

# DER Models

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## 1 Without Storage

### 1.1 Disaggregation

$$\max \sum_{t \in T} (P_t^{DA} x_{it} + \mathbb{E} [P_t^{RT}(\xi) y_{it}^+(\xi) - P_t^{PN} y_{it}^-(\xi)]) \quad (1a)$$

$$\text{s.t. } R_{it}(\xi) - x_{it} = y_{it}^+(\xi) - y_{it}^-(\xi) \quad \forall t \in T \quad (1b)$$

$$R_{it}(\xi) \geq y_{it}^+(\xi) \quad \forall t \in T \quad (1c)$$

$$y_{it}^+(\xi) \leq M z_{it}(\xi), \quad y_{it}^-(\xi) \leq M(1 - z_{it}(\xi)) \quad \forall t \in T \quad (1d)$$

$$x_{it}^{DA} \geq 0, y_{it}^+(\xi) \geq 0, y_{it}^-(\xi) \geq 0, z_{it}(\xi) \in \{0, 1\} \quad \forall t \in T \quad (1e)$$

### 1.2 Aggregation

$$\max \sum_{t \in T} (P_t^{DA} \alpha_t + \mathbb{E} [P_t^{RT}(\xi) \beta_t^+(\xi) - P_t^{PN} \beta_t^-(\xi)]) \quad (2a)$$

$$\text{s.t. } \sum_{i \in I} R_{it}(\xi) - \alpha_t = \beta_t^+(\xi) - \beta_t^-(\xi) \quad \forall t \in T \quad (2b)$$

$$\sum_{i \in I} R_{it}(\xi) \geq \beta_t^+(\xi) \quad \forall t \in T \quad (2c)$$

$$\beta_t^+(\xi) \leq M z_t(\xi), \quad \beta_t^-(\xi) \leq M(1 - z_t(\xi)) \quad \forall t \in T \quad (2d)$$

$$\alpha_t^{DA} \geq 0, \beta_t^+(\xi) \geq 0, \beta_t^-(\xi) \geq 0, z_t(\xi) \in \{0, 1\} \quad \forall t \in T \quad (2e)$$

### 1.3 Settlement

$$\max \sum_{t \in T} (P_t^{DA} \alpha_t + \mathbb{E} [P_t^{RT}(\xi) \beta_t^+(\xi) - P_t^{PN} \beta_t^-(\xi)]) \quad (3a)$$

$$\text{s.t. } \sum_{i \in I} R_{it}(\xi) - \alpha_t^{DA} = \beta_t^+(\xi) - \beta_t^-(\xi) \quad \forall t \in T \quad (3b)$$

$$\sum_{i \in I} R_{it}(\xi) \geq \beta_t^+(\xi) \quad \forall t \in T \quad (3c)$$

$$\beta_t^+(\xi) \leq M z_t(\xi), \quad \beta_t^-(\xi) \leq M(1 - z_t(\xi)) \quad \forall t \in T \quad (3d)$$

$$\alpha_t = \sum_{i \in I} x_{it}(\xi), \quad \beta_t^+(\xi) = \sum_{i \in I} e_{it}^+(\xi), \quad \beta_t^-(\xi) = \sum_{i \in I} e_{it}^-(\xi) \quad \forall t \in T \quad (3e)$$

$$R_{it}(\xi) - x_{it}(\xi) = y_{it}^+(\xi) - y_{it}^-(\xi) \quad \forall t \in T \quad (3f)$$

$$R_{it}(\xi) \geq y_{it}^+(\xi) \quad \forall t \in T \quad (3g)$$

$$y_{it}^+(\xi) \leq M z_{it}(\xi), \quad y_{it}^-(\xi) \leq M(1 - z_{it}(\xi)) \quad \forall t \in T \quad (3h)$$

$$\sum_{j \in I, j \neq i} d_{ijt}(\xi) \leq y_{it}^+(\xi), \quad \sum_{j \in I, j \neq i} d_{jit}(\xi) \leq y_{it}^-(\xi) \quad \forall t \in T \quad (3i)$$

$$d_{iit}(\xi) = 0 \quad \forall t \in T \quad (3j)$$

$$e_{it}^+(\xi) = y_{it}^+(\xi) - \sum_{j \in I, j \neq i} d_{ijt}(\xi) \quad \forall t \in T \quad (3k)$$

$$e_{it}^-(\xi) = y_{it}^-(\xi) - \sum_{j \in I, j \neq i} d_{jit}(\xi) \quad \forall t \in T \quad (3l)$$

