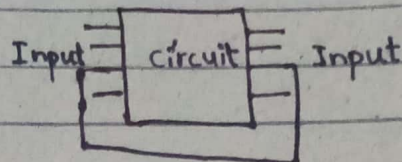


If any other grouping of implicants is not possible and there is only one F, then all the prime implicants are essential.

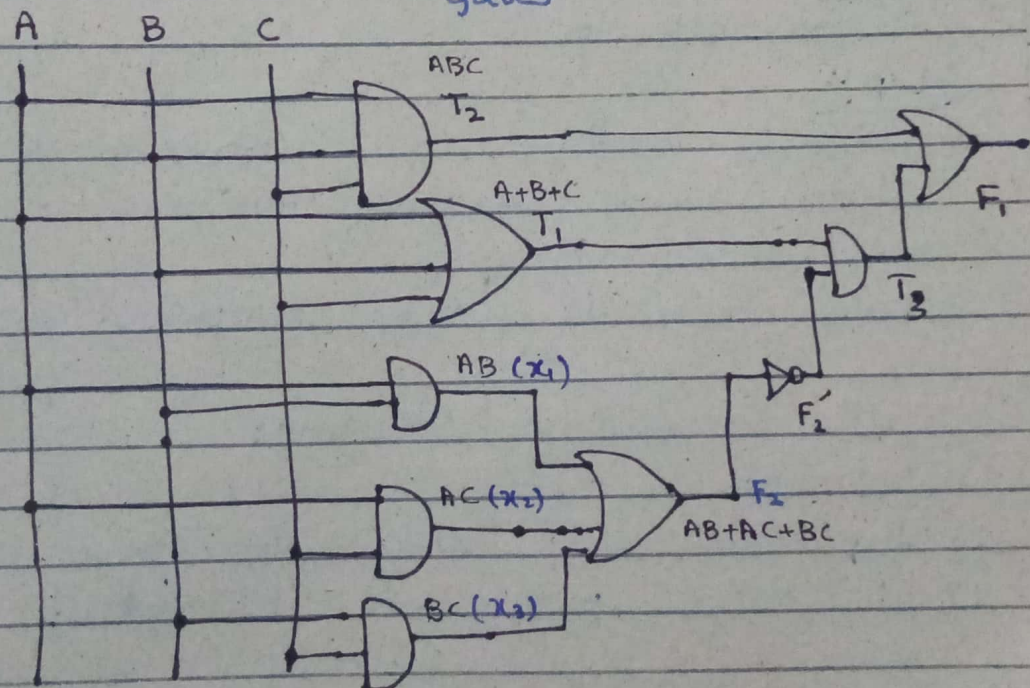
## Combinational Circuits:

(other type of circuit is sequential circuit)



\* Output is dependent on input and input is dependent on output.

Combinational Circuits: → circuit which generates output from inputs using gates



$$F_1 = T_3 + T_2$$

$$F_1 = (T_1 \cdot F_2') + T_2$$

$$F_1 = (A+B+C) \cdot F_2' + T_2$$

$$F_1 = (A+B+C) \cdot (AB+AC+BC)' + T_2$$

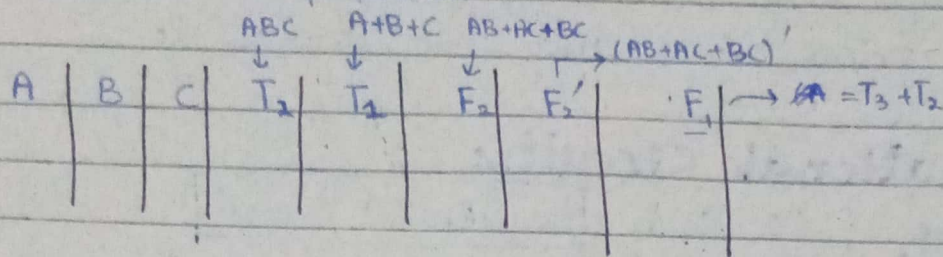
$$F_1 = (A+B+C) \cdot (AB+AC+BC)' + ABC$$

$$F_2 = (AB+AC+BC)$$

$$F_2' = (AB+AC+BC)'$$



Table can be derived direct by circuit or by Boolean expression.



Requirement  $\rightarrow$  Circuit Design.

$\hookrightarrow$  See:  
Input

$\hookrightarrow$  Output

Step 1:

$\rightarrow$  Input, output (Numbers)

Step 2:

$\rightarrow$  Assign values (names)  $x_1, x_2, x_3, \dots$

Step 3:

$\rightarrow$  Design Truth Table

Step 4:

$\rightarrow$  Simplify output from truth table by writing terms against minterms and then simplify it.

