Section 3

Electromagnetism

* The study of magnetic fields and magnetic interactions due to moving charges.

Chapter

Magnetic field of a current.

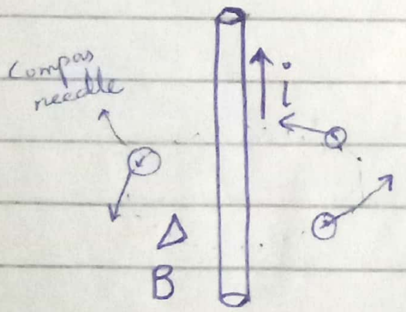
2nd right hand rule → b/w $\vec{B} \longleftrightarrow \vec{i}$

↓ Current & Magnetic field

→ b/w two quantities.

→ Thumb ← Straight } → Right hand

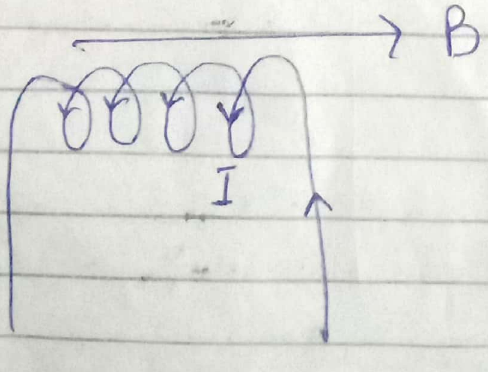
Fingers ← curl



→ Here Current → Straight

↓ So thumb is used

While B → fingers (Curly)



I → Curly

B → Straight

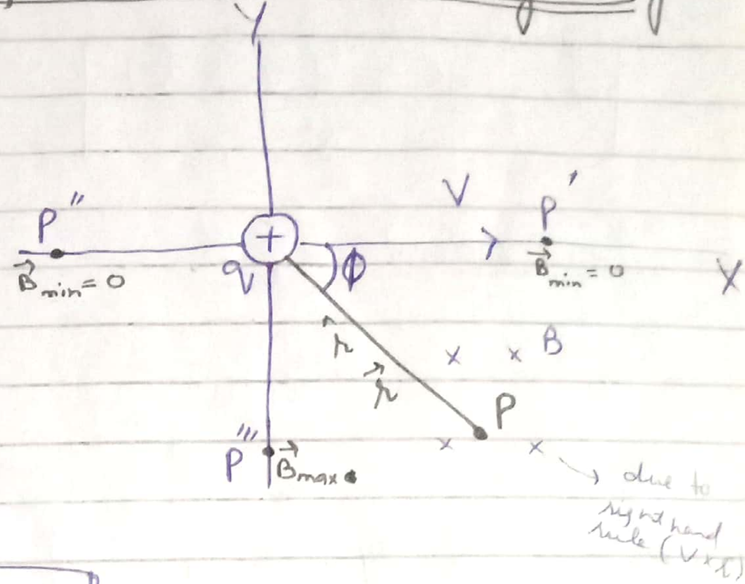
⇨ The Magnetic field due to a moving charge:

$B \propto q$

$B \propto V$

$B \propto \frac{1}{r^2}$

$B \propto \sin\phi$



$B \propto \frac{q v \sin\phi}{r^2}$

$$\boxed{B = \frac{K q v \sin\phi}{r^2}}$$

K = magnetic constant.