## 2 With Storage

### 2.1 Disaggregation

$$\max \sum_{t \in T} (P_t^{DA} x_{it} + \mathbb{E} [P_t^{RT}(\xi) y_{it}^+(\xi) - P_t^{PN} y_{it}^-(\xi)]) \quad (4a)$$

$$\text{s.t. } R_{it}(\xi) - x_{it} = y_{it}^+(\xi) - y_{it}^-(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi) \quad \forall i, t \quad (4b)$$

$$R_{it}(\xi) \geq y_{it}^+(\xi) \quad \forall i, t \quad (4c)$$

$$z_{i,t+1}(\xi) = z_{it}(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi) \quad \forall i, t \quad (4d)$$

$$z_{it}^D(\xi) \leq z_{it}(\xi) \quad \forall i, t \quad (4e)$$

$$z_{it}^C(\xi) \leq K_i - z_{it}(\xi) \quad \forall i, t \quad (4f)$$

$$0 \leq z_{it}(\xi) \leq K_i \quad \forall i, t \quad (4g)$$

$$y_{it}^+(\xi) \leq M_1 \rho_{it}(\xi), \quad y_{it}^-(\xi) \leq M_1 (1 - \rho_{it}(\xi)) \quad \forall i, t \quad (4h)$$

$$y_{it}^-(\xi) \leq M_1 \delta_{it}(\xi), \quad z_{it}^C(\xi) \leq M_1 (1 - \delta_{it}(\xi)) \quad \forall i, t \quad (4i)$$

$$z_{it}^C(\xi) \leq M_1 \zeta_{it}(\xi), \quad z_{it}^D(\xi) \leq M_1 (1 - \zeta_{it}(\xi)) \quad \forall i, t \quad (4j)$$

### 2.2 Aggregation

$$\max \sum_{t \in T} (P_t^{DA} \alpha_t + \mathbb{E} [P_t^{RT}(\xi) \beta_t^+(\xi) - P_t^{PN} \beta_t^-(\xi)]) \quad (5a)$$

$$\text{s.t. } \sum_{i \in I} R_{it}(\xi) - \alpha_t = \beta_t^+(\xi) - \beta_t^-(\xi) + \gamma_t^C(\xi) - \gamma_t^D(\xi) \quad \forall t \quad (5b)$$

$$\sum_{i \in I} R_{it}(\xi) \geq \beta_t^+(\xi) \quad \forall t \quad (5c)$$

$$\gamma_t^D(\xi) \leq \gamma_t(\xi) \quad \forall t \quad (5d)$$

$$\gamma_t^C(\xi) \leq \sum_{i \in I} K_i - \gamma_t(\xi) \quad \forall t \quad (5e)$$

$$0 \leq \gamma_t(\xi) \leq \sum_{i \in I} K_i \quad \forall t \quad (5f)$$

$$\gamma_{t+1}(\xi) = \gamma_t(\xi) + \gamma_t^C(\xi) - \gamma_t^D(\xi) \quad \forall t \quad (5g)$$

$$\beta_t^+(\xi) \leq M_2 \mu_t(\xi), \quad \beta_t^-(\xi) \leq M_2 (1 - \mu_t(\xi)) \quad \forall t \quad (5h)$$

$$\beta_t^-(\xi) \leq M_2 \eta_t(\xi), \quad \gamma_t^C(\xi) \leq M_2 (1 - \eta_t(\xi)) \quad \forall t \quad (5i)$$

$$\gamma_t^C(\xi) \leq M_2 \lambda_t(\xi), \quad \gamma_t^D(\xi) \leq M_2 (1 - \lambda_t(\xi)) \quad \forall t \quad (5j)$$

