LCM & HCF

Least Common Multiple (LCM):

The least number which is exactly divisble by each one of the given numbers is called L.C.M.

Example: LCM of 6 and 8 is

6 Multiples: 6,12, 18, 24, 30, 36, 42, 48, 54,...

8-Multiples: 8, 16, 24, 32, 40, 48, 56,....

Here the least common multiples (LCM) of 6 & 8:24

Highest Common Factor (HCF):

As factors of a number are finite, we can think of the **Highest Common Factor** of numbers, shortly denoted as HCF. The Highest Common Factor (HCF) is also called as the Greatest Common Divisor (GCD) or the Greatest Common Factor (GCF).

- HCF of (1, Z) = 1
- HCF (x, y) = x, if y is a multiple of x. For example, HCF (4, 8) = 4
- HCF of two numbers is 1, then the numbers are said to be co-primes or relatively prime. Here, the two numbers can both be primes as (5, 7) or both can be composites as (14, 27) or one can be a prime and other a composite as (11, 12).

Relation between HCF and LCM:

- The product of two given numbers is equal to the product of their HCF and LCM.
- LCM is always greater than or equal to the largest of the given numbers.
- LCM will always be a multiple of HCF.
- A natural number having more than two factors is called a composite number.
- A pair of prime numbers whose difference is 2 is called as twin primes.
- Every composite number can be expressed as a product of prime numbers in a unique way.

HCF and LCM of fractions:

- H.C.F = HCF of Numerators / LCM of Denominators
- L.C.M = LCM of Numerators / HCF of Denominators

Problems:

- Q.1) The greatest number that will divide 64, 80 and 111 leaving remainders 4, 5 and 6 respectively is
 - A) 12
 - B) 13
 - C) 14
 - D) 15

Ans: D

Solution: Get all the common factors of 64 - 4, 80 - 5 and 111 - 6, i.e., 60, 75 and 105 and see that the common factors will divide them all. The greatest number is the H.C.F of 60, 75 and 105.

Hence, the HCF is $3 \times 5=15$, which is the greatest number that will divide 64, 80, 111 leaving remainders 4, 5 and 6 respectively.

- Q.2) The Product of two numbers is 5712 and their LCM and HCF are 1428 and 4 respectively. If one of the numbers are 84, then find the other number?
 - A) 72
 - B) 69
 - C) 68
 - D) 70

Ans: C

Solution: HCF * LCM = Product of the two numbers

$$4 * 1428 = 84 * X$$

$$X = 68$$

- Q.3) HCF and LCM of 55, 75 and 'x' are 5 and 5775 respectively. The least value of 'x' is
 - A) 39
 - B) 37
 - C) 35
 - D) 34

Ans: C

Solution: As the HCF of the numbers is 5, obviously 5 must be a factor of each of these numbers.

Now, LCM of 55, 75, X = 11* 15 * X But LCM is already given 5775 Hence, 11* 15 * X = 5775

Therefore, least value of X = 35

- Q.4) An electronic device makes a beep after every 30 seconds another device makes a beep after 72 seconds. They beeped together at 8 am. When they will make a next beep together at the earliest?
 - A) 8.06 am
 - B) 8.05 am
 - C) 8.09 am
 - D) 8.10 am

Ans: A

Solution: LCM of (30, 72) is 360

360sec/60= 6 mins,. So they beep together at 8.06 am

- Q.5) What is the least number which when divided by 12, 18, 24 and 30 leaves the same remainder 5 in each case.
 - A) 722
 - B) 730
 - C) 825
 - D) 725

Ans: D

Solution: LCM of 12, 18,24 and 30 is 360.

According to the 725 is the answer. Because 720 is multiple of 360 (360*2) If we divide 725 by any given number remainder is '5'.

- Q.6) Three numbers are in the ratio 6: 8: 10 and their LCM is 1200. Their HCF is
 - A) 40
 - B) 80
 - C) 10
 - D) 200

Ans: C

Solution: Let the numbers be 6x, 8x and 10x.

Then, their L.C.M. = 120x.

So, 120x = 1200 or x = 10.

The numbers are (6 x 10), (8 x 10) and (10 x 10) i.e. 60, 80, 100

Hence, required H.C.F. = 10.

