Question # 1:

$(i) \qquad f(x) = 2x^2 + x - 5 \qquad\qquad c = 1$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (2x^2 + x - 5) = 2(1)^2 + 1 - 5 = 2 + 1 - 5 = -2$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (2x^2 + x - 5) = 2(1)^2 + 1 - 5 = 2 + 1 - 5 = -2$$

$$\Rightarrow \qquad \lim_{x \to 1^{-1}} f(x) = \lim_{x \to 1^+} f(x) = -2 \qquad\qquad \therefore \qquad \lim_{x \to 1} f(x) = -2$$

$(ii) \qquad f(x) = \dfrac{x^2 - 9}{x - 3} \qquad\qquad C = -3$

$$\lim_{x \to -3^-} f(x) = \lim_{x \to -3^-} \frac{x^2 - 9}{x - 3} = \frac{\lim\limits_{x \to -3^-} (x^2 - 9)}{\lim\limits_{x \to -3^-} (x - 3)} = \frac{(-3)^2 - 9}{-3 - 3} = \frac{9 - 9}{-6} = \frac{0}{-6} = 0$$

$$Now \quad \lim_{x \to -3^+} f(x) = \lim_{x \to -3^+} \frac{x^2 - 9}{x - 3} = \frac{\lim\limits_{x \to -3^+} (x^2 - 9)}{\lim\limits_{x \to -3^+} (x - 3)} = \frac{(-3)^2 - 9}{-3 - 3} = \frac{9 - 9}{-6} = \frac{0}{-6} = 0$$

$$\Rightarrow \qquad \lim_{x \to -3^-} f(x) = \lim_{x \to -3^+} f(x) = 0 \qquad\qquad \therefore \qquad \lim_{x \to -3} f(x) = 0$$

$(iii) \qquad f(x) = |x - 5| \qquad\qquad C = 5$

$$\lim_{x \to 5^-} f(x) = \lim_{x \to 5^-} |x - 5| \qquad\qquad\qquad\qquad\qquad |x - 5| = \pm(x - 5)$$

$$\begin{array}{ccc} & -(x-5) & +(x-5) \\ \hline -\infty & 5 & +\infty \end{array}$$

$$= \lim_{x \to 5^-} \left[-(x - 5) \right] = -\lim_{x \to 5} (x - 5) = -(5 - 5) = 0$$

$$\lim_{x \to 5^+} f(x) = \lim_{x \to 5^+} |x - 5| = \lim_{x \to 5^+} (x - 5) = 5 - 5 = 0$$

$$\Rightarrow \qquad \lim_{x \to 5^-} f(x) = \lim_{x \to 5^+} f(x) = 0$$

$$\lim_{x \to 5} f(x) = 0$$

Question # 2:

$Discuss\ the\ continuity\ of \qquad f(x) \quad at \quad x = c$

$(i) \qquad f(x) = \begin{cases} 2x + 5 & if \ \ x \le 2 \\ 4x + 1 & if \ \ x > 2 \\ & c = 2 \end{cases}$

$We\ have\ to\ discuss\ the\ continuity\ of\ f(x) \qquad at \quad x = 2$

$(a) \qquad f(2) = 2(2) + 5 = 4 + 5 = 9 \qquad\qquad ....(1)$

$(b) \qquad \lim\limits_{x \to 2} f(x) = ?$

$$\begin{array}{ccc} f(x) = 2x + 5 & & f(x) = 4x + 1 \\ \hline -\infty & 2 & +\infty \end{array}$$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2} (2x + 5) = 2(2) + 5 = 4 + 5 = 9$$

$$and \quad \lim_{x \to 2^+} f(x) = \lim_{x \to 2} (4x + 1) = 4(2) + 1 = 8 + 1 = 9$$

$$\therefore \qquad \lim_{x \to 2} f(x) = 9 \ \ ....(2)$$

(c) *from* (1) *and* (2) *we get*

$$\lim_{x \to 2} f(x) = f(2)$$

$\therefore$ $f(x)$ *is continous at* $x = 2$

(ii) $f(x) = \begin{cases} 3x-1 & if \quad x < 1 \\ 4 & x = 1 \quad c = 2 \\ 2x & x > 1 \end{cases}$

if $c = 2$ $f(c) = f(2)$

is not defined so given function is discontinous

(ii) *Correction*

$$f(x) = \begin{cases} 3x-1 & if \quad x < 1 \\ 4 & if \quad x = 1 \\ 2x & if \quad x > 1 \end{cases}$$