## 2.3 Settlement

$$\begin{aligned}
& \max \quad \sum_{t \in T} \left( P_t^{DA} \alpha_t + \mathbb{E} \left[ P_t^{RT}(\xi) \beta_t^+(\xi) - P_t^{PN} \beta_t^-(\xi) \right] \right) \\
& \text{s.t.} \quad R_{it}(\xi) - x_{it} = y_{it}^+(\xi) - y_{it}^-(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi) \quad \forall i, t \\
& \quad R_{it}(\xi) \geq y_{it}^+(\xi) \quad \forall i, t \\
& \quad z_{it}^D(\xi) \leq z_{it}(\xi) \quad \forall i, t \\
& \quad z_{it}^C(\xi) \leq K_i - z_{it}(\xi) \quad \forall i, t \\
& \quad 0 \leq z_{it}(\xi) \leq K_i \quad \forall i, t \\
& \quad y_{it}^+(\xi) \leq M_1 \rho_{it}(\xi), \quad y_{it}^-(\xi) \leq M_1(1 - \rho_{it}(\xi)) \quad \forall i, t \\
& \quad y_{it}^-(\xi) \leq M_1 \delta_{it}(\xi), \quad z_{it}^C(\xi) \leq M_1(1 - \delta_{it}(\xi)) \quad \forall i, t \\
& \quad z_{it}^C(\xi) \leq M_1 \zeta_{it}(\xi), \quad z_{it}^D(\xi) \leq M_1(1 - \zeta_{it}(\xi)) \quad \forall i, t \\
& \quad e_{it}^+(\xi) = y_{it}^+(\xi) - d_{it}^+(\xi) \\
& \quad e_{it}^-(\xi) = y_{it}^-(\xi) - d_{it}^-(\xi) \\
& \quad e_{it}^C(\xi) = z_{it}^C(\xi) - \widehat{d}_{it}^C(\xi) + \widetilde{d}_{it}^C(\xi) \\
& \quad \widehat{d}_{it}^C(\xi) \leq M_1 q_{it}^3(\xi), \quad \widetilde{d}_{it}^C(\xi) \leq M_1(1 - q_{it}^3(\xi)) \\
& \quad e_{it}^C(\xi) \leq K_i - z_{it}(\xi) \\
& \quad e_{it}^D(\xi) = z_{it}^D(\xi) - \widehat{d}_{it}^D(\xi) + \widetilde{d}_{it}^D(\xi) \\
& \quad \widehat{d}_{it}^D(\xi) \leq M_1 q_{it}^4(\xi), \quad \widetilde{d}_{it}^D(\xi) \leq M_1(1 - q_{it}^4(\xi)) \\
& \quad e_{it}^D(\xi) \leq z_{it}(\xi) \\
& \quad e_{it}^+(\xi) \leq M_1 q_{it}^5(\xi), \quad e_{it}^-(\xi) \leq M_1(1 - q_{it}^5(\xi)) \\
& \quad e_{it}^-(\xi) \leq M_1 q_{it}^6(\xi), \quad e_{it}^C(\xi) \leq M_1(1 - q_{it}^6(\xi)) \\
& \quad e_{it}^C(\xi) \leq M_1 q_{it}^6(\xi), \quad e_{it}^D(\xi) \leq M_1(1 - q_{it}^6(\xi)) \\
& \quad z_{i,t+1}(\xi) = z_{it}(\xi) + e_{it}^C(\xi) - e_{it}^D(\xi) \quad \forall i, t \\
& \quad \alpha_t = \sum_{i \in I} x_{it}, \quad \beta_t^+(\xi) = \sum_{i \in I} e_{it}^+(\xi), \quad \beta_t^-(\xi) = \sum_{i \in I} e_{it}^-(\xi) \\
& \quad \gamma_t(\xi) = \sum_{i \in I} z_{it}(\xi), \quad \gamma_t^C(\xi) = \sum_{i \in I} e_{it}^C(\xi), \quad \gamma_t^D(\xi) = \sum_{i \in I} e_{it}^D(\xi) \\
& \quad \sum_{i \in I} R_{it}(\xi) - \alpha_t = \beta_t^+(\xi) - \beta_t^-(\xi) + \gamma_t^C(\xi) - \gamma_t^D(\xi) \quad \forall t \\
& \quad \sum_{i \in I} R_{it}(\xi) \geq \beta_t^+(\xi) \quad \forall t \\
& \quad \gamma_t^D(\xi) \leq \gamma_t(\xi) \quad \forall t \\
& \quad \gamma_t^C(\xi) \leq \sum_{i \in I} K_i - \gamma_t(\xi) \quad \forall t \\
& \quad 0 \leq \gamma_t(\xi) \leq \sum_{i \in I} K_i \quad \forall t \\
& \quad \gamma_{t+1}(\xi) = \gamma_t(\xi) + \gamma_t^C(\xi) - \gamma_t^D(\xi) \quad \forall t \\
& \quad \beta_t^+(\xi) \leq M_2 \mu_t(\xi), \quad \beta_t^-(\xi) \leq M_2(1 - \mu_t(\xi)) \quad \forall t \\
& \quad \beta_t^-(\xi) \leq M_2 \eta_t(\xi), \quad \gamma_t^C(\xi) \leq M_2(1 - \eta_t(\xi)) \quad \forall t \\
& \quad \gamma_t^C(\xi) \leq M_2 \lambda_t(\xi), \quad \gamma_t^D(\xi) \leq M_2(1 - \lambda_t(\xi)) \quad \forall t \\
& \quad \sum_{i \in I} \widehat{d}_{it}^C(\xi) \leq \sum_{i \in I} y_{it}^-(\xi) \\
& \quad \sum_{i \in I} \widetilde{d}_{it}^D(\xi) \leq \sum_{i \in I} y_{it}^-(\xi) \\
& \quad \left( d_{it}^+(\xi) - \sum_{i \in I} y_{it}^-(\xi) \right) - (K_i - z_{it}(\xi)) \leq M_1(1 - q_{it}^6(\xi)) - \epsilon \\
& \quad (K_i - z_{it}(\xi)) - e_{it}^C(\xi) \leq M_1 q_{it}^6(\xi) \\
& \quad (K_i - z_{it}(\xi)) - \left( d_{it}^+(\xi) - \sum_{i \in I} y_{it}^-(\xi) \right) \leq M_1(1 - q_{it}^7(\xi)) - \epsilon \\
& \quad \left( e_{it}^C(\xi) \right) - (d_{it}^+(\xi) + z_{it}^C(\xi)) \leq M_1 q_{it}^7(\xi)
\end{aligned}$$

