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Quantum teleportation with partially entangled states via noisy channels

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Abstract Using a partially entangled EPR-type state as quantum channel, we investigate quantum teleportation (QT) of a qubit state in noisy environments by solving the master equation in the Lindblad form. We analyze the different influence for the partially entangled EPR-type channel and the EPR channel on the fidelity and the average fidelity of the QT process in the presence of Pauli noises. It is found that the fidelity depends on the type and the strength of the noise, and the initial state to be teleported. Moreover, the EPR channel is more robust than the partially entangled EPR-type channel against the influence of the noises. It is also found that the partially entangled EPR-type channel enables the average fidelity as a function of the decoherence parameter kt to decay with different velocities for different Pauli noises.

Keywords Quantum teleportation · Noisy environments · Fidelity · EPR-type state

1 Introduction

Quantum teleportation (QT), firstly proposed by Bennett et al. in [1], is one of the most important information transmission protocols in the field of quantum information science. It sheds light on an interesting and unusual method for implementing rapid, safe and secret communication. In QT, an unknown quantum state can be transmitted from a sender Alice to a remote recipient Bob via dual classical and quantum channel without

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physically sending the particle of the state. It is well known that quantum entanglement, acting as the entangled channel, plays a crucial role in the process of QT. In Bennett et al.'s protocol, a maximally entangled EPR state was adopted to realize the perfect QT and an accurate copy of the unknown state is transferred successfully from the sender's side to the recipient's side in the ideal quantum system without any interaction with its external environment. Subsequently, the other usual entangled states, such as W state [2], GHZ state [3] and Cluster state [4], were employed to accomplish the probabilistic even perfect teleportation process [5–10]. On the other hand, partially entangled states are also important resources for quantum communication. Based on partially entangled channels, a great deal of QT protocols are presented in Refs. [11–17]. In these protocols, the relations between quantum teleportation and the entanglement amount of the quantum channel were taken into account. For instance, Bennett et al. [11] found that the quantum channel with less entanglement reduces the fidelity of the teleportation. Li et al. [12] examined the entanglement matching of probabilistic teleportation through a partially entangled state. Banaszek [17] investigated the fidelity of quantum teleportation with non-maximally entangled states and discussed the trade-off between the information gain and the quantum state disturbance. So far, a lot of attention has been paid to implement QT both theoretically and experimentally in various physical systems, such as optical systems [18–20], nuclear magnetic resonance technique [21], and trapped ions [22,23], and so on.

Actually, one quantum system will unavoidably interact with the real world so that quantum states exposed to the external environment will gradually lose their coherence. So it is necessary to study and explore the quantum information processes in the presence of noisy environments. In recent years, quantum systems subject to Markovian reservoirs and their time evolution described by a general master equation in the Lindblad form, have been appropriately considered [24–32]. Oh et al. [31] calculated the fidelity and the average fidelity for the standard QT protocol through noisy EPR channel. Jung et al. [32] analyzed the effect of the noises on the process of QT via the noisy GHZ and W channels, and discussed the capacity of the quantum channel to resist the influence of the noisy environment. Furthermore, Ishizaka [33] examined the quantum channel subject to local interaction with a two-level environment. Hao et al. [34] investigated the effects of amplitude damping in quantum noise channels on every stage of teleportation and found that the non-maximally entangled channels are immune to quantum noise of teleportation to some degree. Hu et al. [35] studied the noise effect on the fidelity of two-qubit teleportation and concluded that the inseparable channels never give a higher teleportation fidelity than Bell pairs even in the presence of collective noise.

We notice that the initial quantum channels previously used to achieve the QT protocols in noisy environments are often the maximally entangled states [30–32]. Generally, the maximally entangled states will reduce to the partially entangled or mixed states due to the ambient noise influence. If the partially entangled states are used as the initial channels of the QT protocol in noisy conditions, what behaviors will these channels reveal in the QT process? To the best of our knowledge, seldom studies [34] have involved with this problem.

To some extent, Pauli noises, namely, the bit flip noises, the dephasing noises, and the bit-phase flip noises, are the fundamental errors among the various types of

Markovian noises [28–32]. In this paper, we investigate the influence of Pauli noises on the QT process. By employing a bipartite EPR-type state as quantum channel, we study quantum teleportation of a qubit state in noisy environment by solving the master equation in the Lindblad form. The fidelity is used to characterize the quality of the QT process. We give the physical quantities affecting the teleportation fidelity and reveal the main difference for the noisy partially entangled EPR-type channel and the noisy EPR channel on the influence of the fidelity. We also remark on how to design the QT scheme for stronger ability to resist the noise influence.

This paper is organized as follows. In Sect. 2, we first describe the scheme for teleporting a qubit state via an EPR-type state in ideal condition. Then, we analyze the effects of noises on the fidelity of the QT process in view of three different situations that the initial state, the EPR-type channel, or both the initial state and the EPR-type channel is subject to the Pauli noises. Finally, conclusions are given in Sect. 3.

2 Teleporting a qubit state with an EPR-type state in noisy environments

In this section, we first briefly describe the teleportation of a qubit state by means of a partially entangled state as the quantum channel without considering any interaction with the external environment [12].

The single-qubit state that Alice wants to teleport to Bob is given by

$$|\psi\rangle_1 = \cos\frac{\theta}{2}\,|0\rangle_1 + \sin\frac{\theta}{2}e^{i\phi}\,|1\rangle_1\,, \tag{1}$$

where $\theta \in [0, \pi]$ and $\phi \in [0, 2\pi)$ are the polar and azimuthal angles of the state to be teleported respectively. Suppose that Alice and Bob share a bipartite partially entangled EPR-type pure state taking the following form of

$$|\psi\rangle_{23} = a\,|00\rangle_{23} + b\,|11\rangle_{23} \quad (|b| \leq |a|\,,\ |a|^2 + |b|^2 = 1), \tag{2}$$

where a and b are real, as well as Alice possesses particle 2 and Bob particle 3. As a fact of matter, if the bipartite EPR-type pure state is in complex Hilbert space, it can be written as the form of $|\psi_c\rangle_{23} = a|00\rangle_{23} + be^{i\zeta}|11\rangle_{23}$ with ζ being real. After Alice makes a unitary transformation $|0\rangle\langle 0| + e^{-i\zeta}|1\rangle\langle 1|$ on particle 2 or Bob makes the same unitary transformation on particle 3, $|\psi_c\rangle_{23}$ can be conveniently changed into the state $|\psi\rangle_{23}$ in real space since Alice and Bob have the complete knowledge of their own shared EPR-type state. So we only consider the state $|\psi\rangle_{23}$ in real space as the entangled resource for QT in the subsequent discussions of this paper.

