11 SAMPLING AND SAMPLING DISTRIBUTIONS

11.1(a) Here population size $N = 4$ and sample size $n = 2$

Number of possible samples $= (N)^n = (4)^2 = 16$.

These samples are: (1,1) (1,3) (1,5) (1,7) (3,1) (3,3) (3,5) (3,7) (5,1) (5,5) (5,7) (7,1) (7,3) (7,5) (7,7)

The corresponding sample means are:

1 2 3 4 2 3 4 5 3 4 5 6 4 5 6 7 .

The sampling distribution of the mean is given in the following table.

(a) Frequency Distribution			(b) Probability Distribution	
$\bar{X}$	Tally	Frequency (f)	$\bar{x}$	$f(\bar{x})$
1	/	1	1	1/16
2	//	2	2	2/16
3	///	3	3	3/16
4	////	4	4	4/16
5	///	3	5	3/16
6	//	2	6	2/16
7	/	1	7	1/16
		$\Sigma f = 16$		$\Sigma f(\bar{x}) = 1$

Calculation of the mean and variance of the sampling distribution of the mean is outlined below:

$\bar{x}$	1	2	3	4	5	6	7	
$f(\bar{x})$	1/16	2/16	3/16	4/16	3/16	2/16	1/16	
$\bar{x}f(\bar{x})$	1/16	4/16	9/16	16/16	15/16	12/16	7/16	$\Sigma \bar{x} f(\bar{x}) = 64/16$
$x^{-2}f(\bar{x})$	1/16	8/16	27/16	64/16	75/16	72/16	49/16	$\Sigma x^{-2} f(\bar{x}) =$ 296/16 = 18.5

(i) $\mu_{\bar{X}} = \Sigma \bar{x} f(\bar{x}) = 64/16 = 4$

$\mu = \frac{\Sigma X}{N} = \frac{1+3+5+7}{4} = \frac{16}{4} = 4.$ Hence $\mu_{\bar{X}} = \mu$

$\sigma_{\bar{X}}^2 = \Sigma \bar{x}^2 f(\bar{x}) - (\mu_{\bar{X}})^2 = \frac{296}{16} - (4)^2 = 18.5 - 16 = 2.5$

$\sigma^2 = \frac{\Sigma(X-\mu)^2}{N}$

$= \frac{(1-4)^2 + (3-4)^2 + (5-4)^2 + (7-4)^2}{4} = \frac{20}{4} = 5$

$\sigma_{\bar{X}}^2 = \sigma^2/n$ or $\sigma^2 = n\sigma_{\bar{X}}^2 = 2(2.5) = 5$

Thus the population variance is twice the variance of sample means.

(ii) The number of samples is ${}^4C_2 = 6$. These samples are (1, 3), (1, 5), (1, 7), (3, 5), (3, 7), (5, 7). the corresponding sample means are 2, 3, 4, 4, 5, 6.

$\mu_{\bar{X}} = (2+3+4+4+5+6)/6 = 24/6 = 14$

$\sigma_{\bar{X}}^2 = [(2-4)^2 + (3-4)^2 + (4-4)^2 + (5-4)^2 + (6-4)^2]/6$

$= 10/6 = 1.67$

11.1(b) No. of possible samples $= (N)^n = (3)^2 = 9$

$\mu = \frac{\Sigma X}{N} = \frac{1+3+5}{3} = \frac{9}{3} = 3$

$\sigma^2 = \frac{\Sigma(X-\mu)^2}{N}$

$= \left[\frac{(1-3)^2 + (3-3)^2 + (5-3)^2}{3}\right] = \left[\frac{4+0+4}{3}\right]$

$= \frac{8}{3} = 2.67$

$\sigma = 1.63$

Sampling distribution of sample mean.

$\bar{X}$	Tally	f	$f\bar{X}$	$f\bar{X}^2$
1	/	1	1	1
2	//	2	4	8
3	///	3	9	27
4	//	2	8	32
5	/	1	5	25
		$\Sigma f = 9$	27	93

$$\mu_{\bar{X}} = \frac{\Sigma f\bar{X}}{\Sigma f} = \frac{27}{9} = 3$$

1. $\mu_{\bar{X}} = \mu$

$3 = 3$

2. $$\sigma^2_{\bar{X}} = \frac{\Sigma f\bar{X}^2}{\Sigma f} - \left(\frac{\Sigma f\bar{X}}{\Sigma f}\right)^2 = \frac{93}{9} - (3)^2 = 10.33 - 9$$

$$\sigma^2_{\bar{X}} = 1.33 \Rightarrow \sigma_{\bar{X}} = 1.153$$

$$\sigma_{\bar{X}} = \sigma/\sqrt{n}$$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = 1.63/\sqrt{2} = 1.1$$

11.2 Here $N = 6$ and $n = 2$. Number of possible samples with replacement = (6)(6) = 36. These samples are:

(2, 2) (2, 4) (2, 6) (2, 8) (2, 10) (2, 12) (4,2), (4, 4) (4, 6) (4, 8) (4, 10) (4, 12) (6, 2) (6, 4) (6, 6) (6, 8) (6, 10) (6, 12) (8, 2) (8, 4) (8, 6) (8, 8) (8, 10) (8, 12) (10, 2) (10, 4) (10,. 6) (10, 8) (10, 10) (10, 12) (12, 2) (12, 4) (12, 6) (12, 8) (12, 10) (12, 12).

The corresponding sample means are:

2, 3, 4, 5, 6, 7, 3, 4, 5, 6, 7, 8, 4, 5, 6, 7, 8, 9, 5, 6, 7, 8, 9, 10, 6, 7, 8, 9, 10, 11, 7, 8, 9, 10, 11, 12.

The sampling distribution of the mean is given below. Last two columns have been added for calculation of the mean and variance of the sampling distribution.

$\bar{X}$	Tally	f	$\bar{x}$	$f(\bar{x})$	$\bar{x}\,f(\bar{x})$	$\bar{x}^2 f(\bar{x})$
2	/	1	2	1/36	2/36	4/36
3	//	2	3	2/36	6/36	18/36
4	///	3	4	3/36	12/36	48/36
5	////	4	5	4/36	20/36	100/36
6	𝍸	5	6	5/36	30/36	180/36
7	𝍸 /	6	7	6/36	42/36	294/36
8	𝍸	5	8	5/36	40/36	320/36
9	////	4	9	4/36	36/36	324/36
10	///	3	10	3/36	30/36	300/36
11	//	2	11	2/36	22/36	242/36
12	/	1	12	1/36	12/36	144/36
	$\Sigma f = 36$		$\Sigma f(\bar{x}) = 1$		$\Sigma \bar{x} f(\bar{x}) =$ 252/36 = 7	$\Sigma \bar{x}^2 f(\bar{x}) =$ 1974/36

$$\mu_{\bar{X}} = \Sigma \bar{x} f(\bar{x}) = \frac{252}{36} = 7$$

$$\sigma_{\bar{X}}^2 = \Sigma \bar{x}^2 f(\bar{x}) - \mu_{\bar{X}}^2 = \frac{1974}{36} - (7)^2$$

$$= 54.833 - 49 = 5.883$$

(ii) Number of possible samples without replacement $= {}^N C_n = {}^6 C_2 = 15$. These samples are:

(2, 4) (2, 6) (2, 8) (2, 10) (2, 12) (4, 6) (4, 8) (4, 10)
(4, 12) (6, 8) (6, 10) (6, 12) (8, 10) (8, 12) (10, 12).

The corresponding sample means are:

3 4 5 6 7 5 6 7 8 7 8 9 9 10 11

The sampling distribution of the mean is given below where the mean and variance of the sampling distribution of means are calculated.

$\bar{x}$	3	4	5	6	7	8	9	10	11	
$f(\bar{x})$	1/15	1/15	2/15	2/15	3/15	2/15	2/15	1/15	1/15	$\Sigma f(\bar{x}) = 1$
$\bar{x}f(\bar{x})$	3/15	4/15	10/15	12/15	21/15	16/15	18/15	10/15	11/15	$\Sigma \bar{x} f(\bar{x}) = 105/15 = 7$
$\bar{x}^2 f(\bar{x})$	9/15	16/15	50/15	72/15	147/15	128/15	162/15	100/15	121/15	$\Sigma \bar{x}^2 f(\bar{x}) = 805/15$

$$\mu_{\bar{X}} = \Sigma \bar{x} f(\bar{x}) = 105/15 = 7$$

$$\sigma_{\bar{X}}^2 = \Sigma \bar{x}^2 f(\bar{x}) - (\mu_{\bar{X}})^2 = \frac{805}{15} - (7)^2 = 53.67 - 49 = 4.67$$

11.3(a) $N = 2$

2, 4, $n = 3$

No. of possible samples $= (N)^n = (2)^3 = 8$

Sr. No.	Samples	$\bar{X}$
1	2, 2, 2	2
2	2, 2, 4	2.67
3	2, 4, 2	2.67
4	2, 4, 4	3.33
5	4, 2, 2	2.67
6	4, 2, 4	3.33
7	4, 4, 2	3.33
8	4, 4, 4	4

Sampling distribution of sample means.

$\bar{X}$	Tally	f	$f\bar{X}$	$f\bar{X}^2$
2	/	1	2	4
2.67	///	3	8.01	21.3867
3.33	///	3	9.99	33.2667
4	/	1	4	16
		$\Sigma f = 8$	24	74.5534

$$\mu_{\bar{X}} = \frac{\Sigma f \bar{X}}{\Sigma f} = \frac{24}{8} = 3$$

$$\sigma_{\bar{X}}^2 = \left[\frac{\Sigma f \bar{X}^2}{\Sigma f} - \left(\frac{\Sigma f \bar{X}}{\Sigma f} \right)^2 \right]$$

$$\sigma_{\bar{X}}^2 = \left[\frac{74.6534}{8} - (3)^2 \right]$$

$$= 9.331675 - 9 = 0.331675$$

$$\mu = \frac{\Sigma X}{N} = \frac{2+4}{2} = \frac{6}{2} = 3$$

$$\sigma^2 = \frac{\Sigma (X-\mu)^2}{N}$$

$$= \left[\frac{(2-3)^2 + (4-3)^2}{2} \right]$$

$$= \frac{(1)^2 + (1)^2}{2} = \frac{2}{2} = 1$$

$$\sigma_{\bar{X}}^2 = \frac{\sigma^2}{n}$$

$$0.331675 = \frac{\sigma^2}{3}$$

$3(0.331675) = \sigma^2$ Hence proved.

11.3(b) Here $n = 5$, $\mu_{\bar{X}} = 20500$, $\sigma_{\bar{X}}^2 = 25$

A large number of samples from a $N(\mu, \sigma^2)$ is considered as sampling with replacement. Thus

$\mu_{\bar{X}} = \mu$ or $\mu = 20500$.

$\sigma_{\bar{X}}^2 = \sigma^2/n$ or $\sigma^2 = n\sigma_{\bar{X}}^2 = 5(25) = 125$.

11.4(a) Here $N = 6$, $n = 2$. Number of possible samples without replacement $= {}^6C_2 = 15$. These samples are:

(4, 5) (4, 6) (4, 7) (4, 8) (4, 9) (5, 6) (5, 7) (5, 8) (5, 9) (6,7) (6, 8) (6, 9) (7, 8) (7, 9) (8, 9).

The corresponding sample means are:

4.5 5 5.5 6 6.5 5.5 6 6.5 7 6.5 7 7.5 7.5 8 8.5.

