DER Models

Seohyun Jang

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1 Without Storage

1.1 Disaggregation

$$\max \quad \sum_{t\in T}\left(P_t^{DA}x_{it}+\mathbb{E}\left[P_t^{RT}(\xi)y_{it}^{+}(\xi)-P_t^{PN}y_{it}^{-}(\xi)\right]\right) \tag{1a}$$

$$\text{s.t.} \quad R_{it}(\xi)-x_{it}=y_{it}^{+}(\xi)-y_{it}^{-}(\xi) \quad \forall t\in T \tag{1b}$$

$$R_{it}(\xi)\geqslant y_{it}^{+}(\xi) \quad \forall t\in T \tag{1c}$$

$$y_{it}^{+}(\xi)\leqslant Mz_{it}(\xi), \quad y_{it}^{-}(\xi)\leqslant M(1-z_{it}(\xi)) \quad \forall t\in T \tag{1d}$$

$$x_{it}^{DA}\geqslant 0, y_{it}^{+}(\xi)\geqslant 0, y_{it}^{-}(\xi)\geqslant 0, z_{it}(\xi)\in\{0,1\} \quad \forall t\in T \tag{1e}$$

1.2 Aggregation

$$\max \quad \sum_{t\in T}\left(P_t^{DA}\alpha_t+\mathbb{E}\left[P_t^{RT}(\xi)\beta_t^{+}(\xi)-P_t^{PN}\beta_t^{-}(\xi)\right]\right) \tag{2a}$$

$$\text{s.t.} \quad \sum_{i\in I}R_{it}(\xi)-\alpha_t=\beta_t^{+}(\xi)-\beta_t^{-}(\xi) \quad \forall t\in T \tag{2b}$$

$$\sum_{i\in I}R_{it}(\xi)\geqslant \beta_t^{+}(\xi) \quad \forall t\in T \tag{2c}$$

$$\beta_t^{+}(\xi)\leqslant Mz_t(\xi), \quad \beta_t^{-}(\xi)\leqslant M(1-z_t(\xi)) \quad \forall t\in T \tag{2d}$$

$$\alpha_t^{DA}\geqslant 0, \beta_t^{+}(\xi)\geqslant 0, \beta_t^{-}(\xi)\geqslant 0, z_t(\xi)\in\{0,1\} \quad \forall t\in T \tag{2e}$$

1.3 Settlement

$$\max \quad \sum_{t\in T}\left(P_t^{DA}\alpha_t+\mathbb{E}\left[P_t^{RT}(\xi)\beta_t^{+}(\xi)-P_t^{PN}\beta_t^{-}(\xi)\right]\right) \tag{3a}$$

$$\text{s.t.} \quad \sum_{i\in I}R_{it}(\xi)-\alpha_t^{DA}=\beta_t^{+}(\xi)-\beta_t^{-}(\xi) \quad \forall t\in T \tag{3b}$$

$$\sum_{i\in I}R_{it}(\xi)\geqslant \beta_t^{+}(\xi) \quad \forall t\in T \tag{3c}$$

$$\beta_t^{+}(\xi)\leqslant Mz_t(\xi), \quad \beta_t^{-}(\xi)\leqslant M(1-z_t(\xi)) \quad \forall t\in T \tag{3d}$$

$$\alpha_t=\sum_{i\in I}x_{it}(\xi), \quad \beta_t^{+}(\xi)=\sum_{i\in I}e_{it}^{+}(\xi), \quad \beta_t^{-}(\xi)=\sum_{i\in I}e_{it}^{-}(\xi) \quad \forall t\in T \tag{3e}$$

$$R_{it}(\xi)-x_{it}(\xi)=y_{it}^{+}(\xi)-y_{it}^{-}(\xi) \quad \forall t\in T \tag{3f}$$

$$R_{it}(\xi)\geqslant y_{it}^{+}(\xi) \quad \forall t\in T \tag{3g}$$

$$y_{it}^{+}(\xi)\leqslant Mz_{it}(\xi), \quad y_{it}^{-}(\xi)\leqslant M(1-z_{it}(\xi)) \quad \forall t\in T \tag{3h}$$

$$\sum_{j\in I,j\neq i}d_{ijt}(\xi)\leqslant y_{it}^{+}(\xi), \quad \sum_{j\in I,j\neq i}d_{jit}(\xi)\leqslant y_{it}^{-}(\xi) \quad \forall t\in T \tag{3i}$$

$$d_{iit}(\xi)=0 \quad \forall t\in T \tag{3j}$$

$$e_{it}^{+}(\xi)=y_{it}^{+}(\xi)-\sum_{j\in I,j\neq i}d_{ijt}(\xi) \quad \forall t\in T \tag{3k}$$

$$e_{it}^{-}(\xi)=y_{it}^{-}(\xi)-\sum_{j\in I,j\neq i}d_{jit}(\xi) \quad \forall t\in T \tag{3l}$$

