DER Models

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1 Without Storage

1.1 Disaggregation

$$
\begin{aligned}
\text{max} \quad & \sum_{t \in T} \left(P_t^{DA} x_{it} + \mathbb{E}\left[P_t^{RT}(\xi) y_{it}^{+}(\xi) - P_t^{PN} y_{it}^{-}(\xi) \right] \right) && \text{(1a)} \\
\text{s.t.} \quad & R_{it}(\xi) - x_{it} = y_{it}^{+}(\xi) - y_{it}^{-}(\xi) \quad \forall t \in T && \text{(1b)} \\
& R_{it}(\xi) \geqslant y_{it}^{+}(\xi) \quad \forall t \in T && \text{(1c)} \\
& y_{it}^{+}(\xi) \leqslant M z_{it}(\xi), \quad y_{it}^{-}(\xi) \leqslant M(1 - z_{it}(\xi)) \quad \forall t \in T && \text{(1d)} \\
& x_{it}^{DA} \geqslant 0, y_{it}^{+}(\xi) \geqslant 0, y_{it}^{-}(\xi) \geqslant 0, z_{it}(\xi) \in \{0,1\} \quad \forall t \in T && \text{(1e)}
\end{aligned}
$$

1.2 Aggregation

$$
\begin{aligned}
\text{max} \quad & \sum_{t \in T} \left(P_t^{DA} \alpha_t + \mathbb{E}\left[P_t^{RT}(\xi) \beta_t^{+}(\xi) - P_t^{PN} \beta_t^{-}(\xi) \right] \right) && \text{(2a)} \\
\text{s.t.} \quad & \sum_{i \in I} R_{it}(\xi) - \alpha_t = \beta_t^{+}(\xi) - \beta_t^{-}(\xi) \quad \forall t \in T && \text{(2b)} \\
& \sum_{i \in I} R_{it}(\xi) \geqslant \beta_t^{+}(\xi) \quad \forall t \in T && \text{(2c)} \\
& \beta_t^{+}(\xi) \leqslant M z_t(\xi), \quad \beta_t^{-}(\xi) \leqslant M(1 - z_t(\xi)) \quad \forall t \in T && \text{(2d)} \\
& \alpha_t^{DA} \geqslant 0, \beta_t^{+}(\xi) \geqslant 0, \beta_t^{-}(\xi) \geqslant 0, z_t(\xi) \in \{0,1\} \quad \forall t \in T && \text{(2e)}
\end{aligned}
$$

1.3 Settlement

$$
\begin{aligned}
\text{max} \quad & \sum_{t \in T} \left(P_t^{DA} \alpha_t + \mathbb{E}\left[P_t^{RT}(\xi) \beta_t^{+}(\xi) - P_t^{PN} \beta_t^{-}(\xi) \right] \right) && \text{(3a)} \\
\text{s.t.} \quad & \sum_{i \in I} R_{it}(\xi) - \alpha_t^{DA} = \beta_t^{+}(\xi) - \beta_t^{-}(\xi) \quad \forall t \in T && \text{(3b)} \\
& \sum_{i \in I} R_{it}(\xi) \geqslant \beta_t^{+}(\xi) \quad \forall t \in T && \text{(3c)} \\
& \beta_t^{+}(\xi) \leqslant M z_t(\xi), \quad \beta_t^{-}(\xi) \leqslant M(1 - z_t(\xi)) \quad \forall t \in T && \text{(3d)} \\
& \alpha_t = \sum_{i \in I} x_{it}(\xi), \quad \beta_t^{+}(\xi) = \sum_{i \in I} e_{it}^{+}(\xi), \quad \beta_t^{-}(\xi) = \sum_{i \in I} e_{it}^{-}(\xi) \quad \forall t \in T && \text{(3e)} \\
& R_{it}(\xi) - x_{it}(\xi) = y_{it}^{+}(\xi) - y_{it}^{-}(\xi) \quad \forall t \in T && \text{(3f)} \\
& R_{it}(\xi) \geqslant y_{it}^{+}(\xi) \quad \forall t \in T && \text{(3g)} \\
& y_{it}^{+}(\xi) \leqslant M z_{it}(\xi), \quad y_{it}^{-}(\xi) \leqslant M(1 - z_{it}(\xi)) \quad \forall t \in T && \text{(3h)} \\
& \sum_{j \in I, j \neq i} d_{ijt}(\xi) \leqslant y_{it}^{+}(\xi), \quad \sum_{j \in I, j \neq i} d_{jit}(\xi) \leqslant y_{it}^{-}(\xi) \quad \forall t \in T && \text{(3i)} \\
& d_{iit}(\xi) = 0 \quad \forall t \in T && \text{(3j)} \\
& e_{it}^{+}(\xi) = y_{it}^{+}(\xi) - \sum_{j \in I, j \neq i} d_{ijt}(\xi) \quad \forall t \in T && \text{(3k)} \\
& e_{it}^{-}(\xi) = y_{it}^{-}(\xi) - \sum_{j \in I, j \neq i} d_{jit}(\xi) \quad \forall t \in T && \text{(3l)}
\end{aligned}
$$

