

EXERCISES

- 7.1 (a) Find the probability of each of the following.
(i) A head appears in tossing a fair coin. (ii) A '5' appears in rolling a six-faced cubical die. (iii) An even number appears when a perfect cubical die is rolled. (B.I.S.E., Sargodha 1983)
- (b) A pair of die is rolled. Let A be the event that prime number appears and B be the event that even number appears. Write down the elements of the following events and find their probabilities. (i) A (ii) A' (iii) B (iv) B' (v) $A \cup B$ (vi) $A \cap B$ (vii) $(A \cup B)'$. (B.I.S.E., Sargodha 2003)
- 7.2 (a) From a well-shuffled pack of 52 cards, a card is drawn at random. What is the probability that it is (i) a card of diamonds (ii) an ace (iii) a king of hearts (iv) a pictured card (v) a black card? (B.I.S.E., Lahore 1971, Gujranwala 2003; Multan 1999, 2003, 2008)
- (b) Two coins are tossed. Write down the sample space S and the following events: A = both coins have the same faces, B = a head appears on either coin, C = head appears only on first coin. (B.I.S.E., Gujranwala 2002)
- 7.3 (a) A bag contains 5 white and 4 black balls. Two balls are drawn. Find the probability that (i) both are white (ii) one is white and the other black.
- (b) Six persons are chosen at random from a group containing 5 men, 4 women and 6 children. Find the probability that (i) at least one is a child (ii) exactly two are women (iii) at most one is a man. (B.I.S.E., Lahore 1998)
- 7.4 Show that in a single throw with two dice, the chance of throwing more than 7 is equal to that of throwing less than 7. (B.I.S.E., Lahore 1969; Bahawalpur 2011)
- 7.5 (a) Two dice are thrown. What is the probability that the total score is more than 8?
- (b) A bag contains 12 balls of which 3 are marked. If 5 balls are drawn out together, what is the probability that 3 of the marked balls are among these 5 balls? (B.I.S.E., Lahore 1967, 1982; Multan 2010)
- (c) Two fair dice are thrown once. Find the probability that (i) the sum of two dots is 12 or the two dots are same (ii) the product of two dots is a complete square (iii) the sum of two dots is a complete square. (B.I.S.E., Sargodha 2002; Multan 201; Gujranwala 2011; Lahore 2015)
- 7.6 (a) A well-shuffled pack of 52 cards is dealt among four players, each receiving 13 cards. What is the probability that one of them has all the four aces?
- (b) What is the probability of throwing either "Sum 7" or "Sum more than 10" with two dice? (B.I.S.E., Lahore 1982)
- 7.7 A bag contains 2 red, 3 green, 5 blue and 2 yellow balls. Find the probability that balls of all the colours are represented if a sample of four balls is selected at random. (B.I.S.E., Sargodha 1974, 1983, Lahore 1987, Rawalpindi 1987)

- 7.8 (a) Determine the probability for each of the following events.
- Sum 8 appears in a single toss of a pair of fair dice. 5/36
 - Sum 7 or 11 comes up in a single toss of a pair of fair dice. (B.I.S.E., Lahore 2015)
 - Two dice show the same number. (B.I.S.E., Lahore 2013, 2014)
 - A ball drawn at random from a bag containing 5 red, 6 white, 4 blue and 3 orange balls. Find the probability that it is either red or blue. (B.I.S.E., Lahore 2014)

(b) A bag contains 5 white and 7 black balls. If 3 balls are drawn, what is the probability that (i) all are white, (ii) two are white and one is black,

(iii) all are of the same colour? (B.I.S.E., Lahore 1995, 2018, Multan 1984, 1985, 2014)

(c) Consider two events A and B such that $P(A) = 5/9$, $P(B) = 4/9$ and $P(\bar{B}|A) = 2/3$. Find $P(A|B)$. (B.I.S.E., Lahore 2002)