$c = 1$ $(correction)$

$$f(1) = 4$$

$-\infty$	1	$+\infty$
$f(x) = 3x-1$		$f(x) = 2x$

(a) $f(1) = 4$ $(given)$

(b) $\lim_{x \to 1} f(x) = ?$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1}(3x-1) = 3(1) - 1 = 2$$

and $\lim_{x \to 1^+} f(x) = \lim_{x \to 1}(2x) = 2(1) = 2$

$\Rightarrow$ $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = 2$

$\therefore$ $\lim_{x \to 1} f(x) = 2$ ....(2)

(c) *From* (1) *and* (2) *we get*

$\lim_{x \to 1} f(x) \neq f(1)$

$\therefore$ $f(x)$ *is discontinous at* $x = 1$

(iii) $f(x) = \begin{cases} 3x-1 & if \quad x < 1 \\ \\ 2x & if \quad x > 1 \end{cases}$ $c = 1$

(a) $f(1)$ *is not defined*

$\therefore$ $f(x)$ *is discontinous at* $x = 1$

Question # 3:

Given that

$$f(x) = \begin{cases} 3x & if & x \leq -2 \\ x^2 - 1 & if & -2 < x < 2 \\ 3 & if & x \geq 2 \end{cases}$$

$-\infty$				$+\infty$
$f(x) = 3x$	-2	$f(x) = x^2 - 1$	2	$f(x) = 3$

(i) *We check continuity at* $x = 2$

(a) $f(2) = 3$ ....(1) $(given)$

(b) $\qquad \lim\limits_{x \to 2} f(x) = ?$

$\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2}\left(x^2 - 1\right) = (2)^2 - 1 = 4 - 1 = 3$

$\lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2}(3) = 3$

$\Rightarrow \qquad \lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^+} f(x) = 3$

$\therefore \qquad \lim\limits_{x \to 2} f(x) = 3 \qquad ....(2)$

$(c) \qquad From\,(1)\,and\,(2), \qquad we\,get$

$\lim\limits_{x \to 2} f(x) = f(2)$

$\therefore \qquad f(x)$ *is continuous at* $\qquad x = 2$

$(ii) \qquad At \qquad x = -2$

$(a) \qquad f(-2) = 3(-2) = -6 \qquad ....(1)$

$(b) \qquad \lim\limits_{x \to -2} f(x) = ?$

$\lim\limits_{x \to -2^-} f(x) = \lim\limits_{x \to -2}(3x) = 3(-2) = -6$

$and \qquad \lim\limits_{x \to -2^+} f(x) = \lim\limits_{x \to -2}\left(x^2 - 1\right) = (-2)^2 - 1 = 4 - 1 = 3$

$\Rightarrow \qquad \lim\limits_{x \to -2^-} f(x) \neq \lim\limits_{x \to -2^+} f(x) \qquad\qquad \Rightarrow \lim\limits_{x \to -2} f(x)$ *does not exist*

$\therefore \qquad f(x)$ *is discontinuous at* $\qquad x = -2$

Question # 4:

Given that

$$f(x) = \begin{cases} x + 2 & x \leq -1 \\ c + 2 & x > -1 \end{cases}$$

$c = ?$

$$\begin{array}{ccc} -\infty & & +\infty \\ \hline f(x) = x + 2 & -1 & f(x) = c + 2 \end{array}$$

$\because \qquad \lim\limits_{x \to -1} f(x) \qquad exists$

$\therefore \qquad \lim\limits_{x \to -1^-} f(x) = \lim\limits_{x \to -1^+} f(x)$

$\Rightarrow \qquad \lim\limits_{x \to -1}(x + 2) = \lim\limits_{x \to -1}(c + 2)$

