Deduction

$$\lim_{x \to 0} \left(\frac{e^x - 1}{x}\right) = \log_e e = 1$$

We know that

$$\lim_{x \to 0} \left(\frac{a^x - 1}{x}\right) = \log_e a$$

Put $a = e$

$$\lim_{x \to 0} \left(\frac{e^x - 1}{x}\right) = \log_e e = 1$$

Important results to remember

(i) $\lim_{x \to +\infty} (e^x) = \infty$ (ii) $\lim_{x \to -\infty} (e^x) = \lim_{x \to -\infty} \left(\frac{1}{e^{-x}}\right) = 0$

(iii) $\lim_{x \to \pm\infty} \left(\frac{a}{x}\right) = 0$, where a is any real number.

EXERCISE 1.3

Q.1 Evaluate each limit by using theorems of limits.

(i) $\lim_{x\to3} (2x + 4)$ **(ii)** $\lim_{x\to1} (3x^2 - 2x + 4)$

(iii) $\lim_{x\to3} \sqrt{x^2 + x + 4}$ **(iv)** $\lim_{x\to2} x\sqrt{x^2 - 4}$

(v) $\lim_{x\to2} \left(\sqrt{x^3 + 1} - \sqrt{x^2 + 5}\right)$ **(iv)** $\lim_{x\to2} \frac{2x^3 + 5x}{3x - 2}$

Solution:

(i) $\lim_{x\to3} (2x + 4) = \lim_{x\to3} (2x) + \lim_{x\to3} (4)$

$= 2 \lim_{x\to3} x + 4$

$= 2(3) + 4 = 6 + 4 = 10$ Ans.

(ii) $\lim_{x\to1} (3x^2 - 2x + 4) = \lim_{x\to1} (3x^2) - \lim_{x\to1} (2x) + \lim_{x\to1} (4)$

$= 3 \lim_{x\to1} x^2 - 2 \lim_{x\to1} x + 4$

$= 3(1)^2 - 2(1) + 4$

$= 3 - 2 + 4$

$= 5$ Ans.

(iii) $\lim_{x\to3} \sqrt{x^2 + x + 4} = [\lim_{x\to3} (x^2 + x + 4)]^{1/2}$

Important Limits

I. $\lim\limits_{x\to a}\dfrac{x^n-a^n}{x-a}=na^{n-1}$, where n is integer and $a>0$.

II. $\lim\limits_{x\to 0}\dfrac{\sqrt{x+a}-\sqrt{a}}{x}=\dfrac{1}{2\sqrt{a}}$.

III. $\lim\limits_{n\to 0}\left(1+\dfrac{1}{n}\right)^n=e$.

IV. $\lim\limits_{x\to \infty}\left(1+x\right)^{\frac{1}{x}}=e$.

V. $\lim\limits_{x\to 0}\dfrac{a^x-1}{x}=\ln a$, where $a>0$.

VI. $\lim\limits_{x\to 0}\dfrac{e^x-1}{x}=\ln e=1$.

VII. If θ is measured in radian, then $\lim\limits_{\theta\to 0}\dfrac{\sin\theta}{\theta}=1$.

Question # 1

(i) $\lim\limits_{x\to 3}(2x+4)\;=\lim\limits_{x\to 3}(2x)+\lim\limits_{x\to 3}(4)\;=2\lim\limits_{x\to 3}(x)+4\;=2(3)+4\;=10$.

(ii) $\lim\limits_{x\to 1}\left(3x^2-2x+4\right)\;=3(1)^2-2(1)+4\;=3-2+4=5$.

(iii) $\lim\limits_{x\to 3}\sqrt{x^2+x+4}\;=\sqrt{(3)^2+(3)+4}\;=\sqrt{9+3+4}\;=\sqrt{16}=4$.

(iv) $\lim\limits_{x\to 2}x\sqrt{x^2-4}\;=2\sqrt{2^2-4}=0$.

(v) $\lim\limits_{x\to 2}\left(\sqrt{x^3+1}-\sqrt{x^2+5}\right)\;=\lim\limits_{x\to 2}\left(\sqrt{x^3+1}\right)-\lim\limits_{x\to 2}\left(\sqrt{x^2+5}\right)$

$$=\left(\sqrt{(2)^3+1}\right)-\left(\sqrt{(2)^2+5}\right)$$

$$=\sqrt{8+1}-\sqrt{4+5}\;=\sqrt{9}-\sqrt{9}\;=0.$$

(vi) $\lim\limits_{x\to -2}\dfrac{2x^3+5x}{3x-2}\;=\dfrac{2(-2)^3+5(-2)}{3(-2)-2}\;=\dfrac{-16-10}{-6-2}\;=\dfrac{-26}{-8}\;=\dfrac{13}{4}$.

