EXERCISES

8.1 It is believed that 80% of the homes in a certain area are insured against fire. In conducting a survey of the area, an insurance company selects three homes at random. Let the random variable X represent the number of homes insured against fire. Construct the probability function of X.

8.2 (a) The probability distribution of a random variable X is given in the table below.

x	1	2	3	4	5
$P(X = x)$	0.1	0.3	y	0.2	0.1

Find (i) the value of y, (ii) the probability distribution of $(3X + 5)$, (iii) the mean, variance and s.d. of X.
(B.I.S.E., Faisalabad 2000; Rawalpindi 2002, 2003; Gujranwala 2005; Multan 2010, 2011; Lahore 2018)

(b) Find the missing value k such that the given distribution is a probability distribution of X.

x	2	3	4	5	6
$P(x)$	0.01	0.25	0.40	k	0.04

If $Y = 2X - 8$, then show that (i) $E(Y) = 2E(X) - 8$ (ii) $\text{Var}(Y) = 4\,\text{Var}(X)$.
(B.I.S.E., Gujranwala 2002)

8.3 (a) A bag contains 4 red and 6 black balls. A sample of 4 balls is selected from the bag without replacement. Let X be the number of red balls. Find the probability distribution of X. (B.I.S.E., Lahore 2005; Multan 2008)

(b) A fair die is rolled 5 times. Let X denote the number of times the face 3 turns up. Obtain a probability distribution of X. (B.I.S.E., Sargodha 2001)

8.4 A continuous random variable has a density function.

$f(x) = \frac{2}{27}(x + 1)$ $2 \le x \le 6$. Find (i) $P(X < 4)$ (ii) $P(X > 3.5)$
(B.I.S.E., Lahore 2015; Gujranwala 2015; Faisalabad 2015)

8.5 (a) A continuous random variable X that can assume values between $X = 1$ and $X = 4$ has a density function given by $f(x) = 1/3$.
(i) Show that the area under the curve is equal to 1.
(ii) Find $P(1.5 < X < 3)$ (iii) $P(X \le 2.2)$.
(B.I.S.E., Multan 2000, 2003; Sargodha 2001; Gujranwala 2009)

(b) A continuous random variable X has probability function $f(x) = cx$ for $0 < x < 2$. Determine (i) c (ii) $P(1 < X < 1.5)$ (iii) $P(X < 1.5)$ (iv) $P(X > 3)$ (v) $P(0 < X < 1)$.
(B.I.S.E., Lahore 1994; Gujranwala 2002; Sargodha 2002, 2003; Bahawalpur 2003, 2008; Multan 2014)

8.6 (a) A function is given by $f(x) = (3 + 2x)/18$, $2 \le x \le 4$. Show that it is probability density function and find the probability that (i) $X \ge 2.5$ (ii) $2 \le X \le 3$ (iii) $X < 3.5$. (B.I.S.E., Lahore 2002; Bahawalpur 2002; Rawalpindi 2003)

(b) Find the constant k so that the function $f(x)$ defined by $f(x) = 1/k$, $a \le x \le b$ is a density function. (B.I.S.E., Bahawalpur 2002)

8.7 Find expectation of X for the following probability distribution.

x	0	1	2	3	4
$P(x)$	15/210	80/210	90/210	24/210	1/210

(B.I.S.E., Lahore 1976)

8.8 (a) Let X be a random variable with the following probability distribution. Find (i) $E(X)$ (ii) $Var(X)$.

x	−1	0	1	2	3
$P(x)$	0.125	0.50	0.20	0.05	0.125

(B.I.S.E., Gujranwala 2012; Lahore 2014)

(b) Find the probability distribution of (i) $(2X - 3)$ (ii) $(X^2 - 1)$ from the probability distribution of (a). (Federal Board, Islamabad 2002; B.I.S.E., Rawalpindi 2010)

8.9 (a) A random variable X takes values $-3, -2, 2, 3, 4$ with probabilities $P(x)$ equal to 1/5, 1/10, 1/10, 1/5, 2/5 respectively. Compute $E(X)$ and show that $E(5X + 10) = 5E(X) + 10$ where $E(X)$ denotes the expectation of X. (B.I.S.E., Rawalpindi 2003)

(b) Compute the standard deviations of X and $(5X + 10)$ in (a). Find the ratio of two standard deviations.

