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Assignment -2 : Induction

Practice problem

1) $f(n) = n-10 \quad g(n) = n+10$

$f(n) = \Theta(g(n))$

Soln :- $f(n) = \Theta(g(n)) \rightarrow$ BigO, Omega

BigΘ

$f(n) \leq C \cdot g(n)$

Substitution in equation.

$n-10 \leq C \cdot (n+10)$

if $C = 1$

$\boxed{f(n) = O(g(n))}$

Omega (Ω)

$f(n) \geq C \cdot g(n)$

$n-10 \geq C \cdot (n+10)$

$C = \frac{1}{2}$ Substituting the value

$n = 100$

$100 - 2 \geq \frac{1}{2}(100+10)$

$90 \geq 55$ - (True)

$\boxed{f(n) = \Omega(g(n))}$

2) $f(n) = n$

$g(n) = n$

$f(n) = \Theta(g(n))$

Soln :- Big O

$f(n) <= c.g(n)$

$n <= cn$ - (true)

$f(n) <= c.g(n)$

Omega

$f(n) >= g(n)$

$n >= c.n$

$c = 1$

$n >= n$

$f(n) >= c.g(n)$

3.) $64^{\log_2 n} \cdot 32^{\log_2 n} = O(n^5)$

Soln :- $\log_2^{64} = \log_2^{2^6}$,

$= 6$

$\log_2^{32} = \log_2^{2^5}$

$= 5$

Taking the value of n

$n^{\log_2^{64}} \cdot n^{\log_2^{32}}$

Substituting the value

$n^6 \cdot n^5 = n^{11}$ $\quad | x^n \cdot x^m = x^{n+m}$

$f(n) <= c.g(n)$

$n^{11} <= c.n^5$ (False)

$c = 1$

Not a valid BigO

4) $\frac{4^n}{2^n} = O(2^n)$

$\frac{4^n}{2^n} = \frac{2^n \times 2^n}{2^n}$,

$2^n <= c.2^n$

$c = 1$

Valid a BigO

5) $128^{\log_2^n} . n^2 = \Theta(n^9)$

$n^{\log_2^{27}} . n^2$

$n^7 . n^2 = n^9$ $\left| x^n . x^m = x^{n+m} \right|$ $\left| \begin{matrix} 128^{\log_2^n} \\ \log_2^{27}, \end{matrix} \right.$

$n^9 = n^9$

Big Θ O

$f(n) <= c(g(n)$

$n^9 <= c.n^9$

$c = 1$ valid BigO

Omega

$n^9 >= c.n^9$

$c = 1$

valid Omega