COT 5600 Quantum Computing
Spring 2019

Homework 3

Out: Wed 04/08

Due: Wed 04/19

Problem 1 (Quantum Fourier transform)

Let $N = 2^n$, $[N] = \{0, \ldots, N-1\}$, and $\omega = e^{2\pi i/N}$ be an Nth root of unity. The Quantum Fourier transform F_N of size N is

$$F_N = \frac{1}{\sqrt{N}} \sum_{k,\ell \in [N]} \omega^{k \cdot \ell} |k\rangle\langle \ell| \, .$$

Show that F_N is unitary.

To show that F_N is unitary, we have to show that $F^\dagger F = 1$

$F = \frac{1}{\sqrt{N}} \sum\limits_{k,\ell \in [N]} e^{\frac{2\pi i k \ell}{N}} |k\rangle\langle \ell|$

$F^\dagger = \frac{1}{\sqrt{N}} \sum\limits_{k^*,\ell^* \in [N]} e^{\frac{-2\pi i k^* \ell^*}{N}} |\ell^*\rangle\langle k^*|$

$F^\dagger F = \frac{1}{\sqrt{N}} \frac{1}{\sqrt{N}} \sum\limits_{k,\ell,k^*,\ell^* \in [N]} e^{\frac{2\pi i (\ell^* k^* - \ell k)}{N}} |\ell\rangle\langle \ell^*| \delta_{kk^*}$

$F^\dagger F = \frac{1}{N} \sum \ell k \ell^* e^{\frac{2\pi i (\ell^* - \ell) k}{N}} |\ell\rangle\langle \ell^*|$

$F^\dagger F = \sum\limits_{\ell \ell^*} |\ell\rangle\langle \ell^*| \delta_{\ell \ell^*}$

$F^\dagger F = I$

Problem 2 (Quantum Phase estimation)

Let $\varphi \in [0, 1)$ be arbitrary and

$$|\varphi\rangle = \bigotimes_{k=n-1,\dots,0} \frac{1}{\sqrt{2}} \big(|0\rangle + \exp(2\pi i 2^k \varphi)|1\rangle\big) \ .$$

Create a Python notebook that lets you compute and plot the probabilities for measuring $x \in \{0, 1\}^n$ when the state is

$$F_N^\dagger |\varphi\rangle$$

for different N and φ. The plot should look similar to the plots on the slides depicting the different probability distributions. Do not forget about the bit-reversal that we talked about in class.