

DER Models

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1 Without Storage

1.1 Disaggregation

$$\max \sum_{t \in T} (P_t^{DA} x_{it} + \mathbb{E} [P_t^{RT}(\xi) y_{it}^+(\xi) - P_t^{PN} y_{it}^-(\xi)]) \quad (1a)$$

$$\text{s.t. } R_{it}(\xi) - x_{it} = y_{it}^+(\xi) - y_{it}^-(\xi) \quad \forall t \in T \quad (1b)$$

$$R_{it}(\xi) \geq y_{it}^+(\xi) \quad \forall t \in T \quad (1c)$$

$$y_{it}^+(\xi) \leq M z_{it}(\xi), \quad y_{it}^-(\xi) \leq M(1 - z_{it}(\xi)) \quad \forall t \in T \quad (1d)$$

$$x_{it}^{DA} \geq 0, y_{it}^+(\xi) \geq 0, y_{it}^-(\xi) \geq 0, z_{it}(\xi) \in \{0, 1\} \quad \forall t \in T \quad (1e)$$

1.2 Aggregation

$$\max \sum_{t \in T} (P_t^{DA} \alpha_t + \mathbb{E} [P_t^{RT}(\xi) \beta_t^+(\xi) - P_t^{PN} \beta_t^-(\xi)]) \quad (2a)$$

$$\text{s.t. } \sum_{i \in I} R_{it}(\xi) - \alpha_t = \beta_t^+(\xi) - \beta_t^-(\xi) \quad \forall t \in T \quad (2b)$$

$$\sum_{i \in I} R_{it}(\xi) \geq \beta_t^+(\xi) \quad \forall t \in T \quad (2c)$$

$$\beta_t^+(\xi) \leq M z_t(\xi), \quad \beta_t^-(\xi) \leq M(1 - z_t(\xi)) \quad \forall t \in T \quad (2d)$$

$$\alpha_t^{DA} \geq 0, \beta_t^+(\xi) \geq 0, \beta_t^-(\xi) \geq 0, z_t(\xi) \in \{0, 1\} \quad \forall t \in T \quad (2e)$$

1.3 Settlement

$$\max \sum_{t \in T} (P_t^{DA} \alpha_t + \mathbb{E} [P_t^{RT}(\xi) \beta_t^+(\xi) - P_t^{PN} \beta_t^-(\xi)]) \quad (3a)$$

$$\text{s.t. } \sum_{i \in I} R_{it}(\xi) - \alpha_t^{DA} = \beta_t^+(\xi) - \beta_t^-(\xi) \quad \forall t \in T \quad (3b)$$

$$\sum_{i \in I} R_{it}(\xi) \geq \beta_t^+(\xi) \quad \forall t \in T \quad (3c)$$

$$\beta_t^+(\xi) \leq M z_t(\xi), \quad \beta_t^-(\xi) \leq M(1 - z_t(\xi)) \quad \forall t \in T \quad (3d)$$

$$\alpha_t = \sum_{i \in I} x_{it}(\xi), \quad \beta_t^+(\xi) = \sum_{i \in I} e_{it}^+(\xi), \quad \beta_t^-(\xi) = \sum_{i \in I} e_{it}^-(\xi) \quad \forall t \in T \quad (3e)$$

$$R_{it}(\xi) - x_{it}(\xi) = y_{it}^+(\xi) - y_{it}^-(\xi) \quad \forall t \in T \quad (3f)$$

$$R_{it}(\xi) \geq y_{it}^+(\xi) \quad \forall t \in T \quad (3g)$$

$$y_{it}^+(\xi) \leq M z_{it}(\xi), \quad y_{it}^-(\xi) \leq M(1 - z_{it}(\xi)) \quad \forall t \in T \quad (3h)$$

$$\sum_{j \in I, j \neq i} d_{ijt}(\xi) \leq y_{it}^+(\xi), \quad \sum_{j \in I, j \neq i} d_{jit}(\xi) \leq y_{it}^-(\xi) \quad \forall t \in T \quad (3i)$$

$$d_{iit}(\xi) = 0 \quad \forall t \in T \quad (3j)$$

$$e_{it}^+(\xi) = y_{it}^+(\xi) - \sum_{j \in I, j \neq i} d_{ijt}(\xi) \quad \forall t \in T \quad (3k)$$

$$e_{it}^-(\xi) = y_{it}^-(\xi) - \sum_{j \in I, j \neq i} d_{jit}(\xi) \quad \forall t \in T \quad (3l)$$

