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(ii) $P(\text{rain falls on just 3 days}) = P(X=3) = {}^7C_3\, p^3 q^4$

$= 35(0.4)^3(0.6)^4 = \mathbf{0.2903}.$

9.10(a) Let p denote the probability of an odd number on the die. The probability of an even number on the die is $2p$. Then $p + 2p = 1$ or $p = 1/3$. Thus $P(\text{odd}) = 1/3$ and $P(\text{even}) = 2/3$.

Let X denote the number of times even number appears.

(i) $P(\text{no even number}) = P(X=0) = {}^5C_0\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^5 = \mathbf{\frac{1}{243}}$

(ii) $P(\text{all even numbers}) = P(X=5) = {}^5C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^0 = \mathbf{\frac{32}{243}}$

(b) Let p denote the probability that a car stolen will be recovered. Thus $p = 0.6$, $q = 1 - 0.6 = 0.4$. Here $n = 10$. Let X denote the number of cars recovered.

$P(X=3) = {}^{10}C_3\, p^3 q^7 = 120(0.6)^3(0.4)^7 = \mathbf{0.04247}.$

(c) Let p denote the probability that a college students passes a subject. Then $p = 3/5$ so that $q = 1 - p = 1 - 3/5 = 2/5$. Here $n = 5$. Let X denote the number of students who pass.

(i) $P(\text{at least 3 students fail}) = P(3, 4 \text{ or 5 students fail})$

$= P(2, 1, 0 \text{ student pass}) = P(X=2) + P(X=1) + P(X=0)$

$= {}^5C_2\left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right)^3 + {}^5C_1\left(\frac{3}{5}\right)\left(\frac{2}{5}\right)^4 + {}^5C_0\left(\frac{3}{5}\right)^0\left(\frac{2}{5}\right)^5$

$= \frac{720}{3125} + \frac{240}{3135} + \frac{32}{3125} = \mathbf{\frac{992}{3125}}.$

(ii) $P(\text{at most 3 students pass}) = P(X=0)+P(X=1)+P(X=2)+P(X=3)$

$= {}^5C_0\left(\frac{2}{5}\right)^5 + {}^5C_1\left(\frac{3}{5}\right)\left(\frac{2}{5}\right)^4 + {}^5C_2\left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right)^3 + {}^5C_3\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^2$

$= \frac{32 + 240 + 720 + 1080}{3125} = \mathbf{\frac{2072}{3125}}$

9.11(a) Here $p = 1/4$ so that $q = 1 - p = 1/4 = 3/4$, $n = 5$.

(i) $P(X=0) = {}^5C_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^5 = \mathbf{\frac{243}{1024}}.$

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(ii) $P(X \le 3) = {}^5C_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^5 + {}^5C_1\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^4 + {}^5C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3$

$+ {}^5C_3\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2 = \frac{243+405+270+90}{1024} = \frac{1008}{1024}$

Alternatively, $P(X \le 3) = 1 - P(X > 3) = 1 - [P(X=4) + P(X=5)]$

$= 1 - \left[{}^5C_4\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right) + {}^5C_5\left(\frac{1}{4}\right)^5\right] = 1 - \frac{16}{1024} = \frac{1008}{1024}$

(b) If $p = P(\text{six}) = 1/6$, $q = 5/6$, $n = 5$. Let X denote the numbe of the times 'six' occurs. Then X takes the values 0, 1, 2, 3 4, 5. The probabilities of 0, 1, 2, 3, 4, 5, sixes are given b the successive terms in the expansion of $\left(\frac{1}{6}+\frac{5}{6}\right)^5$ i.e. $\left(\frac{5}{6}\right)^5$ ${}^5C_1\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^4, {}^5C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3, {}^5C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2, {}^5C_4\left(\frac{1}{6}\right.$ $\left(\frac{5}{6}\right), \left(\frac{1}{6}\right)^5$ or $\frac{3125}{7776}, \frac{3125}{7776}, \frac{1250}{7776}, \frac{250}{7776}, \frac{25}{7776}, \frac{1}{7776}$.