(d) Consider two events A and B such that $P(\bar{A}) = 1/3$, $P(B|A) = 3/5$ and $P(B|\bar{A}) = 4/5$. Find $P(B)$. (B.I.S.E., Lahore 2002)

7.9 A pair of dice is rolled. List the elements of the sample space S . Let A denote the event "the sum is less than 8" and B the event "a 6 occurs on either die." List the events $A, B, A \cap B, A \cup B, A - B, B - A, (A \cup B) \cup A'$. (B.I.S.E., Gujranwala 1997; Sargodha 2002)

7.10 (a) Two cards are drawn at random from a well-shuffled pack of 52 cards. Find the probability that (i) one is a king and the other a queen, (ii) both are of the same colour, (iii) both are of different colours. (B.I.S.E., Lahore 1969, Rawalpindi 2001; Faisalabad 2017)

(b) A pair of dice is thrown. If two numbers appearing are different, find the probability that (i) the sum is 6 (ii) the sum is 4 or less (iii) the sum is even. (B.I.S.E., Lahore 1994, Gujranwala 1997, Multan 2001)

7.11 A bag contains 9 white and 12 black balls. Find the probability of drawing 5 black balls. (B.I.S.E., Lahore 1965)

7.12 From a bag containing 5 white and 3 black balls, 2 are drawn at random. Find the chance that they are both of the same colour. (B.I.S.E., Lahore 1968, 2011)

7.13 The probability that the brother and sister will pass an entry exam is 0.8 and 0.7 respectively. Find the probability that (i) both will pass (ii) only one will pass (iii) at least one will pass (iv) at most one of them will pass the entry exam. (B.I.S.E., Multan 2002)

7.14 In a poker hand consisting of 5 cards, what is the probability of holding (i) 2 aces and 2 kings (ii) 5 spades? (B.I.S.E., Lahore 1976; Gujranwala 2014)

- 7.15 A bag contains 14 identical balls, 4 of which are red, 5 black and 5 white. Six balls are drawn from the bag. Find the probability that (i) 3 are red, (ii) at least two are white. (B.I.S.E., Sargodha 1983; Multan 2013)
- 7.16 A marble is drawn at random from a box containing 10 red, 30 white, 20 blue and 15 orange marbles. Find the probability that it is (i) orange or red (ii) not 'red or blue' (iii) not blue (iv) white (v) red, white or blue. (B.I.S.E., Lahore 1969, Sargodha 1987)
- 7.17 (a) A and B can solve 70% and 80% of the problems in a book respectively. Find the chance that a problem chosen at random will be solved by at least one of them. (B.I.S.E., Lahore 2012, 2017; Faisalabad 2015; Gujranwala 2018)
- (b) (i) For any two events A and B , it is known that $P(A) = 2/3$, $P(A \cup B) = 7/12$ and $P(A \cap B) = 5/12$. Find $P(B)$. (B.I.S.E., Multan 2010)
- (ii) If A and B are not mutually exclusive events and $P(A) = 0.60$ and $P(B) = 0.08$, then find $P(A \cup B)$. (B.I.S.E., Bahawalpur 2002; Gujranwala 2010)
- 7.18 (a) A class contains 10 men and 20 women, of which half the men and half the women have brown eyes. Find the probability that a person chosen at random is a man or has brown eyes. (B.I.S.E., Sargodha 2001; Gujranwala 2015)
- (b) In a group of 20 adults, 4 out of 7 women and 2 out of 13 men wear glasses. What is the probability that a person chosen at random from the group is a woman or someone who wears glasses? (B.I.S.E., Faisalabad 1993, Lahore 1997)
- 7.19 An integer is chosen at random from the first 50 digits. What is the probability that it is divisible by 6 or 8? (B.I.S.E., Bahalpur 2003; Gujranwala 2017)
- 7.20 In a consignment of bolts, 25% of the bolts are rusted and 30% are defective. Find the probability that a bolt selected at random is rusted or defective.
- 7.21 (a) A coin is tossed twice. Find the conditional probability of getting two heads given at least one head. (B.I.S.E., Lahore 2003)
- (b) A box contains 6 good and 4 defective tubes. Two tubes are drawn. One of them is tested and found to be good. What is the probability that the other one is also good. (B.I.S.E., Lahore 1995, 2003)
- 7.22 (a) A pair of dice is thrown. Find the conditional probability of getting a total of 7 given that the sum is (i) odd (ii) more than 6 (iii) 8 or more. (B.I.S.E., Gujranwala 2002)
- (b) A pair of dice is rolled. It is known that one die has shown a 5. Find the probability that (i) the other die will show a 4. (ii) sum of both the dice will exceed 7.
- 7.23 (a) The probability that a regular scheduled flight departs in time is 0.83, the probability that it arrives in time is 0.92 and the probability that it departs and arrives in time is 0.78. Find the probability that a plane (i) arrives in time given that it departed in time. (ii) departs in time given that it arrived in time.