$K = \frac{\mu_0}{4\pi} = 10^{-7} \frac{T \cdot m}{A}$

$\mu_0 \rightarrow$ Permeability of free space

$\mu_0 = 4\pi \times 10^{-7} \frac{T \cdot m}{A}$

$C = \frac{\quad}{4\pi}$ $\cancel{C = \sqrt{\quad}}$

$$\boxed{C = \frac{1}{\sqrt{\mu_0 \epsilon_0}}}$$

B in vector form

$\cancel{\vec{B}} = \cancel{\frac{k(q \vec{v})}{r^2}}$ $B =$

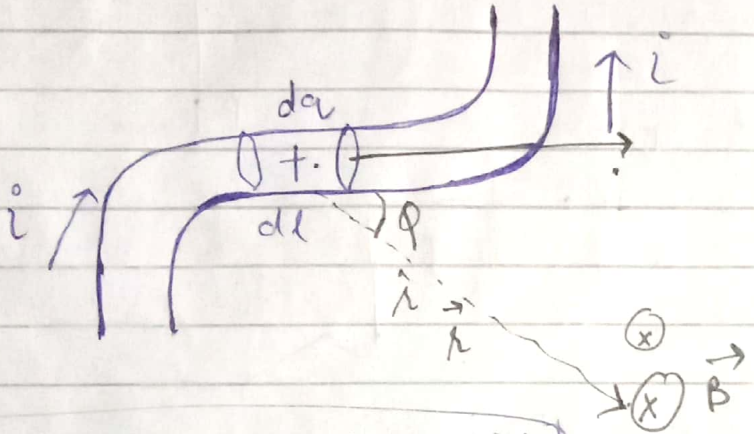
- Here q is not vector so not $q \times v$
- θ is angle b/w V & r
- $\vec{V} \times \hat{r} = V(1)\sin\theta$
- $\vec{V} \times \hat{r} = V\sin\theta$

$$\vec{B} = \frac{k q V \sin\theta}{r^2}$$

$$\boxed{\vec{B} = \frac{k q (\vec{V} \times \hat{r})}{r^2}}$$

$$\boxed{B = \frac{k q (\vec{V} \times \vec{r})}{r^3}}$$

⇒ The Magnetic field of a current :-



As $$\boxed{\vec{B} = \frac{k q (\vec{V} \times \hat{r})}{r^2}}$$ → for single charge.

So for this

$$d\vec{B} = \frac{k\,dq\,(\vec{V} \times \hat{r})}{r^2}$$

As $i = \frac{dq}{dt}$, $\vec{V} = \frac{dl}{dt}$

$$d\vec{B} = \frac{k(i\,dt)\left(\frac{dl}{dt} \times \hat{r}\right)}{r^2}$$

$$\int d\vec{B} = \int \frac{k(i\,\cancel{dt})\left(\frac{dl}{\cancel{dt}} \times \hat{r}\right)}{r^2}$$

