DER Models

Seohyun Jang

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1 Without Storage

1.1 Disaggregation

$$\max \quad \sum_{t\in T}\left(P_t^{DA}x_{it}+\mathbb{E}\left[P_t^{RT}(\xi)y_{it}^{+}(\xi)-P_t^{PN}y_{it}^{-}(\xi)\right]\right) \tag{1a}$$

$$\text{s.t.} \quad R_{it}(\xi)-x_{it}=y_{it}^{+}(\xi)-y_{it}^{-}(\xi) \quad \forall t\in T \tag{1b}$$

$$R_{it}(\xi)\geqslant y_{it}^{+}(\xi) \quad \forall t\in T \tag{1c}$$

$$y_{it}^{+}(\xi)\leqslant Mz_{it}(\xi), \quad y_{it}^{-}(\xi)\leqslant M(1-z_{it}(\xi)) \quad \forall t\in T \tag{1d}$$

$$x_{it}^{DA}\geqslant 0, y_{it}^{+}(\xi)\geqslant 0, y_{it}^{-}(\xi)\geqslant 0, z_{it}(\xi)\in\{0,1\} \quad \forall t\in T \tag{1e}$$

1.2 Aggregation

$$\max \quad \sum_{t\in T}\left(P_t^{DA}\alpha_t+\mathbb{E}\left[P_t^{RT}(\xi)\beta_t^{+}(\xi)-P_t^{PN}\beta_t^{-}(\xi)\right]\right) \tag{2a}$$

$$\text{s.t.} \quad \sum_{i\in I}R_{it}(\xi)-\alpha_t=\beta_t^{+}(\xi)-\beta_t^{-}(\xi) \quad \forall t\in T \tag{2b}$$

$$\sum_{i\in I}R_{it}(\xi)\geqslant \beta_t^{+}(\xi) \quad \forall t\in T \tag{2c}$$

$$\beta_t^{+}(\xi)\leqslant Mz_t(\xi), \quad \beta_t^{-}(\xi)\leqslant M(1-z_t(\xi)) \quad \forall t\in T \tag{2d}$$

$$\alpha_t^{DA}\geqslant 0, \beta_t^{+}(\xi)\geqslant 0, \beta_t^{-}(\xi)\geqslant 0, z_t(\xi)\in\{0,1\} \quad \forall t\in T \tag{2e}$$

1.3 Settlement

$$\max \quad \sum_{t\in T}\left(P_t^{DA}\alpha_t+\mathbb{E}\left[P_t^{RT}(\xi)\beta_t^{+}(\xi)-P_t^{PN}\beta_t^{-}(\xi)\right]\right) \tag{3a}$$

$$\text{s.t.} \quad \sum_{i\in I}R_{it}(\xi)-\alpha_t^{DA}=\beta_t^{+}(\xi)-\beta_t^{-}(\xi) \quad \forall t\in T \tag{3b}$$

$$\sum_{i\in I}R_{it}(\xi)\geqslant \beta_t^{+}(\xi) \quad \forall t\in T \tag{3c}$$

$$\beta_t^{+}(\xi)\leqslant Mz_t(\xi), \quad \beta_t^{-}(\xi)\leqslant M(1-z_t(\xi)) \quad \forall t\in T \tag{3d}$$

$$\alpha_t=\sum_{i\in I}x_{it}(\xi), \quad \beta_t^{+}(\xi)=\sum_{i\in I}e_{it}^{+}(\xi), \quad \beta_t^{-}(\xi)=\sum_{i\in I}e_{it}^{-}(\xi) \quad \forall t\in T \tag{3e}$$

$$R_{it}(\xi)-x_{it}(\xi)=y_{it}^{+}(\xi)-y_{it}^{-}(\xi) \quad \forall t\in T \tag{3f}$$

$$R_{it}(\xi)\geqslant y_{it}^{+}(\xi) \quad \forall t\in T \tag{3g}$$

$$y_{it}^{+}(\xi)\leqslant Mz_{it}(\xi), \quad y_{it}^{-}(\xi)\leqslant M(1-z_{it}(\xi)) \quad \forall t\in T \tag{3h}$$

$$\sum_{j\in I,j\neq i}d_{ijt}(\xi)\leqslant y_{it}^{+}(\xi), \quad \sum_{j\in I,j\neq i}d_{jit}(\xi)\leqslant y_{it}^{-}(\xi) \quad \forall t\in T \tag{3i}$$

$$d_{iit}(\xi)=0 \quad \forall t\in T \tag{3j}$$

$$e_{it}^{+}(\xi)=y_{it}^{+}(\xi)-\sum_{j\in I,j\neq i}d_{ijt}(\xi) \quad \forall t\in T \tag{3k}$$

$$e_{it}^{-}(\xi)=y_{it}^{-}(\xi)-\sum_{j\in I,j\neq i}d_{jit}(\xi) \quad \forall t\in T \tag{3l}$$

