

3**MEASURES OF LOCATION**

3.1 $\bar{X} = \sum X/n = (3.88 + 4.09 + 3.92 + 3.97 + 4.02 + 3.95 + 4.03 + 3.92 + 3.98 + 4.06)/10 = 39.82/10 = 3.98$ inches.

3.2

Student	History	Statistics	Economics
1	41	46	50
2	35	50	52
3	38	39	41
4	34	50	46
5	30	38	39
Total	178	223	228
Mean	$178/5 = 35.6$	$223/5 = 44.6$	$228/5 = 45.6$

Since mean for Economics (45.6) is the highest, so level of knowledge in Economics is highest.

3.3(a) Average charges for 12 residents (\bar{X}) = 93.50.

$$\bar{X} = \sum X/n \text{ or } 93.50 = \sum X/12 \text{ or } \sum X = 12(93.50) = 1122.$$

Sum of charges paid by 11 residents:

$$\sum X' = 110+90+68+95+118+85+78+87+20+82+75 = 1008.$$

Charges paid by the 12th resident

$$= \sum X - \sum X' = 1122 - 1008 = \text{Rs. 114.}$$

Q. 3.3(b) Average wage of 4 men = 17

$$\bar{X} = \frac{\text{Sum of the wages of 4 men}}{4}$$

$$17 = \frac{\sum X}{4}$$

$$17 \times 4 = \sum X$$

$$68 = \sum X$$

Average wage of 10 men = 20

$$\bar{X} = \frac{\sum X}{n}$$

$$20 = \frac{\sum X}{10}$$

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$$20 \times 10 = \Sigma X$$

$$200 = \Sigma X$$

Sum of the wages of 10 men = 200

Sum of the wages of 6 men = $200 - 68 = 132$

$$\text{Average of 6 men} = \frac{132}{6} = 22$$

3.4

Class	Frequency (f)	Class mark (X)	fX
1 - 5	7	3	21
6 - 10	10	8	80
11 - 15	16	13	208
16 - 20	32	18	576
21 - 25	24	23	552
26 - 30	18	28	504
31 - 35	10	33	330
36 - 40	5	38	190
41 - 45	1	43	43
	$\sum f = 123$		$\sum fX = 2504$

$$\bar{X} = \frac{\sum fX}{\sum f} = \frac{2504}{123} = 20.358 \text{ or } 20.36.$$

$$3.5(a) \quad \bar{X} = \frac{6(6) + 7(7) + 8(8) + 9(9) + 10(10)}{6 + 7 + 8 + 9 + 10} = \frac{330}{40} = 8.25.$$

Array: 6, 6, 6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 8, 8, 8, 8, 8, 8, 8, 8, 8,
9, 9, 9, 9, 9, 9, 9, 9, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10.

Since $n = 40$ is even, median is the mean of the 20th and 21st items in the array, i.e. median = $(8 + 8)/2 = 8$.

Since 10 occurs the maximum number of times in the series, mode = 10.

(b)

Marks	Class mark (X)	Class A		Class B	
		f	fX	f	fX
10 - 20	15	100	1500	90	1350
20 - 30	25	125	3125	140	3500
30 - 40	35	86	3010	75	2625
40 - 50	45	45	2025	50	2250

Marks	Class mark (X)	Class A		Class B	
		f	fX	f	fX
50 – 60	55	18	990	15	825
60 – 70	65	12	780	10	650
		$\sum f = 386$	$\sum fX = 11430$	$\sum f = 380$	$\sum fX = 11200$

$$\text{A.M. for class } A: \bar{X}_A = \frac{\sum fX}{\sum f} = \frac{11430}{386} = 29.61$$

$$\text{A.M. for class } B: \bar{X}_B = \frac{\sum fX}{\sum f} = \frac{11200}{380} = 29.47$$

Class A is better on the average.

3.6(a) (i) $\bar{X} = A + \frac{\sum D}{n} = 39 + \frac{240}{10} = 63.$

(ii) $\bar{X} = A + \frac{\sum u}{n} \times h = 57 + \frac{(23)(5)}{20} = 62.75.$

(iii) $\bar{X} = A + \frac{\sum u}{n} \times h = 120 + \frac{(25)(3)}{15} = 125.$

(iv) Here $A = 15$, $n = 12$ and $\sum D = 27$.

$$\bar{X} = A + \frac{\sum D}{n} = 15 + \frac{27}{12} = 17.25.$$

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(b) (i) $\bar{X} = \sum X/n = 1552/16 = 97.$

(ii) Taking $D = X - 100$, the values of D are 21, 15, -21, -48, 2, 26, -19, -35, 9, 19, 15, 21, 3, -25, -41, 10 and $\sum D = -48$.

$$\bar{D} = \sum D/n = -48/16 = -3 \text{ and } \bar{X} = -3 + 100 = 97.$$

This shows that the mean is affected by change of origin.
We have applied the short method to calculate the mean.

3.7 (i) $\bar{X} = A + \frac{\sum fd}{n} = 18 + \frac{150}{25} = 24.$

(ii) $\bar{X} = A + \left(\frac{\sum fu}{n} \right) h = 124.5 + \left(\frac{50}{150} \right) 3 = 125.5.$

(iii) $\bar{X} = A + \left(\frac{\sum fu}{n} \right) h = 10 + \left(\frac{46}{125} \right) 5 = 11.84.$

3.8(a) Mean of 20 observations is 42.

$$\bar{X} = \sum X/n \text{ or } 42 = \sum X/20 \text{ or } \sum X = 840.$$

Corrected $\sum X = 840 - (45 + 38) + (35 + 58) = 840 - 83 + 93$
 $= 850.$

Corrected \bar{X} = Corrected $\sum X/n = 850/20 = 42.5$.

