

# DER Models

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June 3, 2025

## 1 Without Storage

### 1.1 Disaggregation

$$\max \sum_{t \in T} (P_t^{DA} x_{it} + \mathbb{E} [P_t^{RT}(\xi) y_{it}^+(\xi) - P_t^{PN} y_{it}^-(\xi)]) \quad (1a)$$

$$\text{s.t. } R_{it}(\xi) - x_{it} = y_{it}^+(\xi) - y_{it}^-(\xi) \quad \forall t \in T \quad (1b)$$

$$R_{it}(\xi) \geq y_{it}^+(\xi) \quad \forall t \in T \quad (1c)$$

$$y_{it}^+(\xi) \leq M z_{it}(\xi), \quad y_{it}^-(\xi) \leq M(1 - z_{it}(\xi)) \quad \forall t \in T \quad (1d)$$

$$x_{it}^{DA} \geq 0, y_{it}^+(\xi) \geq 0, y_{it}^-(\xi) \geq 0, z_{it}(\xi) \in \{0, 1\} \quad \forall t \in T \quad (1e)$$

### 1.2 Aggregation

$$\max \sum_{t \in T} (P_t^{DA} \alpha_t + \mathbb{E} [P_t^{RT}(\xi) \beta_t^+(\xi) - P_t^{PN} \beta_t^-(\xi)]) \quad (2a)$$

$$\text{s.t. } \sum_{i \in I} R_{it}(\xi) - \alpha_t = \beta_t^+(\xi) - \beta_t^-(\xi) \quad \forall t \in T \quad (2b)$$

$$\sum_{i \in I} R_{it}(\xi) \geq \beta_t^+(\xi) \quad \forall t \in T \quad (2c)$$

$$\beta_t^+(\xi) \leq M z_t(\xi), \quad \beta_t^-(\xi) \leq M(1 - z_t(\xi)) \quad \forall t \in T \quad (2d)$$

$$\alpha_t^{DA} \geq 0, \beta_t^+(\xi) \geq 0, \beta_t^-(\xi) \geq 0, z_t(\xi) \in \{0, 1\} \quad \forall t \in T \quad (2e)$$

### 1.3 Settlement

$$\max \sum_{t \in T} (P_t^{DA} \alpha_t + \mathbb{E} [P_t^{RT}(\xi) \beta_t^+(\xi) - P_t^{PN} \beta_t^-(\xi)]) \quad (3a)$$

$$\text{s.t. } \sum_{i \in I} R_{it}(\xi) - \alpha_t^{DA} = \beta_t^+(\xi) - \beta_t^-(\xi) \quad \forall t \in T \quad (3b)$$

$$\sum_{i \in I} R_{it}(\xi) \geq \beta_t^+(\xi) \quad \forall t \in T \quad (3c)$$

$$\beta_t^+(\xi) \leq M z_t(\xi), \quad \beta_t^-(\xi) \leq M(1 - z_t(\xi)) \quad \forall t \in T \quad (3d)$$

$$\alpha_t = \sum_{i \in I} x_{it}(\xi), \quad \beta_t^+(\xi) = \sum_{i \in I} e_{it}^+(\xi), \quad \beta_t^-(\xi) = \sum_{i \in I} e_{it}^-(\xi) \quad \forall t \in T \quad (3e)$$

$$R_{it}(\xi) - x_{it}(\xi) = y_{it}^+(\xi) - y_{it}^-(\xi) \quad \forall t \in T \quad (3f)$$

$$R_{it}(\xi) \geq y_{it}^+(\xi) \quad \forall t \in T \quad (3g)$$

$$y_{it}^+(\xi) \leq M z_{it}(\xi), \quad y_{it}^-(\xi) \leq M(1 - z_{it}(\xi)) \quad \forall t \in T \quad (3h)$$