Question # 2

(i) $\lim\limits_{x\to -1}\dfrac{x^3-x}{x+1}\;=\lim\limits_{x\to -1}\dfrac{x(x^2-1)}{x+1}\;=\lim\limits_{x\to -1}\dfrac{x(x+1)(x-1)}{x+1}$

$$=\lim_{x\to -1}x(x-1)\;=(-1)(-1-1)\;=2$$

(ii) $\lim\limits_{x\to 0}\left(\dfrac{3x^3+4x}{x^2+x}\right)\;=\lim\limits_{x\to 0}\dfrac{x(3x^2+4)}{x(x+1)}\;=\lim\limits_{x\to 0}\dfrac{3x^2+4}{x+1}\;=\dfrac{3(0)+4}{0+1}\;=4$.

(iii) $\lim_{x\to 2}\dfrac{x^3-8}{x^2+x-6}$

$=\lim_{x\to 2}\dfrac{x^3-(2)^3}{x^2+3x-2x-6} \quad =\lim_{x\to 2}\dfrac{(x-2)(x^2+2x+4)}{x(x+3)-2(x+3)}$

$=\lim_{x\to 2}\dfrac{(x-2)(x^2+2x+4)}{(x+3)(x-2)} \quad =\lim_{x\to 2}\dfrac{(x^2+2x+4)}{(x+3)}$

$=\dfrac{(2)^2+2(2)+4)}{(2+3)} \quad =\dfrac{12}{5}$

(iv) $\lim_{x\to 1}\dfrac{x^3-3x^2+3x-1}{x^3-x}$

$=\lim_{x\to 1}\dfrac{(x-1)^3}{x(x^2-1)} \quad =\lim_{x\to 1}\dfrac{(x-1)^3}{x(x-1)(x+1)}$

$=\lim_{x\to 1}\dfrac{(x-1)^2}{x(x+1)} \quad =\lim_{x\to 1}\dfrac{(1-1)^2}{(1)(1+1)} \quad =0$

(v) $\lim_{x\to -1}\left(\dfrac{x^3+x^2}{x^2-1}\right) \quad =\lim_{x\to -1}\dfrac{x^2(x+1)}{(x+1)(x-1)} \quad =\lim_{x\to -1}\dfrac{x^2}{(x-1)}$

$=\dfrac{(-1)^2}{(-1-1)}=-\dfrac{1}{2}$

(vi) $\lim_{x\to 4}\dfrac{2x^2-32}{x^3-4x^2} \quad =\lim_{x\to 4}\dfrac{2(x^2-16)}{x^2(x-4)}$

$=\lim_{x\to 4}\dfrac{2(x+4)(x-4)}{x^2(x-4)} \quad =\lim_{x\to 4}\dfrac{2(x+4)}{x^2}$

$=\dfrac{2(4+4)}{4^2} \quad =\dfrac{16}{16} \quad = 1.$

(vii) $\lim_{x\to 2}\dfrac{\sqrt{x}-\sqrt{2}}{x-2} \quad =\lim_{x\to 2}\dfrac{\sqrt{x}-\sqrt{2}}{x-2}\left(\dfrac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}}\right)$

$=\lim_{x\to 2}\dfrac{\left(\sqrt{x}\right)^2-\left(\sqrt{2}\right)^2}{(x-2)\left(\sqrt{x}+\sqrt{2}\right)} \quad =\lim_{x\to 2}\dfrac{x-2}{(x-2)\left(\sqrt{x}+\sqrt{2}\right)}$

$=\lim_{x\to 2}\dfrac{1}{\sqrt{x}+\sqrt{2}} \quad =\dfrac{1}{\sqrt{2}+\sqrt{2}} \quad =\dfrac{1}{2\sqrt{2}}$

(viii) $\lim_{h\to 0}\dfrac{\sqrt{x+h}-\sqrt{x}}{h} \quad =\lim_{h\to 0}\dfrac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\dfrac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

