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Calculus and Analytic Geometry, MATHEMATICS 12

Question # 1
Find by making suitable substitution in the following functions defined as:

(i) $y = \sqrt{\dfrac{1-x}{1+x}}$

(ii) $y = \sqrt{x+\sqrt{x}}$

(iii) $y = x\sqrt{\dfrac{a+x}{a-x}}$

(iv) $y=\left(3x^2-2x+7\right)^6$

(v) $\dfrac{\sqrt{a^2+x^2}}{\sqrt{a^2-x^2}}$

Solution

(i)

$$y = \sqrt{\frac{1-x}{1+x}}$$

Put $u = \dfrac{1-x}{1+x}$

So $y = \sqrt{u} \qquad \Rightarrow \; y = u^{\frac{1}{2}}$

Now diff. u w.r.t. x

$$\frac{du}{dx} = \frac{d}{dx}\left(\frac{1-x}{1+x}\right)$$

$$= \frac{(1+x)\frac{d}{dx}(1-x)-(1-x)\frac{d}{dx}(1+x)}{(1+x)^2}$$

$$= \frac{(1+x)(-1)-(1-x)(1)}{(1+x)^2}$$

$$= \frac{-1-x-1+x}{(1+x)^2}$$

$$\Rightarrow \frac{du}{dx} = \frac{-2}{(1+x)^2}$$

Now diff. y w.r.t. u

$$\frac{dy}{du} = \frac{d}{du}u^{\frac{1}{2}}$$

$$= \frac{1}{2}u^{-\frac{1}{2}} = \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-\frac{1}{2}}$$

$$\Rightarrow \frac{dy}{du} = \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$$

Now by chain rule

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

$$= \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}\cdot\frac{-2}{(1+x)^2}$$

$$= \frac{(1+x)^{\frac{1}{2}}}{(1-x)^{\frac{1}{2}}}\cdot\frac{-1}{(1+x)^2}$$

$$= \frac{-1}{(1-x)^{\frac{1}{2}}(1+x)^{2-\frac{1}{2}}}$$

$$= \frac{-1}{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}} \qquad \textit{Answer}$$

(ii)

$$y = \sqrt{x+\sqrt{x}}$$

Let $u = x+\sqrt{x} = x+x^{\frac{1}{2}}$

$\Rightarrow y = \sqrt{u} = u^{\frac{1}{2}}$

Diff. u w.r.t. x

$$\frac{du}{dx} = \frac{d}{dx}\left(x+x^{\frac{1}{2}}\right)$$

$$= 1+\frac{1}{2}x^{-\frac{1}{2}} = 1+\frac{1}{2\sqrt{x}}$$

$$= \frac{2\sqrt{x}+1}{2\sqrt{x}}$$

Now diff. y w.r.t. x

$$\frac{dy}{du} = \frac{d}{du}u^{\frac{1}{2}}$$

$$= \frac{1}{2}u^{-\frac{1}{2}} = \frac{1}{2u^{\frac{1}{2}}} = \frac{1}{2\left(x+\sqrt{x}\right)^{\frac{1}{2}}}$$

$$\Rightarrow \frac{dy}{du} = \frac{1}{2\sqrt{x+\sqrt{x}}}$$

Now by chain rule

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

$$= \frac{1}{2\sqrt{x+\sqrt{x}}}\cdot\frac{2\sqrt{x}+1}{2\sqrt{x}}$$

$$= \frac{2\sqrt{x}+1}{4\sqrt{x}\cdot\sqrt{x+\sqrt{x}}} \quad \textit{Answer}$$

(iii)

$$y = x\sqrt{\frac{a+x}{a-x}}$$

Put $u = \frac{a+x}{a-x}$

So $y = x\sqrt{u} = x(u)^{\frac{1}{2}}$

Diff. w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}x(u)^{\frac{1}{2}}$$

$$= x\frac{d}{dx}(u)^{\frac{1}{2}} + (u)^{\frac{1}{2}}\frac{d}{dx}x$$

$$= x\frac{1}{2}(u)^{-\frac{1}{2}}\frac{du}{dx} + (u)^{\frac{1}{2}}(1)$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{2}(u)^{-\frac{1}{2}}\frac{du}{dx} + (u)^{\frac{1}{2}} \ \ldots\ldots\ (i)$$