The EPR-type state $|\psi\rangle_{23}$ can be transformed into an EPR state by Bob performing a unitary-reduction process. To be specific, Bob first introduces an auxiliary qubit with the original state $|0\rangle_4$, the total state of the particles 2, 3, and 4 can be expressed as

$$|\psi\rangle_{234} = |\psi\rangle_{23} \otimes |0\rangle_4 = a\,|000\rangle_{234} + b\,|110\rangle_{234}\,. \tag{3}$$

Then Bob makes a unitary transformation U_{34} on particles 3 and 4, which leads to

$$U_{34}\left|\psi\right\rangle_{234} = b\left(\left|00\right\rangle_{23} + \left|11\right\rangle_{23}\right)\left|0\right\rangle_4 + \sqrt{a^2 - b^2}\left|00\right\rangle_{23}\left|1\right\rangle_4, \tag{4}$$

where

$$U_{34} = \left(\frac{b}{a}\left|00\right\rangle\left\langle 00\right| - \sqrt{1 - \frac{b^2}{a^2}}\left|00\right\rangle\left\langle 01\right| + \sqrt{1 - \frac{b^2}{a^2}}\left|01\right\rangle\left\langle 00\right| \right.$$
$$\left. + \frac{b}{a}\left|01\right\rangle\left\langle 01\right| + \left|10\right\rangle\left\langle 10\right| + \left|11\right\rangle\left\langle 11\right|\right)_{34}. \tag{5}$$

Now Bob measures the state of the auxiliary particle 4 with the basis vectors $\{\left|0\right\rangle, \left|1\right\rangle\}$. If he gets $\left|0\right\rangle_4$, particles 2 and 3 are collapsed to the maximally entangled state $\left|\psi\right\rangle'_{23} = \frac{1}{\sqrt{2}}\left(\left|00\right\rangle_{23} + \left|11\right\rangle_{23}\right)$ with the success probability of $2b^2$. Next the unknown state $\left|\psi\right\rangle_1$ can be teleported from Alice to Bob via the EPR state $\left|\psi\right\rangle'_{23}$ following the procedure of the standard QT protocol stated in Ref. [1]. Otherwise, the particles 2 and 3 are collapsed to the state $\left|00\right\rangle_{23}$ with the probability of $a^2 - b^2$, which means the failure of the teleportation. Therefore, the initial single-qubit state can be obtained successfully in Bob's location with the probability of $2b^2$. If $a = b = \frac{1}{\sqrt{2}}$, namely, $\left|\psi\right\rangle_{23}$ becomes the maximally entangled EPR state, in this case, the perfect QT can be achieved.

In the following, we will consider the above teleportation protocol under the influence of the surrounding environments. To this end, the quantum circuit of the teleportation protocol is first constructed in Fig. 1, where Alice holds the initial state $\left|\psi\right\rangle_1$, Alice and Bob pre-share the EPR-type state $\left|\psi\right\rangle_{23}$.

Generally, an open quantum system interacts inevitably with the external environment. This interaction is usually looked as noise in quantum optics. Under the assumption of Markov and Born approximations, the dynamics of an open quantum

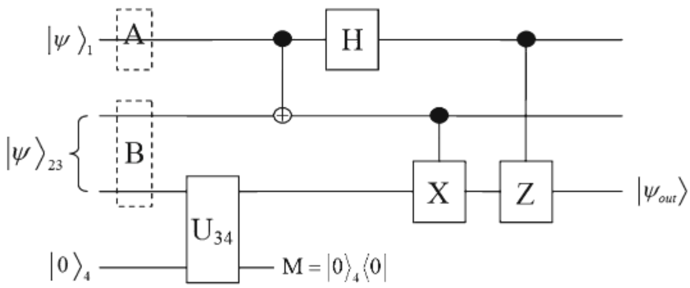


Fig. 1 (Color online) The quantum circuit for the teleportation of a qubit state by means of a noisy EPR-type state. The *top line* stands for the initial state to be teleported at Alice's side, the *second line* for the state of the first particle of the EPR-type channel belonging to Alice, the *third line* for the state of the second particle of the EPR-type channel belonging to Bob, and the *bottom line* for the state of the auxiliary particle. The *dashed box A* and *B* represent the noises acting on the input state and the quantum channel, respectively

system can be described by the following master equation in Lindblad form [24,25]

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[H, \rho] + \sum_{j,\alpha} \left(L_{j,\alpha} \rho L_{j,\alpha}^{\dagger} - \frac{1}{2} L_{j,\alpha}^{\dagger} L_{j,\alpha} \rho - \frac{1}{2} \rho L_{j,\alpha}^{\dagger} L_{j,\alpha} \right), \tag{6}$$

where H is the Hamiltonian of the quantum system. The Lindblad operator $L_{j,\alpha} = \sqrt{k_{j,\alpha}} \sigma_{\alpha}^{j}$ describes the coupling of the system to its environment with σ_{α}^{j} $(\alpha = x, y, z)$ denoting the Pauli operator of the jth qubit and $k_{j,\alpha}$ being the strength of decoherence rates acting on the jth qubit. According to the above master equation, we can analytically calculate the time evolution of the state $|\psi\rangle_1$ to be teleported and that of the EPR-type state $|\psi\rangle_{23}$ employed in our teleportation protocol.