The sampling distribution of the mean and computations needed for finding standard error of the mean are given below:

$\bar{x}$	f	$f(\bar{x})$	$\bar{x}f(\bar{x})$	$\bar{x}^2 f(\bar{x})$
4.5	1	1/15	4.5/15	20.25/15
5	1	1/15	5/15	25/15
5.5	2	2/15	11/15	60.5/15
6	2	2/15	12/15	72/15
6.5	3	3/15	19.5/15	126.75/15
7	2	2/15	14/15	98/15
7.5	2	2/15	15/15	112.5/15
8	1	1/15	8/15	64/15
8.5	1	1/15	8.5/15	72.25/15
	$\Sigma f = 15$		$\Sigma\bar{x}f(\bar{x}) =$ 97.5/15	$\Sigma\bar{x}^2 f(\bar{x}) =$ 651.25/15

$$\mu_{\bar{X}} = \Sigma\bar{x}f(\bar{x}) = 97.5/15 = 6.5$$

$$\sigma_{\bar{X}} = \sqrt{\Sigma\bar{x}^2 f(\bar{x}) - [\Sigma\bar{x}f(\bar{x})]^2}$$

$$\sqrt{\frac{651.25}{15} - \left(\frac{97.5}{15}\right)^2} = \sqrt{43.4167 - 42.25} = \sqrt{1.1667} = 1.08$$

$$\mu = \frac{\Sigma X}{N} = \frac{4+5+6+7+8+9}{6} = \frac{39}{6} = 6.5$$

$$\sigma = \sqrt{\frac{\Sigma(X-\mu)^2}{N}}$$

$$= \sqrt{\frac{(4-6.5)^2 + (5-6.5)^2 + (6-6.5)^2 + (7-6.5)^2 + (8-6.5)^2 + (9-6.5)^2}{6}}$$

$$= \sqrt{17.5/6} = \sqrt{2.9167} = 1.7078$$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$$

$$1.08 = \frac{1.7078}{\sqrt{2}}\sqrt{\frac{6-2}{6-1}} = (1.7078)\,(0.6325) = 1.08.$$

(b) Here $N = 5$, $n = 2$. Number of possible samples with replacement $= (N)^n = (5)^2 = 25$. These samples are:

(3, 3) (3,7) (3, 11) (3, 15) (3, 19) (7, 3) (7, 7) (7, 11) (7, 15) (7, 19) (11, 3) (11, 7) (11, 11) (11, 15) (11, 19) (15, 3) (15, 7) (15, 11) (15, 15) (15, 19) (19, 3) (19, 7) (19, 11) (19, 15) (19, 19).

The corresponding samples means are:

3 5 7 9 11 5 7 9 11 13 7 9 11 13 15 9 11 13 15 17 11 13 15 17 19.

The sampling distribution of mean and computations needed for finding the mean and standard deviation of the sampling distribution are given below.

$\bar{x}$	Tally	f	$f(\bar{x})$	$\bar{x}f(\bar{x})$	$\bar{x}^2 f(\bar{x})$
3	/	1	1/25	3/36	9/25
5	//	2	2/25	10/25	50/25
7	///	3	3/25	21/25	147/25
9	////	4	4/25	36/25	324/25
11	~~////~~	5	5/25	55/25	605/25
13	////	4	4/25	52/25	676/25
15	///	3	3/25	45/25	675/25
17	//	2	2/25	34/25	578/25
19	/	1	1/25	19/25	361/25
		$\Sigma f = 25$		$\Sigma \bar{x} f(\bar{x}) =$ 275/25	$\Sigma \bar{x}^2 f(\bar{x}) =$ 3425/25

$\mu_{\bar{X}} = \Sigma \bar{x} f(\bar{x}) = 275/25 = 11.$

$\sigma_{\bar{X}} = \sqrt{\Sigma \bar{x}^2 f(\bar{x}) - (\mu_{\bar{X}})^2}$

$= \sqrt{3425/25 - (11)^2} = \sqrt{137 - 121} = \sqrt{16} = 4$

$\mu = \Sigma X/N = (3 + 7 + 11 + 15 + 19)/5 = 55/5 = 11$

$$\sigma = \sqrt{\frac{\Sigma(X-\mu)^2}{N}}$$

$$= \sqrt{\frac{1}{5}[(3-11)^2 + (7-11)^2 + (11-11)^2 + (15-11)^2 + (19-11)^2]}$$

$$= \sqrt{160/5} = \sqrt{32} = 5.65685$$

Verification 1. $\mu_{\bar{X}} = \mu$ 2. $\sigma_{\bar{X}}^2 = \sigma^2/n = 32/2 = 16$ and $\sigma_{\bar{X}} = \sqrt{16} = 4$

11.5 Here $N = 5$, $n = 2$. Number of possible samples without replacement $m = {}^5C_2 = 10$.

These samples are:

(15, 16) (15, 20) (15, 32) (15, 40) (20, 16) (20, 32) (20, 40) (16, 32) (16, 40) (32, 40).

Corresponding sample means are:

15.5 17.5 23.5 27.5 18 26 30 24 28 36.

$$\mu_{\bar{X}} = \Sigma \bar{x}/m$$

$$= (15.5 + 17.5 + 23.5 + 27.5 + 18 + 26 + 30 + 24 + 28 + 36)/10$$

$$= 246/10 = 24.6$$

$$\sigma_{\bar{X}}^2 = \frac{\Sigma \bar{X}^2}{m} - \mu_{\bar{X}}^2 = \frac{6411}{10} - (24.6)^2 = 641.1 - 605.16 = 35.94$$

$$\mu = \Sigma X/N = (15 + 20 + 16 + 32 + 40)/5 = 123/5 = 24.6$$

$$\sigma^2 = \frac{\Sigma X^2}{N} - \left(\frac{\Sigma X}{N}\right)^2 = \frac{3505}{5} - (24.6)^2 = 701 - 605.16 = 95.84$$

(i) $\mu_{\bar{X}} = \mu = 24.6$

(ii) $\sigma_{\bar{X}}^2 = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right) = \frac{95.84}{2}\left(\frac{5-2}{5-1}\right) = (95.84)(0.375) = 35.94$

11.6(a) Here $N = 5$ and $n = 2$. Number of possible samples with replacement $= (5)^2 = 25$. These samples are:

(20, 20) (20, 24) (20, 28) (20, 32) (20, 36) (24, 20) (24, 24) (24, 28) (24, 32) (24, 36) (28, 20) (28, 24) (28, 28) (28, 32) (28, 36) (32, 20) (32, 24) (32, 28) (32, 32) (32, 36) (36, 20) (36, 24) (36, 28) (36, 32) (36, 36)

Corresponding sample means are:

20 22 24 26 28 22 24 26 28 30 24 26 28 30 32 26 28 30 32 34 28 30 32 34 36.

The sampling distribution of means and computations needed for finding the mean and standard deviation of the sampling distribution of means are given below:

$\bar{x}$	Tally	f	$f(\bar{x})$	$\bar{x}f(\bar{x})$	$\bar{x}^2 f(\bar{x})$
20	/	1	1/25	20/36	400/25
22	//	2	2/25	44/25	968/25
24	///	3	3/25	72/25	1728/25
26	////	4	4/25	104/25	2704/25
28	~~////~~	5	5/25	140/25	3920/25
30	////	4	4/25	120/25	3600/25
32	///	3	3/25	96/25	3072/25
34	//	2	2/25	68/25	2312/25
36	/	1	1/25	36/25	1296/25
		$\Sigma f = 25$		$\Sigma\bar{x}f(\bar{x}) =$ 700/25	$\Sigma\bar{x}^2 f(\bar{x}) =$ 20000/25

$$\mu_{\bar{X}} = \Sigma\bar{x}f(\bar{x}) = \frac{700}{25} = 28$$

$$\sigma_{\bar{X}} = \Sigma\bar{x}^2 f(\bar{x}) - (\mu_{\bar{X}})^2 = \sqrt{\frac{20000}{25} - (28)^2} = \sqrt{800 - 784} = \sqrt{16} = 4.$$

(b) Here $N = 3$, $n = 3$. The number of possible samples with replacement = (3)(3)(3) = 27. These samples are: (3, 3, 3), (3, 3, 6), (3, 3, 9), (3, 6, 3), (3, 6, 6), (3, 6, 9), (3, 9, 3), (3, 9, 6), (3, 9, 9), (6, 6, 6), (6, 6, 3), (6, 6, 9), (6, 3, 6), (6, 9, 6), (6, 3, 9), (6, 9, 3), (6, 3, 3), (6, 9, 9), (9, 9, 3), (9, 9, 6), (9, 9, 9), (9, 3, 9), (9, 6, 9), (9, 3, 3), (9, 6, 6), (9, 3, 6), (9, 6, 3).

The corresponding sample means are 3, 4, 5, 4, 5, 6, 5, 6, 7, 6, 5, 7, 5, 7, 6, 6, 4, 8, 7, 8, 9, 7, 8, 5, 7, 6, 6.

The sampling distribution of the mean and necessary calculations for the mean and variance of the sampling distribution are given in the following table:

$\bar{x}$	f	$f(\bar{x})$	$\bar{x}f(\bar{x})$	$\bar{x}^2 f(\bar{x})$
3	1	1/27	3/27	9/27
4	3	3/27	12/27	48/27
5	6	6/27	30/27	150/27
6	7	7/27	42/27	252/27
7	6	6/27	42/27	294/27
8	3	3/27	24/27	192/27
9	1	1/27	9/27	81/27
	$\sum f = 27$		$\sum \bar{x}f(\bar{x}) = 162/27$	$\sum \bar{x}^2 f(\bar{x}) = 1026/27$

$\mu_{\bar{X}} = E(\bar{X}) = \sum \bar{x}\, f(\bar{x}) = 162/27 = 6$

$\sigma_{\bar{X}}^2 = \text{Var}(\bar{X}) = \sum \bar{x}^2 f(\bar{x}) - (\mu_{\bar{X}})^2 = \dfrac{1026}{27} - (6)^2 = (38-36) = 2$

$\mu = \sum X/n = (3+6+9)/3 = 18/3 = 6$

$\sigma^2 = \dfrac{\sum X^2}{N} - \mu^2 = \dfrac{126}{3} - (6)^2 = 42 - 36 = 6$

(i) $E(\bar{X}) = \mu = 6.$

(ii) $\text{Var}(\bar{X}) = \sigma^2/n = 6/3 = 2.$

11.7(a) Here N=6 and n=2. Number of possible samples $= {}^6C_2 = 15$.

These samples are:

(1, 5) (1, 7) (1, 11) (1, 15) (1, 17) (5, 7) (5, 11) (5, 15) (5, 17)
(7, 11) (7, 15) (7, 17) (11, 15) (11, 17) (15, 17).

The corresponding sample means are:

3 4 6 8 9 6 8 10 11 9 11 12 13 14 16.

(ii) The sampling distribution of means and computations needed for finding the mean and standard deviation of the sampling distribution of the mean are given below:

$\bar{x}$	f	$f(\bar{x})$	$\bar{x}\,f(\bar{x})$	$\bar{x}^2\,f(\bar{x})$
3	1	1/15	3/15	9/15
4	1	1/15	4/15	16/15
6	2	2/15	12/15	72/15
8	2	2/15	16/15	128/15
9	2	2/15	18/15	162/15
10	1	1/15	10/15	100/15
11	2	2/15	22/15	242/45
12	1	1/15	12/15	144/15
13	1	1/15	13/15	169/15
14	1	1/15	14/15	196/15
16	1	1/15	16/15	256/15
	$\Sigma f = 15$	$\Sigma f(\bar{x}) = 1$	$\Sigma \bar{x}\,f(\bar{x}) =$ 140/15	$\Sigma \bar{x}^2\,f(\bar{x}) =$ 1494/15

$$\mu_{\bar{X}} = \Sigma \bar{x}\,f(\bar{x}) = 140/15 = 9.33$$

$$\sigma_{\bar{X}} = \sqrt{\Sigma \bar{x}^2\,f(\bar{x}) - (\mu_{\bar{X}})^2} = \sqrt{\frac{1494}{15} - (9.33)^2}$$

$$= \sqrt{99.6 - 87.0489} = \sqrt{12.5511} = 3.543$$

(iii) $\mu = \Sigma X/N = (1 + 5 + 7 + 11 + 15 + 17)/6 = 56/6 = 9.33$

$$\sigma_{\bar{X}} = \frac{\Sigma X^2}{N} - (\mu)^2 = \frac{710}{6} - (9.33)^2 = 118.3333 - 87.0489 = 31.2844$$

$$\sigma = \sqrt{31.2844} = 5.5932$$

(iv) $\mu_{\bar{X}} = \mu = 9.33$

$$\sigma_{\bar{X}}^2 = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right) = \frac{31.2844}{2}\left(\frac{6-2}{6-1}\right) = 12.5136$$

$$\sigma_{\bar{X}} = \sqrt{12.5136} = 3.537$$

11.7(b) 2, 4, 6, 8

$N = 4, \quad n = 2$

No. of possible samples $= (N)^n = (4)^2 = 16$

Sr. No.	Samples	$\bar{X}$
1	2, 2	2
2	2, 4	3
3	2, 6	4
4	2, 8	5
5	4, 2	3
6	4, 4	4
7	4, 6	5
8	4, 8	6
9	6, 2	4
10	6, 4	5
11	6, 6	6
12	6, 8	7
13	8, 2	5
14	8, 4	6
15	8, 6	7
16	8, 8	8