$=\lim_{h\to 0}\dfrac{\left(\sqrt{x+h}\right)^2-\left(\sqrt{x}\right)^2}{h\left(\sqrt{x+h}+\sqrt{x}\right)} \quad =\lim_{h\to 0}\dfrac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$

$$= \lim_{h\to 0} \frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)} \qquad = \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}}$$

$$= \frac{1}{\sqrt{x+0}+\sqrt{x}} \qquad = \frac{1}{2\sqrt{x}}$$

(ix) $\displaystyle \lim_{x\to a} \frac{x^n - a^n}{x^m - a^m}$

$$= \lim_{x\to a} \frac{(x-a)\left(x^{n-1}+x^{n-2}a+x^{n-3}a^2+....+a^{n-1}\right)}{(x-a)\left(x^{m-1}+x^{m-2}a+x^{m-3}a^2+....+a^{m-1}\right)}$$

$$= \lim_{x\to a} \frac{\left(x^{n-1}+x^{n-2}a+x^{n-3}a^2+....+a^{n-1}\right)}{\left(x^{m-1}+x^{m-2}a+x^{m-3}a^2+....+a^{m-1}\right)}$$

$$= \frac{a^{n-1}+a^{n-2}a+a^{n-3}a^2+....+a^{n-1}}{a^{m-1}+a^{m-2}a+a^{m-3}a^2+....+a^{m-1}}$$

$$= \frac{a^{n-1}+a^{n-1}+a^{n-1}+....+a^{n-1} \quad (n \text{ terms})}{a^{m-1}+a^{m-1}+a^{m-1}+....+a^{m-1} \quad (m \text{ terms})}$$

$$= \frac{n\,a^{n-1}}{m\,a^{m-1}} \qquad = \frac{n}{m}a^{n-1-m+1} \qquad = \frac{n}{m}a^{n-m}$$

Law of Sine

If θ is measured in radian, then $\displaystyle \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$

See proof on book at page 25

Question # 3

(i) $\displaystyle \lim_{x\to 0} \frac{\sin 7x}{x}$

Put $t = 7x \;\Rightarrow\; \dfrac{t}{7} = x$

When $x \to 0$ then $t \to 0$, so

$$\lim_{x\to 0} \frac{\sin 7x}{x} = \lim_{t\to 0} \frac{\sin t}{t/7}$$

$$= 7\lim_{t\to 0} \frac{\sin t}{t} = 7(1) = 7$$ By law of sine.

(ii) $\displaystyle \lim_{x\to 0} \frac{\sin x^\circ}{x}$

Since $180^\circ = \pi$ rad $\quad \Rightarrow 1^\circ = \dfrac{\pi}{180}$ rad $\quad \Rightarrow x^\circ = \dfrac{x\pi}{180}$ rad

So $\displaystyle \lim_{x\to 0} \frac{\sin x^\circ}{x} = \lim_{x\to 0} \frac{\sin \pi x/180}{x}$

Now put $\dfrac{\pi x}{180} = t$ i.e. $x = \dfrac{180t}{\pi}$

When $x \to 0$ then $t \to 0$, so

$$\lim_{x\to 0}\frac{\sin \pi x/180}{x} = \lim_{x\to 0}\frac{\sin t}{180t/\pi}$$

$$= \frac{\pi}{180}\lim_{x\to 0}\frac{\sin t}{t} = \frac{\pi}{180}(1) = \frac{\pi}{180}$$ by law of sine

(iii) $\displaystyle\lim_{\theta\to 0}\frac{1-\cos\theta}{\sin\theta} = \lim_{\theta\to 0}\frac{1-\cos\theta}{\sin\theta}\cdot\frac{1+\cos\theta}{1+\cos\theta}$

$$= \lim_{\theta\to 0}\frac{1-\cos^2\theta}{\sin\theta(1+\cos\theta)} = \lim_{\theta\to 0}\frac{\sin^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \lim_{\theta\to 0}\frac{\sin\theta}{(1+\cos\theta)} = \frac{\sin(0)}{1+\cos(0)} = \frac{0}{1+1} = 0$$

(iv) $\displaystyle\lim_{x\to\pi}\frac{\sin x}{\pi - x}$

Put $t = \pi - x \Rightarrow x = \pi - t$

When $x \to \pi$ then $t \to 0$, so

$$\lim_{x\to\pi}\frac{\sin x}{\pi - x} = \lim_{t\to 0}\frac{\sin(\pi - t)}{t}$$

$$= \lim_{t\to 0}\frac{\sin t}{t}$$ $\because \sin(\pi - t) = \sin\left(2\cdot\frac{\pi}{2} - t\right) = \sin t$

$= 1$ By law of sine.