2 With Storage

2.1 Disaggregation

$$\max \sum_{t \in T} (P_t^{DA} x_{it} + \mathbb{E} [P_t^{RT}(\xi) y_{it}^+(\xi) - P_t^{PN} y_{it}^-(\xi)]) \quad (4a)$$

$$\text{s.t. } R_{it}(\xi) - x_{it} = y_{it}^+(\xi) - y_{it}^-(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi) \quad \forall i, t \quad (4b)$$

$$R_{it}(\xi) \geq y_{it}^+(\xi) \quad \forall i, t \quad (4c)$$

$$z_{i,t+1}(\xi) = z_{it}(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi) \quad \forall i, t \quad (4d)$$

$$z_{it}^D(\xi) \leq z_{it}(\xi) \quad \forall i, t \quad (4e)$$

$$z_{it}^C(\xi) \leq K_i - z_{it}(\xi) \quad \forall i, t \quad (4f)$$

$$0 \leq z_{it}(\xi) \leq K_i \quad \forall i, t \quad (4g)$$

$$y_{it}^+(\xi) \leq M_1 \rho_{it}(\xi), \quad y_{it}^-(\xi) \leq M_1 (1 - \rho_{it}(\xi)) \quad \forall i, t \quad (4h)$$

$$y_{it}^-(\xi) \leq M_1 \delta_{it}(\xi), \quad z_{it}^C(\xi) \leq M_1 (1 - \delta_{it}(\xi)) \quad \forall i, t \quad (4i)$$

$$z_{it}^C(\xi) \leq M_1 \zeta_{it}(\xi), \quad z_{it}^D(\xi) \leq M_1 (1 - \zeta_{it}(\xi)) \quad \forall i, t \quad (4j)$$

2.2 Aggregation

$$\max \sum_{t \in T} (P_t^{DA} \alpha_t + \mathbb{E} [P_t^{RT}(\xi) \beta_t^+(\xi) - P_t^{PN} \beta_t^-(\xi)]) \quad (5a)$$

$$\text{s.t. } \sum_{i \in I} R_{it}(\xi) - \alpha_t = \beta_t^+(\xi) - \beta_t^-(\xi) + \gamma_t^C(\xi) - \gamma_t^D(\xi) \quad \forall t \quad (5b)$$

$$\sum_{i \in I} R_{it}(\xi) \geq \beta_t^+(\xi) \quad \forall t \quad (5c)$$

$$\gamma_t^D(\xi) \leq \gamma_t(\xi) \quad \forall t \quad (5d)$$

$$\gamma_t^C(\xi) \leq \sum_{i \in I} K_i - \gamma_t(\xi) \quad \forall t \quad (5e)$$

$$0 \leq \gamma_t(\xi) \leq \sum_{i \in I} K_i \quad \forall t \quad (5f)$$

$$\gamma_{t+1}(\xi) = \gamma_t(\xi) + \gamma_t^C(\xi) - \gamma_t^D(\xi) \quad \forall t \quad (5g)$$

$$\beta_t^+(\xi) \leq M_2 \mu_t(\xi), \quad \beta_t^-(\xi) \leq M_2 (1 - \mu_t(\xi)) \quad \forall t \quad (5h)$$

$$\beta_t^-(\xi) \leq M_2 \eta_t(\xi), \quad \gamma_t^C(\xi) \leq M_2 (1 - \eta_t(\xi)) \quad \forall t \quad (5i)$$

$$\gamma_t^C(\xi) \leq M_2 \lambda_t(\xi), \quad \gamma_t^D(\xi) \leq M_2 (1 - \lambda_t(\xi)) \quad \forall t \quad (5j)$$