8.10 A and B throw a die for a prize of Rs. 11 which is to be won by the player who first throws 6. If A has the first throw, what are their respective expectations? (B.I.S.E., Lahore 1966; Sargodha 2003)

8.11 A man draws 2 balls from a box containing 3 white and 2 black balls. If he receives Rs. 70 for each white ball and Rs. 7 for each black ball, find his expectation. (B.I.S.E., Bahawalpur 2003)

8.12 In summer season a dealer of desert room coolers can earn rupees 500 per day if the day is hot and rupees 200 per day it is fair and loses rupees 60 per day if it is cloudy. Find his mathematical expectation if the probability of the day being hot is 0.80 and for being cloudy is 0.05. (B.I.S.E., Lahore 1995; Rawalpindi 2002)

8.13 (a) In 900 trials of a throw of two dice, what is the expected number of times that the sum will be less than 5?

(b) What is the expectation of a person who is to get Rs. 80 if he obtains 3 heads in a single toss of 3 coins?

8.14 Five balls are drawn from a box containing 4 white and 7 black balls. If X denotes the number of black balls drawn, then obtain the probability

distribution of X. Find mean and variance of this distribution. (B.I.S.E., Bahawalpur 2003; Lahore 2003)

8.15 (a) A coin is tossed 4 times. If X denotes the number of tails, what is the probability distribution of X? Draw a probability histogram.

(b) If $Y = 2X$, find $E(Y)$ and $\text{Var}(Y)$, where X is the number of tails as in (a). (B.I.S.E., Lahore 1994)

8.16 A coin is so biased that a head is twice as likely to occur as a tail. Find the probability distribution of heads and also find the mean and variance of this distribution when the coin is tossed 4 times. (B.I.S.E., Lahore 1993)

8.17 (a) If $f(x) \begin{cases} \dfrac{6 - |7 - x|}{36} & \text{for } x = 2, 3, 4, \ldots, 12, \text{ or} \\ (x - 1)/36 & \text{for } x = 2, 3, \ldots, 9 \\ (13 - x)/36 & \text{for } x = 8, 9, \ldots, 12 \end{cases}$ then find the mean and variance of the random variable X.

(B.I.S.E., Bahawalpur 2002; Lahore 2005, 2006, 2014; Gujranwala 1993, 2010, 2015)

(b) Two fair dice are rolled where X denotes the number of dots on the first die and Y denotes the number of dots on the second die respectively. Find the probability distribution of (i) sum of dots (ii) difference of dots on the two dice. (Federal Board, Islamabad 2002)

8.18 A random variable 'x' has the following probability distribution.

x	1	2	3
P(x)	6/9	2/9	1/9

Find mean and standard deviation of x.

8.19 Given the following probability distribution.

x_i	0	1	2	3	4
$P(x_i)$	1/126	20/126	60/126	40/126	5/126

Verify that $E(2X + 3) = 2E(X) + 3$.

(B.I.S.E., Gujranwala 1994, 2005, 2014; Multan 2002, 2018; Lahore 2001, 2013)

8.20 A pair of fair dice is tossed. Let X represent the sum of the outcomes on both dice. Find (i) range of X (ii) probability distribution of X (iii) mean and variance of the distribution. (B.I.S.E., Sargodha 2001)

8.21 (a) If $f(x) = A(2x^3 + 1)$, $0 \le x \le 2$ and $= 0$ elsewhere. Find (i) A (ii) $P(X > 1.5)$ (iii) $P(X < 1.2)$ (iv) $P(1 < X \le 2)$. (B.I.S.E., Rawalpindi 2001)

(b) A continuous variable X which assumes values between $X = 2$ and $X = 8$ has a density function $f(x) = k(2x + 3)$. Find (i) k (ii) $P(X > 6)$ (iii) $P(X \le 6)$. (B.I.S.E., Multan 2002; Lahore 2005)

8.22 (a) Find the coefficient of variation for the following probability distribution.

X	−5	−1	0	1	5
$P(X)$	0.20	0.30	0.05	0.15	0.30

(b) Find the probability distribution of the number of girls in families with three children, assuming $P(\text{girl}) = 1/3$ and $P(\text{boy}) = 2/3$. (B.I.S.E., Rawalpindi 2001)

8.23 Four coins are tossed. A tail is recorded as 2 and head as 1. The variable of interest is the product of records. Find the probability distribution of this random variable. Also find its mean and variance. (B.I.S.E., Sargodha 2002)

8.24 Which of the following functions $f(x)$ is a valid probability function?

(i) $f(x) = (4 - x)/4$, $1 \le x \le 3$
(ii) $f(x) = 1/4$, $-2 < x \le 3$
(iii) $f(x) = (x + 1)/8$, $2 \le x \le 4$ (B.I.S.E., Gujranwala 2002)

8.25 If $f(x) = 2(5 - x)/25$, $0 < x < 5$. Show that it is a p.d.f. Also find $P(X < 4)$, $P(X > 6)$, $P(2 < X < 3)$, $P(1 < X < 4)$. (B.I.S.E., Rawalpindi 2003)

8.26 The random variable X has the following probability distribution. It is known that $P(X<4) = P(X>4)$ and $P(X \le 5) = 2P(X>5)$. Find the values of a, b, and c.