### 3 With Individual Storage

#### 3.1 RT Storage Dispatching: for each $i$

$$\max \sum_{t \in T} (P_t^{DA} x_{it} + \mathbb{E} [P_t^{RT}(\xi) y_{it}^+(\xi) - P_t^{PN} y_{it}^-(\xi)]) \quad (7a)$$

$$\text{s.t. } R_{it}(\xi) - x_{it} = y_{it}^+(\xi) - y_{it}^-(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi) \quad \forall i, t \quad (7b)$$

$$R_{it}(\xi) \geq y_{it}^+(\xi) \quad \forall i, t \quad (7c)$$

$$z_{i,t+1}(\xi) = z_{it}(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi) \quad \forall i, t \quad (7d)$$

$$z_{it}^D(\xi) \leq z_{it}(\xi) \quad \forall i, t \quad (7e)$$

$$z_{it}^C(\xi) \leq K_i - z_{it}(\xi) \quad \forall i, t \quad (7f)$$

$$0 \leq z_{it}(\xi) \leq K_i \quad \forall i, t \quad (7g)$$

$$y_{it}^+(\xi) \leq M_1 \rho_{it}(\xi), \quad y_{it}^-(\xi) \leq M_1 (1 - \rho_{it}(\xi)) \quad \forall i, t \quad (7h)$$

$$y_{it}^-(\xi) \leq M_1 \delta_{it}(\xi), \quad z_{it}^C(\xi) \leq M_1 (1 - \delta_{it}(\xi)) \quad \forall i, t \quad (7i)$$

$$z_{it}^C(\xi) \leq M_1 \zeta_{it}(\xi), \quad z_{it}^D(\xi) \leq M_1 (1 - \zeta_{it}(\xi)) \quad \forall i, t \quad (7j)$$

