2nd Homework

Due: May 21th 18:00

1. A process is in statistical control with $\bar{\bar{x}} = 199$ and $\bar{R} = 3.5$. The control chart uses a sample size of $n = 4$. Specifications are at 200 ± 8. The quality characteristic is normally distributed.

a. Estimate the potential capability of the process.

$$\hat{\sigma} = \bar{R}/d_2 = 3.5/2.059 = 1.7$$

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{208-192}{6 \times 1.7} = 1.569$$

b. Estimate the actual process capability.

$$C_{pu} = \frac{208-199}{3 \times 1.7} = 1.765$$

$$C_{pl} = \frac{199-192}{3 \times 1.7} = 1.373$$

$$C_{pk} = \min(C_{pu}, C_{pl}) = 1.373$$

c. How much improvement could be made in process performance if the mean could be centered at the nominal value?

$$1.569 - 1.373 = 0.196$$

2. A normally distributed process has specifications of $LSL = 75$ and $USL = 85$ on the output. A random sample of 25 parts indicates that the process is centered at the middle of the specification band, and the standard deviation is $s = 1.5$. Calculate a point estimate of the process capability index of C_p.

$$C_p = \frac{85-75}{6 \times 1.5} = 1.11$$

3. The molecular weight of a particular polymer should fall between 2,100 and 2,350. Fifty samples of this material were analyzed with the results $\overline{x} = 2{,}275$ and $s = 60$. Assume that the molecular weight is normally distributed. Calculate a point estimate of C_{pk}.

$$C_{pu} = \frac{2350 - 2275}{3 \times 60} = 0.417$$

$$C_{pl} = \frac{2275 - 2100}{3 \times 60} = 0.972$$

$$C_{pk} = \min(C_{pu}, C_{pl}) = 0.417$$

4. A machine is used to fill cans with motor oil additive. A single sample can is selected every hour, and the weight of the can is obtained. Since the filling process is automated, it has very stable variability, and long experience indicates that $\sigma = 0.05$ oz.

The individual observations for 24 hours of operation are shown in the following Table.

Sample Number	x	Sample Number	x
1	8	13	8.05
2	8.01	14	8.04
3	8.02	15	8.03
4	8.01	16	8.05
5	8	17	8.06
6	8.01	18	8.04
7	8.06	19	8.05
8	8.07	20	8.06
9	8.01	21	8.04
10	8.04	22	8.02
11	8.02	23	8.03
12	8.01	24	8.05

a. Assuming that the process target is 8.02 oz, set up a tabular CUSUM for this process. Design the CUSUM using the standardized values $h = 4.77$ and $k = 0.5$.

$$H = 4.77 \times 0.05 = 0.2385$$

$$K = 0.5 \times 0.05 = 0.025$$

#4	H	0.2385	K	0.025			
	x	x - 8.045	C+	N+	7.995 - x	C-	N-
			0			0	
1	8	-0.045	0	0	-0.005	0	0
2	8.01	-0.035	0	0	-0.015	0	0
3	8.02	-0.025	0	0	-0.025	0	0
4	8.01	-0.035	0	0	-0.015	0	0
5	8	-0.045	0	0	-0.005	0	0
6	8.01	-0.035	0	0	-0.015	0	0
7	8.06	0.015	0.015	1	-0.065	0	0
8	8.07	0.025	0.04	2	-0.075	0	0
9	8.01	-0.035	0.005	3	-0.015	0	0
10	8.04	-0.005	0	0	-0.045	0	0
11	8.02	-0.025	0	0	-0.025	0	0
12	8.01	-0.035	0	0	-0.015	0	0
13	8.05	0.005	0.005	1	-0.055	0	0
14	8.04	-0.005	0	0	-0.045	0	0
15	8.03	-0.015	0	0	-0.035	0	0
16	8.05	0.005	0.005	1	-0.055	0	0
17	8.06	0.015	0.02	2	-0.065	0	0
18	8.04	-0.005	0.015	3	-0.045	0	0
19	8.05	0.005	0.02	4	-0.055	0	0
20	8.06	0.015	0.035	5	-0.065	0	0
21	8.04	-0.005	0.03	6	-0.045	0	0
22	8.02	-0.025	0.005	7	-0.025	0	0
23	8.03	-0.015	0	0	-0.035	0	0
24	8.05	0.005	0.005	1	-0.055	0	0

b. Does the value of $\sigma = 0.05$ seem reasonable for this process?

Yes.

filling process가 stable variability와 long experience를 가지고 있기 때문.

5. Rework Problem 4 using the standardized CUSUM parameters of $h = 8.01$ and $k = 0.25$. Compare the results with those obtained previously in Exercise **4.a**. What can you say about the performance of those two CUSUM schemes?