Sampling distribution of $\bar{X}$

$\bar{X}$	Tally	f	$f\bar{X}$	$f\bar{X}^2$
2	/	1	2	4
3	//	2	6	18
4	///	3	12	48
5	////	4	20	100
6	///	3	18	108
7	//	2	14	98
8	/	1	8	64
		$\Sigma f = 16$	80	440

$$\mu_{\bar{X}} = \frac{\Sigma f\bar{X}}{\Sigma f} = \frac{18}{60} = 5$$

$$\sigma^2_{\bar{X}} = \frac{\Sigma f\bar{X}^2}{\Sigma f} - \left(\frac{\Sigma f\bar{X}^2}{\Sigma f}\right)^2$$

$$\sigma_{\bar{X}}^2 = \left[\frac{440}{16} - \left(\frac{80}{16}\right)^2\right]$$

$$= 27.5 - 25 = 2.5$$

$$\mu = \frac{\Sigma X}{N} = \frac{2 = 4 + 6 + 8}{4} = \frac{20}{4} = 5$$

$$\sigma^2 = \frac{\Sigma(X - \mu)^2}{N}$$

$$= \frac{[(2-5)^2 + (4-5)^2 + (6-5)^2 + (8-5)^2]}{2}$$

$$= \frac{(-3)^2 + (1)^2 + (1)^2 + (3)^2}{4}$$

$$= \frac{9 = 1 + 1 + 9}{4} = \frac{20}{4} = 5$$

(i) $\mu_{\bar{X}} = \mu$

$5 = 5$

(ii) $\sigma_{\bar{X}}^2 = \sigma^2/n$

$\sigma_{\bar{X}}^2 = 2.5$

$\sigma_{\bar{X}}^2 = \frac{5}{2}$

$= 2.5$ Hence proved.

11.8(a) $\mu = \frac{\Sigma X}{N} = \left(\frac{3 + 7 + 8 + 12 + 15}{6}\right) = 45/6 = 9$

$$\sigma = \sqrt{\frac{\Sigma(X - \mu)^2}{N}}$$

$$= \sqrt{[(3-9)^2 + (7-9)^2 + (8-9)^2 + (12-9)^2 + (15-9)^2]/5}$$

$$= \sqrt{\frac{86}{5}} = \sqrt{17.2} = 4.1473$$

Here $N = 5$ and $n = 2$. Number of possible samples with replacement = 5(5) = 25. These samples are:

(3, 3) (3, 7) (3, 8) (3, 12) (3, 15) (7, 3) (7, 7) (7, 8) (7, 12) (7, 15) (8, 3) (8, 7) (8, 8) (8, 12) (8, 15) (12, 3) (12, 7) (12, 8) (12, 12) (12, 15) (15, 3) (15, 7) (15, 8) (15, 12) (15,15)

The corresponding sample means are:

3, 5, 5.5, 7.5, 9, 5, 7, 7.5, 9.5, 11 5.5, 7.5, 8, 10, 11.5 7.5, 9.5, 10, 12, 13.5, 9, 11, 11.5, 13.5, 15

The sampling distribution of the mean and computations needed for finding the mean and standard deviation of the sampling distribution of the mean are given below:

$\bar{x}$	f	$f(\bar{x})$	$\bar{x}\,f(\bar{x})$	$\bar{x}^2 f(\bar{x})$
3	1	1/25	3/25	9/25
5	2	2/25	10/25	50/25
5.5	2	2/25	11/25	60.5/25
7	1	1/25	7/25	49/25
7.5	4	4/25	30/25	225/25
8	1	1/25	8/25	64/25
9	2	2/25	18/25	162/25
9.5	2	2/25	19/25	180.5/25
10	2	2/25	20/25	200/25
11	2	2/25	22/25	242/25
11.5	2	2/25	23/25	264.5/25
12	1	1/25	12/25	144/25
13.5	2	2/25	27/25	364.5/25
15	1	1/25	15/25	225/25
	$\Sigma f = 25$		$\Sigma \bar{x}\,f(\bar{x}) =$ 225/25	$\Sigma \bar{x}^2 f(\bar{x}) =$ 2240/25

$$\mu_{\bar{X}} = \Sigma \bar{x}\,f(\bar{x}) = 225/25 = 9$$

$$\sigma_{\bar{X}} = \Sigma \bar{x}^2 f(\bar{x}) - (\mu_{\bar{X}})^2 = \frac{2440}{25} - (9)^2 = \sqrt{89.6 - 81} = \sqrt{8.6} = 2.93$$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{4.1473}{\sqrt{2}} = \frac{4.1473}{1.4142} = 2.93$$

(b) $N = 6$ and the population values are 4, 8, 8, 12, 12, 16.

Number of possible samples of size $n = 2$ which can be drawn without replacement is ${}^6C_2 = 15$. These samples are

(4, 8), (4, 8), (4, 12), (4, 12), (4, 16), (8, 8), (8, 12), (8, 12), (8, 12) (8, 16), (8, 12), (8, 12), (8, 16), (12, 12), (12, 16), (12, 16).

The corresponding sample means are

6, 6, 8, 8, 10, 8, 10, 10, 12, 10, 10, 12, 12, 14, 14

The sampling distribution of the mean and the computations of the mean and variance of the sampling distribution is shown in the following table:

$\bar{x}$	f	$f(\bar{x})$	$\bar{x} f(\bar{x})$	$\bar{x}^2 f(\bar{x})$
6	2	2/15	12/15	72/15
8	3	3/15	24/15	192/15
10	5	5/15	50/15	500/15
12	3	3/15	36/15	432/15
14	2	2/15	28/15	392/15
	$\Sigma f = 15$	$\Sigma f(\bar{x}) = 1$	$\Sigma \bar{x} f(\bar{x}) = 150/15$	$\Sigma \bar{x}^2 f(\bar{x}) = 1588/15$

$\mu_{\bar{X}} = 150/15 = 10$

$$\sigma_{\bar{X}}^2 = \Sigma\, \bar{x}^2 f(\bar{x}) - (\mu_{\bar{X}})^2 = \frac{1588}{15} - (10)^2 = 105.8667 - 100 = 5.8667$$

$\mu = \Sigma x\, f(x) = (4 + 16 + 24 + 16)/6 = 60/6 = 10$

$\sigma^2 = \Sigma x^2 f(x) - \mu^2 = (16 + 128 + 288 + 256)/6 - (10)^2$

$= 688/6 - 100 = 114.6667 - 100 = 14.6667$

$\mu_{\bar{X}} = \mu = 10$

$$\sigma_{\bar{X}}^2 = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right) = \frac{14.6667}{2}\left(\frac{6-2}{6-1}\right) = 5.8667$$

(c) Here $N = 7$ and $n = 2$. Number of possible samples without replacement is ${}^7C_2 = 21$. These samples are

(2, 3), (2, 4), (2, 5), (2, 7), (2, 8), (2, 10), (3, 4), (3, 5), (3, 7), (3, 8), (3, 10), (4, 5), (4, 7), (4, 8), (4, 10), (5, 7), (5, 8), (5, 10), (7, 8), (7, 10), (8, 10).

The corresponding sample means are

2.5, 3, 3.5, 4.5, 5, 6, 3.5, 4, 5, 5.5, 6.5, 4.5, 5.5, 6, 7, 6, 6.5, 7.5, 7.5, 8.5, 9.

The sampling distribution of the mean and computations of its mean and variance are shown in the following table.

$\bar{x}$	f	$f(\bar{x})$	$\bar{x}f(\bar{x})$	$\bar{x}^2 f(\bar{x})$
2.5	1	1/21	2.5/21	6.25/21
3.0	1	1/21	3.0/21	9.0/21
3.5	2	2/21	7.0/21	24.5/21
4.0	1	1/21	4.0/21	16.0/21
4.5	2	2/21	9.0/21	40.5/21
5.0	2	2/21	10.0/21	50.0/21
5.5	2	2/21	11.0/21	60.5/21
6.0	3	3/21	18.0/21	108.0/21
6.5	2	2/21	13.0/21	84.5/21
7.0	1	1/21	7.0/21	49.0/21
7.5	2	2/21	15.0/21	112.5/21
8.5	1	1/21	8.5/21	72.25/21
9.0	1	1/21	9.0/21	81.0/21
	$\Sigma f = 21$	$\Sigma f(\bar{x}) = 1$	$\Sigma\bar{x}f(\bar{x}) = 117/21$	$\Sigma\bar{x}^2 f(\bar{x}) = 714/21$

$$\mu_{\bar{X}} = \Sigma\bar{x}f(\bar{x}) = 117/21 = 5.57$$

$$\sigma_{\bar{X}}^2 = \Sigma\bar{x}^2 f(\bar{x}) - \mu_{\bar{X}}^2 = 714/21 - (5.57)^2 = 34 - 31.0249 = 2.9751$$

$$\mu = \Sigma X/N = 39/7 = 5.57$$

$$\sigma^2 = \sum X^2/N - \mu^2 = 267/7 - (5.57)^2 = 38.1429 - 31.0249 = 7.1180$$

(i) $\mu_{\bar{X}} = \mu = 5.57$

(ii) $\sigma_{\bar{X}}^2 = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right) = \frac{7.1180}{2}\left(\frac{7-2}{7-1}\right) = 2.9658$

11.9(a) $P = 0.36,\ n = 100$

(i) $\mu_{\hat{p}} = p$

$\mu_{\hat{p}} = 0.36$

(ii) $\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.36)(1-0.36)}{100}} = 0.048$

(b) $P = 0.65$

$n = 200$

(i) $\mu_{\hat{p}} = P$

$= 0.65$

(ii) $\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.65)(1-0.65)}{200}} = 0.0337$

11.10(i) $\mu = \frac{\Sigma X}{N} = \frac{2+2+4+6+8}{5} = \frac{22}{5} = 4.4$

$$\sigma^2 = \frac{\Sigma X^2}{N} - \mu^2 = \frac{124}{5} - (4.4)^2 = 24.8 - 19.36 = 5.44$$

(ii) Possible samples of two children without replacement $= {}^5C_2 = 10$. These samples are:

(2, 2) (2, 4) (2, 6) (2, 8) (2, 4) (2, 6) (2, 8) (4, 6) (4, 8) (6, 8).

The corresponding sample means are:

2 3 4 5 3 4 5 5 6 7.

The sampling distribution of the mean and the computations needed to find its mean and variance are shown in the following table.