- (b) A man opens a general store and a book shop. In any year the probability of a robbery in the general store is 0.10 and the probability of a robbery in the book shop is 0.02. For any one year, what is the probability that (i) neither business will be robbed (ii) both will be robbed (iii) either one or the other (but not both) will be robbed? (B.I.S.E., Rawalpindi 2002)
- 7.24 A card is drawn from an ordinary deck. If A is the event of getting a red card and B the event that the card is greater than 2 but less than 9. Find $P(A)$, $P(B)$, $P(A \text{ and } B)$ and $P(A \text{ or } B)$. (B.I.S.E., Lahore 1993)
- 7.25 A box contains nine tickets numbered 1 to 9. If 3 tickets are drawn from the box one at a time, find the probability that they are alternately either odd, even, odd or even, odd, even. (B.I.S.E., Lahore 1974)
- 7.26 (a) Two drawings, each of three balls, are made from a bag containing 5 white and 8 black balls, the balls are not being replaced before the next trial. What is the probability that the first drawing will give 3 white balls and the second 3 black balls? (B.I.S.E., Rawalpindi 1987, Multan 1993)
 (b) Three balls are drawn successively from a box containing 6 red balls, 4 white balls and 5 blue balls. Find the probability that they are drawn in the order red, white, blue if each ball is (i) replaced (ii) not replaced. (B.I.S.E., Sargodha 2003; Multan 2009)
- 7.27 One bag contains 4 white balls and 3 black balls. A second bag contains 3 white balls and 5 black balls. One ball is drawn from the first bag and placed unseen in the second bag. What is the probability that a ball now drawn from the second bag is black? (B.I.S.E., Lahore 1976)
- 7.28 One bag contains 4 white and 2 black balls and another bag contains 3 white and 5 black balls. If one ball is drawn from each bag, find the probability that (i) both are white (ii) both are black (iii) one is white and one is black. (B.I.S.E., Lahore 1967)
- 7.29 A box contains three red, four white and five blue balls. Another box contains five red, six white and seven blue balls. One ball is drawn from each box. Find the probability that both balls are of the same colour.
- 7.30 Two urns contain respectively 3 white, 7 red, 15 black balls; and 10 white, 6 red, 9 black balls. One ball is taken from each urn. What is the probability that both will be of the same colour? (B.I.S.E., Lahore 1978)
- 7.31 Two pairs of dice are rolled once. Find the probability that (i) the absolute difference of the two dots is 2 or 3 (ii) the product of the two dots is 6 or 12 (iii) the sum of the two dots is 7 or 11. (B.I.S.E., Bahawalpur 2002; Gujranwala 2004)
- 7.32 (a) The probability that a man will be alive in 25 years is $\frac{3}{5}$ and that his wife will be alive in 25 years is $\frac{2}{3}$. Find the probability that (i) both will be alive, (ii) only the man will be alive, (iii) only the wife will be alive, (iv) at least one will be alive, (v) neither will be alive in 25 years. (B.I.S.E., Lahore 1974)
 (b) A fair die is tossed twice. Find the probability of getting a 4, 5, or 6 on the first throw and a 1, 2, 3, or 4 on the second throw.