2 With Storage

2.1 Disaggregation

$$\max \quad \sum_{t\in T} \left(P_t^{DA} x_{it} + \mathbb{E}\left[P_t^{RT}(\xi) y_{it}^{+}(\xi) - P_t^{PN} y_{it}^{-}(\xi)\right]\right) \tag{4a}$$

$$\text{s.t.} \quad R_{it}(\xi) - x_{it} = y_{it}^{+}(\xi) - y_{it}^{-}(\xi) + z_{it}^{C}(\xi) - z_{it}^{D}(\xi) \tag{4b}$$

$$R_{it}(\xi) \geqslant y_{it}^{+}(\xi) \tag{4c}$$

$$z_{i,t+1}(\xi) = z_{it}(\xi) + z_{it}^{C}(\xi) - z_{it}^{D}(\xi) \quad \forall i, t \tag{4d}$$

$$z_{it}^{D}(\xi) \leqslant z_{it}(\xi), \quad z_{it}^{C}(\xi) \leqslant K_i - z_{it}(\xi), \quad 0 \leqslant z_{it}(\xi) \leqslant K_i \tag{4e}$$

$$y_{it}^{+}(\xi) \leqslant M_1 \phi_{it}^{1}(\xi), \quad y_{it}^{-}(\xi) \leqslant M_1(1 - \phi_{it}^{1}(\xi)) \tag{4f}$$

$$y_{it}^{-}(\xi) \leqslant M_1 \phi_{it}^{2}(\xi), \quad z_{it}^{C}(\xi) \leqslant M_1(1 - \phi_{it}^{2}(\xi)) \tag{4g}$$

$$z_{it}^{C}(\xi) \leqslant M_1 \phi_{it}^{3}(\xi), \quad z_{it}^{D}(\xi) \leqslant M_1(1 - \phi_{it}^{3}(\xi)) \tag{4h}$$

2.2 Aggregation with BTM storage control

$$\max \quad \sum_{t\in T} \left(P_t^{DA} \sum_{i\in I} x_{it} + \mathbb{E}\left[P_t^{RT}(\xi) \sum_{i\in I} e_{it}^{+}(\xi) - P_t^{PN} \sum_{i\in I} e_{it}^{-}(\xi)\right]\right) \tag{5a}$$

$$\tag{5b}$$

$$\text{s.t.} \quad R_{it}(\xi) - x_{it} = y_{it}^{+}(\xi) - y_{it}^{-}(\xi) + z_{it}^{C}(\xi) - z_{it}^{D}(\xi) \tag{5c}$$

$$R_{it}(\xi) \geqslant y_{it}^{+}(\xi) \tag{5d}$$

$$z_{i,t+1}(\xi) = z_{it}(\xi) + z_{it}^{C}(\xi) - z_{it}^{D}(\xi) \tag{5e}$$

$$z_{it}^{D}(\xi) \leqslant z_{it}(\xi), \quad z_{it}^{C}(\xi) \leqslant K_i - z_{it}(\xi), \quad 0 \leqslant z_{it}(\xi) \leqslant K_i \tag{5f}$$

$$e_{it}^{+}(\xi) = y_{it}^{+}(\xi) - \sum_{j\in I} d_{ijt}(\xi), \quad e_{it}^{-}(\xi) = y_{it}^{-}(\xi) - \sum_{j\in I} d_{jit}(\xi) \tag{5g}$$

$$d_{iit}(\xi) = 0 \tag{5h}$$

$$y_{it}^{+}(\xi) \leqslant M_1 \phi_{it}^{1}(\xi), \quad y_{it}^{-}(\xi) \leqslant M_1(1 - \phi_{it}^{1}(\xi)) \tag{5i}$$