Now diff. u w.r.t. x

$$\frac{du}{dx} = \frac{d}{dx}\left(\frac{a+x}{a-x}\right)$$

$$= \frac{(a-x)\frac{d}{dx}(a+x)-(a+x)\frac{d}{dx}(a-x)}{(a-x)^2}$$

$$= \frac{(a-x)(0+1)-(a+x)(0-1)}{(a-x)^2}$$

$$= \frac{(a-x)(1)-(a+x)(-1)}{(a-x)^2}$$

$$= \frac{a-x+a+x}{(a-x)^2} = \frac{2a}{(a-x)^2}$$

Using value of u and $\frac{du}{dx}$ in eq. (i)

$$\frac{dy}{dx} = \frac{x}{2}\left(\frac{a+x}{a-x}\right)^{-\frac{1}{2}}\frac{2a}{(a-x)^2} + \left(\frac{a+x}{a-x}\right)^{\frac{1}{2}}$$

$$= \frac{(a+x)^{-\frac{1}{2}}}{(a-x)^{-\frac{1}{2}}}\cdot\frac{ax}{(a-x)^2} + \frac{(a+x)^{\frac{1}{2}}}{(a-x)^{\frac{1}{2}}}$$

$$= \frac{ax}{(a+x)^{\frac{1}{2}}(a-x)^{2-\frac{1}{2}}} + \frac{(a+x)^{\frac{1}{2}}}{(a-x)^{\frac{1}{2}}}$$

$$= \frac{ax}{(a+x)^{\frac{1}{2}}(a-x)^{\frac{3}{2}}} + \frac{(a+x)^{\frac{1}{2}}}{(a-x)^{\frac{1}{2}}}$$

$$= \frac{ax+(a+x)(a-x)}{(a+x)^{\frac{1}{2}}(a-x)^{\frac{3}{2}}}$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{ax+a^2-x^2}{(a+x)^{\frac{1}{2}}(a-x)^{\frac{3}{2}}}}$$

(iv)

Do yourself as above

(v)

Do yourself as above

Question # 2

Find $\frac{dy}{dx}$ if:

(i) $3x+4y+7 = 0$

(ii) $xy+y^2 = 2$

(iii) $x^2-4xy-5y=0$

(iv) $4x^2+2hxy+by^2+2gx+2fy+c = 0$

(v) $x\sqrt{1+y}+y\sqrt{1+x} = 0$

(vi) $y\left(x^2-1\right)=x\sqrt{x^2+4}$

Solution

(i)

$$3x+4y+7 = 0$$

Diff. w.r.t. x.

$$\frac{d}{dx}(3x+4y+7) = \frac{d}{dx}(0)$$

$$\Rightarrow 3(1)+4\frac{dy}{dx}+0 = 0 \quad \Rightarrow 4\frac{dy}{dx} = -3$$

$$\Rightarrow \boxed{\frac{dy}{dx} = -\frac{3}{4}}$$

(ii) $\quad xy+y^2 = 2$

Differentiating w.r.t. x

$$\frac{d}{dx}\left(xy+y^2\right) = \frac{d}{dx}(2)$$

$$\Rightarrow \frac{d}{dx}(xy)+\frac{d}{dx}y^2 = 0$$

$$\Rightarrow x\frac{dy}{dx}+y\frac{dx}{dx}+2y\frac{dy}{dx} = 0$$

$$\Rightarrow (x+2y)\frac{dy}{dx}+y(1) = 0$$

$$\Rightarrow (x+2y)\frac{dy}{dx} = -y$$

$$\Rightarrow (x+2y)\frac{dy}{dx} = -y$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{-y}{x+2y}}$$

(iii)

Do yourself

(iv)

