discrete sum i.e $1+2+3+4\cdots\cdots$

integration $\int$ = continous sum

lect # 4

→ Superposition of force

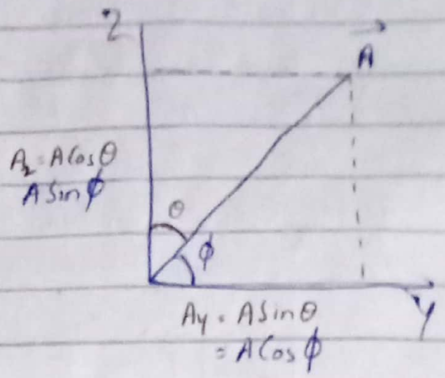
$$\vec{F_1} = \vec{F_{12}} + \vec{F_{13}} + \vec{F_{14}} + \cdots \vec{F_{1N}}$$

$$F_1 = \sum_{i=2}^{N} \vec{F_{1i}}$$

$$\vec{F_1} = \frac{kq_1q_2}{r_{12}^2}\hat{r}_{12} + \frac{kq_1q_3}{r_{13}^2}\hat{r}_{13} + \cdots\cdots \frac{kq_1q_n}{r_{1N}^2}\hat{r}_{1N}$$

$$\boxed{\vec{F_1} = K\sum_{i=2}^{N} \frac{q_1q_i}{r_{1i}^2}\hat{r}_{1i}}$$

Resolution of Vectors into its rectangular components



$\boxed{\begin{array}{c} \text{Base} \rightarrow \cos\theta \\ \theta \text{ [illegible]} \rightarrow \text{Base} \end{array}}$

∴ if $\theta + \phi = 90$ then

→ ~~A_y~~ $A\sin\theta = A\cos\phi$

→ $\underline{A\cos\phi}$

→ $A\cos\theta = A\sin\phi$.

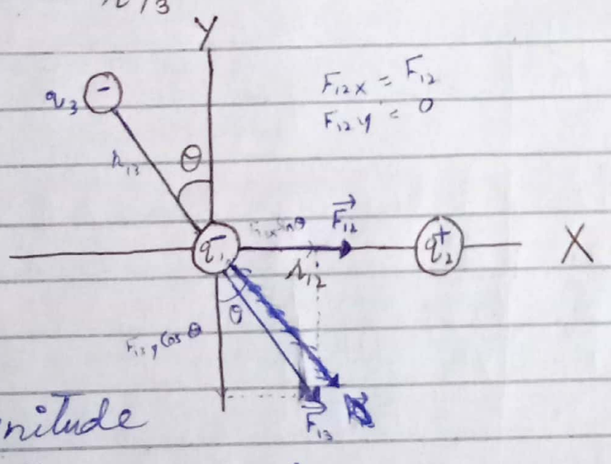
Prob 25.3

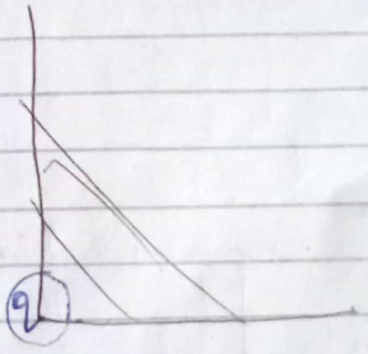
Statment:- figure show three charged particles held in place by forces not shown. What electrostatic force acts on q_1 due to other Two charges.

Data:- Take $q_1 = -1.2\mu C$, $q_2 = +3.7\mu C$

$q_3 = -2.3\mu C$, $r_{12} = 15\,cm$

$r_{13} = 10\,cm$, $\theta = 32°$





Magnitude

$$|F_{12}| = k\frac{q_1 q_2}{r_{12}^2}$$

$$= 9\times10^{9}(1.2\times10^{-6})(3.7\times10^{-6}C)$$

$|\vec{F}_{12}| = 1.77N$

$|\vec{F}_{13}| = 2.48N$

$\therefore F_{1x} = F_{12x} + F_{13x}$

$= F_{12} + F_{13}\sin\theta \Rightarrow \quad 3.08N$

$\therefore F_{1y} = F_{12y} + F_{13y}$

$= 0 - F_{13}\cos\theta \Rightarrow \quad -2.10N$

where -ve sign shows that the component of force is in -ve y direction

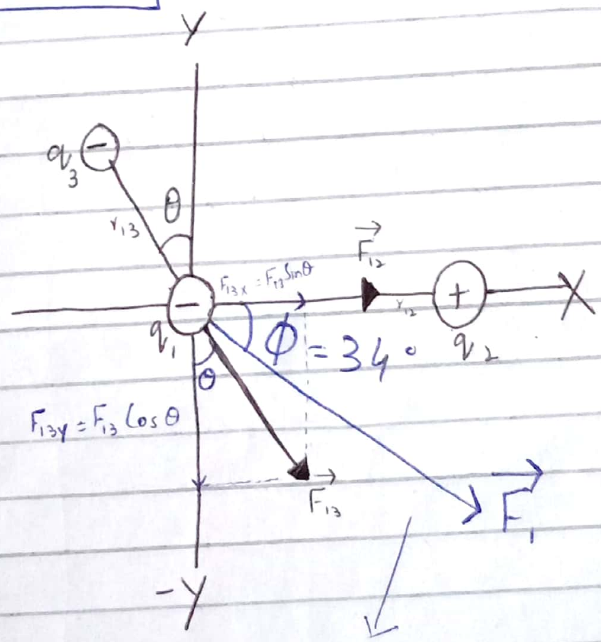
$$F_1 = \sqrt{F_{1x}^2 + F_{1y}^2}$$

$$= \sqrt{(3.08)^2 + (-2.10)^2}$$

$$\boxed{F_1 = 3.73N}$$

$$\phi = \tan^{-1}\frac{F_{1y}}{F_{1x}}$$

$$\boxed{\phi = -34^\circ}$$



This resultant is greater in magnitude than first ones.

(Exercises)
1 - 8

Pg 581, MCQs (1-5) 8, 10, 11, 13

Quantization ⟶ $Q = Ne$

⇒ Conservation of Charges

. "Net charge before a process is always equal to net charge after the process."

$$\Sigma q = \text{Constant}$$

$$q_i = q_f$$

Example

$$e^- + e^+ \longrightarrow \underbrace{\gamma + \gamma}_{\gamma\text{-rays} = \text{neutral}}$$

electron, positron

$-1.6 \times 10^{-19}\,C$ + $+1.6 \times 10^{-19}\,C$

net charge = 0   net charge = 0

Example

Neutron decay.

$$n \longrightarrow e^- + p^+ + \bar{\nu}$$

neutrino, neutral

neutral   +ve   -ve   neutral

net charge = 0   net charge = 0

Chapter # 2

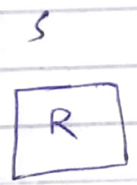
Electric field

→ What is a field?

A field is the propeity of individual particle wheather it influences or not

→ "Force is the property of system and field ~~field~~ is the property of individual particles."

"If we can associate a physical quantity at each point in a region "R" of space "S", then we can say that a field exists in that region



Scaler field:

e.g Temprature field

- **Static Scaler**
  - that does not changes with time
  - $T(x, y, z)$
- **Time vary Scaler**
  - that changes with time
  - e.g Temprature
  - $T(x, y, z, t)$

Vector field

Electric field, All force related fields.

- **Static Vector**
  - e.g Electric field around $\bar{e}$ is same
  - $\vec{V}(x, y, z)$
- **Time Vary Vector**
  - $\vec{V}(x, y, z, t)$
  - e.g Around moving charges

And this concept was wrong

⇒ Electric field

• "Concept of Action at a distance" is the principle on which coulumbic force is based

★ "Action at a distance means direct and instantaneous interactions without any mediator"

⇒ mediator ⟶ Carry information from one point to other e.g E.M Waves.