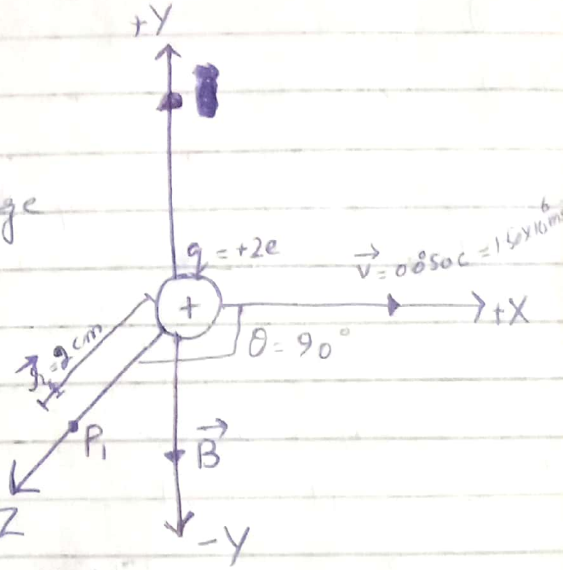
Pg 752 → Sample Problem 33-1

$$\boxed{\vec{B} = ki \int \frac{d\vec{l} \times \hat{r}}{r^2}}$$

Biot - Swart law

→ We can find the magnetic field of any current sphere

Sample Problem #1



*a) Formula for B of a single charge

$$B = K \frac{q v \sin\theta}{r_1^2}$$

$$K = \frac{\mu_0}{4\pi}$$

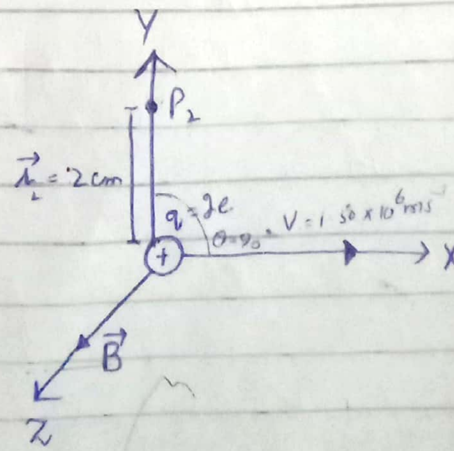
$$B = \frac{\mu_0}{4\pi} \times \frac{q v \sin\theta}{r_1^2}$$

$$\frac{\cancel{4\pi} \times 10^{-7}}{\cancel{4\pi}} \times \frac{2(1.6 \times 10^{-19})(1.50 \times 10^{6})(\sin 90)}{(0.020)^2}$$

$$\boxed{B = 1.2 \times 10^{-16}\, T}$$

∴ The direction of B is in -y axis (According to right hand rule).

*b)



$$B = \frac{k q v \sin\theta}{r_2^2}$$

$$K = \frac{\mu_0}{4\pi}$$

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Putting the values

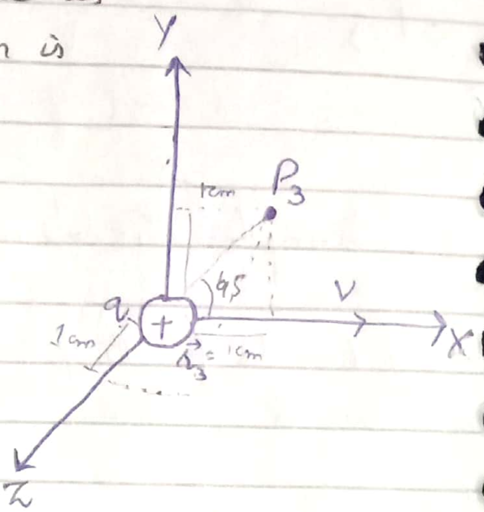
$$= \frac{\mu_0}{4\pi} \times \frac{q, v \sin\phi}{r^2}$$

$$= \frac{\cancel{4\pi} \times 10^{-7}}{\cancel{4\pi}} \times \frac{2(1.6 \times 10^{-19})(1.50 \times 10^{6})(\sin 90)}{(0.020)^2}$$

$$\boxed{B = 1.2 \times 10^{-16}\, T}$$

∴ The magnitude of $\vec{B}$ is same here as in first case. But its direction is towards +ve Z-axis.

*c)



Here $\vec{r}_3'$ is find by the vector sum of three vectors

where $r_1 = 1\,cm$, $r_2 = 1\,cm$, $r_3 = 1\,cm$

$$\vec{r}_3' = \sqrt{r_1^2 + r_2^2 + r_3^2}$$

$$\vec{r}_3 = \sqrt{1 + 1 + 1}$$

$$\vec{r}_3 = \sqrt{3} \Rightarrow \boxed{\vec{r}_3 = 1.73\, cm}$$

$$B = \frac{\mu_0}{4\pi} \times q v \sin\theta$$

$$= 1 \times 10^{-7} \times \frac{2(1.6 \times 10^{-19})(1.50 \times 10^{6})(\sin 57.3^\circ)}{(0.0173)^2}$$

$B =$

The direction of B is in XYZ space.