3.8(b) $n = 1000$

$$\bar{X} = 130.40$$

$$X_w = 297 \text{ and } 165$$

$$X_{corr} = 197 \text{ and } 185$$

$$\bar{X}_w = \frac{\Sigma X_w}{n}$$

$$130.40 = \frac{\Sigma X_w}{1000}$$

$$\Sigma X_w = 130.40 \times 1000 = 130400$$

So correct total

$$\begin{aligned}\Sigma X_{corr} &= 130400 - (297 + 165) + (197 + 185) \\ &= 130320\end{aligned}$$

$$\bar{X}_{corr} = \frac{130320}{1000} = 130.22$$

3.9

Class	f	X	fX
10 — 15	3	12.5	37.5
15 — 20	7	17.5	122.5
20 — 25	16	22.5	360
25 — 30	12	27.5	330
30 — 35	9	32.5	292.5
35 — 40	5	37.5	187.5
40 — 45	2	42.5	85
	$\Sigma f = 54$		1415

$$\bar{X} = \frac{\Sigma fX}{\Sigma f} = \frac{45}{54} = 26.204$$

3.10 The weighted mean is given by

$$\begin{aligned}\bar{X}_W &= \frac{\Sigma WX}{\Sigma W} = \frac{4(73) + 3(82) + 3(80) + 2(57) + 2(62)}{4 + 3 + 3 + 2 + 2} \\ &= (292 + 246 + 240 + 114 + 124)/14 = 1016/14 \\ &= 72.57 \text{ marks.}\end{aligned}$$

- 3.11 Putting $X = -3, -2, -1, 0, 1, 2, 3, 4, 5$ in the equation $Y = 10 - 4X$, we get values of Y . These values are shown below:

X	-3	-2	-1	0	1	2	3	4	5	$\Sigma X = 9$
Y	22	18	14	10	6	2	-2	-6	-10	$\Sigma Y = 54$

$$\bar{X} = \sum X/n = 9/9 = 1 \text{ and } \bar{Y} = \sum Y/n = 54/9 = 6.$$

$$\bar{Y} = 10 - 4\bar{X} \text{ or } 6 = 10 - 4(1) = 6.$$

- 3.12(a) The combined mean $\bar{\bar{X}}$ is given by $\bar{\bar{X}} = \frac{\sum n \bar{X}}{\sum n}$

$$= \frac{45(68) + 42(58) + 38(52)}{45 + 42 + 38} = \frac{3060 + 2436 + 1976}{125} = \frac{7472}{125} \\ = 59.78 \text{ marks.}$$

- (b) Mean weight of whole class $\bar{\bar{X}}$ is given by

$$\bar{\bar{X}} = \frac{45(120.4) + 37(116.7) + 5(122.3)}{45 + 37 + 5} \\ = \frac{5418 + 4317.9 + 611.5}{87} = \frac{10347.4}{87} = 118.94 \text{ lb.}$$

3.13

u	-3	-2	-1	0	1	2	3	4	
f	2	15	38	65	39	21	13	7	$n = \sum f = 200$
fu	-6	-30	-38	0	39	42	39	28	$\sum fu = 148 - 74 = 74$
X	120	130	140	150	160	170	180	190	
fX	240	1950	5320	9750	6240	3570	2340	1330	$\sum f = 30740$
$\log X$	2.0792	2.1139	2.1461	2.1761	2.2041	2.2304	2.2553	2.2788	$\sum f \log X =$
$f \log X$	4.1584	31.7085	81.5518	141.4405	85.0509	46.8384	29.3180	15.9916	436.9340

$$(i) \quad \bar{X} = A + \left(\frac{\sum fu}{\sum f} \right) h = 150 + \left(\frac{74}{200} \right) 10 = 153.7.$$

- (ii) Since $A = 150$ and $h = 10$, the column X is obtained from $X = 150 + 10u$ by substituting $u = -3, -2, -1, 0, 1, 2, 3, 4$.

$$\bar{X} = \frac{\sum fX}{n} = \frac{30740}{200} = 153.7.$$

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$$\log GM = \frac{1}{n} \sum f \log X = \frac{436.9340}{200} = 2.18467$$

$$G.M. = \text{antilog}(2.18467) = 152.99$$

$$3.14 \text{ G.M.} = [0.5 \times 1.0 \times 2.7 \times 3.48 \times 4.71]^{1/5} = (22.12758)^{1/5}$$

$$= 1.8577 = 1.86. \text{ (Using scientific calculator).}$$

3.15

Marks	No. of students (f)	Class mark (X)	$\log X$	$f \log X$
10 – 19	5	14.5	1.16137	5.80685
20 – 29	25	24.5	1.38917	34.72925
30 – 39	40	34.5	1.53782	61.5128
40 – 49	20	44.5	1.64836	32.9672
50 – 59	10	54.5	1.73640	17.364
	$n = \sum f = 100$		$\sum f \log X = 152.3801$	

$$\text{Log G.M.} = \frac{\sum f \log X}{n} = \frac{152.3801}{100} = 1.5238$$

$$\text{G.M.} = \text{antilog}(1.5238) = 33.404.$$

3.16(a)