x	$P(x)$	$xP(x)$	$x^2P(x)$
0	$\frac{3125}{7776}$	0	0
1	$\frac{3125}{7776}$	$\frac{3125}{7776}$	$\frac{3125}{7776}$
2	$\frac{1250}{7776}$	$\frac{2500}{7776}$	$\frac{5000}{7776}$
3.	$\frac{250}{7776}$	$\frac{750}{7776}$	$\frac{2250}{7776}$
4	$\frac{25}{7776}$	$\frac{100}{7776}$	$\frac{400}{7776}$
5	$\frac{1}{7776}$	$\frac{5}{7776}$	$\frac{25}{7776}$
		$\Sigma xP(x) = 6480/7776$	$\Sigma x^2(x) = 10800/7776$

$\mu = E(X) = \Sigma xP(x) = 6480/7776 = 0.83333$

$E(X^2) = 10800/7776 = 1.38889$

$\sigma^2 = E(X^2) - \mu^2 = 1.38889 - (0.83333)^2 = 0.69445.$

(d) Here $N = 400$, $n = 10$, $p = 1/2$ and $q = 1/2$.

Let X denote the number of people who smoke.

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$P(X \le 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= \left(\frac{1}{2}\right)^{10} + {}^{10}C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^9 + {}^{10}C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^8 + {}^{10}C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^7$

$= (1 + 10 + 45 + 120)/1024 = 176/1024$

Number of investigators = 400(176/1024) = 68.75 or 69.

9.12(a) $N(p+q)^n = 600(0.3+0.7)^6 = 600[(0.3)^6 + {}^6C_1\,(0.3)^5(0.7)$
$+ {}^6C_2\,(0.3)^4(0.7)^2 + {}^6C_3\,(0.3)^3(0.7)^3 + {}^6C_4\,(0.3)^2(0.7)^4$
$+ {}^6C_5\,(0.3)(0.7)^5 + (0.7)^6] = 600[0.000729 + 0.010206$
$+ 0.059535 + 0.18522 + 0.324135 + 0.302526 + 0.1176491]$

The successive terms are 0.4374, 6.1236. 35.7210. 111.1320. 194.4810. 181.5156 and 70.5894.

(b) Here $p = P(\text{success}) = P(4, 5 \text{ or } 6) = 3/6 = 1/2$, $q = 1/2$, $n = 5$, and $N = 96$. Expected frequencies are given by the successive terms in the expansion of $N(p+q)^n$.

$96\left(\frac{1}{2}+\frac{1}{2}\right)^5 = 96\left[\left(\frac{1}{2}\right)^5 + {}^5C_1\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right) + {}^5C_2\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2\right.$
$\left. + {}^5C_3\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^3 + {}^5C_4\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^5\right]$

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$= \frac{96}{32}[1 + 5 + 10 + 5 = 1] = [13 + 15 + 30 + 30 + 15 + 3]$

Thus the successive terms are 3, 15, 30, 30, 15 and 3.

9.13(a) For $n = 2$, $\mu = np = 2p$ and $\sigma = \sqrt{npq} = \sqrt{2pq}$ For solution see Section 9.4 in the Text.

(b) Let $p = P(\text{six}) = 1/6$, $q = 5/6$, $n = 4$, $N = 108$. Let X denote the number of times 'six' occurs. The theoretical frequencies for $X = 0, 1, 2, 3, 4$ are given by the successive terms in the expansion of $108\left(\frac{5}{6}+\frac{1}{6}\right)^4$, i.e.

$108\left[\left(\frac{5}{6}\right)^4, {}^4C_1\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^3, {}^4C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2, {}^4C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right),\right.$
$\left.{}^4C_4\left(\frac{1}{6}\right)^4\right]$ or $108\left[\frac{625}{1296}, \frac{500}{1296}, \frac{150}{1296}, \frac{20}{1296}, \frac{1}{1296}\right]$

or = 52.08, 41.67, 12.50, 1.67, 0.08 or = 52, 42, 13, 2, 0.