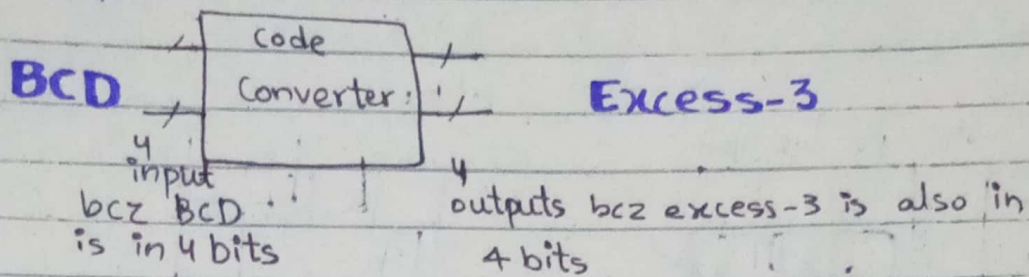
Draw Circuit from simplified output.

Circuit Correctness Checking:

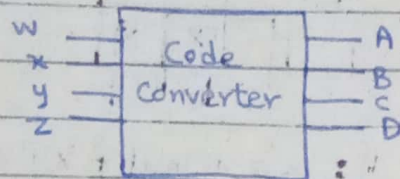
(a) Manually.

(b) By comparing input values from truth table with circuit's output

Step 1:



Step #2:



Step 3: Truth Table

	BCD				Excess-3			
	w	x	y	z	A	B	C	D
0	0	0	0	0	0	0	1	1
1	0	0	0	1	0	1	0	0
2	0	0	1	0	0	1	0	1
3	0	0	1	1	0	1	1	0
4	0	1	0	0	0	1	1	1
5	0	1	0	1	1	0	0	0
6	0	1	1	0	1	0	0	1
7	0	1	1	1	1	0	1	0
8	1	0	0	0	1	0	1	1
9	1	0	0	1	1	1	0	0

Min-Terms against A:

$$A(w, x, y, z) = \sum (5, 6, 7, 8, 9)$$

B:

$$B(w, x, y, z) = \sum (1, 2, 3, 4, 9)$$

C:

$$C(w, x, y, z) = \sum (0, 3, 4, 7, 8)$$

D:

$$D(w, x, y, z) = \sum (0, 2, 4, 6, 8)$$

Don't Care functions:

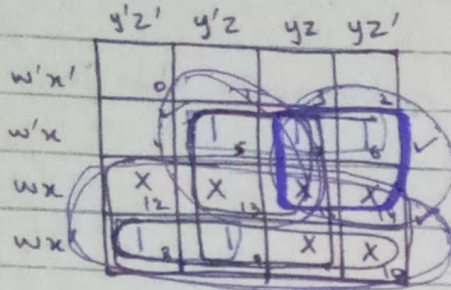
$$d(w, x, y, z) = \sum (10, 11, 12, 13, 14, 15)$$



Step 4:

Simplify by K-Map  $\rightarrow$  (to reduce the no. of literals) then implement circuit.

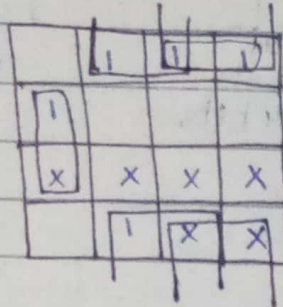
A:



$$A = wx'y' + w'xz + w'xy$$

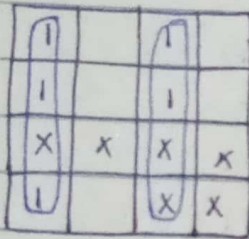
$$A = xz + xy + w$$

B:



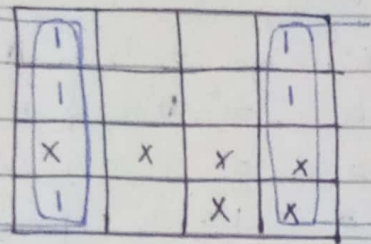
$$B = x'z + x'y + xy'z'$$

C:



$$C = y'z' + yz$$

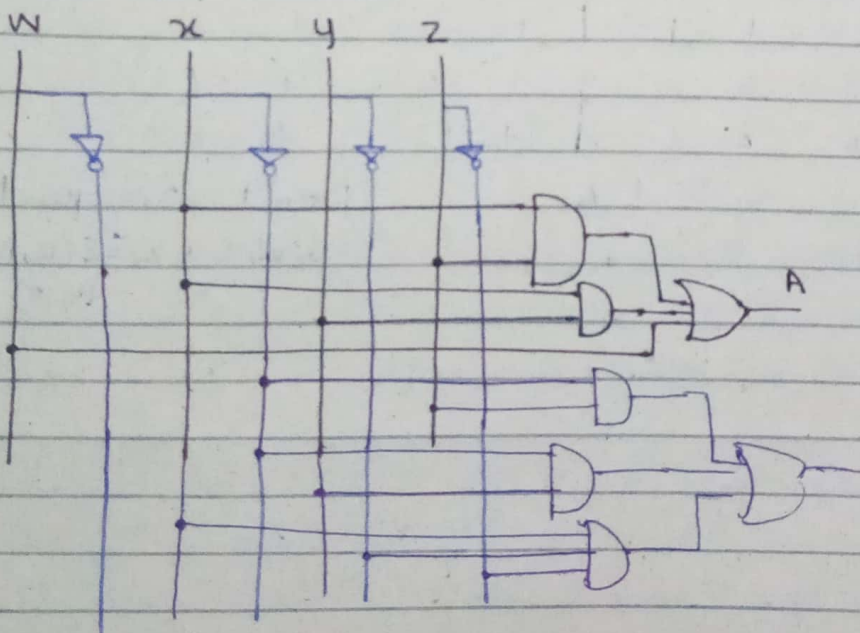
D:



$$D = x'y'z' + yz'$$

$$D = z'$$

Designing Circuit:



$$\begin{array}{r}
 1 \\
 4 \uparrow \\
 0
 \end{array}
 \quad
 \begin{array}{r}
 100 \\
 8000 \\
 000
 \end{array}
 \quad
 \begin{array}{r}
 100 \text{ (3)} \\
 8011 \\
 000
 \end{array}$$

scanf("%d", &x)

while(1)

"Mostly codings contain 6 don't care combinations"

Design a code converter from 84-2-1 to BCD.

8	4	2	1		8	4	2	1	(BCD)
A	B	C	D		A	B	C	D	
0	0	0	0	m <sub>0</sub>	0	0	0	0	
0	1	1	1	m <sub>7</sub>	0	0	0	1	
0	1	1	0	m <sub>6</sub>	0	0	1	0	
0	1	0	1	m <sub>5</sub>	0	0	1	1	
0	1	0	0	m <sub>4</sub>	0	1	0	0	
1	0	1	1	m <sub>11</sub>	0	1	0	1	
1	0	1	0	m <sub>10</sub>	0	1	1	0	
1	0	0	1	m <sub>9</sub>	0	1	1	1	
1	0	0	0	m <sub>8</sub>	1	0	0	0	
1	1	1	1	m <sub>15</sub>	1	0	0	1	

Min-terms against A:

$$A = \sum(8, 15)$$

Against B:

$$B = \sum(4, 11, 10, 9)$$

$$C = \sum(6, 5, 10, 9)$$

$$D = \sum(7, 3, 11, 9, 15)$$

$$d = \sum(1, 2, 3, 12, 13, 14)$$



A:

	$y'z'$	$y'z$	$yz$	$yz'$
$w'x'$	$m_0$	$X_{m_1}$	$X_{m_2}$	$X_{m_3}$
$w'x$	4	5	7	6
$wx$	$X_{12}$	$X_{13}$	15	$X_{14}$
$wx'$	1	9	11	10

B:

	$y'z'$	$y'z$	$yz$	$yz'$
$w'x'$	0	$X_1$	$X_3$	$X_2$
$w'x$	4	5	7	6
$wx$	$X_{12}$	$X_{13}$	15	$X_{14}$
$wx'$	8	9	11	10

$$A = wy'z' + wx$$

$$B = wx'z + wx'y + xy'z'$$

C:

	$y'z'$	$y'z$	$yz$	$yz'$
$w'x'$	0	$X_1$	$X_3$	$X_2$
$w'x$	4	5	7	6
$wx$	$X_{12}$	$X_{13}$	15	$X_{14}$
$wx'$	8	9	11	10

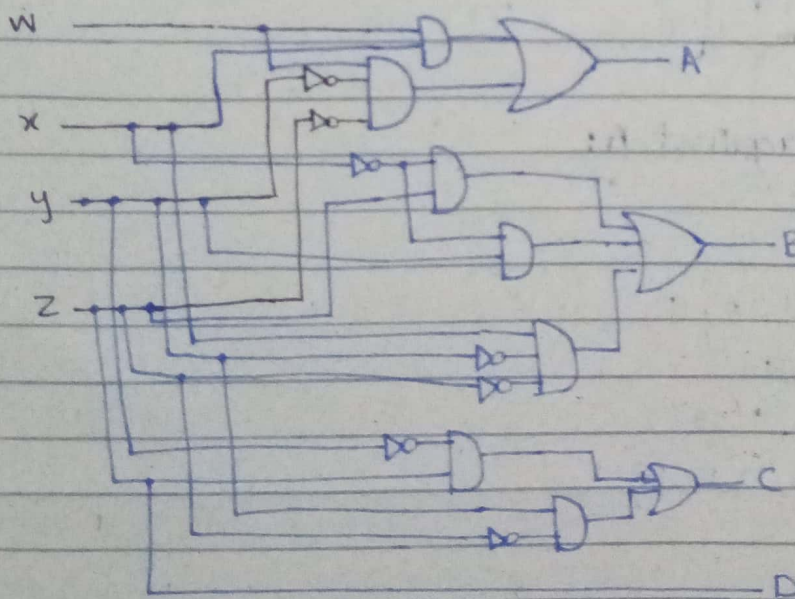
D:

	$y'z'$	$y'z$	$yz$	$yz'$
$w'x'$	0	$X_1$	$X_3$	$X_2$
$w'x$	4	5	7	6
$wx$	$X_{12}$	$X_{13}$	15	$X_{14}$
$wx'$	8	9	11	10

$$C = y'z + yz'$$

$$D = Z$$

Circuit Diagram:



Design a Half Adder Circuit.

## Binary Adder:

① This circuit is called half adder bcz it has 2 inputs and 2 outputs

x	y	S (x+y)	C (carry)
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

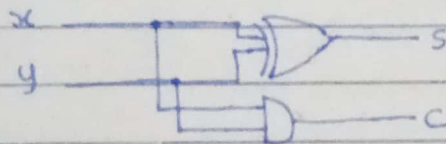
$$\begin{array}{r} 1 \\ + 1 \\ \hline \textcircled{1} 0 \\ \uparrow \\ \text{carry} \end{array}$$

For S output and C output, make min-terms by K-map:

$$S = x'y + x.y' = x \oplus y$$

$$C = x.y$$

### Half-Adder Circuit:



## Full Adder Circuit:

3 inputs → 2 outputs (that's why it is called full-adder Circuit)

x	y	z		S (x+y+z)	C (carry)
0	0	0	$m_0$	0	0
0	0	1	$m_1$	1	0
0	1	0	$m_2$	1	0
0	1	1	$m_3$	0	1
1	0	0	$m_4$	1	0
1	0	1	$m_5$	0	1
1	1	0	$m_6$	0	1
1	1	1	$m_7$	1	1



$$S = \Sigma(1, 2, 4, 7)$$

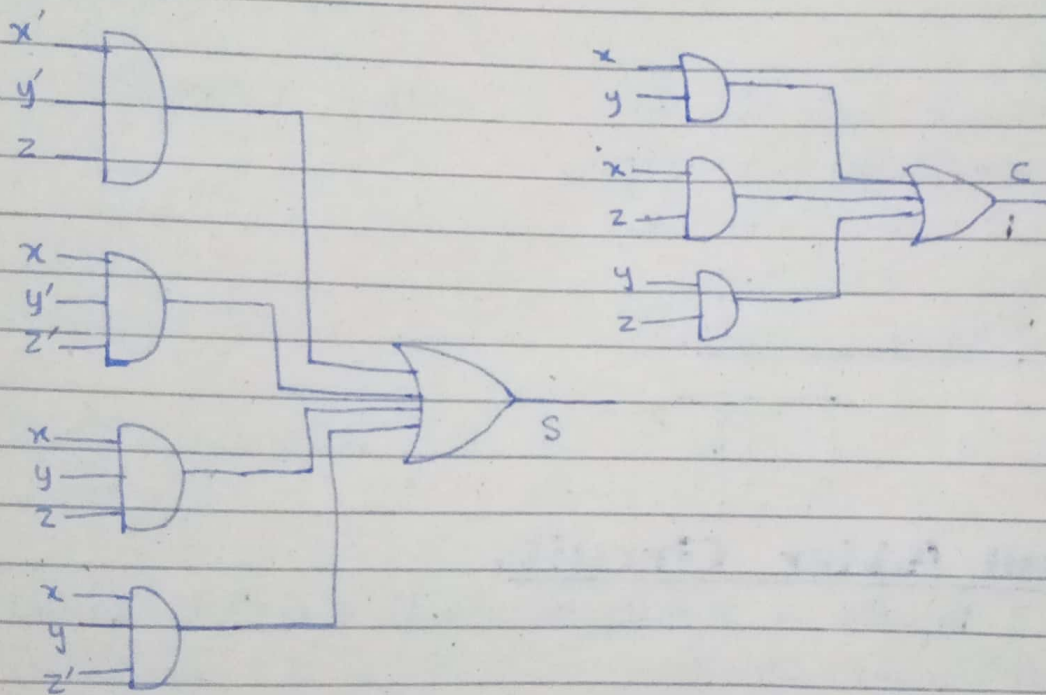
$$C = \Sigma(3, 5, 6, 7)$$

	$y'z'$	$y'z$	$yz$	$yz'$
$x'$	0	1	0	1
$x$	1	0	1	0

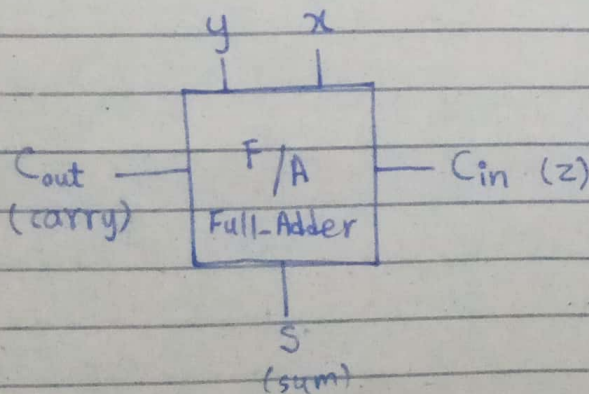
	$y'z'$	$y'z$	$yz$	$yz'$
$x'$	0	0	1	0
$x$	0	1	1	1

