DYNAMICS OF NON-LINEAR ROBOTIC SYSTEMS

Assignment 4

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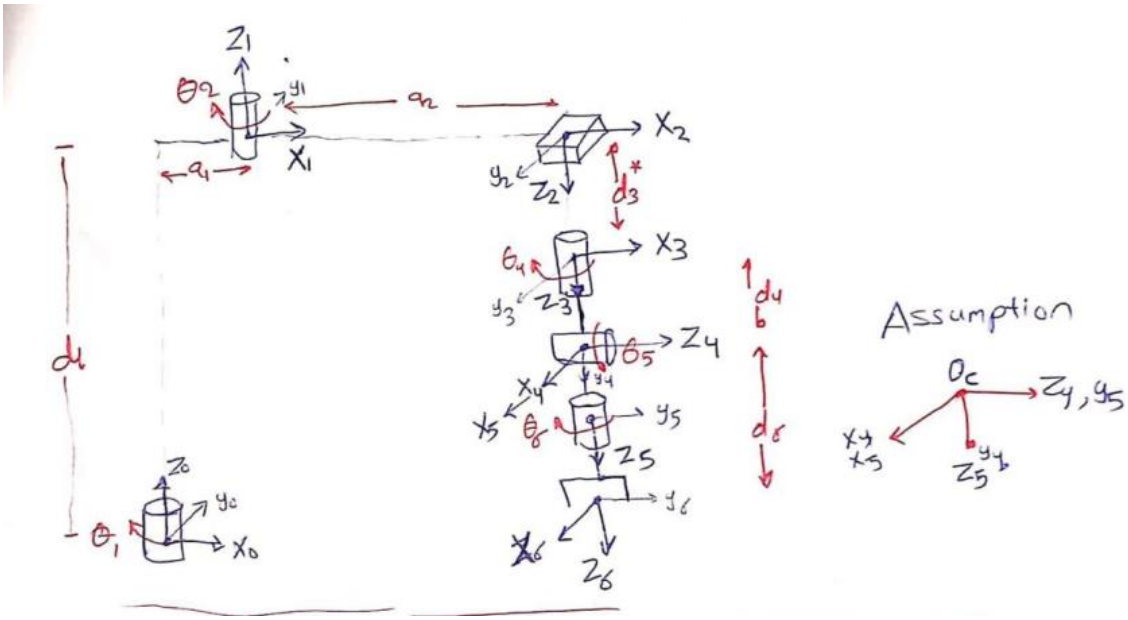
Recall Robot Configuration:



Assumption:

Links are represented with lines; geometry is not considered for simplification when calculating CoM.

Assignment $\boxed{4}$

Walid Shaker

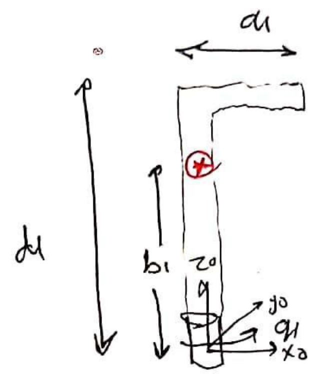
Recall Robot Configuration to locate CoM

Link $\boxed{1}$

$X_1 = 0$

$y_1 = 0$

$Z_1 = b_1$

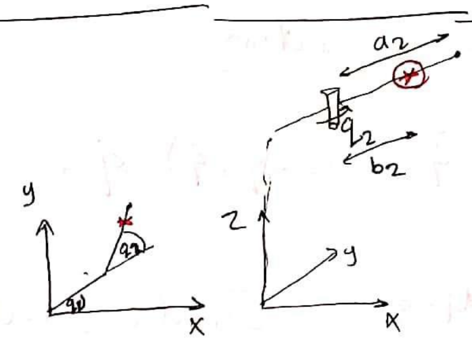


Link $\boxed{2}$

$X_2 = a_1 Cq_1 + b_2 C(q_1+q_2)$

$y_2 = a_1 Sq_1 + b_2 S(q_1+q_2)$

$Z_2 = d_1$

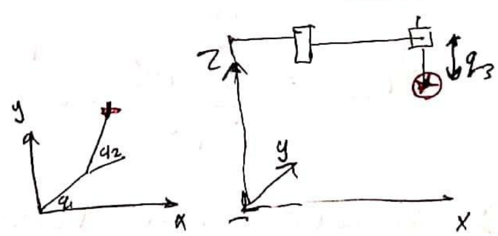


Link $\boxed{3}$

$X_3 = a_1 Cq_1 + a_2 C(q_1+q_2)$

$y_3 = a_1 Sq_1 + a_2 S(q_1+q_2)$

$Z_3 = d_1 - q_3$

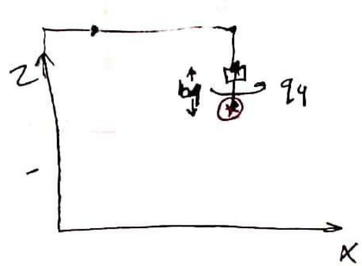


Link $\boxed{4}$

$X_4 = X_3$

$y_4 = y_3$

$Z_4 = Z_3 - b_4$

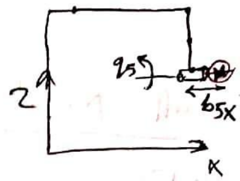


CS Scanned with CamScanner

$x_5 = x_4 + b_{5x}$ Link (5)

