Integers Mathematics and Algorithms Number Theory Algorithms

What is the most efficient algorithm to find the Pisano period for a given integer (even for large integers)?

1 Answer

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Answered Sep 28, 2016

The Pisano period $\pi(n)$ is a multiplicative function, that is if a and b are coprime than $\pi(ab) = \pi(a)\pi(b)$. So we need only concern ourselves with the value of $\pi(p^k)$ for prime p. (Factoring even a large number is still better than brute force periodicity search.)

It is hypothesized that $\pi(p^k) = p^{k-1}\pi(p)$ and since no counterexamples are known to exist, you might as well use that in your algorithm.

So, how to calculate $\pi(p)$ efficiently? There are two special cases and two general cases

$\pi(2^k) = 3 \cdot 2^{k-1}$

$\pi(5^k) = 4 \cdot 5^k$

If $p \equiv 1$ or $p \equiv 9 \pmod{10}$ then $\pi(p) \mid p - 1$

If $p \equiv 3$ or $p \equiv 7 \pmod{10}$ then $\pi(p) \mid 2(p + 1)$, and by an odd divisor too.

The last two statements give us a relatively small number of cases to try (after factoring $p - 1$ or $2(p + 1)$.) Now use your favorite formula to calculate large values of the Fibonacci numbers $F(x) \bmod p$. See Michal Forišek's answer to What's a fast algorithm to find the remainder of the division of a huge Fibonacci number by some big integer? To test a candidate period R, calculate $F(R) \bmod p$ and $F(R + 1) \bmod p$. If these are equal to $F(0) = 0$ and $F(1) = 1$, then $\pi(p) \mid R$.

It might be that $p - 1$ or $2(p + 1)$ have a lot of divisors, but we don't need to try them all. Suppose $q^k \mid R$ for some prime q. Then test R/q. If that doesn't produce a cycle, then $\pi(p)$ must have factor q^k, and we can leave it in and go on to other factors. Otherwise, we can use R/q as our new starting point and repeat the process. Thus we have to do a number of checks proportional to $\Omega(2(p + 1))$, not $d(2(p + 1))$.

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