$$C = yz + xy'z + xy$$

$$S = x'y'z + xy'z' + xyz + xyz'$$



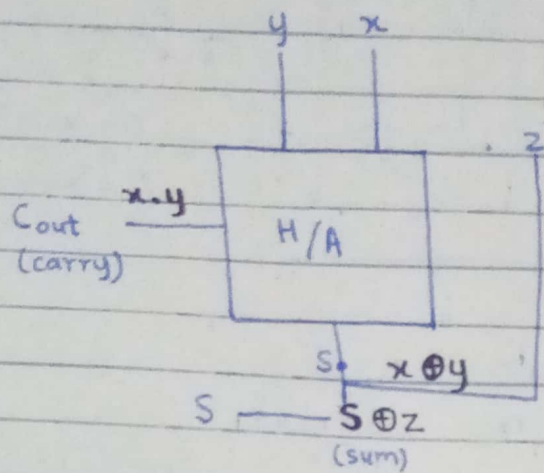
**Black Box Representation of full-Adder Circuit:**



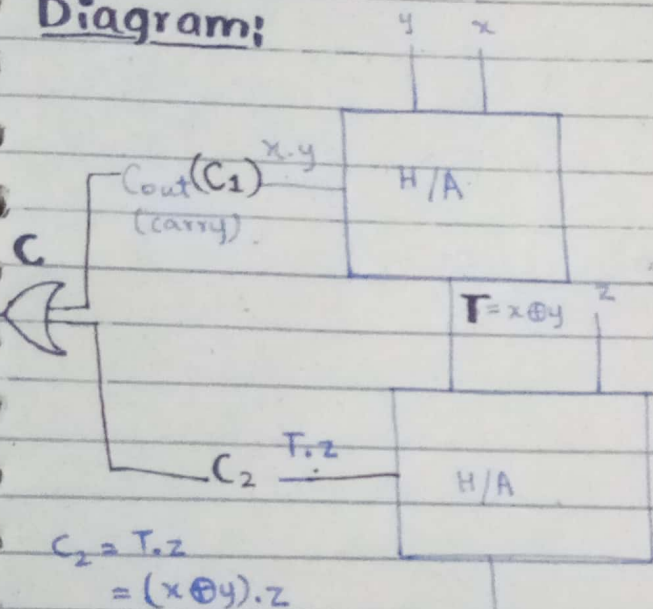


## Working of full Adder Using half-Adder

(2 half-Adders will be used, first for 2 inputs, and H/A for 'S and Z (as input)



### Block-Diagram:



$$0+0+1$$

$$\downarrow$$

$$S+1$$

$$\downarrow$$

$$S$$

$$C = C_1 + C_2$$

$$C = Tz + x.y$$

$$C = (x \oplus y)z + x.y$$

Why OR?

to generate 1 as C

only when  $C_1=1$

and  $C_2=1$  or  $C_1=0$

and  $C_2=1$  or  $C_1=1$

and  $C_2=0$

$$S = T \oplus z = (x \oplus y) \oplus z$$

$$S = x'y'z + x'yz' + xy'z + xy'z'$$

$$= x'y'z' + x'y'z + x'y'z + xyz$$

$$= z'(x'y + xy') + z(xy + x'y')$$

XOR

XNOR

XNOR = (XOR)'

