Chem 30324, Spring 2025, Homework 2

Due: January 31, 2025

Problem 1. Gases on a table top

In class we derived the properties of a 3-dimensional gas from the Boltzmann distribution and three postulates, and you studied a 1-dimensional gas in Homework 1. Suppose you were interested instead in a 2-dimensional gas, for example gas molecules able to freely skate around on a surface but that couldn't escape the surface.

1. Derive the Maxwell-Boltzman speed distribution for a 2-dimensional gas. (*Hint*: Think polar coordinates.)

2. Plot this 2-dimensional speed distribution for O_2 molecules at 200, 400 and 600 K.

3. Calculate the mean (expected value) of the speed of a 2-dimensional gas of molecules. How does your answer compare to a 3-dimensional gas?

4. Calculate the mean kinetic energy of a 2-dimensional gas. How does your answer compare to a one and 3-dimensional gas?

Problem 2. (Kinetics and Transport)

In junior Chemical Engineering laboratory, you might study the diffusion and solubility of CO_2 in an organic solvent using a diffusion cell like the one sketched here. No description has been provided for this imageLet's suppose that the gas space at the top of the diffusion cell is approximately 2 cm high and 4 cm in diameter. Further suppose that the gas in the head of the cell is pure CO_2 at 298 K and 1 bar pressure. Note that CO_2 has a collision diameter d of 0.40 nm.

5. What does gas kinetic theory predict for the gas self-diffusion constant D_{11} of CO_2 gas in the cell, in cm^{2}s^{-1}?

6. Use the Stokes-Einstein relationship to estimate the diffusion constant of CO$_2$ in the organic solvent (take it to be Stoddard solvent, a relatively high molecular weight petroleum distillate). How does this compare with the diffusion constant in the gas phase? Why?

7. We found in class that the probability for a molecule to diffuse a distance x in time t is Gaussian with mean 0 and standard deviation $\sigma = \sqrt{2D_{11}t}$. About how long will it take for 1/3 of the molecules starting at the center of the gas space to diffuse all the way to the surface of the liquid, i.e., > 1 cm?

8. How many collisions on average does one of these diffusing CO$_2$ molecule make with other gas molecules on the way from the middle of the vapor space to the surface of the solvent?

9. How far in total distance does this typical CO$_2$ molecule travel in the time it takes to reach the solvent surface from the middle of the cell?

10. How many CO$_2$ molecules impinge on the surface of the Stoddard solvent in one second?

11. Using your estimate of the diffusion constant in the solvent, how long does it take the same fraction CO$_2$ molecules to diffuse a similar distance in 1-dimension in the liquid phase?

12. Is it safe to assume that any CO$_2$ that travels from the surface into the bulk of the solvent is rapidly replaced from the gas phase?

13. Suppose the volume of the gas-handling manifold is 100 cm^3 and is pressurized with CO$_2$ in the morning to 1.1 atm. What will the pressure in the manifold be four hours later, when lab starts, if the manifold has a pinhole of 1 µm^2?