$$\sum_{j \in I, j \neq i} d_{ijt}(\xi) \leq y_{it}^+(\xi), \quad \sum_{j \in I, j \neq i} d_{jit}(\xi) \leq y_{it}^-(\xi) \quad \forall t \in T \quad (3i)$$

$$d_{iit}(\xi) = 0 \quad \forall t \in T \quad (3j)$$

$$e_{it}^+(\xi) = y_{it}^+(\xi) - \sum_{j \in I, j \neq i} d_{ijt}(\xi) \quad \forall t \in T \quad (3k)$$

$$e_{it}^-(\xi) = y_{it}^-(\xi) - \sum_{j \in I, j \neq i} d_{jit}(\xi) \quad \forall t \in T \quad (3l)$$

## 2 With Storage

### 2.1 Disaggregation

$$\max \sum_{t \in T} (P_t^{DA} x_{it} + \mathbb{E} [P_t^{RT}(\xi) y_{it}^+(\xi) - P_t^{PN} y_{it}^-(\xi)]) \quad (4a)$$

$$\text{s.t. } R_{it}(\xi) - x_{it} = y_{it}^+(\xi) - y_{it}^-(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi) \quad (4b)$$

$$R_{it}(\xi) \geq y_{it}^+(\xi) \quad (4c)$$

$$z_{i,t+1}(\xi) = z_{it}(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi) \quad \forall i, t \quad (4d)$$

$$z_{it}^D(\xi) \leq z_{it}(\xi), \quad z_{it}^C(\xi) \leq K_i - z_{it}(\xi), \quad 0 \leq z_{it}(\xi) \leq K_i \quad (4e)$$

$$y_{it}^+(\xi) \leq M_1 \phi_{it}^1(\xi), \quad y_{it}^-(\xi) \leq M_1(1 - \phi_{it}^1(\xi)) \quad (4f)$$

$$y_{it}^-(\xi) \leq M_1 \phi_{it}^2(\xi), \quad z_{it}^C(\xi) \leq M_1(1 - \phi_{it}^2(\xi)) \quad (4g)$$

$$z_{it}^C(\xi) \leq M_1 \phi_{it}^3(\xi), \quad z_{it}^D(\xi) \leq M_1(1 - \phi_{it}^3(\xi)) \quad (4h)$$

### 2.2 Aggregation with BTM storage control

$$\max \sum_{t \in T} \left( P_t^{DA} \sum_{i \in I} x_{it} + \mathbb{E} \left[ P_t^{RT}(\xi) \sum_{i \in I} e_{it}^+(\xi) - P_t^{PN} \sum_{i \in I} e_{it}^-(\xi) \right] \right) \quad (5a)$$

$$(5b)$$

$$\text{s.t. } R_{it}(\xi) - x_{it} = y_{it}^+(\xi) - y_{it}^-(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi) \quad (5c)$$

$$R_{it}(\xi) \geq y_{it}^+(\xi) \quad (5d)$$

$$z_{i,t+1}(\xi) = z_{it}(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi) \quad (5e)$$

$$z_{it}^D(\xi) \leq z_{it}(\xi), \quad z_{it}^C(\xi) \leq K_i - z_{it}(\xi), \quad 0 \leq z_{it}(\xi) \leq K_i \quad (5f)$$

$$e_{it}^+(\xi) = y_{it}^+(\xi) - \sum_{j \in I} d_{ijt}(\xi), \quad e_{it}^-(\xi) = y_{it}^-(\xi) - \sum_{j \in I} d_{jit}(\xi) \quad (5g)$$

$$d_{iit}(\xi) = 0 \quad (5h)$$

$$y_{it}^+(\xi) \leq M_1 \phi_{it}^1(\xi), \quad y_{it}^-(\xi) \leq M_1(1 - \phi_{it}^1(\xi)) \quad (5i)$$

$$y_{it}^-(\xi) \leq M_1 \phi_{it}^2(\xi), \quad z_{it}^C(\xi) \leq M_1(1 - \phi_{it}^2(\xi)) \quad (5j)$$

$$z_{it}^C(\xi) \leq M_1 \phi_{it}^3(\xi), \quad z_{it}^D(\xi) \leq M_1(1 - \phi_{it}^3(\xi)) \quad (5k)$$

$$\sum_{i \in I} e_{it}^+(\xi) \leq M_2 \phi_t^4(\xi), \quad \sum_{i \in I} e_{it}^-(\xi) \leq M_2(1 - \phi_t^4(\xi)) \quad (5l)$$

