DER Models

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1 Without Storage

1.1 Disaggregation

$$\begin{aligned}
\text{max} \quad & \sum_{t\in T} \left(P_t^{DA} x_{it} + \mathbb{E}\left[P_t^{RT}(\xi) y_{it}^+(\xi) - P_t^{PN} y_{it}^-(\xi) \right] \right) && \text{(1a)} \\
\text{s.t.} \quad & R_{it}(\xi) - x_{it} = y_{it}^+(\xi) - y_{it}^-(\xi) \quad \forall t \in T && \text{(1b)} \\
& R_{it}(\xi) \geqslant y_{it}^+(\xi) \quad \forall t \in T && \text{(1c)} \\
& y_{it}^+(\xi) \leqslant M z_{it}(\xi), \quad y_{it}^-(\xi) \leqslant M(1 - z_{it}(\xi)) \quad \forall t \in T && \text{(1d)} \\
& x_{it}^{DA} \geqslant 0, y_{it}^+(\xi) \geqslant 0, y_{it}^-(\xi) \geqslant 0, z_{it}(\xi) \in \{0,1\} \quad \forall t \in T && \text{(1e)}
\end{aligned}$$

1.2 Aggregation

$$\begin{aligned}
\text{max} \quad & \sum_{t\in T} \left(P_t^{DA} \alpha_t + \mathbb{E}\left[P_t^{RT}(\xi) \beta_t^+(\xi) - P_t^{PN} \beta_t^-(\xi) \right] \right) && \text{(2a)} \\
\text{s.t.} \quad & \sum_{i\in I} R_{it}(\xi) - \alpha_t = \beta_t^+(\xi) - \beta_t^-(\xi) \quad \forall t \in T && \text{(2b)} \\
& \sum_{i\in I} R_{it}(\xi) \geqslant \beta_t^+(\xi) \quad \forall t \in T && \text{(2c)} \\
& \beta_t^+(\xi) \leqslant M z_t(\xi), \quad \beta_t^-(\xi) \leqslant M(1 - z_t(\xi)) \quad \forall t \in T && \text{(2d)} \\
& \alpha_t^{DA} \geqslant 0, \beta_t^+(\xi) \geqslant 0, \beta_t^-(\xi) \geqslant 0, z_t(\xi) \in \{0,1\} \quad \forall t \in T && \text{(2e)}
\end{aligned}$$

1.3 Settlement

$$\begin{aligned}
\text{max} \quad & \sum_{t\in T} \left(P_t^{DA} \alpha_t + \mathbb{E}\left[P_t^{RT}(\xi) \beta_t^+(\xi) - P_t^{PN} \beta_t^-(\xi) \right] \right) && \text{(3a)} \\
\text{s.t.} \quad & \sum_{i\in I} R_{it}(\xi) - \alpha_t^{DA} = \beta_t^+(\xi) - \beta_t^-(\xi) \quad \forall t \in T && \text{(3b)} \\
& \sum_{i\in I} R_{it}(\xi) \geqslant \beta_t^+(\xi) \quad \forall t \in T && \text{(3c)} \\
& \beta_t^+(\xi) \leqslant M z_t(\xi), \quad \beta_t^-(\xi) \leqslant M(1 - z_t(\xi)) \quad \forall t \in T && \text{(3d)} \\
& \alpha_t = \sum_{i\in I} x_{it}(\xi), \quad \beta_t^+(\xi) = \sum_{i\in I} e_{it}^+(\xi), \quad \beta_t^-(\xi) = \sum_{i\in I} e_{it}^-(\xi) \quad \forall t \in T && \text{(3e)} \\
& R_{it}(\xi) - x_{it}(\xi) = y_{it}^+(\xi) - y_{it}^-(\xi) \quad \forall t \in T && \text{(3f)} \\
& R_{it}(\xi) \geqslant y_{it}^+(\xi) \quad \forall t \in T && \text{(3g)} \\
& y_{it}^+(\xi) \leqslant M z_{it}(\xi), \quad y_{it}^-(\xi) \leqslant M(1 - z_{it}(\xi)) \quad \forall t \in T && \text{(3h)} \\
& \sum_{j\in I, j\neq i} d_{ijt}(\xi) \leqslant y_{it}^+(\xi), \quad \sum_{j\in I, j\neq i} d_{jit}(\xi) \leqslant y_{it}^-(\xi) \quad \forall t \in T && \text{(3i)} \\
& d_{iit}(\xi) = 0 \quad \forall t \in T && \text{(3j)} \\
& e_{it}^+(\xi) = y_{it}^+(\xi) - \sum_{j\in I, j\neq i} d_{ijt}(\xi) \quad \forall t \in T && \text{(3k)} \\
& e_{it}^-(\xi) = y_{it}^-(\xi) - \sum_{j\in I, j\neq i} d_{jit}(\xi) \quad \forall t \in T && \text{(3l)}
\end{aligned}$$

