DER Models

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1 Without Storage

1.1 Disaggregation

$$\max \quad \sum_{t\in T}\left(P_t^{DA}x_{it}+\mathbb{E}\left[P_t^{RT}(\xi)y_{it}^{+}(\xi)-P_t^{PN}y_{it}^{-}(\xi)\right]\right) \tag{1a}$$

$$\text{s.t.} \quad R_{it}(\xi)-x_{it}=y_{it}^{+}(\xi)-y_{it}^{-}(\xi) \quad \forall t\in T \tag{1b}$$

$$R_{it}(\xi)\geqslant y_{it}^{+}(\xi) \quad \forall t\in T \tag{1c}$$

$$y_{it}^{+}(\xi)\leqslant Mz_{it}(\xi), \quad y_{it}^{-}(\xi)\leqslant M(1-z_{it}(\xi)) \quad \forall t\in T \tag{1d}$$

$$x_{it}^{DA}\geqslant 0, y_{it}^{+}(\xi)\geqslant 0, y_{it}^{-}(\xi)\geqslant 0, z_{it}(\xi)\in\{0,1\} \quad \forall t\in T \tag{1e}$$

1.2 Aggregation

$$\max \quad \sum_{t\in T}\left(P_t^{DA}\alpha_t+\mathbb{E}\left[P_t^{RT}(\xi)\beta_t^{+}(\xi)-P_t^{PN}\beta_t^{-}(\xi)\right]\right) \tag{2a}$$

$$\text{s.t.} \quad \sum_{i\in I}R_{it}(\xi)-\alpha_t=\beta_t^{+}(\xi)-\beta_t^{-}(\xi) \quad \forall t\in T \tag{2b}$$

$$\sum_{i\in I}R_{it}(\xi)\geqslant \beta_t^{+}(\xi) \quad \forall t\in T \tag{2c}$$

$$\beta_t^{+}(\xi)\leqslant Mz_t(\xi), \quad \beta_t^{-}(\xi)\leqslant M(1-z_t(\xi)) \quad \forall t\in T \tag{2d}$$

$$\alpha_t^{DA}\geqslant 0, \beta_t^{+}(\xi)\geqslant 0, \beta_t^{-}(\xi)\geqslant 0, z_t(\xi)\in\{0,1\} \quad \forall t\in T \tag{2e}$$

1.3 Settlement

$$\max \quad \sum_{t\in T}\left(P_t^{DA}\alpha_t+\mathbb{E}\left[P_t^{RT}(\xi)\beta_t^{+}(\xi)-P_t^{PN}\beta_t^{-}(\xi)\right]\right) \tag{3a}$$

$$\text{s.t.} \quad \sum_{i\in I}R_{it}(\xi)-\alpha_t^{DA}=\beta_t^{+}(\xi)-\beta_t^{-}(\xi) \quad \forall t\in T \tag{3b}$$

$$\sum_{i\in I}R_{it}(\xi)\geqslant \beta_t^{+}(\xi) \quad \forall t\in T \tag{3c}$$

$$\beta_t^{+}(\xi)\leqslant Mz_t(\xi), \quad \beta_t^{-}(\xi)\leqslant M(1-z_t(\xi)) \quad \forall t\in T \tag{3d}$$

$$\alpha_t=\sum_{i\in I}x_{it}(\xi), \quad \beta_t^{+}(\xi)=\sum_{i\in I}e_{it}^{+}(\xi), \quad \beta_t^{-}(\xi)=\sum_{i\in I}e_{it}^{-}(\xi) \quad \forall t\in T \tag{3e}$$

$$R_{it}(\xi)-x_{it}(\xi)=y_{it}^{+}(\xi)-y_{it}^{-}(\xi) \quad \forall t\in T \tag{3f}$$

