Documentation of Decomposition and Closure Rules

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1 Decomposition Rules

In this section, γ is an expectation prefix, possibly empty. φ, φ_1 and φ_2 are formulae of propositional partial information ionic logic and $\Vdash$ stands for potential truth turnstile. Rules for formulae with the soft turnstile is similar.

1.1 Negation

1.1.1 $-$Connective

$\gamma \models \neg\varphi$ $\mid$ $\gamma \nVdash \varphi$	$\gamma \not\models \neg\varphi$ $\mid$ $\gamma \Vdash \varphi$	$\gamma \Vdash \neg\varphi$ $\mid$ $\gamma \not\models \varphi$	$\gamma \nVdash \neg\varphi$ $\mid$ $\gamma \models \varphi$

1.1.2 $\sim$ Connective

$\gamma \models \sim \varphi$ $\mid$ $\gamma \not\models \varphi$	$\gamma \not\models \sim \varphi$ $\mid$ $\gamma \models \varphi$	$\gamma \Vdash \sim \varphi$ $\mid$ $\gamma \not\models \varphi$	$\gamma \nVdash \sim \varphi$ $\mid$ $\gamma \models \varphi$

1.1.3 $\sim'$Connective

$\gamma \models \sim' \varphi$ $\mid$ $\gamma \nVdash \varphi$	$\gamma \not\models \sim' \varphi$ $\mid$ $\gamma \Vdash \varphi$	$\gamma \Vdash \sim' \varphi$ $\mid$ $\gamma \nVdash \varphi$	$\gamma \nVdash \sim' \varphi$ $\mid$ $\gamma \Vdash \varphi$

1.2 Bottom Function

<table>
<tr><td rowspan="2">$\gamma \models bot(\varphi)$
$|$
$\gamma \Vdash \varphi$
$\gamma \not\models \varphi$</td><td colspan="2">$\gamma \not\models bot(\varphi)$</td></tr>
<tr><td>$\gamma \models \varphi$</td><td>$\gamma \nVdash \varphi$</td></tr>
</table>

1.3 Conjunction

<table>
<tr><td rowspan="2">$\gamma \models \varphi_1 \wedge \varphi_2$
$|$
$\gamma \models \varphi_1$
$\gamma \models \varphi_2$</td><td colspan="2">$\gamma \not\models \varphi_1 \wedge \varphi_2$</td><td rowspan="2">$\gamma \Vdash \varphi_1 \wedge \varphi_2$
$|$
$\gamma \Vdash \varphi_1$
$\gamma \Vdash \varphi_2$</td><td colspan="2">$\gamma \nVdash \varphi_1 \wedge \varphi_2$</td></tr>
<tr><td>$\gamma \not\models \varphi_1$</td><td>$\gamma \not\models \varphi_2$</td><td>$\gamma \nVdash \varphi_1$</td><td>$\gamma \nVdash \varphi_2$</td></tr>
</table>

1.4 Disjunction

<table>
<tr><td colspan="2">$\gamma \models \varphi_1 \vee \varphi_2$</td><td rowspan="2">$\gamma \not\models \varphi_1 \vee \varphi_2$
$|$
$\gamma \not\models \varphi_1$
$\gamma \not\models \varphi_2$</td><td colspan="2">$\gamma \Vdash \varphi_1 \vee \varphi_2$</td><td rowspan="2">$\gamma \nVdash \varphi_1 \vee \varphi_2$
$|$
$\gamma \nVdash \varphi_1$
$\gamma \nVdash \varphi_2$</td></tr>
<tr><td>$\gamma \models \varphi_1$</td><td>$\gamma \models \varphi_2$</td><td>$\gamma \Vdash \varphi_1$</td><td>$\gamma \Vdash \varphi_2$</td></tr>
</table>

1.5 Implication

$\gamma \models \varphi_1 \to \varphi_2$	$\gamma \not\models \varphi_1 \to \varphi_2$	$\gamma \Vdash \varphi_1 \to \varphi_2$	$\gamma \nVdash \varphi_1 \to \varphi_2$
	$\vert$		$\vert$
$\gamma \nVdash \varphi_1$ $\gamma \models \varphi_2$	$\gamma \Vdash \varphi_1$	$\gamma \not\models \varphi_1$ $\gamma \Vdash \varphi_2$	$\gamma \models \varphi_1$
	$\gamma \not\models \varphi_2$		$\gamma \nVdash \varphi_2$