Electric Charge $\rightleftharpoons$ Electric Charge

No Mediator

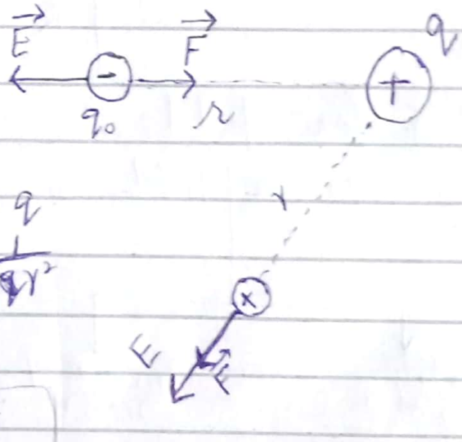
According to Coulomb. (Wrong Concept)

Electric Charge $\xrightarrow{E}$ Electric field $\xrightarrow{F}$ Electric charge

Electric Charge $\xleftarrow{F}$ Electric field $\xleftarrow{E}$ Electric charge

$F = \frac{kq_1q_2}{r^2}$

$E = \frac{F}{q_0} = \frac{kq}{r^2}$



$\boxed{E = \frac{kq}{r^2}}$ — (1)

$E \propto q$

$E \propto \frac{1}{r^2}$

$\boxed{E = \frac{F}{q_0}}$ — (2)

$\boxed{E = \frac{\cancel{2}F}{\cancel{2}q_0}}$

E does not depends on q_0 in any case (directly or inversly) as when q_0 is decreased or increased force will all ~~d~~ Increase or decrease correspondingly

★ Field is not dependent on force and q_0

$E \xrightarrow{\text{does not depend}} q_0$

$E \not\rightarrow F$

But $F \xrightarrow{\text{depends}} E$

$F \propto q q_0 \rightarrow$ force is the property of system

i.e depends on more than one particle

$E \propto q \rightarrow$ field is the not the property of system

↳ not q_0

$$\boxed{\vec{E} = \frac{kq}{r^2}\hat{r}} \rightarrow \text{vector form}$$

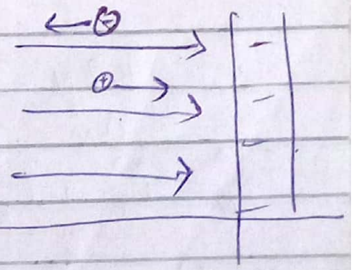
$$\boxed{\vec{E} = \lim_{q \to 0} \frac{\vec{F}}{q_0}}$$

min $e = 1.6 \times 10^{-19}\,C$

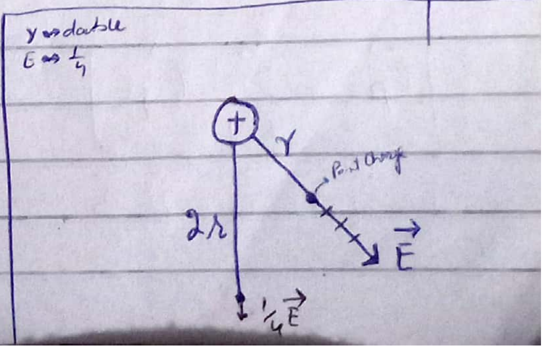
Test charge can be positive or negative. But it can be of very small value so that it cannot disturb the field of other charge.

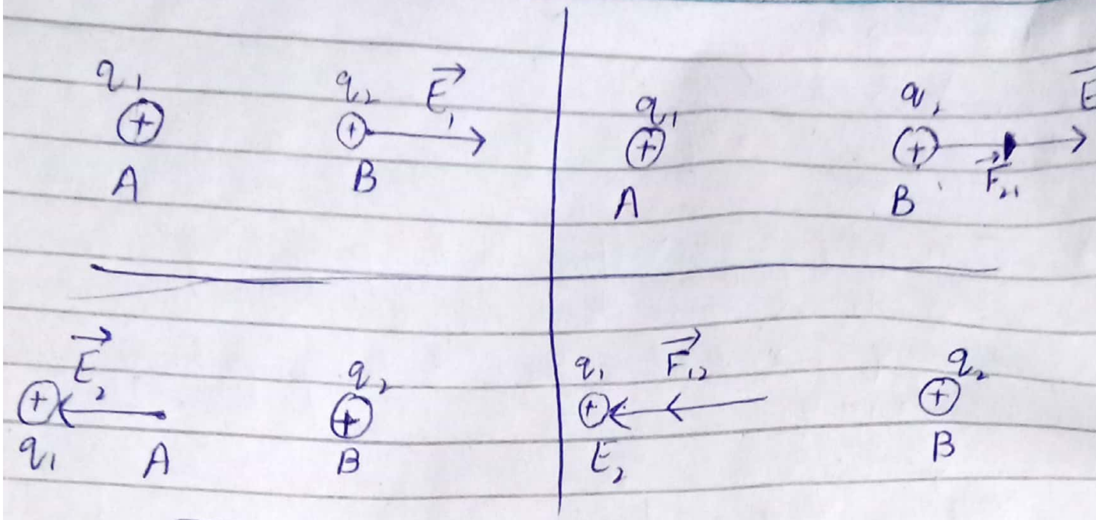
" if the charge is positive then the force is same to the direction of field

if the charge is -ve then the force is opposite to the direction of field



$$\boxed{\vec{F} = q_0 \vec{E}}$$





$$\boxed{\vec{F}_{21} = q_2 \vec{E}_1}$$ اوپر والی figure

$$\boxed{\vec{F}_{12} = q_1 \vec{E}_2}$$ نیچے والی figure

⇒ Electric field of point charges :-

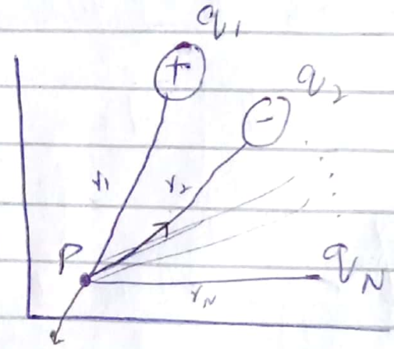
Principle of Superposition.

$$E_1 = \frac{kq_1}{r_1^2}$$

$$E_2 = \frac{kq_2}{r_2^2}$$

$$\vdots$$

$$E_N = \frac{kq_N}{r_N^2}$$



$$\vec{E}_P = \vec{E}_1 + \vec{E}_2 + \dots + \vec{E}_N$$

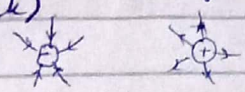
$$\boxed{\vec{E}_P = \sum_{i=1}^{N} \vec{E}_i}$$

$i = 1, 2, 3 \dots, N$

⇒ Electric field lines

i) The electric field lines start on +ve charge (Source) and end on −ve charge (Sink)

• Electric monopole exists i e



• Magnetic monopole does not exists i.e magnetic field lines never start or end.

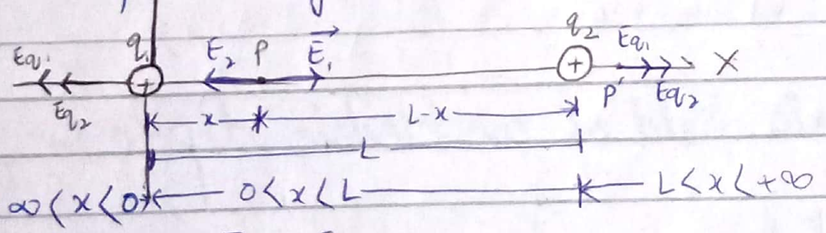
We can find the strength by calculating the number of field lines passing through unit area held perpendicular to field lines

• Calculate the no. of field lines passing through unit area held perpendicular to the field lines.
↳ max. Flux.

Sample Problem:-

26.3 figure shows a charge $q_1 = +1.5\mu C$ ($x=0$), $q_2 = +2.3\mu C$ ($x = L$)
$L = 13$ cm.