#5	H	0.4005	K	0.0125			
	x	x - 8.0225	C+	N+	8.0175 - x	C-	N-
			0			0	
1	8	-0.0225	0	0	0.0175	0.0175	1
2	8.01	-0.0125	0	0	0.0075	0.025	2
3	8.02	-0.0025	0	0	-0.0025	0.0225	3
4	8.01	-0.0125	0	0	0.0075	0.03	4
5	8	-0.0225	0	0	0.0175	0.0475	5
6	8.01	-0.0125	0	0	0.0075	0.055	6
7	8.06	0.0375	0.0375	1	-0.0425	0.0125	7
8	8.07	0.0475	0.085	2	-0.0525	0	0
9	8.01	-0.0125	0.0725	3	0.0075	0.0075	1
10	8.04	0.0175	0.09	4	-0.0225	0	0
11	8.02	-0.0025	0.0875	5	-0.0025	0	0
12	8.01	-0.0125	0.075	6	0.0075	0.0075	1
13	8.05	0.0275	0.1025	7	-0.0325	0	0
14	8.04	0.0175	0.12	8	-0.0225	0	0
15	8.03	0.0075	0.1275	9	-0.0125	0	0
16	8.05	0.0275	0.155	10	-0.0325	0	0
17	8.06	0.0375	0.1925	11	-0.0425	0	0
18	8.04	0.0175	0.21	12	-0.0225	0	0
19	8.05	0.0275	0.2375	13	-0.0325	0	0
20	8.06	0.0375	0.275	14	-0.0425	0	0
21	8.04	0.0175	0.2925	15	-0.0225	0	0
22	8.02	-0.0025	0.29	16	-0.0025	0	0
23	8.03	0.0075	0.2975	17	-0.0125	0	0
24	8.05	0.0275	0.325	18	-0.0325	0	0

둘 다 in-control을 나타내지만,
#5에서 K가 감소해서 더 작은 upward shift를 빠르게 감지할 것이고,
K가 큰 #4가 더 보수적인 감지를 할 것이다.
#5는 아직 out-of-control인 포인트가 발생하지 않았지만,
누적값이 계속 증가하고 있는 것을 확인할 수 있다.

6. Reconsider the data in Problem 4. Suppose the data there represent observations taken immediately after a process adjustment that was intended to reset the process to a target of $\mu_0 = 8.00$. Set up and apply an FIR CUSUM to monitor this process

#6	H	0.2385	K	0.025			
	x	x - 8.025	C+	N+	7.975 - x	C-	N-
			0.11925			0.11925	
1	8	-0.025	0.09425	1	-0.025	0.09425	1
2	8.01	-0.015	0.07925	2	-0.035	0.05925	2
3	8.02	-0.005	0.07425	3	-0.045	0.01425	3
4	8.01	-0.015	0.05925	4	-0.035	0	0
5	8	-0.025	0.03425	5	-0.025	0	0
6	8.01	-0.015	0.01925	6	-0.035	0	0
7	8.06	0.035	0.05425	7	-0.085	0	0
8	8.07	0.045	0.09925	8	-0.095	0	0
9	8.01	-0.015	0.08425	9	-0.035	0	0
10	8.04	0.015	0.09925	10	-0.065	0	0
11	8.02	-0.005	0.09425	11	-0.045	0	0
12	8.01	-0.015	0.07925	12	-0.035	0	0
13	8.05	0.025	0.10425	13	-0.075	0	0
14	8.04	0.015	0.11925	14	-0.065	0	0
15	8.03	0.005	0.12425	15	-0.055	0	0
16	8.05	0.025	0.14925	16	-0.075	0	0
17	8.06	0.035	0.18425	17	-0.085	0	0
18	8.04	0.015	0.19925	18	-0.065	0	0
19	8.05	0.025	0.22425	19	-0.075	0	0
20	8.06	0.035	**0.25925**	20	-0.085	0	0
21	8.04	0.015	**0.27425**	21	-0.065	0	0
22	8.02	-0.005	**0.26925**	22	-0.045	0	0
23	8.03	0.005	**0.27425**	23	-0.055	0	0
24	8.05	0.025	**0.29925**	24	-0.075	0	0

FIR을 적용하면 더욱 빠르게 out-of-control을 감지할 수 있다.

∴ process adjustment fail

7. Bath concentrations are measured hourly in a chemical process. Data (in ppm) for the last 32 hours are shown in Table (read down from left).