Q.7) The Product of two numbers is 16,428 and their H.C.F is 74. The larger number is

- A) 185
- B) 222
- C) 107
- D) 101

Ans: b

Solution: Let the numbers be 74a and 74b.

Then, $74a \times 74b = 4107$

Now, co-primes with product 3 are (1, 3).

So, the required numbers are (74 x 1, 74 x 3) i.e., (74, 222).

Greater number 222.

Q.8) The greatest number less than 10, 000 which is exactly divisible by 48, 60, 65 is

- A) 9000
- B) 9600
- C) 4800
- D) 9650

Ans: B

Solution: 48= 2x2x2x2x3, 60= 2x2x3x5, 62= 2x2x2x2x2x2

L.C.M. 0f48, 60 and 64= 2^6 x 3x 5= 960 x 10 = 9600

9600 < 10000,

9600 is the greatest number less than 10000 which is exactly divisible by 48, 60 and 64.

Q.9) Find the LCM of 3(a-1), $2(a-1)^2$, (a^2-1)

- A) $(a + 1)^2 (a + 1)$
- B) $(a 1)^2 (a + 1)$
- C) 6 (a 1) $(a + 1)^2$
- D) 6 $(a 1)^2 (a + 1)$

Ans: B

Solution: LCM of 3(a-1), $2(a-1)^2$, (a^2-1) is **6** $(a-1)^2$ (a+1)

Q.10) Find the LCM 35a²c³b, 42a³cb², 30acb²

- a) 215 a3c3b3
- b) 210 a3c3b3
- c) 210 a2c3b
- d) 315 a3c b2

Ans: B

Solution: LCM of 35a²c³b, 42a³cb², 30acb² is **210 a3c3b3**

Q.11) Find the LCM $(a-1)^5 (a+3)^2$, $(a-2)^2 (a-1)^3 (a+3)^4$

- a) (a 1)3 (a 3)4 (a 2)2
- b) (a + 1)3 (a + 2)4 (a + 2)2
- c) (a 1)5 (a + 3)4 (a 2)2
- d) (a 1)5 (a 3)2 (a + 3)4

Ans: C

Solution: LCM of $(a-1)^5 (a+3)^2$, $(a-2)^2 (a-1)^3 (a+3)^4$ is $(a-1)^5 (a+3)^4 (a-2)^2$

Q.12) The HCF and LCM of two numbers m and n are 6 and 210 respectively. If m+n=72 then 1/m+ 1/n is equal to

- A) 1/35
- B) 2/35
- C) 3/35
- D) 5/37

Ans: B

Solution:
$$(1/m) + (1/n) = (m+n)/mn$$

 $(m+n)/mn = 72/(6*210)$
 $(m+n)/mn = 2/25$

Q.13) If m and n are two numbers, then their LCM is _____ (Let HCF (m, n) = HCF).

- a) mn/HCF
- b) mn/LCM
- c) m+n/HCF
- d) m-n/HCF

Ans: A

Solution: LCM of m and n is 'mn'

Product of numbers =LCM *HCF

LCM= Product of numbers / HCF = mn/ HCF

- Q.14) In a BurjKhalifa consisting of 216 floors, two lifts A & B starting from the ground floor, stop at every 6th and 10th floors respectively. On which floors, will both of them stop together?
 - a) 30, 45, 50, 65, 70, 80, 85
 - b) 30, 60, 90, 120, 150, 180, 210
 - c) 15, 30, 45, 60, 75, 90, 105
 - d) 15, 30, 45, 60, 72, 80, 85

Ans: B

Solution: LCM of 6 and 10 is 30.

Every 30th floor they stop together, i.e, 30, 60, 90, 120, 150, 180, 210

- Q.15) The sum of two numbers is 45. Their difference is 1/9 of their sum. Their L.C.M is
 - a) 100
 - b) 150
 - c) 200
 - d) 250

Ans: A

Solution: Let the number be x and y where x > y

According to the question,

$$x + y = 45 - - - (1)$$

$$x-y=1/9 (x+y) = (1/9) *45=5----(2)$$

From (1) and (2)

$$x = 20$$

Hence, LCM of 20 and 25

- Q. 16) The HCF of 20 successive even numbers is.....
 - a) 3
 - b) 2
 - c) 4
 - d) 1

Ans: B

Solution: HCF of successive even numbers always 2.