At what point P along the X-axis is the electric field zero.



$$E_1 = E_2$$

$$\frac{kq_1}{x^2} = \frac{kq_2}{(L-x)^2}$$

$$q_1(L-x)^2 = q_2 x^2$$

$$\sqrt{\frac{(L-x)^2}{x^2}} = \pm\sqrt{\frac{q_2}{q_1}}$$

$$\frac{L-x}{x} = \pm\sqrt{\frac{q_2}{q_1}}$$

$$L - x = \pm x\sqrt{q_2/q_1}$$

$$L = x \pm x\sqrt{\frac{q_2}{q_1}}$$

$$L = x\left(1 \pm \sqrt{\frac{q_2}{q_1}}\right)$$

$$x \Rightarrow \frac{L}{1 \pm \sqrt{\frac{q_2}{q_1}}}$$

Putting Values

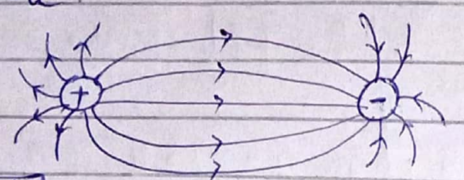
$x = +5.8$ cm

$$\boxed{x = -54.6\text{cm}}$$

* At the distance of -54.6 cm, we get a point on $-ve$ x-axis. Where the two fields are equal in magnitude but not opposite in direction. In same direction so they can't give the zero resultant.

The Electric Dipole:-

Two equal and opposite charges separated by a certain distance is called an electric dipole.



$d <<< x$

d → separation b/w poles

x → distance of field point from the center of the dipole

$\boxed{p = qd}$

↳ dipole moment
↳ Vector → direction from ($-ve$ to $+ve$)

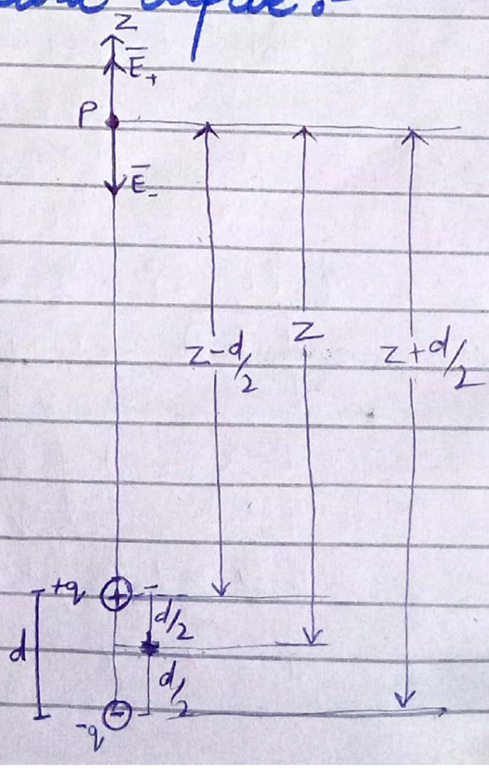
Electric field of an Electric dipole:-

$d <<< z$

$\vec{E} = \vec{E}_+ + \vec{E}_-$

$$\boxed{|\bar{E}_+| = \frac{kq}{(z-\frac{d}{2})^2}} \quad \text{--- (1)}$$

$$\boxed{E_- = \frac{kq}{(z+\frac{d}{2})^2}} \quad \text{--- (2)}$$



Put in (a)

$$E = \frac{kq}{(z-\frac{d}{2})^2} - \frac{kq}{(z+\frac{d}{2})^2}$$

where -ve sign shows that E_- is pointing in the -ve z direction.

$$E = kq\left(\frac{1}{(z-\frac{d}{2})^2} - \frac{1}{(z+\frac{d}{2})^2}\right)$$

$$= kq\left[\left(z-\frac{d}{2}\right)^{-2} - \left(z+\frac{d}{2}\right)^{-2}\right]$$

Binomial formula

$(1+x)^n$ n is negative

$$(1+n)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \ldots\ldots$$

$$E = kq\left[z^{-2}\left(1-\frac{d}{2z}\right)^{-2} - \left(1+\frac{d}{2z}\right)^{z^{-2}}\right]$$

$$E = kq\,z^{-2}\left[\left(1-\frac{d}{2z}\right)^{-2} - \left(1+\frac{d}{2z}\right)^{-2}\right]$$

$$= \frac{kq}{z^2}\left[\left(1+(-\cancel{2})\left(\frac{-d}{\cancel{2}z}\right)\right) - \left(1+(\cancel{2})\left(\frac{d}{\cancel{2}z}\right)\right)\right]$$

$$= \frac{kq}{z^2}\left[\left(1+\frac{d}{z}\right) - \left(1-\frac{d}{z}\right)\right]$$

$$= \frac{kq}{z^2}\left(\right.$$

$$= \frac{kq}{z^2}\left(\cancel{1}+\frac{d}{z} - \cancel{1} + \frac{d}{z}\right]$$

$$E = \frac{2kqd}{z^3}$$

$$E = \frac{2kp}{z^3}$$

$$\Rightarrow \boxed{E \propto \frac{1}{z^3}}$$

Scanned with CamScanner

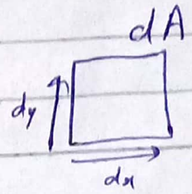
Continous Charge Distribution:-

"Coulumb's law is only valid for point charges This is the drawback of coulumb's law."

integration $\longrightarrow$ Sum of all the points

$dA \rightarrow$ is not d multiplied by A its only one term diffrential Area. i.e smallest part of a particular Area.

$\int dx = x \qquad \int dy = y \qquad \int dz = z \qquad \int dA = A$



e.g

$$\int_{2m}^{3.2m} A\,dL = AL\Big|_{2m}^{3.2m}$$

limit is always larger to smaller.

$$= A(3.2 - 2m).$$

Power Rule of Integration

$$\boxed{\int x^n dx = \frac{x^{n+1}}{n+1}}$$

Drawbacks

$\Rightarrow$ Coulombs law is only valid for insulators.

* Gauss's law is more apropriate & easy as compared to Coulombs law.

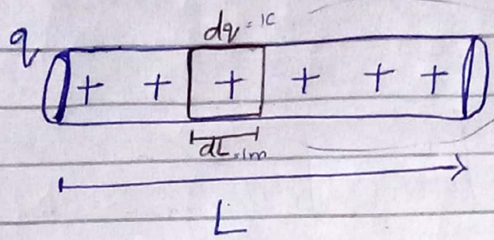
Charge density:-

1 ⇒ linear Charge Density:

Defined for 1D objects.

$$\lambda = \frac{\text{Charge}}{\text{Length}} = \frac{q}{L}$$

unit = Cm^{-1}



so ratio of $\frac{q}{L}$ is always 1

If their is uniform charge distribution then $\frac{dq}{dL} = \frac{q}{L}$ so

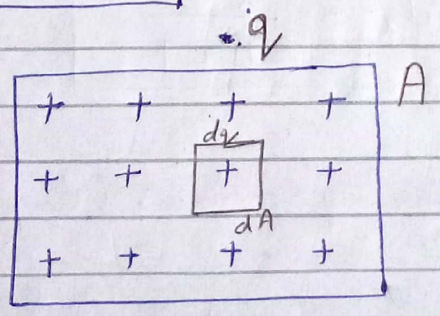
i.e ratio is always 1

2 ⇒ Surface Charge Density:-

Defined for 2D objects

$$\sigma = \frac{\text{Charge}}{\text{Area}} = \frac{q}{A}$$

$\boxed{\text{Unit} = Cm^{-2}}$



Here ratio also remains 1 if the distribution is uniform.

Volume Charge Density

Defined for 3D objects

$$\rho = \frac{q}{V} = \frac{dq}{dV}$$

$$\boxed{\text{Unit} = Cm^{-3}}$$

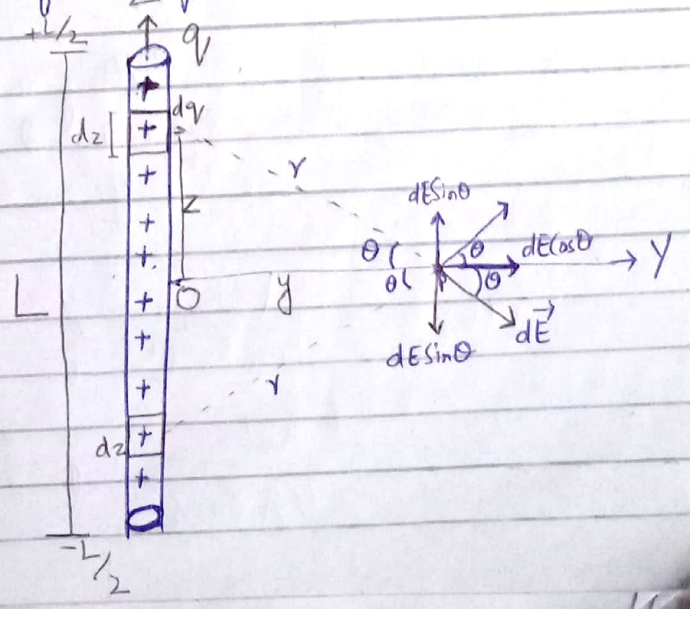
A Uniform line Of charge:-

Example of uniform charge distribution

$$\boxed{E = \frac{q}{4\pi\varepsilon_0 r^2}} \longrightarrow \text{Electric field of a point charge}$$

$$\boxed{dE = \frac{dq}{4\pi\varepsilon_0 r^2}} \longrightarrow (1)$$

means part of a charge creates particular (smaller) field.