- 7.33 Let A and B be the events with $P(A \cup B) = 7/8$, $P(A \cap B) = 1/4$, $P(A') = 5/8$. Find $P(A)$, $P(B)$ and $P(A \cap B')$. (B.I.S.E., Rawalpindi 2002)
- 7.34 (a) Three cards are drawn at random from an ordinary pack of 52 cards. Find the probability that they will consist of a knave, a queen and a king. (B.I.S.E., Lahore 1987)
- (b) Four persons are chosen at random from a group containing 3 men, 2 women and 4 children. Find the probability that exactly two of them will be children.
- 7.35 A , B and C are taking part in a race. The chance of winning the race by A is half of that of B and B winning the race is half of that of C . Find their respective chances of winning the race. (Federal Board, Islamabad 2002)
- 7.36 Urn A contains 5 red balls and 3 white balls and urn B contains 2 red balls and 6 white balls. (i) If a ball is drawn from each urn, what is the probability that they are both of the same colour? (ii) If two balls are drawn from each urn, what is the probability that all four balls are of the same colour? (B.I.S.E., Lahore 1979)
- 7.37 (a) A can hit a target three times in 5 shots, B two times in 5 shots, C three times in 4 shots. They fire a volley. Find the probability that (i) 2 shots hit (ii) at least 2 shots hit. (B.I.S.E., Multan 1984; Gujranwala 2005)
- (b) Three missiles are fired at a target. The probabilities of hitting the target are 0.4, 0.5 and 0.6 respectively. If the missiles are fired independently, what is the probability that at least two missiles hit the target? (B.I.S.E., Lahore 1994; Rawalpindi 2003)
- (c) The probabilities of hitting the target by three men A , B , C are $3/5$, $2/5$, $1/5$ respectively. Find the probability that (i) two shots hit (ii) one shot hits (iii) all the three shots hit (iv) none hits. (Federal Board, Islamabad 2002)
- 7.38 (a) There are three events A , B , C one of which must and only one can happen. If the odds are 8 to 3 against A , 5 to 2 against B , then find the odds against C . (B.I.S.E., Lahore 1988)
- (b) Supposing that it is 9 to 7 against a person A who is now 35 years of age living till he is 65, and 3 to 2 against a person B now 45 living till he is 75. Find the probability that at least one of these persons will be alive 30 years hence. (B.I.S.E., Lahore 1988)
- 7.39 (a) A bag contains 3 red and 5 black balls, and another bag contains 5 red and 3 black balls. A ball is drawn from a bag selected at random. Find the probability that it is red. (B.I.S.E., Multan 2001)
- (b) A purse contains 2 silver and 4 copper coins, and a second purse contains 4 silver and 3 copper coins. If a coin is selected at random from one of the purses, what is the probability that it is a (i) silver coin (ii) copper coin. (B.I.S.E., Lahore 1966)
- 7.40 (a) One purse contains 1 sovereign and 3 shillings, a second purse contains 2 sovereigns and 4 shillings and a third contains 3 sovereigns and 1 shilling.

If a coin is taken out of the purse selected at random, find the chance that it is a sovereign.

(B.I.S.E., Lahore 1987)

- (b) A set of eight corals contains one joker. Player A chosen 5 cards at random and player B taken the remaining 3 cards. What is the probability that (i) A has a joker (ii) B has a joker (iii) not A has a joker (iv) not B has a joker.

The coloured balls are distributed in three indistinguishable boxes as

7.41

Box	1	2	3
Red	2	4	3
White	3	1	4
Blue	5	3	3

- (i) A box is selected at random from which a ball is selected at random. What is the probability that the selected ball is white?
- (ii) If the ball drawn from the box selected at random is observed to be red; what is the probability that it came from box three?
- 7.42 (a) The probabilities that a student will get an A, a B or a C in statistics course are 0.09, 0.15 and 0.53 respectively. What is the probability that the student will get a grade lower than C?
- (b) If A and B are the events that a consumer testing service will rate a given stereo system very good or good. If $P(A) = 0.22$ and $P(B) = 0.35$, find $P(A \cup B)$.