End of	Increase	Income
1st year	10%	110
2nd year	20%	120
3rd year	25%	125

$$\text{G.M. of incomes} = (110 \times 120 \times 125)^{1/3} = (1650000)^{1/3}$$

$$= \text{Rs. } 118.17.$$

$$\text{Average increase} = 118.17 - 100 = 18.17\%.$$

(b) We have the mean of 3 numbers a, b, c , i.e.

$$(a + b + c)/3 = 72$$

$$a + b + c = 3(72) = 216 \quad (\text{i})$$

$$\text{G.M.} = [abc]^{1/3} = 60 \text{ or } abc = (60)^3 = 216000.$$

$$\text{We have the mean } a \text{ and } b, \text{ i.e. } (a + b)/2 = 45.5$$

$$\text{or } a + b = 2(45.5) = 91 \quad (\text{ii})$$

$$\text{From (i) and (ii), we get } c = 216 - 91 = 125.$$

$$ab = abc/c = 216000/125 = 1728.$$

Two numbers a and b whose sum is 91 and whose product is 1728 are 27 and 64.

(c)	End of	Depreciation	Price
	1st year	20%	80
	2nd year	15%	85
	3rd year	10%	90
	4th year	10%	90

$$\text{G.M. of prices} = (80 \times 85 \times 90 \times 90)^{1/4} = 86.15$$

$$\text{Average depreciation} = 100 - 86.15 = 13.85\%.$$

3.17

D	X = 25.5 + D	log X
-15.4	10.1	1.00432
-1.9	23.6	1.37291
6.2	31.7	1.50106
13.7	39.2	1.59329
24.6	50.1	1.69984
25.5	51.0	1.70757
3.8	29.3	1.46687
-7.9	17.6	1.24551
-13.6	11.9	1.07555
21.1	46.6	1.66839
16.4	41.9	1.62221
18.7	44.2	1.64542
-14.3	11.2	1.04922
-9.8	15.7	1.19590
4.9	30.4	1.48287
	$\Sigma X = 454.5$	$\Sigma \log X = 21.33093$

$$\bar{X} = \sum X/n = 454.5/15 = 30.3.$$

$$\log \text{G.M.} = \sum \log X/n = 21.33093/15 = 1.422062.$$

$$\text{G.M.} = \text{antilog}(1.422062) = 26.42786.$$

3.18 Suppose the numbers are a , b and c . Then we have

$$(a+b+c)/3 = 7 \text{ or } a+b+c = 21 \quad (1)$$

$$\text{and } (abc)^{1/3} = 4 \text{ or } abc = 64 \quad (2)$$

$$\text{We also have } (a+b)/2 = 10 \text{ or } a+b = 20 \quad (3)$$

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From Equation (1) and (3), we get $c = 1$

Equation (2) becomes $ab = 64$ (4)

From Equation (3) and (4), $a(20-a) = 64$ or $a^2 - 20a + 64 = 0$

or $(a-4)(a-16) = 0$ or $a = 4$ or 16 . Substitution in

Equation (3) gives $b = 16$ or 4 . Thus the numbers are $16, 4$ and 1 .

3.19(a). Since $D = X - 18$, $X = 18 + D$.

D	f	X	f/D	$\log X$	$f \log X$
-12	2	6	-24	0.77815	1.55630
-8	5	10	-40	1.00000	5.00000
-4	8	14	-32	1.14613	9.16904
0	18	18	0	1.25527	22.59486
4	22	22	88	1.34242	29.53324
8	13	26	104	1.41497	18.39461
12	8	30	96	1.47712	11.81696
16	4	34	64	1.53148	6.12592
$\sum f = 80$		$\sum f/D = 256$		$\sum f \log X = 104.19093$	

$$\bar{X} = A + \frac{\sum f}{\sum f} = 18 + \frac{256}{80} = 21.2.$$

$$\text{log G.M.} = \frac{\sum f \log X}{\sum f} = \frac{104.19093}{80} = 1.30239$$

G.M. = antilog (1.30239) = 20.0627.

(b) $u = (X - 136.5)/2$ or $X = 136.5 + 2u$

Corresponding to $x = -4, -3, -2, -1, 0, 1, 2, 3$, the X values are determined in the following table along with other computations.

u	f	fu	$X = 136.5 + 2u$	$\log X$	$f \log X$	f/X
-4	2	-8	128.5	2.10890	4.21780	0.01556
-3	5	-15	130.5	2.11561	10.57805	0.03831
-2	8	-16	132.5	2.12222	16.97776	0.06038
-1	18	-18	134.5	2.12872	38.31696	0.13383
0	22	0	136.5	2.13513	46.97286	0.16117

u	f	fu	$X = 136.5 + 2u$	$\log X$	$f \log X$	f/X
1	13	13	138.5	2.14145	27.83885	0.09386
2	8	16	140.5	2.14768	17.18144	0.05694
3	4	12	142.5	2.15381	8.61524	0.02807
$n = \sum f = 80$		$\sum fu = 41 - 57 = -16$		$\sum f \log X = 170.69896$	$\sum(f/X) = 0.58812$	

$$\bar{X} = A + \left(\frac{\sum fu}{\sum f} \right) h = 136.5 + \left(\frac{-16}{80} \right) 2 \\ = 136.5 - 0.4 = 136.1$$

$$\text{log G.M.} = \frac{\sum f \log X}{\sum f} = \frac{170.69896}{80} = 2.133737$$

$$\text{G.M.} = \text{antilog}(2.133737) = 136.062$$

$$\text{H.M.} = \frac{\sum f}{\sum(f/X)} = \frac{80}{0.58812} = 136.027$$

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3.20(a)