Now we suppose that the Lindblad operator of the noises acting on the initial state $|\psi\rangle_1$ takes the form of

$$L_{1,x} = \sqrt{\lambda_1} \sigma_x, \quad L_{1,y} = \sqrt{\lambda_2} \sigma_y, \quad L_{1,z} = \sqrt{\lambda_3} \sigma_z. \tag{7}$$

By switching $\lambda_i (i = 1, 2, 3)$ on and off, we can control the type of the noises affecting the input state. Based on Eqs. (1, 6) and (7) with the assumption of $H = 0$ here and after, the density matrix elements of the initial state $\rho_{\text{in}} = |\psi\rangle_1 \langle \psi|$ can be evolved into the state $\rho_{\text{in}}^{\text{evo}}$ taking the following form

$$\begin{aligned}
\rho_{\text{in}}^{11} &= \frac{1}{2} \left[1 + \cos\theta e^{-2(\lambda_1 + \lambda_2)t} \right], \\
\rho_{\text{in}}^{22} &= \frac{1}{2} \left[1 - \cos\theta e^{-2(\lambda_1 + \lambda_2)t} \right], \\
\rho_{\text{in}}^{12} &= \frac{1}{2} \left[-i \sin\theta \sin\phi e^{-2(\lambda_1 + \lambda_3)t} + \sin\theta \cos\phi e^{-2(\lambda_2 + \lambda_3)t} \right], \\
\rho_{\text{in}}^{21} &= \frac{1}{2} \left[i \sin\theta \sin\phi e^{-2(\lambda_1 + \lambda_3)t} + \sin\theta \cos\phi e^{-2(\lambda_2 + \lambda_3)t} \right].
\end{aligned} \tag{8}$$

On the other hand, the Lindblad operators acting on the quantum channel $|\psi\rangle_{23}$ are assumed to be

$$L_{j,x} = \sqrt{k_1} \sigma_x, \quad L_{j,y} = \sqrt{k_2} \sigma_y, \quad L_{j,z} = \sqrt{k_3} \sigma_z, \quad (j = 2, 3). \tag{9}$$

Similarly, by switching $k_i (i = 1, 2, 3)$ on and off, we can manipulate the type of various noises affecting the quantum channel. According to Eqs. (2, 6) and (9), the density matrix of the time evolution of the initial channel, ρ_{23}^{evo}, can be calculated as

$$\begin{aligned}
\rho_{23}^{11} &= \frac{1}{4} \left[1 + 2(a^2 - b^2) e^{-2(k_1 + k_2)t} + e^{-4(k_1 + k_2)t} \right], \\
\rho_{23}^{22} &= \frac{1}{4} \left[1 - e^{-4(k_1 + k_2)t} \right], \\
\rho_{23}^{33} &= \frac{1}{4} \left[1 - e^{-4(k_1 + k_2)t} \right],
\end{aligned}$$

$$\begin{aligned}
\rho_{23}^{44} &= \frac{1}{4}\left[1-2(a^2-b^2)e^{-2(k_1+k_2)t}+e^{-4(k_1+k_2)t}\right],\\
\rho_{23}^{14} &= \rho_{23}^{41}=\frac{1}{2}ab\left[e^{-4(k_1+k_3)t}+e^{-4(k_2+k_3)t}\right],\\
\rho_{23}^{23} &= \rho_{23}^{32}=\frac{1}{2}ab\left[-e^{-4(k_1+k_3)t}+e^{-4(k_2+k_3)t}\right],\\
\rho_{23}^{mn} &= 0,\ \text{for the other values of}\, m, n.
\end{aligned} \tag{10}$$

In this situation, from Fig. 1, the output state at Bob's side can be expressed as

$$\rho_{\text{out}} = Tr_{1,2}\left\{U_{\text{tel}}(\rho_{\text{in}}^{\text{evo}}\otimes\rho_{23}')U_{\text{tel}}^{\dagger}\right\}, \tag{11}$$

where $U_{\text{tel}} = (\sigma_z)_{1\to 3}(\sigma_x)_{2\to 3}H_1(\sigma_x)_{1\to 2}$, $\rho_{23}' = Tr_4\{M[I_2\otimes U_{34}(\rho_{23}^{\text{evo}}\otimes |0\rangle_4\langle 0|)(I_2\otimes U_{34})^{\dagger}]M^{+}\}$. Here H stands for the Hadamard operation, I_2 for the identity operation, $M = |0\rangle_4\langle 0|$ for the projective measurement on particle 4, and $(\sigma_w)_{u\to v}$ for the two-qubit controlled-σ_w $(w = x, z)$ gate with particle u controlling particle v $(u = 1, 2$, and $v = 2, 3)$.

Due to the noise influence, the input state cannot be accurately reconstructed at Bob's side, how can we quantify the QT process? It is well accepted that fidelity is a useful tool to describe how much quantum information is transferred from the sender to the receiver. Generally, fidelity describing the degree of closeness between the input state and the output state is defined as

$$F \equiv \langle\psi_{\text{in}}|\,\rho_{\text{out}}\,|\psi_{\text{in}}\rangle, \tag{12}$$

where $|\psi_{\text{in}}\rangle$ and ρ_{out} denote the input state and the output state, respectively. The smaller the value of the fidelity is, the more quantum information is lost during the transfer process. And $F = 1$ means the perfect transmission of quantum state. To analyze the influence of the noises on the QT, we only take into account the process corresponding to the successful output states in the rest of this paper. Substituting Eqs. (1) and (11) into (12), through some calculations, the fidelity of the teleportation process can be derived as

$$\begin{aligned}
F = \frac{C}{4}&\Big[1-\left(a^2-b^2\right)^2 e^{-2(k_1+k_2)t}-\left(a^2-b^2\right)^2\cos^2\theta\cdot e^{-2(k_1+k_2)t}\cdot e^{-2(\lambda_1+\lambda_2)t}\\
&+\cos^2\theta\cdot e^{-4(k_1+k_2)t}\cdot e^{-2(\lambda_1+\lambda_2)t}\Big]\\
&+C\cdot a^2b^2\sin^2\theta\Big[\sin^2\phi\cdot e^{-4(k_1+k_3)t}\cdot e^{-2(\lambda_1+\lambda_3)t}\\
&+\cos^2\phi\cdot e^{-4(k_2+k_3)t}\cdot e^{-2(\lambda_2+\lambda_3)t}\Big],
\end{aligned} \tag{13}$$

with

$$\frac{1}{C} = \frac{1}{2}-\frac{1}{2}\left(a^2-b^2\right)^2 e^{-2(k_1+k_2)t}. \tag{14}$$