E.g. HCF of (2, 4, 6, 8, 10, ...) always 2.

Q. 17)L.C.M of 16/24 and 40/72 is

- a)80/24
- b)40/12
- c)20/6
- d)10/3

Ans: D

Solution: LCM of fraction = (LCM of Numerators)/ (HCF of Denominator) LCM of 16 & 24= 80, HCF of 3 & 9= 24 LCM = 80/24 = 10/3.

Q.18) What is the L.C.M. of 50, 60, 70 and 80?

- a) 8800
- b) 8200
- c) 8400
- d) 8200

Ans: C

Solution: We can write 50 as $5 \times 5 \times 2$

Similarly, $60 = 2 \times 3 \times 5 \times 2$

 $70=5 \times 7 \times 2$, $80=2 \times 2 \times 2 \times 5 \times 2$

so, L.C.M.=5 x 5 x 3 x 7 x 2 x 2 x 2 x 2 =8400

Q.19) In a school 782 boys and 646 girls have been divided into the largest possible equal classes. So that there are equal number of boys and girls in each class. What is the number of classes?

- a) 23 girls classes, 19 boys classes
 - b) 34 boys classes, 34 girls classes
 - c) 23 boys classes, 19 girls classes
 - d) 23 boys classes, 17 girls classes

Ans: B

Solution: HCF of 782 and 646 is 34. So, in each class 34 girls and 34 boys are there.

- Q.20) A merchant has three different types of milk: 870 liters, 986 liters and 1102 liters. Find the least number of casks of equal size required to store all the milk without mixing.
 - a) 51
 - b) 47
 - c) 45
 - d) 61

Ans: A

Solution: Finding the HCF of three quantities -

870=3 x 5 x 29 x 2, 986= 17 x 29 x 2,1102= 19 x 29 x 2

therefore, the HCF will be 58 (29*2) as it is common in all the numbers.

Now, Number of caskets = 870/58 + 986/58 + 1102/58

$$=15 + 17 + 19$$

=51

- Q.21) The HCF of two numbers is 18. Which one of the following can never be their LCM?
 - A) 36
 - B) 108
 - C) 198
 - D) 380

Ans: D

Solution: Here 36, 108 and 198 are multiples of 18.

So, 380 is never be their LCM.

- Q.22) About the number of pairs which have 16 as their HCF and 136 as their LCM, we can definitely say that
 - A) No such pairs exist
 - B) Only one such pair exists
 - C) Only two such pair exists
 - D) Many such pair exist

Ans: A

Solution: No such pair exist. Because 136 is not multiple of 16.

Q.23) Find the smallest number by which 10985 should be divided so that the quotient is a perfect cube

- A) 5
- B) 9
- C) 15
- D) 25

Ans: A

Solution: We have $10985 = 5 \times 13 \times 13 \times 13$

we have a triplet of 13 and we are left over with 5.

If we divide 10985 by 5, the new number will be a perfect cube.

The required number is 5.

Q .24)A room has its length, breath and height as 1650cm, 1350cm and 900cm respectively. Find the maximum length to measure all these measurements?

- a) 160cm
- b) 150cm
- c) 150cm
- d) 140cm

Ans: B

Solution: HCF of 1650, 1350 and 900 is 150

Q.25) If Z is a prime number. What is the LCM of Z and Z+1?

- A) Z^2+Z
- B) Z(Z+1)/2
- C) $(Z + 1)^2$
- D) Z(Z + 1)

Ans: A

Solution: If p is a prime number,

Let us take the example to prove the above statement.

Let
$$Z = 3$$
, $Z+1 = 4$

LCM of 3 & 4 is 12, which means 3^2+3 . So, option ' Z^2+Z ' is correct.