Speed (X)	10	15	20	25	20	30	40	50	30	40	
$\frac{1}{X}$	0.1	0.06667	0.05	0.04	0.05	0.03333	0.025	0.02	0.03333	0.025	$\sum\left(\frac{1}{X}\right) = 0.44333$

The average speed is given by the harmonic mean

$$\text{H.M.} = n / \sum\left(\frac{1}{X}\right) = 10 / 0.44333 = 22.56 \text{ mph.}$$

(b)

$\frac{1}{X}$	0.05	0.0454	0.0400	0.0333	0.0285	0.0232	0.0213	0.0200	0.0182	0.0151	0.0143
X	20	22.0264	25	30.03	35.0877	43.1034	46.9484	50	54.9451	66.2252	69.9301
											$\sum\left(\frac{1}{X}\right) = 0.3093$
											$\sum X = 463.2963$

$$\text{H.M.} = n / \sum\left(\frac{1}{X}\right) = \frac{11}{0.3093} = 35.5642.$$

$$\text{A.M.} = \frac{\sum X}{n} = \frac{463.2963}{11} = 42.1178.$$

3.21

X	f	$\log X$	$f \log X$	$f\left(\frac{1}{X}\right)$
12	1	1.0792	1.0792	0.0833
14	4	1.1461	4.5844	0.2857
16	6	1.2041	7.2246	0.3750
18	10	1.2553	12.5530	0.5556
20	7	1.3010	9.1070	0.3500
22	2	1.3424	2.6848	0.0909
$n = \sum f = 30$		$\sum f \log X = 37.2330$		$\sum f\left(\frac{1}{X}\right) = 1.7405$

$$\log G.M. = \frac{\sum f \log X}{\sum f} = \frac{37.2330}{30} = 1.2411$$

$$G.M. = \text{antilog}(1.2411) = 17.422.$$

$$H.M. = \sum f / \sum f\left(\frac{1}{X}\right) = \frac{30}{1.7405} = 17.236.$$

3.22(a) Let the three numbers be X, Y, Z . The mean and the G.M. are given by

$$\bar{X} = (X + Y + Z)/3$$

$$34 = (X + Y + Z)/3 \text{ or } X + Y + Z = 102$$

$$G.M. = (XYZ)^{1/3} = 18 \text{ or } XYZ = (18)^3 = 5832.$$

The G.M. of the first two numbers X and Y is

$$G.M. = \sqrt{XY} = 9 \text{ or } XY = (9)^2 = 81.$$

$$Z = \frac{XYZ}{XY} = \frac{5832}{81} = 72.$$

$$X + Y = 102 - 72 = 30.$$

Two numbers (X and Y) whose sum is 30 and whose product is 81 are 3 and 27. Thus the three numbers are 3, 27 and 72.

(b) $\log G = \sum \log X/n = (1 + 1.2 + 1.3 + 2 + 4)/5 = 9.5/5 = 1.9$

$$G = \text{antilog}(1.9) = 79.4328$$

Antilog of 1, 1.2, 1.3, 2 and 4 are 10, 15.8489, 19.9526, 100 are 10000 and their reciprocals are 0.1, 0.0631, 0.0501, 0.01 and 0.0001.

$$\text{H.M.} = \frac{5}{(0.1 + 0.0631 + 0.0501 + 0.01 + 0.0001)} = \frac{5}{0.2233} \\ = 22.3914$$

3.23 Average rate of motion is the harmonic mean of 6, 8 and 10.

$$\text{H.M.} = \frac{3}{\frac{1}{10} + \frac{1}{8} + \frac{1}{6}} = \frac{3}{0.39167} = 7.66 \text{ mph.}$$

3.24(a) Array: 61, 62, 64, 68, 72, 76, 80, 82, 85, 91. Since $n = 10$, the median is the mean of 5th and 6th items. Thus Median = $(72 + 76)/2 = 74$.

- (b) Array: 21, 31, 36, 41, 46, 51, 54, 58, 59, 65, 68. Since $n = 11$, the median is the 6th item. Thus median = 51.
- (c) Array: 20.6, 28.3, 28.8, 29.7, 30.2, 39.3, 40.2, 40.2, 42.6. Since $n = 9$, the median is 5th item. Thus median = 30.2.

3.25(a)

Class Boundary	X	f	$\sum fX$	Cumulative Frequency
59.5—62.5	61	5	305	5
62.5—65.5	64	18	1152	23
65.5—68.5	67	42	2814	65 ←
68.5—71.5	70	27	1890	92
71.5—74.5	73	8	584	100
$n = \sum f = 100$		$\sum fX = 6745$		

$$\bar{X} = \frac{\sum fX}{n} = \frac{6745}{100} = 67.45 \text{ inches}$$

$$\text{Median} = l + \frac{c}{f} \left(\frac{n}{2} - F \right) = 65.5 + \frac{3}{42} (50 - 23) \\ = 65.5 + 1.93 = 67.43 \text{ inches}$$

$$\text{Mode} = l + \frac{c(f_m - f_{m-1})}{(f_m - f_{m-1}) + (f_m - f_{m+1})} \\ = 65.5 + \frac{3(42 - 18)}{(42 - 18) + (42 - 27)} \\ = 65.5 + 1.846 = 67.346 \text{ inches}$$

(b) For missing values see Answer of Exercise 2.14(b).