1.6 Interjunction

$\gamma \models \varphi_1 \sqcap \varphi_2$	$\gamma \not\models \varphi_1 \sqcap \varphi_2$	$\gamma \Vdash \varphi_1 \sqcap \varphi_2$	$\gamma \nVdash \varphi_1 \sqcap \varphi_2$
$\vert$			$\vert$
$\gamma \models \varphi_1$	$\gamma \not\models \varphi_1$ $\gamma \not\models \varphi_2$	$\gamma \Vdash \varphi_1$ $\gamma \Vdash \varphi_2$	$\gamma \nVdash \varphi_1$
$\gamma \models \varphi_2$			$\gamma \nVdash \varphi_2$

1.7 Conditional Ion

1.7.1 Generic Operator

$\gamma \models *(\varphi_1, \varphi_2)$	$\gamma \not\models *(\varphi_1, \varphi_2)$ $\vert$ $\overline{-*}\varphi_1$
$\gamma + *\varphi_1$ $\gamma - *\varphi_1$ $\gamma \models_{soft} \varphi_2$	$\gamma\overline{+*}\varphi_1$ $\not\models_{soft} \varphi_2$ $\gamma \models_{soft} \varphi_2$
$\gamma \Vdash *(\varphi_1, \varphi_2)$	$\gamma \nVdash *(\varphi_1, \varphi_2)$ $\vert$
$\gamma\overline{+*}\varphi_1$ $\gamma \Vdash_{soft} \varphi_2$ $\gamma \nVdash_{soft} \varphi_2$	$\gamma + *\varphi_1$ $\gamma \nVdash_{soft} \varphi_2$

1.7.2 Diamondsuit Operator

	$\gamma \nvDash \diamondsuit(\varphi_1, \varphi_2)$
$\gamma \vDash \diamondsuit(\varphi_1, \varphi_2)$	$\mid$
$\gamma \Vdash \varphi_1 \forall$ $\quad$ $\gamma \nVDash \varphi_1 \forall$	$\gamma \Vdash \varphi_1 \exists$
$\gamma \vDash_{soft} \varphi_2$	$\gamma \nVDash \varphi_1 \exists$ $\quad$ $\nvDash_{soft} \varphi_2$
	$\gamma \vDash_{soft} \varphi_2$

	$\gamma \nVDash \diamondsuit(\varphi_1, \varphi_2)$
$\gamma \Vdash \diamondsuit(\varphi_1, \varphi_2)$	$\mid$
$\gamma \nVDash \varphi_1 \exists$ $\quad$ $\gamma \Vdash_{soft} \varphi_2$	$\gamma \Vdash \varphi_1 \forall$
$\gamma \nVDash_{soft} \varphi_2$	$\gamma \nVDash_{soft} \varphi_2$

1.7.3 Heartsuit Operator

	$\gamma \nvDash \heartsuit(\varphi_1, \varphi_2)$
$\gamma \vDash \heartsuit(\varphi_1, \varphi_2)$	$\mid$
$\gamma \vDash \varphi_1 \forall$ $\quad$ $\gamma \nvDash \varphi_1 \forall$	$\gamma \vDash \varphi_1 \exists$
$\gamma \vDash_{soft} \varphi_2$	$\gamma \nvDash \varphi_1 \exists$ $\quad$ $\nvDash_{soft} \varphi_2$
	$\gamma \vDash_{soft} \varphi_2$

	$\gamma \nVDash \heartsuit(\varphi_1, \varphi_2)$
$\gamma \Vdash \heartsuit(\varphi_1, \varphi_2)$	$\mid$
$\gamma \nvDash \varphi_1 \exists$ $\quad$ $\gamma \Vdash_{soft} \varphi_2$	$\gamma \vDash \varphi_1 \forall$
$\gamma \nVDash_{soft} \varphi_2$	$\gamma \nVDash_{soft} \varphi_2$

1.7.4 Circle Operator

$\gamma \models \bigcirc(\varphi_1, \varphi_2)$ $\gamma \models \varphi_1 \forall \qquad \gamma \nVDash \varphi_1 \forall$ $\gamma \models_{soft} \varphi_2$	$\gamma \not\models \bigcirc(\varphi_1, \varphi_2)$ $\vert$ $\gamma \Vdash \varphi_1 \exists$ $\gamma \not\models \varphi_1 \exists \qquad \not\models_{soft} \varphi_2$ $\gamma \models_{soft} \varphi_2$
$\gamma \Vdash \bigcirc(\varphi_1, \varphi_2)$ $\gamma \not\models \varphi_1 \exists \qquad \gamma \Vdash_{soft} \varphi_2$ $\gamma \nVDash_{soft} \varphi_2$	$\gamma \nVDash \bigcirc(\varphi_1, \varphi_2)$ $\vert$ $\gamma \models \varphi_1 \forall$ $\gamma \nVDash_{soft} \varphi_2$