⇒ Ampere's law :-

The closed line integral of dot product of magnetic field and vector length is equal to μ_0 times the current enclosed by ~~amp~~ Amperian loop.

$$\oint_l \vec{B}\cdot d\vec{l} = \mu_0\, i_{enclosed}$$

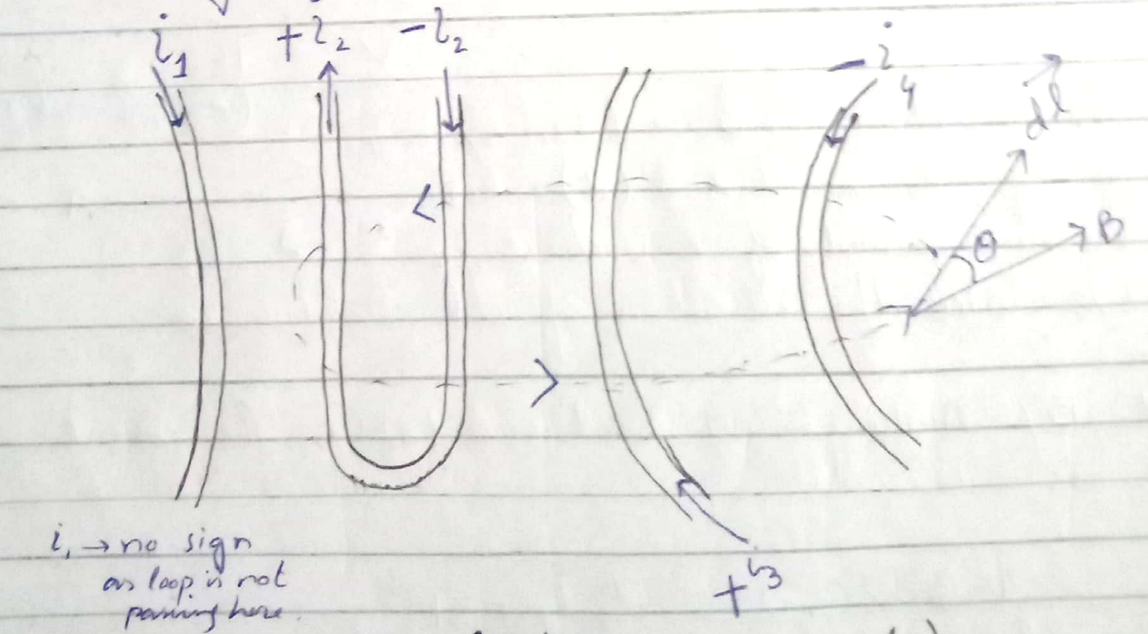
$\vec{B}$ ↓ On the Amperian loop

$d\vec{l}$ ↓ length of Amperian loop

$i_{enclosed}$ ↓ Current enclosed by the Amperian loop

○ → closed
l → line
∮ : limited closed linear

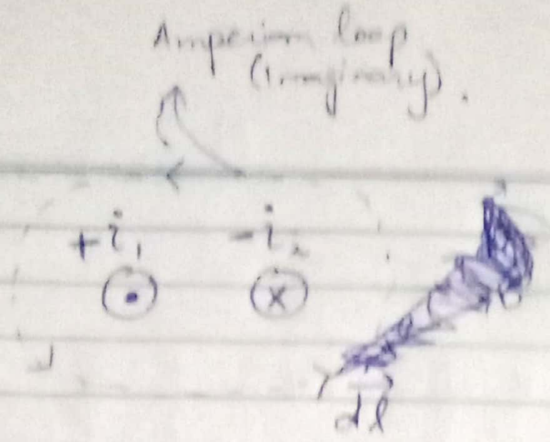
* The direction of length ($\vec{l}$) (vector) of Amperian loop is arbitrary (Clockwise & Anticlockwise).



i_1 → no sign as loop is not passing here.

Sign of Current :- (3rd Right hand rule)
~~Take~~

Place thumb towards Current if finger curl in the direction of dl then current is taken as +ve.