Class Boundary	X	f	fX	Cum Frequency
61.5—64.5	63	6	378	6
64.5—67.5	66	23	1518	29
67.5—70.5	69	34	2346	63 ←
70.5—73.5	72	17	1224	80
73.5—76.5	75	12	900	92
76.5—79.5	78	6	468	98
79.5—82.5	81	2	162	100
$n = \sum f = 100$		$\sum fX = 6996$		

$$\bar{X} = \frac{\sum fX}{n} = \frac{6996}{100} = 69.96 \text{ kg}$$

$$\begin{aligned}\text{Median} &= l + \frac{c}{f} \left(\frac{n}{2} - F \right) = 67.5 + \frac{3}{34} (50 - 29) \\ &= 67.5 + 1.85 = 69.35 \text{ kg}\end{aligned}$$

$$\begin{aligned}\text{Mode} &= l + \frac{c(f_m - f_{m-1})}{(f_m - f_{m-1}) + (f_m - f_{m+1})} \\ &= 67.5 + \frac{3(34 - 23)}{(34 - 23) + (34 - 17)} \\ &= 67.5 + 1.18 = 68.68 \text{ kg}\end{aligned}$$

3.26(a) 127, 129, 127, 125, 124, 127, 125, 129.

Mode = 127

101, 106, 99, 108, 76, 87, 102, 93

(b) No mode because no value is repeated.

(c) 14, 16, 14, 18, 19, 12, 14, 12, 16, 17, 18, 16, 15, 15, 15, 14, 16.

Mode = 14 and 16

3.27(b)

Maximum load (short tons)	No. of cables (f)	Class boundary	C.F.	Class mark (X)	fX
9.3 – 9.7	2	9.25 – 9.75	2	9.5	19
9.8 – 10.2	5	9.75 – 10.25	7	10.0	50
10.3 – 10.7	12	10.25 – 10.75	19	10.5	126
10.8 – 11.2	17	10.75 – 11.25	36	11.0	187
11.3 – 11.7	14	11.25 – 11.75	50	11.5	161

Maximum load (short tons)	No. of cables (f)	Class boundary	C.F.	Class mark (X)	fX
11.8 – 12.2	6	11.75 – 12.25	56	12.0	72
12.3 – 12.7	3	12.25 – 12.75	59	12.5	37.5
12.8 – 13.2	1	12.75 – 13.25	60	13.0	13
$n = \sum f = 60$				$\sum fX = 665.5$	

$$\bar{X} = \sum fX / \sum f = 665.5 / 60 = 11.09 \text{ short tons.}$$

$n/2 = 60/2 = 30$ in the class 10.8 – 11.2, which is the median class.

$$\text{Median: } \tilde{X} = l + \frac{h}{f} \left(\frac{n}{2} - F \right) = 10.75 + \frac{0.5}{17} \left(\frac{60}{2} - 19 \right)$$

$$= 10.75 + \frac{0.5}{17} (11) = 10.75 + 0.32 = 11.07 \text{ short tons.}$$

$$\begin{aligned} \text{Mode: } \hat{X} &= l + \frac{(f_m - f_1)h}{(f_m - f_1) + (f_m - f_2)} \\ &= 10.75 + \frac{(17 - 12)0.5}{(17 - 12) + (17 - 14)} = 10.75 + \frac{5(0.5)}{5 + 3} = 10.75 + \frac{2.5}{8} \\ &= 11.06 \text{ short tons.} \end{aligned}$$

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3.28

Weight (Pound)	Frequency (f)	Class mark (X)	fX	Class boundary	Cumulative frequency
118 – 126	3	122	366	117.5–126.5	3
127 – 135	5	131	655	126.5–135.5	8
136 – 144	9	140	1260	135.5–144.5	17
145 – 153	12	149	1788	144.5–153.5	29
154 – 162	5	158	790	153.5–162.5	34
163 – 171	4	167	668	162.5–171.5	38
172 – 180	2	176	352	171.5–180.5	40
$n = \sum f = 40$			$\sum fX = 5879$		

$$\bar{X} = \frac{\sum fX}{\sum f} = \frac{5879}{40} = 146.975 \text{ pounds.}$$

$n/2 = 40/2 = 20$ lies in the class 145 – 153, which is the median class.

$$\text{Median: } \tilde{X} = l + \frac{h}{f} \left(\frac{n}{2} - F \right) = 144.5 + \frac{9}{12} \left(\frac{40}{2} - 17 \right)$$

$$= 144.5 + \frac{9}{12} (3) = 144.5 + 2.25 = 146.75 \text{ pounds.}$$

$$\begin{aligned}\text{Mode: } \hat{X} &= l + \frac{(f_m - f_1)}{(f_m - f_1) + (f_m - f_2)} \times h \\ &= 144.5 + \frac{(12 - 9)}{(12 - 9) + (12 - 5)} \times 9 = 144.5 + \frac{(3)9}{3 + 7} \\ &= 144.5 + \frac{27}{10} = 147.2 \text{ pounds.}\end{aligned}$$