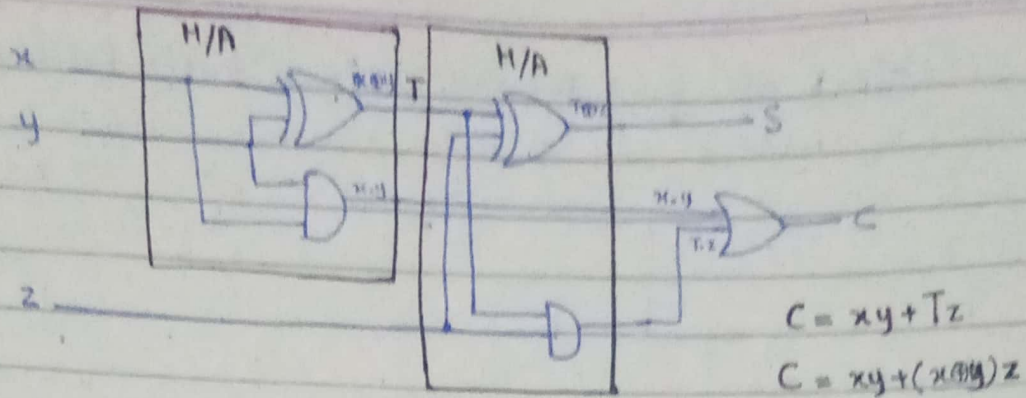
$$= z'(x'y + xy') + z(x'y + xy')$$

T =  $x \oplus y$

$$= z'T + zT'$$

$$= z \oplus T = \boxed{z \oplus x \oplus y} \text{ Proved}$$

## Circuit Diagram:



## Half Subtractor:

2 inputs → 2 outputs

① Truth Table:

x	y	Difference (x-y) D	Borrow In Bin
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

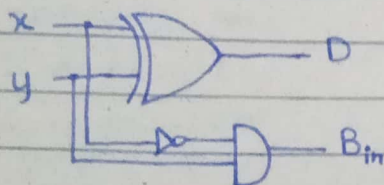
$\begin{array}{r} 0 \\ - 0 \\ \hline 0 \\ 0 \end{array}$   
 $B_{in} \text{ (1)} \rightarrow 0$   
 $\begin{array}{r} -1 \\ - 1 \\ \hline 1 \end{array}$   
 diff.

② Simplification:

$$D = x'y + y'x = x \oplus y \text{ (XOR)}$$

$$B_{in} = x'y$$

③ Circuit Diagram:



## Full Subtractor:

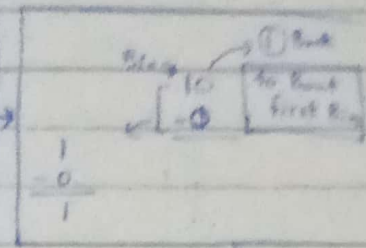
3 inputs generally but here 3 inputs are x, y, Bout (Borrow Out).

1 gives borrow which is called Bout  
 0 takes borrow which is called Bin  
 $\begin{array}{r} 1 \\ - 0 \\ \hline 1 \end{array}$



## 1. Truth Table

x	y	B <sub>out</sub>	D	B <sub>in</sub>
0	0	0 m <sub>0</sub>	0	0
0	0	1 m <sub>1</sub>	1	1
0	1	0 m <sub>2</sub>	1	1
0	1	1 m <sub>3</sub>	0	1
1	0	0 m <sub>4</sub>	0	0
1	0	1 m <sub>5</sub>	0	0
1	1	0 m <sub>6</sub>	0	0
1	1	1 m <sub>7</sub>	1	1



## 2. Simplification:

D:

	y'B'	y'B	yB'	yB
x'	0	1	0	1
x	1	0	1	0

B<sub>in</sub>:

	y'B'	y'B	yB'	yB
x'	0	1	1	1
x	0	0	1	0

$$D = x'y'B + x'yB' + x'yB + xyB$$

↓

$$B_{in} = x'E_{out} + yB'_{out} + x'y$$

For that combination, there is no grouping in k-map.

Then the combination is either exclusive NOR or exclusive OR.

↓

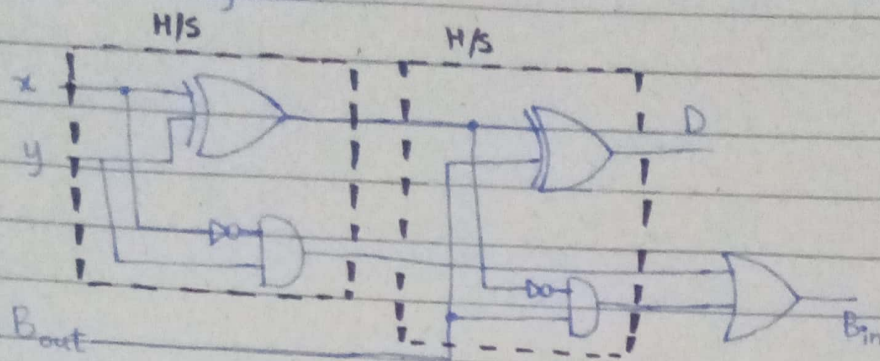
also filling of K-maps is in alternative boxes, even in 4 inputs/4 variables' K-map.

$$D = x'y'B + x'yB' + x'yB + xyB$$

$$D = x \oplus y \oplus B$$

### 3. Circuit Diagram:

2 H/S Combine to give the working of F/S



### Binary Adder: → Full adder adds only 1 bits of 4 bit - Binary Adder

A 1 1 0 1  
B 0 1 0 0  
① ← 0 0 0 1  
carry

Bits of A =  $A_3 A_2 A_1 A_0$   
Bits of B =  $B_3 B_2 B_1 B_0$   
 $C_4 \leftarrow S_3 S_2 S_1 S_0$

