# **DER Models**

Seohyun Jang

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# 1 Without Storage

## 1.1 Disaggregation

$$\max \sum_{t \in T} \left( P_t^{DA} x_{it} + \mathbb{E} \left[ P_t^{RT}(\xi) y_{it}^+(\xi) - P_t^{PN} y_{it}^-(\xi) \right] \right)$$
s.t.  $R_{it}(\xi) - x_{it} = y_{it}^+(\xi) - y_{it}^-(\xi) \quad \forall t \in T$  (1b)

$$R_{it}(\xi) \geqslant y_{it}^+(\xi) \quad \forall t \in T$$
 (1c)

$$y_{it}^+(\xi) \leqslant M z_{it}(\xi), \quad y_{it}^-(\xi) \leqslant M (1 - z_{it}(\xi)) \quad \forall t \in T$$

$$x_{it}^{DA} \geqslant 0, y_{it}^{+}(\xi) \geqslant 0, y_{it}^{-}(\xi) \geqslant 0, z_{it}(\xi) \in \{0, 1\} \quad \forall t \in T$$
 (1e)

## 1.2 Aggregation

$$\max \sum_{t \in T} \left( P_t^{DA} \alpha_t + \mathbb{E} \left[ P_t^{RT}(\xi) \beta_t^+(\xi) - P_t^{PN} \beta_t^-(\xi) \right] \right)$$
 (2a)

s.t. 
$$\sum_{i \in I} R_{it}(\xi) - \alpha_t = \beta_t^+(\xi) - \beta_t^-(\xi) \quad \forall t \in T$$
 (2b)

$$\sum_{i \in I} R_{it}(\xi) \geqslant \beta_t^+(\xi) \quad \forall t \in T$$
 (2c)

$$\beta_t^+(\xi) \leqslant M z_t(\xi), \quad \beta_t^-(\xi) \leqslant M(1 - z_t(\xi)) \quad \forall t \in T$$
(2d)

$$\alpha_t^{DA} \ge 0, \beta_t^+(\xi) \ge 0, \beta_t^-(\xi) \ge 0, z_t(\xi) \in \{0, 1\} \quad \forall t \in T$$
 (2e)

#### 1.3 Settlement

$$\max \sum_{t \in T} \left( P_t^{DA} \alpha_t + \mathbb{E} \left[ P_t^{RT}(\xi) \beta_t^+(\xi) - P_t^{PN} \beta_t^-(\xi) \right] \right)$$
 (3a)

s.t. 
$$\sum_{i \in I} R_{it}(\xi) - \alpha_t^{DA} = \beta_t^+(\xi) - \beta_t^-(\xi) \quad \forall t \in T$$
 (3b)

$$\sum_{i \in I} R_{it}(\xi) \geqslant \beta_t^+(\xi) \quad \forall t \in T$$
 (3c)

$$\beta_t^+(\xi) \leqslant M z_t(\xi), \quad \beta_t^-(\xi) \leqslant M (1 - z_t(\xi)) \quad \forall t \in T$$
 (3d)

$$\alpha_t = \sum_{i \in I} x_{it}(\xi), \quad \beta_t^+(\xi) = \sum_{i \in I} e_{it}^+(\xi), \quad \beta_t^-(\xi) = \sum_{i \in I} e_{it}^-(\xi) \quad \forall t \in T$$
 (3e)

$$R_{it}(\xi) - x_{it}(\xi) = y_{it}^{+}(\xi) - y_{it}^{-}(\xi) \quad \forall t \in T$$
 (3f)

$$R_{it}(\xi) \geqslant y_{it}^+(\xi) \quad \forall t \in T$$
 (3g)

$$y_{it}^+(\xi) \leqslant M z_{it}(\xi), \quad y_{it}^-(\xi) \leqslant M (1 - z_{it}(\xi)) \quad \forall t \in T$$

$$\sum_{j \in I, j \neq i} d_{ijt}(\xi) \leqslant y_{it}^{+}(\xi), \quad \sum_{j \in I, j \neq i} d_{jit}(\xi) \leqslant y_{it}^{-}(\xi) \quad \forall t \in T$$
(3i)

$$d_{iit}(\xi) = 0 \quad \forall t \in T$$

$$e_{it}^{+}(\xi) = y_{it}^{+}(\xi) - \sum_{j \in I, j \neq i} d_{ijt}(\xi) \quad \forall t \in T$$
 (3k)

$$e_{it}^{-}(\xi) = y_{it}^{-}(\xi) - \sum_{j \in I, j \neq i} d_{jit}(\xi) \quad \forall t \in T$$
 (31)