3.29

No. of leaves (X)	No. of Branches	D	fD	Cumulative frequency
5	3	-3	-9	3
6	8	-2	-16	11
7	11	-1	-11	22
A → 8	18	0	0	40
9	20	1	20	60
10	13	2	26	73
11	9	3	27	82
12	5	4	20	87

$$\Sigma Df = 93 - 36 = 57$$

$$\begin{aligned}\bar{X} &= A + \frac{\Sigma fD}{n} = 8 + \frac{57}{87} \\ &= 8 + 0.56 = 8.56\end{aligned}$$

Median \bar{X} = value of $\left(\frac{n+1}{2}\right)$ th

$$= \frac{88}{2} = 44\text{th} = 9$$

Mode of \hat{X} = No. of leaves corresponding to maximum frequency = 9

3.30

Subject	X Marks	W Weight	WX
English	73	3	219
Urdu	85	3	255

Subject	X Marks	W Weight	WX
Math	92	4	368
Economics	65	3	195
		13	1037

$$\bar{X}_w = \frac{\sum wX}{\sum w} = \frac{1037}{13} = 79.77$$

3.31

Marks	Frequency	Class boundary	Cumulative frequency
30 – 39	2	29.5 – 39.5	2
40 – 49	3	39.5 – 49.5	5
50 – 59	11	49.5 – 59.5	16
60 – 69	20	59.5 – 69.5	36
70 – 79	32	69.5 – 79.5	68
80 – 89	25	79.5 – 89.5	93
90 – 99	7	89.5 – 99.5	100
	$n = \sum f = 100$		

$n/2 = 100/2 = 50$ lies in the class 70 – 79, which is the median class.

$$\text{Median: } \tilde{X} = l + \frac{h}{f} \left(\frac{n}{2} - F \right) = 69.5 + \frac{10}{32} (50 - 36) \\ = 69.5 + 4.375 = 73.875.$$

$n/4 = 100/4 = 25$ lies in the class 60 – 69, which is the class where Q_1 lies.

$$Q_1 = l + \frac{h}{f} \left(\frac{n}{4} - F \right) = 59.5 + \frac{10}{20} (25 - 16) \\ = 59.5 + 4.5 = 64.$$

$3n/4 = 3(100)/n = 75$ lies in the class 80 – 89, which is the class where Q_3 lies.

$$Q_3 = l + \frac{h}{f} \left(\frac{3n}{4} - F \right) = 79.5 + \frac{10}{25} (75 - 68) \\ = 79.5 + 2.8 = 82.3.$$

3.32

Marks	No. of students	Class boundary	Frequency (f)
Less than 30	3	20 – 30	3
Less than 40	17	30 – 40	14
Less than 50	43	40 – 50	26
Less than 60	58	50 – 60	15
Less than 70	67	60 – 70	9
Less than 80	75	70 – 80	8
			$n = \sum f = 75$

$n/2 = 75/2 = 37.5$ lies in the class 40 – 50, which is the median class.

$$\text{Median: } \tilde{X} = l + \frac{h}{f} \left(\frac{n}{2} - F \right) = 40 + \frac{10}{26} (37.5 - 17) = 40 + 7.88 \\ = 47.88 \text{ or } 48 \text{ marks.}$$

- 3.33 The marks of 49 students are arranged in ascending order as 15 17 18 18 20 24 24 26 27 28 29 29 30 30 30 30 32 35 36 36 37 38 39 40 40 40 40 40 41 42 42 42 43 45 45 45 50 50 50 53 53 54 54 56 56 56 58 60

$$\text{Median } (\tilde{X}) = \text{value of } \left(\frac{n+1}{2} \right) \text{th or } \left(\frac{49+1}{2} \right) \text{th or 25th item}$$

from below. The value of the 25th item is 40, thus $\tilde{X} = 40$.

$$Q_1 = \text{value of } \left(\frac{n+1}{4} \right) \text{th or } \left(\frac{49+1}{4} \right) \text{th or 12.5th item from below.}$$

The value of the 12th item is 29 and that of the 13th item is 30. Thus Q_1 is a value 0.5th of the way between 29 and 30, which is 29.5. Thus $Q_1 = 29.5$.

$$Q_3 = \text{value of } \frac{3(n+1)}{4} \text{ th or } \frac{3(49+1)}{4} \text{ th or 37.5th item from below.}$$

The value of the 37th item is 50 and that of the 38th item is also 50. Hence $Q_3 = 50$.

$$D_7 = \text{value of } \frac{7(n+1)}{10} \text{ th or } \frac{7(49+1)}{10} \text{ th or 35th item from below. The value of the 35th item is 45. Thus } D_7 = 45.$$

$$P_{84} = \text{value of } \frac{84(n+1)}{100} \text{ th or } \frac{84(49+1)}{100} \text{ th or 42nd item from below. The value of the 42nd item is 53. Thus } P_{84} = 53.$$

3.34 Mode = 3 Median - 2 Mean or 2 Mean = 3 Median - Mode
 or Mean = (3 Median - Mode)/2 = (3(65) - 85)/2 = 110/2 = 55.

3.35

Hourly Wages	Number of Employees
4 — 6	13
6 — 8	111
8 — 10	182
10 — 12	105
12 — 14	19
14 — 16	7

$$\begin{aligned} \text{Mode} &= l + \frac{f_m - f_{m-1}}{(f_m - f_{m-1}) + (f_m - f_{m-2})} \times h \\ &= 8 + \frac{(182 - 111)}{(182 - 111) + (182 - 105)} \times 2 \\ &= 8 + \frac{71}{71 + 77} \times 2 \\ &= 8 + 0.9595 = 8.9595 \end{aligned}$$

Mode = 9

3.36

X	f	log X	f log X
2	5	0.3010	1.505
3	7	0.4771	3.3397
4	8	0.6021	4.8168
5	3	0.6991	2.0973
6	2	0.7782	1.5564
	$\Sigma f = 25$		13.3152

$$\log G = \frac{\sum f \log X}{\sum f} = \frac{13.3152}{25} = 0.532608$$

$$G = \text{Anti log } (0.532608) = 3.409$$

3.37(a)

1/X	0.667	0.1111	0.0833	0.0566	0.0500	0.0357	0.0278	0.02222
X	15	9	12	18	20	28	36	45

Array: 9, 12, 15, 18, 20, 28, 36, 45

Median = 19.