(v) $\displaystyle\lim_{x\to 0}\frac{\sin ax}{\sin bx} = \lim_{x\to 0}\sin ax\cdot\frac{1}{\sin bx}$

$$= \lim_{x\to 0}\sin ax\cdot\left(\frac{ax}{ax}\right)\frac{1}{\sin bx\cdot\left(\frac{bx}{bx}\right)} = \lim_{x\to 0}\frac{\sin ax}{ax}\cdot ax\frac{1}{\frac{\sin bx}{bx}\cdot bx}$$

$$= \frac{a}{b}\lim_{x\to 0}\frac{\sin ax}{ax}\cdot\frac{1}{\lim\limits_{x\to 0}\frac{\sin bx}{bx}} = \frac{a}{b}\cdot(1)\cdot\frac{1}{(1)} = \frac{a}{b}$$ by law of sine

(vi) $\displaystyle\lim_{x\to 0}\frac{x}{\tan x} = \lim_{x\to 0}\frac{x}{\frac{\sin x}{\cos x}} = \lim_{x\to 0}\frac{x}{\sin x}\cdot\cos x$

$$= \lim_{x\to 0}\frac{1}{\frac{\sin x}{x}}\cdot\cos x = \frac{1}{\lim\limits_{x\to 0}\frac{\sin x}{x}}\cdot\lim_{x\to 0}\cos x = \frac{1}{1}\cdot 1 = 1$$

(vii) $\displaystyle\lim_{x\to 0}\frac{1-\cos 2x}{x^2}$

$$= \lim_{x\to 0}\frac{2\sin^2 x}{x^2}$$

$\because \sin^2 x = \dfrac{1-\cos 2x}{2}$

$\therefore 2\sin^2 x = 1 - \cos 2x$

$$= 2\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2 = 2\left(\lim_{x\to 0}\frac{\sin x}{x}\right)^2 = 2(1)^2 = 2$$

(vii) *Do yourself by rationalizing*

(viii) $\lim\limits_{\theta\to 0}\dfrac{\sin^2\theta}{\theta} = \lim\limits_{\theta\to 0}\dfrac{\sin\theta}{\theta}\cdot\sin\theta$

$$= \lim_{\theta\to 0}\frac{\sin\theta}{\theta}\cdot\lim_{\theta\to 0}\sin\theta = (1)\cdot(0) = 0$$

(x) $\lim\limits_{x\to 0}\dfrac{\sec x - \cos x}{x}$

$$= \lim_{x\to 0}\frac{\frac{1}{\cos x} - \cos x}{x} = \lim_{x\to 0}\frac{\frac{1-\cos^2 x}{\cos x}}{x}$$

$$= \lim_{x\to 0}\frac{1-\cos^2 x}{x\cos x} = \lim_{x\to 0}\frac{\sin^2 x}{x\cos x} = \lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{\sin x}{\cos x}$$

$$= \lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{\sin x}{\cos x} = (1)\frac{\sin(0)}{\cos(0)} = (1)\cdot\frac{0}{1} = 0$$

(xi) $\lim\limits_{x\to 0}\dfrac{1-\cos p\theta}{1-\cos q\theta}$

$$= \lim_{x\to 0}\frac{2\sin^2\frac{p\theta}{2}}{2\sin^2\frac{q\theta}{2}} \qquad \because\ \sin^2\frac{x}{2} = \frac{1-\cos x}{2}$$

$$= \lim_{x\to 0}\ \sin^2\frac{p\theta}{2}\cdot\frac{1}{\sin^2\frac{q\theta}{2}} = \lim_{x\to 0}\ \sin^2\frac{p\theta}{2}\cdot\frac{\left(\frac{p\theta}{2}\right)^2}{\left(\frac{p\theta}{2}\right)^2}\cdot\frac{1}{\sin^2\frac{q\theta}{2}\cdot\frac{\left(\frac{q\theta}{2}\right)^2}{\left(\frac{q\theta}{2}\right)^2}}$$

$$= \lim_{x\to 0}\ \frac{\sin^2\frac{p\theta}{2}}{\left(\frac{p\theta}{2}\right)^2}\cdot\left(\frac{p\theta}{2}\right)^2\cdot\frac{1}{\frac{\sin^2\frac{q\theta}{2}}{\left(\frac{q\theta}{2}\right)^2}\cdot\left(\frac{q\theta}{2}\right)^2} = \lim_{x\to 0}\left(\frac{\sin\frac{p\theta}{2}}{\frac{p\theta}{2}}\right)^2\cdot\frac{1}{\left(\frac{\sin\frac{q\theta}{2}}{\frac{q\theta}{2}}\right)^2}\cdot\frac{p^2\theta^2/4}{q^2\theta^2/4}$$

