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What's a fast algorithm to find the remainder of the division of a huge Fibonacci number by some big integer?



Michal Forišek, Ph. D. in theoretical Computer Science Updated Apr 25, 2014

Each recurrence similar to Fibonacci numbers can be expressed in terms of matrix multiplication as follows:

In order to compute the next Fibonacci number F_{n+2} , we need two previous ones: (F_n, F_{n+1}) . The transformation that changes (F_n, F_{n+1}) to (F_{n+1}, F_{n+2}) is linear, and therefore we can find a matrix that performs it:

$$\forall a,b:(a,b)\cdot\left(egin{smallmatrix}0&1\1&1\end{smallmatrix}
ight)=(b,a+b).$$

Let's call the above matrix A. Obviously, for any n we have:

$$(F_n, F_{n+1}) \cdot A = (F_{n+1}, F_n + F_{n+1})$$

which is precisely (F_{n+1}, F_{n+2}) .

We can use A multiple times. For example:

$$\Big(\Big((F_0,F_1)\cdot A\Big)\cdot A\Big)\cdot A=(F_3,F_4)$$











$$(F_0, F_1) \cdot A^3 = (F_3, F_4)$$

And, in general:

$$(0,1)\cdot A^n = (F_n,F_{n+1})$$

The above equation is valid in integers, therefore it is valid modulo any m.

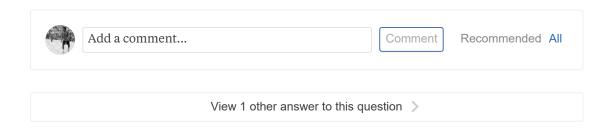
Computing modulo m, we can use Exponentiation by squaring to compute A^n using $O(\log n)$ operations on numbers smaller than m^2 .

In terms of a practical implementation: as long as m fits into a 32-bit integer variable, all intermediate values will fit into 64-bit variables, and you can use the above formula to compute $F_n \mod m$ even for $n = 10^{1000}$.

(Or use a language with arbitrary precision integers if you need larger values of m. For example, $n=m=10^{1000}$ is still perfectly feasible.)

Sample implementation: http://ideone.com/fP8krp

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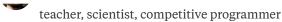
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Works at Comenius University in Bratislava

Studied at Comenius University in Bratislava

Lives in Bratislava

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