$$y_{it}^{-}(\xi) \leqslant M_1 \phi_{it}^{2}(\xi), \quad z_{it}^{C}(\xi) \leqslant M_1(1 - \phi_{it}^{2}(\xi)) \tag{5j}$$

$$z_{it}^{C}(\xi) \leqslant M_1 \phi_{it}^{3}(\xi), \quad z_{it}^{D}(\xi) \leqslant M_1(1 - \phi_{it}^{3}(\xi)) \tag{5k}$$

$$\sum_{i\in I} e_{it}^{+}(\xi) \leqslant M_2 \phi_t^{4}(\xi), \quad \sum_{i\in I} e_{it}^{-}(\xi) \leqslant M_2(1 - \phi_t^{4}(\xi)) \tag{5l}$$

2.3 Aggregation with direct control over storage

$$\max \quad \sum_{t\in T} \left(P_t^{DA} \alpha_t + \mathbb{E}\left[P_t^{RT}(\xi) \beta_t^{+}(\xi) - P_t^{PN} \beta_t^{-}(\xi))\right]\right) \tag{6a}$$

$$\text{s.t.} \quad \sum_{i\in I} R_{it}(\xi) - \alpha_t = \beta_t^{+}(\xi) - \beta_t^{-}(\xi) + \gamma_t^{C}(\xi) - \gamma_t^{D}(\xi) \quad \forall t \tag{6b}$$

$$\sum_{i\in I} R_{it}(\xi) \geqslant \beta_t^{+}(\xi) \tag{6c}$$

$$\gamma_t^{D}(\xi) \leqslant \gamma_t(\xi), \quad \gamma_t^{C}(\xi) \leqslant \sum_{i\in I} K_i - \gamma_t(\xi), \quad 0 \leqslant \gamma_t(\xi) \leqslant \sum_{i\in I} K_i \quad \forall t \tag{6d}$$

$$\gamma_{t+1}(\xi) = \gamma_t(\xi) + \gamma_t^{C}(\xi) - \gamma_t^{D}(\xi) \quad \forall t \tag{6e}$$

$$\beta_t^{+}(\xi) \leqslant M_2 \mu_t(\xi), \quad \beta_t^{-}(\xi) \leqslant M_2(1 - \mu_t(\xi)) \quad \forall t \tag{6f}$$

$$\beta_t^{-}(\xi) \leqslant M_2 \eta_t(\xi), \quad \gamma_t^{C}(\xi) \leqslant M_2(1 - \eta_t(\xi)) \quad \forall t \tag{6g}$$

$$\gamma_t^{C}(\xi) \leqslant M_2 \lambda_t(\xi), \quad \gamma_t^{D}(\xi) \leqslant M_2(1 - \lambda_t(\xi)) \quad \forall t \tag{6h}$$

3 Individual

3.1 Different Internal Price (Non-linear)

$$\begin{aligned}
\textit{max}\quad & \sum_{t\in T}\left(P_t^{DA}\cdot x_t+\mathbb{E}\left[P_t^{RT}(\xi)\cdot y_t^+(\xi)-P_t^{PN}\cdot y_t^-(\xi)+\rho_t^+(d)\cdot d_t^+(\xi)-\rho_t^-(d)\cdot d_t^-(\xi)\right]\right) && \textit{(7a)}\\
\textit{s.t.}\quad & R_t(\xi)-x_t=y_t^+(\xi)-y_t^-(\xi)+d_t^+(\xi)-d_t^-(\xi)+z_t^C(\xi)-z_t^D(\xi) && \textit{(7b)}\\
& R_t(\xi)\geqslant y_t^+(\xi)+d_{it}^+(\xi) && \textit{(7c)}\\
& x_t-R_t(\xi)\geqslant y_t^-(\xi)+d_t^-(\xi) && \textit{(7d)}\\
& z_{t+1}(\xi)=z_t(\xi)+z_t^C(\xi)-z_t^D(\xi) && \textit{(7e)}\\
& z_t^D(\xi)\leqslant z_t(\xi),\quad z_t^C(\xi)\leqslant K-z_t(\xi),\quad 0\leqslant z_t(\xi)\leqslant K && \textit{(7f)}\\
& y_t^+(\xi)\leqslant M\phi_t^1(\xi),\quad y_t^-(\xi)\leqslant M(1-\phi_t^1(\xi)) && \textit{(7g)}\\
& y_t^-(\xi)\leqslant M\phi_t^2(\xi),\quad z_t^C(\xi)\leqslant M(1-\phi_t^2(\xi)) && \textit{(7h)}\\
& z_t^C(\xi)\leqslant M\phi_t^3(\xi),\quad z_t^D(\xi)\leqslant M(1-\phi_t^3(\xi)) && \textit{(7i)}\\
& d_t^+(\xi)\leqslant M\phi_t^4(\xi),\quad d_t^-(\xi)\leqslant M(1-\phi_t^4(\xi)) && \textit{(7j)}
\end{aligned}$$