$$R_{it}(\xi)\geqslant y_{it}^{+}(\xi) \quad \forall t\in T \tag{3g}$$

$$y_{it}^{+}(\xi)\leqslant Mz_{it}(\xi), \quad y_{it}^{-}(\xi)\leqslant M(1-z_{it}(\xi)) \quad \forall t\in T \tag{3h}$$

$$\sum_{j\in I,j\neq i}d_{ijt}(\xi)\leqslant y_{it}^{+}(\xi), \quad \sum_{j\in I,j\neq i}d_{jit}(\xi)\leqslant y_{it}^{-}(\xi) \quad \forall t\in T \tag{3i}$$

$$d_{iit}(\xi)=0 \quad \forall t\in T \tag{3j}$$

$$e_{it}^{+}(\xi)=y_{it}^{+}(\xi)-\sum_{j\in I,j\neq i}d_{ijt}(\xi) \quad \forall t\in T \tag{3k}$$

$$e_{it}^{-}(\xi)=y_{it}^{-}(\xi)-\sum_{j\in I,j\neq i}d_{jit}(\xi) \quad \forall t\in T \tag{3l}$$

2 With Storage

2.1 Disaggregation

$$\max \quad \sum_{t\in T}\left(P_t^{DA}x_{it}+\mathbb{E}\left[P_t^{RT}(\xi)y_{it}^{+}(\xi)-P_t^{PN}y_{it}^{-}(\xi)\right]\right) \tag{4a}$$

$$\text{s.t.} \quad R_{it}(\xi)-x_{it}=y_{it}^{+}(\xi)-y_{it}^{-}(\xi)+z_{it}^{C}(\xi)-z_{it}^{D}(\xi) \quad \forall i,t \tag{4b}$$

$$R_{it}(\xi)\geqslant y_{it}^{+}(\xi) \quad \forall i,t \tag{4c}$$

$$z_{i,t+1}(\xi)=z_{it}(\xi)+z_{it}^{C}(\xi)-z_{it}^{D}(\xi) \quad \forall i,t \tag{4d}$$

$$z_{it}^{D}(\xi)\leqslant z_{it}(\xi) \quad \forall i,t \tag{4e}$$

$$z_{it}^{C}(\xi)\leqslant K_i-z_{it}(\xi) \quad \forall i,t \tag{4f}$$

$$0\leqslant z_{it}(\xi)\leqslant K_i \quad \forall i,t \tag{4g}$$

$$y_{it}^{+}(\xi)\leqslant M_1\rho_{it}(\xi), \quad y_{it}^{-}(\xi)\leqslant M_1(1-\rho_{it}(\xi)) \quad \forall i,t \tag{4h}$$

$$y_{it}^{-}(\xi)\leqslant M_1\delta_{it}(\xi), \quad z_{it}^{C}(\xi)\leqslant M_1(1-\delta_{it}(\xi)) \quad \forall i,t \tag{4i}$$

$$z_{it}^{C}(\xi)\leqslant M_1\zeta_{it}(\xi), \quad z_{it}^{D}(\xi)\leqslant M_1(1-\zeta_{it}(\xi)) \quad \forall i,t \tag{4j}$$

2.2 Aggregation

$$\max \quad \sum_{t\in T}\left(P_t^{DA}\alpha_t+\mathbb{E}\left[P_t^{RT}(\xi)\beta_t^{+}(\xi)-P_t^{PN}\beta_t^{-}(\xi)\right]\right) \tag{5a}$$

$$\text{s.t.} \quad \sum_{i\in I}R_{it}(\xi)-\alpha_t=\beta_t^{+}(\xi)-\beta_t^{-}(\xi)+\gamma_t^{C}(\xi)-\gamma_t^{D}(\xi) \quad \forall t \tag{5b}$$

$$\sum_{i\in I}R_{it}(\xi)\geqslant \beta_t^{+}(\xi) \quad \forall t \tag{5c}$$

$$\gamma_t^{D}(\xi)\leqslant \gamma_t(\xi) \quad \forall t \tag{5d}$$

$$\gamma_t^{C}(\xi)\leqslant \sum_{i\in I}K_i-\gamma_t(\xi) \quad \forall t \tag{5e}$$

