IIE2104 Deterministic Models in OR
2024-1 Term Project

Group 1
Kwon Minjae, Kim Wonjun, Kim Hyunjin, Jang Seohyun

[Credits]

Kwon Minjae :	LP formulation, Additional Analysis (i), Writing Review
Kim Wonjun :	Further Analysis, Software Development, Report Writing
Kim Hyunjin :	Further Analysis, Software Development, Writing Review
Jang Seohyun :	LP formulation, Additional Analysis (ii), Report Writing

I. Problem Settings - “Basic Model”

For the given two planning months, our goal is to manage operations at Golf-Sports at an optimal level, leading to maximize the profits of the Golf-Sports. We have used cvxpy packages to maximize profit, which is hindered by several constraints.

The constraints include:

- Labor availability
- Packing availability
- Assembly time limit
- Advertising availability
- Demand of each products (and sets)
- Graphite supply
- Production and Inventory Chain

Following indexes will be used for formulating LP optimization models.

- Product = $i \in \{ \ 0, 1, 2, 3, 4, 5, 6, 7, 8 \ \}$

i = 0	Product - Steel shafts
i = 1	Product - Graphite shafts
i = 2	Product - Forged iron heads
i = 3	Product - Metal wood heads
i = 4	Product - Titanium insert heads
i = 5	Set - Steel, Iron, Metal
i = 6	Set - Steel, Iron, Insert
i = 7	Set - Graphite, Iron, Metal
i = 8	Set - Graphite, Iron, Insert

- Factory = $j \in \{ \ 0, 1, 2 \ \}$

j = 0	Chandler Plant
j = 1	Glendale Plant
j = 2	Tucson Plant

- Period = $t \in \{ \ 0, 1 \ \}$

t = 0	Month 1
t = 1	Month 2

Note that zero-based numbering was used for programming convenience.

Based on these indexes, let's denote the decision variables and parameters of the LP model.

- Decision Variables

manufacture$_{ijt}$ (m_{ijt})	Number of units of product (set) i produced at plant j at month t
sold$_{ijt}$ (s_{ijt})	Number of units of product (set) i sold at plant j at month t
inventory$_{ijt}$ (i_{ijt})	Nuber of inventory of product (set) i left at plant j at month t

- Parameters

price$_{ij}$ (R_{ij}) [$ / unit]	Revenue per product i in plant j
cost$_{ijt}$ (C_{ijt}) [$ / unit]	Material, Production, and Assembly Costs per product i in plant j
inventory_cost$_{ijt}$ (I_{ijt}) [$ / unit]	Inventory costs which is based on inventory for each product i in plant j at the end of month t
advertise_cost$_{ij}$ (A_{ij}) [$ / unit]	Advertising costs needed for product i in plant j
advertise_limit (AL) [$]	Monthly available advertising cost
labor_time$_{ij}$ (L_{ij}) [min / unit]	Labor needed to produce product i in plant j
labor_limit$_j$ (LL_j) [min]	Monthly available labor in plant j
packing_time$_{ij}$ (P_{ij}) [min / unit]	Packing needed to produce product i in plant j
packing_limit$_j$ (PL_j) [min]	Monthly available packing in plant j
assmebly_time$_{ij}$ (AS_{ij}) [min / set]	Time needed to assemble a set i in plant j
assembly_limit$_j$ (ASL_j) [min]	Total time available for assembling set in plant j
demand_min$_{ij}$ $(D_{\min,ij})$ [unit (s)]	Minimum amount of demand for each product i - plant j pair
demand_max$_{ij}$ $(D_{\max,ij})$ [unit (s)]	Maximum amount of demand for each product i - plant j pair

Note that the values of these parameters are also given in Excel file "OR.xlsx".

II. Objective Function

Objective function of the model is to maximize Golf-Sport company's profit, which refers to the total amount of revenue substracted by costs of production and inventory. Using the notations from (I), objective function can be expressed as following.