Table Bath Concentration

160	186	190	206
158	195	189	210
150	179	185	216
151	184	182	212
153	175	181	211
154	192	180	202
158	186	183	205
162	197	186	197

The process target is $\mu_0 = 175$ ppm.

a. Estimate the process standard deviation.

process mean $= \Sigma x_i / n = 183.594$

process std $= \sqrt{\dfrac{\Sigma(x_i-\bar{x})^2}{n-1}} = 19.434$

b. Construct a tabular CUSUM for this process using standardized values of $h = 5$ and $k = 1$

#7	target	175	std	19.43	H	97.15	K	19.43
	x	x - 194.43	C+	N+	155.57 - x	C-	N-	
			0			0		
1	160	-34.43	0	0	-4.43	0	0	
2	158	-36.43	0	0	-2.43	0	0	
3	150	-44.43	0	0	5.57	5.57	1	
4	151	-43.43	0	0	4.57	10.14	2	
5	153	-41.43	0	0	2.57	12.71	3	
6	154	-40.43	0	0	1.57	14.28	4	
7	158	-36.43	0	0	-2.43	11.85	5	
8	162	-32.43	0	0	-6.43	5.42	6	
9	186	-8.43	0	0	-30.43	0	0	
10	195	0.57	0.57	1	-39.43	0	0	
11	179	-15.43	0	0	-23.43	0	0	
12	184	-10.43	0	0	-28.43	0	0	
13	175	-19.43	0	0	-19.43	0	0	
14	192	-2.43	0	0	-36.43	0	0	
15	186	-8.43	0	0	-30.43	0	0	
16	197	2.57	2.57	1	-41.43	0	0	
17	190	-4.43	0	0	-34.43	0	0	
18	189	-5.43	0	0	-33.43	0	0	
19	185	-9.43	0	0	-29.43	0	0	
20	182	-12.43	0	0	-26.43	0	0	
21	181	-13.43	0	0	-25.43	0	0	
22	180	-14.43	0	0	-24.43	0	0	
23	183	-11.43	0	0	-27.43	0	0	
24	186	-8.43	0	0	-30.43	0	0	
25	206	11.57	11.57	1	-50.43	0	0	
26	210	15.57	27.14	2	-54.43	0	0	
27	216	21.57	48.71	3	-60.43	0	0	
28	212	17.57	66.28	4	-56.43	0	0	
29	211	16.57	82.85	5	-55.43	0	0	
30	202	7.57	90.42	6	-46.43	0	0	
31	205	10.57	**100.99**	7	-49.43	0	0	
32	197	2.57	**103.56**	8	-41.43	0	0	

8. Consider the "minute clinic" waiting time data in the following table

Observation	Waiting Time	Observation	Waiting Time	Observation	Waiting Time
1	2.49	11	1.34	21	1.14
2	3.39	12	0.5	22	2.66
3	7.41	13	4.35	23	4.67
4	2.88	14	1.67	24	1.54
5	0.76	15	1.63	25	5.06
6	1.32	16	4.88	26	3.4
7	7.05	17	15.19	27	1.39
8	1.37	18	0.67	28	1.11
9	6.17	19	4.14	29	6.92
10	5.12	20	2.16	30	36.99

a. These data may not be normally distributed. Set up a CUSUM chart for monitoring this process. Does the process seem to be in statistical control? Yes. C+, C-가 H보다 작다

#8							
	h	5	k	0.5			
	x	x - 8.049	C+	N+	1.242 - x	C-	N-
			0			0	
1	2.49	-5.559	0	0	-1.248	0	0
2	3.39	-4.659	0	0	-2.148	0	0
3	7.41	-0.639	0	0	-6.168	0	0
4	2.88	-5.169	0	0	-1.638	0	0
5	0.76	-7.289	0	0	0.482	0.482	1
6	1.32	-6.729	0	0	-0.078	0.404	1
7	7.05	-0.999	0	0	-5.808	0	0
8	1.37	-6.679	0	0	-0.128	0	0
9	6.17	-1.879	0	0	-4.928	0	0
10	5.12	-2.929	0	0	-3.878	0	0
11	1.34	-6.709	0	0	-0.098	0	0
12	0.5	-7.549	0	0	0.742	0.742	1
13	4.35	-3.699	0	0	-3.108	0	0
14	1.67	-6.379	0	0	-0.428	0	0
15	1.63	-6.419	0	0	-0.388	0	0
16	4.88	-3.169	0	0	-3.638	0	0
17	15.19	7.141	7.141	1	-13.948	0	0
18	0.67	-7.379	0	0	0.572	0.572	1
19	4.14	-3.909	0	0	-2.898	0	0
20	2.16	-5.889	0	0	-0.918	0	0
21	1.14	-6.909	0	0	0.102	0.102	1
22	2.66	-5.389	0	0	-1.418	0	0
23	4.67	-3.379	0	0	-3.428	0	0
24	1.54	-6.509	0	0	-0.298	0	0
25	5.06	-2.989	0	0	-3.818	0	0
26	3.4	-4.649	0	0	-2.158	0	0
27	1.39	-6.659	0	0	-0.148	0	0
28	1.11	-6.939	0	0	0.132	0.132	1
29	6.92	-1.129	0	0	-5.678	0	0
30	36.99	28.941	28.941	1	-35.748	0	0
x_bar	4.6456667						
sigma	6.8073						
H	34.0365						

b. These data may not be normally distributed. Set up an EWMA control chart using $\lambda = 0.1$ for monitoring this process. Does the process seem to be in statistical control?