$$0\leqslant \gamma_t(\xi)\leqslant \sum_{i\in I}K_i \quad \forall t \tag{5f}$$

$$\gamma_{t+1}(\xi)=\gamma_t(\xi)+\gamma_t^{C}(\xi)-\gamma_t^{D}(\xi) \quad \forall t \tag{5g}$$

$$\beta_t^{+}(\xi)\leqslant M_2\mu_t(\xi), \quad \beta_t^{-}(\xi)\leqslant M_2(1-\mu_t(\xi)) \quad \forall t \tag{5h}$$

$$\beta_t^{-}(\xi)\leqslant M_2\eta_t(\xi), \quad \gamma_t^{C}(\xi)\leqslant M_2(1-\eta_t(\xi)) \quad \forall t \tag{5i}$$

$$\gamma_t^{C}(\xi)\leqslant M_2\lambda_t(\xi), \quad \gamma_t^{D}(\xi)\leqslant M_2(1-\lambda_t(\xi)) \quad \forall t \tag{5j}$$

2.3 Settlement

$$\max \quad \sum_{t\in T}\left(P_t^{DA}\alpha_t + \mathbb{E}\left[P_t^{RT}(\xi)\beta_t^+(\xi) - P_t^{PN}\beta_t^-(\xi))\right]\right)$$