1.7.5 Spadesuit Operator

$\gamma \models \spadesuit(\varphi_1, \varphi_2)$ $\gamma \Vdash \varphi_1 \exists \qquad \gamma \nVDash \varphi_1 \forall$ $\gamma \models_{soft} \varphi_2$	$\gamma \not\models \spadesuit(\varphi_1, \varphi_2)$ $\vert$ $\gamma \Vdash \varphi_1 \exists$ $\gamma \not\models_{soft} \varphi_2$
$\gamma \Vdash \spadesuit(\varphi_1, \varphi_2)$ $\gamma \nVDash \varphi_1 \forall \qquad \gamma \Vdash_{soft} \varphi_2$ $\gamma \nVDash_{soft} \varphi_2$	$\gamma \nVDash \spadesuit(\varphi_1, \varphi_2)$ $\vert$ $\gamma \Vdash \varphi_1 \exists$ $\gamma \nVDash_{soft} \varphi_2$

1.7.6 Clubsuit Operator

$\gamma \models \clubsuit(\varphi_1, \varphi_2)$ $\gamma \models \varphi_1 \exists$ $\quad$ $\gamma \not\models \varphi_1 \forall$ $\gamma \models_{soft} \varphi_2$	$\gamma \not\models \clubsuit(\varphi_1, \varphi_2)$ $\mid$ $\gamma \models \varphi_1 \exists$ $\gamma \not\models_{soft} \varphi_2$
$\gamma \Vdash \clubsuit(\varphi_1, \varphi_2)$ $\gamma \not\models \varphi_1 \forall$ $\quad$ $\gamma \Vdash_{soft} \varphi_2$ $\gamma \nVdash_{soft} \varphi_2$	$\gamma \nVdash \clubsuit(\varphi_1, \varphi_2)$ $\mid$ $\gamma \models \varphi_1 \exists$ $\gamma \nVdash_{soft} \varphi_2$

1.7.7 Blackfly Operator

$\gamma \models \bullet(\varphi_1, \varphi_2)$ $\gamma \models \varphi_1 \exists$ $\quad$ $\gamma \nVdash \varphi_1 \forall$ $\gamma \models_{soft} \varphi_2$	$\gamma \not\models \bullet(\varphi_1, \varphi_2)$ $\mid$ $\gamma \Vdash \varphi_1 \exists$ $\gamma \not\models \varphi_1 \forall$ $\quad$ $\not\models_{soft} \varphi_2$ $\gamma \models_{soft} \varphi_2$
$\gamma \Vdash \bullet(\varphi_1, \varphi_2)$ $\gamma \not\models \varphi_1 \forall$ $\quad$ $\gamma \Vdash_{soft} \varphi_2$ $\gamma \nVdash_{soft} \varphi_2$	$\gamma \nVdash \bullet(\varphi_1, \varphi_2)$ $\mid$ $\gamma \models \varphi_1 \exists$ $\gamma \nVdash_{soft} \varphi_2$

1.7.8 Spadesuit-twin Operator

$\gamma \vDash \triangle(\varphi_1, \varphi_2)$	$\gamma \nvDash \triangle(\varphi_1, \varphi_2)$
$\gamma \Vdash \varphi_1\forall$ $\gamma \nVDash \varphi_1\exists$	$\vert$
$\gamma \vDash_{soft} \varphi_2$	$\gamma \Vdash \varphi_1\forall$
	$\gamma \nvDash_{soft} \varphi_2$

$\gamma \Vdash \triangle(\varphi_1, \varphi_2)$	$\gamma \nVDash \triangle(\varphi_1, \varphi_2)$
$\gamma \nVDash \varphi_1\exists$ $\gamma \Vdash_{soft} \varphi_2$	$\vert$
$\gamma \nVDash_{soft} \varphi_2$	$\gamma \Vdash \varphi_1\forall$
	$\gamma \nVDash_{soft} \varphi_2$

1.7.9 Clubsuit-twin Operator

$\gamma \vDash \nabla(\varphi_1, \varphi_2)$	$\gamma \nvDash \nabla(\varphi_1, \varphi_2)$
$\gamma \vDash \varphi_1\forall$ $\gamma \nvDash \varphi_1\exists$	$\vert$
$\gamma \vDash_{soft} \varphi_2$	$\gamma \vDash \varphi_1\forall$
	$\gamma \nvDash_{soft} \varphi_2$