X	1	5	9
P(x)	a	b	c

(B.I.S.E., Gujranwala 2004)

8.27 A discrete random variable has probability function $P(x)$ defined by $P(0) = P(6) = 1/16$, $P(1) = P(4) = 1/4$, $P(3) = 3/8$, $P(0) = 0$ elsewhere. Find (i) $E(X)$ and Var (X). (i) Mean and Variance of Y where $Y = 2X - 5$. (B.I.S.E., Gujranwala, 2004)

8.28 A random variable X has the probability distribution

X	0	1	2	3
$P(x)$	0.1	0.2	0.3	0.4

(i) Find $E(X)$ and Var (X) (ii) Show that $E(5X + 8) = 5E(X) + 8$. (B.I.S.E., Gujranwala, 2003)

8.29 For a continuous random variable X, $f(x) = cx$, $0 \le x \le 2$. Find (i) c (ii) $P(X<1)$ (iii) $P\left(\frac{1}{2} \le X \le \frac{3}{2}\right)$ (iv) $P(X>3)$. (B.I.S.E., Gujranwala 2002, 2017; Multan 2011)

8.30 Show that $f(x) = (3 + 2x)/8$, $2 \le x \le 4$ is a proper density function and find the probability that (i) $X \ge 2.5$ (ii) $2 \le X \le 3$ (iii) $X < 3.5$. (B.I S.E., Lahore, 2002)

8.31 A continuous random variable X has density function $f(x) = a(x + 3)$, $2<x<8$. Find (i) a (ii) $P(X<6)$. (B.I.S.E., Gujranwala, 2001, Rawalpindi 2008)

sed random numbers are:
numbers by Tippett (1927)

8.32 (a) A coin is so biased that a head is twice as likely to occur as a tail. Find the probability distribution of heads and also find the mean and variance of the distribution when it is tossed 4 times. (B.I.S.E., Gujranwala 2001)

(b) A state lottery is conducted in which six winning numbers are selected from a total of 54 numbers. What is the probability that if six numbers are randomly selected (i) non of the numbers will be winning numbers? At least one number will be winning numbers? (B.I.S.E., Rawalpindi, 2008)

8.33 A continuous *r.v.* 'X' has a *p.d.f.*, $f(x) = 2x,\ 0 \le x \le 1$.

Find (i) $P\left(X \le \frac{1}{2}\right)$ (i) $P\left(X > \frac{1}{4}\right)$ (iii) $P\left(\frac{1}{4} < X < \frac{1}{2}\right)$.

(B.I.S.E., Rawalpindi 2007; Lahore 2015; Faisalabad 2017; Gujranwala 2018)

8.34 A random variable X has the following probability distribution.

X	-2	-1	0	1	2	3
P(X)	0.1	K	0.2	2K	0.3	3K

Find (i) K (ii) $P(X < 2)$ (iii) $P(X > 2)$ (iv) $P(-2, < X < 2)$.
(B.I.S.E., Multan, 2008; Lahore 2011; Gujranwala 2011)

8.35 (a) Determine value of C so that the function can serve as a probability function of random variable. (B.I.S.E. Gujranwala, 2013)

1. C_y for y = 1, 2, 3, 4.
2. $(1 - C)C^y$ for y = 0, 1, 2, ...

8.36 Calculate Mean and Variance of the following probability distribution.

x	0	1	2	3	4	5
$P(x)$	0.1	0.2	0.3	0.2	0.1	0.1

(B.I.S.E., Gujranwala 2013, 2017)

8.37 Find the mean, variance and $E(3X - 2)$ from the following probability distribution.

X	3	5	7	9	11
f(x)	0.1	0.2	0.4	0.2	0.1

(Gujranwala Board, 2014)

8.38 Let X has the following probability distribution.

X	1	2	3	4	5
P(X)	0.05	0.04	0.10	0.24	0.05

Find E(X) and $E(X)^2$. (B.I.S.E., Lahore 2015)

8.39. A random variable X has the probability distribution.

X	0	1	2	3
f(x)	0.1	0.2	0.3	0.4

Show that E(5X + 8) = 5E(X) + 8. (B.I.S.E., Lahore 2017)

8.40. A continuous random variable X has p.d.f.

f(x) = K(4 – x) = 0 for $1 \le X \le 3$

= 0 else where

(B.I.S.E., Lahore 2017)