DER Models

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1 Without Storage

1.1 Disaggregation

$$\max \quad \sum_{t \in T} \left(P_t^{DA} x_{it} + \mathbb{E}\left[P_t^{RT}(\xi) y_{it}^+(\xi) - P_t^{PN} y_{it}^-(\xi) \right] \right) \tag{1a}$$

$$\text{s.t.} \quad R_{it}(\xi) - x_{it} = y_{it}^+(\xi) - y_{it}^-(\xi) \quad \forall t \in T \tag{1b}$$

$$R_{it}(\xi) \geqslant y_{it}^+(\xi) \quad \forall t \in T \tag{1c}$$

$$y_{it}^+(\xi) \leqslant M z_{it}(\xi), \quad y_{it}^-(\xi) \leqslant M(1 - z_{it}(\xi)) \quad \forall t \in T \tag{1d}$$

$$x_{it}^{DA} \geqslant 0, y_{it}^+(\xi) \geqslant 0, y_{it}^-(\xi) \geqslant 0, z_{it}(\xi) \in \{0,1\} \quad \forall t \in T \tag{1e}$$

1.2 Aggregation

$$\max \quad \sum_{t \in T} \left(P_t^{DA} \alpha_t + \mathbb{E}\left[P_t^{RT}(\xi) \beta_t^+(\xi) - P_t^{PN} \beta_t^-(\xi) \right] \right) \tag{2a}$$

$$\text{s.t.} \quad \sum_{i \in I} R_{it}(\xi) - \alpha_t = \beta_t^+(\xi) - \beta_t^-(\xi) \quad \forall t \in T \tag{2b}$$

$$\sum_{i \in I} R_{it}(\xi) \geqslant \beta_t^+(\xi) \quad \forall t \in T \tag{2c}$$

$$\beta_t^+(\xi) \leqslant M z_t(\xi), \quad \beta_t^-(\xi) \leqslant M(1 - z_t(\xi)) \quad \forall t \in T \tag{2d}$$

$$\alpha_t^{DA} \geqslant 0, \beta_t^+(\xi) \geqslant 0, \beta_t^-(\xi) \geqslant 0, z_t(\xi) \in \{0,1\} \quad \forall t \in T \tag{2e}$$

1.3 Settlement

$$\max \quad \sum_{t \in T} \left(P_t^{DA} \alpha_t + \mathbb{E}\left[P_t^{RT}(\xi) \beta_t^+(\xi) - P_t^{PN} \beta_t^-(\xi) \right] \right) \tag{3a}$$

$$\text{s.t.} \quad \sum_{i \in I} R_{it}(\xi) - \alpha_t^{DA} = \beta_t^+(\xi) - \beta_t^-(\xi) \quad \forall t \in T \tag{3b}$$

$$\sum_{i \in I} R_{it}(\xi) \geqslant \beta_t^+(\xi) \quad \forall t \in T \tag{3c}$$

$$\beta_t^+(\xi) \leqslant M z_t(\xi), \quad \beta_t^-(\xi) \leqslant M(1 - z_t(\xi)) \quad \forall t \in T \tag{3d}$$

$$\alpha_t = \sum_{i \in I} x_{it}(\xi), \quad \beta_t^+(\xi) = \sum_{i \in I} e_{it}^+(\xi), \quad \beta_t^-(\xi) = \sum_{i \in I} e_{it}^-(\xi) \quad \forall t \in T \tag{3e}$$

$$R_{it}(\xi) - x_{it}(\xi) = y_{it}^+(\xi) - y_{it}^-(\xi) \quad \forall t \in T \tag{3f}$$

$$R_{it}(\xi) \geqslant y_{it}^+(\xi) \quad \forall t \in T \tag{3g}$$

$$y_{it}^+(\xi) \leqslant M z_{it}(\xi), \quad y_{it}^-(\xi) \leqslant M(1 - z_{it}(\xi)) \quad \forall t \in T \tag{3h}$$

$$\sum_{j \in I, j \neq i} d_{ijt}(\xi) \leqslant y_{it}^+(\xi), \quad \sum_{j \in I, j \neq i} d_{jit}(\xi) \leqslant y_{it}^-(\xi) \quad \forall t \in T \tag{3i}$$

$$d_{iit}(\xi) = 0 \quad \forall t \in T \tag{3j}$$

$$e_{it}^+(\xi) = y_{it}^+(\xi) - \sum_{j \in I, j \neq i} d_{ijt}(\xi) \quad \forall t \in T \tag{3k}$$

$$e_{it}^-(\xi) = y_{it}^-(\xi) - \sum_{j \in I, j \neq i} d_{jit}(\xi) \quad \forall t \in T \tag{3l}$$