2 With Storage

2.1 Disaggregation

$$\max \quad \sum_{t\in T}\left(P_t^{DA}x_{it}+\mathbb{E}\left[P_t^{RT}(\xi)y_{it}^{+}(\xi)-P_t^{PN}y_{it}^{-}(\xi)\right]\right) \tag{4a}$$

$$\text{s.t.} \quad R_{it}(\xi)-x_{it}=y_{it}^{+}(\xi)-y_{it}^{-}(\xi)+z_{it}^{C}(\xi)-z_{it}^{D}(\xi) \tag{4b}$$

$$R_{it}(\xi)\geqslant y_{it}^{+}(\xi) \tag{4c}$$

$$z_{i,t+1}(\xi)=z_{it}(\xi)+z_{it}^{C}(\xi)-z_{it}^{D}(\xi) \quad \forall i,t \tag{4d}$$

$$z_{it}^{D}(\xi)\leqslant z_{it}(\xi), \quad z_{it}^{C}(\xi)\leqslant K_i-z_{it}(\xi), \quad 0\leqslant z_{it}(\xi)\leqslant K_i \tag{4e}$$

$$y_{it}^{+}(\xi)\leqslant M_1\phi_{it}^{1}(\xi), \quad y_{it}^{-}(\xi)\leqslant M_1(1-\phi_{it}^{1}(\xi)) \tag{4f}$$

$$y_{it}^{-}(\xi)\leqslant M_1\phi_{it}^{2}(\xi), \quad z_{it}^{C}(\xi)\leqslant M_1(1-\phi_{it}^{2}(\xi)) \tag{4g}$$

$$z_{it}^{C}(\xi)\leqslant M_1\phi_{it}^{3}(\xi), \quad z_{it}^{D}(\xi)\leqslant M_1(1-\phi_{it}^{3}(\xi)) \tag{4h}$$

2.2 Aggregation with individual BTM storage control

$$\max \quad \sum_{t\in T}\left(P_t^{DA}\sum_{i\in I}x_{it}+\mathbb{E}\left[P_t^{RT}(\xi)\sum_{i\in I}e_{it}^{+}(\xi)-P_t^{PN}\sum_{i\in I}e_{it}^{-}(\xi)\right]\right) \tag{5a}$$

$$\text{s.t.} \quad R_{it}(\xi)-x_{it}=y_{it}^{+}(\xi)-y_{it}^{-}(\xi)+z_{it}^{C}(\xi)-z_{it}^{D}(\xi) \tag{5b}$$

$$R_{it}(\xi)+z_{it}^{D}(\xi)\geqslant y_{it}^{+}(\xi)+z_{it}^{C}(\xi) \tag{5c}$$

$$z_{i,t+1}(\xi)=z_{it}(\xi)+z_{it}^{C}(\xi)-z_{it}^{D}(\xi) \tag{5d}$$

$$z_{it}^{D}(\xi)\leqslant z_{it}(\xi), \quad z_{it}^{C}(\xi)\leqslant K_i-z_{it}(\xi), \quad 0\leqslant z_{it}(\xi)\leqslant K_i \tag{5e}$$

$$e_{it}^{+}(\xi)=y_{it}^{+}(\xi)-\sum_{j\in I}d_{ijt}(\xi), \quad e_{it}^{-}(\xi)=y_{it}^{-}(\xi)-\sum_{j\in I}d_{jit}(\xi) \tag{5f}$$

$$d_{iit}(\xi)=0 \tag{5g}$$

$$y_{it}^{+}(\xi)\leqslant M_1\phi_{it}^{1}(\xi), \quad y_{it}^{-}(\xi)\leqslant M_1(1-\phi_{it}^{1}(\xi)) \tag{5h}$$