$\bar{x}$	f	$f(\bar{x})$	$\bar{x} f(\bar{x})$	$\bar{x}^2 f(\bar{x})$
2	1	1/10	2/10	4/10
3	2	2/10	6/10	18/10
4	2	2/10	8/10	32/10

$\bar{x}$	f	$f(\bar{x})$	$\bar{x}f(\bar{x})$	$\bar{x}^2 f(\bar{x})$
5	3	3/10	15/10	75/10
6	1	1/10	6/10	36/10
7	1	1/10	7/10	49/10
	$\Sigma f = 10$		$\Sigma\bar{x}f(\bar{x}) = 44/10$	$\Sigma\bar{x}^2 f(\bar{x}) = 214/10$

(iii) $\mu_{\bar{X}} = \Sigma\bar{x}f(\bar{x}) = 44/10 = 4.4$

$$\sigma_{\bar{X}} = \sqrt{\Sigma\bar{x}^2 f(\bar{x}) - (\mu_{\bar{X}})^2} = \sqrt{(214/10) - (4.4)^2}$$

$$= \sqrt{21.4 - 19.36} = \sqrt{2.04} = 1.428$$

11.11 We have $\mu_{\bar{X}} = 10$, $\sigma_{\bar{X}}^2 = 2.5$ and $n = 2$

$$\mu = \mu_{\bar{X}} = 10$$

$$\sigma_{\bar{X}}^2 = \sigma^2/n \text{ or } \sigma^2 = n\sigma_{\bar{X}}^2 = 2(2.5) = 5.$$

11.12 Here $N = 6$ and $n = 3$. Number of possible samples without replacement $= {}^6C_3 = 20$. These samples are:

(13, 15, 25) (13, 15, 40) (13, 15, 42) (13, 15, 48) (13, 25, 40)
(13, 25, 42) (13, 25, 48) (13, 40, 42) (13, 40, 48) (13, 42, 48)
(15, 25, 40) (15, 25, 42) (15, 25, 48) (15, 40, 42) (15, 40, 48)
(15, 42, 48) (25, 40, 42) (25, 40, 48) (25, 42, 48) (40, 42, 48)

Corresponding sample means are:

17.67 22.67 23.33 25.33 26.00 26.67 28.67 31.67
33.67 34.33 26.67 27.33 29.33 32.33 34.33 35.00
37.67 35.67 38.33 43.33

The sampling distribution of the mean and the computations needed for finding the mean of the sampling distribution are given below:

$\bar{x}$	f	$f(\bar{x})$	$\bar{x}$	f	$f(\bar{x})$	$\bar{x}$	f	$f(\bar{x})$
17.67	1	1/20	27.33	1	1/20	34.33	2	2/20
22.67	1	1/20	28.67	1	1/20	35.00	1	1/20

$\bar{x}$	f	$f(\bar{x})$	$\bar{x}$	f	$f(\bar{x})$	$\bar{x}$	f	$f(\bar{x})$
23.33	1	1/20	29.33	1	1/20	35.67	1	1/20
25.33	1	1/20	31.67	1	1/20	37.67	1	1/20
26.00	1	1/20	32.33	1	1/20	38.33	1	1/20
26.67	2	2/20	33.67	1	1/20	43.33	1	1/20
							$\Sigma f=20$	

$\mu = \Sigma X/N = (13 + 15 + 25 + 40 + 42 + 48)/6 = 183/6 = 30.5$

$\sigma^2 = (\Sigma X^2/N) - \mu^2 = 6687/6 - (30.5)^2 = 1114.5 - 930.25 = 184.25$

$\mu_{\bar{X}} = \Sigma \bar{x} f(\bar{x}) = 610.01/20 = 30.5$

$$\sigma_{\bar{X}}^2 = \frac{\Sigma \bar{x}^2}{20} - \left(\frac{\Sigma x}{20}\right)^2$$

$$\sigma_{\bar{X}}^2 = \Sigma \bar{x}^2 f(\bar{x}) - (\mu_{\bar{X}})^2 = \frac{19342.2}{20} - (30.5)^2$$

$= 967.11 - 930.25 = 36.86$

(i) $\mu_{\bar{X}} = \mu = 30.5$

(ii) $\sigma_{\bar{X}}^2 = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right) = \frac{184.25}{3}\left(\frac{6-3}{6-1}\right) = 36.85.$

11.13(b)(i) $\mu = \Sigma X/N = (3 + 3 + 7 + 9 + 14)/5 = 36/5 = 7.2$

$$\sigma^2 = \frac{\Sigma(X-\mu)^2}{N}$$

$= [(3-7.2)^2 + (3-7.2)^2 + (7-7.2)^2 + (9-7.2)^2 + (14-7.2)^2]/5$

$= 84.8/5 = 16.96$

(ii) Number of possible samples without replacement $= {}^5C_3 = 10.$
These sample are:
(3, 3, 7) (3, 3, 9) (3, 3, 14) (3, 7, 9) (3, 7, 14) (3, 9, 14) (3, 7, 9) (3, 7, 14) (3, 9, 14) (7, 9, 14).
The corresponding sample means are:
4.33 5 6.67 6.33 8 8.67 6.33 8 8.67 10.

(iii) The sampling distribution of the mean of tyre lives and the computations needed for finding the mean and variance of the sampling distribution are given below:

$\bar{x}$	f	$f(\bar{x})$	$\bar{x}\,f(\bar{x})$	$\bar{x}^2 f(\bar{x})$
4.33	1	1/10	4.33/10	18.7489/10
5.00	1	1/10	5/10	25/10
6.33	2	2/10	12.66/10	80.1378/10
6.67	1	1/10	6.67/10	44.4889/10
8.00	2	2/10	16/10	128/10
8.67	2	2/10	17.34/10	150.3378/10
10.00	1	1/10	10/10	100/10
	$\Sigma f = 10$		$\Sigma \bar{x}\,f(\bar{x}) = 72/10$	$\Sigma \bar{x}^2 f(\bar{x}) =$ 546.7134/10

$$\mu_{\bar{X}} = \Sigma \bar{x}\,f(\bar{x}) = 72/10 = 7.2$$

$$\sigma_{\bar{X}}^{2} = \Sigma \bar{x}^2 f(\bar{x}) - (\mu_{\bar{X}})^2 = \frac{546.7134}{10} - (7.2)^2 = 54.67 - 51.84 = 2.83$$

(iv) $\mu_{\bar{X}} = \mu = 7.2$

$$\sigma_{\bar{X}}^{2} = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right) = \frac{16.96}{3}\left(\frac{5-3}{5-1}\right) = 2.83$$

11.14(i) $\mu = \Sigma X/N = (4 + 5 + 7 + 9 + 10)/5 = 35/5 = 7$

$$\sigma^2 = \frac{\Sigma(X-\mu)^2}{N}$$

$$= [(4-7)^2 + (5-7)^2 + (7-7)^2 + (9-7)^2 + (10-7)^2]/5$$

$$= 26/5 = 5.2$$

(ii) For sampling with replacement

$$\sigma_{\bar{X}}^{2} = \sigma^2/n = \frac{5.2}{3} = 1.7333$$

For sampling without replacement

$$\sigma_{\bar{X}}^{2} = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right) = \frac{5.2}{3}\left(\frac{5-3}{5-1}\right) = 0.8667.$$

(iii) Number of possible samples with replacement = $(5)^3 = 125$.

These samples and the sample means ($\bar{X}$) are given in the following table.

Sample	$\bar{X}$	Sample	$\bar{X}$	Sample	$\bar{X}$
4,4,4	4	4,10,10	8.00	5,10,9	8.00
4,4,5	4.33	5,4,4	4.33	5,10,10	8.33
4,4,7	5.00	5,4,5	4.67	7,4,4	5.00
4,4,9	5.67	5,4,7	5.33	7,4,5	5.33
4,4,10	6.00	5,4,9	6.00	7,4,7	6.00
4,5,4	4.33	5,4,10	6.33	7,4,9	6.67
4,5,5	4.67	5,5,4	4.67	7,4,10	7.00
4,5,7	5.33	5,5,5	5.00	7,5,4	5.33
4,5,9	6.00	5,5,7	5.67	7,5,5	5.67
4,5,10	6.33	5,5,9	6.33	7,5,7	6.33
4,7,4	5.00	5,5,10	6.67	7,5,9	7.00
4,7,5	5.33	5,7,4	5.33	7,5,10	7.33
4,7,7	6.00	5,7,5	5.67	7,7,4	6.00
4,7,9	6.67	5,7,7	6.33	7,7,5	6.33
4,7,10	7.00	5,7,9	7.00	7,7,7	7.00
4,9,4	5.67	5,7,10	7.33	7,7,9	7.67
4,9,5	6.00	5,9,4	6.00	7,7,10	8.00
4,9,7	6.67	5,9,5	6.33	7,9,4	6.67
4,9,9	7.33	5,9,7	7.00	7,9,5	7.00
4,9,10	7.67	5,9,9	7.67	7,9,7	7.67
4,10,4	6.00	5,9,10	8.00	7,9,9	8.33
4,10,5	6.33	5,10,4	6.33	7,9,10	8.67
4,10,7	7.00	5,10,5	6.67	7,10,4	7.00
4,10,9	7.67	5,10,7	7.33	7,10,5	7.33

Sample	$\bar{X}$	Sample	$\bar{X}$	Sample	$\bar{X}$
7,10,7	8.00	9,9,4	7.33	10,5,9	8.00
7,10,9	8.67	9,9,5	7.67	10,5,10	8.33
7,10,10	9.00	9,9,7	8.33	10,7,4	7.00
9,4,4	5.67	9,9,9	9.00	10,7,5	7.33
9,4,5	6.00	9,9,10	9.33	10,7,7	8.00
9,4,7	6.67	9,10,4	7.67	10,7,9	8.67
9,4,9	7.33	9,10,5	8.00	10,7,10	9.00
9,4,10	7.67	9,10,7	8.67	10,9,4	7.67
9,5,4	6.00	9,10,9	9.33	10,9,5	8.00
9,5,5	6.33	9,10,10	9.67	10,9,7	8.67
9,5,7	7.00	10,4,4	6.00	10,9,9	9.33
9,5,9	7.67	10,45	6.33	10,9,10	9.67
9,5,10	8.00	10,4,7	7.00	10,10,4	8.00
9,7,4	6.67	10,4,9	7.67	10,10,5	8.33
9,7,5	7.00	10,4,10	8.00	10,10,7	9.00
9,7,7	7.67	10,5,4	6.33	10,10,9	9.67
9,7,9	8.33	10,5,5	6.67	10,10,10	10.00
9,7,10	8.67	10,5,7	7.33		

The sampling distribution of the mean and the computations needed for finding $\sigma_{\bar{X}}^2$ are given below:

$\bar{x}$	Tally	f	$f(\bar{x})$	$\bar{x}\,f(\bar{x})$	$\bar{x}^2\,f(\bar{x})$
4.00	/	1	1/125	4/125	16/125
4.33	///	3	3/125	12.99/125	56.2467/125
4.67	///	3	3/125	14.01/125	65.4267/125
5.00	////	4	4/125	20/125	100/125
5.33	~~////~~ /	6	6/125	31.98/125	170.4534/125
5.67	~~////~~ /	6	6/125	34.02/125	192.8934/125
6.00	~~////~~ ~~////~~ //	12	12/125	72/125	432/125

$\bar{x}$	Tally	f	$f(\bar{x})$	$\bar{x}f(\bar{x})$	$\bar{x}^2 f(\bar{x})$
6.33	~~IIII~~ ~~IIII~~ II	12	12/125	75.96/125	480.8268/125
6.67	~~IIII~~ IIII	9	9/125	60.03/125	400.4001/125
7.00	~~IIII~~ ~~IIII~~ III	13	13/125	91/125	637/125
7.33	~~IIII~~ IIII	9	9/125	65.97/125	483.5601/125
7.67	~~IIII~~ ~~IIII~~ II	12	12/125	92.04/125	705.9468/125
8.00	~~IIII~~ ~~IIII~~ II	12	12/125	96/125	768/125
8.33	~~IIII~~ I	6	6/125	49.98/125	416.3334/125
8.67	~~IIII~~ I	6	6/125	52.02/125	451.0134/125
9.00	IIII	4	4/125	36/125	324/125
9.33	III	3	3/125	27.99/125	261.1467/125
9.67	III	3	3/125	29.01/125	280.5267/125
10.00	I	1	1/125	10/125	100/125
		$\Sigma f = 125$		$\Sigma\bar{x}f(\bar{x}) =$ 875/125	$\Sigma\bar{x}^2 f(\bar{x}) =$ 6341.7742/125

$\mu_{\bar{X}} = \Sigma\bar{x}f(\bar{x}) = 875/125 = 7.$

$\sigma_{\bar{X}}^2 = \Sigma\bar{x}^2 f(\bar{x}) - \mu_{\bar{X}}^2 = 6341.7742/125 - (7)^2 = 50.7342 - 49$

$= 1.7342.$

Number of possible samples without replacement

$m = {}^5C_3 = 10$. These samples are:

(4, 5, 7) (4, 5, 9) (4, 5, 10) (4, 7, 9) (4, 7, 10) (4, 9, 10)
(5, 7, 9) (5, 7, 10) (5, 9, 10) (7, 9, 10).