	Lambda	0.1	L	3
	x	z	UCL	LCL
		4.646		
1	2.49	4.4304	6.6878567	2.6034767
2	3.39	4.32636	7.3931523	1.898181
3	7.41	4.634724	7.8526853	1.438648
4	2.88	4.4592516	8.1813936	1.1099398
5	0.76	4.0893264	8.4267572	0.8645762
6	1.32	3.8123938	8.6143973	0.676936
7	7.05	4.1361544	8.7601176	0.5312157
8	1.37	3.859539	8.8744717	0.4168616
9	6.17	4.0905851	8.9648798	0.3264536
10	5.12	4.1935266	9.036746	0.2545874
11	1.34	3.9081739	9.0941066	0.1972268
12	0.5	3.5673565	9.1400321	0.1513013
13	4.35	3.6456209	9.1768905	0.1144428
14	1.67	3.4480588	9.2065275	0.0848058
15	1.63	3.2662529	9.230393	0.0609403
16	4.88	3.4276276	9.2496334	0.0416999
17	15.19	4.6038649	9.2651594	0.026174
18	0.67	4.2104784	9.2776973	0.0136361
19	4.14	4.2034305	9.2878281	0.0035052
20	2.16	3.9990875	9.2960179	-0.004685
21	1.14	3.7131787	9.3026411	-0.011308
22	2.66	3.6078609	9.307999	-0.016666
23	4.67	3.7140748	9.3123344	-0.021001
24	1.54	3.4966673	9.3158431	-0.02451
25	5.06	3.6530006	9.3186833	-0.02735
26	3.4	3.6277005	9.3209825	-0.029649
27	1.39	3.4039305	9.322844	-0.031511
28	1.11	3.1745374	9.3243514	-0.033018
29	6.92	3.5490837	9.3255719	-0.034239
30	36.99	6.8931753	9.3265604	-0.035227
x_bar	4.6456667			
sigma	6.8073			

Yes.
모든점이 UCL과 LCL 안에 있다.

c. Set up an EWMA control chart using $\lambda = 0.4$ for monitoring this process. Compare this EWMA chart to the one from Problem 8.b

	Lambda	0.4	L	3
	x	z	UCL	LCL
		4.646		
1	2.49	3.7836	12.814427	-3.523093
2	3.39	3.62616	14.171996	-4.880663
3	7.41	5.139696	14.61557	-5.324237
4	2.88	4.2358176	14.770501	-5.479168
5	0.76	2.8454906	14.825699	-5.534366
6	1.32	2.2352943	14.845497	-5.554164
7	7.05	4.1611766	14.852615	-5.561282
8	1.37	3.044706	14.855176	-5.563843
9	6.17	4.2948236	14.856098	-5.564765
10	5.12	4.6248941	14.85643	-5.565097
11	1.34	3.3109365	14.856549	-5.565216
12	0.5	2.1865619	14.856592	-5.565259
13	4.35	3.0519371	14.856608	-5.565275
14	1.67	2.4991623	14.856614	-5.56528
15	1.63	2.1514974	14.856616	-5.565282
16	4.88	3.2428984	14.856616	-5.565283
17	15.19	8.0217391	14.856617	-5.565283
18	0.67	5.0810434	14.856617	-5.565283
19	4.14	4.7046261	14.856617	-5.565283
20	2.16	3.6867756	14.856617	-5.565283
21	1.14	2.6680654	14.856617	-5.565283
22	2.66	2.6648392	14.856617	-5.565283
23	4.67	3.4669035	14.856617	-5.565283
24	1.54	2.6961421	14.856617	-5.565283
25	5.06	3.6416853	14.856617	-5.565283
26	3.4	3.5450112	14.856617	-5.565283
27	1.39	2.6830067	14.856617	-5.565283
28	1.11	2.053804	14.856617	-5.565283
29	6.92	4.0002824	14.856617	-5.565283
30	**36.99**	**17.196169**	**14.856617**	**-5.565283**
x_bar	4.6456667			
sigma	6.8073			

statistically out of control.
λ가 큰 c가 b보다 작은 변화에 민감하다.
그래서 b에서는 in control이라고 모니터되지만,
c에서는 즉각적으로 변화를 감지해서 out of control을
모니터할 수 있다.