# 2 With Storage

### 2.1 Disaggregation

$$\max \quad \sum_{t \in T} \left( P_t^{DA} x_{it} + \mathbb{E} \left[ P_t^{RT}(\xi) y_{it}^+(\xi) - P_t^{PN} y_{it}^-(\xi) \right] \right)$$
 (4a) 
$$\text{s.t.} \quad R_{it}(\xi) - x_{it} = y_{it}^+(\xi) - y_{it}^-(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi) \quad \forall i, t$$
 (4b) 
$$R_{it}(\xi) \geqslant y_{it}^+(\xi) \quad \forall i, t$$
 (4c) 
$$z_{i,t+1}(\xi) = z_{it}(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi) \quad \forall i, t$$
 (4d) 
$$z_{it}^D(\xi) \leqslant z_{it}(\xi) \quad \forall i, t$$
 (4e) 
$$z_{it}^C(\xi) \leqslant K_i - z_{it}(\xi) \quad \forall i, t$$
 (4f) 
$$0 \leqslant z_{it}(\xi) \leqslant K_i \quad \forall i, t$$
 (4g) 
$$y_{it}^+(\xi) \leqslant M_1 \rho_{it}(\xi), \quad y_{it}^-(\xi) \leqslant M_1 (1 - \rho_{it}(\xi)) \quad \forall i, t$$
 (4h) 
$$y_{it}^C(\xi) \leqslant M_1 \delta_{it}(\xi), \quad z_{it}^D(\xi) \leqslant M_1 (1 - \delta_{it}(\xi)) \quad \forall i, t$$
 (4j) 
$$z_{it}^C(\xi) \leqslant M_1 \zeta_{it}(\xi), \quad z_{it}^D(\xi) \leqslant M_1 (1 - \zeta_{it}(\xi)) \quad \forall i, t$$
 (4j)

### 2.2 Aggregation

$$\max \quad \sum_{t \in T} \left( P_t^{DA} \alpha_t + \mathbb{E} \left[ P_t^{RT}(\xi) \beta_t^+(\xi) - P_t^{PN} \beta_t^-(\xi) \right) \right] \right) \tag{5a}$$

$$\text{s.t.} \quad \sum_{i \in I} R_{it}(\xi) - \alpha_t = \beta_t^+(\xi) - \beta_t^-(\xi) + \gamma_t^C(\xi) - \gamma_t^D(\xi) \quad \forall t \tag{5b}$$

$$\sum_{i \in I} R_{it}(\xi) \geqslant \beta_t^+(\xi) \quad \forall t \tag{5c}$$

$$\gamma_t^D(\xi) \leqslant \gamma_t(\xi) \quad \forall t \tag{5d}$$

$$\gamma_t^C(\xi) \leqslant \sum_{i \in I} K_i - \gamma_t(\xi) \quad \forall t \tag{5e}$$

$$0 \leqslant \gamma_t(\xi) \leqslant \sum_{i \in I} K_i \quad \forall t \tag{5f}$$

$$\gamma_{t+1}(\xi) = \gamma_t(\xi) + \gamma_t^C(\xi) - \gamma_t^D(\xi) \quad \forall t \tag{5g}$$

$$\beta_t^+(\xi) \leqslant M_2 \mu_t(\xi), \quad \beta_t^-(\xi) \leqslant M_2 (1 - \mu_t(\xi)) \quad \forall t \tag{5h}$$

$$\beta_t^-(\xi) \leqslant M_2 \eta_t(\xi), \quad \gamma_t^C(\xi) \leqslant M_2 (1 - \eta_t(\xi)) \quad \forall t \tag{5i}$$

(5j)

 $\gamma_t^C(\xi) \leqslant M_2 \lambda_t(\xi), \quad \gamma_t^D(\xi) \leqslant M_2 (1 - \lambda_t(\xi)) \quad \forall t$ 

#### 2.3 Settlement

$$\max \quad \sum_{t \in T} \left( P_t^{DA} \alpha_t + \mathbb{E} \left[ P_t^{RT}(\xi) \beta_t^+(\xi) - P_t^{PN} \beta_t^-(\xi) ) \right] \right)$$

s.t. 
$$R_{it}(\xi) - x_{it} = y_{it}^{+}(\xi) - y_{it}^{-}(\xi) + z_{it}^{C}(\xi) - z_{it}^{D}(\xi) \quad \forall i, t$$
  
