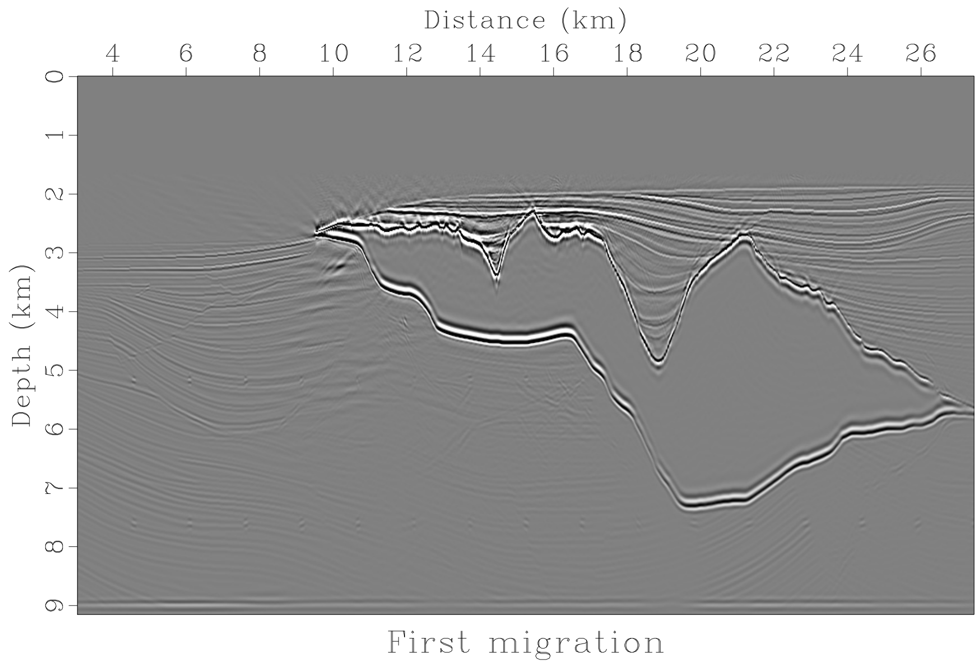
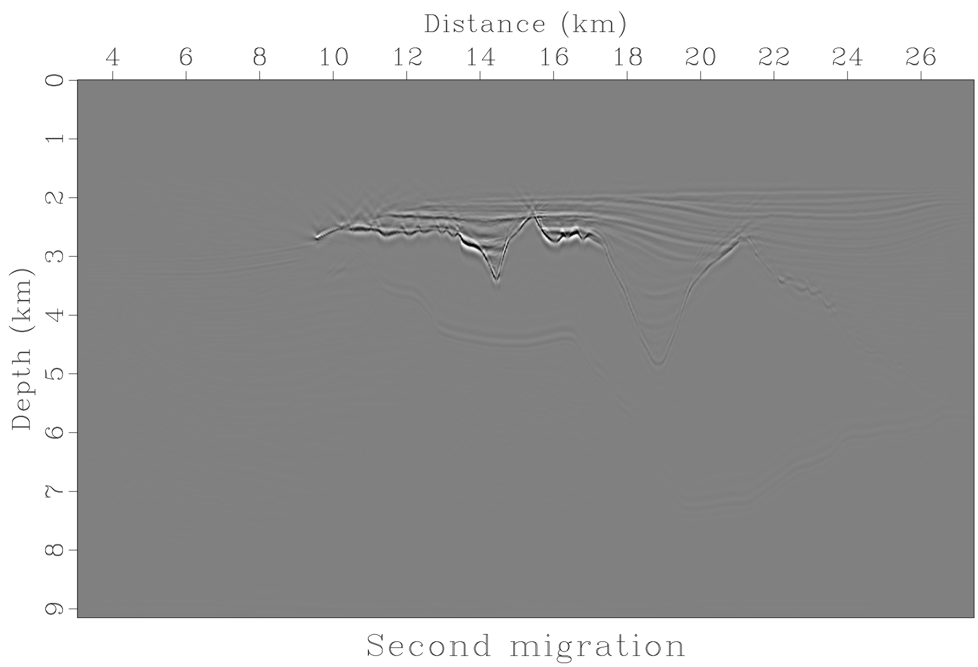


