Jacobian Matrix of Stanford Manipulator with Spherical Wrist in ZXZ Euler angle arrangement.

$$J(q) = \left[\begin{array}{c|c|c|c|c|c} z_0 \times (P_{ee}-P_0) & x_1 \times (P_{ee}-P_1) & z_2 & z_3 \times (P_{ee}-P_c) & x_4 \times (P_{ee}-P_c) & z_5 \times (P_{ee}-P_c) \\ z_0 & x_1 & 0 & z_3 & x_4 & z_5 \end{array}\right]$$

→ Geometrical Approach :

We know ~~that~~ $T_1^0, T_2^0, T_3^0, T_4^0, T_5^0, T_6^0$ from forward Kinematics

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad P_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad x_1 = \begin{bmatrix} R_1^0 \rightarrow x \text{ vector} \\ (1^{st} \text{ Column}) \end{bmatrix} \quad z_2 = \begin{bmatrix} R_2^0 \rightarrow z \text{ vector} \\ (3^{rd} \text{ Column}) \end{bmatrix}$$

$$z_3 = \begin{bmatrix} R_3^0 \rightarrow z \text{ Vector} \\ (3^{rd} \text{ Column}) \end{bmatrix} \quad x_4 = \begin{bmatrix} R_4^0 \Rightarrow x \text{ vector} \\ (1^{st} \text{ Column}) \end{bmatrix} \quad z_5 = \begin{bmatrix} R_5^0 \rightarrow z \text{ vector} \\ (3^{rd} \text{ Column}) \end{bmatrix}$$

$$\underbrace{P_{ee} = \begin{bmatrix} P_{eex} \\ P_{eey} \\ P_{eez} \end{bmatrix}}_{\text{End effector Position}} \quad P_1 = \begin{bmatrix} T_1^0 \rightarrow \text{Translation Vector} \\ (4^{th} \text{ Column}) \end{bmatrix} \quad \underbrace{P_2 = \begin{bmatrix} T_2^0 \rightarrow \text{Translation Vector} \end{bmatrix}}_{\text{Not Needed}}$$

$$P_c = \begin{bmatrix} T_3^0 \text{ (or) } T_4^0 \text{ or } T_6^0 \rightarrow \text{Translation Vector} \\ (4^{th} \text{ Column}) \end{bmatrix}$$

$$J = \begin{bmatrix} J_1 & J_2 & J_3 & J_4 & J_5 & J_6 \end{bmatrix}_{6\times 6}$$

Singularities Check :-

The Jacobian Matrix Contains information about the Singularities

We can find the determinant of the J matrix

$\boxed{\det(J) = 1 \text{ for angles/coordinates } [90,90,1,90,90,90]}$

If we Find the Jacobian Matrix in Symbolic form then we get the Coordinates (Joint Coordinates) at which the Singularities Occur.

So in the case of Stanford Manipulator in ZXZ Configuration

We get $\boxed{d3=0,\ t2=0,\ t5=0,\ t5=pi}$

Singularities occurs at these Joint Coordinates (These values are derived from the code)

Bonus task 2 :-

From T_6^0 (Symbolic) we can extract the position and orientation:

X = equation ....

Y = equation ...

Z = equation ..

ThetaX = equation ..

ThetaY = equation ... ThetaZ = equation ...

Now we do the derivative for each of these functions and with respect to each joint coordinate.

<u>for example:</u> Δq = small change $\approx 10^{-10}$

$$J_1 = \begin{bmatrix} \dfrac{\partial x}{\partial q_1} = \dfrac{x(q_1+\Delta q) - x(q_1)}{\Delta q} \\[2ex] \dfrac{\partial y}{\partial q_1} = \dfrac{y(q_1+\Delta q) - y(q_1)}{\Delta q} \\[2ex] \dfrac{\partial z}{\partial q_1} = \dfrac{z(q_1+\Delta q) - z(q_1)}{\Delta q} \\[2ex] \dfrac{\partial\, thetaX}{\partial q_1} = \dfrac{thetaX(q_1+\Delta q) - thetaX(q_1)}{\Delta q} \\[2ex] \dfrac{\partial\, thetaY}{\partial q_1} = \dfrac{thetaY(q_1+\Delta q) - thetaY(q_1)}{\Delta q} \\[2ex] \dfrac{\partial\, thetaZ}{\partial q_1} = \dfrac{thetaZ(q_1+\Delta q) - thetaZ(q_1)}{\Delta q} \end{bmatrix}_{6\times 1} \longrightarrow J_1$$

(This is Just J_1)

Now Similarly find the J_2, J_3, J_4, J_5, J_6

finally we get the Jacobian Matrix Numerically.