$y_5 = y_4 + b_{5y}$

$z_5 = z_4 - b_{5z}$

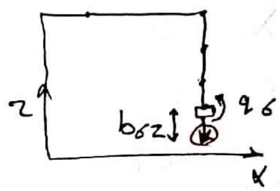


Link (6)

$x_6 = x_4$

$y_6 = y_4$

$z_6 = z_5 - b_{6z}$



Dynamics Equation

$$M(q)\,\ddot{q} + C(q,\dot{q})\,\dot{q} + g(q) = \tau$$

First: Mass or Inertia Matrix

$$M(q) = \sum_{i=1}^{n} m_i\, J_v^{i\,T} J_v^i + J_\omega^{i\,T} R_i\, I_i\, R_i^T J_\omega^i$$

(1) → Calculate linear velocity Jacobian (COM)

$$J_v^i = \begin{bmatrix} \frac{\partial x_i}{\partial q_1} & \frac{\partial x_i}{\partial q_2} & \cdots & \frac{\partial x_i}{\partial q_6} \\ \frac{\partial y_i}{\partial q_1} & \frac{\partial y_i}{\partial q_2} & \cdots & \frac{\partial y_i}{\partial q_6} \\ \frac{\partial z_i}{\partial q_1} & \frac{\partial z_i}{\partial q_2} & \cdots & \frac{\partial z_i}{\partial q_6} \end{bmatrix}_{3\times 6}$$

Scanned with CamScanner

Code is performed in symbolic way:

```
J1v =

[ 0, 0, 0, 0, 0, 0]
[ 0, 0, 0, 0, 0, 0]
[ 0, 0, 0, 0, 0, 0]


J2v =

[ - b2*sin(q1 + q2) - a1*sin(q1), -b2*sin(q1 + q2), 0, 0, 0, 0]
[   b2*cos(q1 + q2) + a1*cos(q1),  b2*cos(q1 + q2), 0, 0, 0, 0]
[                             0,                 0, 0, 0, 0, 0]


J3v =

[ - a2*sin(q1 + q2) - a1*sin(q1), -a2*sin(q1 + q2),  0, 0, 0, 0]
[   a2*cos(q1 + q2) + a1*cos(q1),  a2*cos(q1 + q2),  0, 0, 0, 0]
[                             0,                 0, -1, 0, 0, 0]


J4v =

[ - a2*sin(q1 + q2) - a1*sin(q1), -a2*sin(q1 + q2),  0, 0, 0, 0]
[   a2*cos(q1 + q2) + a1*cos(q1),  a2*cos(q1 + q2),  0, 0, 0, 0]
[                             0,                 0, -1, 0, 0, 0]


J5v =

[ - a2*sin(q1 + q2) - a1*sin(q1), -a2*sin(q1 + q2),  0, 0, 0, 0]
[   a2*cos(q1 + q2) + a1*cos(q1),  a2*cos(q1 + q2),  0, 0, 0, 0]
[                             0,                 0, -1, 0, 0, 0]


J6v =

[ - a2*sin(q1 + q2) - a1*sin(q1), -a2*sin(q1 + q2),  0, 0, 0, 0]
[   a2*cos(q1 + q2) + a1*cos(q1),  a2*cos(q1 + q2),  0, 0, 0, 0]
[                             0,                 0, -1, 0, 0, 0]
```

② Calculate Jacobian of angular velocity of the CM

Note for prismatic joint $Z_{i-1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

for revolute joint $Z_{i-1} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$
(rotation around Z)

$$J_{\omega}^{1} = \begin{bmatrix} Z_0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{3\times 6}$$

$$J_{\omega}^{2} = \begin{bmatrix} Z_0 & Z_1 & 0 & 0 & 0 & 0 \end{bmatrix}_{3\times 6}$$

$$\vdots$$

$$J_{\omega}^{6} = \begin{bmatrix} Z_0 & Z_1 & Z_2 & Z_3 & Z_4 & Z_5 \end{bmatrix}_{3\times 6}$$

Code is performed in symbolic way:

```
J1w =

[ 0, 0, 0, 0, 0, 0]
[ 0, 0, 0, 0, 0, 0]
[ 1, 0, 0, 0, 0, 0]


J2w =

[ 0, 0, 0, 0, 0, 0]
[ 0, 0, 0, 0, 0, 0]
[ 1, 1, 0, 0, 0, 0]


J3w =

[ 0, 0, 0, 0, 0, 0]
[ 0, 0, 0, 0, 0, 0]
[ 1, 1, 0, 0, 0, 0]
```

```
J4w =

[ 0, 0, 0, 0, 0, 0]
[ 0, 0, 0, 0, 0, 0]
[ 1, 1, 0, 1, 0, 0]


J5w =

[ 0, 0, 0, 0, 0, 0]
[ 0, 0, 0, 0, 0, 0]
[ 1, 1, 0, 1, 1, 0]


J6w =

[ 0, 0, 0, 0, 0, 0]
[ 0, 0, 0, 0, 0, 0]
[ 1, 1, 0, 1, 1, 1]
```