Max. number in 4 bits = 15

$A + B = 4 + 4 \text{ bits}$

$= 15 + 15 = 30$

Binary adder is

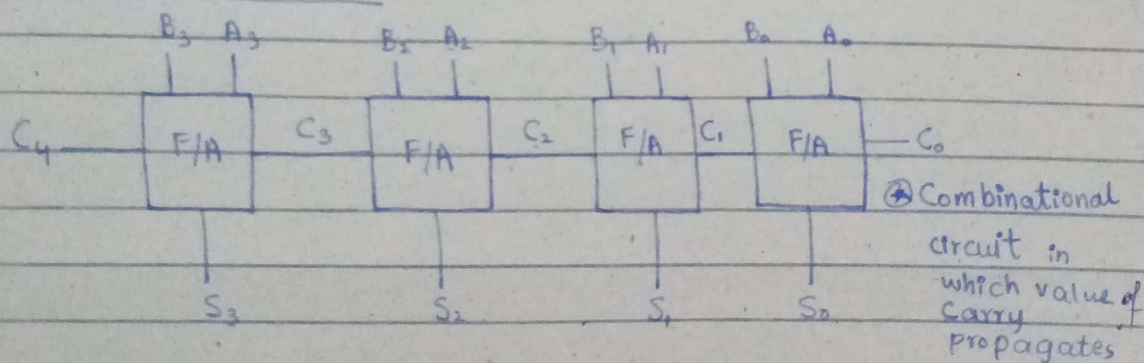
Bits of output = 5 =  $S_3 S_2 S_1 S_0$  and  $C_4$

Solved by taking 4 full

adders: why 4? bcz input numbers (A and B) are 4 bit numbers.

If inputs were 8 bit number, then 8 full adders will be used.

### Circuit Diagram:



⊕  $C_1$  is carry out of  $A_0, B_0$  and carry in of  $A_1, B_1$ . Same goes for  $C_2$  and  $C_3$ .

$C_4$  is final carry out.  $C_0$  is carry in of  $A_0, B_0$ . →



General form to understand Sum and  $C_{in}/C_{out}$ :

$$A_i + B_i$$

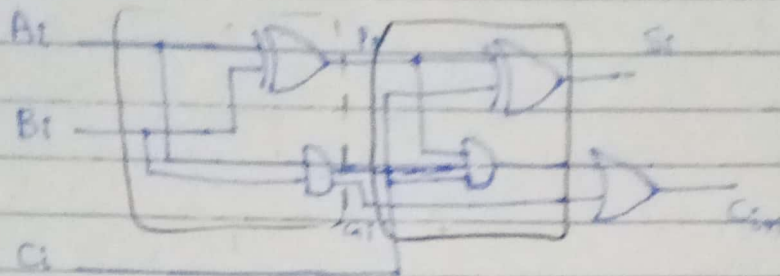
$$+ B_i$$

$$S_i$$

$$C_{i+1}$$

$$C_{out}$$

• This circuit is a combinational circuit which don't produce value of next carry and sum until the carry from previous F/A <sup>don't</sup> propagate into the next F/A.



$$P_i = A_i \oplus B_i$$

$$G_i = A_i \cdot B_i$$

$$S_i = P_i \oplus C_i$$

$$C_{i+1} = P_i C_i + G_i$$

$P_i$  can be computed at same time bcz  
All 4-bits have diff.  $A_i$  and  $B_i$ .  
Same goes with  $G_i$

Each full adder will produce output ( $S_i$ ) at the ~~Binary~~ same time if we can compute value of carry ( $C_i$ ) and don't let F/A to compute it. We can calculate  $C_i$  by above 4 equations. This will make parallel binary adder.

