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FKP Blatt 9

$$x=\frac{a}{2} \xRightarrow{\text{in } \psi(x)} \psi(\tfrac{a}{2}) = At e^{ik\frac{a}{2}} + B(e^{-ik\frac{a}{2}} + re^{ik\frac{a}{2}})$$

$$x=-\frac{a}{2} \xRightarrow{\text{in } \psi(x+a)} \psi(\tfrac{a}{2}) = e^{iKa}\left[A(e^{-ika} + re^{ik\frac{a}{2}}) + Bt e^{ik\frac{a}{2}}\right]$$

$$(1) \Rightarrow \text{„}\psi(\tfrac{a}{2}) - \psi(\tfrac{a}{2})\text{"} = A\left[te^{ik\frac{a}{2}} - e^{iKa}(e^{-ik\frac{a}{2}} + re^{ik\frac{a}{2}})\right] + B\left[e^{ik\frac{a}{2}} + re^{ik\frac{a}{2}} - e^{iKa}te^{ik\frac{a}{2}}\right]$$

$$x=\frac{a}{2} \text{ in } \psi'(x) \Rightarrow \psi'(\tfrac{a}{2}) = ik\left[Ate^{ik\frac{a}{2}} + B(re^{ik\frac{a}{2}} - e^{ik\frac{a}{2}})\right]$$

$$x=\frac{a}{2} \text{ in } \psi'(x+a) \Rightarrow \psi'(\tfrac{a}{2}) = e^{iKa}\left[ik(A(e^{-ik\frac{a}{2}} - re^{ik\frac{a}{2}}) - Bte^{ik\frac{a}{2}})\right]$$

$$(2) \Rightarrow \frac{\text{„}\psi'(\tfrac{a}{2}) - \psi'(\tfrac{a}{2})\text{"}}{ik} = A\left[te^{ik\frac{a}{2}} - e^{iKa}(e^{ik\frac{a}{2}} - re^{ik\frac{a}{2}})\right] + B\left[re^{ik\frac{a}{2}} - e^{-ik\frac{a}{2}} + te^{iKa}e^{ik\frac{a}{2}}\right] = 0$$

Was sind α & β? Einmal hinschr. −0.5

Aus (1) & (2):

$$\underbrace{\begin{pmatrix} te^{\alpha} - e^{\beta}(e^{-\alpha} - re^{\alpha}) & re^{\alpha} - e^{-\alpha} + e^{\beta}e^{\alpha} \\ te^{\alpha} - e^{\beta}(e^{-\alpha} + re^{\alpha}) & e^{-\alpha} + re^{\alpha} - e^{\beta}te^{\alpha} \end{pmatrix}}_{M} \begin{pmatrix} A \\ B \end{pmatrix} \overset{!}{=} 0$$

$\det M \overset{!}{=} 0$ Sehr gut!

$$\Rightarrow \left[te^{\alpha} - e^{\beta}(e^{-\alpha} - re^{\alpha})\right]\left[e^{-\alpha} + re^{\alpha} - \beta te^{\alpha}\right] - \left[te^{\alpha} - e^{\beta}(e^{-\alpha} + re^{\alpha})\right]\left[re^{\alpha} - e^{-\alpha} + e^{\beta}te^{\alpha}\right] = 0$$

$$\Rightarrow t + rte^{2\alpha} - t^2e^{\beta}e^{2\alpha} - e^{\beta}(e^{-2\alpha} - r) - e^{\beta}(r - r^2e^{2\alpha}) + e^{2\beta}(t - rte^{2\alpha}) - rte^{2\alpha} + t - t^2e^{\beta}e^{2\alpha} + e^{\beta}(r + r^2e^{2\alpha}) - e^{\beta}(e^{-2\alpha} + r) + e^{2\beta}(t + rte^{2\alpha}) = 0$$

$$\Rightarrow 2t - 2t^2e^{\beta}e^{2\alpha} \cancel{+ e^{\beta}\ldots} - 2e^{\beta}e^{-2\alpha} + 2e^{\beta}r^2e^{2\alpha} + 2e^{2\beta}t = 0$$

$$\Rightarrow t - t^2e^{\beta}e^{2\alpha} - e^{\beta}e^{-2\alpha} + e^{\beta}r^2e^{2\alpha} + e^{2\beta}t$$

$$\Rightarrow e^{2\alpha}e^{\beta}(r^2 - t^2) + t - e^{\beta}e^{-2\alpha} + e^{2\beta}t = 0$$

$$\Rightarrow te^{-\beta} + te^{\beta} = e^{-2\alpha} - e^{2\alpha}(r^2 - t^2)$$

$$\Rightarrow \cos(Ka) = \frac{e^{-2\alpha} - e^{-2\alpha}(t^2 - r^2)e^{-Ka}}{2t}$$

Ersetz doch nächstes Mal das α wieder zurück

~~Sonst super~~ ⇒ ~~4,5P~~ r=0 ? −1P ⇒ 3.5P b)

$x \le -\frac{a}{2}$

$$W(\psi,\psi^*) = ik(e^{ikx} - re^{-ikx})(e^{-ikx} + r^* e^{ikx})$$
$$- (e^{ikx} + re^{-ikx})(-ik)(e^{-ikx} - r^* e^{ikx})$$
$$= ik(e^{ikx} - re^{-ikx})(e^{-ikx} + r^* e^{ikx}) + ik(e^{ikx} + re^{-ikx}) \cdot (e^{-ikx} - r^* e^{ikx})$$
$$= 2ik(1 - |r^2|)$$

$x \ge \frac{a}{2}$

$$W(\psi,\psi^*) = ik(te^{ikx} \cdot t^* e^{-ikx}) - (ik)te^{ikx} t^* e^{-ikx}$$
$$= ik|t|^2$$

Letzter Schritt fehlt $\Rightarrow -1$

$\Rightarrow 2P.$

c)

$$W(\psi_L, \psi_r^*) = ikte^{ikx}(e^{ikx} + r^* e^{-ikx}) + te^{ikx}(ik)(e^{ikx} - r^* e^{-ikx})t^*$$
$$= -2ikr^* t$$

2. Teil d. Rechnung fehlt... $-1.5P$

Unabh. von x: $rt^* = -r^* t$

$$\Rightarrow rt^* = -r^* t$$
$$(rt^*) = -(rt^*)^* \Rightarrow r = \pm i|r|e^{i\alpha}$$

$\Rightarrow 0.5P$

$\Rightarrow$ 6P. ges