$$4x^2+2hxy+by^2+2gx+2fy+c = 0$$

Differentiating w.r.t. x

$$\frac{d}{dx}\left(4x^2+2hxy+by^2+2gx+2fy+c\right) = \frac{d}{dx}(0)$$

$$\Rightarrow 4\frac{d}{dx}(x^2)+2h\frac{d}{dx}(xy)+b\frac{d}{dx}(y^2)$$

$$+2g\frac{d}{dx}(x)+2f\frac{d}{dx}(y)+\frac{d}{dx}(c) = 0$$

$$\Rightarrow 4(2x)+2h\left(x\frac{dy}{dx}+y(1)\right)+b\cdot 2y\frac{dy}{dx}$$

$$+2g(1)+2f\frac{dy}{dx}+0 = 0$$

$$\Rightarrow 8x+2hx\frac{dy}{dx}+2hy+2by\frac{dy}{dx}$$

$$+2g+2f\frac{dy}{dx} = 0$$

$$\Rightarrow 2(hx+by+f)\frac{dy}{dx}+2(4x+hy++g)=0$$

$$\Rightarrow 2(hx+by+f)\frac{dy}{dx}=-2(4x+hy++g)$$

$$\Rightarrow (hx+by+f)\frac{dy}{dx}=-(4x+hy++g)$$

$$\Rightarrow \boxed{\frac{dy}{dx} = -\frac{4x+hy++g}{hx+by+f}}$$

(v)

$$x\sqrt{1+y}+y\sqrt{1+x} = 0 \quad \Rightarrow x(1+y)^{\frac{1}{2}}+y(1+x)^{\frac{1}{2}} = 0$$

Differentiating w.r.t. x

$$\Rightarrow \frac{d}{dx}\left[x(1+y)^{\frac{1}{2}}\right]+\frac{d}{dx}\left[y(1+x)^{\frac{1}{2}}\right] = \frac{d}{dx}(0)$$

$$\Rightarrow x\frac{d}{dx}(1+y)^{\frac{1}{2}}+(1+y)^{\frac{1}{2}}\frac{dx}{dx} + y\frac{d}{dx}(1+x)^{\frac{1}{2}}+(1+x)^{\frac{1}{2}}\frac{dy}{dx} = 0$$

$$\Rightarrow\ x\cdot\frac{1}{2}(1+y)^{-\frac{1}{2}}\frac{dy}{dx}+(1+y)^{\frac{1}{2}}(1)\ +y\cdot\frac{1}{2}(1+x)^{-\frac{1}{2}}(1)+(1+x)^{\frac{1}{2}}\frac{dy}{dx}\ =\ 0$$

$$\Rightarrow\ \frac{x}{2(1+y)^{\frac{1}{2}}}\frac{dy}{dx}+(1+y)^{\frac{1}{2}}\ +\frac{y}{2(1+x)^{\frac{1}{2}}}+(1+x)^{\frac{1}{2}}\frac{dy}{dx}\ =\ 0$$

$$\Rightarrow\ \left[\frac{x}{2(1+y)^{\frac{1}{2}}}+(1+x)^{\frac{1}{2}}\right]\frac{dy}{dx}\ =-\left[(1+y)^{\frac{1}{2}}+\frac{y}{2(1+x)^{\frac{1}{2}}}\right]$$

$$\Rightarrow\ \left[\frac{x+2(1+x)^{\frac{1}{2}}(1+y)^{\frac{1}{2}}}{2(1+y)^{\frac{1}{2}}}\right]\frac{dy}{dx}\ =-\left[\frac{2(1+x)^{\frac{1}{2}}(1+y)^{\frac{1}{2}}+y}{2(1+x)^{\frac{1}{2}}}\right]$$

$$\Rightarrow\ \left[\frac{x+2\sqrt{(1+x)(1+y)}}{2\sqrt{1+y}}\right]\frac{dy}{dx}\ =-\left[\frac{2\sqrt{(1+x)(1+y)}+y}{2\sqrt{1+x}}\right]$$

$$\Rightarrow\ \frac{dy}{dx}=-\frac{2\sqrt{(1+x)(1+y)}+y}{2\sqrt{1+x}}\cdot\frac{2\sqrt{1+y}}{x+2\sqrt{(1+x)(1+y)}}$$

$$\Rightarrow\ \frac{dy}{dx}=-\frac{\sqrt{1+y}\left(2\sqrt{(1+x)(1+y)}+y\right)}{\sqrt{1+x}\left(x+2\sqrt{(1+x)(1+y)}\right)}\qquad \textit{Answer}$$