$$C_0 = \text{input carry}$$

$$C_1 = P_0 C_0 + G_0$$

$$C_1 = (A_0 \oplus B_0) C_0 + A_0 \cdot B_0$$

$$C_2 = P_1 C_1 + G_1 = (A_1 \oplus B_1) C_1 + A_1 \cdot B_1$$

$$C_2 = P_1 (P_0 C_0 + G_0) + G_1$$

$$C_3 = P_2 C_2 + G_2 = (A_2 \oplus B_2) C_2 + A_2 \cdot B_2$$

$$C_3 = P_2 (P_1 (P_0 C_0 + G_0) + G_1) + G_2$$

$$C_3 = P_2 (P_1 (P_0 C_0 + G_0) + G_1) + G_2 = P_2 P_1 P_0 C_0 + P_2 P_1 G_0 + P_2 G_1 + G_2$$

$$C_4 = \underbrace{P_3 C_3}_{\downarrow} + G_3 = (A_3 \oplus B_3) C_3 + G_3$$

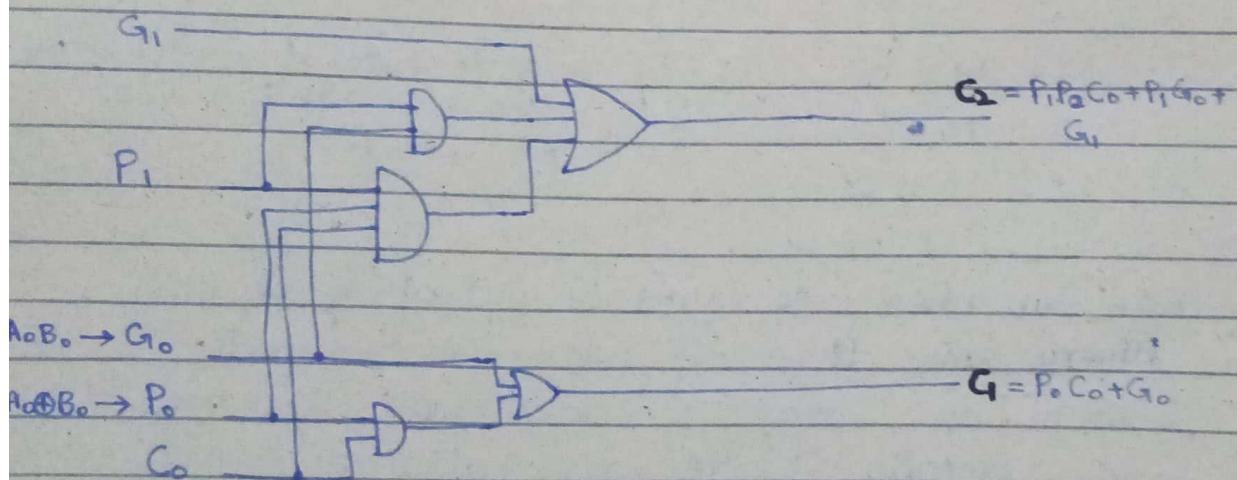
$$= P_3 (\underbrace{P_2 C_2}_{\downarrow} + G_2) + G_3$$

$$= P_3 (P_2 (P_1 C_1 + G_1) + G_2) + G_3$$

$$= P_3 (P_2 (P_1 (\underbrace{P_0 C_0}_{\downarrow} + G_0) + G_1) + G_2) + G_3$$

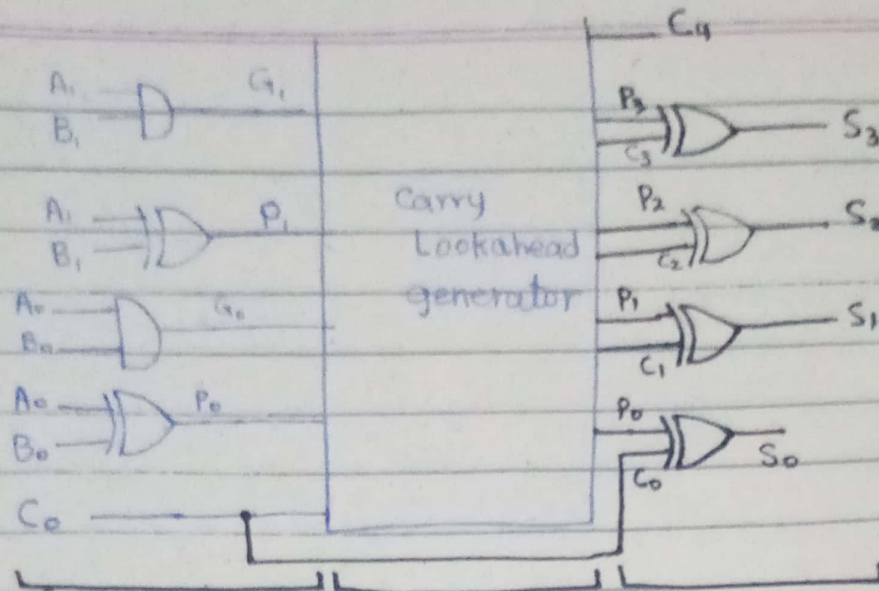
$$C_4 = P_3 P_2 P_1 P_0 G_0 + P_3 P_2 P_1 G_1 + P_3 P_2 G_2 + P_3 G_3 + P_0 G_0$$

\* **Carry Lookahead Generator** (generates all carries in parallel way at the same time)





⊛ It gives output in milliseconds.



→ No time delay  
1st level

2nd level

3rd level

due to AND/OR

4th level

$$P_i = A_i \oplus B_i$$

$$G_i = A_i B_i$$

$$S_i = P_i \oplus C_i$$

$$C_{i+1} = P_i C_i + G_i$$

Draw carry lookahead generator for 2 bits/3bits?