Clearly, the fidelity described in Eq. (13) depends on the type and the strength of the noise, the EPR-type state, and the initial state to be teleported. We notice that the input state is an unknown arbitrary single-qubit state. To measure how much information is transmitted from Alice to Bob in the noisy environment, it is more convictive to calculate the average fidelity over all possible input states, which is expressed as

$$F_{\mathrm{av}} \equiv \frac{1}{4\pi}\int_0^{2\pi} \mathrm{d}\phi \int_0^{\pi} \mathrm{d}\theta \sin\theta\, F(\theta,\phi). \tag{15}$$

Thus we can analyze the fidelity and the average fidelity of the QT process under different noisy situations according to Eqs. (13) and (15). In what follows, we turn to study the influence of the noises on the QT protocol in view of three different cases. For simplicity we will only take into account the situations of the successful teleported states in the rest of this paper.

Case A: The initial state to be teleported is subject to the noises

Here we assume that the quantum channel does not interact with its environment (say, $k_1 = k_2 = k_3 = 0$ in Eq. 9) and the initial state at Alice's side is subject to one of the following noises:

(i) Bit flip noise, i.e., $\lambda_2 = \lambda_3 = 0$ and $\lambda_1 = \lambda$.
(ii) Dephasing noise, i.e., $\lambda_1 = \lambda_2 = 0$ and $\lambda_3 = \lambda$.
(iii) Isotropic noise, i.e., $\lambda_1 = \lambda_2 = \lambda_3 = \lambda/3$, here and after.

It should be pointed out that to conveniently compare the noise influence on the QT process, the above three types of noises are normalized with the same strength of the particles coupling to their respective environments throughout this paper. We notice that the coupling strength in the case of the isotropic noise channel takes the form of $\lambda_1 = \lambda_2 = \lambda_3 = \lambda$ in Refs. [31,32]

Through straightforward calculations, we can obtain the fidelity and the average fidelity of the teleportation process corresponding to the initial state subject to the above three types of noises, and the calculation results are depicted in Table 1. Obviously, it can be seen from Table 1 that the average fidelity is always larger than $\frac{2}{3}$ no matter whether the initial state is subject to the bit flip noise or the dephasing noise, but the average fidelity is only more than $\frac{1}{2}$ if the initial state is infected by the isotropic noise. Our results regarding the fidelity and the average fidelity are the same as those described in Ref. [31], which is due to the fact that the quantum channel consisting

Table 1 The fidelity and the average fidelity for the initial state under the influence of the bit flip noise, the dephasing noise, and the isotropic noise

Noise type	Fidelity	Average fidelity
Bit flip noise	$\frac{1}{2}\left[1+\sin^2\theta\cos^2\phi + e^{-2\lambda t}(\cos^2\theta + \sin^2\theta\sin^2\phi)\right]$	$\frac{2}{3}+\frac{1}{3}e^{-2\lambda t}$
Dephasing noise	$\frac{1}{2}\left(1+\cos^2\theta + \sin^2\theta e^{-2\lambda t}\right)$	$\frac{2}{3}+\frac{1}{3}e^{-2\lambda t}$
Isotropic noise	$\frac{1}{2}+\frac{1}{2}e^{-4\lambda t/3}$	$\frac{1}{2}+\frac{1}{2}e^{-4\lambda t/3}$

of the EPR-type state becomes the maximally entangled EPR state after the unitary-reduction process. Nevertheless, the success probability of our QT protocol depends on the parameter b, while the success probability of the standard QT protocol is 100 %.

Case B: The quantum channel is subject to the noises

Now we assume that the initial state to be teleported is not disturbed by the noises (say, $\lambda_1 = \lambda_2 = \lambda_3 = 0$ in Eq. 7) while the quantum channel is subject to various noises.

(i) If the quantum channel is infected by the bit flip noise, namely, $k_2 = k_3 = 0$ and $k_1 = k$. By direct calculations, the fidelity can be obtained from Eq. (13) as

$$F'_x = C_1 \left\{ \frac{1}{4} + a^2 b^2 \sin^2\theta \cos^2\phi - \frac{1}{4}\left(1 - 4a^2b^2\right)\left(1 + \cos^2\theta\right) e^{-2kt} + \left(\frac{1}{4}\cos^2\theta + a^2b^2 \sin^2\theta \sin^2\phi\right) e^{-4kt} \right\}, \tag{16}$$

with $\frac{1}{C_1} = \frac{1}{2} - \frac{1}{2}\left(1 - 4a^2b^2\right) e^{-2kt}$, and the corresponding average fidelity can be calculated as

$$F'_{x,\mathrm{av}} = C_1 \left[\frac{1}{4} + \frac{1}{3}a^2b^2 - \frac{1}{3}\left(1 - 4a^2b^2\right) e^{-2kt} + \left(\frac{1}{12} + \frac{1}{3}a^2b^2\right) e^{-4kt} \right]. \tag{17}$$

(ii) If the quantum channel is infected by the dephasing noise, namely, $k_1 = k_2 = 0$ and $k_3 = k$. Similarly, the fidelity and the average fidelity can be derived respectively as

$$F'_z = \frac{1}{2}\left(1 + \cos^2\theta + \sin^2\theta \cdot e^{-4kt}\right), \tag{18}$$

$$F'_{z,\mathrm{av}} = \frac{2}{3} + \frac{1}{3} e^{-4kt}. \tag{19}$$

In this situation, both the fidelity and the average fidelity are independent of the parameters a and b of the initial EPR-type state $|\psi\rangle_{23}$. In what follows, we briefly describe the calculation process of the fidelity F'_z. According to Eq. (10), the density matrix elements ρ_{23}^{evo} under the action of dephasing noise are given by $\rho_{23}^{11} = a^2$, $\rho_{23}^{44} = b^2$, $\rho_{23}^{14} = \rho_{23}^{41} = ab \cdot e^{-4k_3 t}$, and $\rho_{23}^{mn} = 0$ (for the other values of m, n). After introducing the auxiliary qubit 4 with the original state $|0\rangle_4$, Bob performs the unitary transformation U_{34} on particles 3 and 4, and makes the projective measurement $M = |0\rangle_4\langle 0|$. As a result, the density matrix of the EPR-type state infected by the dephasing noise is transformed into $\rho'_{23} = \frac{1}{2}\left(|00\rangle\langle 00| + e^{-4k_3 t}|00\rangle\langle 11| + e^{-4k_3 t}|11\rangle\langle 00| + |11\rangle\langle 11|\right)$ with the probability of $2b^2$.