2 With Storage

2.1 Disaggregation

$$
\begin{aligned}
\textit{max} \quad & \sum_{t \in T} \left(P_t^{DA} x_{it} + \mathbb{E}\left[P_t^{RT}(\xi) y_{it}^{+}(\xi) - P_t^{PN} y_{it}^{-}(\xi) \right] \right) && \text{(4a)} \\
\textit{s.t.} \quad & R_{it}(\xi) - x_{it} = y_{it}^{+}(\xi) - y_{it}^{-}(\xi) + z_{it}^{C}(\xi) - z_{it}^{D}(\xi) && \text{(4b)} \\
& R_{it}(\xi) \geqslant y_{it}^{+}(\xi) && \text{(4c)} \\
& z_{i,t+1}(\xi) = z_{it}(\xi) + z_{it}^{C}(\xi) - z_{it}^{D}(\xi) \quad \forall i, t && \text{(4d)} \\
& z_{it}^{D}(\xi) \leqslant z_{it}(\xi), \quad z_{it}^{C}(\xi) \leqslant K_i - z_{it}(\xi), \quad 0 \leqslant z_{it}(\xi) \leqslant K_i && \text{(4e)} \\
& y_{it}^{+}(\xi) \leqslant M_1 \phi_{it}^{1}(\xi), \quad y_{it}^{-}(\xi) \leqslant M_1 (1 - \phi_{it}^{1}(\xi)) && \text{(4f)} \\
& y_{it}^{-}(\xi) \leqslant M_1 \phi_{it}^{2}(\xi), \quad z_{it}^{C}(\xi) \leqslant M_1 (1 - \phi_{it}^{2}(\xi)) && \text{(4g)} \\
& z_{it}^{C}(\xi) \leqslant M_1 \phi_{it}^{3}(\xi), \quad z_{it}^{D}(\xi) \leqslant M_1 (1 - \phi_{it}^{3}(\xi)) && \text{(4h)}
\end{aligned}
$$

2.2 Aggregation with individual BTM storage control

$$
\begin{aligned}
\textit{max} \quad & \sum_{t \in T} \left(P_t^{DA} \sum_{i \in I} x_{it} + \mathbb{E}\left[P_t^{RT}(\xi) \sum_{i \in I} e_{it}^{+}(\xi) - P_t^{PN} \sum_{i \in I} e_{it}^{-}(\xi) \right] \right) && \text{(5a)} \\
\textit{s.t.} \quad & R_{it}(\xi) - x_{it} = y_{it}^{+}(\xi) - y_{it}^{-}(\xi) + z_{it}^{C}(\xi) - z_{it}^{D}(\xi) && \text{(5b)} \\
& R_{it}(\xi) + z_{it}^{D}(\xi) \geqslant y_{it}^{+}(\xi) + z_{it}^{C}(\xi) && \text{(5c)} \\
& z_{i,t+1}(\xi) = z_{it}(\xi) + z_{it}^{C}(\xi) - z_{it}^{D}(\xi) && \text{(5d)} \\
& z_{it}^{D}(\xi) \leqslant z_{it}(\xi), \quad z_{it}^{C}(\xi) \leqslant K_i - z_{it}(\xi), \quad 0 \leqslant z_{it}(\xi) \leqslant K_i && \text{(5e)} \\
& e_{it}^{+}(\xi) = y_{it}^{+}(\xi) - \sum_{j \in I} d_{ijt}(\xi), \quad e_{it}^{-}(\xi) = y_{it}^{-}(\xi) - \sum_{j \in I} d_{jit}(\xi) && \text{(5f)} \\
& d_{iit}(\xi) = 0 && \text{(5g)} \\
& y_{it}^{+}(\xi) \leqslant M_1 \phi_{it}^{1}(\xi), \quad y_{it}^{-}(\xi) \leqslant M_1 (1 - \phi_{it}^{1}(\xi)) && \text{(5h)} \\
& y_{it}^{-}(\xi) \leqslant M_1 \phi_{it}^{2}(\xi), \quad z_{it}^{C}(\xi) \leqslant M_1 (1 - \phi_{it}^{2}(\xi)) && \text{(5i)} \\
& z_{it}^{C}(\xi) \leqslant M_1 \phi_{it}^{3}(\xi), \quad z_{it}^{D}(\xi) \leqslant M_1 (1 - \phi_{it}^{3}(\xi)) && \text{(5j)} \\
& \sum_{i \in I} e_{it}^{+}(\xi) \leqslant M_2 \phi_{t}^{4}(\xi), \quad \sum_{i \in I} e_{it}^{-}(\xi) \leqslant M_2 (1 - \phi_{t}^{4}(\xi)) && \text{(5k)}
\end{aligned}
$$

