0.1 Probability Theory

Theorem 0.1.1. *Given a sample space $\mathcal{S}$ and an associated σ-algebra $\mathcal{B}$, a probability function is a function P with domain $\mathcal{B}$ that satisfies*

1. $P(A) \geq 0$ *for all* $A \in \mathcal{B}$
2. $P(\mathcal{S}) = 1$
3. *if* $A_1, A_2, \dots \in \mathcal{B}$ *are pairwise disjoint, then* $P(\bigcup_{i=1}^{\infty} A_i) = \Sigma_{i=1}^{\infty} P(A_i)$.

0.2 Random Variables

Definition 0.2.1 (Random Variable)**.** *A* ***random variable*** *is a function from a sample space $\mathcal{S}$ into the real numbers.*

Assume we have a sample space $S = \{s_1, \dots, s_n\}$ with a probability function P. We can define a random variable X with range $\mathcal{X} = \{x_1, \dots, x_m\}$. We can define a probability function P_x on $\mathcal{X}$ in the following way: $X = x_i$ if and only if the outcome of the random experiment is an $s_j \in \mathcal{S}$ such that $X(s_j) = x_i$.

$$P_X(X = x_i) = P(\{s_j \in \mathcal{S} : X(s_j) = x_i\})$$

P_X is the induced probability function on $\mathcal{X}$, defined in terms of the original function P. We can show that the induced probability function satisfies the Kolmogorov axioms.

Proof. Wtf $\square$