 $R_{it}(\xi) \geqslant y_{it}^{+}(\xi) \quad \forall i, t$ 

$$z_{it}^D(\xi) \leqslant z_{it}(\xi) \quad \forall i, t$$

$$z_{it}^D(\xi) \leqslant z_{it}(\xi) \quad \forall i, t$$

$$z_{it}^C(\xi) \leqslant K_i - z_{it}(\xi) \quad \forall i, t$$

$$0 \leqslant z_{it}(\xi) \leqslant K_i \quad \forall i, t$$

$$y_{it}^{+}(\xi) \leqslant M_1 \rho_{it}(\xi), \quad y_{it}^{-}(\xi) \leqslant M_1 (1 - \rho_{it}(\xi)) \quad \forall i, t$$

$$y_{it}^-(\xi) \leqslant M_1 \delta_{it}(\xi), \quad z_{it}^C(\xi) \leqslant M_1 (1 - \delta_{it}(\xi)) \quad \forall i, t$$

$$z_{it}^C(\xi) \leqslant M_1 \zeta_{it}(\xi), \quad z_{it}^D(\xi) \leqslant M_1 (1 - \zeta_{it}(\xi)) \quad \forall i, t$$

$$e_{it}^+(\xi) = y_{it}^+(\xi) - d_{it}^+(\xi)$$

$$e_{it}^{-}(\xi) = y_{it}^{-}(\xi) - d_{it}^{-}(\xi)$$

$$e_{it}^{C}(\xi) = z_{it}^{C}(\xi) - \widehat{d_{it}^{C}}(\xi) + \widecheck{d_{it}^{C}}(\xi)$$

$$\widehat{d_{it}^C}(\xi) \le M_1 q_{it}^3(\xi), \quad \widecheck{d_{it}^C}(\xi) \le M_1 (1 - q_{it}^3(\xi))$$

$$e_{it}^C(\xi) \leqslant K_i - z_{it}(\xi)$$

$$e_{it}^{D}(\xi) = z_{it}^{D}(\xi) - \widehat{d_{it}^{D}}(\xi) + \widecheck{d_{it}^{D}}(\xi)$$

$$\widehat{d_{it}^D}(\xi) \leqslant M_1 q_{it}^4(\xi), \quad \widecheck{d_{it}^D}(\xi) \leqslant M_1 (1 - q_{it}^4(\xi))$$

$$e_{it}^D(\xi) \leqslant z_{it}(\xi)$$

$$e_{it}^+(\xi) \leqslant M_1 q_{it}^5(\xi), \quad e_{it}^-(\xi) \leqslant M_1 (1 - q_{it}^5(\xi))$$

$$e_{it}^{-}(\xi) \leq M_1 q_{it}^{6}(\xi), \quad e_{it}^{C}(\xi) \leq M_1 (1 - q_{it}^{6}(\xi))$$

$$e_{it}^C(\xi) \leq M_1 q_{it}^6(\xi), \quad e_{it}^D(\xi) \leq M_1 (1 - q_{it}^6(\xi))$$

$$z_{i,t+1}(\xi) = z_{it}(\xi) + e_{it}^{C}(\xi) - e_{it}^{D}(\xi) \quad \forall i, t$$

$$\alpha_t = \sum_{i \in I} x_{it}, \quad \beta_t^+(\xi) = \sum_{i \in I} e_{it}^+(\xi), \quad \beta_t^-(\xi) = \sum_{i \in I} e_{it}^-(\xi)$$

$$\gamma_t(\xi) = \sum_{i \in I} z_{it}(\xi), \quad \gamma_t^C(\xi) = \sum_{i \in I} e_{it}^C(\xi), \quad \gamma_t^D(\xi) = \sum_{i \in I} e_{it}^D(\xi)$$

$$\sum_{i \in I} R_{it}(\xi) - \alpha_t = \beta_t^+(\xi) - \beta_t^-(\xi) + \gamma_t^C(\xi) - \gamma_t^D(\xi) \quad \forall t$$