(3) Calculate $J_v^{i\,T} * J_v^i$

(4) Calculate $J_\omega^{i\,T} * R_i$

(5) Calculte $J_\omega^{i\,T} * R_i * I * \underbrace{(R_i^T J_\omega^i)}_{\text{Transpose of step (4)}}$

(6) Calculate $M_i(q) = \left[\;\right]_{6\times 6}$

(7) Calcualte $\boxed{M(q) = \sum_{i=1}^{n} M_i}$

Code is performed in symbolic way: Please check M matrix symbolic in MATLAB.

Second: Coriolis forces

not always symmetric exists only when combined motion (T+R)

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix}_{6\times 6}$$

$$C_{ij} = \sum_{k=1}^{n} C_{ijk}\,\dot{q}_k \quad , \quad C_{ijk} = \frac{1}{2}\left(\frac{\partial m_{ij}}{\partial q_k} + \frac{\partial m_{ik}}{\partial q_j} - \frac{\partial m_{jk}}{\partial q_i}\right)$$

e.g. $$C_{11} = \sum_{k=1}^{6} C_{11k}\,\dot{q}_k = C_{111}\dot{q}_1 + C_{112}\dot{q}_2 + \cdots + C_{116}\dot{q}_6$$

$$\boxed{C_{ijk} = C_{ikj}} \rightarrow \text{property}$$

Code is performed in symbolic way:

```
C matrix symbolic:
[ -a1*dq2*sin(q2)*(a2*m3 + a2*m4 + a2*m5 + a2*m6 + b2*m2), -a1*sin(q2)*(dq1 + dq2)*(a2*m3 + a2*m4 + a2*m5 + a2*m6 + b2*m2), 0, 0, 0, 0]
[  a1*dq1*sin(q2)*(a2*m3 + a2*m4 + a2*m5 + a2*m6 + b2*m2),                                                          0, 0, 0, 0, 0]
[                                                        0,                                                          0, 0, 0, 0, 0]
[                                                        0,                                                          0, 0, 0, 0, 0]
[                                                        0,                                                          0, 0, 0, 0, 0]
[                                                        0,                                                          0, 0, 0, 0, 0]
```

Third: Gravity vector

$$g = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \\ g_5 \\ g_6 \end{bmatrix}_{6\times 1} \qquad g_0 = \begin{bmatrix} 0 \\ -9.81 \\ 0 \end{bmatrix}$$

$$g_i = - \sum_{k=1}^{n} \left(J_{v_i}^{k} \right)^T * m_k * g_0$$

$$J_v^1 = \left[J_{v_1}^1 \; J_{v_1}^2 \; J_{v_1}^3 \; — \; J_{v_1}^6 \right]$$

$J_{v_1}^1$, $k=1$ $\qquad$ $J_{v_1}^2$, $k=2$

e.g. $$g_1 = - \sum_{k=1}^{6} \left(J_{v_1}^{k} \right)^T * m_1 * g_0$$

Code is performed in symbolic way: Please check g vector symbolic in MATLAB.

For input:

```
d1 = 0.6;
a1 = 0.1;
a2 = 0.4;
b1 = 0.35;
b2 = 0.2;
b4z = 0.1;
b5x = 0.1;
b5y = 0.1;
b5z = 0.1;
b6z = 0.1;
gs = 9.81;
m1 = 3;
m2 = 2;
m3 = 1.5;
m4 = 1;
m5 = 0.9;
m6 = 0.8;
q1 = deg2rad(50);
q2 = deg2rad(30);
q3 = 0.15;
q4 = deg2rad(90);
q5 = deg2rad(-30);
q6 = deg2rad(10);
```

```
dq1 = dq(1);
dq2 = dq(2);
dq3 = dq(3);
dq4 = dq(4);
dq5 = dq(5);
dq6 = dq(6);
Izz1 = 0.01;
Izz2 = 0.015;
Izz3 = 0.001;
Izz4 = 0.0015;
Izz5 = 0.02;
Izz6 = 0.002;
```

This output is obtained:

```
M matrix:
    1.2238    0.9716         0    0.0235    0.0220    0.0020
    0.9716    0.7915         0    0.0235    0.0220    0.0020
         0         0    4.2000         0         0         0
    0.0235    0.0235         0    0.0235    0.0220    0.0020
    0.0220    0.0220         0    0.0220    0.0220    0.0020
    0.0020    0.0020         0    0.0020    0.0020    0.0020

C matrix:
   -0.0312   -0.0520         0         0         0         0
    0.0208         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0

g matrix:
    7.4528
    3.5433
         0
         0
         0
         0
```

```
Dynamics Equation: M(q)ddq + C(q,dq)dq + g(q) = T
Desired Torques:
    7.7023
    3.7665
    0.4200
    0.0091
    0.0086
    0.0010

>>
```