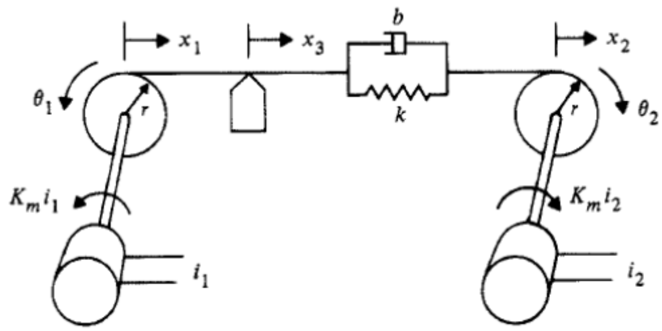
[F22] Fundamentals of Robot Control
Assignment 1: Linear Analysis and Control

Problem №1: Regulation of magnetic tape system via pole-placement

Consider the magnetic tape system:



where:

- θ_1, θ_2, [rad] angular position of motor capstan assembly
- x_1, x_2, [mm] position of tape at capstan assembly
- i_1, i_2, [A] current into drive motors 1 and 2
- x_3,[mm] position of tape over read head
- T_e,[N] tension in tape
- $J = 0.006375$ [kg m^2] motor and capstan inertia
- $K_m = 0.544$ [Nm/A] motor torque constant
- $k = 2113$ [N/m] tape spring constant
- $b = 3.75$ [Ns/m] tape damping constant
- $r = 0.05$ [m] capstan radius

The behavior of this system is governed by following differential equations:

$$\begin{cases} J\ddot{\theta}_1 = -T_e r + K_m i_1 \\ J\ddot{\theta}_2 = -T_e r + K_m i_2 \\ T_e = k(x_2 - x_1) + b(\dot{x}_2 - \dot{x}_1) \\ x_3 = \frac{x_1 + x_2}{2} \end{cases}$$

The goal is to design feedback controller, $\mathbf{u} = -\mathbf{K}\mathbf{x}$ to **regulate to constant position of magnetic tape** $x_3 = x_d$ [mm] over the read head of a magnetic tape drive **while maintaining a specified tension** $T_e = 10$, namely:

- **[10 points]** Rewrite system in **linear state space form**: note that position and motor angle are coupled with: $x_j = r\theta_j$ and take control input as current: $u_j = i_j$
- **[20 points]** Test the **stability** and **controllability**:
  - Answer either this system stable and controllable
  - Check controllability just with one actuator ($u = i_1$, $i_2 = 0$)
  - What is the minimal number of control channels for this system to be controllable/stabilizable
- **[35 points] Place poles of closed loop system** (find feedback gains) such that tension T_e and position x_3 - converges to desired values x_d, T_d, simulate the response in the colab.

Problem №2: Pole Placement Design of Linear Observers

The drawback of full state feedback controllers is that the measurements of system state should be measurable. However, in most practical cases, the physical state of the system cannot be determined by direct observation. Instead, **indirect effects of the internal state are estimated by way of the system outputs**. The algorithm (dynamical system) that supports such estimation is called the **state observer** (state estimator).

More specifically we consider the LTI system in state space form:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$
$$\mathbf{y} = \mathbf{C}\mathbf{x}$$

Where the last equation describe your actual measurements (output) as some linear combination of states $\mathbf{y} = \mathbf{C}\mathbf{x}$. Goal is to estimate the full state $\hat{\mathbf{x}}$, based on known measurements $\mathbf{y}$ and control signal $\mathbf{u}$ assuming that $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are known.

Consider the state observer algorithm is given as follows:

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}\mathbf{u} + \mathbf{L}(\mathbf{y} - \hat{\mathbf{y}})$$

Do the following:

- **[15 Points]** Find the closed loop response of estimation error $\mathbf{e} = \hat{\mathbf{x}} - \mathbf{x}$ in terms of $\mathbf{A}, \mathbf{C}, \mathbf{L}, \mathbf{e}(t)$ (equations for $\dot{\mathbf{e}}$).
- **[20 Points]** What are the conditions on $\mathbf{L}$ that implies the convergence of estimates $\hat{\mathbf{x}}$ to the actual state $\mathbf{x}$, describe how would you use pole placement routine to tune observer gains $\mathbf{L}$
- **[BONUS]** Implement the proposed observer in order to estimate the motor states $\theta_j, \dot{\theta}_j$ of magnetic tape system given the measurements of x_3 and T_e and use these estimate for the feedback instead of $\mathbf{x}$, ($\mathbf{u} = -\mathbf{K}\hat{\mathbf{x}}$)