Maximize profit $z = \sum_{t=0}^{1} \sum_{j=0}^{2} \sum_{i=0}^{8} [\ R_{ij}s_{ijt} - C_{ijt}m_{ijt} - I_{ijt}i_{ijt}\ ]$

Matrix form of each parameter are as following.

$$R = \begin{bmatrix} 10 & 10 & 12 \\ 25 & 25 & 30 \\ 8 & 8 & 10 \\ 18 & 18 & 22 \\ 40 & 40 & 45 \\ 290 & 290 & 310 \\ 380 & 380 & 420 \\ 560 & 560 & 640 \\ 650 & 650 & 720 \end{bmatrix} \quad C_1 = \begin{bmatrix} 6 & 5 & 7 \\ 19 & 18 & 20 \\ 4 & 5 & 5 \\ 10 & 11 & 12 \\ 26 & 24 & 27 \\ 178 & 175 & 180 \\ 228 & 220 & 240 \\ 350 & 360 & 370 \\ 420 & 435 & 450 \end{bmatrix} \quad C_2 = \begin{bmatrix} 6.72 & 5.6 & 7.84 \\ 21.28 & 20.16 & 22.4 \\ 4.48 & 5.6 & 5.6 \\ 11.2 & 12.32 & 13.44 \\ 29.12 & 26.88 & 30.24 \\ 199.36 & 196 & 201.6 \\ 255.36 & 246.4 & 268.8 \\ 392 & 403.2 & 414.4 \\ 470.4 & 487.2 & 504 \end{bmatrix}$$

$$S_1 = \begin{bmatrix} 0.48 & 0.4 & 0.56 \\ 1.52 & 1.44 & 1.6 \\ 0.32 & 0.4 & 0.4 \\ 0.8 & 0.88 & 0.96 \\ 2.08 & 1.92 & 2.16 \\ 14.24 & 14 & 14.4 \\ 18.24 & 17.6 & 19.2 \\ 28 & 28.8 & 29.6 \\ 33.6 & 34.8 & 36 \end{bmatrix} \quad S_2 = \begin{bmatrix} 0.5376 & 0.448 & 0.627 \\ 1.7024 & 1.6128 & 1.792 \\ 0.3584 & 0.448 & 0.448 \\ 0.896 & 0.9856 & 1.0752 \\ 2.3296 & 2.1504 & 2.4192 \\ 15.9488 & 15.68 & 16.128 \\ 20.4288 & 19.712 & 21.504 \\ 31.36 & 32.256 & 33.152 \\ 37.632 & 38.976 & 40.32 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1.1 & 1.3 \\ 1.5 & 1.1 & 1.3 \\ 1.1 & 1.1 & 1.3 \\ 1.5 & 1.2 & 1.3 \\ 1.9 & 1.9 & 1.9 \\ 28.5 & 28.9 & 33.8 \\ 29.7 & 31 & 35.6 \\ 35 & 28.9 & 33.8 \\ 36.2 & 31 & 35.6 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 3.5 & 3 \\ 1.5 & 3.5 & 3.5 \\ 1.5 & 4.5 & 4 \\ 3 & 4.5 & 4.5 \\ 4 & 5 & 5.5 \\ 37 & 104 & 92.5 \\ 40 & 105.5 & 95.5 \\ 43.5 & 104 & 99 \\ 46.5 & 105.5 & 102 \end{bmatrix} \quad P = \begin{bmatrix} 4 & 7 & 7.5 \\ 4 & 7 & 7.5 \\ 5 & 8 & 8.5 \\ 6 & 9 & 9.5 \\ 6 & 7 & 8 \\ 120 & 198 & 211 \\ 120 & 192 & 206.5 \\ 120 & 198 & 211 \\ 120 & 192 & 206.5 \end{bmatrix}$$