(a)



(b)

Figure 5.2: The first migrated image, $\mathbf{m}_0$ (a), and the second migrated image, $\mathbf{m}_1$ (b). chapter-mighes/sigsbee image0,image1

Because the primary differences between conventionally migrated images and least-squares migrated images amount to amplitude and frequency variations, we use two separate non-stationary operators to represent the forward Hessian—one to account for amplitude variations, and the other to account for frequency variations. Therefore, our approximation of $\mathbf{H}$ can be calculated from some application of a non-stationary amplitude balancing operator, $\mathbf{A}$, and a non-stationary frequency balancing operator, $\mathbf{S}$. We first calculate the forward Hessian because the forward frequency balancing operation, $\mathbf{S}$, is well defined and simple to calculate. We then invert our approximation to the Hessian and apply it to $\mathbf{m}_0$ to correct the amplitude and frequency content of the migrated image.

Amplitude operator

First, we choose to find an amplitude balancing operation that, when applied to $\mathbf{m}_0$, balances the amplitudes of $\mathbf{m}_0$ with respect to $\mathbf{m}_1$. This operation can be equated to a trace-by-trace multiplication of a diagonal matrix to each trace, where the matrix changes for every trace. We estimate it by first calculating the amplitude envelope of the traces in $\mathbf{m}_0$ and $\mathbf{m}_1$, and then smoothly dividing them. The corresponding diagonal weighting operator, $\mathbf{A}$, can be applied such that $\mathbf{m}_1 \approx \mathbf{A}\mathbf{m}_0$ to balance the amplitudes of each trace. Because this is a linear operation that only has diagonal elements, finding the inverse operator, $\mathbf{A}^{-1}$ is trivial.

Frequency operator

In addition to amplitude corrections, we also attempt to account for the decrease in resolution of a conventional migrated image compared to its corresponding least-squares migrated image. This loss in resolution can be equated to the fact that

$\mathbf{L}^\mathsf{T}\mathbf{L}$ acts as a blurring operator, where the conventional migrated image is a blurred version of the ideal least-squares migrated image (Hu et al., 2001). We choose to approximate this blurring using non-stationary triangle smoothing.

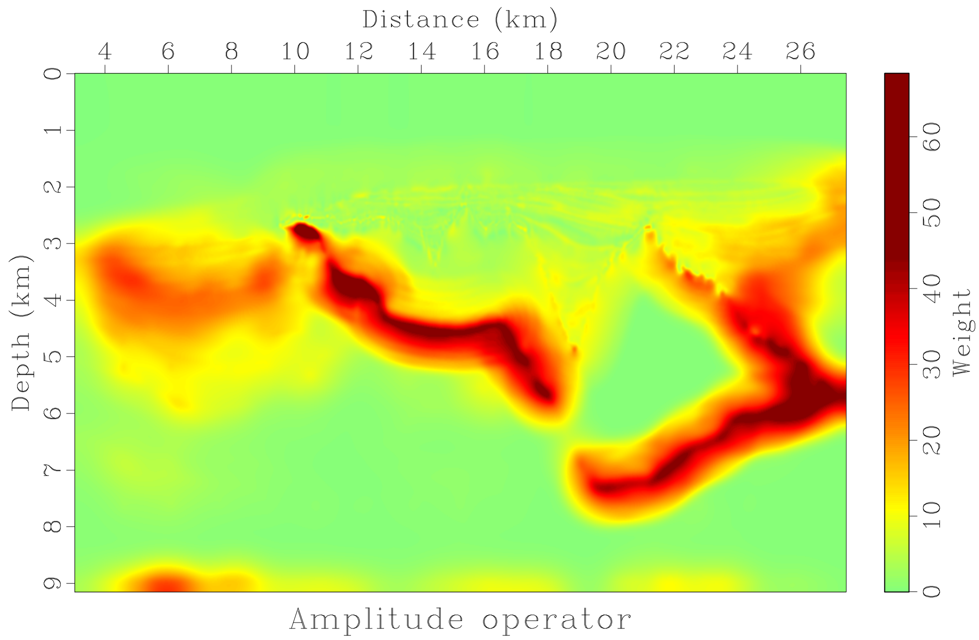
We begin by first calculating the local frequency of the two initial images (Fomel, 2007a). Local frequency is a temporally and spatially varying frequency attribute that smoothly varies across the image without windows. Our goal is to find a transformation that we can apply to $\mathbf{m}_0$ that matches the local frequency content with $\mathbf{m}_1$. To do this, we propose using non-stationary triangle smoothing. This approach involves finding and applying a non-stationary smoothing operator, which is the number of samples, in both dimensions, that $\mathbf{m}_0$ will be averaged over in a triangle weight, to balance the local frequency content with $\mathbf{m}_1$.