### 3.2 RT Storage Dispatching with aggregation

$$\begin{aligned}
\max \quad & \sum_{t \in T} (P_t^{DA} \alpha_t + \mathbb{E} [P_t^{RT}(\xi) \beta_t^+(\xi) - P_t^{PN} \beta_{it}^-(\xi)]) \\
\text{s.t.} \quad & R_{it}(\xi) - x_{it} = y_{it}^+(\xi) - y_{it}^-(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi), \quad R_{it}(\xi) \geq y_{it}^+(\xi) \\
& y_{it}^+(\xi) \leq M_1 \rho_{it}(\xi), \quad y_{it}^-(\xi) \leq M_1 (1 - \rho_{it}(\xi)) \\
& y_{it}^-(\xi) \leq M_1 \delta_{it}(\xi), \quad z_{it}^C(\xi) \leq M_1 (1 - \delta_{it}(\xi)) \\
& z_{it}^C(\xi) \leq M_1 \zeta_{it}(\xi), \quad z_{it}^D(\xi) \leq M_1 (1 - \zeta_{it}(\xi)) \\
& z_{it}^D(\xi) \leq z_{it}(\xi), \quad z_{it}^C(\xi) \leq K_i - z_{it}(\xi), \quad 0 \leq z_{it}(\xi) \leq K_i \\
\\
& e_{it}^+(\xi) = y_{it}^+(\xi) - \sum_{j \in I} d_{ijt}^+(\xi), \quad e_{it}^-(\xi) = y_{it}^-(\xi) - d_{it}^-(\xi) \\
& e_{it}^C(\xi) = z_{it}^C(\xi) - \sum_{j \in I} \widehat{d_{ijt}^C}(\xi) + \widetilde{d_{it}^C}(\xi), \quad e_{it}^D(\xi) = z_{it}^D(\xi) - \widehat{d_{it}^D}(\xi) + \sum_{j \in I} \widetilde{d_{ijt}^D}(\xi) \\
& e_{it}^C(\xi) \leq K_i - z_{it}(\xi), \quad e_{it}^D(\xi) \leq z_{it}(\xi), \quad z_{i,t+1}(\xi) = z_{it}(\xi) + e_{it}^C(\xi) - e_{it}^D(\xi) \\
\\
& \sum_{j \in I} \widehat{d_{ijt}^C}(\xi) \leq M_1 q_{it}^1(\xi), \quad \widetilde{d_{it}^C}(\xi) \leq M_1 (1 - q_{it}^1(\xi)) \\
& \widehat{d_{it}^D}(\xi) \leq M_1 q_{it}^2(\xi), \quad \sum_{j \in I} \widetilde{d_{ijt}^D}(\xi) \leq M_1 (1 - q_{it}^2(\xi)) \\
& e_{it}^+(\xi) \leq M_1 q_{it}^3(\xi), \quad e_{it}^-(\xi) \leq M_1 (1 - q_{it}^3(\xi)) \\
& e_{it}^-(\xi) \leq M_1 q_{it}^4(\xi), \quad e_{it}^C(\xi) \leq M_1 (1 - q_{it}^4(\xi)) \\
& e_{it}^C(\xi) \leq M_1 q_{it}^5(\xi), \quad e_{it}^D(\xi) \leq M_1 (1 - q_{it}^5(\xi)) \\
\\
& y_{it}^+(\xi) \geq \sum_{j \in I} d_{ijt}^+(\xi), \quad y_{it}^-(\xi) \geq d_{it}^-(\xi), \quad z_{it}^C(\xi) \geq \sum_{j \in I} \widehat{d_{ijt}^C}(\xi), \quad z_{it}^D(\xi) \geq \widehat{d_{it}^D}(\xi) \\
& z_{it}^D(\xi) + \widetilde{d_{it}^D}(\xi) \geq \min\{y_{it}^-(\xi), z_{it}(\xi)\} \\
& \sum_{j \in I} d_{ijt}^+(\xi) - \sum_{j \in I} d_{jit}^+(\xi) \leq d_{it}^+(\xi), \quad d_{it}^+(\xi) \geq \widetilde{d_{it}^C}(\xi) + \widehat{d_{it}^D}(\xi) \\
& d_{it}^-(\xi) \geq \left( d_{jit}^+(\xi) + \widehat{d_{jit}^C}(\xi) + \widetilde{d_{jit}^D}(\xi) \right) + \left( \widehat{d_{it}^C}(\xi) + \widetilde{d_{it}^D}(\xi) \right) \quad \forall j \in I, j \neq i \\
\\
& \alpha_t = \sum_{i \in I} x_{it}, \quad \beta_t^+(\xi) = \sum_{i \in I} e_{it}^+(\xi), \quad \beta_t^-(\xi) = \sum_{i \in I} e_{it}^-(\xi), \quad \gamma_t^C(\xi) = \sum_{i \in I} e_{it}^C(\xi), \quad \gamma_t^D(\xi) = \sum_{i \in I} e_{it}^D(\xi) \\
& \sum_{i \in I} R_{it}(\xi) - \alpha_t = \beta_t^+(\xi) - \beta_t^-(\xi) + \gamma_t^C(\xi) - \gamma_t^D(\xi), \quad \sum_{i \in I} R_{it}(\xi) \geq \beta_t^+(\xi) \\
& \beta_t^+(\xi) \leq M_2 \mu_t(\xi), \quad \beta_t^-(\xi) \leq M_2 (1 - \mu_t(\xi)) \\
& \beta_t^-(\xi) \leq M_2 \eta_t(\xi), \quad \gamma_t^C(\xi) \leq M_2 (1 - \eta_t(\xi))
\end{aligned}$$