$\Rightarrow \qquad -1 + 2 = c + 2$

$\Rightarrow \qquad 1 = c + 2$

$\Rightarrow \qquad c = 1 - 2 \qquad \Rightarrow \qquad c = -1$

Question # 5:

(i)

$$f(x) = \begin{cases} mx & if & x < 3 \\ n & if & x = 3 \\ -2x + 9 & if & x > 3 \end{cases}$$

$here \qquad f(3) = n \qquad (given)$

$\because \qquad f(x)$ *is continuous at* $\qquad x = 3$

$\therefore \qquad \lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^+} f(x) = f(3)$

$\Rightarrow \qquad \lim\limits_{x \to 3}(mx) = \lim\limits_{x \to 3}(-2x + 9) = n$

$\Rightarrow \quad (m)(3) = -2(3) + 9 = n$

$\Rightarrow \quad 3m = -6 + 9 = n$

$\Rightarrow \quad 3m = 3 = n$

$\Rightarrow \quad 3m = 3 \qquad , \qquad n = 3$

$\Rightarrow \quad m = 1 \qquad , \qquad n = 3$

$(ii) \quad f(x) = \begin{cases} mx & if & x < 4 \\ x^2 & if & x \geq 4 \end{cases}$

here $\quad f(4) = (4)^2 = 16$

$\because \quad f(x)$ *is continuous at* $\quad x = 4$

$\because \quad \lim\limits_{x \to 4^-} f(x) = \lim\limits_{x \to 4^+} f(x) = f(4)$

$\Rightarrow \quad \lim\limits_{x \to 4}(mx) = \lim\limits_{x \to 4}(x^2) = 16$

$\Rightarrow \quad 4m = (4)^2 = 16$

$\Rightarrow \quad 4m = 16 = 16 \qquad \Rightarrow \qquad 4m = 16$

$\Rightarrow \quad m = 4$

Question # 6:

Given that

$$f(x) = \begin{cases} \dfrac{\sqrt{2x+5} - \sqrt{x+7}}{x-2} & x \neq 2 \\ K & x = 2 \end{cases}$$

$$K = ?$$

here $\quad f(2) = K \qquad$ *given*

$\because \quad f(x)$ *is continuous at* $\quad x = 2$

$\because \quad \lim\limits_{x \to 2} f(x) = f(2)$

$\lim\limits_{x \to 2} \dfrac{\sqrt{2x+5} - \sqrt{x+7}}{x-2} = K \quad \Rightarrow \quad \lim\limits_{x \to 2} \dfrac{\sqrt{2x+5} - \sqrt{x+7}}{x-2} \dfrac{\sqrt{2x+5} + \sqrt{x+7}}{\sqrt{2x+5} + \sqrt{x+7}} = K$

$\Rightarrow \quad \lim\limits_{x \to 2} \dfrac{\left(\sqrt{2x+5}\right)^2 - \left(\sqrt{x+7}\right)^2}{(x-2)\left(\sqrt{2x+5} + \sqrt{x+7}\right)} = K \quad \Rightarrow \quad \dfrac{(2x+5) - (x+7)}{(x-2)\left(\sqrt{2x+5} + \sqrt{x+7}\right)} = K$

$\Rightarrow \quad \dfrac{2x+5-x-7}{(x-2)\left(\sqrt{2x+5} + \sqrt{x+7}\right)} = K \quad \Rightarrow \quad \dfrac{(x-2)}{(x-2)\left(\sqrt{2x+5} + \sqrt{x+7}\right)} = K$

$\Rightarrow \quad \lim\limits_{x \to 2} \dfrac{1}{\sqrt{2x+5} + \sqrt{x+7}} = K \quad \Rightarrow \quad \dfrac{1}{\lim\limits_{x \to 2}\left[\sqrt{2x+5} + \sqrt{x+7}\right]} = K$

$\Rightarrow \quad \dfrac{1}{\sqrt{2(2)+5} + \sqrt{2+7}} = K \quad \Rightarrow \quad \dfrac{1}{\sqrt{9} + \sqrt{9}} = K$

$\Rightarrow \quad \dfrac{1}{3+3} = K$

$\Rightarrow \quad K = \dfrac{1}{6}$