We find the smoothing radius iteratively using the method of Greer and Fomel (2017a) from Chapter 3, with a modification that allows the smoothing radius to be calculated in both spatial directions. Essentially, this is found by choosing an initial guess of a smoothing radius, $\mathbf{R}^{(0)}$, and updating it iteratively such that

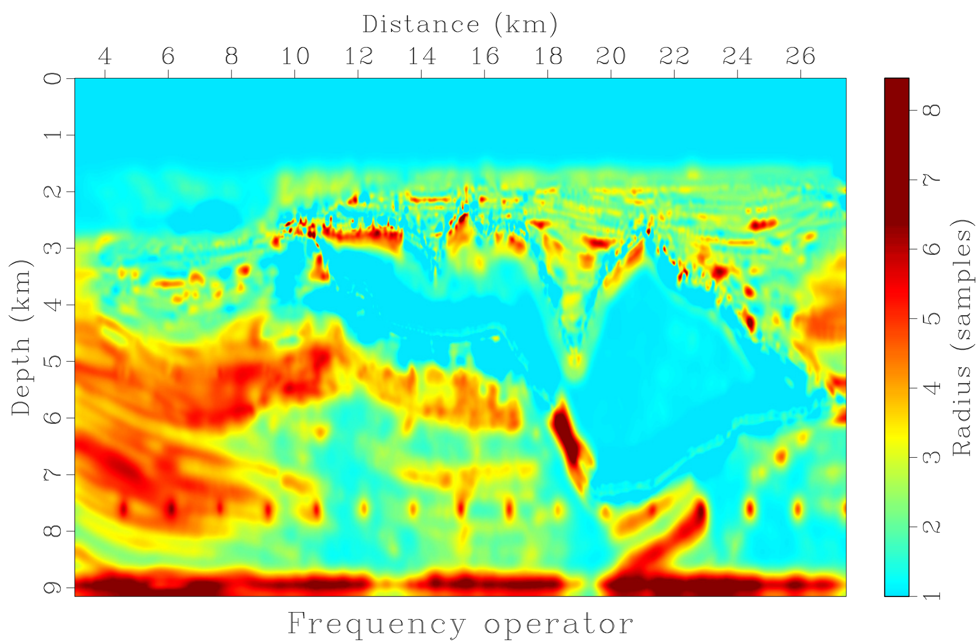
$$\mathbf{R}^{(i+1)} = \mathbf{R}^{(i)} + \alpha \left[\mathbf{F}[\mathbf{S}_{\mathbf{R}^{(i)}}\mathbf{m}_0] - \mathbf{F}[\mathbf{m}_1]\right] \ , \tag{5.8}$$

where $\mathbf{F}$ is the local frequency operator, $\mathbf{S}_{\mathbf{R}^{(i)}}$ is the smoothing operator of radius $\mathbf{R}$ at the ith iteration, and α is a scalar constant that represents the step length. After a small number of iterations, the smoothing operator is found that, once applied to $\mathbf{m}_0$, balances local frequency content with $\mathbf{m}_1$.

For this particular application using depth migration, this operator should technically be specified to balance *wavenumber* instead of *frequency.* However, it is kept as frequency to keep consistent terminology with the algorithm developed in Chapter 3.



(a)



(b)

Figure 5.3: The forward amplitude balancing weight, $\mathbf{A}$ (a) and the smoothing radius (b), which represents the number of samples in both dimensions that $\mathbf{m}_0$ must be smoothed over in a triangle weight to balance the local frequency content with $\mathbf{m}_1$. This represents the forward smoothing operation, $\mathbf{S}$.
chapter-mighes/sigsbee a0,rect10b