### 2.3 Aggregation with direct control over storage

$$\max \sum_{t \in T} (P_t^{DA} \alpha_t + \mathbb{E} [P_t^{RT}(\xi) \beta_t^+(\xi) - P_t^{PN} \beta_t^-(\xi)]) \quad (6a)$$

$$\text{s.t. } \sum_{i \in I} R_{it}(\xi) - \alpha_t = \beta_t^+(\xi) - \beta_t^-(\xi) + \gamma_t^C(\xi) - \gamma_t^D(\xi) \quad \forall t \quad (6b)$$

$$\sum_{i \in I} R_{it}(\xi) \geq \beta_t^+(\xi) \quad (6c)$$

$$\gamma_t^D(\xi) \leq \gamma_t(\xi), \quad \gamma_t^C(\xi) \leq \sum_{i \in I} K_i - \gamma_t(\xi), \quad 0 \leq \gamma_t(\xi) \leq \sum_{i \in I} K_i \quad \forall t \quad (6d)$$

$$\gamma_{t+1}(\xi) = \gamma_t(\xi) + \gamma_t^C(\xi) - \gamma_t^D(\xi) \quad \forall t \quad (6e)$$

$$\beta_t^+(\xi) \leq M_2 \mu_t(\xi), \quad \beta_t^-(\xi) \leq M_2(1 - \mu_t(\xi)) \quad \forall t \quad (6f)$$

$$\beta_t^-(\xi) \leq M_2 \eta_t(\xi), \quad \gamma_t^C(\xi) \leq M_2(1 - \eta_t(\xi)) \quad \forall t \quad (6g)$$

$$\gamma_t^C(\xi) \leq M_2 \lambda_t(\xi), \quad \gamma_t^D(\xi) \leq M_2(1 - \lambda_t(\xi)) \quad \forall t \quad (6h)$$

### 3 Individual

#### 3.1 Different Internal Price (Non-linear)

$$\max \sum_{t \in T} (P_t^{DA} \cdot x_t + \mathbb{E} [P_t^{RT}(\xi) \cdot y_t^+(\xi) - P_t^{PN} \cdot y_t^-(\xi) + \rho_t^+(d) \cdot d_t^+(\xi) - \rho_t^-(d) \cdot d_t^-(\xi)]) \quad (7a)$$

$$\text{s.t. } R_t(\xi) - x_t = y_t^+(\xi) - y_t^-(\xi) + d_t^+(\xi) - d_t^-(\xi) + z_t^C(\xi) - z_t^D(\xi) \quad (7b)$$

$$R_t(\xi) \geq y_t^+(\xi) + d_{it}^+(\xi) \quad (7c)$$

$$x_t - R_t(\xi) \geq y_t^-(\xi) + d_t^-(\xi) \quad (7d)$$

$$z_{t+1}(\xi) = z_t(\xi) + z_t^C(\xi) - z_t^D(\xi) \quad (7e)$$

$$z_t^D(\xi) \leq z_t(\xi), \quad z_t^C(\xi) \leq K - z_t(\xi), \quad 0 \leq z_t(\xi) \leq K \quad (7f)$$

$$y_t^+(\xi) \leq M\phi_t^1(\xi), \quad y_t^-(\xi) \leq M(1 - \phi_t^1(\xi)) \quad (7g)$$

$$y_t^-(\xi) \leq M\phi_t^2(\xi), \quad z_t^C(\xi) \leq M(1 - \phi_t^2(\xi)) \quad (7h)$$

$$z_t^C(\xi) \leq M\phi_t^3(\xi), \quad z_t^D(\xi) \leq M(1 - \phi_t^3(\xi)) \quad (7i)$$

$$d_t^+(\xi) \leq M\phi_t^4(\xi), \quad d_t^-(\xi) \leq M(1 - \phi_t^4(\xi)) \quad (7j)$$