$$= \frac{p^2}{q^2}\left(\lim_{x\to 0}\frac{\sin\frac{p\theta}{2}}{\frac{p\theta}{2}}\right)^2\cdot\frac{1}{\left(\lim\limits_{x\to 0}\frac{\sin\frac{q\theta}{2}}{\frac{q\theta}{2}}\right)^2} = \frac{p^2}{q^2}(1)^2\cdot\frac{1}{(1)^2} = \frac{p^2}{q^2}$$

(xii) $\lim\limits_{\theta\to 0}\dfrac{\tan\theta-\sin\theta}{\sin^3\theta}$

$$= \lim_{\theta\to 0}\frac{\frac{\sin\theta}{\cos\theta}-\sin\theta}{\sin^3\theta} \quad = \lim_{\theta\to 0}\frac{\frac{\sin\theta-\sin\theta\cos\theta}{\cos\theta}}{\sin^3\theta}$$

$$= \lim_{\theta\to 0}\frac{\sin\theta-\sin\theta\cos\theta}{\sin^3\theta\cos\theta} \quad = \lim_{\theta\to 0}\frac{\sin\theta\left(1-\cos\theta\right)}{\sin^3\theta\cos\theta}$$

$$= \lim_{\theta\to 0}\frac{1-\cos\theta}{\sin^2\theta\cos\theta} \quad = \lim_{\theta\to 0}\frac{1-\cos\theta}{\sin^2\theta\cos\theta}\cdot\frac{1+\cos\theta}{1+\cos\theta}$$

$$= \lim_{\theta\to 0}\frac{1-\cos^2\theta}{\sin^2\theta\cos\theta\left(1+\cos\theta\right)} \quad = \lim_{\theta\to 0}\frac{\sin^2\theta}{\sin^2\theta\cos\theta\left(1+\cos\theta\right)}$$

$$= \lim_{\theta\to 0}\frac{1}{\cos\theta\left(1+\cos\theta\right)} \quad = \lim_{x\to 0}\frac{1}{\cos\theta\left(1+\cos\theta\right)}$$

$$= \frac{1}{\cos(1)\left(1+\cos(1)\right)} \quad = \frac{1}{1\cdot\left(1+1\right)} \quad = \frac{1}{2}$$

Note:

a) $\lim\limits_{n\to\infty}\left(1+\dfrac{1}{n}\right)^n = e$

b) $\lim\limits_{x\to 0}\left(1+x\right)^{\frac{1}{x}} = e$ where $e = 2.718281...$

See proof of (a) and (b) on book at page 23

c) $\lim\limits_{x\to 0}\dfrac{a^x-1}{x} = \log_e a$ or $\ln a$

Proof:

Put $y = a^x - 1$ .... (i)

When $x\to 0$ then $y\to 0$

Also from (i) $\quad 1+y = a^x$

Taking log on both sides

$$\ln\left(1+y\right) = \ln a^x \quad \Rightarrow \quad \ln(1+y) = x\ln a \qquad \because \ln x^m = m\ln x$$

$$\Rightarrow \quad x = \frac{\ln\left(1+y\right)}{\ln a}$$

Now $\lim\limits_{x\to 0}\dfrac{a^x-1}{x} = \lim\limits_{y\to 0}\dfrac{y}{\frac{\ln\left(1+y\right)}{\ln a}}$

$$= \lim_{y\to 0}\frac{y\ln a}{\ln\left(1+y\right)} \quad = \lim_{y\to 0}\frac{\ln a}{\frac{1}{y}\ln\left(1+y\right)}$$

$$= \lim_{y\to 0}\frac{\ln a}{\ln(1+y)^{\frac{1}{y}}} = \frac{\ln a}{\lim_{y\to 0}\ln(1+y)^{\frac{1}{y}}} \qquad \because \ln x^m = m\ln x$$

$$= \frac{\ln a}{\ln\left(\lim_{y\to 0}(1+y)^{\frac{1}{y}}\right)} = \frac{\ln a}{\ln(e)} \qquad \because \lim_{x\to 0}(1+x)^{\frac{1}{x}} = e$$

$$= \frac{\ln a}{1} = \ln a \qquad \because \ln e = 1$$

Question # 4

(i) $\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^{2n} = \left[\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^{n}\right]^2 = e^2$