2 With Storage

2.1 Disaggregation

$$\max \quad \sum_{t \in T} \left(P_t^{DA} x_{it} + \mathbb{E}\left[P_t^{RT}(\xi) y_{it}^+(\xi) - P_t^{PN} y_{it}^-(\xi) \right] \right) \tag{4a}$$

$$\text{s.t.} \quad R_{it}(\xi) - x_{it} = y_{it}^+(\xi) - y_{it}^-(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi) \tag{4b}$$

$$R_{it}(\xi) \geqslant y_{it}^+(\xi) \tag{4c}$$

$$z_{i,t+1}(\xi) = z_{it}(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi) \quad \forall i, t \tag{4d}$$

$$z_{it}^D(\xi) \leqslant z_{it}(\xi), \quad z_{it}^C(\xi) \leqslant K_i - z_{it}(\xi), \quad 0 \leqslant z_{it}(\xi) \leqslant K_i \tag{4e}$$

$$y_{it}^+(\xi) \leqslant M_1 \phi_{it}^1(\xi), \quad y_{it}^-(\xi) \leqslant M_1(1 - \phi_{it}^1(\xi)) \tag{4f}$$

$$y_{it}^-(\xi) \leqslant M_1 \phi_{it}^2(\xi), \quad z_{it}^C(\xi) \leqslant M_1(1 - \phi_{it}^2(\xi)) \tag{4g}$$

$$z_{it}^C(\xi) \leqslant M_1 \phi_{it}^3(\xi), \quad z_{it}^D(\xi) \leqslant M_1(1 - \phi_{it}^3(\xi)) \tag{4h}$$

2.2 Aggregation with individual BTM storage control

$$\max \quad \sum_{t \in T} \left(P_t^{DA} \sum_{i \in I} x_{it} + \mathbb{E}\left[P_t^{RT}(\xi) \sum_{i \in I} e_{it}^+(\xi) - P_t^{PN} \sum_{i \in I} e_{it}^-(\xi) \right] \right) \tag{5a}$$

$$\text{s.t.} \quad R_{it}(\xi) - x_{it} = y_{it}^+(\xi) - y_{it}^-(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi) \tag{5b}$$

$$R_{it}(\xi) + z_{it}^D(\xi) \geqslant y_{it}^+(\xi) + z_{it}^C(\xi) \tag{5c}$$

$$z_{i,t+1}(\xi) = z_{it}(\xi) + z_{it}^C(\xi) - z_{it}^D(\xi) \tag{5d}$$

$$z_{it}^D(\xi) \leqslant z_{it}(\xi), \quad z_{it}^C(\xi) \leqslant K_i - z_{it}(\xi), \quad 0 \leqslant z_{it}(\xi) \leqslant K_i \tag{5e}$$

$$e_{it}^+(\xi) = y_{it}^+(\xi) - \sum_{j \in I} d_{ijt}(\xi), \quad e_{it}^-(\xi) = y_{it}^-(\xi) - \sum_{j \in I} d_{jit}(\xi) \tag{5f}$$

$$d_{iit}(\xi) = 0 \tag{5g}$$

$$y_{it}^+(\xi) \leqslant M_1 \phi_{it}^1(\xi), \quad y_{it}^-(\xi) \leqslant M_1(1 - \phi_{it}^1(\xi)) \tag{5h}$$

$$y_{it}^-(\xi) \leqslant M_1 \phi_{it}^2(\xi), \quad z_{it}^C(\xi) \leqslant M_1(1 - \phi_{it}^2(\xi)) \tag{5i}$$

$$z_{it}^C(\xi) \leqslant M_1 \phi_{it}^3(\xi), \quad z_{it}^D(\xi) \leqslant M_1(1 - \phi_{it}^3(\xi)) \tag{5j}$$

$$\sum_{i \in I} e_{it}^+(\xi) \leqslant M_2 \phi_t^4(\xi), \quad \sum_{i \in I} e_{it}^-(\xi) \leqslant M_2(1 - \phi_t^4(\xi)) \tag{5k}$$