2 With Storage

2.1 Disaggregation

$$\max \quad \sum_{t\in T}\left(P_t^{DA}x_{it}+\mathbb{E}\left[P_t^{RT}(\xi)y_{it}^{+}(\xi)-P_t^{PN}y_{it}^{-}(\xi)\right]\right) \tag{4a}$$

$$\text{s.t.} \quad R_{it}(\xi)-x_{it}=y_{it}^{+}(\xi)-y_{it}^{-}(\xi)+z_{it}^{C}(\xi)-z_{it}^{D}(\xi) \quad \forall i,t \tag{4b}$$

$$R_{it}(\xi)\geqslant y_{it}^{+}(\xi) \quad \forall i,t \tag{4c}$$

$$z_{i,t+1}(\xi)=z_{it}(\xi)+z_{it}^{C}(\xi)-z_{it}^{D}(\xi) \quad \forall i,t \tag{4d}$$

$$z_{it}^{D}(\xi)\leqslant z_{it}(\xi) \quad \forall i,t \tag{4e}$$

$$z_{it}^{C}(\xi)\leqslant K_i-z_{it}(\xi) \quad \forall i,t \tag{4f}$$

$$0\leqslant z_{it}(\xi)\leqslant K_i \quad \forall i,t \tag{4g}$$

$$y_{it}^{+}(\xi)\leqslant M_1\rho_{it}(\xi), \quad y_{it}^{-}(\xi)\leqslant M_1(1-\rho_{it}(\xi)) \quad \forall i,t \tag{4h}$$

$$y_{it}^{-}(\xi)\leqslant M_1\delta_{it}(\xi), \quad z_{it}^{C}(\xi)\leqslant M_1(1-\delta_{it}(\xi)) \quad \forall i,t \tag{4i}$$

$$z_{it}^{C}(\xi)\leqslant M_1\zeta_{it}(\xi), \quad z_{it}^{D}(\xi)\leqslant M_1(1-\zeta_{it}(\xi)) \quad \forall i,t \tag{4j}$$

2.2 Aggregation

$$\max \quad \sum_{t\in T}\left(P_t^{DA}\alpha_t+\mathbb{E}\left[P_t^{RT}(\xi)\beta_t^{+}(\xi)-P_t^{PN}\beta_t^{-}(\xi)\right]\right) \tag{5a}$$

$$\text{s.t.} \quad \sum_{i\in I}R_{it}(\xi)-\alpha_t=\beta_t^{+}(\xi)-\beta_t^{-}(\xi)+\gamma_t^{C}(\xi)-\gamma_t^{D}(\xi) \quad \forall t \tag{5b}$$

$$\sum_{i\in I}R_{it}(\xi)\geqslant \beta_t^{+}(\xi) \quad \forall t \tag{5c}$$

$$\gamma_t^{D}(\xi)\leqslant \gamma_t(\xi) \quad \forall t \tag{5d}$$

$$\gamma_t^{C}(\xi)\leqslant \sum_{i\in I}K_i-\gamma_t(\xi) \quad \forall t \tag{5e}$$

$$0\leqslant \gamma_t(\xi)\leqslant \sum_{i\in I}K_i \quad \forall t \tag{5f}$$

$$\gamma_{t+1}(\xi)=\gamma_t(\xi)+\gamma_t^{C}(\xi)-\gamma_t^{D}(\xi) \quad \forall t \tag{5g}$$

$$\beta_t^{+}(\xi)\leqslant M_2\mu_t(\xi), \quad \beta_t^{-}(\xi)\leqslant M_2(1-\mu_t(\xi)) \quad \forall t \tag{5h}$$

$$\beta_t^{-}(\xi)\leqslant M_2\eta_t(\xi), \quad \gamma_t^{C}(\xi)\leqslant M_2(1-\eta_t(\xi)) \quad \forall t \tag{5i}$$

$$\gamma_t^{C}(\xi)\leqslant M_2\lambda_t(\xi), \quad \gamma_t^{D}(\xi)\leqslant M_2(1-\lambda_t(\xi)) \quad \forall t \tag{5j}$$