$\gamma \Vdash \nabla(\varphi_1, \varphi_2)$	$\gamma \nVDash \nabla(\varphi_1, \varphi_2)$
$\gamma \nvDash \varphi_1\exists$ $\gamma \Vdash_{soft} \varphi_2$	$\vert$
$\gamma \nVDash_{soft} \varphi_2$	$\gamma \vDash \varphi_1\forall$
	$\gamma \nVDash_{soft} \varphi_2$

1.7.10 Butterfly Operator

$\gamma \models_{\bowtie} (\varphi_1, \varphi_2)$	$\gamma \nvDash_{\bowtie} (\varphi_1, \varphi_2)$ | $\gamma \Vdash \varphi_1 \forall$
$\gamma \models \varphi_1 \forall$ $\gamma \nVDash \varphi_1 \exists$ $\gamma \models_{soft} \varphi_2$	$\gamma \nvDash \varphi_1 \exists$ $\nvDash_{soft} \varphi_2$ $\gamma \models_{soft} \varphi_2$

$\gamma \Vdash_{\bowtie} (\varphi_1, \varphi_2)$	$\gamma \nVDash_{\bowtie} (\varphi_1, \varphi_2)$ | $\gamma \models \varphi_1 \forall$ $\gamma \nVDash_{soft} \varphi_2$
$\gamma \nvDash \varphi_1 \exists$ $\gamma \Vdash_{soft} \varphi_2$ $\gamma \nVDash_{soft} \varphi_2$	

1.8 No Good Formula

1.8.1 Universal Ions

$\gamma \models$ $\Diamond(\varphi, False)$ | $\gamma \nVDash \varphi \forall$	$\gamma \nvDash$ $\Diamond(\varphi, False)$ | $\gamma \Vdash \varphi \exists$	$\gamma \Vdash$ $\Diamond(\varphi, False)$ | $\gamma \nVDash \varphi \exists$	$\gamma \nVDash$ $\Diamond(\varphi, False)$ | $\gamma \Vdash \varphi \forall$

$\gamma \models$ $\heartsuit(\varphi, False)$ | $\gamma \nvDash \varphi \forall$	$\gamma \nvDash$ $\heartsuit(\varphi, False)$ | $\gamma \models \varphi \exists$	$\gamma \Vdash$ $\heartsuit(\varphi, False)$ | $\gamma \nvDash \varphi \exists$	$\gamma \nVDash$ $\heartsuit(\varphi, False)$ | $\gamma \models \varphi \forall$

$\gamma \models$ $\bigcirc(\varphi, False)$ | $\gamma \nVDash \varphi \forall$	$\gamma \nvDash$ $\bigcirc(\varphi, False)$ | $\gamma \Vdash \varphi \exists$	$\gamma \Vdash$ $\bigcirc(\varphi, False)$ | $\gamma \nvDash \varphi \exists$	$\gamma \nVDash$ $\bigcirc(\varphi, False)$ | $\gamma \models \varphi \forall$

1.8.2 Universal-existential Ions

$\gamma \models$ $\triangle(\varphi, False)$ $\mid$ $\gamma \nVdash \varphi\exists$	$\gamma \not\models$ $\triangle(\varphi, False)$ $\mid$ $\gamma \Vdash \varphi\forall$	$\gamma \Vdash$ $\triangle(\varphi, False)$ $\mid$ $\gamma \nVdash \varphi\exists$	$\gamma \nVdash$ $\triangle(\varphi, False)$ $\mid$ $\gamma \Vdash \varphi\forall$
$\gamma \models$ $\nabla(\varphi, False)$ $\mid$ $\gamma \not\models \varphi\exists$	$\gamma \not\models$ $\nabla(\varphi, False)$ $\mid$ $\gamma \models \varphi\forall$	$\gamma \Vdash$ $\nabla(\varphi, False)$ $\mid$ $\gamma \not\models \varphi\exists$	$\gamma \nVdash$ $\nabla(\varphi, False)$ $\mid$ $\gamma \models \varphi\forall$
$\gamma \models\bowtie$ $(\varphi, False)$ $\mid$ $\gamma \nVdash \varphi\exists$	$\gamma \not\models\bowtie$ $(\varphi, False)$ $\mid$ $\gamma \Vdash \varphi\forall$	$\gamma \Vdash\bowtie$ $(\varphi, False)$ $\mid$ $\gamma \not\models \varphi\exists$	$\gamma \nVdash\bowtie$ $(\varphi, False)$ $\mid$ $\gamma \models \varphi\forall$