2.3 Aggregation with direct control over storage

$$\max \quad \sum_{t \in T} \left(P_t^{DA} \alpha_t + \mathbb{E}\left[P_t^{RT}(\xi) \beta_t^+(\xi) - P_t^{PN} \beta_t^-(\xi) \right] \right) \tag{6a}$$

$$\text{s.t.} \quad \sum_{i \in I} R_{it}(\xi) - \alpha_t = \beta_t^+(\xi) - \beta_t^-(\xi) + \gamma_t^C(\xi) - \gamma_t^D(\xi) \quad \forall t \tag{6b}$$

$$\sum_{i \in I} R_{it}(\xi) \geqslant \beta_t^+(\xi) \tag{6c}$$

$$\gamma_t^D(\xi) \leqslant \gamma_t(\xi), \quad \gamma_t^C(\xi) \leqslant \sum_{i \in I} K_i - \gamma_t(\xi), \quad 0 \leqslant \gamma_t(\xi) \leqslant \sum_{i \in I} K_i \quad \forall t \tag{6d}$$

$$\gamma_{t+1}(\xi) = \gamma_t(\xi) + \gamma_t^C(\xi) - \gamma_t^D(\xi) \quad \forall t \tag{6e}$$

$$\beta_t^+(\xi) \leqslant M_2 \mu_t(\xi), \quad \beta_t^-(\xi) \leqslant M_2(1 - \mu_t(\xi)) \quad \forall t \tag{6f}$$

$$\beta_t^-(\xi) \leqslant M_2 \eta_t(\xi), \quad \gamma_t^C(\xi) \leqslant M_2(1 - \eta_t(\xi)) \quad \forall t \tag{6g}$$

$$\gamma_t^C(\xi) \leqslant M_2 \lambda_t(\xi), \quad \gamma_t^D(\xi) \leqslant M_2(1 - \lambda_t(\xi)) \quad \forall t \tag{6h}$$

3 Individual

3.1 Different Internal Price (Non-linear)

$$\max \quad \sum_{t \in T} \left(P_t^{DA} \cdot x_t + \mathbb{E}\left[P_t^{RT}(\xi) \cdot y_t^+(\xi) - P_t^{PN} \cdot y_t^-(\xi) + \rho_t^+(d) \cdot d_t^+(\xi) - \rho_t^-(d) \cdot d_t^-(\xi) \right] \right) \tag{7a}$$

$$\text{s.t.} \quad R_t(\xi) - x_t = y_t^+(\xi) - y_t^-(\xi) + d_t^+(\xi) - d_t^-(\xi) + z_t^C(\xi) - z_t^D(\xi) \tag{7b}$$

$$R_t(\xi) \geqslant y_t^+(\xi) + d_{it}^+(\xi) \tag{7c}$$

$$x_t - R_t(\xi) \geqslant y_t^-(\xi) + d_t^-(\xi) \tag{7d}$$

$$z_{t+1}(\xi) = z_t(\xi) + z_t^C(\xi) - z_t^D(\xi) \tag{7e}$$

$$z_t^D(\xi) \leqslant z_t(\xi), \quad z_t^C(\xi) \leqslant K - z_t(\xi), \quad 0 \leqslant z_t(\xi) \leqslant K \tag{7f}$$

$$y_t^+(\xi) \leqslant M\phi_t^1(\xi), \quad y_t^-(\xi) \leqslant M(1 - \phi_t^1(\xi)) \tag{7g}$$

$$y_t^-(\xi) \leqslant M\phi_t^2(\xi), \quad z_t^C(\xi) \leqslant M(1 - \phi_t^2(\xi)) \tag{7h}$$

$$z_t^C(\xi) \leqslant M\phi_t^3(\xi), \quad z_t^D(\xi) \leqslant M(1 - \phi_t^3(\xi)) \tag{7i}$$

$$d_t^+(\xi) \leqslant M\phi_t^4(\xi), \quad d_t^-(\xi) \leqslant M(1 - \phi_t^4(\xi)) \tag{7j}$$