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x	f	fx
0	52.08	0
1	41.67	41.67
2	12.50	25.00
3	1.67	5.01
4	0.08	0.32
	$n = \Sigma f = 108$	$\Sigma fx = 72$

$\mu = \Sigma fx/n = 72/108 = \mathbf{0.667}$

(c) $\mu = np = 6$ (1) $\sigma = npq = 2.4$ (2)

From equations (1) and (2), $6q = 2.4$ or $q = 0.40$ and $p = \mathbf{0.60}$.

From equation (1), $(0.60)n = 6$ or $n = \mathbf{10}$.

(d) Here $n = 5$, $p = 1/6$, $q = 5/6$. Let X denote the number of times '3' appears.

(i) Using the binomial formula: $P(X = x) = {}^nC_x\, p^x\, q^{n-x}$.

$$P(X=0) = {}^5C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{5-0} = \frac{3125}{7776}$$

$$P(X=1) = {}^5C_1\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{5-1} = \frac{3125}{7776}$$

$$P(X=2) = {}^5C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{5-2} = \frac{1250}{7776}$$

9.14(b) Here $n = 5$. According to the given conditions

$P(X = 0) = P(X = 1)$ or ${}^5C_0\, q^5 = {}^5C_1\, pq^4$ or $q^5 = 5pq^4$ or

$q^5/q^4 = 5p$ or $q = 5p$ or $1 - p = 5p$

or $p = 1/6$ and $q = 1 - p = 5/6$

$$\mu_x = np = 5\left(\frac{1}{6}\right) = \frac{5}{6},\ \sigma_x^2 = npq = 5\left(\frac{1}{6}\right)\left(\frac{5}{6}\right) = \mathbf{\frac{25}{36}}$$

(c) Here $n = 10$, $p = 0.4$ so that $q = 1 - p = 1 - 0.4 = 0.6$.

$\mu_x = np = 10(0.4) = 4$, $\sigma_x^2 = npq = 10(0.4)(0.6) = 2.4$

$$\mu_y = \frac{\mu_x - 10}{6} = \frac{4-10}{6} = -1,\ \sigma_y^2 = \frac{\sigma_x^2}{(6)^2} = \frac{2.4}{36} = \mathbf{0.067}$$

9.15(a) For solution see Section 9.4 in the Text.

(b) $p = 4/5$ $q = \frac{1}{}$

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(i) $P(X=3) = {}^5C_3\,(4/5)^3\,(1/5)^{5-3} = 10(4/5)^3\,(1/5)^2 = \frac{640}{3/25}$

$= 0.2048$

ii) $P(X=5) = n = 10$

$= {}^{10}C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^{10-5} = 252\left(\frac{1}{2}\right)^{10} = \frac{252}{1024}$

$= 0.2461$

9.16(b) $p = P(3 \text{ or } 4) = 2/6 = 1/3$, $q = 1 - p = 2/3$, $n = 4$ and $N = 405$. Let X denote the number times '3 or 4' occurs. The expected frequencies are given by the successive terms in the expansion of

$$405\left(\frac{2}{3}+\frac{1}{3}\right)^4 = 405\left[{}^4C_0\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^0 + {}^4C_1\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)\right.$$

$$\left. + {}^4C_2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right) + {}^4C_3\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^3 + {}^4C_4\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^4\right]$$

$$= 405\left(\frac{16}{81}+\frac{32}{81}+\frac{24}{81}+\frac{8}{81}+\frac{1}{81}\right)$$

The expected frequencies are 80, 160, 120, 40 and 5.

9.17(b) Here $\mu = np = 5$ and $\sigma = \sqrt{npq} = 2.5$, $npq = 6.25$. Thus

$q = \frac{npq}{np} = \frac{6.25}{5} = 1.25$ and $p = 1 - q = 1 - 1.25 = -0.25$.