(ii) $\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^{\frac{n}{2}} = \left[\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^{n}\right]^{\frac{1}{2}} = e^{\frac{1}{2}} = \sqrt{e}$

(iii) $\lim_{n\to+\infty}\left(1-\frac{1}{n}\right)^{n} = \left[\lim_{n\to+\infty}\left(1-\frac{1}{n}\right)^{-n}\right]^{-1} = e^{-1} = \frac{1}{e}$

(iv) $\lim_{n\to+\infty}\left(1+\frac{1}{3n}\right)^{n}$

$$= \lim_{n\to+\infty}\left(1+\frac{1}{3n}\right)^{\frac{3n}{3}} = \left[\lim_{n\to+\infty}\left(1+\frac{1}{3n}\right)^{3n}\right]^{\frac{1}{3}} = e^{\frac{1}{3}}$$

(v) $\lim_{n\to+\infty}\left(1+\frac{4}{n}\right)^{n} = \lim_{n\to+\infty}\left(1+\frac{4}{n}\right)^{\frac{4n}{4}} = \left[\lim_{n\to+\infty}\left(1+\frac{4}{n}\right)^{\frac{n}{4}}\right]^4 = e^4.$

(vi) $\lim_{x\to 0}(1+3x)^{\frac{2}{x}}$

$$= \lim_{x\to 0}(1+3x)^{\frac{6}{3x}} = \left[\lim_{x\to 0}(1+3x)^{\frac{1}{3x}}\right]^6 = e^6$$

(vii) $\lim_{x\to 0}\left(1+2x^2\right)^{\frac{1}{x^2}} = \lim_{x\to 0}\left(1+2x^2\right)^{\frac{2}{2x^2}} = \left[\lim_{x\to 0}\left(1+2x^2\right)^{\frac{1}{2x^2}}\right]^2 = e^2$

(viii) $\lim_{h\to 0}(1-2h)^{\frac{1}{h}} = \lim_{h\to 0}(1-2h)^{\frac{-2}{-2h}} = \left[\lim_{h\to 0}(1-2h)^{\frac{1}{-2h}}\right]^{-2} = e^{-2} = \frac{1}{e^2}$

(ix) $\lim_{x\to\infty}\left(\frac{x}{1+x}\right)^{x}$

$$= \lim_{x\to\infty}\left(\frac{1+x}{x}\right)^{-x} = \lim_{x\to\infty}\left(\frac{1}{x}+\frac{x}{x}\right)^{-x} = \lim_{x\to\infty}\left(\frac{1}{x}+1\right)^{-x}$$

$$=\left[\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}\right]^{-1} = e^{-1} = \frac{1}{e}$$

(x) $\lim_{x\to 0}\dfrac{e^{1/x}-1}{e^{1/x}+1}$; $x<0$

Put $x=-t$ where $t>0$

When $x\to 0$ then $t\to 0$, so

$$\lim_{x\to 0}\frac{e^{1/x}-1}{e^{1/x}+1} = \lim_{t\to 0}\frac{e^{-1/t}-1}{e^{-1/t}+1} = \frac{e^{-1/0}-1}{e^{-1/0}+1}$$

$$=\frac{e^{-\infty}-1}{e^{-\infty}+1} = \frac{0-1}{0+1} \qquad \because\ e^{-\infty}=\frac{1}{e^{\infty}}=\frac{1}{\infty}=0$$

$$=-1$$

(xi) $\lim_{x\to 0}\dfrac{e^{1/x}-1}{e^{1/x}+1}$; $x>0$

$$=\lim_{x\to 0}\frac{e^{1/x}\left(1-\dfrac{1}{e^{1/x}}\right)}{e^{1/x}\left(1+\dfrac{1}{e^{1/x}}\right)} = \lim_{x\to 0}\frac{\left(1-\dfrac{1}{e^{1/x}}\right)}{\left(1+\dfrac{1}{e^{1/x}}\right)}$$

$$=\frac{1-\dfrac{1}{e^{1/0}}}{1+\dfrac{1}{e^{1/0}}} = \frac{1-\dfrac{1}{e^{\infty}}}{1+\dfrac{1}{e^{\infty}}} = \frac{1-\dfrac{1}{\infty}}{1+\dfrac{1}{\infty}} = \frac{1-0}{1+0} = 1$$

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<u>*Book:*</u> *Exercise 1.3 (Page 27)*
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