2.3 Settlement

$$\begin{aligned}
& \max \sum_{t \in T} \left(P_t^{DA} \alpha_t + \mathbb{E} \left[P_t^{RT}(\xi) \beta_t^+(\xi) - P_t^{PN} \beta_t^-(\xi) \right] \right) \\
& \text{s.t. } R_{it}(\xi) - x_{it} = y_{it}^+(\xi) - y_{it}^-(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi) \quad \forall i, t \\
& R_{it}(\xi) \geq y_{it}^+(\xi) \quad \forall i, t \\
& z_{it}^D(\xi) \leq z_{it}(\xi) \quad \forall i, t \\
& z_{it}^C(\xi) \leq K_i - z_{it}(\xi) \quad \forall i, t \\
& 0 \leq z_{it}(\xi) \leq K_i \quad \forall i, t \\
& y_{it}^+(\xi) \leq M_1 \rho_{it}(\xi), \quad y_{it}^-(\xi) \leq M_1 (1 - \rho_{it}(\xi)) \quad \forall i, t \\
& y_{it}^-(\xi) \leq M_1 \delta_{it}(\xi), \quad z_{it}^C(\xi) \leq M_1 (1 - \delta_{it}(\xi)) \quad \forall i, t \\
& z_{it}^C(\xi) \leq M_1 \zeta_{it}(\xi), \quad z_{it}^D(\xi) \leq M_1 (1 - \zeta_{it}(\xi)) \quad \forall i, t \\
& e_{it}^+(\xi) = y_{it}^+(\xi) - d_{it}^+(\xi) \\
& e_{it}^-(\xi) = y_{it}^-(\xi) - d_{it}^-(\xi) \\
& e_{it}^C(\xi) = z_{it}^C(\xi) - \widehat{d}_{it}^C(\xi) + \widetilde{d}_{it}^C(\xi) \\
& \widehat{d}_{it}^C(\xi) \leq M_1 q_{it}^3(\xi), \quad \widetilde{d}_{it}^C(\xi) \leq M_1 (1 - q_{it}^3(\xi)) \\
& e_{it}^C(\xi) \leq K_i - z_{it}(\xi) \\
& e_{it}^D(\xi) = z_{it}^D(\xi) - \widehat{d}_{it}^D(\xi) + \widetilde{d}_{it}^D(\xi) \\
& \widehat{d}_{it}^D(\xi) \leq M_1 q_{it}^4(\xi), \quad \widetilde{d}_{it}^D(\xi) \leq M_1 (1 - q_{it}^4(\xi)) \\
& e_{it}^D(\xi) \leq z_{it}(\xi) \\
& e_{it}^+(\xi) \leq M_1 q_{it}^5(\xi), \quad e_{it}^-(\xi) \leq M_1 (1 - q_{it}^5(\xi)) \\
& e_{it}^-(\xi) \leq M_1 q_{it}^6(\xi), \quad e_{it}^C(\xi) \leq M_1 (1 - q_{it}^6(\xi)) \\
& e_{it}^C(\xi) \leq M_1 q_{it}^6(\xi), \quad e_{it}^D(\xi) \leq M_1 (1 - q_{it}^6(\xi)) \\
& z_{i,t+1}(\xi) = z_{it}(\xi) + e_{it}^C(\xi) - e_{it}^D(\xi) \quad \forall i, t \\
& \alpha_t = \sum_{i \in I} x_{it}, \quad \beta_t^+(\xi) = \sum_{i \in I} e_{it}^+(\xi), \quad \beta_t^-(\xi) = \sum_{i \in I} e_{it}^-(\xi) \\
& \gamma_t(\xi) = \sum_{i \in I} z_{it}(\xi), \quad \gamma_t^C(\xi) = \sum_{i \in I} e_{it}^C(\xi), \quad \gamma_t^D(\xi) = \sum_{i \in I} e_{it}^D(\xi) \\
& \sum_{i \in I} R_{it}(\xi) - \alpha_t = \beta_t^+(\xi) - \beta_t^-(\xi) + \gamma_t^C(\xi) - \gamma_t^D(\xi) \quad \forall t \\
& \sum_{i \in I} R_{it}(\xi) \geq \beta_t^+(\xi) \quad \forall t \\
& \gamma_t^D(\xi) \leq \gamma_t(\xi) \quad \forall t \\
& \gamma_t^C(\xi) \leq \sum_{i \in I} K_i - \gamma_t(\xi) \quad \forall t \\
& 0 \leq \gamma_t(\xi) \leq \sum_{i \in I} K_i \quad \forall t \\
& \gamma_{t+1}(\xi) = \gamma_t(\xi) + \gamma_t^C(\xi) - \gamma_t^D(\xi) \quad \forall t \\
& \beta_t^+(\xi) \leq M_2 \mu_t(\xi), \quad \beta_t^-(\xi) \leq M_2 (1 - \mu_t(\xi)) \quad \forall t \\
& \beta_t^-(\xi) \leq M_2 \eta_t(\xi), \quad \gamma_t^C(\xi) \leq M_2 (1 - \eta_t(\xi)) \quad \forall t \\
& \gamma_t^C(\xi) \leq M_2 \lambda_t(\xi), \quad \gamma_t^D(\xi) \leq M_2 (1 - \lambda_t(\xi)) \quad \forall t \\
& \sum_{i \in I} \widehat{d}_{it}^C(\xi) \leq \sum_{i \in I} y_{it}^-(\xi) \\
& (K_i - z_{it}(\xi) - z_{it}^C(\xi)) - \left(d_{it}^+(\xi) - \sum_{i \in I} y_{it}^-(\xi) \right) \leq M_1 (1 - q_{it}^6(\xi)) - \epsilon \\
& \left(d_{it}^+(\xi) - \sum_{i \in I} y_{it}^-(\xi) \right) - e_{it}^C(\xi) \leq M_1 q_{it}^6(\xi) \\
& \left(d_{it}^+(\xi) - \sum_{i \in I} y_{it}^-(\xi) \right) - (K_i - z_{it}(\xi)) \leq M_1 (1 - q_{it}^7(\xi)) - \epsilon \\
& (K_i - z_{it}(\xi)) - e_{it}^C(\xi) \leq M_1 q_{it}^7(\xi)
\end{aligned}$$