* For each charge element on +ve Z-side there is a corresponding charge element on the -ve side. They establish their electric fields in such a way that

$$\int dE_z = 0$$

$$E_z = 0$$

$$E_x = 0$$

$$\boxed{dE_y = dE\cos\theta} \quad (2)$$

$$\boxed{\lambda = \frac{q}{L} = \frac{dq}{dz}} \quad (3)$$

Eq 1 becomes

$$\boxed{dE = \frac{\lambda dz}{4\pi\varepsilon_0 r^2}} \quad (4)$$

From figure:- $\boxed{\cos\theta = \frac{y}{r}}$ (5)

by pythagorus theorm

$$r^2 = y^2 + z^2$$

$\cancel{\frac{\lambda dz}{4\pi\varepsilon_0 r^2} \times \frac{y}{r}}$

$\cancel{\frac{\lambda dz \times y}{4\pi\varepsilon_0 r^3}}$

Putting value of dE from eq 4 in 2 and also putting value of cos θ from eq 5 then

$$dE_y = \frac{\lambda dz}{4\pi\varepsilon_0 r^2} \times \frac{y}{r}$$

$$= \frac{\lambda dz}{4\pi\varepsilon_0 r^3} \times y$$

for r^3

$$\sqrt{r^2} = \sqrt{y^2 + z^2}$$

$$r = (y^2 + z^2)^{\frac{1}{2}}$$

~~p~~ taking cube root

$$\boxed{r^3 = (y^2 + z^2)^{3/2}}$$

$$\boxed{dE_y = \frac{\lambda dz}{4\pi \varepsilon_0 (y^2+z^2)^{3/2}} \times y}$$

$$\boxed{dE_y = \frac{\lambda y}{4\pi \varepsilon_0} \frac{dz}{(y^2+z^2)^{3/2}}}$$

z is variable and y is constant

By Applying integration on variables

$$\boxed{\int dE_y = \frac{\lambda y}{4\pi \varepsilon_0} \int_{-L/2}^{+L/2} \frac{dz}{(y^2+z^2)^{3/2}}}$$

After integrating we get

$$E_y = \frac{\lambda \cancel{y}}{4\pi \varepsilon_0} \frac{z}{y^{\cancel{2}} \sqrt{y^2+z^2}} \Bigg|_{-L/2}^{+L/2}$$

$$E_y = \frac{\lambda}{4\pi \varepsilon_0 y} \frac{z}{\sqrt{y^2+z^2}} \Bigg|_{-L/2}^{+L/2}$$

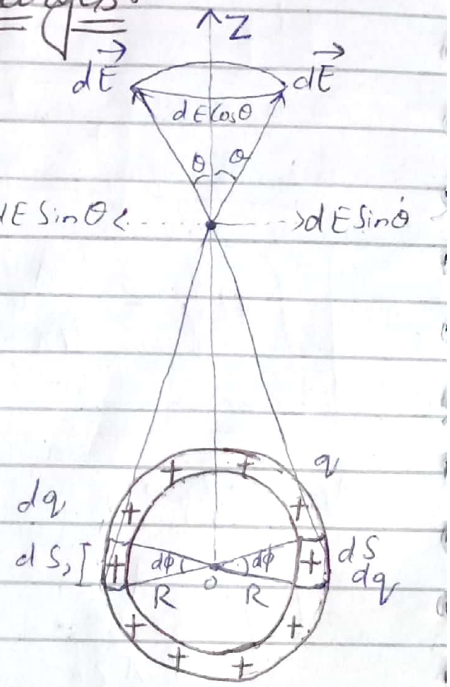
$$E_y = \frac{\lambda}{4\pi \varepsilon_0 y} \left[\frac{L/2}{\sqrt{y^2 + \left(\frac{L}{2}\right)^2}} - \frac{-L/2}{\sqrt{y^2 + \left(-\frac{L}{2}\right)^2}} \right]$$

$$\boxed{E_y = \frac{\lambda}{4\pi\varepsilon_0 y}\left[\frac{L}{\sqrt{y^2+\left(\frac{L}{2}\right)^2}}\right]}$$

$$\vec{E} = \cancel{\vec{E_x}\hat{i}} + E_y\hat{j} + \cancel{E_z\hat{k}}$$

$$\boxed{\vec{E} = \left[\frac{\lambda}{4\pi\varepsilon_0 y}\left[\frac{L}{\sqrt{y^2+\left(\frac{L}{2}\right)^2}}\right]\right]\hat{j}}$$

⇒ A Uniform ring of Charges:-



$$\boxed{dE = \frac{dq}{4\pi\varepsilon_0 r^2}} \text{ — } (1)$$

$$\boxed{\lambda = \text{density} = \frac{q}{2\pi r}} \text{ — } 2(A)$$

↳ For Whole ring

ϕ is variable that is varying from 0° to 360°

z, r, R are all constant

z is distance of point P from origin

$$\boxed{\lambda = \frac{dq}{ds}} \text{ — } 2(B)$$

$$\boxed{dS = R\,d\phi}$$

Substituting Value

$$\boxed{\lambda = \frac{dq}{R\,d\phi}} \quad (3)$$

$$\boxed{dq = \lambda R\,d\phi} \quad (4)$$

Eq 1 becomes

$$\boxed{dE = \frac{\lambda R}{4\pi \varepsilon_0 r^2}\, d\phi} \quad (5)$$

For each ~~side~~ charge element on one side of xy plane there is a corresponding charge element on other side of xy plane that establish the electric field in such a way that.

$$\int dE_x = E_x = 0$$

$$\int dE_y = E_y = 0$$

$$\boxed{\int dE_z = dE\cos\theta} \quad (6)$$

From figure

$$\boxed{\cos\theta = \frac{z}{r}} \quad (7)$$

$$r^2 = z^2 + R^2$$

Putting values from Eq 5 & 7 to eq 6

$$\int dE_z = \frac{\lambda R}{4\pi \varepsilon_0 r^2}\, d\phi\, \frac{z}{r}$$

$$dE_z = \frac{\lambda R z}{4\pi \varepsilon_0 r^3}\, d\phi$$

Scanned with CamScanner

$$r^2 = z^2 + R^2$$
$$\sqrt{r^2} = (z^2+R^2)^{1/2}$$
$$r^3 = (z^2+R^2)^{3/2}$$

$$dE_z = \frac{\lambda R z}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}}\, d\phi$$

$$\int dE_z = \frac{\lambda R z}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}} \int d\phi$$

$$\boxed{\int dE_z = \frac{\lambda R z}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}} \left[\phi\right]_0^{2\pi}}$$

$$\therefore \int dE_z = E_z$$

$$\boxed{E_z = \frac{\lambda R z}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}} \int_0^{2\pi} \phi}$$

$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$
$$= 0 + 0 + \left[\frac{\lambda R z}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}}\right] \int_0^{2\pi} \phi$$

$$\boxed{\vec{E} = \left[\frac{\lambda R z}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}} \int_0^{2\pi} \phi\right] \hat{k}}$$

$$\bar{E}_z = \frac{\lambda R z}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}} (2\pi - 0)$$

$$\therefore \int_0^{2\pi} \phi = 2\pi - 0$$

$$E_z = \frac{\lambda R z}{\cancel{4\pi}_{\,2\pi}\, \varepsilon_0 (z^2+R^2)^{3/2}} \cancel{2\pi}$$

$$\boxed{E_z = \frac{\lambda R z}{2\pi\varepsilon_0 (z^2+R^2)^{3/2}}}$$

Scanned with CamScanner

$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

$$0 + 0 + E_z$$

$$\boxed{\vec{E} = \left[\frac{\lambda R z}{2\pi\varepsilon_0 (z^2+R^2)^{3/2}}\right]\hat{k}}$$

A Uniform disk of Charges:-

for ring

$$\vec{E} = \left[\frac{\overbrace{\lambda R z\, 2\pi}^{dq\, z}}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}}\right]$$

as $\lambda = \dfrac{q}{2\pi R}$

$$\boxed{q = 2\pi R\lambda}$$

A disk

An Infinite sheet of Charge.

if we take $R \rightarrow$ apraches to infinity then disc becomes infinite sheet of charge.