#### 3.2 Different Internal Price (Piecewise-Linear)

$$\max \sum_{t \in T} \left( P_t^{DA} \cdot x_t + \mathbb{E} \left[ P_t^{RT}(\xi) \cdot y_t^+(\xi) - P_t^{PN} \cdot y_t^-(\xi) + \sum_{b^+ \in B^+} \rho_{t,b^+}^+ \cdot (w_{t,b^+}^+ + u_{t,b^+}^+ \cdot d_{t,b^+}^{min+}) - \sum_{b^- \in B^-} \rho_{t,b^-}^- \cdot (w_{t,b^-}^- + u_{t,b^-}^- \cdot d_{t,b^-}^{min-}) \right] \right) \quad (8a)$$

$$\text{s.t. } d_t^+ = \sum_{b^+ \in B^+} (w_{t,b^+}^+ + u_{t,b^+}^+ \cdot d_{t,b^+}^{min+}) \quad (8b)$$

$$0 \leq w_{t,b^+}^+ \leq u_{t,b^+}^+ \cdot w_{t,b^+}^{max+} \quad (8c)$$

$$\sum_{b^+ \in B^+} u_{t,b^+}^+ = 1 \quad (8d)$$

$$d_t^- = \sum_{b^- \in B^-} (w_{t,b^-}^- + u_{t,b^-}^- \cdot d_{t,b^-}^{min-}) \quad (8e)$$

$$0 \leq w_{t,b^-}^- \leq u_{t,b^-}^- \cdot w_{t,b^-}^{max-} \quad (8f)$$

$$\sum_{b^- \in B^-} u_{t,b^-}^- = 1 \quad (8g)$$

$$R_t(\xi) - x_t = y_t^+(\xi) - y_t^-(\xi) + d_t^+(\xi) - d_t^-(\xi) + z_t^C(\xi) - z_t^D(\xi) \quad (8h)$$

$$R_t(\xi) \geq y_t^+(\xi) + d_t^+(\xi) + z_t^C(\xi) \quad (8i)$$

$$z_{t+1}(\xi) = z_t(\xi) + z_t^C(\xi) - z_t^D(\xi) \quad (8j)$$

$$z_t^D(\xi) \leq z_t(\xi), \quad z_t^C(\xi) \leq K - z_t(\xi), \quad 0 \leq z_t(\xi) \leq K \quad (8k)$$

$$y_t^+(\xi) \leq M\phi_t^1(\xi), \quad y_t^-(\xi) \leq M(1 - \phi_t^1(\xi)) \quad (8l)$$

$$y_t^-(\xi) \leq M\phi_t^2(\xi), \quad z_t^C(\xi) \leq M(1 - \phi_t^2(\xi)) \quad (8m)$$

$$z_t^C(\xi) \leq M\phi_t^3(\xi), \quad z_t^D(\xi) \leq M(1 - \phi_t^3(\xi)) \quad (8n)$$

$$d_t^+(\xi) \leq M\phi_t^4(\xi), \quad d_t^-(\xi) \leq M(1 - \phi_t^4(\xi)) \quad (8o)$$

$$y_t^+(\xi) \leq M\phi_t^5(\xi), \quad d_t^-(\xi) \leq M(1 - \phi_t^5(\xi)) \quad (8p)$$

$$y_t^-(\xi) \leq M\phi_t^6(\xi), \quad d_t^+(\xi) \leq M(1 - \phi_t^6(\xi)) \quad (8q)$$