2.3 Aggregation with direct control over storage

$$
\begin{aligned}
\textit{max} \quad & \sum_{t \in T} \left(P_t^{DA} \alpha_t + \mathbb{E}\left[P_t^{RT}(\xi) \beta_t^{+}(\xi) - P_t^{PN} \beta_t^{-}(\xi)) \right] \right) && \text{(6a)} \\
\textit{s.t.} \quad & \sum_{i \in I} R_{it}(\xi) - \alpha_t = \beta_t^{+}(\xi) - \beta_t^{-}(\xi) + \gamma_t^{C}(\xi) - \gamma_t^{D}(\xi) \quad \forall t && \text{(6b)} \\
& \sum_{i \in I} R_{it}(\xi) \geqslant \beta_t^{+}(\xi) && \text{(6c)} \\
& \gamma_t^{D}(\xi) \leqslant \gamma_t(\xi), \quad \gamma_t^{C}(\xi) \leqslant \sum_{i \in I} K_i - \gamma_t(\xi), \quad 0 \leqslant \gamma_t(\xi) \leqslant \sum_{i \in I} K_i \quad \forall t && \text{(6d)} \\
& \gamma_{t+1}(\xi) = \gamma_t(\xi) + \gamma_t^{C}(\xi) - \gamma_t^{D}(\xi) \quad \forall t && \text{(6e)} \\
& \beta_t^{+}(\xi) \leqslant M_2 \mu_t(\xi), \quad \beta_t^{-}(\xi) \leqslant M_2 (1 - \mu_t(\xi)) \quad \forall t && \text{(6f)} \\
& \beta_t^{-}(\xi) \leqslant M_2 \eta_t(\xi), \quad \gamma_t^{C}(\xi) \leqslant M_2 (1 - \eta_t(\xi)) \quad \forall t && \text{(6g)} \\
& \gamma_t^{C}(\xi) \leqslant M_2 \lambda_t(\xi), \quad \gamma_t^{D}(\xi) \leqslant M_2 (1 - \lambda_t(\xi)) \quad \forall t && \text{(6h)}
\end{aligned}
$$

3 Individual

3.1 Different Internal Price (Non-linear)

$$
\begin{aligned}
\textit{max} \quad & \sum_{t\in T} \left(P_t^{DA}\cdot x_t + \mathbb{E}\left[P_t^{RT}(\xi)\cdot y_t^+(\xi) - P_t^{PN}\cdot y_t^-(\xi) + \rho_t^+(d)\cdot d_t^+(\xi) - \rho_t^-(d)\cdot d_t^-(\xi)\right]\right) && \text{(7a)}\\
\text{s.t.} \quad & R_t(\xi) - x_t = y_t^+(\xi) - y_t^-(\xi) + d_t^+(\xi) - d_t^-(\xi) + z_t^C(\xi) - z_t^D(\xi) && \text{(7b)}\\
& R_t(\xi) \geqslant y_t^+(\xi) + d_{it}^+(\xi) && \text{(7c)}\\
& x_t - R_t(\xi) \geqslant y_t^-(\xi) + d_t^-(\xi) && \text{(7d)}\\
& z_{t+1}(\xi) = z_t(\xi) + z_t^C(\xi) - z_t^D(\xi) && \text{(7e)}\\
& z_t^D(\xi) \leqslant z_t(\xi), \quad z_t^C(\xi) \leqslant K - z_t(\xi), \quad 0 \leqslant z_t(\xi) \leqslant K && \text{(7f)}\\
& y_t^+(\xi) \leqslant M\phi_t^1(\xi), \quad y_t^-(\xi) \leqslant M(1-\phi_t^1(\xi)) && \text{(7g)}\\
& y_t^-(\xi) \leqslant M\phi_t^2(\xi), \quad z_t^C(\xi) \leqslant M(1-\phi_t^2(\xi)) && \text{(7h)}\\
& z_t^C(\xi) \leqslant M\phi_t^3(\xi), \quad z_t^D(\xi) \leqslant M(1-\phi_t^3(\xi)) && \text{(7i)}\\
& d_t^+(\xi) \leqslant M\phi_t^4(\xi), \quad d_t^-(\xi) \leqslant M(1-\phi_t^4(\xi)) && \text{(7j)}
\end{aligned}
$$