$$AS = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 65 & 60 & 65 \\ 65 & 60 & 65 \\ 65 & 60 & 65 \\ 65 & 60 & 65 \end{bmatrix} \quad D_{\min} = \begin{bmatrix} 0 & 0 & 0 \\ 100 & 100 & 50 \\ 200 & 200 & 100 \\ 30 & 30 & 15 \\ 100 & 100 & 100 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad D_{\max} = \begin{bmatrix} 2000 & 2000 & 2000 \\ 2000 & 2000 & 2000 \\ 2000 & 2000 & 2000 \\ 2000 & 2000 & 2000 \\ 2000 & 2000 & 2000 \\ 200 & 200 & 200 \\ 100 & 100 & 100 \\ 300 & 300 & 300 \\ 400 & 400 & 400 \end{bmatrix}$$

$LL = [\ 12000 \quad 15000 \quad 22000\ ]$ $\qquad PL = [\ 20000 \quad 40000 \quad 35000\ ]$

$ASL = [\ 5500 \quad 5000 \quad 6000\ ]$ $\qquad AL = [\ 20000\ ]$

III. Constraints

Constraints can be expressed as following.

- Labor availability

$\sum_{i=0}^{8} L_{ij} m_{ijt} \le LL_j$ for each $j \in \{0, 1, 2\}$ and $t \in \{0, 1\}$

- Packing availability

$\sum_{i=0}^{8} P_{ij} m_{ijt} \le PL_j$ for each $j \in \{0, 1, 2\}$ and $t \in \{0, 1\}$

- Assembly time limit

$\sum_{i=0}^{8} AS_{ij} m_{ijt} \le ASL_j$ for each $j \in \{0, 1, 2\}$ and $t \in \{0, 1\}$

Precisely, we can also use $\sum_{i=5}^{8} AS_{ij} m_{ijt} \le ASL_j$ since $\sum_{i=0}^{4} AS_{ij} m_{ijt} = 0$.

- Advertising availability

$\sum_{j=0}^{2} \sum_{i=0}^{8} A_{ij} m_{ijt} \le AL$ for each $t \in \{0, 1\}$

- Demand of each products (and sets)

$s_{ijt} \ge D_{\min,ij}$ for each $i \in \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$, $j \in \{0, 1, 2\}$ and $t \in \{0, 1\}$

$s_{ijt} \le D_{\max,ij}$ for each $i \in \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$, $j \in \{0, 1, 2\}$ and $t \in \{0, 1\}$

- Graphite supply

$\sum_{j=0}^{2} [\ m_{1jt} + 13m_{7jt} + 13m_{8jt}\ ] \le 4000$ for each $t \in \{0, 1\}$

1,000 pounds are equal to 16,000 ounces, therefore we can produce 4,000 graphite shafts per month.

- Production and Inventory Chain

$m_{ij1} = i_{ij1} + s_{ij1}$ for each $i \in \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$, $j \in \{0, 1, 2\}$

$m_{ij1} + i_{ij1} = i_{ij2} + s_{ij2}$ for each $i \in \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$, $j \in \{0, 1, 2\}$

Since inventory after Month 2 does not have to be considered, $m_{ij1} + i_{ij1} = s_{ij2}$ for each $i \in \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$, $j \in \{0, 1, 2\}$ is also acceptable.

IV. Results

Through our basic model, the following results could be obtained.

Optimal Value :

	Profit
Advertising cost excluded	$ 258,326.76
Advertising cost included	$ 221,659.88

Because it is ambiguous whether advertising cost should be included, both values, with and without advertising cost were calculated. For further convenience, we used the "Excluded" optimal value $ 258,326.76 in this project.