3.2 Different Internal Price (Stepwise-Linear)

$$\max \quad \sum_{t \in T} \left(P_t^{DA} \cdot x_t + \mathbb{E}\left[P_t^{RT}(\xi) \cdot y_t^+(\xi) - P_t^{PN} \cdot y_t^-(\xi) \right.\right.$$

$$\left.\left. + \sum_{b^+ \in B_t^+} \rho_{t,b^+}^+ \cdot (w_{t,b^+}^+(\xi) + u_{t,b^+}^+(\xi) \cdot D_{t,b^+}^{\min +}) - \sum_{b^- \in B_t^-} \rho_{t,b^-}^- \cdot (w_{t,b^-}^-(\xi) + u_{t,b^-}^-(\xi) \cdot D_{t,b^-}^{\min -}) \right] \right) \tag{8a}$$

$$\text{s.t.} \quad d_t^+(\xi) = \sum_{b^+ \in B_t^+} (w_{t,b^+}^+(\xi) + u_{t,b^+}^+(\xi) \cdot D_{t,b^+}^{\min +}) \qquad \forall t \tag{8b}$$

$$0 \leqslant w_{t,b^+}^+(\xi) \leqslant u_{t,b^+}^+(\xi) \cdot W_{t,b^+}^{\max +} \qquad \forall t, b^+ \tag{8c}$$

$$\sum_{b^+ \in B_t^+} u_{t,b^+}^+(\xi) \leqslant 1 \qquad \forall t \tag{8d}$$

$$d_t^-(\xi) = \sum_{b^- \in B_t^-} (w_{t,b^-}^-(\xi) + u_{t,b^-}^-(\xi) \cdot D_{t,b^-}^{\min -}) \qquad \forall t \tag{8e}$$

$$0 \leqslant w_{t,b^-}^-(\xi) \leqslant u_{t,b^-}^-(\xi) \cdot W_{t,b^-}^{\max -} \qquad \forall t, b^- \tag{8f}$$

$$\sum_{b^- \in B_t^-} u_{t,b^-}^-(\xi) \leqslant 1 \qquad \forall t \tag{8g}$$

$$R_t(\xi) - x_t = y_t^+(\xi) - y_t^-(\xi) + d_t^+(\xi) - d_t^-(\xi) + z_t^C(\xi) - z_t^D(\xi) \qquad \forall t \tag{8h}$$

$$R_t(\xi) + z_t^D(\xi) \geqslant y_t^+(\xi) + d_t^+(\xi) + z_t^C(\xi) \qquad \forall t \tag{8i}$$

$$z_{t+1}(\xi) = z_t(\xi) + z_t^C(\xi) - z_t^D(\xi) \qquad \forall t \tag{8j}$$

$$z_t^D(\xi) \leqslant z_t(\xi), \quad z_t^C(\xi) \leqslant K - z_t(\xi), \quad 0 \leqslant z_t(\xi) \leqslant K \qquad \forall t \tag{8k}$$

$$y_t^+(\xi) \leqslant M\phi_t^1(\xi), \quad y_t^-(\xi) \leqslant M(1 - \phi_t^1(\xi)) \qquad \forall t \tag{8l}$$

$$y_t^-(\xi) \leqslant M\phi_t^2(\xi), \quad z_t^C(\xi) \leqslant M(1 - \phi_t^2(\xi)) \qquad \forall t \tag{8m}$$

$$z_t^C(\xi) \leqslant M\phi_t^3(\xi), \quad z_t^D(\xi) \leqslant M(1 - \phi_t^3(\xi)) \qquad \forall t \tag{8n}$$

$$d_t^+(\xi) \leqslant M\phi_t^4(\xi), \quad d_t^-(\xi) \leqslant M(1 - \phi_t^4(\xi)) \qquad \forall t \tag{8o}$$