$$\sum_{i=1}^{n} R_{it}(\xi) \geqslant \beta_t^+(\xi) \quad \forall t$$

$$\gamma_t^D(\xi) \leqslant \gamma_t(\xi) \quad \forall t$$

$$\gamma_t^C(\xi) \leqslant \sum_{i \in I} K_i - \gamma_t(\xi) \quad \forall t$$

$$0 \leqslant \gamma_t(\xi) \leqslant \sum_{i \in I} K_i \quad \forall t$$

$$\gamma_{t+1}(\xi) = \gamma_t(\xi) + \gamma_t^C(\xi) - \gamma_t^D(\xi) \quad \forall t$$

$$\beta_t^+(\xi) \leqslant M_2 \mu_t(\xi), \quad \beta_t^-(\xi) \leqslant M_2 (1 - \mu_t(\xi)) \quad \forall t$$

$$\beta_t^-(\xi) \leqslant M_2 \eta_t(\xi), \quad \gamma_t^C(\xi) \leqslant M_2 (1 - \eta_t(\xi)) \quad \forall t$$

$$\gamma_t^C(\xi) \leqslant M_2 \lambda_t(\xi), \quad \gamma_t^D(\xi) \leqslant M_2(1 - \lambda_t(\xi)) \quad \forall t$$

$$\sum_{i \in I} \widehat{d_{it}^C}(\xi) \leqslant \sum_{i \in I} y_{it}^-(\xi)$$

$$\sum_{i \in I} \widecheck{d_{it}^D}(\xi) \leqslant \sum_{i \in I} y_{it}^-(\xi)$$

$$\left(d_{it}^{+}(\xi) - \sum_{i \in I} y_{it}^{-}(\xi)\right) - (K_i - z_{it}(\xi)) \leqslant M_1(1 - q_{it}^6(\xi)) - \epsilon$$

$$(K_i - z_{it}(\xi)) - e_{it}^C(\xi) \le M_1 q_{it}^6(\xi)$$

$$(K_i - z_{it}(\xi)) - \left(d_{it}^+(\xi) - \sum_{i \in I} y_{it}^-(\xi)\right) \le M_1(1 - q_{it}^7(\xi)) - \epsilon$$

$$\left(e_{it}^C(\xi)\right) - \left(d_{it}^+(\xi) + z_{it}^c(\xi)\right) \leqslant M_1 q_{it}^7(\xi)$$

# 3 With Individual Storage

# 3.1 RT Storage Dispatching: for each i

$$\max \quad \sum_{t \in T} \left( P_t^{DA} x_{it} + \mathbb{E} \left[ P_t^{RT}(\xi) y_{it}^+(\xi) - P_t^{PN} y_{it}^-(\xi) \right] \right)$$
 (7a) 
$$\text{s.t.} \quad R_{it}(\xi) - x_{it} = y_{it}^+(\xi) - y_{it}^-(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi) \quad \forall i, t$$
 (7b) 
$$R_{it}(\xi) \geqslant y_{it}^+(\xi) \quad \forall i, t$$
 (7c) 
$$z_{i,t+1}(\xi) = z_{it}(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi) \quad \forall i, t$$
 (7d) 
$$z_{it}^D(\xi) \leqslant z_{it}(\xi) \quad \forall i, t$$
 (7e) 
$$z_{it}^C(\xi) \leqslant K_i - z_{it}(\xi) \quad \forall i, t$$
 (7f) 
$$0 \leqslant z_{it}(\xi) \leqslant K_i \quad \forall i, t$$
 (7g) 
$$y_{it}^+(\xi) \leqslant M_1 \rho_{it}(\xi), \quad y_{it}^-(\xi) \leqslant M_1 (1 - \rho_{it}(\xi)) \quad \forall i, t$$
 (7h) 
$$y_{it}^-(\xi) \leqslant M_1 \delta_{it}(\xi), \quad z_{it}^C(\xi) \leqslant M_1 (1 - \delta_{it}(\xi)) \quad \forall i, t$$
 (7j) 
$$z_{it}^C(\xi) \leqslant M_1 \zeta_{it}(\xi), \quad z_{it}^D(\xi) \leqslant M_1 (1 - \zeta_{it}(\xi)) \quad \forall i, t$$
 (7j)