So from the previous cases, Ampere's law is written as

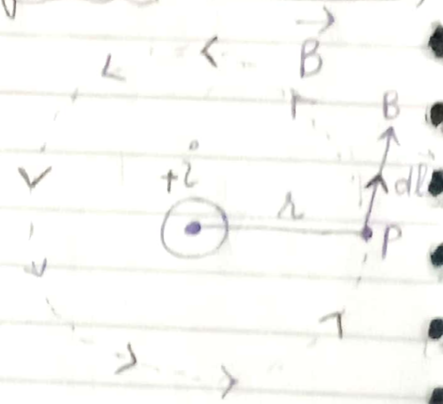
$$\oint_l B\,dl\cos\theta = \mu_0(+i_2 - i_2 + i_3 - i_4)$$

$$\oint_l B\,dl\cos\theta = \mu_0(i_3 - i_4)$$

⇒ **Applications of Ampere's law:-**

* **A long straight (Current Carrying) Wire:- (External Point)**

$$\oint_l \vec{B}\cdot\vec{dl} = \mu_0 i_{enclosed}$$



* The direction of $\vec{B}$ is taken by Applying right hand rule (2). This direction is not of our choice.
* We take the direction of $\vec{dl}$ Anticlockwise

* By Applying right hand rule (3), The current is +ve.

$$\oint_l B\,dl\cos\theta = \mu_0 i_{enclosed}$$

$$\oint_l B\,dl\cos 0^\circ = \mu_0(+i)$$

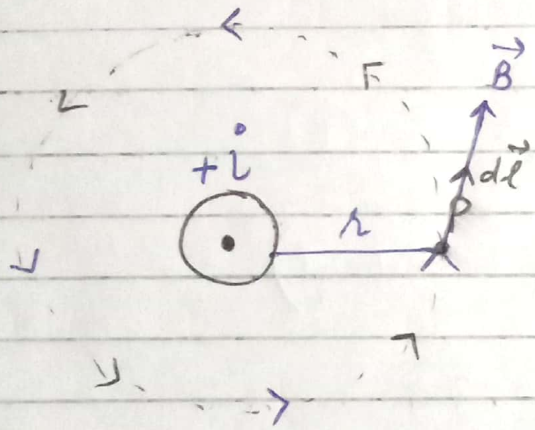
$$B\oint_l dl = \mu_0 i$$

$$B(2\pi r) = \mu_0 i$$

$$\boxed{B = \frac{\mu_0 i}{2\pi r}} \longrightarrow B \propto \frac{1}{r}$$

* Prove that the direction of Amperian loop or $(\vec{dl})$ vector is arbitrary?

Direction Anticlockwise



$$\oint B \cdot \vec{dl} = \mu_0 i_{enclosed}$$

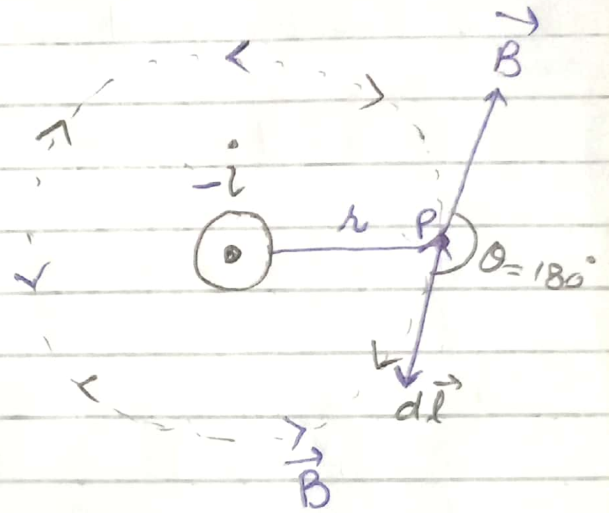
$$\oint B \cdot dl \cos 0 = \mu_0 (+i)$$

$$B \oint dl = \mu_0 (+i)$$

$$B(2\pi r) = \mu_0 i$$

$$\boxed{B = \frac{\mu_0 i}{2\pi r}}$$

Direction Clockwise



$$\oint B \cdot \vec{dl} = \mu_0 i$$

$$\oint B dl \cos 180 = \mu_0 (-i)$$

$$-B \oint dl = -\mu_0 i$$

$$B (2\pi r) = \mu_0 i$$

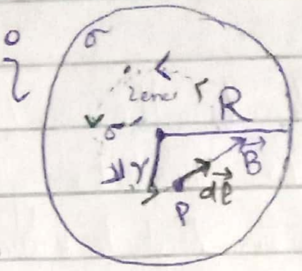
$$\boxed{B = \frac{\mu_0 i}{2\pi r}}$$

* A long straight (Current Carrying) Wire (Internal Points)