(vi)

$$y\left(x^2-1\right)=x\sqrt{x^2+4}$$

Differentiating w.r.t x

$$\frac{d}{dx}y\left(x^2-1\right)=\frac{d}{dx}x\left(x^2+4\right)^{\frac{1}{2}}$$

$$\Rightarrow\ y\frac{d}{dx}\left(x^2-1\right)+\left(x^2-1\right)\frac{dy}{dx}=x\frac{d}{dx}\left(x^2+4\right)^{\frac{1}{2}}+\left(x^2+4\right)^{\frac{1}{2}}\frac{dx}{dx}$$

$$\Rightarrow\ y(2x)+\left(x^2-1\right)\frac{dy}{dx}=x\frac{1}{2}\left(x^2+4\right)^{-\frac{1}{2}}(2x)+\left(x^2+4\right)^{\frac{1}{2}}(1)$$

$$\Rightarrow\ 2xy+\left(x^2-1\right)\frac{dy}{dx}=\frac{x^2}{\left(x^2+4\right)^{\frac{1}{2}}}+\left(x^2+4\right)^{\frac{1}{2}}$$

$$\Rightarrow\ \left(x^2-1\right)\frac{dy}{dx}=\frac{x^2}{\left(x^2+4\right)^{\frac{1}{2}}}+\left(x^2+4\right)^{\frac{1}{2}}-2xy$$

$$\Rightarrow\ \left(x^2-1\right)\frac{dy}{dx}=\frac{x^2}{\left(x^2+4\right)^{\frac{1}{2}}}+\left(x^2+4\right)^{\frac{1}{2}}-2xy$$

$$\Rightarrow\ \left(x^2-1\right)\frac{dy}{dx}=\frac{x^2+x^2+4-2xy\left(x^2+4\right)^{\frac{1}{2}}}{\left(x^2+4\right)^{\frac{1}{2}}}\qquad\Rightarrow\ \boxed{\frac{dy}{dx}=\frac{2x^2+4-2xy\sqrt{x^2+4}}{\left(x^2-1\right)\sqrt{x^2+4}}}$$

Question # 3

Find $\frac{dy}{dx}$ of the following parametric functions:

(i) $x = \theta + \frac{1}{\theta}$ and $y = \theta + 1$

(ii) $x = \frac{a(1-t^2)}{1+t^2}$, $y = \frac{2bt}{1+t^2}$

Solution

(i) Since $x = \theta + \frac{1}{\theta}$

$\Rightarrow x = \theta + \theta^{-1}$

Differentiating x w.r.t. θ

$$\frac{dx}{d\theta} = \frac{d}{d\theta}\left(\theta + \theta^{-1}\right)$$

$$= 1 - \theta^{-2} = 1 - \frac{1}{\theta^2} = \frac{\theta^2 - 1}{\theta^2}$$

$$\Rightarrow \frac{d\theta}{dx} = \frac{\theta^2}{\theta^2 - 1}$$

Now $y = \theta + 1$

Diff. w.r.t. θ

$$\frac{dy}{d\theta} = \frac{d}{d\theta}(\theta + 1) \Rightarrow \frac{dy}{d\theta} = 1$$

Now by chain rule

$$\frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx}$$

$$= \frac{dy}{d\theta} \cdot \frac{d\theta}{dx} = 1 \cdot \frac{\theta^2}{\theta^2 - 1}$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{\theta^2}{\theta^2 - 1}}$$

(ii) Since $x = \frac{a(1-t^2)}{1+t^2}$

Diff. w.r.t. t

$$\frac{dx}{dt} = a\frac{d}{dt}\left(\frac{1-t^2}{1+t^2}\right)$$

$$= a\,\frac{(1+t^2)\frac{d}{dt}(1-t^2) - (1-t^2)\frac{d}{dt}(1+t^2)}{(1+t^2)^2}$$

$$= a\frac{(1+t^2)(-2t) - (1-t^2)(2t)}{(1+t^2)^2}$$

$$= a\frac{-2t - 2t^3 - 2t + 2t^3}{(1+t^2)^2}$$

$$\Rightarrow \frac{dx}{dt} = \frac{-4at}{(1+t^2)^2} \Rightarrow \frac{dt}{dx} = \frac{(1+t^2)^2}{-4at}$$