Sold quantities (Quantities of product that should be sold) :

	Month 1			Month 2		
	Chandler	Glendale	Tucson	Chandler	Glendale	Tucson
Steel shafts	0	0	0	0	0	0
Graphite shafts	100	100	2,000	100	100	50
Forged iron heads	200	200	100	200	200	100
Metal wood heads	30	30	15	30	30	15
Titanium insert heads	2,000	2,000	2,000	2,000	2,000	2,000
Set : Steel, metal	0	0	0	0	0	0
Set : Steel, insert	0	0	0	0	0	0
Set : Graphite, metal	0	0	0	0	0	83.57
Set : Graphite, insert	53.50	34.27	14.56	53.50	34.27	0

Manufacture quantities (Quantities of product that should be produced) :

The table is on the next page.

	Month 1			Month 2		
	Chandler	Glendale	Tucson	Chandler	Glendale	Tucson
Steel shafts	0	0	0	0	0	0
Graphite shafts	100	100	2,000	100	100	50
Forged iron heads	200	200	100	200	200	100
Metal wood heads	30	30	15	30	30	15
Titanium insert heads	2,000	2,000	2,000	2,000	2,000	2,000
Set : Steel, metal	0	0	0	0	0	0
Set : Steel, insert	0	0	0	0	0	0
Set : Graphite, metal	0	0	0	0	0	83.57
Set : Graphite, insert	53.50	34.27	14.56	53.50	34.27	0

Inventory quantities (Amount of inventory that should be stored after Month 1) :

	Month 1		
	Chandler	Glendale	Tucson
Steel shafts	0	0	0
Graphite shafts	0	0	0
Forged iron heads	0	0	0
Metal wood heads	0	0	0
Titanium insert heads	0	0	0
Set : Steel, metal	0	0	0
Set : Steel, insert	0	0	0
Set : Graphite, metal	0	0	0
Set : Graphite, insert	0	0	0

Resource usage (The usages of labor, packing, advertising and graphite) :

	Month 1		
	Chandler	Glendale	Tucson
Labor	11,027.75	15,000.00	19,953.04
Packing	20,000.00	23,148.96	35,000.00
Advertising	6,151.70	5,228.23	7,067.98
Graphite	795.50	545.45	2,189.33

	Month 2		
	Chandler	Glendale	Tucson
Labor	11,027.75	15,000.00	19,915.57
Packing	20,000.00	23,148.96	35,000.00
Advertising	6,151.70	5,228.23	6,839.04
Graphite	795.50	545.45	1,136.36

V. Further Analysis

i. Graphite vs. Advertising Cash

Question : If you could get more graphite or advertising cash, what would you be willing to pay?

Since the original optimum point did not reach the maximum point of both the graphite usage and advertising cost constraint, the optimum solution will not change even if we get more access to graphite or advertisement cash. Thus, since the optimum value of profit won't change, we will not be willing to pay anymore to earn additional graphite and advertising cash.

ii. Lifting Limitations for Extra Production

Question : At what site (s) would you like to add extra packing machine hours, assembly hours, and / or extra labor hours? How much would you be willing to pay per hour?

To determine which plant site to extend additional packing / assembly / labor hours, we remodeled the original problem by removing packing / assembly / labor hours respectively in each distinctive plant site.

1. Removing **labor** constraints in **Chandler**

Newly obtained optimal value : $ 258,326.76

Difference of optimal value between original and current cost : $ 0.00

Since the optimal value has not changed, dual price of labor in Chandler : $ 0.00

2. Removing **labor** constraints in **Glendale**

Newly obtained optimal value : $ 288,668.07

Difference of optimal value between original and current cost : $ 30,341.30

Additional labor time required for the optimal value :

	Glendale
Month 1	8,891.67
Month 2	8,891.67

Glendale's labor time dual price : 30,431.30 / 17,783.34 = $ 1.71

3. Removing **labor** constraints in **Tucson**

Newly obtained optimal value : $ 258,326.76

Difference of optimal value between original and current cost : $ 0.00

Since the optimal value has not changed, dual price of labor in Tucson : $ 0.00

4. Removing **packing** constraints in **Chandler**

Newly obtained optimal value : $ 270,682.68

Difference of optimal value between original and current cost : $ 12,355.91

Additional packing time required for the optimal value :