# 3.2 RT Storage Dispatching with aggregation

$$\begin{split} \max \quad & \sum_{t \in T} \left( P_t^{DA} \alpha_t + \mathbb{E} \left[ P_t^{RT}(\xi) \beta_t^+(\xi) - P_t^{PN} \beta_{it}^-(\xi) \right] \right) \\ \text{s.t.} \quad & R_{it}(\xi) - x_{it} = y_{it}^+(\xi) - y_{it}^-(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi), \quad R_{it}(\xi) \geqslant y_{it}^+(\xi) \\ & y_{it}^+(\xi) \leqslant M_1 \rho_{it}(\xi), \quad y_{it}^-(\xi) \leqslant M_1 (1 - \rho_{it}(\xi)) \end{split}$$

$$g_{it}(\zeta) < M_1 p_{it}(\zeta), \quad g_{it}(\zeta) < M_1 (1 - p_{it}(\zeta)),$$
 $g_{it}(\zeta) < M_2 (\zeta) < M_3 (\zeta) < M_4 (1 - \delta_3(\zeta)),$ 

$$y_{it}^-(\xi) \leqslant M_1 \delta_{it}(\xi), \quad z_{it}^C(\xi) \leqslant M_1 (1 - \delta_{it}(\xi))$$

$$z_{it}^C(\xi) \leqslant M_1 \zeta_{it}(\xi), \quad z_{it}^D(\xi) \leqslant M_1 (1 - \zeta_{it}(\xi))$$

$$z_{it}^D(\xi) \leqslant z_{it}(\xi), \quad z_{it}^C(\xi) \leqslant K_i - z_{it}(\xi), \quad 0 \leqslant z_{it}(\xi) \leqslant K_i$$

$$e_{it}^+(\xi) = y_{it}^+(\xi) - \sum_{i \in I} d_{ijt}^+(\xi), \quad e_{it}^-(\xi) = y_{it}^-(\xi) - \sum_{i \in I} d_{ijt}^-(\xi)$$

$$e^C_{it}(\xi) = z^C_{it}(\xi) - \sum_{i \in I} \widehat{d^C_{ijt}}(\xi) + \sum_{i \in I} \widecheck{d^C_{ijt}}(\xi), \quad e^D_{it}(\xi) = z^D_{it}(\xi) - \sum_{i \in I} \widehat{d^D_{ijt}}(\xi) + \sum_{i \in I} \widecheck{d^D_{ijt}}(\xi)$$

$$e_{it}^C(\xi) \leqslant K_i - z_{it}(\xi), \quad e_{it}^D(\xi) \leqslant z_{it}(\xi), \quad e_{it}^D(\xi) \geqslant \min\{y_{it}^-(\xi), z_{it}(\xi)\}, \quad z_{i,t+1}(\xi) = z_{it}(\xi) + e_{it}^C(\xi) - e_{it}^D(\xi)\}$$

$$y_{it}^+(\xi) \geqslant \sum_{i \in I} d_{ijt}^+(\xi), \quad y_{it}^-(\xi) \geqslant \sum_{i \in I} d_{ijt}^-(\xi), \quad z_{it}^C(\xi) \geqslant \sum_{i \in I} \widehat{d_{ijt}^C}(\xi), \quad z_{it}^D(\xi) \geqslant \sum_{i \in I} \widehat{d_{ijt}^D}(\xi)$$

$$\sum_{i \in I} y_{it}^-(\xi) \geqslant \sum_{j \in I} \widehat{d_{jit}^C}(\xi), \quad \sum_{i \in I} y_{it}^-(\xi) \geqslant \sum_{j \in I} \widecheck{d_{jit}^D}(\xi)$$

$$\sum_{i \in I} d_{ijt}^{+}(\xi) - \sum_{j \in I, j \neq i} d_{jit}^{+}(\xi) = d_{iit}^{+}(\xi)$$

$$d_{iit}^+(\xi) \geqslant \widetilde{d_{iit}^C}(\xi) + \widehat{d_{iit}^D}(\xi)$$

$$\widetilde{d_{iit}^C}(\xi) \geqslant \min\{d_{iit}^+(\xi) - \widehat{d_{iit}^D}(\xi), K_i - z_{it}(\xi) - z_{it}^C(\xi)\}$$

$$d_{jit}^{-}(\xi) \geqslant d_{jit}^{+}(\xi) + \widehat{d_{jit}^{C}}(\xi) + \widecheck{d_{jit}^{D}}(\xi) \quad \forall j \in I, j \neq i$$

$$\sum_{i \in I} \widehat{d_{ijt}^C}(\xi) \leqslant M_1 q_{it}^1(\xi), \quad \widecheck{d_{iit}^C}(\xi) \leqslant M_1 (1 - q_{it}^1(\xi))$$