Now $y = \frac{2bt}{1+t^2}$ y

Diff. w.r.t. t

$$\frac{dy}{dt} = \frac{d}{dt}\left(\frac{2bt}{1+t^2}\right)$$

$$= \frac{(1+t^2)\frac{d}{dt}2bt - 2bt\frac{d}{dt}(1+t^2)}{(1+t^2)^2}$$

$$= \frac{(1+t^2)2b(1) - 2bt(2t)}{(1+t^2)^2}$$

$$= \frac{2b + 2bt^2 - 4bt^2}{(1+t^2)^2} = \frac{2b - 2bt^2}{(1+t^2)^2}$$

$$= \frac{2b(1-t^2)}{(1+t^2)^2}$$

Now by chain rule

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$= \frac{2b(1-t^2)}{(1+t^2)^2} \cdot \frac{(1+t^2)^2}{-4at}$$

$$\Rightarrow \boxed{\frac{dy}{dx} = -\frac{b(1-t^2)}{2at}}$$

Question # 4

Prove that $y\dfrac{dy}{dx}+x=0$ if $x = \dfrac{1-t^2}{1+t^2}$, $y = \dfrac{2t}{1+t^2}$

Solution Since $x = \dfrac{1-t^2}{1+t^2}$

Differentiating w.r.t. t, we get (*solve yourself as above*)

$$\frac{dx}{dt} = \frac{-4t}{\left(1+t^2\right)^2} \Rightarrow \frac{dt}{dx} = \frac{\left(1+t^2\right)^2}{-4t}$$

Now $y = \dfrac{2t}{1+t^2}$

Differentiating w.r.t. t, we get (*solve yourself as above*)

$$\frac{dy}{dt} = \frac{2\left(1-t^2\right)}{\left(1+t^2\right)^2}$$

Now by chain rule

$$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx}$$

$$= \frac{2\left(1-t^2\right)}{\left(1+t^2\right)^2}\cdot\frac{\left(1+t^2\right)^2}{-4t}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1-t^2}{2t}$$

Multiplying both sides by y

$$\Rightarrow y\frac{dy}{dx} = -y\cdot\frac{1-t^2}{2t}$$

$$= -\frac{2t}{1+t^2}\cdot\frac{1-t^2}{2t}$$

$$\Rightarrow y\frac{dy}{dx} = -\frac{1-t^2}{1+t^2}$$

$$\Rightarrow y\frac{dy}{dx} = -x \qquad \because x = \frac{1-t^2}{1+t^2}$$

$$\Rightarrow y\frac{dy}{dx}+x = 0$$ *Proved.*

Question # 5

Differentiate

(i) $x^2-\dfrac{1}{x^2}$ w.r.t. x^4

(ii) $\left(1+x^2\right)^n$ w.r.t. x^2

(iii) $\dfrac{x^2+1}{x^2-1}$ w.r.t. $\dfrac{x-1}{x+1}$

(iv) $\dfrac{ax+b}{cx+d}$ w.r.t. $\dfrac{ax^2+b}{ax^2+d}$

(v) $\dfrac{x^2+1}{x^2-1}$ w.r.t. x^3

Solution

(i) Suppose $y = x^2-\dfrac{1}{x^2}$ and $u=x^4$

Diff. y w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^2-\frac{1}{x^2}\right)$$

$$= \frac{d}{dx}\left(x^2-x^{-2}\right) = 2x+2x^{-3}$$

$$= 2\left(x+\frac{1}{x^3}\right)$$

$$\Rightarrow \frac{dy}{dx} = 2\left(\frac{x^4+1}{x^3}\right)$$

Now diff. u w.r.t x

$$\frac{du}{dx} = \frac{d}{dx}\left(x^4\right)$$

$$\Rightarrow \frac{du}{dx} = 4x^3$$

Now by chain rule

$$\frac{dy}{du} = \frac{dy}{dx}\cdot\frac{dx}{du}$$

$$= \frac{dy}{dx}\cdot\frac{1}{\frac{du}{dx}}$$

$$\Rightarrow \frac{dy}{du} = 2\left(\frac{x^4+1}{x^3}\right)\cdot\frac{1}{4x^3}$$

$$\Rightarrow \boxed{\frac{dy}{du} = \frac{x^4+1}{2x^6}}$$

(ii) Let $y=\left(1+x^2\right)^n$ and $u = x^2$

Differentiation y w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}\left(1+x^2\right)^n$$