### 3.3 RT Storage Dispatching with aggregation (2)

$$\max \sum_{t \in T} (P_t^{DA} \alpha_t + \mathbb{E} [P_t^{RT}(\xi) \beta_t^+(\xi) - P_t^{PN} \beta_{it}^-(\xi)])$$

$$\text{s.t. } R_{it}(\xi) - x_{it} = y_{it}^+(\xi) - y_{it}^-(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi), \quad R_{it}(\xi) \geq y_{it}^+(\xi)$$

$$y_{it}^+(\xi) \leq M_1 \rho_{it}(\xi), \quad y_{it}^-(\xi) \leq M_1 (1 - \rho_{it}(\xi))$$

$$y_{it}^-(\xi) \leq M_1 \delta_{it}(\xi), \quad z_{it}^C(\xi) \leq M_1 (1 - \delta_{it}(\xi))$$

$$z_{it}^C(\xi) \leq M_1 \zeta_{it}(\xi), \quad z_{it}^D(\xi) \leq M_1 (1 - \zeta_{it}(\xi))$$

$$z_{it}^D(\xi) \leq z_{it}(\xi), \quad z_{it}^C(\xi) \leq K_i - z_{it}(\xi), \quad 0 \leq z_{it}(\xi) \leq K_i$$

$$e_{it}^+(\xi) = y_{it}^+(\xi) - \sum_{j \in I} d_{ijt}^+(\xi), \quad e_{it}^-(\xi) = y_{it}^-(\xi) - \sum_{j \in I} d_{ijt}^-(\xi)$$

$$e_{it}^C(\xi) = z_{it}^C(\xi) - \sum_{j \in I} \widehat{d_{ijt}^C}(\xi) + \sum_{j \in I} \widetilde{d_{ijt}^C}(\xi), \quad e_{it}^D(\xi) = z_{it}^D(\xi) - \sum_{j \in I} \widehat{d_{ijt}^D}(\xi) + \sum_{j \in I} \widetilde{d_{ijt}^D}(\xi)$$