- Q.26) What is the maximum length of scale which is placed in 10m*10m* 5m room?
 - a) 15.00 m
 - b) 13.75 m
 - c) 14.35 m
 - d) 12.55 m

Ans: A

Solution: Diagonal of room = $\sqrt{(l^2+b^2+h^2)} = \sqrt{(10^2+10^2+5^2)} = 15$

- Q.27) Three planets revolve round the Sun once in 200, 250 and 300 days, respectively in their own orbits. When do they all come relatively to the same position as at a certain point of time in their orbits?
 - A) After 3000 days
 - B) After 2000 days
 - C) After 1500 days
 - D) After 1200 days

Ans: A

Solution: LCM of 200,250 and 300 is 3000.

- Q .28) The maximum number of students among whom 3003 pens and 2730 pencils can be distributed in such a way that each student gets same number of pens and same number of pencils, is:
 - A) 273
 - B) 910
 - C) 300
 - D) 293

Ans: A

Solution: Total number of pens=3003, Total number of pencils= 2730

maximum number of students who get the same number of pens and same number of pencils HCF (3003 and 2730)

using Prime factorization: 2730=2x5x7x13 x2, 3003 = 7 x 11 x 13 x2Now HCF (3003 and 2730) = 273

Q.29) The greatest number that divides 411, 684, 821 and leaves 3, 4 and 5 as remainders, respectively, is

- A) 254
- B) 146
- C) 136
- C) 204

Ans: C

Solution: Since the remainders are 3, 4 and 5 respectively

We need the HCF of the below numbers.

$$821-5=816, 480=2*2*2*3*17, 680=2*2*2*5*17$$

816=2*2*2*2*3*17

Q .30) Find the greatest number which will exactly divide 200 and 320.

- (A) 10
- (B) 20
- (C) 16
- (D) 40

Ans: D

Solution: Greatest number which will exactly divide 200 and 320 = HCF (200, 320)

- = HCF (40 x 5, 40 x 8)
- = 40

Q .31) What is the least number of square tiles required to pave the floor of a room 15 m 17 cm long and 9m 2cm broad?

- (A) 840
- (B) 841
- (C) 820
- (D) 814

Ans: D

Solution: Length of largest tile = H.C.F. of 1517 cm and 902 cm = 41 cm.

Area of each tile= (41 x 41) cm²

Required number of tiles = (1517*902)/(41*41) = 814

Q.32) The LCM and the HCF of the numbers 28 and 42 are in the ratio:

- (A) 6:1
- (B) 2:3
- (C) 3:2
- (D) 7:2

Ans: A

Solution: L.C.M. of 28 and 42 = 84

H.C.F. of 28 and 42

Required ratio = 84: 14 = 6:1

Q.33) Two numbers are in the ratio 3:4. Their L.C.M. is 84. The greater number is

- (A) 21
- (B) 24
- (C) 28
- (D) 84

Ans: C

Solution: The numbers are 21 and 28.

Q .34) The ratio of two numbers is 4:5 and their H.C.F. is 8. Then their L.C.M. is

- (A) 130
- (B) 140
- (C) 150
- (D) 160

Ans: D

Solution: Let the numbers be 4p and 5p.

H.C.F. =
$$p = 8$$

Numbers =
$$4p = 4 \times 8 = 32$$
 and $5p = 5 \times 8 = 40$

Their LCM = 160

Q.35) The number between 4000 and 5000 that is divisible by each of 12, 18, 21 and 32 is

- (A) 4023
- (B) 4032
- (C) 4302
- (D) 4203

Ans: C

Solution: LCM of (12, 18, 21, 32) is 2016

 $2016 \times k = 2016k$

Let
$$k = 2$$

2016 x 2 = 4032

- Q.36) The LCM of two multiples of 12 is 1056. If one of the number is 132, the other number is
 - (A) 12
 - (B) 72
 - (C) 96
 - (D) 132

Ans: C

Solution: HCF (a, b) x LCM(a, b) = a x b $132b = 12 \times 1056$ b= 96

- Q.37) The smallest square number divisible by 10, 16 and 24 is
 - (A) 900
 - (B) 1600
 - (C) 2500
 - (D) 3600

Ans: D

Solution: $10=2^1 \times 51$, $16=2^4$, $24=2^3 \times 3^1$ $3600=2^4 \times 3^2 \times 5^2$

- Q.38) What is the smallest number which leaves remainder 3 when divided by any of the numbers 5, 6 or 8 but leaves no remainder when it is divided by 9?
 - (A) 123
 - (B) 603
 - (C) 723
 - (D) 363

Ans: D

Solution: LCM (5, 6, 8, 9)

LCM = 2x3x5x4x3 = 360

required number = 360 + 3 = 363

363 is the least number divided by 5, 6, 8, and 9 gives remainder 3.