3.37(b)

X	Y = 2X + 5
3	11
6	17
2	9
1	7
7	19
5	15
24	78

The means of the two sets are connected by the relation.

$$\bar{Y} = 2 \bar{X} + 5$$

$$13 = 2(4) + 5$$

$$13 = 8 + 5$$

$$13 = 13$$

$$\bar{X} = \frac{24}{6} = 4$$

$$\bar{Y} = \frac{78}{6} = 13$$

3.38

X	f	C.F.
10	3	3
20	8	11
30	16	27
40	26	53
50	20	73
60	18	91
70	6	97
80	4	101

$$\text{Median} = \left(\frac{n+1}{2} \right) \text{th item} = 51\text{th item} = 40$$

3.39

X No. of leaves	f No. of branches	Xf	C.F.
1	5	5	5
2	23	46	28
3	34	102	62
4	50	200	112
5	98	490	210
6	75	450	285
7	4	28	289
8	2	16	291
	291	1337	

$$\text{Mean} = \bar{X} = \frac{\sum Xf}{\sum f} = \frac{1337}{291} = 4.5945 \quad \text{or} \quad 5$$

Median class $\left(\frac{n+1}{2}\right)$ th value

$$\frac{292}{2} = 146$$

$$\text{Median} = \tilde{X} = 5.$$

3.41(a)

Time	f No. of customers	X	fX	C.F.	C.B.
10 - 19	4	14.5	58	4	9.5 - 19.5
20 - 29	8	24.5	196	12	19.5 - 29.5
30 - 39	16	34.5	552	28	29.5 - 39.5
40 - 49	20	44.5	890	48	39.5 - 49.5
50 - 59	10	54.5	545	58	49.5 - 59.5
60 - 69	2	64.5	129	60	59.5 - 69.5
$\sum f = 60$		$\sum fX = 2370$			

$$\bar{X} = \frac{\sum fX}{\sum f} = \frac{2370}{60} = 39.5$$

$$\begin{aligned}\text{Median} &= l + \frac{h}{f} \left(\frac{n}{2} - C \right) = 39.5 + \frac{10}{20} (30 - 28) = 39.5 + \frac{20}{20} \\ &= 39.5\end{aligned}$$

$$\begin{aligned}\text{Mode} &= l + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h \\ &= 39.5 + \frac{(20 - 16)}{(20 - 16) + (20 - 10)} \times 10 = 39.5 + \frac{4}{4 + 10} \times 10\end{aligned}$$

$$\text{Mode} = 39.5 + \frac{4}{14} \times 10 = 39.5 + 0.2041 = 39.7041$$

$$Q_1 = l + \frac{h}{f} \left(\frac{n}{4} - \text{C.F.} \right)$$

$$\frac{n}{4} = \frac{60}{4} = 15$$

$$\begin{aligned}Q_1 &= 29.5 + \frac{10}{16} (15 - 12) = 29.5 + \frac{10}{16} (3) = 29.5 + \frac{30}{16} \\ &= 29.5 + 1.875 = 31.375\end{aligned}$$

$$Q_3 = l + \frac{h}{f} \left(\frac{3n}{4} - \text{C.F.} \right)$$

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$$\frac{3n}{4} = \frac{3(6)}{4} = 45$$

$$Q_3 = 39.5 + \frac{10}{20} (45 - 28) = 39.5 + 8.5 = 48$$

$$D_6 = l + \frac{h}{f} \left(\frac{6n}{10} - C.F. \right)$$

$$\frac{6n}{10} = \frac{6 \times 60}{10} = \frac{360}{10} = 36$$

$$D_6 = 39.5 + \frac{10}{20} (36 - 28) = 39.5 + \frac{10}{20} (8) = 39.5 + 4 = 43.5$$

$$P_{37} = l + \frac{h}{f} \left(\frac{37n}{100} - C.F. \right)$$

$$\frac{37n}{100} = \frac{37(6)}{100} = 22.2$$

$$P_{37} = 29.5 + \frac{10}{16} (22.2 - 12) = 29.5 + 6.375 = 35.875$$

$$P_{67} = l + \frac{h}{f} \left(\frac{67n}{100} - C.F. \right)$$

$$\frac{67n}{100} = \frac{67(6)}{100} = 40.2 = 39.5 + \frac{10}{20} (40.2 - 28) = 39.5 + 6.1 = 45.6$$

3.41(b)