3.2 Different Internal Price (Piecewise-Linear)

$$\begin{aligned}
\textit{max}\quad & \sum_{t\in T}\Bigg(P_t^{DA}\cdot x_t+\mathbb{E}\Big[P_t^{RT}(\xi)\cdot y_t^+(\xi)-P_t^{PN}\cdot y_t^-(\xi)\\
&\quad+\sum_{b^+\in B^+}\rho_{t,b^+}^+\cdot(w_{t,b^+}^+ + u_{t,b^+}^+\cdot d_{t,b^+}^{\mathit{min}+})-\sum_{b^-\in B^-}\rho_{t,b^-}^-\cdot(w_{t,b^-}^- + u_{t,b^-}^-\cdot d_{t,b^-}^{\mathit{min}-})\Big]\Bigg) && \textit{(8a)}
\end{aligned}$$

$$\begin{aligned}
\textit{s.t.}\quad & d_t^+=\sum_{b^+\in B^+}(w_{t,b^+}^+ + u_{t,b^+}^+\cdot d_{t,b^+}^{\mathit{min}+}) && \textit{(8b)}\\
& 0\leqslant w_{t,b^+}^+\leqslant u_{t,b^+}^+\cdot w_{t,b^+}^{\mathit{max}+} && \textit{(8c)}\\
& \sum_{b^+\in B^+}u_{t,b^+}^+=1 && \textit{(8d)}
\end{aligned}$$

$$\begin{aligned}
& d_t^-=\sum_{b^-\in B^-}(w_{t,b^-}^- + u_{t,b^-}^-\cdot d_{t,b^-}^{\mathit{min}-}) && \textit{(8e)}\\
& 0\leqslant w_{t,b^-}^-\leqslant u_{t,b^-}^-\cdot w_{t,b^-}^{\mathit{max}-} && \textit{(8f)}\\
& \sum_{b^-\in B^-}u_{t,b^-}^-=1 && \textit{(8g)}
\end{aligned}$$

$$\begin{aligned}
& R_t(\xi)-x_t=y_t^+(\xi)-y_t^-(\xi)+d_t^+(\xi)-d_t^-(\xi)+z_t^C(\xi)-z_t^D(\xi) && \textit{(8h)}\\
& R_t(\xi)+z_t^D(\xi)\geqslant y_t^+(\xi)+d_t^+(\xi)+z_t^C(\xi) && \textit{(8i)}\\
& z_{t+1}(\xi)=z_t(\xi)+z_t^C(\xi)-z_t^D(\xi) && \textit{(8j)}\\
& z_t^D(\xi)\leqslant z_t(\xi),\quad z_t^C(\xi)\leqslant K-z_t(\xi),\quad 0\leqslant z_t(\xi)\leqslant K && \textit{(8k)}\\
& y_t^+(\xi)\leqslant M\phi_t^1(\xi),\quad y_t^-(\xi)\leqslant M(1-\phi_t^1(\xi)) && \textit{(8l)}\\
& y_t^-(\xi)\leqslant M\phi_t^2(\xi),\quad z_t^C(\xi)\leqslant M(1-\phi_t^2(\xi)) && \textit{(8m)}\\
& z_t^C(\xi)\leqslant M\phi_t^3(\xi),\quad z_t^D(\xi)\leqslant M(1-\phi_t^3(\xi)) && \textit{(8n)}\\
& d_t^+(\xi)\leqslant M\phi_t^4(\xi),\quad d_t^-(\xi)\leqslant M(1-\phi_t^4(\xi)) && \textit{(8o)}\\
& y_t^+(\xi)\leqslant M\phi_t^5(\xi),\quad d_t^-(\xi)\leqslant M(1-\phi_t^5(\xi)) && \textit{(8p)}\\
& y_t^-(\xi)\leqslant M\phi_t^6(\xi),\quad d_t^+(\xi)\leqslant M(1-\phi_t^6(\xi)) && \textit{(8q)}
\end{aligned}$$