$\cancel{E = \frac{\sigma}{2n}}$ $\quad E = \dfrac{\sigma}{2\varepsilon_0}$

Chapter 3

Gauss's law

The flux of an Electric field:-

flux:-

Number of field lines passing through the area (which is vector Area) is called the flux.

• Dependence

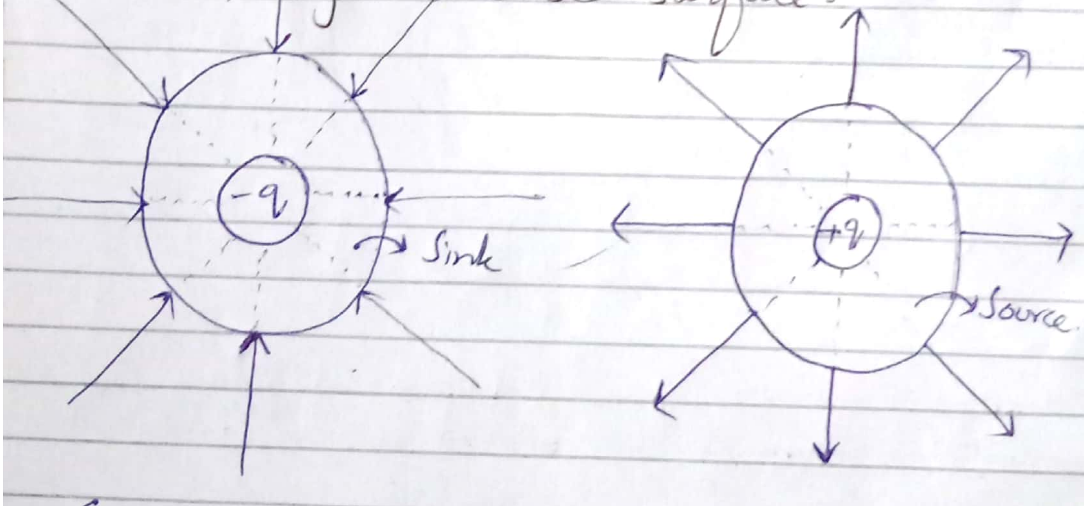
i) Magnitude of flux.

ii) Strength of Electric field

iii) Orientation of Area, inside the field (θ)

$$\phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta$$

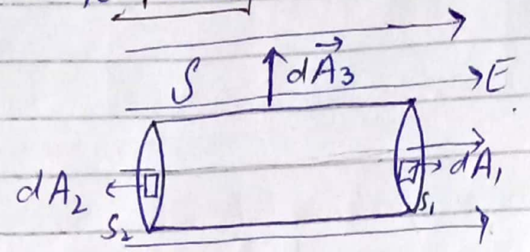
⇒ Flux through a closed surface:-



flux → -ve → -6 Units

flux → +ve → +6 Units

* $\boxed{\text{Flux}}$ through a $\boxed{\text{closed surface}}$ placed in a uniform electric field is always equal to $\boxed{\text{zero}}$



$$\phi_s = \oint_s \vec{E}\cdot d\vec{A} = ?$$

$$\oint_s \vec{E}\cdot d\vec{A} = \int_{s_1} \vec{E}\cdot dA_1 + \int_{s_2} \vec{E}\cdot d\vec{A}_2 + \int_{s_3} \vec{E}\cdot d\vec{A}_3$$

$$= \int_{s_1} E\,dA\cos 0^\circ + \int_{s_2} E\,dA_2\cos 180^\circ + \int_{s_3} E\,dA_3 \cos 90^\circ$$

$\because (\cos 180 = -1)$

$$= E\int_{s_1} dA_1 - E\int_{s_2} dA_2 + 0$$

$$= EA - EA$$

$$= 0$$

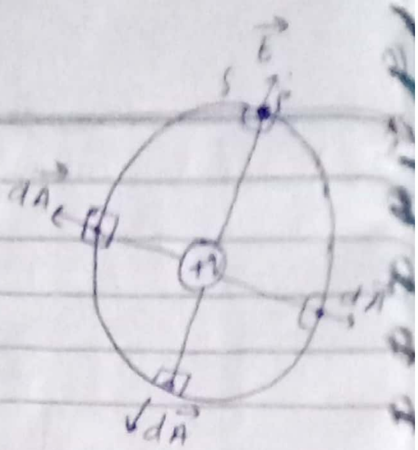
$\because \int_{s_1} d\vec{A}_1 + \int_{2} dA = A = \pi r^2$

Gauss' law:-

Flux through a closed surface is equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the closed surface (Gausian Surface)

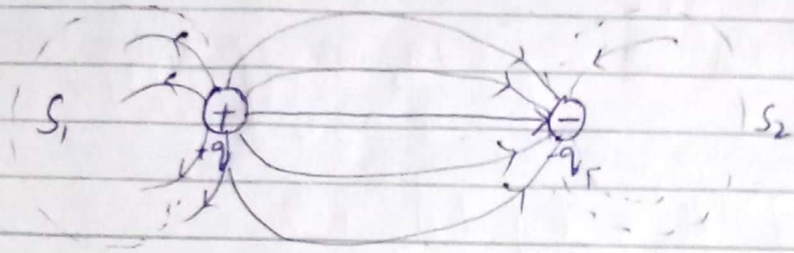
$$\oint_s \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

Proof:-



$$\oint_s \vec{E}\cdot d\vec{A} = \oint E\, dA \cos 0^\circ$$
$$= E \oint dA$$
$$= \left(\frac{q}{4\pi\varepsilon_0 r^2}\right)(4\pi r^2)$$
$$= \frac{q}{\varepsilon_0}$$

* Gauss's law is more fundamental more basic, and more appliable as compared to coulumb's law due to its higher symmetry.
* "Gauss's law is valid for insulators as well as for conductors."



$$\phi_1 = \frac{+q}{\varepsilon_0}$$

$$\phi_2 = \frac{-q}{\varepsilon_0}$$

(When you have to deal with scalar quantity you have to deal with sign of charge)

$$\phi_3 = \frac{+q-q}{\varepsilon_0} = 0 \quad \text{(No net Charge)}.$$

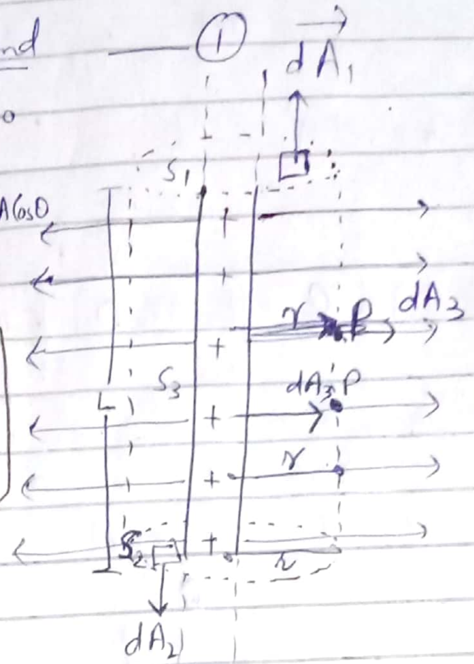
$$\phi_3 = \text{(No charge) } 0$$

Applications of Gauss's law:-

λ = linear charge density

i) An Infinite line of charges:

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_0} \quad \text{—— (1)}$$



$$\cancel{\int_{S1} E\,dA_1 \cos 90^\circ} + \cancel{\int_{S2} E\,dA_2 \cos 90^\circ} + \int_{S3} E\,dA \cos 0$$

$$= \frac{q}{\varepsilon_0}$$

$$E \int_{S3} dA_3 = \frac{\lambda L}{\varepsilon_0}$$

$$\boxed{\lambda = \frac{q}{L} \quad q = \lambda L}$$

Area of cylinder ~~spheres~~ $2\pi r l$ + height

$$E\,(2\pi r \cancel{L}) = \frac{\lambda \cancel{L}}{\varepsilon_0}$$

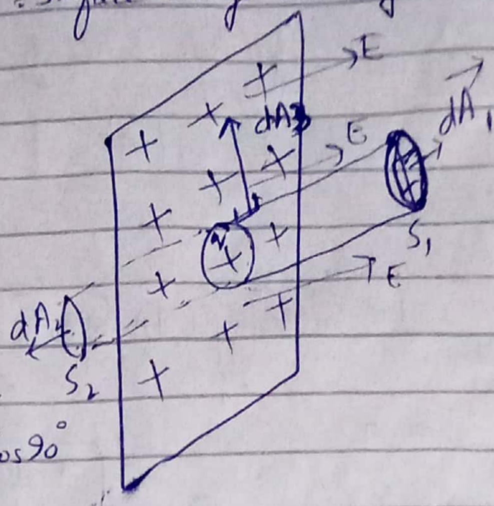
$$E = \frac{\lambda}{2\pi \varepsilon_0 r}$$

$$\boxed{\because \text{Area of cylinder} = 2\pi r h + 2\pi r^2}$$

$$\boxed{\because \text{Area of Circle} = \pi r^2}$$

2) An infinite sheet of Charges.

σ = Surface charge density



$$\sigma = \frac{q}{A}$$

$$q = \sigma A$$

$$\oint_3 E \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

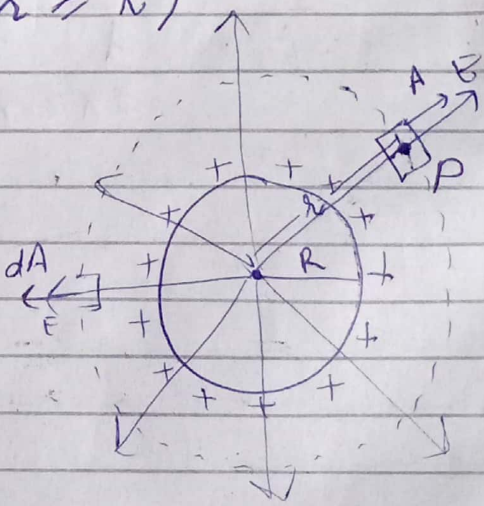
$$\int_{S1} E\,dA_1 \cos 0^\circ + \int_{S2} E\,dA_2 \cos 0^\circ + \cancel{\int_{S3} E\,dA_3 \cos 90^\circ}^{\,0} = \frac{q}{\varepsilon_0}$$

$$2E\cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

3) A Spherical Shell of Charge:-

For external points:- $(r > R)$



$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

$$\oint_s E\,dA \cos 0^\circ = \frac{q}{\varepsilon_0}$$

$$E \oint_s dA = \frac{q}{\varepsilon_0}$$

$$E(4\pi r^2) = q/\varepsilon_0$$

$$\boxed{E = \frac{q}{4\pi\varepsilon_0 r^2}}$$

$(r \geq R)$

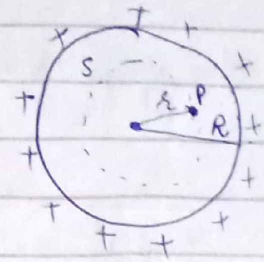
$E \propto \frac{1}{r^2}$

~~Shell's law~~ Spherical Shell's law

For external points a spherical shell behaves like a point charge.

For internal Points ($r < R$)

$$\oint_s \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$



$$\oint_s E\,dA \cos\theta = 0$$

No charge enclosed in Gaussian Surface.

So

$$\oint_s dA \neq 0$$

$$\cos\theta \neq 0$$

So

$$\Rightarrow \vec{E} = 0$$

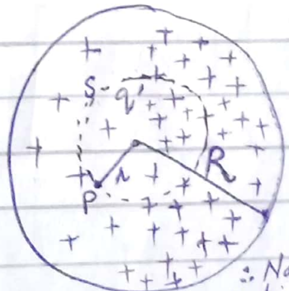
No force is exerted on a Test charge placed inside a shell i.e as their is no ~~E~~ field inside it.

4) Spherically Symmetrical Charge distribution.

- For internal Points :-

$\rho = \frac{q}{V}$ ρ = Volume charge density

"Spherical Symmetry means the density is same when we move spherically and otherwise not"



∴ Not a circle but its a sphere.

★ q' → charge in Gausian Surface

↳ "Is ka Matlab Ye ha k Agar Wese dekha jai to charge ki distribution unequal ha. lekin jb hm gausian surface assume krein ge to us main kisi side charge kam ho ga or doosri side zyada lekin overall charge ki distribution uniform ho jai gi.

From Application No 3.

$$\boxed{E = \frac{q'}{4\pi\varepsilon_0 r^2}} \quad (2)$$

Eq 2 showing external behaviour and only covering gaussian surface.

∴ As Volume charge density (ρ) overall is symmetrical, So.

$$\rho = \rho'$$

$$\frac{q}{\cancel{\frac{4}{3}}\pi R^3} = \frac{q'}{\cancel{\frac{4}{3}}\pi r^3}$$

$\boxed{\because \text{Volume of Sphere} = \frac{4}{3}\pi r^3}$

ρ = Volume charge density of whole sphere

ρ' = Volume charge density of Gaussian Surface

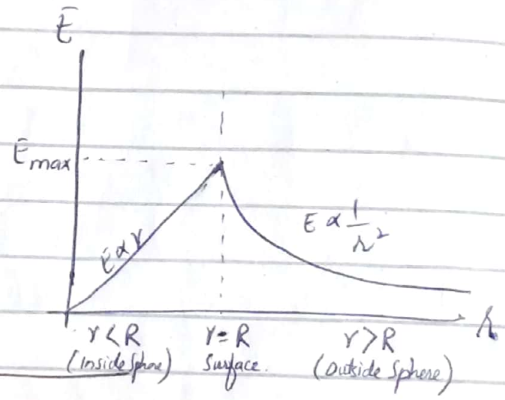
$$\boxed{q' = q\frac{r^3}{R^3}} \quad (3)$$

Putting the value of q' of eq 3 in Eq 2, So eq becomes.

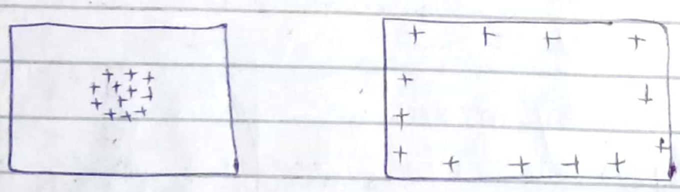
$$\boxed{E = \frac{q r}{4\pi\varepsilon_0 R^3}} \qquad r < R$$

$$E \propto r$$

∴ Same calculations for external points as done in previous page



Gauss's law & Conductors :-



"An excess charge placed on an isolated conductor moves entirely to the outer surface of the conductor."

mcqs → pg 626 (1-7)
Assignment #3 628 (Ex Q 1 to 5)

Chapter 4:- The Electric Potential Energy & Electric Potential.

- F & U → Property of system
- E & V → Property of individual charge.

- F & E differs due to a "q"
- U & V differs due to "r".