- 7.43 A and B play 12 games of chess of which 6 are won by A, 4 are won by B and two games end in a tie. They agree to play a tournament consisting of 3 games. Find the probability that (i) A wins all the three games (ii) Two games end in a tie (iii) A and B win alternately (iv) B wins at least one game. (B.I.S.E., Lahore 1966)

- 7.44 (a) Find the probability of boys and girls in families with 3 children, assuming equal probabilities for boys and girls.
- (b) Find the probability of a 'four' turning up at least once in two tosses of a fair die. (B.I.S.E., Lahore 1975)

- 7.45 (a) If A and B are independent events and $P(A) = 0.25$ and $P(B) = 0.60$, then find $P(A \cup B)$.
- (b) For two rolls of a balanced die, find the probability of getting first a '5' and then a number less than 5.
- (c) If two cards are drawn from an ordinary deck of 52 playing cards, what is the probability that they will both be diamonds if the drawing is without replacement? (B.I.S.E., Lahore 1989)

- 7.46 A, B and C in the order cut a pack of cards, replacing them after each cut, on the condition that the first who cuts a heart shall win a prize. Find their respective chances. (B.I.S.E., Lahore 1989)

7.47 A and B throw two dice on the understanding that the first to throw a total of 9 will win. Find their respective chances of success if A takes the first turn.

7.48 Given $P(A) = 0.60$, $P(B) = 0.40$, $P(A \cap B) = 0.24$. Find $P(A|B)$, $P(B|A)$ and $P(A \cup B)$. What is the relationship between A and B?
(B.I.S.E., Lahore 1976, 1994; Rawalpindi 2003)

7.49 A random sample of 200 adults are classified below according to sex and level of education attained. If a person is selected at random from this group, find the probability that (i) the person is a male given that he has secondary education (ii) the person has a college education given that the person is female.

	Male	Female
Elementary	38	45
Secondary	28	50
College	22	17

(B.I.S.E., Lahore 2002)

7.50 (a) An urn contains 6 discs numbered 1 to 6. Two discs are drawn at random from the urn. What is the probability that the sum of the numbers is 8, if they are drawn (i) one after the other with replacement (ii) one after the other without replacement (iii) both are drawn at a time?

(b) There are 2 red, 3 green and 4 black balls in a bag. Another bag contains 3 red, 4 green and 2 black balls. One ball is drawn from the bag selected at random. Find the probability that (i) ball is red, (ii) ball is green (iii) ball is black.
(B.I.S.E., Lahore 2001)

7.51 (a) Five cards are drawn from a pack of 52 cards. Find the probability that (i) there is at least one red card (ii) there is at most one picture card (iii) there are one heart, one diamond, one spade and two clubs.
(b) Three balls are drawn at random from a bag containing 3 red, 3 white and 4 blue balls. Find the probability of (i) no red ball (ii) one red and 2 blue balls (iii) at least two white balls.
(B.I.S.E., Faisalabad 2000)

7.52 (b) Bag A contains 5 white and 7 black balls, bag B contains 4 white and 6 black balls and bag C contains 2 white and 4 black balls. A ball is drawn from a bag selected at random. Find the probability that it is white.

7.53 A bag contains 12 balls of which 3 are marked. If 5 balls are drawn out altogether, what is the probability that 2 are, 3 are, 4 are, 5 are marked balls.
(B.I.S.E., Lahore 2001, 2003)

7.54 Consider two events A and B such that $P(A) = 5/9$, $P(B) = 4/9$ and $P(B|A) = 2/3$. Find $P(A|B)$.
(B.I.S.E., Lahore, 2002)

7.55 Consider two events A and B such that $P(A) = 1/3$, $P(B|A) = 3/5$, $P(B|A) = 4/5$. Find $P(B)$.
(B.I.S.E., Lahore, 2002)