The corresponding sample means are:

5.33 6 6.33 6.67 7 7.67 7 7.33 8 8.67

$\mu_{\bar{X}} = \Sigma\bar{x}/m = (5.33 + 6 + 6.33 + 6.67 + 7 + 7.67 + 7 + 7.33$

$+ 8 + 8.67)/10 = 70/10 = 7.$

$\sigma_{\bar{X}}^2 = \Sigma\bar{x}^2/m - \mu_{\bar{X}}^2 = 498.6934/10 - (7)^2 = 0.8693.$

11.15(i) Here $N_1 = N_2 = 3$ and $n_1 = n_2 = 2$. There are (3)(3) = 9 possible samples which can be drawn with replacement from each population. These samples and their corresponding means are:

Population 1 (4, 4) (4, 6) (4, 8) (6, 4) (6, 6) (6, 8) (8, 4) (8, 6) (8, 8) and $\bar{X}_1$: 4, 5, 6, 5, 6, 7, 6, 7, 8.

Population 2 (1, 1) (1, 1) (1, 3) (1, 1) (1, 1) (1, 3) (3, 1) (3, 1) (3, 3) and $\bar{X}_2$: 1, 1, 2, 1, 1, 2, 2, 2, 3

Possible differences between the sample means $(\bar{X}_1 - \bar{X}_2)$ are given in the following table.

$\bar{X}_2$ \ $\bar{X}_1$	4	5	6	5	6	7	6	7	8
1	3	4	5	4	5	6	5	6	7
1	3	4	5	4	5	6	5	6	7
2	2	3	4	3	4	5	4	5	6
1	3	4	5	4	5	6	5	6	7
1	3	4	5	4	5	6	5	6	7
2	2	3	4	3	4	5	4	5	6
2	2	3	4	3	4	5	4	5	6
2	2	3	4	3	4	5	4	5	6
3	1	2	3	2	3	4	3	4	5

(ii) The sampling distribution of the differences between means and the computations needed for computing the mean and variance of the sampling distribution are given below:

$\bar{x}_1 - \bar{x}_2$	Tally	f	$f(\bar{x}_1 - \bar{x}_2)$	$(\bar{x}_1 - \bar{x}_2)\cdot f(\bar{x}_1 - \bar{x}_2)$	$(\bar{x}_1 - \bar{x}_2)^2\cdot f(\bar{x}_1 - \bar{x}_2)$
1	/	1	1/81	1/81	1/81
2	~~////~~ /	6	6/81	12/81	24/81
3	~~////~~ ~~////~~ ~~////~~	15	15/81	45/81	135/81
4	~~////~~ ~~////~~ ~~////~~ ~~////~~ //	22	22/81	88/81	352/81
5	~~////~~ ~~////~~ ~~////~~ ~~////~~ /	21	21/81	105/81	525/81

$\bar{x}_1 - \bar{x}_2$	Tally	f	$f(\bar{x}_1 - \bar{x}_2)$	$(\bar{x}_1 - \bar{x}_2)\delta \cdot f(\bar{x}_1 - \bar{x}_2)$	$(\bar{x}_1 - \bar{x}_2)^2 \cdot f(\bar{x}_1 - \bar{x}_2)$
6	~~||||~~ ~~||||~~ ||	12	12/81	72/81	432/81
7	||||	4	4/81	28/81	196/81
		$\Sigma f = 81$		$\Sigma(\bar{x}_1 - \bar{x}_2) \cdot f(\bar{x}_1 - \bar{x}_2) = 351/81$	$\Sigma(\bar{x}_1 - \bar{x}_2)^2 \cdot f(\bar{x}_1 - \bar{x}_2) = 1665/81$

$$\mu_{\bar{X}_1 - \bar{X}_2} = \Sigma(\bar{x}_1 - \bar{x}_2) f(\bar{x}_1 - \bar{x}_2) = 351/81 = 4.3333$$

$$\sigma^2_{\bar{X}_1 - \bar{X}_2} = \Sigma(\bar{x}_1 - \bar{x}_2)^2 f(\bar{x}_1 - \bar{x}_2) - (\mu_{\bar{X}_1 - \bar{X}_2})^2$$

$$= 1665/81 - (4.33)^2 = 20.5556 - 18.7775 = 1.7781.$$

$$\mu_1 = \Sigma X_1/N_1 = (4 + 6 + 8)/3 = 18/3 = 6$$

$$\sigma_1^2 = \Sigma X_1^2/N_1 - \mu_1^2 = 116/3 - (6)^2 = 38.6667 - 36 = 2.6667$$

$$\mu_2 = \Sigma X_2/N_2 = (1 + 1 + 3)/3 = 5/3 = 1.6667$$

$$\sigma_2^2 = \Sigma X_2^2/N_2 - \mu_2^2 = 11/3 - (1.6667)^2 = 3.6667 - 2.7779$$

$$= 0.8888.$$

(iii) $\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2 = 6 - 1.6667 = 4.3333$

$$\sigma^2_{\bar{X}_1 - \bar{X}_2} = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} = \frac{2.6667}{2} + \frac{0.8888}{2} = 1.778.$$

11.16 Here $N_1 = 3$, $n_1 = 2$, $N_2 = 2$ and $n_2 = 3$. There are (3)(3) = 9 possible samples with replacement from population 1 and (2)(2)(2) = 8 possible samples with replacement from population 2. These samples are their corresponding means are

Population 1 (3, 3) (3, 4) (3, 5) (4, 3) (4, 4) (4, 5) (5, 3) (5, 4) (5, 5) and $\bar{X}_1$: 3, 3.5, 4, 3.5, 4, 4.5, 4, 4.5, 5

Population 2 (0, 0, 0) (0, 0, 3) (0, 3, 0) (0, 3, 3) (3, 0, 0) (3, 0, 3) (3, 3, 0) (3, 3, 3) and $\bar{X}_2$: 0, 1, 1, 2, 1, 2, 2, 3

(i) Possible differences between sample means ($\bar{X}_1 - \bar{X}_2$) are given in the following table.

$\bar{X}_2$ \ $\bar{X}_1$	3	3.5	4	3.5	4	4.5	4	4.5	5
0	3	3.5	4	3.5	4	4.5	4	4.5	5
1	2	2.5	3	2.5	3	3.5	3	3.5	4
1	2	2.5	3	2.5	3	3.5	3	3.5	4
2	1	1.5	2	1.5	2	2.5	2	2.5	3
1	2	2.5	3	2.5	3	3.5	3	3.5	4
2	1	1.5	2	1.5	2	2.5	2	2.5	3
2	1	1.5	2	1.5	2	2.5	2	2.5	3
3	0	0.5	1	0.5	1	1.5	1	1.5	2

(ii) The sampling distribution of ($\bar{X}_1 - \bar{X}_2$) and computations needed for finding its mean and variance are outlined below:

$(\bar{x}_1-\bar{x}_2)$	Tally	f	$f(\bar{x}_1-\bar{x}_2)$	$(\bar{x}_1-\bar{x}_2)\,f(\bar{x}_1-\bar{x}_2)$	$(\bar{x}_1-\bar{x}_2)^2\,f(\bar{x}_1-\bar{x}_2)$
0	/	1	1/72	0	0
0.5	//	2	2/72	1/72	0.5/72
1.0	|||| /	6	6/72	6/72	6/72
1.5	|||| ///	8	8/72	12/72	18/72
2.0	|||| |||| ///	13	13/72	26/72	52/72
2.5	|||| |||| //	12	12/72	30/72	75/72
3.0	|||| |||| ///	13	13/72	39/72	117/72
3.5	|||| ///	8	8/72	28/72	98/72
4.0	|||| /	6	6/72	24/72	96/72
4.5	//	2	2/72	9/72	40.5/72
5.0	/	1	1/72	5/72	25/72
		$\Sigma f = 72$		$\Sigma(\bar{x}_1-\bar{x}_2)\,f(\bar{x}_1-\bar{x}_2) = 180/72$	$\Sigma(\bar{x}_1-\bar{x}_2)^2\,f(\bar{x}_1-\bar{x}_2) = 528/72$

$$\mu_{\bar{X}_1 - \bar{X}_2} = \Sigma(\bar{x}_1 - \bar{x}_2)\, f(\bar{x}_1 - \bar{x}_2) = \frac{180}{72} = 2.5$$

$$\sigma^2_{\bar{X}_1 - \bar{X}_2} = \Sigma(\bar{x}_1 - \bar{x}_2)^2 f(\bar{x}_1 - \bar{x}_2) - \mu^2_{\bar{X}_1 - \bar{X}_2} = \frac{528}{72} - (2.5)^2$$

$$= 7.33 - 6.25 = 1.08$$

$$\mu_1 = \Sigma X_1/N_1 = (3 + 4 + 5)/3 = 4$$

$$\sigma_1^{\,2} = \Sigma(X_1 - \mu_1)^2/N_1 = [(3-4)^2 + (4-4)^2 + (5-4)^2]/3 = 2/3$$

$$\mu_2 = \Sigma X_2/N_2 = (0 + 3)/2 = 1.5$$

$$\sigma_2^{\,2} = \Sigma(X_2 - \mu_2)^2/N_2 = [(0-1.5)^2 + (3-1.5)^2]/2 = 2.25$$

(ii) $\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2 = 4 - 1.5 = 2.5$

$$\sigma^2_{\bar{X}_1 - \bar{X}_2} = \frac{\sigma_1^{\,2}}{n_1} + \frac{\sigma_2^{\,2}}{n_2} = \frac{1}{2}\cdot\frac{2}{3} + \frac{2.25}{3} = 0.33 + 0.75 = 1.08$$

11.17 Here $N = 6$ and $n = 3$. There are ${}^6C_3 = 20$ possible sample which can be drawn with replacement from this population. These samples are

(2, 3, 4) (2, 3, 5) (2, 3 6) (2, 3, 8) (2, 4, 5) (2, 4, 6) (2, 4, 8)
(2, 5, 6) (2, 5, 8) (2, 6, 8) (3, 4, 5) (3, 4, 6) (3, 4, 8) (3, 5, 6)
(3, 5, 8) (3, 6, 8) (4, 5, 6) (4, 5, 8) (4, 6, 8) (5, 6, 8)

The corresponding sample proportions P are

2/3, 1/3, 2/3, 2/3, 2/3, 1, 1, 2/3, 2/3, 1, 1/3, 2/3, 2/3, 1/3, 1/3, 2/3, 2/3, 2/3, 1, 2/3.

The sampling distribution of the proportion is given in the following table where computation of its mean and variance is also outlined.

$P = \hat{p}$	f	$f(\hat{p})$	$\hat{p}\, f(\hat{p})$	$\hat{p}^2\, f(\hat{p})$
1/3	4	4/20	1/15	1/45
2/3	12	12/20	2/5	4/15
1	4	4/20	1/5	1/5
		$\Sigma f(\hat{p}) = 1$	$\Sigma\hat{p}\, f(\hat{p}) = 10/15$	$\Sigma\hat{p}^2 f(\hat{p}) = 22/45$

$$\mu_P = \Sigma\hat{p}\, f(\hat{p}) = \frac{10}{15} = 0.67$$

$$\text{Var}(P) = \sigma_P^2 = \Sigma\hat{p}^2 f(\hat{p}) - \mu_P^2 = \frac{22}{45} - (0.67)^2$$

$$= 0.4889 - 0.4489 = 0.04$$

There are 4 even numbers in the population

$p = X/N = 4/6 = 0.67$

(i) $\mu_P = p = 0.67$

(ii) $\sigma_P^2 = \frac{p(1-p)}{n}\left(\frac{N-n}{N-1}\right) = \frac{(0.67)(0.33)}{3}\left(\frac{6-3}{6-1}\right) = 0.04$

11.18(i) Since there are $X = 3$ boys in the population of $N = 5$, $p = X/N = 3/5 = 0.6$.

