BME 302
Homework 5: THEORIES OF FAILURE AND FACTOR OF SAFETY
SOLUTION

1. **A plate is made up of AISI 304 stainless steel which has the following material property: Yield strength = 200 MPa, ultimate tensile strength = 552 MPa, modulus of elasticity = 193 GPa. If the plate is subjected to the following combinations of static stresses, determine the factor of safety achieved in each of the following cases.**

(i) σ_x = 150 MPa, σ_y = 0 MPa, and τ_{xy} = 0 MPa, according to the maximum shear stress theory.

(ii) σ_x = 150 MPa, σ_y = 0 MPa, and τ_{xy} = 0 MPa, according to the maximum distortion energy theory.

(iii) σ_x = 150 MPa, σ_y = -50 MPa, and τ_{xy} = 0 MPa, according to the maximum shear stress theory.

(iv) σ_x = 150 MPa, σ_y = -50 MPa, and τ_{xy} = 0 MPa, according to the maximum distortion energy theory.

Formulae used:

$$\tau_{max} = \text{radius} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

σ_1 = center distance of Mohr circle + radius

$$= \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

σ_2 = center distance of Mohr circle - radius

$$= \left(\frac{\sigma_x + \sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

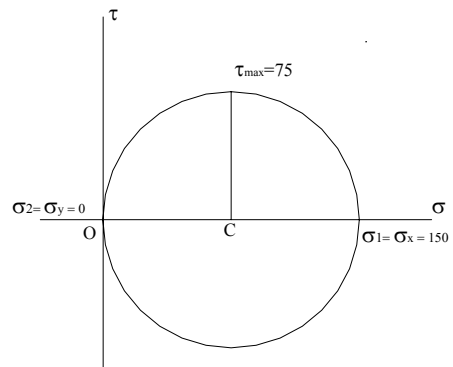**(i) and (ii) SOLUTION**

For the given stress condition,
σ_x = 150, σ_y = 0 and τ_{xy} = 0:

From the above formulae:
        $\tau_{max} = \sigma_x/2 = 150/2 = 75$ MPa.
        $\sigma_1 = \sigma_x = 150$ MPa, and $\sigma_2 = \sigma_y = 0$.

You can also determine τ_{max}, σ_1 and σ_2 easily by drawing a Mohr circle as shown.

(i) According to the maximum shear stress theory, τ_{max} is compared with shear strength $\sigma_{yp}/2$.

Thus the factor of safety (FOS) achieved = $(\sigma_{yp}/2)/\tau_{max}$= 100/75 = 1.33

(ii) According to the max distortion energy theory, $\sqrt{\sigma_1^2 - \sigma_1.\sigma_2 + \sigma_2^2}$ is compared with the limiting σ_{yp} to detect failure.

$$\sqrt{\sigma_1^2 - \sigma_1.\sigma_2 + \sigma_2^2} = \sqrt{150^2} = 150 MPa$$

The limiting strength value = σ_{yp} = 200 MPa

The FOS =200/150 = 1.33

NOTE: For uniaxial stress, the FOS is same from both the failure theories.

(iii) and (iv) SOLUTION

For the given stress condition, σ_x = 150 MPa, σ_y = -50 MPa, and τ_{xy} = 0 MPa:

Using the either formulae or using a Mohr circle:

τ_{max} = $(\sigma_x-\sigma_y)/2$ = (150+50)/2 = 100 MPa.

$\sigma_1 = \sigma_x$ = 150 MPa, and

$\sigma_2 = \sigma_y$ = -50.
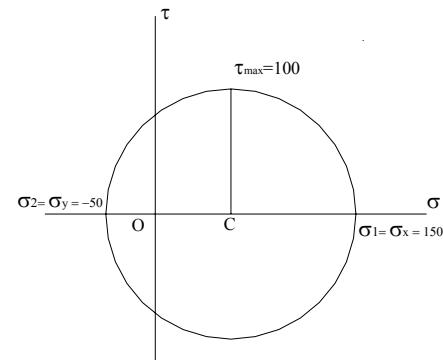

(iii) According to the maximum shear stress theory

FOS = = $(\sigma_{yp}/2)/\tau_{max}$= 100/100 = 1

(iv) According to the maximum distortion energy theory

$$\sqrt{\sigma_1^2 - \sigma_1.\sigma_2 + \sigma_2^2} = \sqrt{150^2 + 150 \times 50 + 50^2} = 180.3 MPa$$

FOS = $\sigma_{yp}/180.3$ = 200/180.3 = 1.11

NOTE: For a bi-axial stress condition the FOS is different from the two failure theories

2. **The critical stresses on a component are σ_x = -80 MPa, σ_y = 0 MPa, and τ_{xy} = 100 MPa. It is made up of cast iron with ultimate compressive strength = 165 MPa. If a factor of safety = 2 is required for this component, would the component be safe to use? Show your calculations.**

As cast iron is a brittle material, we use maximum normal stress theory to predict failure.

The magnitude of the maximum normal compressive stress can be found by using the formulae given in (1) or by drawing a Mohr circle.

σ_2 = center distance of Mohr circle − radius

$$= \left(\frac{\sigma_x}{2}\right) - \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} = \left(\frac{-80}{2}\right) - \sqrt{\left(\frac{-80}{2}\right)^2 + 100^2} = -147.7 MPa$$

With a FOS = 2, allowable stress in compression

$\sigma_{allowable}$= σ_{ult}/FOS = 165/2 = 82.5 MPa (compressive)

Because the maximum normal compressive stress σ_2= 148 MPa is more than the $\sigma_{allowable}$ = 82.5 MPa, the component can fail at the given stress level.