	Chandler
Month 1	9,937.32
Month 2	8,021.92

Chandler's packing time dual price : 12,355.91 / 17,959.24 = $ 0.69

5. Removing **packing** constraints in **Glendale**

Newly obtained optimal value : $ 258,326.76

Difference of optimal value between original and current cost : $ 0.00

Since the optimal value has not changed, dual price of packing in Glendale : $ 0.00

6. Removing **packing** constraints in **Tucson**

Newly obtained optimal value : $ 268,647.63

Difference of optimal value between original and current cost : $ 10,320.87

Additional packing time required for the optimal value :

	Tucson
Month 1	4,521.36
Month 2	4,456.65

Tucson's packing time dual price : 10320.87 / 8978.01 = $ 1.15

7. Removing **assembly** constraints in **Chandler**
   Newly obtained optimal value : $ 258,326.76
   Difference of optimal value between original and current cost : $ 0.00
   Since the optimal value has not changed, dual price of assembly time in Chandler : $ 0.00

8. Removing **assembly** constraints in **Glendale**
   Newly obtained optimal value : $ 258,326.76
   Difference of optimal value between original and current cost : $ 0.00
   Since the optimal value has not changed, dual price of assembly time in Glendale : $ 0.00

9. Removing **assembly** constraints in **Tucson**
   Newly obtained optimal value : $ 258,326.76
   Difference of optimal value between original and current cost : $ 0.00
   Since the optimal value has not changed, dual price of assembly time in Tucson : $ 0.00

From the result, we would like to add extra labor for Glendale, extra packing for Chandler and Tucson. Also, we are willing to pay less than $ 1.71 , $ 0.69 and $ 1.15 (dual prices) for Glendale (the second factory), Chandler (the first) and Tucson (the third) respectively.

iii. Relaxing Maximum Demand

Question : Marketing is trying to get Golf-Sport to consider an advertising program that promises a 50% increase in their maximum demand. Can we handle this with the current system or do we need more resources? How much more is the production going to cost if we take on the additional demand?

If we increase the maximum demand limit to 50%, $D_{\max}$ matrix changes as follows :

$$D_{\max} = \begin{bmatrix} 3000 & 3000 & 3000 \\ 3000 & 3000 & 3000 \\ 3000 & 3000 & 3000 \\ 3000 & 3000 & 3000 \\ 3000 & 3000 & 3000 \\ 300 & 300 & 300 \\ 150 & 150 & 150 \\ 450 & 450 & 450 \\ 600 & 600 & 600 \end{bmatrix}$$

This new constraint of additional demand affects the optimal value to change as follows :

	Profit
Optimal value	$ 282,606.48

Sold and Manufacture quantities also changed as follows :

	Month 1			Month 2		
	Chandler	Glendale	Tucson	Chandler	Glendale	Tucson
Steel shafts	0	0	0	0	0	0
Graphite shafts	100	100	1,334.33	100	100	50
Forged iron heads	200	200	100	200	200	100
Metal wood heads	30	30	15	30	30	15
Titanium insert heads	2,580.45	2,723	3,000	2,580.45	2,723	3,000
Set : Steel, metal	0	0	0	0	0	0
Set : Steel, insert	0	0	0	0	0	0
Set : Graphite, metal	0	0	0	0	0	45.65
Set : Graphite, insert	24.48	0	0	24.48	0	0

The inventory value remains zero for all products, same as the basic model.

Delving into the production status of the three plants respectively, we can conclude that Chandler and Glendale are in need of additional resources while Tucson can bear with its current system.

In the case of Chandler, our original result value shows that Labor and Packing status is both at the limit already without any product reaching its maximum demand. Original production value of Glendale also had its Labor used up to its limit, having no product meeting its maximum demand. Meanwhile, Tucson initially produced to the point of maximum packing time, letting the Titanium insert head produce to its maximum demand. This implies that only the first two plants, Chandler and Glendale, are stifled by the current constraints and thus, has the potential to expand its production only if more resources are supplied.