Cross sectional Area of Wire is shown

R = Radius of wire

$(r < R)$



i = Current enclosed in whole wire

i_{enc} = current enclosed in Amperiam surface

$$\oint_l \vec{B} \cdot d\vec{l} = \mu_0 \, i_{enclosed} \quad \text{--- (1)}$$

$$\oint_l B \, dl \cos\theta = \mu_0 \, (i_{enclosed})$$

$\theta = 0°$

$$B \oint_l dl = \mu_0 \, (i_{enclosed})$$

$$B(2\pi r) = \mu_0 \, (i_{enclosed}) \quad \text{--- (2)}$$

As current is uniform so their surface density remains same so

σ (outside A'loop) $= \sigma'$ (Surface charge density inside the A. loop)

$$\frac{i}{\cancel{\pi} R^2} = \frac{i_{end}}{\cancel{\pi} r^2}$$

$$\boxed{i_{enclosed} = \frac{r^2}{R^2} \, i} \quad \text{--- (3)}$$

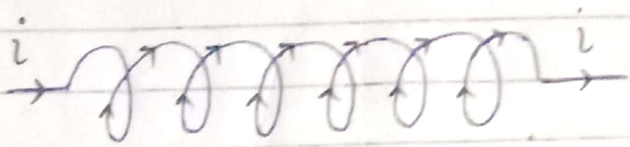
Put eq 3 in 2

$$B(2\pi r) = \mu_0 \frac{r^2}{R^2} \times i$$

$$B(2\pi) = \frac{\mu_0 r i}{R^2}$$

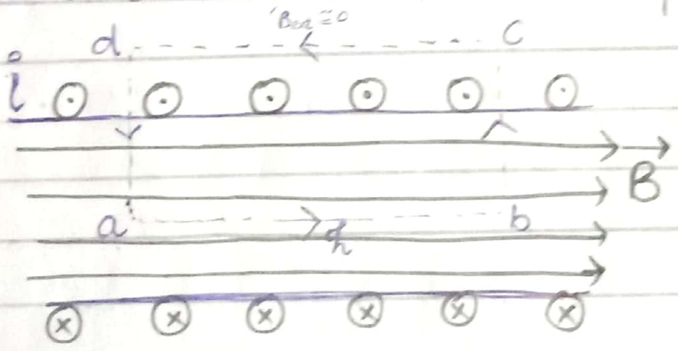
$$\boxed{B = \frac{\mu_0 r i}{2\pi R^2}} \rightarrow \boxed{= \frac{\mu_0 i}{2\pi R^2}\, r = \text{const. } r} \quad \text{so } \boxed{B \propto r}$$

* A Current Carrying solenoid:- (Internal Points)



$B_{ext} \cong 0$

i.e Current outside the solenoid is approximately 0.



$\vec{B}$ is same as of bar magnet

h = length of Amperian loop

$$\int_a^b B\,dl\cos 0^\circ + \int_b^c B\,dl\,\cancel{\cos 90^\circ}^{\,0} + \int_c^d \cancel{B_{ext}}^{\,0} \cdot dl\cos\theta + \int_d^a B\,dl\,\cancel{\cos 90^\circ}^{\,0}$$

$$= \mu_0 i_{enclosed}$$

$$= B\int_a^b dl = \mu_0 i_{enclosed}$$

$$\boxed{Bh = \mu_0 i_{enclosed}} \quad \text{—(2)}$$

N = number of Turns enclosed by the Amperian loop.