According to Eqs. (11) and (12), then we can obtain the expression of the fidelity as Eq. (18). It should be mentioned that although both F'_z and $F'_{z,\mathrm{av}}$ do not depend on the EPR-type state shown in Eq. (2), the corresponding successful probability of $2b^2$

is related to the parameter b (or a). Besides, in this paper we only consider the fidelity and the average fidelity in the case of successful teleportation. And if the unsuccessful teleportation is also taken into account, the total average fidelity is surely related to the parameters a and b.

(iii) If the quantum channel is infected by the isotropic noise, i.e., $k_1 = k_2 = k_3 = k/3$. The fidelity of the QT protocol can be calculated as

$$F'_{\mathrm{iso}} = C_2 \left[\frac{1}{4} - \frac{1}{4} \left(1 - 4a^2b^2\right) \left(1 + \cos^2\theta\right) \cdot e^{-\frac{4}{3}kt} + \left(\frac{1}{4}\cos^2\theta + a^2b^2\sin^2\theta \right) \cdot e^{-\frac{8}{3}kt} \right], \tag{20}$$

with $\frac{1}{C_2} = \frac{1}{2} - \frac{1}{2}\left(1 - 4a^2b^2\right)e^{-\frac{4}{3}kt}$. And, the corresponding average fidelity can be derived as

$$F'_{\mathrm{iso,av}} = C_2 \left[\frac{1}{4} - \frac{1}{3} \left(1 - 4a^2b^2\right) e^{-\frac{4}{3}kt} + \left(\frac{1}{12} + \frac{2}{3}a^2b^2 \right) e^{-\frac{8}{3}kt} \right]. \tag{21}$$

According to Eqs. (16, 18), and (20), we plot the fidelity of the teleportation protocol through the noisy channel versus the parameters θ and ϕ for $b = 0.5$ and $kt = 0.5$. From Fig. 2a, we can see that the fidelity F'_x is always larger than 0.5239 and has several peak values. To be specific, when $\theta = \frac{\pi}{2}$ and $\phi = 0, \pi$, or 2π, the fidelity F'_x reaches the maximum 0.9130. Figure 2b displays that the fidelity F'_z under the action of dephasing noise is only the function of θ, and the value of F'_z varies from 0.5677 to 1. As shown in Fig. 2c, the fidelity influenced by the isotropic noise is also only the function of θ and has the value varying from 0.5776 to 0.6134. Through numerical analysis, it is not difficult to find that the fidelity subject to the three types of noises versus θ and ϕ has the similar change tendency for the other values b and kt.

Next we examine the effects of quantum noises on the average fidelity. Let us first consider the case that the EPR state is used as the quantum channel to achieve the teleportation. According to Eqs. (17) and (19), if $a = b = \sqrt{2}/2$, meaning that $|\psi\rangle_{23}$ is an EPR state, the average fidelity under the influence of bit flip noise has the same expression as that under the influence of dephasing noise, namely, $F'_{x,\mathrm{av}} = F'_{z,\mathrm{av}} = \frac{2}{3} + \frac{1}{3}e^{-4kt}$. Furthermore, if the isotropic noise is applied to the EPR state, the average fidelity takes the form of $F'_{\mathrm{iso,av}} = \frac{1}{2} + \frac{1}{2}e^{-8kt/3}$. These results regarding average fidelity are in good agreement with those stated in Ref. [31] accordingly. When the quantum channel is partially entangled, i.e., $b < \sqrt{2}/2$, what behavior will the average fidelity reveal?

Now we analyze the property of the average teleportation fidelity when the EPR-type channel is affected by various noises. Without loss of generality, we plot the average fidelity varying with kt in Fig. 3a for different types of noises when $b = 0.5$. From the figure, it can be obviously seen that different types of the noises have different influence on the average fidelity. For the same kt, the average fidelity subject to the dephasing noise is always larger than that subject to the bit-flip noise and to the isotropic noise. Moreover, the average fidelity infected by the isotropic noise is more