$$e_{it}^C(\xi) \leq K_i - z_{it}(\xi), \quad e_{it}^D(\xi) \leq z_{it}(\xi), \quad e_{it}^D(\xi) \geq \min\{y_{it}^-(\xi), z_{it}(\xi)\}, \quad z_{i,t+1}(\xi) = z_{it}(\xi) + e_{it}^C(\xi) - e_{it}^D(\xi)$$

$$y_{it}^+(\xi) \geq \sum_{j \in I} d_{ijt}^+(\xi), \quad y_{it}^-(\xi) \geq \sum_{j \in I} d_{ijt}^-(\xi), \quad z_{it}^C(\xi) \geq \sum_{j \in I} \widehat{d_{ijt}^C}(\xi), \quad z_{it}^D(\xi) \geq \sum_{j \in I} \widehat{d_{ijt}^D}(\xi)$$

$$\sum_{i \in I} y_{it}^-(\xi) \geq \sum_{j \in I} \widehat{d_{jit}^C}(\xi), \quad \sum_{i \in I} y_{it}^-(\xi) \geq \sum_{j \in I} \widetilde{d_{jit}^D}(\xi)$$

$$\sum_{j \in I} d_{ijt}^+(\xi) - \sum_{j \in I, j \neq i} d_{jit}^+(\xi) = d_{iit}^+(\xi)$$

$$d_{iit}^+(\xi) \geq \widetilde{d_{iit}^C}(\xi) + \widehat{d_{iit}^D}(\xi)$$

$$e_{it}^C(\xi) \geq \min\{d_{iit}^+(\xi) - \widehat{d_{iit}^D}(\xi), K_i - z_{it}(\xi)\}??$$

$$\widetilde{d_{iit}^C}(\xi) \geq \min\{d_{iit}^+(\xi) - \widehat{d_{iit}^D}(\xi), K_i - z_{it}(\xi) - z_{it}^C(\xi)\}$$

$$d_{jit}^-(\xi) \geq d_{jit}^+(\xi) + \widehat{d_{jit}^C}(\xi) + \widetilde{d_{jit}^D}(\xi) \quad \forall j \in I, j \neq i$$

$$\sum_{j \in I} \widehat{d_{ijt}^C}(\xi) \leq M_1 q_{it}^1(\xi), \quad \widetilde{d_{iit}^C}(\xi) \leq M_1 (1 - q_{it}^1(\xi))$$

$$\widehat{d_{iit}^D}(\xi) \leq M_1 q_{it}^2(\xi), \quad \sum_{j \in I} \widetilde{d_{ijt}^D}(\xi) \leq M_1 (1 - q_{it}^2(\xi))$$

$$e_{it}^+(\xi) \leq M_1 q_{it}^3(\xi), \quad e_{it}^-(\xi) \leq M_1 (1 - q_{it}^3(\xi))$$

$$e_{it}^-(\xi) \leq M_1 q_{it}^4(\xi), \quad e_{it}^C(\xi) \leq M_1 (1 - q_{it}^4(\xi))$$

$$e_{it}^C(\xi) \leq M_1 q_{it}^5(\xi), \quad e_{it}^D(\xi) \leq M_1 (1 - q_{it}^5(\xi))$$

$$\alpha_t = \sum_{i \in I} x_{it}, \quad \beta_t^+(\xi) = \sum_{i \in I} e_{it}^+(\xi), \quad \beta_t^-(\xi) = \sum_{i \in I} e_{it}^-(\xi), \quad \gamma_t^C(\xi) = \sum_{i \in I} e_{it}^C(\xi), \quad \gamma_t^D(\xi) = \sum_{i \in I} e_{it}^D(\xi)$$

$$\sum_{i \in I} R_{it}(\xi) - \alpha_t = \beta_t^+(\xi) - \beta_t^-(\xi) + \gamma_t^C(\xi) - \gamma_t^D(\xi), \quad \sum_{i \in I} R_{it}(\xi) \geq \beta_t^+(\xi)$$

$$\beta_t^+(\xi) \leq M_2 \mu_t(\xi), \quad \beta_t^-(\xi) \leq M_2 (1 - \mu_t(\xi))$$

$$\beta_t^-(\xi) \leq M_2 \eta_t(\xi), \quad \gamma_t^C(\xi) \leq M_2 (1 - \eta_t(\xi))$$