- Q. 39) The least multiple of 7, which leaves the remainder 4, when divided by any of 6, 9, 15 and 18, is
 - (A) 76
 - (B) 94
 - (C) 184
 - (D) 364

Ans: D

Solution: L.C.M. of 6, 9, 15 and 18 is 90. Let required number be 90k + 4, which is a multiple of 7.

Least value of k for which (90k + 4) is divisible by 7 is k = 4. Required number = $90 \times 4 + 4 = 364$.

- Q.40) Three sets of English, Mathematics and Science books containing 336, 240, 96 books respectively have to be stacked in such a way that all the books are stored subjectwise and the height of each stack is the same. Total number of stacks will be
 - (A) 14
 - (B) 21
 - (C) 22
 - (D) 48

Ans: A

Solution: Height of each stack = HCF of (336, 240, 96) books

HCF of (336, 240, 96) = 48

Number of stacks = (336/48) + (240/48) + (96/48)

Number of stacks =7+5+2=14

- Q.41) Find the HCF of 1/2, 3/4, 5/6, 7/8, 9/10 and 1/12
 - A) 1/2
 - B) 1/10
 - C)9/120
 - D)1/120

Ans: D

Solution: HCF of fraction = (HCF of Numerators)/ (LCM of Denominator) = 1/120

Q.42) Find the LCM of 12 $(x-1)^3$ and 15 $(x-1)(x+2)^2$

- A) $60(x-1)^3(x+2)^2$
- B) 60(x-1)
- C) 3(x-1)
- D) $3(x-1)^3(x+2)^2$

Ans: A

Solution: LCM of 12 $(x-1)^3$ and 15 $(x-1)(x+2)^2$ is $60(x-1)^3(x+2)^2$

Q.43) Which of the following pairs is co-prime?

- A) 51,63
- B) 52,91
- C) 71,81
- D) 81, 99

Ans: B

Q.44) The product of any consecutive numbers is always divisible by what number?

- A)3
- B)4
- C)7
- D)6

Ans: B

Q.45) Find the smallest number which is exactly divisible by all the numbers from 1 to 9.

- A) 1620
- B) 2520
- C) 2530
- D) 2540

Ans: B

Solution: To find the smallest number we have to find the LCM (1, 2, 3, 4, 5, 6, 7, 8, 9)

LCM is2x3x2x5x7x2x3= 2520

The required number is 2520

- Q.46) The LCM of two co-prime numbers is 5005. If one of the numbers is 65, then find the other number.
 - A) 99
 - B) 88
 - C) 77
 - D) 66

Ans: C

Solution: let the other number be 'X'

we know that the numbers are co primes so HCF = 1

HCF x LCM =PRODUCT OF TWO NUMBERS

$$1 \times 5005 = 65 \times X$$

X = 77

- Q.47) The product of the LCM and HCF of two numbers is 24. The difference of the two numbers is 2. Find the numbers?
 - (A) 8 and 6
 - (B) 8 and 10
 - (C) 2 and 4
 - (D) 6 and 4

Ans: D

Solution: Let the numbers be p and (p + 2).

Product of numbers = $HCF \times LCM$

$$P * (P + 2) = 24$$

$$P^2+2P-24=0---->(P-4)(P+6)$$

Numbers = p = 4 and P+2=6

- Q.48) If x: y be the ratio of two whole numbers and z be their HCF, then the LCM of those two numbers is
 - (A) yz
 - (B) xz/y
 - (C) xy/z
 - (D) *xyz*

Ans: D

Solution: Ratio of the numbers = x : y

HCF of the numbers = z

So, z is the common factor of the numbers

Then, First number = xz

Second Number = yz

First Number x Second Number = HCF and LCM of the numbers

 $xzyz = z \times LCM$

LCM = xyz

Q.49) The HCF (GCD) of a, b is 12, a,b are positive integers and a > b > 12. The smallest values of (a, b) are respectively