(X) No. of children	(f) No. of families	C.F.
0	4	4
1	7	11
2	14	25
3	22	47
4	12	59
5	6	65

$$\text{Median} = \left(\frac{n+1}{2} \right) \text{th item} = \left(\frac{65+1}{2} \right) \text{th item} = 33 \text{th item}$$

$$= 3$$

Mode = 3

$$Q_1 = \left(\frac{n+1}{4} \right) \text{th item} = \left(\frac{65+1}{4} \right) \text{th item} = 16.5 \text{th item}$$

$$= 2$$

$$Q_3 = 3 \left(\frac{n+1}{4} \right) \text{th item} = 3 \left(\frac{65+1}{4} \right) \text{th item} = 49.5 \text{th item}$$

$$= 4$$

$$D_9 = 9 \left(\frac{n+1}{10} \right) \text{th item} = 9 \left(\frac{65+1}{10} \right) \text{th item} = 59.4 \text{th item}$$

$$= 5$$

$$P_{10} = 10 \left(\frac{n+1}{100} \right) \text{th item} = 10 \left(\frac{65+1}{100} \right) \text{th item} = 6.6 \text{th item}$$

$$= 1$$

$$P_{60} = 60 \left(\frac{n+1}{100} \right) \text{th item} = 60 \left(\frac{65+1}{100} \right) \text{th item} = 39.6 \text{th item}$$

$$= 3$$

$$P_{80} = 80 \left(\frac{n+1}{100} \right) \text{th item} = 80 \left(\frac{65+1}{100} \right) \text{th item} = 52.8 \text{th item}$$

$$= 4$$

3.42(a)

Marks	f	X	$\log X$	$f \log X$	$1/X$	f/X
30 - 39	3	34.5	1.5378	4.6134	0.0291	0.0873
40 - 49	10	44.5	1.6484	16.484	0.0225	0.225
50 - 59	20	54.5	1.7364	34.728	0.0183	0.366
60 - 69	13	64.5	1.8096	23.5248	0.0155	0.2015
70 - 79	4	74.5	1.8722	7.4888	0.0134	0.536
	50			86.839		0.9334

$$\log G = \frac{\sum f \log X}{\sum f} = \frac{86.839}{50} = 1.73678$$

$$G = \text{Antilog}(1.73678) = 54.55$$

$$H.M. = \frac{\Sigma f}{\Sigma(f/X)} = \frac{50}{0.9334} = 53.57$$

G.M. > H.M.

3.42(b)

No. of orders	No. of weeks	C.F.
18	2	
19	2	
20	3	
21	4	
22	6	

No. of orders	No. of weeks	C.F.
23	9	26
24	12	38
25	6	44
26	5	49
27	2	51
28	1	52

Mode = 24

Median = $\left(\frac{n+1}{2}\right)$ th item = 26.5th item = 24

$$3.42(c) \text{ H.M.} = \frac{n}{\sum(1/X)} = \frac{3}{\frac{1}{25} + \frac{1}{30} + \frac{1}{40}} = \frac{3}{0.04 + 0.033 + 0.025}$$

$$= \frac{3}{0.098} = 30.61$$

3.43(a)

D	X = A + D	X
-12	10.5	10.5
-8.5	14	14
3.0	25.5	22.5
0	22.5	22.9
2.5	25	23
6.6	29.1	24.1
9.2	31.7	25
1.6	24.1	25.5
0.5	23	29.1
0.4	22.9	31.7

$Q_1 = \left(\frac{n+1}{4}\right)$ th item

$= \left(\frac{10+1}{4}\right)$ th item

$= 2.75$ th item

$= 14 + 0.75 (22.5 - 14)$

$= 20.375$

3.43(b)

Classes	f	X	1/X	f/X
40 — 50	4	45	0.0222	0.0888
50 — 60	8	55	0.0182	0.1456

Classes	f	X	1/X	f/X
60 — 70	16	65	0.0154	0.2464
70 — 80	8	75	0.0133	0.1064
80 — 90	4	85	0.0118	0.0472
	40			0.6344

$$\text{H.M.} = \frac{\Sigma f}{\Sigma(f/X)} = \frac{40}{0.6344} = 63.05$$

3.44

Marks	f	C.B.
10 — 14	2	9.5 — 14.5
15 — 19	4 f_1	14.5 — 19.5
20 — 24	8 f_m	19.5 — 24.5
25 — 29	6 f_2	24.5 — 29.5
30 — 34	3	29.5 — 34.5

$$\begin{aligned}\text{Mode} &= 1 + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h \\ &= 19.5 + \frac{(8 - 4)}{(8 - 4) + (8.6)} \times 5 \\ &= 19.5 + \frac{4}{4 + 2} \times 5 \\ &= 19.5 + \frac{4}{6} \times 5 \\ &= 19.5 + 3.33\end{aligned}$$

$$\text{Mode} = 22.83$$

3.45(a)

$u = \frac{X-30}{5}$	f	fu
-2	5	-10
-1	8	-8
0	15	0
1	20	20
2	12	24
3	4	12
	$\Sigma f = 64$	$\Sigma fu = 38$

$$\bar{X} = A + \frac{\sum f u}{\sum f} \times h = 30 + \frac{38}{64} \times 5 = 30 + 2.97 = 32.97$$

3.45(b)

Group	<i>f</i>
0 — 5	4
5 — 10	7 <i>f</i> ₁
10 — 15	10 <i>f</i> _m
15 — 20	6 <i>f</i> ₂
20 — 25	4
	31

$$\begin{aligned}
 \text{Mode} &= l + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h \\
 &= 10 + \frac{(10 - 7)}{(10 - 7) + (10 - 6)} \times 5 = 10 + \frac{3}{3 + 4} \times 5 \\
 &= 10 + 2.143 = 12.143
 \end{aligned}$$