$$\widehat{d_{iit}^D}(\xi) \le M_1 q_{it}^2(\xi), \quad \sum_{j \in I} \widecheck{d_{ijt}^D}(\xi) \le M_1 (1 - q_{it}^2(\xi))$$

$$e_{it}^+(\xi) \leqslant M_1 q_{it}^3(\xi), \quad e_{it}^-(\xi) \leqslant M_1 (1 - q_{it}^3(\xi))$$

$$e_{it}^{-}(\xi) \leq M_1 q_{it}^4(\xi), \quad e_{it}^C(\xi) \leq M_1 (1 - q_{it}^4(\xi))$$

$$e_{it}^C(\xi) \leqslant M_1 q_{it}^5(\xi), \quad e_{it}^D(\xi) \leqslant M_1 (1 - q_{it}^5(\xi))$$

$$\alpha_t = \sum_{i \in I} x_{it}, \quad \beta_t^+(\xi) = \sum_{i \in I} e_{it}^+(\xi), \quad \beta_t^-(\xi) = \sum_{i \in I} e_{it}^-(\xi), \quad \gamma_t^C(\xi) = \sum_{i \in I} e_{it}^C(\xi), \quad \gamma_t^D(\xi) = \sum_{i \in I} e_{it}^D(\xi)$$

$$\sum_{i \in I} R_{it}(\xi) - \alpha_t = \beta_t^+(\xi) - \beta_t^-(\xi) + \gamma_t^C(\xi) - \gamma_t^D(\xi), \quad \sum_{i \in I} R_{it}(\xi) \geqslant \beta_t^+(\xi)$$

$$\beta_t^+(\xi) \leqslant M_2 \mu_t(\xi), \quad \beta_t^-(\xi) \leqslant M_2 (1 - \mu_t(\xi))$$

$$\beta_t^-(\xi) \leqslant M_2 \eta_t(\xi), \quad \gamma_t^C(\xi) \leqslant M_2 (1 - \eta_t(\xi))$$

# 3.3 RT Storage Dispatching with aggregation (2)

$$\max \quad \sum_{t \in T} \left( P_t^{DA} \alpha_t + \mathbb{E} \left[ P_t^{RT}(\xi) \beta_t^+(\xi) - P_t^{PN} \beta_{it}^-(\xi) \right] \right)$$

s.t. 
$$R_{it}(\xi) - x_{it} = y_{it}^{+}(\xi) - y_{it}^{-}(\xi) + z_{it}^{C}(\xi) - z_{it}^{D}(\xi), \quad R_{it}(\xi) \ge y_{it}^{+}(\xi)$$
  
 $y_{it}^{+}(\xi) \le M_1 \rho_{it}(\xi), \quad y_{it}^{-}(\xi) \le M_1 (1 - \rho_{it}(\xi))$ 

$$y_{it}(\xi) \leqslant M_1 \rho_{it}(\xi), \quad y_{it}(\xi) \leqslant M_1 (1 - \rho_{it}(\xi))$$

$$y_{it}^-(\xi) \leqslant M_1 \delta_{it}(\xi), \quad z_{it}^C(\xi) \leqslant M_1 (1 - \delta_{it}(\xi))$$

$$z_{it}^C(\xi) \leqslant M_1 \zeta_{it}(\xi), \quad z_{it}^D(\xi) \leqslant M_1 (1 - \zeta_{it}(\xi))$$

$$z_{it}^D(\xi) \leqslant z_{it}(\xi), \quad z_{it}^C(\xi) \leqslant K_i - z_{it}(\xi), \quad 0 \leqslant z_{it}(\xi) \leqslant K_i$$

$$e_{it}^+(\xi) = y_{it}^+(\xi) - \sum_{i \in I} d_{ijt}^+(\xi), \quad e_{it}^-(\xi) = y_{it}^-(\xi) - d_{it}^-(\xi)$$

$$e^C_{it}(\xi) = z^C_{it}(\xi) - \sum_{i \in I} \widehat{d^C_{ijt}}(\xi) + \widecheck{d^C_{it}}(\xi), \quad e^D_{it}(\xi) = z^D_{it}(\xi) - \widehat{d^D_{it}}(\xi) + \sum_{i \in I} \widecheck{d^D_{ijt}}(\xi)$$