Thus the statement is wrong because p or q cannot be greater than 1 or negative.

9.18(a) $\mu = np = 60(0.7) = 42$

$\sigma = \sqrt{npq} = \sqrt{60(0.7)(0.3)} = \sqrt{12.6} = \mathbf{3.55}$.

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(b) $\mu = np = 12$ and $\sigma = \sqrt{npq} = 3$ or $npq = 9$. Thus

$\frac{npq}{np} = \frac{9}{12}$ or $q = 3/4$, $p = 1/4$ and $n = 12/p = 12(4) = \mathbf{48}$.

The binomial distribution is $(q+p)^n = \left(\frac{3}{4}+\frac{1}{4}\right)^{48}$

9.19(a) $p = P(\text{defective}) = 0.1$, $q = 0.9$, $n = 400$

$\mu = np = 400(0.1) = \mathbf{40}$

and $\sigma = \sqrt{npq} = \sqrt{400(0.1)(0.9)} = \sqrt{36} = \mathbf{6}$

(b)(i) $\mu = np = 42$ and $\sigma^2 = npq = 12.6$ or $42q = 12.6$ or $q = 0.3$ and $p = \mathbf{0.7}$

$np = 42$ or $n(0.7) = 42$ or $n = \mathbf{60}$.

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(ii) $\mu = np = 36$ and $\sigma = \sqrt{npq} = 4.8$ or $npq = 23.04$ or
$36q = 23.04$ or $q = 0.64$ and $p = \mathbf{0.36}$.
$np = 36$ or $n(0.36) = 36$ or $n = \mathbf{100}$

9.20(a) $\mu = np = 5$ and $\sigma = \sqrt{npq} = 3$ or $npq = 9$ or $5q = 9$ or $q = 1.8$ and $p = 1 - q = \mathbf{-0.8}$.
This is not possible because p or q cannot be greater than 1 or negative.

(b) $\mu = np = 3$ and $\sigma = \sqrt{npq} = 1.5$ or $npq = 2.25$ or $3q = 2.25$ or $q = 0.75$ and $p = \mathbf{0.25}$.
$np = 3$ or $n(0.25) = 3$ or $n = \mathbf{12}$.

9.21(a) $P(\text{result observed once}) = P(X=1) = {}^5C_1\, pq^4 = 5pq^4$
$P(\text{result observed twice}) = P(X=2) = {}^5C_2\, p^2q^3 = 10p^2q^4$
$1024(5pq^4) = 405$ (1) $1024(10\,p^2q^3) = 270$ (2)
Dividing (1) by (2), we get $\frac{q}{2p} = \frac{405}{270}$ or $\frac{q}{p} = 3$.

That is, p and q are in the ratio 1:3. Hence $p = \frac{1}{4}$ and $q = \frac{3}{4}$

(b) Let $p = P(3 \text{ heads})$ and $q = P(1 \text{ tail})$ Let X denote the number of heads.
$P(3 \text{ heads}) = P(X=3) = {}^5C_3\, p^3q^2 = 10p^3q^2$
$P(1 \text{ tail}) = P(4 \text{ heads}) = P(X=4) = {}^5C_4\, p^4q = 5p^4q$
Since $P(3 \text{ heads})$ is twice $P(1 \text{ tail})$, we have
$10\,p^3q^2 = 2(5\,p^4q)$ or $p/q = 1$. Thus p and q are in the ration $1:1$, i.e. $p = \mathbf{1/2}$ and $q = \mathbf{1/2}$.

c) $P(4 \text{ successes in 10 trials})\ P(X=4) = {}^{10}C_4\, p^4q^6$
$= {}^{10}C_4\,(0.2)^4(0.8)^6 = 210(0.2)^4(0.8)^6 = \mathbf{0.08808}$

x	$P(x)$	$xP(x)$
0	q^4	0
1	$4\,pq^3$	$4pq^3$
2	$6\,p^2q^2$	$12\,p^2q^2$