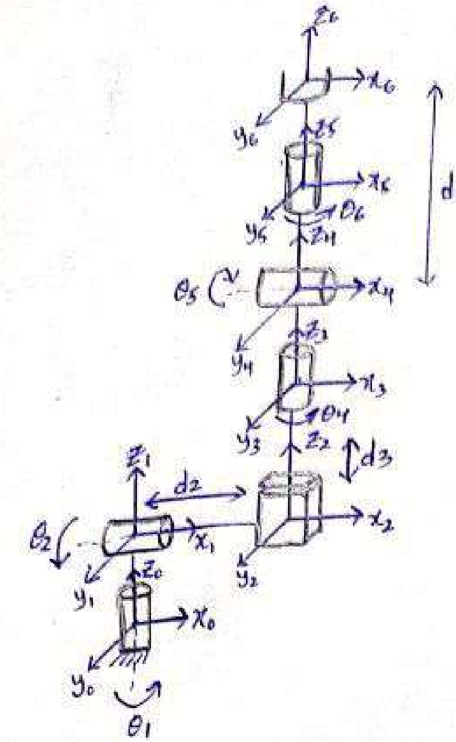
STANFORD MANIPULATOR WITH SPHERICAL WRIST (ZXZ)



I have kept the frame fixed and the Spherical wrist is Satisfing Euler angle arrangment of "ZXZ"

Total 6-DOF including the Spherical arm and Spherical Wrist.

fig. 1

fig. 1 :- Simple graph and Assignment of Coordinate frames.

Forward Kinematic Solution :-

$$[T_6^0] = \Big[\underbrace{R_z(\theta_1)}_{T_1^0}\ \underbrace{R_x(\theta_2)\,T_x(d_2)}_{T_2^1}\ \cancel{R_z}(\ \underbrace{T_z(d_3)}_{T_3^2}\ \underbrace{R_z(\theta_4)}_{T_4^3}\ \underbrace{R_x(\theta_5)}_{T_5^4}\ \underbrace{R_z(\theta_6)\,T_z(d_6)}_{T_6^5}\Big]$$

Inverse Kinematics Solution:-

Pipers Method:-

In our case its z axis vector of the R_6 (or) $P^0_{end\ effector}$

$$P^0_{wrist} = P^0_{end\ effector} - V_{end\ effector} \cdot d^{wrist}_{end\ effector}.$$

Extracted from T^0_3 (symbolic)

$$\underset{(\text{Symbolic})}{P^0_3} = P^0_{wrist}$$

↳ We get the angles for t_1, t_2, ~~t~~ d_3 ie Joint 1, Joint 2, Joint 3

$$\underset{(\text{Symbolic})}{P^0_3} = \begin{bmatrix} d2\cos(t_1) + d3\sin(t_1)\sin(t_2) \\ d2\sin(t_1) - d3\sin(t_2)\cos(t_1) \\ d3\cos(t_2) \end{bmatrix} \qquad P^0_{wrist} = \begin{bmatrix} P^0_{wx} \\ P^0_{wy} \\ P^0_{wz} \end{bmatrix}$$

Solve the following equations to get t_1, t_2 and d_3

eq(1) $d_2\cos(t_1) + d_3\sin(t_1)\sin(t_2) = P^0_{wx}$

eq(2) $d_2\sin(t_1) - d_3\sin(t_2)\cos(t_1) = P^0_{wy}$

eq(3) $d_3\cos(t_2) = P^0_{wz}$

↳ We will get t_1, t_2 and d_3 from these equations

Now we extract R_6^0 from T_6^0

Also we know that $R_6^0 = R_3^0 R_6^3$

We can write it as

$$R_6^3 = (R_3^0)^{-1} R_6^0$$

R_6^3 is the Rotation Matrix of the Spherical Wrist Which Satisfies the Euler arrangment ZXZ

$$R_{Z_4 X_5 Z_6} = \begin{bmatrix} c_4 c_6 - c_5 s_4 s_6 & -c_4 s_6 - c_5 c_6 s_4 & s_4 s_5 \\ c_6 s_4 + c_4 c_5 s_6 & c_4 c_5 c_6 - s_4 s_6 & -c_4 s_5 \\ s_5 s_6 & c_6 s_5 & c_5 \end{bmatrix}$$

$$\therefore \quad q_4 = \arctan\left(\frac{R_{13}}{-R_{23}}\right)$$

$$q_5 = \arctan\left(\frac{\sqrt{1 - R_{33}^2}}{R_{33}}\right)$$

$$q_6 = \arctan\left(\frac{R_{31}}{R_{32}}\right)$$

We get the q_4, q_5 and q_6 angles.

The Resuts and Validation are done Inside the Code.