1.8.3 Existential-universal Ions

$\gamma \models$ $\spadesuit(\varphi, False)$ $\mid$ $\gamma \nVdash \varphi\forall$	$\gamma \not\models$ $\spadesuit(\varphi, False)$ $\mid$ $\gamma \Vdash \varphi\exists$	$\gamma \Vdash$ $\spadesuit(\varphi, False)$ $\mid$ $\gamma \nVdash \varphi\forall$	$\gamma \nVdash$ $\spadesuit(\varphi, False)$ $\mid$ $\gamma \Vdash \varphi\exists$
$\gamma \models$ $\clubsuit(\varphi, False)$ $\mid$ $\gamma \not\models \varphi\forall$	$\gamma \not\models$ $\clubsuit(\varphi, False)$ $\mid$ $\gamma \models \varphi\exists$	$\gamma \Vdash$ $\clubsuit(\varphi, False)$ $\mid$ $\gamma \not\models \varphi\forall$	$\gamma \nVdash$ $\clubsuit(\varphi, False)$ $\mid$ $\gamma \models \varphi\exists$
$\gamma \models \bullet(\varphi, False)$ $\mid$ $\gamma \nVdash \varphi\forall$	$\gamma \not\models \bullet(\varphi, False)$ $\mid$ $\gamma \Vdash \varphi\exists$	$\gamma \Vdash \bullet(\varphi, False)$ $\mid$ $\gamma \not\models \varphi\forall$	$\gamma \nVdash \bullet(\varphi, False)$ $\mid$ $\gamma \models \varphi\exists$

1.9 Canonical Justification

$\gamma\tau\varphi\exists$ $\mid$ $j\gamma\tau\varphi$	$\gamma\tau\varphi\forall$ $\mid$ $J\gamma\tau\varphi$

where j is the first interpretation symbol that has not been used so far in the tableau and J is an interpretation variable that can bind to every interpretation symbol that has occured at the same rank in the current branch of the tableau.

2 Closure Rules

In this section, γ, γ_1 and γ_2 are expectation prefixes and p is a propositional variable.

2.1 Generalized from classical Beth method

$\gamma \models p$ $\gamma \not\models p$	$\gamma \Vdash p$ $\gamma \nVdash p$	$\gamma \models p$ $\gamma \nVdash p$
$\gamma \models_{soft} p$ $\gamma \not\models_{soft} p$	$\gamma \Vdash_{soft} p$ $\gamma \nVdash_{soft} p$	$\gamma \models_{soft} p$ $\gamma \nVdash_{soft} p$

2.2 Justification knowledge extends Kernel knowledge

$\gamma_1 \models p$ $\gamma_2\gamma_1 \nVdash p$	$\gamma_1 \models p$ $\gamma_2\gamma_1 \not\models p$	$\gamma_1 \nVdash p$ $\gamma_2\gamma_1 \Vdash p$	$\gamma_1 \nVdash p$ $\gamma_2\gamma_1 \models p$
$\models \varphi$ $- * \varphi$	$\nVdash \varphi$ $+ * \varphi$	$\models \varphi$ $\overline{+*}\varphi$	$\nVdash \varphi$ $\overline{-*}\varphi$

where $\varphi \in \mathcal{F}_{\mathcal{O}}$and $*$is the generic ionic operator.

2.3 Soft knowledge extends Kernel knowledge

$\gamma_1 \models p$ $\gamma_2\gamma_1 \nVdash_{soft} p$	$\gamma_1 \models p$ $\gamma_2\gamma_1 \not\models_{soft} p$	$\gamma_1 \nVdash p$ $\gamma_2\gamma_1 \Vdash_{soft} p$	$\gamma_1 \nVdash p$ $\gamma_2\gamma_1 \models_{soft} p$

2.4 Justification knowledge closure properties

$+ * a$	$+ * (a \wedge b)$	$- * (a \to b)$	$\overline{+*}(a \to b)$	$- * (a \to b)$
$- * a$	$- * b$	$\overline{-*}b$	$+ * b$	$+ * b$

where a, b are propositional formulae and * is the generic ionic operator.

2.5 Trivial Cases

$\nVdash bot(\varphi)$	$\nvDash True$	$\nVdash True$	$\vDash False$	$\Vdash False$