(ii) There are $M = {}^5C_3 = 10$ possible samples of size $n = 3$ without replacement from this population. These samples are (1, 2, 3) (1, 2, 4) (1, 2, 5) (2, 3, 4) (2, 3, 5) (3, 4, 5) (1, 3, 4) (1, 3, 5) (1, 4, 5) (2, 4, 5)

(iii) The corresponding proportions of boys of these samples are

1/3, 2/3, 2/3, 1/3, 1/3, 2/3, 2/3, 2/3, 1, 2/3

The sampling distribution of the proportion is given in the following table

$P = \hat{p}$	f	$f(\hat{p})$	$\hat{p}\, f(\hat{p})$	$\hat{p}^2 f(\hat{p})$
1/3	3	3/10	1/10	1/30
2/3	6	6/10	4/10	8/30
1	1	1/10	1/10	1/10
	$\Sigma f = 10$		$\Sigma\hat{p}f(\hat{p}) = 6/10$	$\Sigma\hat{p}^2 f(\hat{p}) = 12/30$

11.19 Here $N = 7$ and $n = 3$. Number of possible samples without replacement $= {}^7C_3 = 35$. These samples and the proportions of odd numbers are given in the following table.

Sample	P	Sample	P	Sample	P
1,1,2	2/3	1,4,4	1/3	1,4,5	2/3
1,1,3	1	1,4,5	2/3	2,3,4	1/3

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Chapter # 11 Sampling and Sampling Distributions Solutions

Sample	P	Sample	P	Sample	P
1,1,4	2/3	1,4,5	2/3	2,3,4	1/3
1,1,4	2/3	1,2,3	2/3	2,3,5	2/3
1,1,5	1	1,2,4	1/3	2,4,4	0
1,2,3	2/3	1,2,4	1/3	2,4,5	1/3
1,2,4	1/3	1,2,5	2/3	2,4,5	1/3
1,2,4	1/3	1,3,4	2/3	3,4,4	1/3
1,2,5	2/3	1,3,4	2/3	3,4,5	2/3
1,3,4	2/3	1,3,5	1	3,4,5	2/3
1,3,4	2/3	1,4,4	1/3	4,4,5	1/3
1,3,5	1	1,4,5	2/3		

The sampling distribution of the proportion and the computation of the mean and variance of the sampling distribution is outlined below:

$P=\hat{p}$	Tally	f	$f(\hat{p})$	$\hat{p}f(\hat{p})$	$\hat{p}^2f(\hat{p})$
0	/	1	1/35	0	0
1/3	//// //// //	12	12/35	4/35	4/105
2/3	//// //// //// ///	18	18/35	12/35	8/35
1	////	4	4/35	4/35	4/35
		$\Sigma f = 35$		$\Sigma\hat{p}f(\hat{p}) = 20/35$	$\Sigma P^2 f(\hat{P}) = 40/105$

$$\mu_P = \Sigma\hat{p}\, f(\hat{p}) = 20/35 = 0.571$$

$$\sigma_P{}^2 = \Sigma\hat{p}^2 f(\hat{p}) - \mu_p{}^2 = \frac{40}{105} - \left(\frac{20}{35}\right)^2 = 0.38095 - 0.32653$$

$$= 0.05442$$

There are 4 odd numbers in the population,

$p = X/n = 4/7 = 0.571$

(i) $\mu_{\hat{p}} = p = 0.571$

(ii) $\sigma^2_{\hat{p}} = \frac{p(1-p)}{n}\left(\frac{N-n}{N-1}\right) = \frac{(0.571)(0.429)}{3}\left(\frac{7-3}{7-1}\right) = 0.0544$

11.20(a) Here $N = 5$ and $n = 2$. There are $M = (5)^2 = 25$ possible samples of size 2 which can be drawn with replacement. The possible samples with corresponding means $(\bar{X})$ and variances (S^2) are given in the following table.

Sample	$\bar{X}$	S^2	Sample	$\bar{X}$	S^2
2,2	2	0	6,8	7	1
2,4	3	1	6,10	8	4
2,6	4	4	8,2	5	9
2,8	5	9	8,4	6	4
2,10	6	16	8,6	7	1
4,2	3	1	8,8	8	0
4,4	4	0	8,10	9	1
4,6	5	1	10,2	6	16
4,8	6	4	10,4	7	9
4,10	7	9	10,6	8	4
6,2	4	4	10,8	9	1
6,4	5	1	10,10	10	0
6,6	6	0		$\sum\bar{X} = 150$	$\sum S^2 = 100$

$\mu_{\bar{X}} = \sum\bar{X}/M = 150/25 = 6$

$\mu_{S^2} = \sum S^2/M = 100/25 = 4.$

(b) Here $N = 4$ and $n = 3$. The number of samples of size 3 without replacement $M = {}^4C_3 = 4$. These samples are

(0, 3, 6), (0, 3, 9), (0, 6, 9), (3, 6, 9).

The corresponding sample means are 3, 4, 5, 6.

$\mu_{\bar{X}} = \sum\bar{X}/M = (3+4+5+6)/4 = 18/4 = 4.5$

$\sigma^2_{\bar{X}} = \frac{\sum\bar{X}^2}{M} - \mu^2_{\bar{X}} = \frac{86}{4} - (4.5)^2 = 21.5 - 20.25 = 1.25$

$\mu = \sum X/N = (0+3+6+9)/4 = 18/4 = 4.5$

$$\sigma^2 = \frac{\sum X^2}{N} - \mu^2 = \frac{126}{4} - (4.5)^2 = 31.5 - 20.25 = 11.25$$

(i) $\mu_{\bar{X}} = \mu = 4.5$ (ii) $\sigma_{\bar{X}}^2 = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right) = \frac{11.25}{3}\left(\frac{4-3}{4-1}\right) = 1.25$

11.21(a) Here $N = 4$ and $n = 2$. The $M = {}^4C_2 = 6$ possible samples which can be drawn without replacement are
(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4).

The sample means are $\bar{X}$ 1.5 2 2.5 2.5 3 3.5

The sample variances are S^2 0.25 1 2.25 0.25 1 0.25

$$\mu_{\bar{X}} = \frac{\sum \bar{X}}{M} = \frac{15}{6} = 2.5 \text{ and } \mu_{S^2} = \frac{\sum S^2}{M} = \frac{5}{6} = 0.833$$

(b) Here $N = 5$ and $n = 2$. There are $(5)^2 = 25$ possible samples which can be drawn with replacement. These samples, sample means and unbiased sample variances computed by the formulae $\bar{X} = \sum X/n$ and $s^2 = \sum(X - \bar{X})^2/(n-1)$ respectively are given below:

Sample	$\bar{X}$	s^2	Sample	$\bar{X}$	s^2	Sample	$\bar{X}$	s^2
3,3	3	0	7,19	13	72	15,15	15	0
3,7	5	8	11,3	7	32	15.19	17	8
3,11	7	32	11,7	9	8	19,3	11	128
3,15	9	72	11,11	11	0	19,7	13	72
3,19	11	128	11,15	13	8	19,11	15	32
7,3	5	8	11,19	15	32	19,15	17	8
7,7	7	0	15,3	9	72	19,19	19	0
7,11	9	8	15,7	11	32			
7,15	11	32	15,11	13	8			

$$\mu = \frac{\sum X}{N} = \frac{3+7+11+15+19}{5} = \frac{55}{5} = 11$$

$$\sigma^2 = \frac{(3-11)^2 + (7-11)^2 + (11-11)^2 + (15-11)^2 + (19-11)^2}{5}$$

$$= \frac{160}{5} = 32.$$

11.22(a) Here $N = 4$ and $n = 2$. There are $M = {}^4C_2 = 6$ possible samples of size 2 which can be drawn without replacement. These samples are:

(2, 8), (2, 12), (2, 18), (8, 12), (8, 18), (12, 18)

Sample means ($\bar{X}$) are: 5 7 10 10 13 15

Sample variances (S^2) are: 9 25 64 4 25 9

$$\mu_{\bar{X}} = \frac{\sum \bar{X}}{M} = \frac{5+7+10+10+13+15}{6} = \frac{60}{6} = 10$$

$$\mu_{S^2} = \frac{\sum S^2}{M} = \frac{9+25+64+4+25+9}{6} = \frac{136}{6} = 22.67$$

$$\mu = \frac{\sum X}{N} = \frac{2+8+12+18}{4} = \frac{40}{4} = 10$$

$$\sigma^2 = \frac{\sum (X-\mu)^2}{N} = \frac{(2-10)^2 + (8-10)^2 + (12-10)^2 + (18-10)^2}{4}$$

$$= \frac{136}{4} = 34.$$

(i) $\mu_{\bar{X}} = \mu = 10$

(ii) $\mu_{S^2} = \left(\frac{N}{N-1}\right)\left(\frac{n-1}{n}\right)\sigma^2 = \left(\frac{4}{4-1}\right)\left(\frac{2-1}{2}\right)(34)$

$= 22.67$

11.22(b) 2, 4, 6

$N = 3, n = 2$

No. of possible samples $= (3)^2 = 9$

Sr. No.	Samples	$\bar{X}$	$s^2 = \frac{\sum(X-\bar{X})}{n-1}$
1	2, 2	2	0
2	2, 4	3	2
3	2, 6	4	8
4	4, 2	3	2
5	4, 4	4	0
6	4, 6	5	2
7	6, 2	4	8
8	6, 4	5	2
9	6, 6	6	0

Sampling distribution of sample means and variances.

$\bar{X}$	Tally	f	$f\bar{X}$
2	/	1	2
3	//	2	6
4	///	3	12
5	//	2	10
6	/	1	6
		$\Sigma f = 9$	$\Sigma f\bar{X}$

$$\mu_{\bar{X}} = \frac{36}{9} = 4$$

$$\mu_{s^2} = \frac{24}{9} = 2.67$$

$$\mu = \frac{\Sigma X}{N} = \frac{2+4+6}{3} = \frac{12}{3} = 4$$

(i) $\mu_{\bar{X}} = \mu = 4$

(ii) $E(s^2) = \sigma^2 = 2.67$

$$\sigma^2 = \frac{\Sigma(X-\mu)}{N} = \frac{[(2-4)^2 + (4-4)^2 + (4-6)^2]}{3}$$

$$= \frac{4+0+4}{3} = 8/3 = 2.67$$

11.23 Here $N = 5$ and $n = 2$. There are $M = {}^5C_2 = 10$ possible samples without replacement. These samples are:

(1, 2), (1, 2), (1, 6), (1, 8), (2, 2), (2, 6), (2, 8), (2, 6), (2, 8), (6, 8)

The corresponding sample means $\bar{X}$ and sample variances S^2

$\bar{X}$	1.5	1.5	3.5	4.5	2	4	5	4	5	7
S^2	0.25	0.25	6.25	12.25	0	4	9	4	9	1

$$\mu_{\bar{X}} = \frac{\Sigma\bar{X}}{M} = \frac{38}{10} = 3.8 \text{ and } \mu_{S^2} = \frac{\Sigma S^2}{M} = \frac{47}{10} = 4.7$$

11.25 $n_1 = 2$ 3, 4, 5, W.R.

$n_2 = 2$ 0, 3 W.R.

(i) Construct the simplify dist. of ($\bar{X}1 - \bar{X}2$).

(ii) Show that $\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2$

and $\sigma^2_{\bar{x}_1-\bar{x}_2} = \frac{\sigma_1^{\,2}}{n_1} + \frac{\sigma_2^{\,2}}{n_2}$

Sol.