⇒ The Electric Potential Energy:- (Scaler)

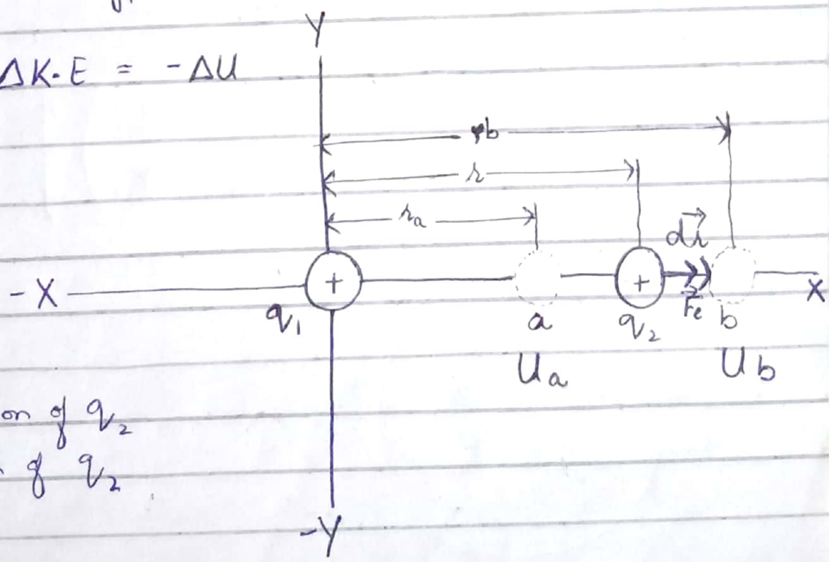
$$E = K.E + U$$

- Total Energy is the combination of K.E & P.E.

$$\Delta E = \Delta K.E + \Delta U$$

Change in total Energy

$$\Delta K.E = -\Delta U$$



a = initial position of q_2
b = Also position of q_2

$$\Delta U = -W_{ab}$$

The change in energy is corresponding to Work done

$$W = \vec{F} \cdot \vec{r}$$

$$= -\int_{r_a}^{r_b} F_e \, dr \cos 0^\circ$$

$$= -kq_1q_2 \int_{r_a}^{r_b} \frac{\cancel{dr}\, dr}{r^2}$$

through Power rule.

$$= -kq_1q_2 \int_{r_a}^{r_b} r^{-2} \, dr$$

$$= -kq_1q_2 \left(\frac{r^{-2+1}}{-2+1} \right) \Bigg|_{r_a}^{r_b}$$

$$= -kq_1q_2 \left(\frac{r^{-1}}{-1} \right) \Bigg|_{r_a}^{r_b}$$

$$= kq_1q_2 \left(\frac{1}{r} \right) \Bigg|_{r_a}^{r_b}$$

$$\boxed{\Delta U = kq_1q_2 \left(\frac{1}{r_b} - \frac{1}{r_a} \right)}$$

As the like charges are moving away so energy must decrease.

$\cancel{r_b > r_a}$ $r_b > r_a$

So $\cancel{\frac{1}{r_b}}$ So $\frac{1}{r_b} < \frac{1}{r_a}$

So in this case $\Delta U \rightarrow -ve$ i.e energy of system decreases.

⇒ Absolute Potential Energy:-

$$U_b - U_a = kq_1q_2\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

At ∞, $r_a \rightarrow \infty \Rightarrow U_a = 0$

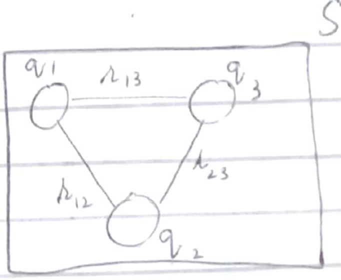
$$U_b - 0 = kq_1q_2\left(\frac{1}{r_b} - \frac{1}{\infty}\right)$$

$$U_b = \frac{kq_1q_2}{r_b}$$

$$\boxed{U = \frac{kq_1q_2}{r}}$$

Energy cannot be of single particles.

Collection of Point Charges:-



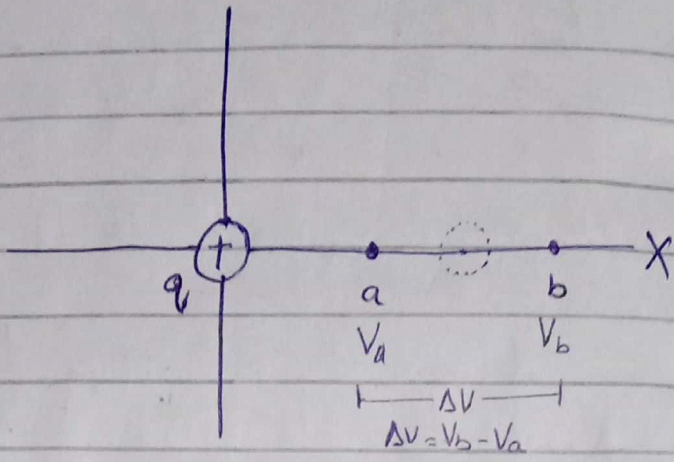
$$U = 0 + 0 + \frac{kq_1q_2}{r_{12}} + \frac{kq_2q_3}{r_{23}} + \frac{kq_1q_3}{r_{13}}$$

⇒ The Electric ~~field~~ Potential

U is the property of system

$$U = \frac{k q q_0}{r}$$

$$\frac{U}{q_0} = \frac{kq}{r}$$



So $\frac{U}{q_0} = V = \frac{kq}{r}$

$\boxed{\text{Derivation of } V = \frac{kq}{r}}$

$$\Delta U = k q q_0 \left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

$$\frac{\Delta U}{q_0} = kq\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

$$\Delta V = kq\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

At ∞ (for Absolute), $V_a = 0V$

$$\cancel{\Delta V} =$$

$$V_b - V_a = kq\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

$$V_b = kq\left(\frac{1}{r_b}\right)$$

$\boxed{V_b = \frac{kq}{r_b}}$ also $\boxed{V = \frac{kq}{r}}$

$$Volt = \frac{Joule}{Coulumb}.$$

$$J = VC$$

$$\Delta U = q_0 \Delta V$$

$$1eV = e\,\Delta V$$

$$1eV = (1.6 \times 10^{-19} C)(1V)$$

$$\boxed{1eV = 1.6 \times 10^{-19} J}$$

eV is the smaller unit of energy, So we mostly use eV instead of J.

* V is also a single particle property.
  - Field = Vector
  - V = Scaler

* Potential due to Point Charges :-

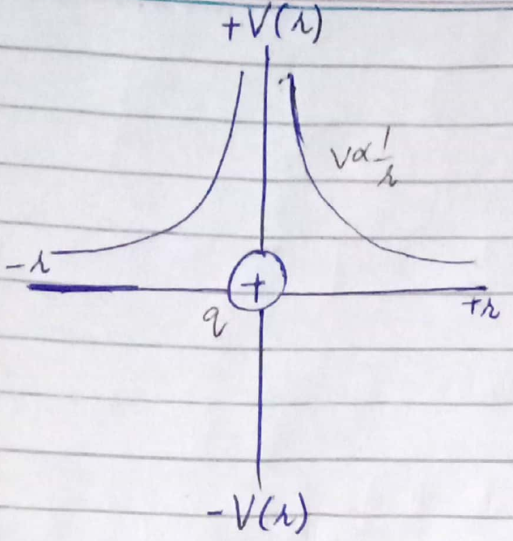
As potential is a single particle property. So if we have to find the potential for multiple charges we have to do superposition. But this superposition is of scaler type.

