

Statistical Inference Notes

Tazmilur Saad

February 9, 2021

My notes based entirely on the book *Statistical Inference* written by George Casella and Roger L. Berger.

Contents

1	Probability Theory	5
1.1	Probability Theory	5
1.2	Random Variables	5

Chapter 1

Probability Theory

1.1 Probability Theory

Theorem 1.1.1. *Given a sample space \mathcal{S} and an associated σ -algebra \mathcal{B} , a probability function is a function P with domain \mathcal{B} that satisfies*

1. $P(A) \geq 0$ for all $A \in \mathcal{B}$
2. $P(\mathcal{S}) = 1$
3. if $A_1, A_2, \dots \in \mathcal{B}$ are pairwise disjoint, then $P(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$.

1.2 Random Variables

Definition 1.2.1 (Random Variable). A **random variable** is a function from a sample space \mathcal{S} into the real numbers.

Assume we have a sample space $\mathcal{S} = \{s_1, \dots, s_n\}$ with a probability function P . We can define a random variable X with range $\mathcal{X} = \{x_1, \dots, x_m\}$. We can define a probability function P_x on \mathcal{X} in the following way: $X = x_i$ if and only if the outcome of the random experiment is an $s_j \in \mathcal{S}$ such that $X(s_j) = x_i$.

$$P_X(X = x_i) = P(\{s_j \in \mathcal{S} : X(s_j) = x_i\})$$

P_X is the induced probability function on \mathcal{X} , defined in terms of the original function P . We can show that the induced probability function satisfies the Kolmogorov axioms.

Proof. Asdas

□