$$= n\left(1+x^2\right)^{n-1}\frac{d}{dx}\left(1+x^2\right)$$

$$= n\left(1+x^2\right)^{n-1}(2x)$$

$$= 2nx\left(1+x^2\right)^{n-1}$$

Now differentiating u w.r.t x

$$\frac{du}{dx} = \frac{d}{dx}x^2$$

$$= 2x \quad \Rightarrow \frac{dx}{du}=\frac{1}{2x}$$

Now by chain rule

$$\frac{dy}{du} = \frac{dy}{dx}\cdot\frac{dx}{du}$$

$$\Rightarrow \frac{dy}{du} = 2nx\left(1+x^2\right)^{n-1}\cdot\frac{1}{2x}$$

$$\Rightarrow \boxed{\frac{dy}{du} = n\left(1+x^2\right)^{n-1}}$$

(iii) Let $y=\dfrac{x^2+1}{x^2-1}$ and $u = \dfrac{x-1}{x+1}$

Diff. y w.r.t x

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x^2+1}{x^2-1}\right)$$

$$= \textit{Solve yourself} = \frac{-4x}{\left(x^2-1\right)^2}$$

Now diff. u w.r.t x

$$\frac{du}{dx} = \frac{d}{dx}\left(\frac{x-1}{x+1}\right)$$

$$= \textit{Solve yourself} = \frac{2}{(x+1)^2}.$$

$$\Rightarrow \frac{dx}{du} = \frac{(x+1)^2}{2}.$$

Now by chain rule

$$\frac{dy}{du} = \frac{dy}{dx}\cdot\frac{dx}{du}$$

$$= \frac{-4x}{\left(x^2-1\right)^2}\cdot\frac{(x+1)^2}{2}$$

$$= \frac{-2x}{(x-1)^2(x+1)^2}\cdot(x+1)^2$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{-2x}{(x-1)^2}}$$

(iv) Let $y=\dfrac{ax+b}{cx+d}$ and $u=\dfrac{ax^2+b}{ax^2+d}$

Diff. y w.r.t. x

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{ax+b}{cx+d}\right)$$

$$=\frac{(cx+d)\frac{d}{dx}(ax+b)-(ax+b)\frac{d}{dx}(cx+d)}{(cx+d)^2}$$

$$= \frac{(cx+d)(a)-(ax+b)(c)}{(cx+d)^2}$$

$$= \frac{acx+ad-acx-bc}{(cx+d)^2}$$

$$\Rightarrow \frac{dy}{dx}= \frac{ad-bc}{(cx+d)^2}$$

Now diff. u w.r.t x

$$\frac{du}{dx} = \frac{d}{dx}\left(\frac{ax^2+b}{ax^2+d}\right)$$

$$=\frac{(ax^2+d)\frac{d}{dx}(ax^2+b)-(ax^2+b)\frac{d}{dx}(ax^2+d)}{\left(ax^2+d\right)^2}$$

$$= \frac{\left(ax^2+d\right)(2ax)-\left(ax^2+b\right)(2ax)}{\left(ax^2+d\right)^2}$$

$$= \frac{2ax\left(ax^2+d-ax^2-b\right)}{\left(ax^2+d\right)^2}$$

$$= \frac{2ax(d-b)}{\left(ax^2+d\right)^2}$$

$$\Rightarrow \frac{dx}{du} = \frac{\left(ax^2+d\right)^2}{2ax(d-b)}$$

Now by chain rule

$$\frac{dy}{du} = \frac{dy}{dx}\cdot\frac{dx}{du}$$

$$= \frac{ad-bc}{(cx+d)^2}\cdot\frac{\left(ax^2+d\right)^2}{2ax(d-b)}$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{(ad-bc)\left(ax^2+d\right)^2}{2ax(cx+d)^2(d-b)}}$$

(v) Let $y = \dfrac{x^2+1}{x^2-1}$ and $u = x^3$

Diff. y w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2+1}{x^2-1}\right)$$

$= $ *Solve yourself*

$$= \frac{-4x}{\left(x^2-1\right)^2}$$

Now diff. u w.r.t x

$$\frac{du}{dx} = \frac{d}{dx}x^3$$

$$= 3x^2$$

$$\Rightarrow \frac{dx}{du} = \frac{1}{3x^2}$$

Now by chain rule

$$\frac{dy}{du} = \frac{dy}{dx}\cdot\frac{dx}{du}$$

$$= \frac{-4x}{\left(x^2-1\right)^2}\cdot\frac{1}{3x^2}$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{-4}{3x\left(x^2-1\right)^2}}$$