$$e_{it}^{C}(\xi) \leqslant K_{i} - z_{it}(\xi), \quad e_{it}^{D}(\xi) \leqslant z_{it}(\xi), \quad z_{i,t+1}(\xi) = z_{it}(\xi) + e_{it}^{C}(\xi) - e_{it}^{D}(\xi)$$

$$y_{it}^+(\xi) \geqslant \sum_{i \in I} d_{ijt}^+(\xi), \quad y_{it}^-(\xi) \geqslant d_{it}^-(\xi), \quad z_{it}^C(\xi) \geqslant \sum_{i \in I} \widehat{d_{ijt}^C}(\xi), \quad z_{it}^D(\xi) \geqslant \widehat{d_{it}^D}(\xi)$$

$$\sum_{i \in I} \widehat{d_{ijt}^C}(\xi) - \sum_{j \in I, j \neq i} \widehat{d_{ijt}^C}(\xi) = \widehat{d_{iit}^C}(\xi)$$

$$\sum_{i \in I} \widetilde{d_{ijt}^D}(\xi) - \sum_{i \in I, i \neq i} \widetilde{d_{ijt}^D}(\xi) = \widetilde{d_{iit}^D}(\xi)$$

$$\sum_{i \in I} d_{ijt}^{+}(\xi) - \sum_{i \in I, i \neq i} d_{ijt}^{+}(\xi) = d_{iit}^{+}(\xi)$$

$$y_{jt}^-(\xi) \geqslant \sum_{i \in I} \left( d_{ijt}^+(\xi) + \widehat{d_{ijt}^C}(\xi) + \widecheck{d_{ijt}^D}(\xi) \right)$$

$$e_{it}^{D}(\xi) \geqslant \min\{y_{it}^{-}(\xi), z_{it}(\xi)\}??$$

$$\widecheck{d_{it}^C}(\xi) \geqslant \min\{d_{iit}^+(\xi) - \widehat{d_{it}^D}(\xi), K_i - z_{it}(\xi) - z_{it}^C(\xi)\}??$$

$$\sum_{j\in I}\widehat{d_{ijt}^C}(\xi)\leqslant M_1q_{it}^1(\xi),\quad \widecheck{d_{it}^C}(\xi)\leqslant M_1(1-q_{it}^1(\xi))$$

$$\widehat{d_{it}^D}(\xi) \leqslant M_1 q_{it}^2(\xi), \quad \sum_{i \in I} \widecheck{d_{ijt}^D}(\xi) \leqslant M_1 (1 - q_{it}^2(\xi))$$

$$e_{it}^+(\xi) \le M_1 q_{it}^3(\xi), \quad e_{it}^-(\xi) \le M_1 (1 - q_{it}^3(\xi))$$

$$e_{it}^{-}(\xi) \leq M_1 q_{it}^4(\xi), \quad e_{it}^C(\xi) \leq M_1 (1 - q_{it}^4(\xi))$$

$$e_{it}^C(\xi) \le M_1 q_{it}^5(\xi), \quad e_{it}^D(\xi) \le M_1 (1 - q_{it}^5(\xi))$$

$$\alpha_t = \sum_{i \in I} x_{it}, \quad \beta_t^+(\xi) = \sum_{i \in I} e_{it}^+(\xi), \quad \beta_t^-(\xi) = \sum_{i \in I} e_{it}^-(\xi), \quad \gamma_t^C(\xi) = \sum_{i \in I} e_{it}^C(\xi), \quad \gamma_t^D(\xi) = \sum_{i \in I} e_{it}^D(\xi)$$

$$\sum_{i \in I} R_{it}(\xi) - \alpha_t = \beta_t^+(\xi) - \beta_t^-(\xi) + \gamma_t^C(\xi) - \gamma_t^D(\xi), \quad \sum_{i \in I} R_{it}(\xi) \geqslant \beta_t^+(\xi)$$

$$\beta_t^+(\xi) \le M_2 \mu_t(\xi), \quad \beta_t^-(\xi) \le M_2 (1 - \mu_t(\xi))$$

$$\beta_t^-(\xi) \leqslant M_2 \eta_t(\xi), \quad \gamma_t^C(\xi) \leqslant M_2 (1 - \eta_t(\xi))$$

#### 3.4 DA + RT Storage Dispatching: for each i

## 3.5 DA + RT Storage Dispatching with aggregation