- (A) 12, 24
- (B) 24, 12
- (C) 24, 36
- (D) 36, 24

Ans: D

Solution: HCF of a, b number is 12

and a > b > 12 (given)

Smallest value of a & b is (36, 24)

Q .50) The number of pair of positive integers whose sum is 99 and HCF is 9 is

- (A) 2
- (B) 3
- (C) 4
- (D) 5

Ans: D

Solution: According to question, HCF = 9

Then the two number will be 9a, 9b

$$9a + 9b = 99$$

Pairs of positive integers (1, 10) (2, 9) (3, 8) (4, 7) (5, 6) = 5

PREVIOUS YEAR QUESTIONS

Q.1)	The LCM of two numbers is 864 and their HCF is 144. If one of the number is 288,
	the other number is:
	(a) 576
	(b) 1296
	(c) 432
	(d) 144
	Ans: (c) 432
Q.2)	LCM of two numbers is 225 andtheir HCF is 5. If one number is 25, the other number
	will be:
	(a) 5
	(b) 45
	(c) 25
	(d) 225
	Ans: (c) 25
Q.3)	The least number which when divided by 4, 6, 8, 12 and 16 leaves a remainder of 2 in
	each case is.
	(a) 46
	(b) 50
	(c) 48
	(d) 56
	Ans: (b) 50
Q.4)	The LCM and the HCF of the numbers 28 and 42 are in the ratio:
	(a) 5
	(b) 15
	(c) 45
	(d) 270
	Ans: (a) 5

Q.5) The LCM of two numbers is 2079 and their HCF is 27. If one of the numbers is 189,

the other number is

(a) 297

- (b) 584
- (c) 189
- (d) 216

Ans: (a) 297

- Q.6) The product of two co-prime numbers is 117. Then their LCM is
 - (a) 117
 - (b) 9
 - (c) 13
 - (d) 39

Ans: (a) 117

- Q.7) The least number which when divided by 16, 18, 20 and 25 leaves 4 as remainder in each case but when divided by 7 leaves no remainder is
 - (a) 17004
 - (b) 18000
 - (c) 18002
 - (d) 18004

Ans: (d) 18004

- Q.8) The least number which when divided by 5, 6, 7 and 8 leaves a remainder 3, but when divided by 9 leaves no remainder is
 - (a) 1677
 - (b) 1683
 - (c) 2523
 - (d) 3363

Ans: (b) 1683

- Q.9) The least number which when divided by 5, 6, 7 and 8 leaves a remainder 3, but when divided by 9 leaves no remainder is
 - (a) 1677
 - (b) 1683
 - (c) 2523
 - (d) 3363

Ans: (b) 1683

Q.10)	A milk vendor has 21 liters of cow milk, 42 liters of toned milk and 63 liters of double
	toned milk. If he wants to pack them in cans so that each can contain the same liters
	of milk and does not want to mix any two kinds of milk in a can, then the least
	number of cans required is

1	`	2
10	ı۱	~
16	ι,	

Ans: (b) 6

Q.11) Which is the least number of square til	es required to pave the floor of a room 15 m
17 cm long and 9 m 2 cm broad?	

- (a) 840
- (b) 841
- (c)820
- (d) 814

Ans: (d) 814

Q.13) Let x be the smallest number, which when added to 2000 makes the resulting number divisible by 12, 16, 18 and 21. The sum of the digits of x is

- (a) 7
- (b) 5
- (c) 6
- (d) 4

Ans: (a) 7

- Q.14) The LCM of two numbers is 44 times of their HCF. The sum of the LCM and HCF is 1125. If one number is 25, then the other number is
 - (a) 1100
 - (b) 975
 - (c) 900
 - (d) 800

Ans: (a) 1100

Q.15) Which least number should be subtracted from the number 36798 so that the new number is completely divisible by 78?

(a) 18

⁽b) 6

⁽c)9

⁽d) 12

- (b) 60
- (c) 38
- (d) 68

Ans: (b) 60

- Q.16) 84 Maths books, 90 Physics books and 120 Chemistry books have to be stacked topicwise. How many books will be there in each stack so that each stack will have the same height too?
 - (a) 12
 - (b) 18
 - (c) 6
 - (d) 21

Ans: (c) 6