~~here we find n because all the turns are not present in A loop.~~

$$i_{enclosed} = Ni$$

Number density $= n = \frac{N}{h}$ (Number of turns Per unit length) $\therefore n = \frac{N}{A}$

$$N = nh$$

$$\boxed{i_{enclosed} = nhi} \quad \text{--- (3)}$$

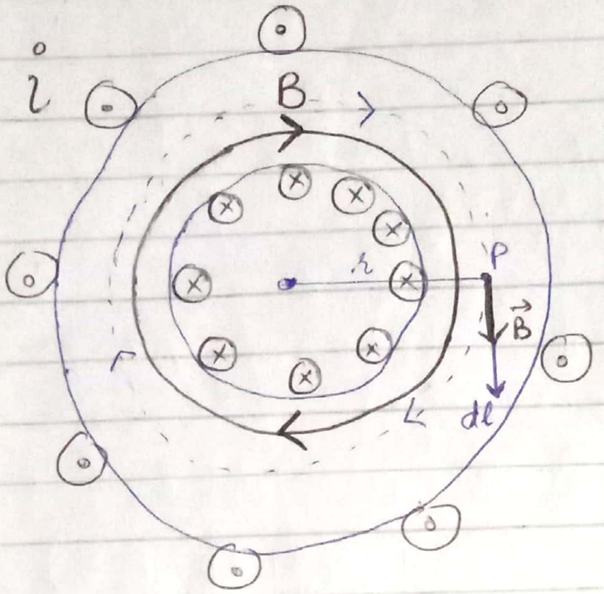
Put 3 in 2

$$B\cancel{h} = \mu_o n\cancel{h}i$$

$$\boxed{B = \mu_o n i}$$

It shows that magnetic field is constant inside a solenoid.

★ Magnetic field of a current carrying toroid:-



The direction of B get by using 3rd Right hand rule
We draw tangents to all the currents and by combining these vectors by combining their tails we get $\vec{B}$ in circular direction.

$$\oint_l \vec{B} \cdot d\vec{l} = \mu_0 i_{enclosed}$$

$$\oint_l B\, dl \cos 0^\circ = \mu_0 i_{enclosed}$$

$$B \int_l dl = \mu_0 N i$$

N = total no of turns enclosed by Amperian loop.
As all the turns are in the Amperian loop so we simply put N in formula

$$B(2\pi r) = \mu_0 N i$$

$$B = \mu_0 \left(\frac{N}{2\pi r}\right) i$$

Where $n = \frac{N}{2\pi r}$

$\therefore n = \frac{N}{A}$

∴ This result also shows that magnetic field is constant.

Chapter No 34

→ Faraday's law of Induction:-

• **Magnetic flux:-**

$$\phi_B = \vec{B} \cdot \vec{A}$$
$$\phi_B = BA\cos\theta$$

* **Dependence**

(i) Strength of field
(ii) Magnitude of Area
(iii) Orientation

Units ⟶ Weber = Tesla . meter^2

1 Wb = $T \cdot m^2$

* Emf is induced when flux is changed.

→ Faraday's Experiments:-

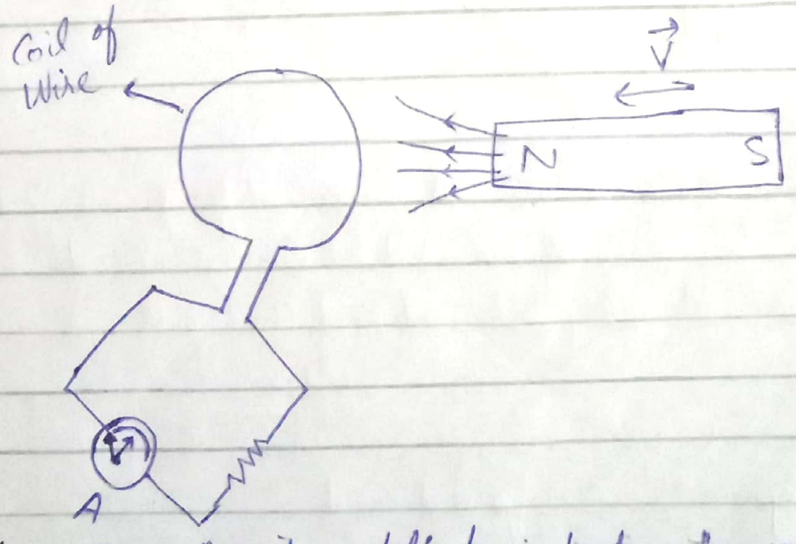


Fig 1:- The Ammeter deflects indicating the current in the circuit when the magnet is moving with respect to coil