Since the original production cost was $ 502,456.92 which was obtained by multiplying products to the cost and new total production cost is $ 535,457.30 using the same way, we can say that the production is going to cost $ 33,000.38 more if we take on the additional demand.

VI. Additional Analysis

i. Simultaneous relaxing

We analyzed the effects of simultaneously unrestricting labor, packing, and assembly time constraints, which is another point of view to interpret the second topic of part (V). With this new model, we will observe the changes in the optimal resource requirements of each factory.

Unlike the original interpretation we illustrated in the second topic of part (V), our goal here is to find the optimal amount of resources rather than an additional amount (dual price). So, instead of removing the constraints for each factory one by one, we modified the model to find the optimal value when the three constraints of the entire factories were removed from the previous optimization problem at the same time.

1. Labor time

Newly obtained optimal value : $ 288,668.07

	Total labor time		
	Chandler	Glendale	Tucson
Month 1	11,027.75	23,891.67	20,352.02
Month 2	11,027.75	23,891.67	19,915.57

	Optimal labor time difference compared with original		
	Chandler	Glendale	Tucson
Month 1	0	8,891.67	398.98
Month 2	0	8,891.67	0

2. Packing time

Newly obtained optimal value : $ 280,384.33

	Total packing time		
	Chandler	Glendale	Tucson
Month 1	28,021.92	23,148.96	39,505.71
Month 2	28,021.92	23,148.96	39,456.65

Other table is on the next page.

	Optimal packing time difference compared with original		
	Chandler	Glendale	Tucson
Month 1	8,021.92	0	4,505.71
Month 2	8,021.92	0	4,456.65

3. Assembly time

Newly obtained optimal value : $ 258,326.76

	Total assembly time		
	Chandler	Glendale	Tucson
Month 1	3,477.50	2,055.92	946.67
Month 2	3,477.50	2,055.92	5,431.81

	Optimal assembly time difference compared with original		
	Chandler	Glendale	Tucson
Month 1	0	0	0
Month 2	0	0	0

ii. Goal Programming

We concluded the second topic of part (V) that it is better off for Chandler to lift packing constraints, Glendale to lift labor constraints, and Tucson to lift packing constraints. Under those circumstances, the advertising costs were used as follows.

	Advertising costs		
	Chandler	Glendale	Tucson
Month 1	4,575.63	7,837.08	7,587.29
Total	20,000		

	Advertising costs		
	Chandler	Glendale	Tucson
Month 2	5,939	6,574.33	7,486.67
Total	20,000		

It is observable that for both months, advertising costs are fully utilized. Yet, in the real world, the advantages of advertising are more noteworthy than in the original problem showcasing remarkable increase in demand, market share expansion, attracting new customers, and so on. So we decided to arbitrarily increase the advertising budget by $ 3,000 to see the potential affects of advertising. Further on, we decided to implement the concept of “Goal Programming’’, having its goal in reaching advertising cost to its maximum availability which is $ 23,000 per month.

The results are as follows :

	Advertising costs		
	Chandler	Glendale	Tucson
Month 1	7,236.67	8,176.04	7,587.29
Total	23,000		

	Advertising costs		
	Chandler	Glendale	Tucson
Month 2	8,102.63	7,310.08	7,587.29
Total	23,000		

	Profit
Optimal value	$ 305,974.52

The optimal value before increasing the advertising budget was $ 281,482.35 , but after raising the advertising cost to $ 23,000, the optimal value increased to $ 305,974.52. From the result, we can conclude that the impact of advertising is evident to have a significantly positive effect on overall profitability. It shows that investing more in advertising leads to greater market reach and bigger customer engagement, ultimately resulting in higher profits for the company.