2.3 Settlement

$$
\begin{aligned}
\max \quad & \sum_{t\in T}\left(P_t^{DA}\alpha_t+\mathbb{E}\left[P_t^{RT}(\xi)\beta_t^+(\xi)-P_t^{PN}\beta_t^-(\xi)\right]\right)\\
\text{s.t.} \quad & R_{it}(\xi)-x_{it}=y_{it}^+(\xi)-y_{it}^-(\xi)+z_{it}^C(\xi)-z_{it}^D(\xi) \quad \forall i,t\\
& R_{it}(\xi)\geqslant y_{it}^+(\xi) \quad \forall i,t\\
& z_{it}^D(\xi)\leqslant z_{it}(\xi) \quad \forall i,t\\
& z_{it}^C(\xi)\leqslant K_i-z_{it}(\xi) \quad \forall i,t\\
& 0\leqslant z_{it}(\xi)\leqslant K_i \quad \forall i,t\\
& z_{i,t+1}(\xi)=z_{it}(\xi)+z_{it}^C(\xi)-z_{it}^D(\xi) \quad \forall i,t\\
& y_{it}^+(\xi)\leqslant M_1\rho_{it}(\xi),\quad y_{it}^-(\xi)\leqslant M_1(1-\rho_{it}(\xi)) \quad \forall i,t\\
& y_{it}^-(\xi)\leqslant M_1\delta_{it}(\xi),\quad z_{it}^C(\xi)\leqslant M_1(1-\delta_{it}(\xi)) \quad \forall i,t\\
& z_{it}^C(\xi)\leqslant M_1\zeta_{it}(\xi),\quad z_{it}^D(\xi)\leqslant M_1(1-\zeta_{it}(\xi)) \quad \forall i,t\\
& e_{it}^+(\xi)=y_{it}^+(\xi)+\widehat{d_{y+}}(\xi)-\widecheck{d_{y+}}(\xi)\\
& \widehat{d_{y+}}(\xi)\leqslant M_1q_{it}^1(\xi),\quad \widecheck{d_{y+}}(\xi)\leqslant M_1(1-q_{it}^1(\xi))\\
& e_{it}^-(\xi)=y_{it}^-(\xi)+\widehat{d_{y-}}(\xi)-\widecheck{d_{y-}}(\xi)\\
& \widehat{d_{y-}}(\xi)\leqslant M_1q_{it}^2(\xi),\quad \widecheck{d_{y-}}(\xi)\leqslant M_1(1-q_{it}^2(\xi))\\
& e_{it}^C(\xi)=z_{it}^C(\xi)+\widehat{d_{z^C}}(\xi)-\widecheck{d_{z^C}}(\xi)\\
& e_{it}^D(\xi)=z_{it}^D(\xi)+\widehat{d_{z^D}}(\xi)-\widecheck{d_{z^D}}(\xi)\\
& \alpha_t=\sum_{i\in I}x_{it},\quad \beta_t^+(\xi)=\sum_{i\in I}e_{it}^+(\xi),\quad \beta_t^-(\xi)=\sum_{i\in I}e_{it}^-(\xi)\\
& \gamma_t(\xi)=\sum_{i\in I}z_{it}(\xi),\quad \gamma_t^C(\xi)=\sum_{i\in I}e_{it}^C(\xi),\quad \gamma_t^D(\xi)=\sum_{i\in I}e_{it}^D(\xi)\\
& \sum_{i\in I}R_{it}(\xi)-\alpha_t=\beta_t^+(\xi)-\beta_t^-(\xi)+\gamma_t^C(\xi)-\gamma_t^D(\xi) \quad \forall t\\
& \sum_{i\in I}R_{it}(\xi)\geqslant\beta_t^+(\xi) \quad \forall t\\
& \gamma_t^D(\xi)\leqslant\gamma_t(\xi) \quad \forall t\\
& \gamma_t^C(\xi)\leqslant\sum_{i\in I}K_i-\gamma_t(\xi) \quad \forall t\\
& 0\leqslant\gamma_t(\xi)\leqslant\sum_{i\in I}K_i \quad \forall t\\
& \gamma_{t+1}(\xi)=\gamma_t(\xi)+\gamma_t^C(\xi)-\gamma_t^D(\xi) \quad \forall t\\
& \beta_t^+(\xi)\leqslant M_2\mu_t(\xi),\quad \beta_t^-(\xi)\leqslant M_2(1-\mu_t(\xi)) \quad \forall t\\
& \beta_t^-(\xi)\leqslant M_2\eta_t(\xi),\quad \gamma_t^C(\xi)\leqslant M_2(1-\eta_t(\xi)) \quad \forall t\\
& \gamma_t^C(\xi)\leqslant M_2\lambda_t(\xi),\quad \gamma_t^D(\xi)\leqslant M_2(1-\lambda_t(\xi)) \quad \forall t
\end{aligned}
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