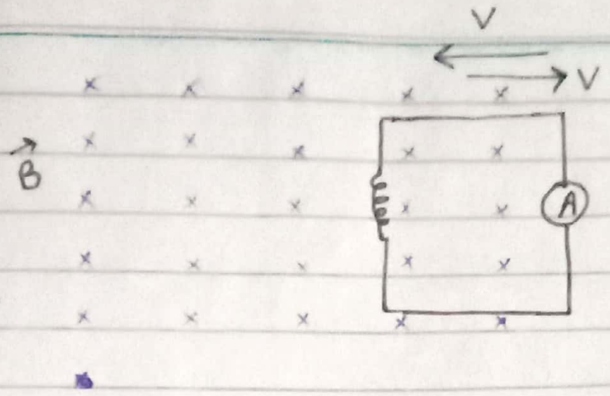


Fig When we move the coil in magnetic field the Ammeter shows deflection According to movement of coil

Motor → Electrical Energy to Mechanical Energy
→ Mechanical Energy to Electrical Energy

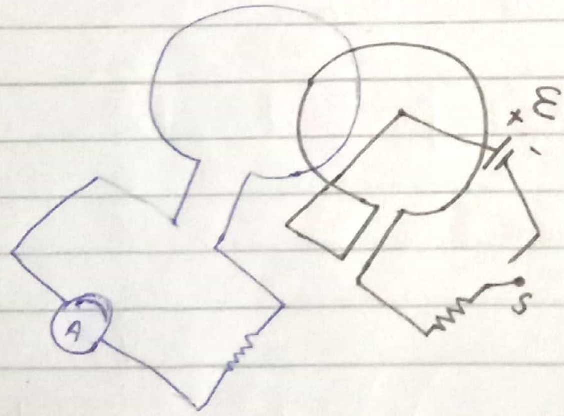


Fig # 3 The Ammeter deflect momentarily When switch S is closed or open
No physical motion of the coils is involved.

ε → Emf induced

$$\varepsilon_{ind} \propto \frac{d\phi_B}{dt}$$

this -ve sign is explained by lenz's law.

$$\boxed{\mathcal{E}_{ind} = -\frac{N\,d\phi_B}{dt}}$$

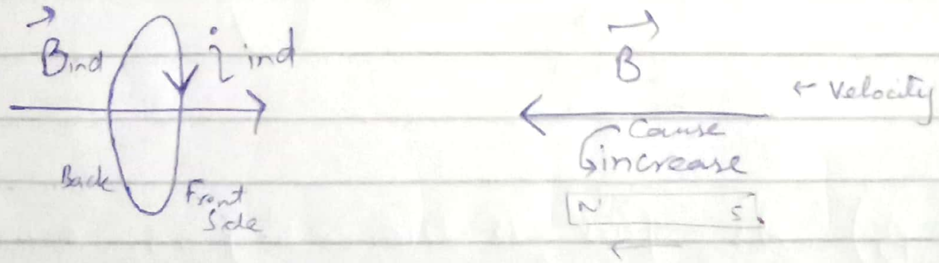
$\mathcal{E}_I$ does not depends on ϕ_B it depends on the change in ϕ_B i.e $d\phi_B$

* "The magnitude of $\mathcal{E}_{ind}$ in a circuit is equal to the rate at which magnetic flux is changing

⇒ <u>Lenz's law:-</u>

The flux of the magnetic field due to induced current opposes the change in the flux that causes the induced current.

* "if the change in flux is increase (cause is increase)"



* "if the change in flux is decrease"