3.2 Different Internal Price (Stepwise-Linear)

$$
\begin{aligned}
\textit{max} \quad & \sum_{t\in T} \Bigg(P_t^{DA}\cdot x_t + \mathbb{E}\Big[P_t^{RT}(\xi)\cdot y_t^+(\xi) - P_t^{PN}\cdot y_t^-(\xi)\\
& \quad + \sum_{b^+\in B_t^+} \rho_{t,b^+}^+ \cdot (w_{t,b^+}^+(\xi) + u_{t,b^+}^+(\xi)\cdot D_{t,b^+}^{\textit{min}\,+}) - \sum_{b^-\in B_t^-} \rho_{t,b^-}^- \cdot (w_{t,b^-}^-(\xi) + u_{t,b^-}^-(\xi)\cdot D_{t,b^-}^{\textit{min}\,-})\Big]\Bigg) && \text{(8a)}
\end{aligned}
$$

$$
\begin{aligned}
\text{s.t.} \quad & d_t^+(\xi) = \sum_{b^+\in B_t^+} (w_{t,b^+}^+(\xi) + u_{t,b^+}^+(\xi)\cdot D_{t,b^+}^{\textit{min}\,+}) && \forall t && \text{(8b)}\\
& 0 \leqslant w_{t,b^+}^+(\xi) \leqslant u_{t,b^+}^+(\xi)\cdot W_{t,b^+}^{\textit{max}\,+} && \forall t, b^+ && \text{(8c)}\\
& \sum_{b^+\in B_t^+} u_{t,b^+}^+(\xi) \leqslant 1 && \forall t && \text{(8d)}\\
\\
& d_t^-(\xi) = \sum_{b^-\in B_t^-} (w_{t,b^-}^-(\xi) + u_{t,b^-}^-(\xi)\cdot D_{t,b^-}^{\textit{min}\,-}) && \forall t && \text{(8e)}\\
& 0 \leqslant w_{t,b^-}^-(\xi) \leqslant u_{t,b^-}^-(\xi)\cdot W_{t,b^-}^{\textit{max}\,-} && \forall t, b^- && \text{(8f)}\\
& \sum_{b^-\in B_t^-} u_{t,b^-}^-(\xi) \leqslant 1 && \forall t && \text{(8g)}\\
\\
& R_t(\xi) - x_t = y_t^+(\xi) - y_t^-(\xi) + d_t^+(\xi) - d_t^-(\xi) + z_t^C(\xi) - z_t^D(\xi) && \forall t && \text{(8h)}\\
& R_t(\xi) + z_t^D(\xi) \geqslant y_t^+(\xi) + d_t^+(\xi) + z_t^C(\xi) && \forall t && \text{(8i)}\\
& z_{t+1}(\xi) = z_t(\xi) + z_t^C(\xi) - z_t^D(\xi) && \forall t && \text{(8j)}\\
& z_t^D(\xi) \leqslant z_t(\xi), \quad z_t^C(\xi) \leqslant K - z_t(\xi), \quad 0 \leqslant z_t(\xi) \leqslant K && \forall t && \text{(8k)}\\
& y_t^+(\xi) \leqslant M\phi_t^1(\xi), \quad y_t^-(\xi) \leqslant M(1-\phi_t^1(\xi)) && \forall t && \text{(8l)}\\
& y_t^-(\xi) \leqslant M\phi_t^2(\xi), \quad z_t^C(\xi) \leqslant M(1-\phi_t^2(\xi)) && \forall t && \text{(8m)}\\
& z_t^C(\xi) \leqslant M\phi_t^3(\xi), \quad z_t^D(\xi) \leqslant M(1-\phi_t^3(\xi)) && \forall t && \text{(8n)}\\
& d_t^+(\xi) \leqslant M\phi_t^4(\xi), \quad d_t^-(\xi) \leqslant M(1-\phi_t^4(\xi)) && \forall t && \text{(8o)}
\end{aligned}
$$