$$y_{it}^{-}(\xi)\leqslant M_1\phi_{it}^{2}(\xi), \quad z_{it}^{C}(\xi)\leqslant M_1(1-\phi_{it}^{2}(\xi)) \tag{5i}$$

$$z_{it}^{C}(\xi)\leqslant M_1\phi_{it}^{3}(\xi), \quad z_{it}^{D}(\xi)\leqslant M_1(1-\phi_{it}^{3}(\xi)) \tag{5j}$$

$$\sum_{i\in I}e_{it}^{+}(\xi)\leqslant M_2\phi_{t}^{4}(\xi), \quad \sum_{i\in I}e_{it}^{-}(\xi)\leqslant M_2(1-\phi_{t}^{4}(\xi)) \tag{5k}$$

2.3 Aggregation with direct control over storage

$$\max \quad \sum_{t\in T}\left(P_t^{DA}\alpha_t+\mathbb{E}\left[P_t^{RT}(\xi)\beta_t^{+}(\xi)-P_t^{PN}\beta_t^{-}(\xi))\right]\right) \tag{6a}$$

$$\text{s.t.} \quad \sum_{i\in I}R_{it}(\xi)-\alpha_t=\beta_t^{+}(\xi)-\beta_t^{-}(\xi)+\gamma_t^{C}(\xi)-\gamma_t^{D}(\xi) \quad \forall t \tag{6b}$$

$$\sum_{i\in I}R_{it}(\xi)\geqslant \beta_t^{+}(\xi) \tag{6c}$$

$$\gamma_t^{D}(\xi)\leqslant \gamma_t(\xi), \quad \gamma_t^{C}(\xi)\leqslant \sum_{i\in I}K_i-\gamma_t(\xi), \quad 0\leqslant \gamma_t(\xi)\leqslant \sum_{i\in I}K_i \quad \forall t \tag{6d}$$

$$\gamma_{t+1}(\xi)=\gamma_t(\xi)+\gamma_t^{C}(\xi)-\gamma_t^{D}(\xi) \quad \forall t \tag{6e}$$

$$\beta_t^{+}(\xi)\leqslant M_2\mu_t(\xi), \quad \beta_t^{-}(\xi)\leqslant M_2(1-\mu_t(\xi)) \quad \forall t \tag{6f}$$

$$\beta_t^{-}(\xi)\leqslant M_2\eta_t(\xi), \quad \gamma_t^{C}(\xi)\leqslant M_2(1-\eta_t(\xi)) \quad \forall t \tag{6g}$$

$$\gamma_t^{C}(\xi)\leqslant M_2\lambda_t(\xi), \quad \gamma_t^{D}(\xi)\leqslant M_2(1-\lambda_t(\xi)) \quad \forall t \tag{6h}$$

3 Individual

3.1 Different Internal Price (Non-linear)

$$\max \quad \sum_{t\in T}\left(P_t^{DA}\cdot x_t+\mathbb{E}\left[P_t^{RT}(\xi)\cdot y_t^+(\xi)-P_t^{PN}\cdot y_t^-(\xi)+\rho_t^+(d)\cdot d_t^+(\xi)-\rho_t^-(d)\cdot d_t^-(\xi)\right]\right) \tag{7a}$$

$$\text{s.t.} \quad R_t(\xi)-x_t=y_t^+(\xi)-y_t^-(\xi)+d_t^+(\xi)-d_t^-(\xi)+z_t^C(\xi)-z_t^D(\xi) \tag{7b}$$

$$R_t(\xi)\geqslant y_t^+(\xi)+d_{it}^+(\xi) \tag{7c}$$

$$x_t-R_t(\xi)\geqslant y_t^-(\xi)+d_t^-(\xi) \tag{7d}$$

$$z_{t+1}(\xi)=z_t(\xi)+z_t^C(\xi)-z_t^D(\xi) \tag{7e}$$

$$z_t^D(\xi)\leqslant z_t(\xi),\quad z_t^C(\xi)\leqslant K-z_t(\xi),\quad 0\leqslant z_t(\xi)\leqslant K \tag{7f}$$

$$y_t^+(\xi)\leqslant M\phi_t^1(\xi),\quad y_t^-(\xi)\leqslant M(1-\phi_t^1(\xi)) \tag{7g}$$

$$y_t^-(\xi)\leqslant M\phi_t^2(\xi),\quad z_t^C(\xi)\leqslant M(1-\phi_t^2(\xi)) \tag{7h}$$

$$z_t^C(\xi)\leqslant M\phi_t^3(\xi),\quad z_t^D(\xi)\leqslant M(1-\phi_t^3(\xi)) \tag{7i}$$

$$d_t^+(\xi)\leqslant M\phi_t^4(\xi),\quad d_t^-(\xi)\leqslant M(1-\phi_t^4(\xi)) \tag{7j}$$