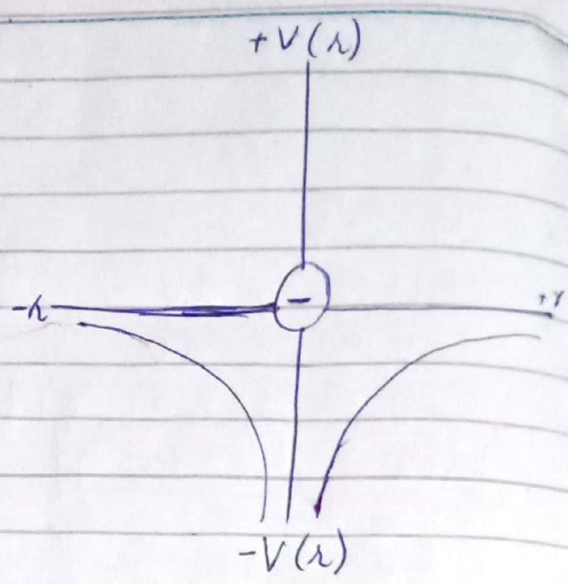
"A positive charge establishes a positive potential in its surroundings and a negative charge establishes -ve potential in its surroundings"

∴ "V is maximum on the surface"
& V is ~~zero~~ infine inside the charge. as $r = 0$ inside the circle

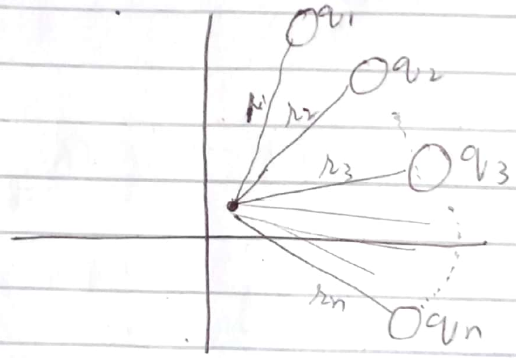
$$V = \frac{kq}{r}$$

$$V \propto \frac{1}{r}$$





⟹ **For "N" number of Charges.** i.e (Superposition)



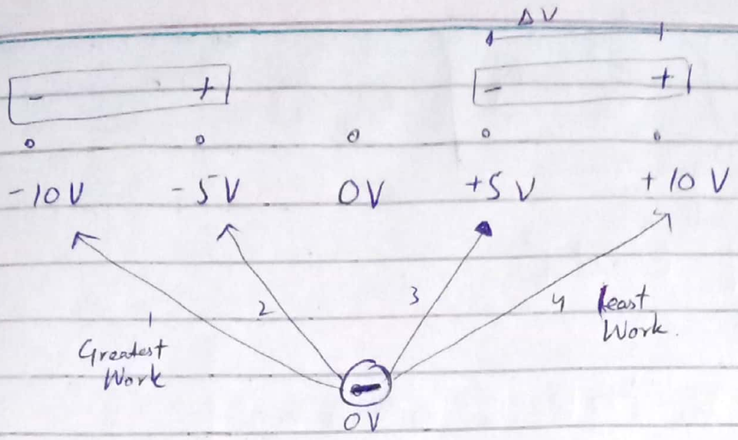
$V_1 = \frac{kq_1}{r_1}$, $V_2 = \frac{kq_2}{r_2}$, $V_3 = \frac{kq_2}{r_2}$ ....

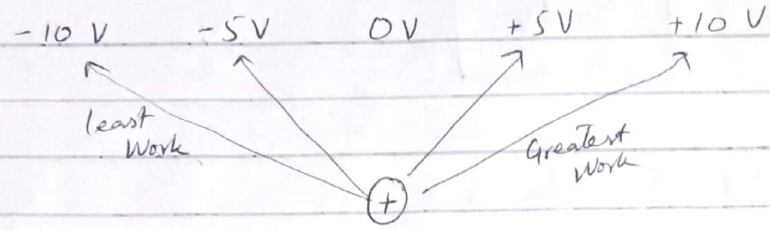
$$V_P = V_1 + V_2 + \ldots\ldots V_n$$

Here no +ve or -ve signs of vector. Only +ve or -ve signs are due to the kind of Charge

$$V_P = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \ldots\ldots \frac{kq_n}{r_n}$$

$$V_N = k \sum_{i=1}^{N} \frac{q_i}{r_i}$$





	Vector Description	Scaler Description
Interaction b/w charges (System's Property)	$\vec{F}\left(\frac{kq_1q_2}{r^2}\right)$	$U\left(\frac{kq_1q_2}{r}\right)$
Effect of one charge of number of charges at a point (Single Particle Property)	$\vec{E}\left(\frac{kq}{r^2}\right)$	$V\left(\frac{kq}{r}\right)$

These rows are ~~rel~~ diffrent due to q_0

⇒ Calculating Potential from field:- (V & E relation)

$$\Delta V = \frac{\Delta U}{q_0}$$

$$\Delta V = -\frac{W_{ab}}{q_0}$$ ⊖ due to work done by system

$$= -\int_a^b \vec{F}\cdot d\vec{r}$$

$$\Delta V = \frac{-\int_a^b \vec{F}\cdot d\vec{r}}{q_0}$$

$$\boxed{\Delta V = -\int_a^b \vec{E}\cdot d\vec{r}}$$ Eq (1)

$$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{r}$$

At infinity $V_a = 0V$, $a \longrightarrow \infty$

$$\boxed{V_b = -\int_\infty^b \vec{E}\cdot d\vec{r}}$$

For U & E relation
Multiply eq (1) by q

$$q \times \Delta V = -q\int_a^b \vec{E}\cdot dr$$

$$\boxed{\Delta U = -q\int_a^b \vec{E}\, dr}$$

This equation will ultimately lead to $V = \frac{kq}{r}$

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⇒ Equipotential Surfaces :-

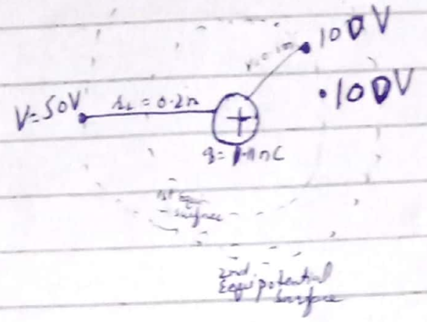
A surface at which potential is same at all the points is called equipotential surface.

$q = 1.11\,nC$

$r = 0.1\,m$

$V = \dfrac{kq}{r}$

$\boxed{V = 100V}$



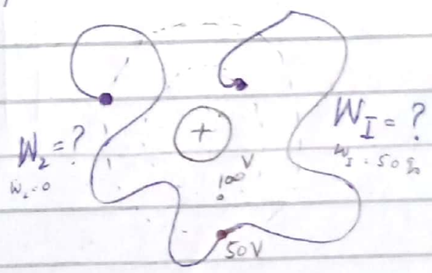
if r gets double i.e $r = 0.2m$
then
V becomes 50V

$$\Delta U = q_0 \Delta V$$

$$-W_{ab} = q_0 \Delta V$$

$$\boxed{W_{ab} = -q_0 \Delta V} = -q(V_b - V_a)$$

Work done will be dependent ~~of~~ on Potential difference. ΔV is required for work done.



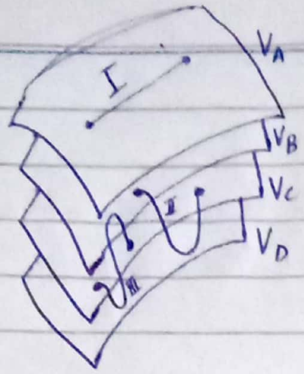
$$W_I = -q_0(50V - 100V)$$

$$\boxed{W_I = 50q_0}$$

+ve and -ve work depends on sign of charge

$$W_{II} = -q_0(50 - 50V)$$

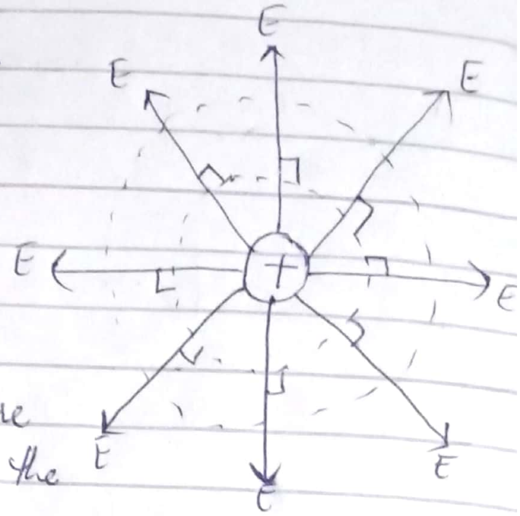
$$\boxed{W_{II} = 0}$$



for I $\longrightarrow$ $V_A - V_A = 0$

for II $\longrightarrow$ $V_C - V_B$

for III $\longrightarrow$ $V_C - V_C = 0$



Property :-

★ Electric field lines are always perpendicular to the equipotential surface