$$\begin{aligned}
\text{s.t.}\quad & R_{it}(\xi) - x_{it} = y_{it}^+(\xi) - y_{it}^-(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi) \quad \forall i,t\\
& R_{it}(\xi) \geqslant y_{it}^+(\xi) \quad \forall i,t\\
& z_{it}^D(\xi) \leqslant z_{it}(\xi) \quad \forall i,t\\
& z_{it}^C(\xi) \leqslant K_i - z_{it}(\xi) \quad \forall i,t\\
& 0 \leqslant z_{it}(\xi) \leqslant K_i \quad \forall i,t\\
& y_{it}^+(\xi) \leqslant M_1\rho_{it}(\xi), \quad y_{it}^-(\xi) \leqslant M_1(1-\rho_{it}(\xi)) \quad \forall i,t\\
& y_{it}^-(\xi) \leqslant M_1\delta_{it}(\xi), \quad z_{it}^C(\xi) \leqslant M_1(1-\delta_{it}(\xi)) \quad \forall i,t\\
& z_{it}^C(\xi) \leqslant M_1\zeta_{it}(\xi), \quad z_{it}^D(\xi) \leqslant M_1(1-\zeta_{it}(\xi)) \quad \forall i,t\\
\\
& e_{it}^+(\xi) = y_{it}^+(\xi) - d_{it}^+(\xi)\\
& e_{it}^-(\xi) = y_{it}^-(\xi) - d_{it}^-(\xi)\\
& e_{it}^C(\xi) = z_{it}^C(\xi) - \widehat{d_{it}^C}(\xi) + \widetilde{d_{it}^C}(\xi)\\
& \widehat{d_{it}^C}(\xi) \leqslant M_1 q_{it}^3(\xi), \quad \widetilde{d_{it}^C}(\xi) \leqslant M_1(1-q_{it}^3(\xi))\\
& e_{it}^C(\xi) \leqslant K_i - z_{it}(\xi)\\
& e_{it}^D(\xi) = z_{it}^D(\xi) - \widehat{d_{it}^D}(\xi) + \widetilde{d_{it}^D}(\xi)\\
& \widehat{d_{it}^D}(\xi) \leqslant M_1 q_{it}^4(\xi), \quad \widetilde{d_{it}^D}(\xi) \leqslant M_1(1-q_{it}^4(\xi))\\
& e_{it}^D(\xi) \leqslant z_{it}(\xi)\\
& e_{it}^+(\xi) \leqslant M_1 q_{it}^5(\xi), \quad e_{it}^-(\xi) \leqslant M_1(1-q_{it}^5(\xi))\\
& e_{it}^-(\xi) \leqslant M_1 q_{it}^6(\xi), \quad e_{it}^C(\xi) \leqslant M_1(1-q_{it}^6(\xi))\\
& e_{it}^C(\xi) \leqslant M_1 q_{it}^6(\xi), \quad e_{it}^D(\xi) \leqslant M_1(1-q_{it}^6(\xi))\\
\\
& z_{i,t+1}(\xi) = z_{it}(\xi) + e_{it}^C(\xi) - e_{it}^D(\xi) \quad \forall i,t\\
& \alpha_t = \sum_{i\in I} x_{it}, \quad \beta_t^+(\xi) = \sum_{i\in I} e_{it}^+(\xi), \quad \beta_t^-(\xi) = \sum_{i\in I} e_{it}^-(\xi)\\
& \gamma_t(\xi) = \sum_{i\in I} z_{it}(\xi), \quad \gamma_t^C(\xi) = \sum_{i\in I} e_{it}^C(\xi), \quad \gamma_t^D(\xi) = \sum_{i\in I} e_{it}^D(\xi)\\
\\
& \sum_{i\in I} R_{it}(\xi) - \alpha_t = \beta_t^+(\xi) - \beta_t^-(\xi) + \gamma_t^C(\xi) - \gamma_t^D(\xi) \quad \forall t\\
& \sum_{i\in I} R_{it}(\xi) \geqslant \beta_t^+(\xi) \quad \forall t\\
& \gamma_t^D(\xi) \leqslant \gamma_t(\xi) \quad \forall t\\
& \gamma_t^C(\xi) \leqslant \sum_{i\in I} K_i - \gamma_t(\xi) \quad \forall t\\
& 0 \leqslant \gamma_t(\xi) \leqslant \sum_{i\in I} K_i \quad \forall t\\
& \gamma_{t+1}(\xi) = \gamma_t(\xi) + \gamma_t^C(\xi) - \gamma_t^D(\xi) \quad \forall t\\
& \beta_t^+(\xi) \leqslant M_2\mu_t(\xi), \quad \beta_t^-(\xi) \leqslant M_2(1-\mu_t(\xi)) \quad \forall t\\
& \beta_t^-(\xi) \leqslant M_2\eta_t(\xi), \quad \gamma_t^C(\xi) \leqslant M_2(1-\eta_t(\xi)) \quad \forall t\\
& \gamma_t^C(\xi) \leqslant M_2\lambda_t(\xi), \quad \gamma_t^D(\xi) \leqslant M_2(1-\lambda_t(\xi)) \quad \forall t\\
\\
& \sum_{i\in I} \widehat{d_{it}^C}(\xi) \leqslant \sum_{i\in I} y_{it}^-(\xi)\\
& \sum_{i\in I} \widetilde{d_{it}^D}(\xi) \leqslant \sum_{i\in I} y_{it}^-(\xi)\\
& \left(d_{it}^+(\xi) - \sum_{i\in I} y_{it}^-(\xi)\right) - (K_i - z_{it}(\xi)) \leqslant M_1(1-q_{it}^6(\xi)) - \epsilon\\
& (K_i - z_{it}(\xi)) - e_{it}^C(\xi) \leqslant M_1 q_{it}^6(\xi)\\
& (K_i - z_{it}(\xi)) - \left(d_{it}^+(\xi) - \sum_{i\in I} y_{it}^-(\xi)\right) \leqslant M_1(1-q_{it}^7(\xi)) - \epsilon\\
& \left(e_{it}^C(\xi)\right) - \left(d_{it}^+(\xi) + z_{it}^c(\xi)\right) \leqslant M_1 q_{it}^7(\xi)
\end{aligned}$$