Sample	$\bar{X}_1$	Sample	$\bar{X}_2$
3, 3	3	0, 0	0
3, 4	3.5	0.3	1.5
3, 5	4	3, 0	1.5
4, 3	3.5	3, 3	3
4, 4	4		
4, 5	4.5		
5, 3	4		
5, 4	4.5		
5, 5	5		

$\bar{X}_2$ / $\bar{X}_1$	$(\bar{X}_1 - \bar{X}_2)$			
	0	1.5	1.5	3.0
3	3	1.5	1.5	0
3.5	3.5	2.0	2.0	0.5
4.0	4.0	2.5	2.5	1.0
3.5	3.5	2.0	2.0	0.5
4.0	4.0	2.5	2.5	1.0
4.5	4.5	3.0	3.0	1.5
4.0	4.0	2.5	2.5	1.0
4.5	4.5	3.0	3.0	1.5
5.0	5.0	3.5	3.5	2.0

$(\bar{X}_1-\bar{X}_2)$	Tally	f	$f(\bar{x})$	$(\bar{x}_1-\bar{x}_2)f(\bar{u})$	$(\bar{x}_1-\bar{x}_2)^2 f(\bar{u})$
0	/	1	1/36	0	0
0.5	//	2	2/36	1/36	0.5/36
1.0	///	3	3/36	3/36	9/36
1.5	////	4	4/36	6/36	20/36
2.0	~~////~~	5	5/36	10/36	37.5/36

$(\bar{X}_1-\bar{X}_2)$	Tally	f	$f(\bar{x})$	$(\bar{x}_1-\bar{x}_2)f(\bar{u})$	$(\bar{x}_1-\bar{x}_2)^2f(\bar{u})$
2.5	𝍸	6	6/36	15/36	45/36
3.0	𝍸	5	5/36	15/36	49/36
3.5	////	4	4/36	14/36	48/36
4.0	///	3	3/36	12/36	40.5/36
4.5	//	2	2/36	9/36	25/36
5.0	//	1	1/36	5/36	
				90/36	277.5/36

$$\mu_{\bar{x}_1-\bar{x}_2} = \Sigma(\bar{x}_1 - \bar{x}_2)f(\bar{u}) = \frac{90}{36} = 2.5$$

$$\sigma^2_{(\bar{x}_1-\bar{x}_2)} = \Sigma(\bar{x}_1 - \bar{x}_2)^2 f(\bar{x}) - (\Sigma \bar{x}_1 - \bar{x}_2) f(\bar{x}))^2$$

$$= \frac{277.5}{36} - \left(\frac{90}{36}\right)^2 = 7.7083 - 6.25$$

$$\sigma^2_{(\bar{x}_1-\bar{x}_2)} = 1.4583 \text{ or } 1.46$$

For population μ and σ^2

$$\mu_1 = \frac{\Sigma X}{N_1} = \frac{12}{3} = 4$$

$$\sigma_1^2 = \frac{\Sigma(X_1-\mu_1)^2}{N_1} = \frac{(3-4)^2 + (4-4)^2 + (5-4)^2}{3}$$

$$\sigma_1^2 = \frac{2}{3} = 0.67$$

$$\mu_2 = \frac{\Sigma X_2}{N_2} = \frac{3}{2} = 1.5$$

$$\sigma_2^2 = \frac{\Sigma(X_2-\mu_2)^2}{N_2} = \frac{(0-1.5)^2 + (3-1.5)^2}{2}$$

$$= \frac{4.5}{2} = 2.25$$

$$\mu_{\bar{x}_1-\bar{x}_2} = \mu_1 - \mu_2$$

$$2.5 = 4 - 1.5 = 0.25$$

$$\sigma^2_{\bar{X}_1-\bar{X}_2} = \frac{\sigma_1{}^2}{n_1} + \frac{\sigma_2{}^2}{n_2} = \frac{0.67}{2} + \frac{2.75}{2} = 1.46$$

11.26 $N = 10, \quad \sigma^2 = 3.15$

$n = 6, \quad \sigma_{\bar{X}} = ? \quad$ W.O.R

Sol. $$\sigma^2_{\bar{X}} = \frac{N-n}{N-1} \times \frac{\sigma^2}{n} = \frac{10-6}{10-1} \times \frac{3.15}{6} = 0.2333$$

$$\sigma^2_{\bar{X}} = 0.4830$$

11.27 $n_1 = 2, \quad \mu_1 = 6, \quad \sigma_1{}^2 = 2.67$

$n_2 = 2, \quad \mu_2 = 2, \quad \sigma_2{}^2 = 0.67$

$$\mu_{\bar{X}_1-\bar{X}_2} = \mu_1 - \mu_2 = 6 - 2 = 4$$

$$\sigma^2_{\bar{X}_1-\bar{X}_2} = \frac{\sigma_1{}^2}{n_1} + \frac{\sigma_2{}^2}{n_2} = \frac{2.67}{2} + \frac{0.67}{2}$$

$$\sigma^2_{\bar{X}_1-\bar{X}_2} = 1.67$$

$$\sigma_{\bar{X}_1-\bar{X}_2} = \sqrt{1.67} = 1.29$$

11.28(a) $\mu = 50, \sigma 2 = 250$

$\bar{X}$ and $\sigma^2_{\bar{X}}$ (i) $n = 25$

(i) $\mu = \mu_{\bar{X}} = 50$

$$\sigma^2_{\bar{X}} = \frac{\sigma^2}{n} = \frac{250}{25} = 10$$

(ii) $n = 100$

$\mu = \mu_{\bar{X}} = 50$

$$\sigma^2_{\bar{X}} = \frac{\sigma^2}{n} = \frac{250}{100} = 2.5$$

(iii) $\mu_{\bar{X}} = 50, \quad n = 1250$

$$\sigma^2_{\bar{X}} = \frac{\sigma^2}{n} = \frac{250}{1250} = 0.2$$

(iv) $n = 225$

$\mu_{\bar{X}} = 50$

$$\sigma^2_{\bar{X}} = \frac{250}{225} = 1.11$$

(b) $\mu = 7, \quad n = 6$

$\sigma^2 = 3.15, \quad N = 10$

$\sigma = 1/7748$ or 1.8

Standard Error

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}} = \frac{1.8}{\sqrt{6}} \cdot \sqrt{\frac{10-6}{10-1}} = \frac{1.8}{2.45}\sqrt{\frac{4}{9}}$$

$$= 0.735 \times 0.67 = 0.492 \quad \text{or} \quad .49$$

11.29(a)

x	f(x)	xf($\bar{x}$)	$\bar{x}$ f(x)
4	0.2	0.8	3.2
5	0.4	2	10
6	0.3	1.8	10.8
7	0.1	0.7	4.9
		5.3	28.9

$\mu = \Sigma xf(x) = 5.3$

$\sigma^2 = \Sigma x^2 f(x) - (\Sigma x\, f(x))^2 = 28.9 - (5.3)^2 = 28.9 - 28.09$

$\sigma^2 = 0.81$

$\sigma = 0.9$

$\mu_{\bar{X}} = \mu = 5.3$

$$\sigma^2_{\bar{X}} = \frac{\sigma^2}{n} = \frac{0.81}{36} = 0.225$$

11.29(b) See Example 11.11

11.30(a) 2, 4, 6,

$n_1 = 2$ W.O.R.

1, 3, 5

$n_2 = 2$ W.O.R.

$$\mu_2 = \frac{1+3+5}{3} = \frac{9}{3} = 3$$

$$\sigma_1^2 = \frac{\Sigma(X_2 - \mu_2)}{N_2} = \frac{(1-3)^2 + (3-3)^2 + (5-3)^2}{3}$$

$$\sigma_2^2 = \frac{4+0+4}{3} = \frac{8}{3}$$

$$\mu_1 = \frac{2+4+6}{3} = \frac{12}{3} = 4$$

$$\sigma_1^2 = \frac{\Sigma(X_1-\mu_1)}{N_2} = \frac{(2-4)^2+(4-4)^2+(4-6)^2}{3}$$

$$= \frac{4+0+4}{3} = 8/3$$

Total Sample $C_2^3 = 3$

Sample	$\bar{X}_1$		Sample	$\bar{X}_2$
2, 4	3	2, 4, 6, $n_1 = 2$	1, 3	2
2, 6	4	1, 3, 5, $n_2 = 2$	1, 5	3
4, 6	5		3, 5	4

$\bar{X}_2$ \ $\bar{X}_1$	3	4	5
2	1	2	3
3	0	1	2
4	–1	0	1

$\bar{X}_1 - \bar{X}_2$	tally	f	$f(\bar{x}_1 - \bar{x}_2)$	$(\bar{X}_1 - \bar{X}_2)\, f(\bar{x}_1 - \bar{x}_2)$
– 1	/	1	1/9	– 1/9
0	//	2	2/9	0
1	///	3	3/9	3/9
2	//	2	2/9	4/9
3	/	1	1/9	3/9
		9		9/9 = 1

$\mu_{\bar{X}_1 - \bar{X}_2} = \Sigma(\bar{X}_1 - \bar{X}_2)\, f(\bar{x}_1 - \bar{x}_2)$

$\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2$

$1 = 4 - 3 = 1$

$1 = 1$ proved

11.30(b) $N = 2500$

$n = 200$

$P = 0.20, \quad q = 1 - p = 1 - 0.20 = 0.80$

$E(\hat{P}) = \mu_{\hat{p}} = p = 0.20$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.20)(0.80)}{200}}$$

$$\sigma_{\hat{p}} = 0.02828 \quad \text{(W.R.)}$$

(ii) $$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{N-n}{N-1}}$$

$$= 0.02828\sqrt{\frac{2500-200}{2500-1}}$$

$$\sigma_{\hat{p}} = 0.0260 \quad \text{(W.O.R.)} = 0.02828 \times 0.9204$$

(c) 3, 7, 18, 2, 40, 42

$$\bar{X}_2 = \frac{\Sigma X}{n} = \frac{3+7+18+22+40+42}{6}$$

$$\bar{X} = \frac{134}{6} = 22.33$$

Unbiased estimate of μ is 22.33 i.e. $\hat{\mu} = 22.33$

$$s^2 = \frac{\Sigma(X-\bar{X})^2}{n-1}$$

$$s^2 = \frac{(3-22.33)^2+(7-22.33)+(18-22.33)+(22-22.33)^2+(40-22.33)^2+(42-22.33)^2}{5}$$

$$= \frac{373.78+235.01+18.75+0.11+312.23+386.91}{5}$$

$$s^2 = \frac{1326.78}{5} = 265.356$$

Unbiased estimate of σ^2 is 265.356 i.e. $\hat{\sigma}^2 = 265.356$

11.31 $p = 0.60$

$q = 1 - p = 0.40$

$n = 150$

Mean $= \mu\hat{p} = p = 0.60$

$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.60)(0.40)}{120}} = \sqrt{\frac{0.24}{120}} = 0.045$$

11.32

X	P(X)
4	0.3 or 3/10
5	0.4 or 5/10
6	0.2 or 2/10

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11.32

X	P(X)
4	0.3 or 3/10
5	0.4 or 5/10
6	0.2 or 2/10

n = 2

W.O.R.