### 3.4 DA + RT Storage Dispatching: for each i

$$\max \sum_{t \in T} (P_t^{DA} x_{it} + \mathbb{E} [P_t^{RT}(\xi) y_{it}^+(\xi) - P_t^{PN} y_{it}^-(\xi)]) \quad (8a)$$

$$\text{s.t. } R_{it}(\xi) - x_{it} = y_{it}^+(\xi) - y_{it}^-(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi) \quad \forall i, t \quad (8b)$$

$$R_{it}(\xi) \geq y_{it}^+(\xi) \quad \forall i, t \quad (8c)$$

$$z_{i,t+1}(\xi) = z_{it}(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi) \quad \forall i, t \quad (8d)$$

$$z_{it}^D(\xi) \leq z_{it}(\xi) \quad \forall i, t \quad (8e)$$

$$z_{it}^C(\xi) \leq K_i - z_{it}(\xi) \quad \forall i, t \quad (8f)$$

$$0 \leq z_{it}(\xi) \leq K_i \quad \forall i, t \quad (8g)$$

$$y_{it}^+(\xi) \leq M_1 \rho_{it}(\xi), \quad y_{it}^-(\xi) \leq M_1 (1 - \rho_{it}(\xi)) \quad \forall i, t \quad (8h)$$

$$y_{it}^-(\xi) \leq M_1 \delta_{it}(\xi), \quad z_{it}^C(\xi) \leq M_1 (1 - \delta_{it}(\xi)) \quad \forall i, t \quad (8i)$$

$$z_{it}^C(\xi) \leq M_1 \zeta_{it}(\xi), \quad z_{it}^D(\xi) \leq M_1 (1 - \zeta_{it}(\xi)) \quad \forall i, t \quad (8j)$$

### 3.5 DA + RT Storage Dispatching with aggregation

$$\max \sum_{t \in T} (P_t^{DA} x_{it} + \mathbb{E} [P_t^{RT}(\xi) y_{it}^+(\xi) - P_t^{PN} y_{it}^-(\xi)]) \quad (9a)$$

$$\text{s.t. } R_{it}(\xi) - x_{it} = y_{it}^+(\xi) - y_{it}^-(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi) \quad \forall i, t \quad (9b)$$

$$R_{it}(\xi) \geq y_{it}^+(\xi) \quad \forall i, t \quad (9c)$$

$$z_{i,t+1}(\xi) = z_{it}(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi) \quad \forall i, t \quad (9d)$$

$$z_{it}^D(\xi) \leq z_{it}(\xi) \quad \forall i, t \quad (9e)$$

$$z_{it}^C(\xi) \leq K_i - z_{it}(\xi) \quad \forall i, t \quad (9f)$$

$$0 \leq z_{it}(\xi) \leq K_i \quad \forall i, t \quad (9g)$$

$$y_{it}^+(\xi) \leq M_1 \rho_{it}(\xi), \quad y_{it}^-(\xi) \leq M_1 (1 - \rho_{it}(\xi)) \quad \forall i, t \quad (9h)$$

$$y_{it}^-(\xi) \leq M_1 \delta_{it}(\xi), \quad z_{it}^C(\xi) \leq M_1 (1 - \delta_{it}(\xi)) \quad \forall i, t \quad (9i)$$

$$z_{it}^C(\xi) \leq M_1 \zeta_{it}(\xi), \quad z_{it}^D(\xi) \leq M_1 (1 - \zeta_{it}(\xi)) \quad \forall i, t \quad (9j)$$