3.2 Different Internal Price (Stepwise-Linear)

$$\max \quad \sum_{t\in T}\Bigg(P_t^{DA}\cdot x_t+\mathbb{E}\Big[P_t^{RT}(\xi)\cdot y_t^+(\xi)-P_t^{PN}\cdot y_t^-(\xi)$$

$$+\sum_{b^+\in B_t^+}\rho_{t,b^+}^+\cdot(w_{t,b^+}^+(\xi)+u_{t,b^+}^+(\xi)\cdot D_{t,b^+}^{\min+})-\sum_{b^-\in B_t^-}\rho_{t,b^-}^-\cdot(w_{t,b^-}^-(\xi)+u_{t,b^-}^-(\xi)\cdot D_{t,b^-}^{\min-})\Big]\Bigg) \tag{8a}$$

$$\text{s.t.} \quad d_t^+(\xi)=\sum_{b^+\in B_t^+}(w_{t,b^+}^+(\xi)+u_{t,b^+}^+(\xi)\cdot D_{t,b^+}^{\min+}) \qquad \forall t \tag{8b}$$

$$0\leqslant w_{t,b^+}^+(\xi)\leqslant u_{t,b^+}^+(\xi)\cdot W_{t,b^+}^{\max+} \qquad \forall t,b^+ \tag{8c}$$

$$\sum_{b^+\in B_t^+}u_{t,b^+}^+(\xi)\leqslant 1 \qquad \forall t \tag{8d}$$

$$d_t^-(\xi)=\sum_{b^-\in B_t^-}(w_{t,b^-}^-(\xi)+u_{t,b^-}^-(\xi)\cdot D_{t,b^-}^{\min-}) \qquad \forall t \tag{8e}$$

$$0\leqslant w_{t,b^-}^-(\xi)\leqslant u_{t,b^-}^-(\xi)\cdot W_{t,b^-}^{\max-} \qquad \forall t,b^- \tag{8f}$$

$$\sum_{b^-\in B_t^-}u_{t,b^-}^-(\xi)\leqslant 1 \qquad \forall t \tag{8g}$$

$$R_t(\xi)-x_t=y_t^+(\xi)-y_t^-(\xi)+d_t^+(\xi)-d_t^-(\xi)+z_t^C(\xi)-z_t^D(\xi) \qquad \forall t \tag{8h}$$

$$R_t(\xi)+z_t^D(\xi)\geqslant y_t^+(\xi)+d_t^+(\xi)+z_t^C(\xi) \qquad \forall t \tag{8i}$$

$$z_{t+1}(\xi)=z_t(\xi)+z_t^C(\xi)-z_t^D(\xi) \qquad \forall t \tag{8j}$$

$$z_t^D(\xi)\leqslant z_t(\xi),\quad z_t^C(\xi)\leqslant K-z_t(\xi),\quad 0\leqslant z_t(\xi)\leqslant K \qquad \forall t \tag{8k}$$

$$y_t^+(\xi)\leqslant M\phi_t^1(\xi),\quad y_t^-(\xi)\leqslant M(1-\phi_t^1(\xi)) \qquad \forall t \tag{8l}$$

$$y_t^-(\xi)\leqslant M\phi_t^2(\xi),\quad z_t^C(\xi)\leqslant M(1-\phi_t^2(\xi)) \qquad \forall t \tag{8m}$$

$$z_t^C(\xi)\leqslant M\phi_t^3(\xi),\quad z_t^D(\xi)\leqslant M(1-\phi_t^3(\xi)) \qquad \forall t \tag{8n}$$

$$d_t^+(\xi)\leqslant M\phi_t^4(\xi),\quad d_t^-(\xi)\leqslant M(1-\phi_t^4(\xi)) \qquad \forall t \tag{8o}$$