$C_2^{10} = 45$

4, 4, 4, 5, 5, 5, 5, 5, 6, 6

4, 4, 4, 5, 5, 5, 5, 6, 6

Sample	$\bar{X}$	Sample	$\bar{X}$
4, 4	4	5, 5	5
4, 4	4	5, 5	5
4, 5	4, 5	5, 5	5
4, 5	4, 5	5, 5	5
4, 5	4, 5	5, 6	5, 5
4, 5	4, 5	5, 6	5, 5
4, 5	4, 5	5, 5	5
4, 6	5	5, 5	5
4, 6	5	5, 5	5
4, 4	4	5, 6	5, 5
4, 5	4, 5	5, 6	5, 5
4, 5	4, 5	5, 5	5
4, 5	4, 5	5, 5	5
4, 5	4, 5	5, 6	5, 5
4, 5	4, 5	5, 6	5, 5
4, 6	5	5, 5	5
4, 6	5	5, 6	5, 5
4, 5	4, 5	5, 6	5, 5
4, 5	4, 5	5, 6	5, 5
4, 5	4, 5	6, 6	6
4, 5	4, 5		
4, 6	5		
4, 6	5		

11.32

$\bar{X} = x$	tally	f	$f(\bar{x})$	$\bar{x}f(\bar{x})$	$\bar{x}^2 f(\bar{x})$
4	///	3	3/45	12/45	48/45
4.5	~~////~~ ~~////~~ ~~////~~	15	15/45	67.5/45	303.75/45
5	~~////~~ ~~////~~ ~~////~~ /	16	16/45	80/45	400/45
5.5	~~////~~ ~~////~~	10	10/45	55/45	302.5/45
6	1	1	1/45	6/45	36/45
		45		220.5/45	1090.25/45

$$\mu_{\bar{X}} = \Sigma\, \bar{x}\, f(\bar{x}) = 4.9$$

$$\sigma^2_{\bar{x}_1 - \bar{x}_2} = \Sigma\, \bar{x}^2 f(\bar{x}) - (\Sigma\, \bar{x}\, f(\bar{x}))^2$$

$$= 24.228 - (4.9)^2 = 0.218$$

$$\mu = \Sigma X/N$$

$$\mu = \frac{4 + 4 + 4 + 5 + 5 + 5 + 5 + 5 + 6 + 6}{10}$$

$$\mu = 4.9$$

$$\sigma^2 = \frac{(X-4)^2}{N}$$

$$= \frac{(4-4.9)^2+(4-4.9)^2+(4-4.9)^2+(5-4.9)^2+(5-4.9)^2+(5-4.9)^2+(5-4.9)^2+(6-4.9)^2+(6-4.9)^2}{10}$$

$$\sigma^2 = \frac{0.81 + 0.81 + 0.81 + 0.01 + 0.01 + 0.01 + 0.01 + 0.01 + 1.21 + 1.21}{10}$$

$$= \frac{4.9}{10} = 0.49$$

$$\sigma^2_{\bar{X}} = \frac{\sigma^2}{n} = \frac{0.49}{2} = 0.245$$

$0.2 \cong 0.245$ proved.

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11.33 $n_1 = 2,$ $n_2 = 2$

$2.4 = 2^2 = 4$ $4.8 = 2^2 = 4$

W.R. W.R.

Sample	$\bar{X}_1$
2, 2	2
2, 4	3
4, 2	3
4, 4	4

Sample	$\bar{X}_2$
4, 4	4
4, 8	6
8, 4	6
8, 8	8

$\bar{X}_1$ \ $\bar{X}_2$	4	6	6	8
2	2	4	4	6
3	1	3	3	5
3	1	3	3	5
4	0	2	2	4

$\bar{X}_2 - \bar{X}_1$	Tally	$f(\bar{x})$	$(\bar{x}_2 - \bar{x}) f(\bar{x})$
0	/	1/16	0
1	//	2/16	2/16
2	///	3/16	6/16
3	////	4/16	12/16
4	///	3/16	12/16
5	//	2/16	10/16
6	/	1/16	6/16
	16		48/16

$$\mu_{(\bar{X}_2 - \bar{X}_1)} = \Sigma(\bar{x}_2 - \bar{x}_1) = \frac{48}{16} = 3$$

$$\mu_1 = \frac{2+4}{2} = \frac{6}{2} = 3$$

$$\mu_2 = \frac{4+8}{2} = \frac{12}{2} = 6$$

$$\mu_2 - \mu_1 = 6 - 3 = 3$$

11.34 "KASHMIR"

$P = 2/7,\ q = 5/7 \quad p = .2857$

$$\sigma_p = \sqrt{\frac{P(1-p)}{n}\left(\frac{N-n}{N-1}\right)} q = 0.7143$$

$$= \sqrt{\frac{\frac{2}{7}(5/7)}{2}\left(\frac{7-2}{7-1}\right)}$$

$C_2^7 = 21$

$$= \sqrt{(0.1020)\ (0.8333)} = \sqrt{0.2916} = 0.2916$$

Sample		P	Sample		P
K	A	1/2	H	M	0
K	S	0	H	I	1/2
K	H	0	H	R	0
K	M	0	M	I	1/2
K	I	1/2	M	I	1/2
K	R	0	M	R	0
A	S	1/2	I	R	1/2
A	H	1/2			
A	M	1/2			
A	I	2/2			
A	R	1/2			
S	H	0			
S	M	0			
S	I	1/2			
S	R	0			

P	Tally	f(P)	pf(p)	$p^2f(p)$
0	𝍸 𝍸	10/21	0	0
1/2	𝍸 𝍸	10/21	10/42	10/84
2/2	1	1/21	2/42	8/84
			12/42	18/84
			0.2857	

$\mu_p = 0.2857$

$\sigma_p^2 = \Sigma p^2 f(p) - (\Sigma p f(p))^2$

$= 0.2143 - 0.0816 = 0.1327$

$\sigma_p = 0.3643$

11.35 $N_1 = 3,\ n_1 = 2,\ p_1 = \frac{1}{2},\ N_2 = 3,\ n_2 = 2,\ p_2 = \frac{1}{3}$

$$p_1 - p_2 = \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}$$

$$\sigma^2_{\hat{p}_1 - \hat{p}_2} = \frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}$$

$$= \frac{\frac{1}{2}(1-1/2)}{2} + \frac{\frac{1}{3}(1-1/3)}{2} = \frac{\frac{1}{2}(1/2)}{2} + \frac{1/3\ (2/3)}{2}$$

$$= 0.125 + 0.1111 = 0.2361 = \sqrt{0.2361}$$

11.36 2, 4, 4, 5

$N = 4$

$\sigma_{\hat{p}}$ when $n = 4$

$$P = \frac{2}{4} = \frac{1}{2}$$

$$q = 1 - 1/2 = \frac{1}{2}$$

(i) $$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}{4}} = \sqrt{\frac{\frac{1}{4}}{4}} = 0.25$$

(ii) $$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n} \times \frac{N-n}{N-1}}$$

$$= \sqrt{\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \times \frac{4-2}{4-1}}{2}} = \sqrt{0.125 \times \frac{2}{3}} = 0.2887$$

11.37(a) $N = 2$

9, 3

$n = 3$

No. of possible samples $= (2)^3 = 8$.

Sr. No.	Samples	$\bar{X}$
1	9, 9, 9	9
2	9, 9, 3	7
3	9, 3, 9	7
4	9, 3, 3	5

5	3, 9, 9	7
6	3, 9, 3	5
7	3, 3, 9	5
8	3, 3, 3	3

Sampling distribution of $\bar{X}$

$\bar{X}$	Tally	f	$f\bar{X}$	$f\bar{X}^2$
3	/	1	3	9
5	///	3	15	75
7	///	3	21	147
9	1	1	9	81
		$\Sigma f = 8$	$\Sigma f\bar{X}$	312

$$\mu_{\bar{X}} = \frac{\Sigma f\bar{X}}{\Sigma f} = \frac{48}{8} = 6.$$

$$\sigma^2_{\bar{X}} = \left[\frac{\Sigma f\bar{X}^2}{\Sigma f} - \left(\frac{\Sigma f\bar{X}}{\Sigma f}\right)^2\right] = \left[\frac{312}{8} - (6)^2\right] = 39 - 36$$

11.38 3, 4, 5, 6

$N = 4$

$n = 3$

No. of possible outcomes = ${}^4C_3 = 4$.

Sr. No.	Samples	$\bar{X}$
1	3, 4, 5	4
2	3, 4, 6	4.33
3	3, 5, 6	4.67
4	4, 5, 6	5

Sampling distribution of means.

$\bar{X}$	Tally	f	$f\bar{X}$	$f\bar{X}^2$
4	/	1	4	16
4.33	/	1	4.33	18.7489
4.67	/	1	4.67	21.8089
5	/	1	5	25
		$\Sigma f = 4$	18	81.5578

$$\mu_{\bar{X}} = \frac{\Sigma f\bar{X}}{\Sigma f} = \frac{18}{4} = 4.5.$$

$$\sigma^2_{\bar{X}} = \left[\frac{\Sigma f\bar{X}^2}{\Sigma f} - \left(\frac{\Sigma f\bar{X}}{\Sigma f}\right)^2\right] = \left[\frac{81.5578}{4} - \left(\frac{18}{4}\right)^2\right]$$

$$= 20.3895 - 20.25 = 0.1395$$

$$\sigma_{\bar{X}} = 0.3734$$

(i) Population Mean $= \frac{\Sigma X}{N}$

$$\mu = \frac{3+4+5+6}{4} = \frac{18}{4} = 4.5$$

Population S.D. $= \sqrt{\frac{\Sigma(X-\mu)^2}{N}}$

$$= \sqrt{\frac{[(3-4.5)^2+(4-4.5)^2+(5-4.5)^2+(6-4.5)^2]}{4}}$$

$$= \sqrt{\frac{[(2.25+0.25)+0.25+2.25]}{4}} = \sqrt{5/4} = \sqrt{1.25} = 1.11803$$

(i) Verification $\mu_{\bar{X}} = \mu = 4.5$

(ii) S.D. $(\bar{X}) = \frac{\sigma}{\sqrt{n}} \times \sqrt{\frac{N-n}{N-1}} = \quad \times \sqrt{\frac{4-3}{4-1}}$

$0.3734 = 0.3727$

Q. 11.39. Draw all possible samples of two letters each without replacement from the word "PUNJAB". Find the proportion of the letter "A" each sample. Make the sampling distribution of sample proportion and verify that $\mu_{\hat{p}} = P$.

Sol. $N = 6,\ n = 2$

No. of possible samples $= {}^6C_2 = 15$

S.No.	Samples	$\hat{p}$
1	P, U	0
2	P, N	0
3	P, J	0
4	P, A	½
5	P, B	0
6	U, N	0
7	U, J	0
8	U, A	½
9	U, B	0
10	N, J	0

Sample proportion of the letter A

$\hat{p}$	Tally	f	$f\hat{p}$
0	~~||||~~ ~~||||~~	10	0
½	~~||||~~	5	5/2
		$\Sigma f = 15$	5/2

$$\mu_{\hat{p}} = \frac{\Sigma f\hat{p}}{\Sigma f} = \frac{\frac{5}{2}}{15} = \frac{5}{2} \times \frac{1}{15} = \frac{1}{6}$$

$$P = \frac{X}{N} = \frac{1}{6}$$

11	N, A	½
12	N, B	0
13	J, A	½
14	J, B	0
15	A, B	½

Verification

$\mu_{\hat{p}} = P$

$\frac{1}{6} = \frac{1}{6}$

Q. 11.40. 2, 4, 5, 7, 10

$N = 5, \quad n = 3$

No. of possible samples $= {}^5C_3 = 10$

S.No.	Samples	$\hat{p}$
1	2, 4, 5	2/3
2	2, 4, 7	2/3
3	2, 4, 10	1/3
4	2, 5, 7	1/3
5	2, 5, 10	3/3 = 1
6	2, 7, 10	2/3
7	4, 5, 7	2/3
8	4, 5, 10	1/3
9	4, 7, 10	1/3
10	5, 7, 10	2/3

Sample proportion of the letter A

$\hat{p}$	Tally	f	$f\hat{p}$	$f\hat{p}^2$
1/3	///			
2/3	~~////~~			
1	1	1	1	1
		$\Sigma f = 10$	$\Sigma\hat{p} = 6$	4

$$\mu_{\hat{p}} = \frac{\Sigma f\hat{p}}{\Sigma f} = \frac{6}{10} = 0.6$$

$$\sigma_{\hat{p}}^2 = \left[\frac{\Sigma f\hat{p}^2}{\Sigma f} - \left(\frac{\Sigma f\hat{p}}{\Sigma f}\right)^2\right] = \left[\frac{4}{10} - (0.6)^2\right] = 0.4 - 0.36 = 0.04$$

$$P = \frac{X}{N} = \frac{3}{5}$$

Verification:

(i) $\mu_{\hat{p}} = P$

$0.6 = 0.6$

(ii) $\sigma_{\hat{p}} = \frac{pq}{n} \times \frac{N-n}{N-1} = \frac{(0.6)(0.4)}{3} \times \frac{5-3}{5-1} = 0.08 \times \frac{2}{4}$

$0.04 = 0.04$

Hence proved.

11.41. $\mu = 20, \sigma^2 = 4, \quad \sigma = 2, \quad n = 5$

(i) $\mu_{\bar{x}} = \mu$

so $\mu_{\bar{x}} = 20$

(ii) S.E($\bar{X}$)

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{5}} = 0.8944$$