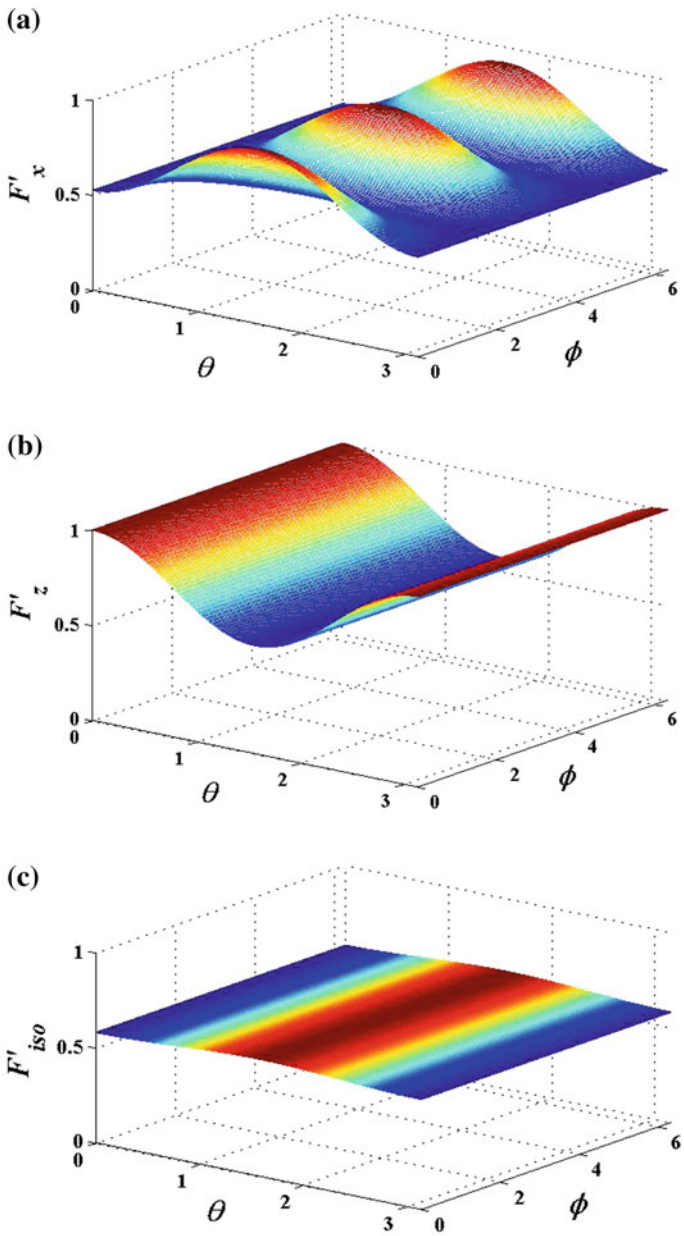
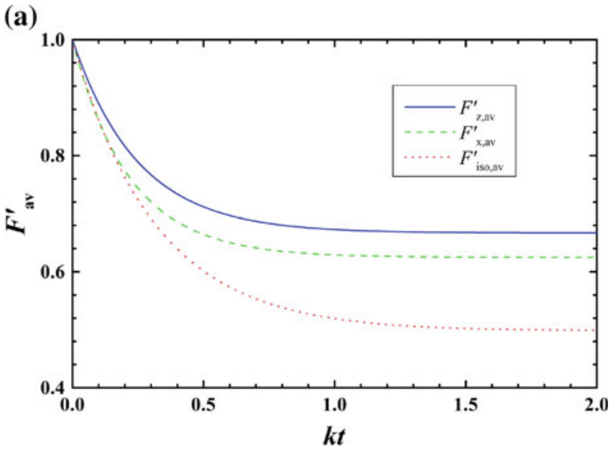


Fig. 2 (color online) The teleportation fidelity under noisy environments is plotted as the function of θ and ϕ for $b = 0.5$ and $kt = 0.5$. Here **a** is subject to the bit flip noise, **b** to the dephasing noise, and **c** to the isotropic noise

than that infected by the bit flip noise for $kt \leq 0.11$, but the former is less than the latter for $kt > 0.11$. If $kt = 0$, *i.e.*, the quantum channel do not begin to interact with the environment, just as we expect, the average fidelities have the identical value



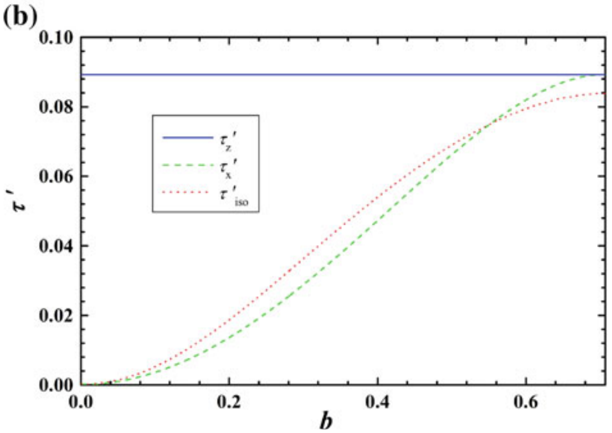


Fig. 3 (color online) **a** The average fidelity of the teleportation is plotted as a function of kt when $b = 0.5$. **b** The decoherence parameter τ' varies with b when the average fidelity decays to 0.9 during the QT process. The *green dashed line* stands for the bit flip noise, the *blue solid line* for the dephasing noise, and the *red dotted line* for the isotropic noise

$F'_{x,\mathrm{av}} = F'_{z,\mathrm{av}} = F'_{\mathrm{iso,av}} = 1$. Whereas for the limit of $kt >> 1$, the average fidelities $F'_{x,av}$, $F'_{z,av}$, and $F'_{\mathrm{iso,av}}$ approach to 0.6250, 0.6667, and 0.5000, respectively.

Figure 3b depicts the decoherence parameter $\tau'(= kt)$ as a function of b when the average fidelity decays to 0.9. We can see from the figure that as b goes from 0 to $\sqrt{2}/2$, τ' is always equal to 0.0892 for the dephasing noise case while τ' increases monotonously for the situation of the bit flip noise and the isotropic noise. Moreover, when $b = 0.5530$, the parameter τ' under the influence of bit flip noise is equal to that under the action of isotropic noise, namely, $\tau'_x = \tau'_{\mathrm{iso}} = 0.0752$. Furthermore, when $b = \sqrt{2}/2$, τ' reaches the maximal values 0.0892 and 0.0838 for the bit flip noise and the isotropic noise, respectively. When b approaches to 0, implying that the entanglement amount of the quantum channel is very small, τ' is also close to 0 for the bit flip noise and the isotropic noise, indicating that there is almost no enough time to keep the coherence of the channel. It should be mentioned that one can obtain the similar result for τ' varying with b if the average fidelity decreases to another value.

Case C: Both the initial state and the quantum channel are subject to noises

Here we suppose that both the initial state and the quantum channel are affected by the noisy environment. For simplicity, the initial state and the quantum channel are assumed to be subject to the noises with the same coupling strength and the same interaction time with the surroundings.

(i) If both the initial state and the quantum channel are subject to the bit flip noise, i.e., $k_2 = k_3 = \lambda_2 = \lambda_3 = 0$ and $k_1 = \lambda_1 = k$, according to Eq. (13), the fidelity can be derived as

$$F''_x = C_3 \left[\frac{1}{4} + a^2 b^2 \sin^2\theta \cos^2\phi - \frac{1}{4}\left(1 - 4a^2b^2\right) e^{-2kt} - \frac{1}{4}\left(1 - 4a^2b^2\right) \cos^2\theta \cdot e^{-4kt} + (a^2 b^2 \sin^2\theta \sin^2\phi + \frac{1}{4}\cos^2\theta) \cdot e^{-6kt} \right], \tag{22}$$

with $\frac{1}{C_3} = \frac{1}{2} - \frac{1}{2}\left(1 - 4a^2b^2\right) e^{-2kt}$, and the corresponding average fidelity reduces to

$$F''_{x,\mathrm{av}} = C_3 \left\{ \frac{1}{4}\left[1 - \left(1 - 4a^2b^2\right) e^{-2kt} - \frac{1}{3}\left(1 - 4a^2b^2\right) e^{-4kt} + \frac{1}{3} e^{-6kt}\right] + \frac{1}{3} a^2 b^2 \left(1 + e^{-6kt}\right) \right\}. \tag{23}$$

If $a = b = \frac{\sqrt{2}}{2}$, then $F''_{x,\mathrm{av}} = \frac{2}{3} + \frac{1}{3} e^{-6kt}$.

(ii) If both the initial state and the quantum channel are subject to the dephasing noise, i.e., $k_1 = k_2 = \lambda_1 = \lambda_2 = 0$ and $k_3 = \lambda_3 = k$. In this case, the fidelity can be expressed as

$$F''_z = \frac{1}{2}\left(1 + \cos^2\theta + \sin^2\theta \cdot e^{-6kt}\right), \tag{24}$$

and the corresponding average fidelity becomes

$$F''_{z,\mathrm{av}} = \frac{2}{3} + \frac{1}{3} e^{-6kt}, \text{ which is the same as } F''_{x,\mathrm{av}} \text{ for } a = b = \frac{\sqrt{2}}{2}. \tag{25}$$

In this case, both the fidelity and the average fidelity do not depend on the parameters a and b of the initial EPR-class state. The reason is the same as that stated in case B (ii).

(iii) If both the initial state and the quantum channel are subject to the isotropic noise, i.e., $k_1 = k_2 = k_3 = \lambda_1 = \lambda_2 = \lambda_3 = k/3$, then the fidelity is given by

$$F''_{\mathrm{iso}} = C_4 \left\{ \frac{1}{4} \left[1 - \left(1 - 4a^2b^2\right) e^{-\frac{4}{3}kt} - \left(1 - 4a^2b^2\right) \cos^2\theta \cdot e^{-\frac{8}{3}kt} \right] + \left(\frac{1}{4}\cos^2\theta + a^2b^2 \sin^2\theta \right) \cdot e^{-4kt} \right\}, \tag{26}$$

with $\frac{1}{C_4} = \frac{1}{2} - \frac{1}{2}\left(1 - 4a^2b^2\right) e^{-\frac{4}{3}kt}$. And the corresponding average fidelity is given by

$$F''_{\mathrm{iso,av}} = C_4 \left\{ \frac{1}{4} \left[1 - \left(1 - 4a^2b^2\right) e^{-\frac{4}{3}kt} - \frac{1}{3}\left(1 - 4a^2b^2\right) e^{-\frac{8}{3}kt} \right] + \left(\frac{1}{12} + \frac{2}{3}a^2b^2 \right) e^{-4kt} \right\}, \tag{27}$$

which reduces to $F''_{\mathrm{iso,av}} = \frac{1}{2} + \frac{1}{2}e^{-4kt}$ for $a = b = \frac{\sqrt{2}}{2}$.

Based on Eqs. (23), (25), and (27), the average fidelity of the teleportation versus kt is depicted in Fig. 4a for $b = 0.5$. Comparing Figs. 4a with 3a, we find that under the influence of the same noises, the average fidelities of case B and case C exhibit the similar evolution behavior except that the average fidelity in Fig. 4a decreases more rapidly than that in Fig. 3a. It can be seen from Fig. 4a that when $kt = 0$, all the average fidelities $F''_{x,\mathrm{av}}$, $F''_{z,\mathrm{av}}$, and $F''_{\mathrm{iso,av}}$ equal 1. Also, for the limit of $kt >> 1$, the average fidelities $F''_{x,\mathrm{av}}$, $F''_{z,\mathrm{av}}$, and $F''_{\mathrm{iso,av}}$ decay to 0.6250, 0.6667, and 0.5000, respectively. In Fig. 4b, we plot the decoherence parameter τ'' varying with b in the case that the average fidelity decreases to 0.9. From the figure, it can be observed that τ'' keeps the fixed value of 0.0596 for the dephasing noise case, while τ'' increases as b goes from 0 to $\sqrt{2}/2$ for both the bit flip noise and the isotropic noise cases. When $b = 0.5202$, the decoherence parameter τ'' in the presence of bit-flip noise is equal to that in the presence of isotropic noise, i.e., $\tau''_x = \tau''_{\mathrm{iso}} = 0.05$. Moreover, when $b = \sqrt{2}/2$, meaning that the EPR-type channel is maximally entangled, τ'' approaches to 0.0596 and 0.0558 for the bit flip noise and the isotropic noise, respectively. Furthermore, if b is close to 0, namely, the EPR-type channel is almost in a separate state, the decoherence parameter τ'' approaches to 0 too. In addition, if the average fidelity decays to the other values, the similar results for τ'' varying with b can be obtained through numerical calculations.

In the above, we have analyzed the average fidelity versus b or kt under various Pauli noisy conditions. In what follows, we further make the comparison of the average fidelity with the above-mentioned three noisy cases taken into account. For the case A that only the initial state $|\psi\rangle_1$ is subject to the noises, with the same kt the average fidelity infected by the isotropic noise is always less than that infected by the bit flip noise and by the dephasing noise, whereas the average fidelities infected by the bit flip noise and by the dephasing noise display the same evolution behavior. For the cases B and C ($b < \sqrt{2}/2$), with the same kt the value of the average fidelity subject to the

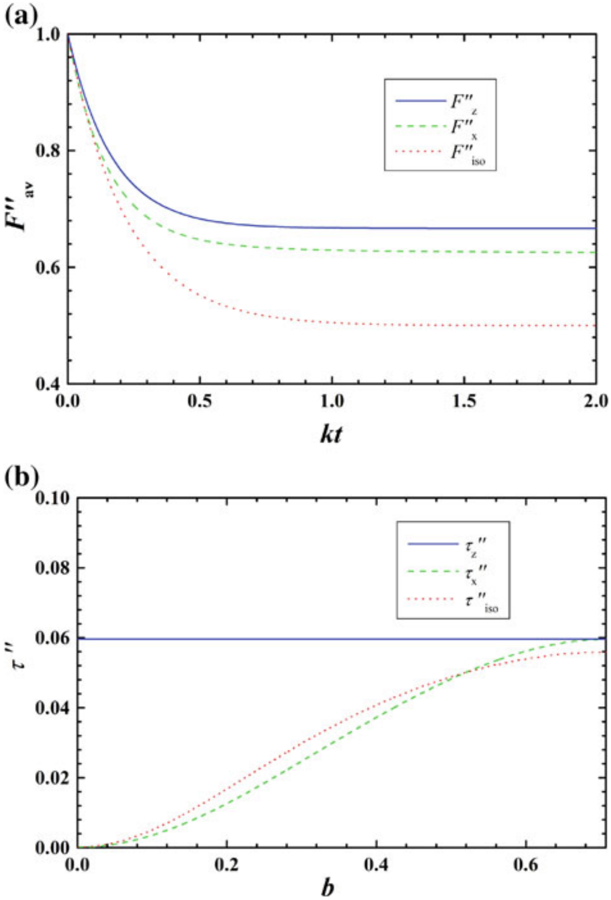


Fig. 4 (color online) **a** The average fidelity of the teleportation is plotted as a function of kt when $b = 0.5$. **b** The decoherence parameter τ'' varies with b when the average fidelity decays to 0.9. The *green dashed line* stands for the bit flip noise, the *blue solid line* for the dephasing noise, and the *red dotted line* for the isotropic noise

dephasing noise is always the largest among the average fidelities subject to the three types of noises, implying that the dephasing noise effect on the teleportation is the weakest. Moreover, with the increase of b, the decoherence parameter in the presence of the isotropic noise is firstly more than that in the presence of the bit flip noise, then the situation becomes completely reversed after a critical value of b. However, when $b = \sqrt{2}/2$, the average fidelity affected by the bit flip noise is the same as that by the dephasing noise.

As we know, the larger the decoherence parameter τ is, the more slowly the velocity of the fidelity decays. As illustrated in Figs. 3b and 4b, τ increases with the increasing b for the cases of bit flip noise and isotropic noise while τ remains constant under the influence of the dephasing noise. Thus the EPR-type channel with the more entanglement amount has a relatively stronger ability to resist the influence of noises.

On the contrary, the EPR-type channel with the less entanglement amount leads to lower fidelity and smaller success probability. Therefore, to some extent, the larger entanglement amount of quantum channel has a better ability to resist the influence of the Pauli noises. In addition, it is not difficult to observe from cases A, B and C that the average fidelity for both the initial state and the quantum channel suffering from the noises decays more rapidly than that for either the initial state or the quantum channel subject to the same noises.

3 Conclusions

In summary, we have investigated the teleportation of a qubit state via a bipartite EPR-type state as quantum channel in the presence of noisy environments by solving the master equation in the Lindblad form. It is found that the fidelity of the teleportation depends on the type of noise, the decoherence rate, and the initial state to be teleported. Moreover, the influence of the dephasing noise on the average teleportation fidelity is the weakest among the influence of the three studied Pauli noises. Furthermore, as b increases, the decoherence parameter under the action of the isotropic noise is first larger than the situation under the bit flip noise, and then there are going in the opposite direction after a critical b. It is also found that the EPR-type channel enables the average fidelity to decay versus the decoherence parameter kt with different velocities for the case of different Pauli noises. Besides, the maximally entangled EPR channel is more robust than the partially entangled EPR-type state against decoherence.

In practical quantum communication, the influence of quantum noises is inevitable. What should we do for the actual teleportation situation exposed to noisy environments? To realize effective teleportation process, we had better choose the initial entangled channel with large entanglement amount, especially with the maximally entangled states. As for the initial channel with small entanglement degree, it can be distilled into the entangled channel possessing the larger and even maximal entanglement amount through the procedure of entanglement purification [36–38].

It deserves mentioning that in contrast to the QT protocols with the maximally entangled states via noisy channels in Refs. [30–32], our paper mainly focuses on the influence of the partially entangled resources on the QT in noisy environments. Moreover, our present work concerning the Pauli noises is different from that of Ref. [34] in which the effects of amplitude damping on every step of teleportation were analyzed. On the other hand, in QT, an unknown quantum state can be transmitted from the sender Alice to the receiver Bob with the assistance of the shared entangled channel and classical information. During the quantum information transmission of QT, the particle carrying the unknown quantum state, however, is not sent from the sending location to the receiving one. So the novel idea of QT is remarkably different from quantum cryptography [39] in which quantum information is transferred by physically sending the informative qubits. Since there are no carriers to send the information, the security of QT is mainly based on the safe establishing of the initial entangled channel between the sender and the receiver. Generally, the secure and faithful establishment of the entangled channel can be verified essentially with the following procedures. First, a great number of copies of the entangled channel are distributed between Alice and

Bob. Second, they perform the orthogonal measurement on their own particle with the priorly promissory measurement bases simultaneously. Last but not least, Alice and Bob can determine whether there exists the behavior of eavesdropping and/or cheating in the entanglement distribution process by publicly checking the measurement results through classical channel. Therefore, the entanglement-based teleportation has been extensively investigated both theoretically and experimentally [20–24, 26–36, 40–43]. In this sense, we believe that our scheme is quite secure.

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