10.7.18

Aufgabe 50

$$H_0 = \frac{1}{2m}(p_x^2+p_y^2) + \frac{1}{2}m\omega^2(x^2+y^2)$$

$$H_1 = g x^2 y^2 \qquad H = H_0 + H_1$$

zu a)
Nicht entartet
p & n müssen 0 sein.
—▷ Nur eine Möglichkeit

$$\Rightarrow H_0 = \hbar\omega(N+P+1), \quad H_0|n,p\rangle = \hbar(n+p+1)|n,p\rangle$$

$$N = a^\dagger a, \quad P = b^\dagger b; \quad H_1 = g\left(\frac{\hbar}{2m\omega}\right)^2 (a^\dagger + a)^2 (b^\dagger + b)^2$$

$$= g\left(\frac{\hbar}{2m\omega}\right)^2 (a^{\dagger 2} + a^\dagger a + a a^\dagger + a^2)(b^{\dagger 2} + b^\dagger b + b b^\dagger + b^2)$$

$$a^\dagger|n,p\rangle = \sqrt{n+1}\,|n+1,p\rangle; \quad a|n,p\rangle = \sqrt{n}\,|n-1,p\rangle$$

$$b^\dagger|n,p\rangle = \sqrt{n+1}\,|n,p+1\rangle; \quad b|n,p\rangle = \sqrt{n}\,|n,p-1\rangle$$

a)

$$E_0^{(1)} = \langle 0,0|H_1|0,0\rangle = g\left(\frac{\hbar}{2m\omega}\right)^2 \langle 0,0|x^2y^2|0,0\rangle$$

$$= g\left(\frac{\hbar}{2m\omega}\right)^2 \langle 0,0|a a^\dagger b b^\dagger|0,0\rangle = g\left(\frac{\hbar}{2m\omega}\right)^2$$

$$E_0^{(2)} = \sum_{n,p} \cancel{E_0^{(0)} - E_{(n,p)}^{(0)}} \frac{\langle n,p|H_1|0,0\rangle}{E_0^{(0)} - E_{(n,p)}^{(0)}}$$

NR: $\langle n,p|H_1|0,0\rangle = g\left(\frac{\hbar}{2m\omega}\right)^2 \langle n,p|a^{\dagger 2}b^{\dagger 2} + a^{\dagger 2}bb^\dagger + aa^\dagger bb^\dagger + aa^\dagger b^{\dagger 2}|0,0\rangle$

(nur Terme mit Erzeugungsoperator relevant)

$$= g\left(\frac{\hbar}{2m\omega}\right)^2 \left(2\delta_{n2}\delta_{p2} + \sqrt{2}(\delta_{n2}\delta_{p0} + \delta_{n0}\delta_{p2}) + \delta_{n0}\delta_{p0}\right)$$

$$= g^2\left(\frac{\hbar}{2m\omega}\right)^4 \left[\frac{4}{\hbar\omega - 5\hbar\omega} + \frac{4}{\hbar\omega - 3\hbar\omega}\right] = g^2\left(\frac{\hbar}{2m\omega}\right)^4 \left(-\frac{1}{\hbar\omega} - \frac{2}{\hbar\omega}\right)$$

$$= -\frac{3g^2\hbar^3}{16m^4\omega^4}$$

b)

1. $2\hbar\omega$: $|1,0\rangle$ & $|0,1\rangle$

$E_1^0 \nearrow$

$$V = \begin{pmatrix} \langle 1,0|H_1|1,0\rangle & \langle 1,0|H_1|0,1\rangle \\ \langle 0,1|H_1|1,0\rangle & \langle 0,1|H_1|0,1\rangle \end{pmatrix}$$

$$\langle 1,0|H_1|1,0\rangle = g\left(\frac{\hbar}{2m\omega}\right)^2 \langle 1,0|a^\dagger a b b^\dagger + a a^\dagger b b^\dagger|1,0\rangle$$

$$= \text{''} \quad \langle 0,0|ab|1,1\rangle + \sqrt{2}\langle 2,0|a^\dagger b|1,1\rangle$$

$$= 3g\left(\frac{\hbar}{2m\omega}\right)^2$$

$$\langle 1,0|H_1|0,1\rangle = 0 \overset{V=V^\dagger}{\Longrightarrow} V = 3g\left(\frac{\hbar}{2m\omega}\right)^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow E_1^1 = 3g\left(\frac{\hbar}{2m\omega}\right)^2$$

2. $E_2^{(0)} = 3\hbar\omega : |2,0\rangle, |1,1\rangle, |0,2\rangle$

$$\langle 2,0|H_1|2,0\rangle = 8\left(\frac{\hbar}{2m\omega}\right)^2 \langle 2,0|a^\dagger a b b^\dagger + a a^\dagger b b^\dagger|2,0\rangle$$
$$= 8\left(\frac{\hbar}{2m\omega}\right)^2 \left(\sqrt{2}\langle 1,0|ab|2,1\rangle + \sqrt{3}\langle 3,0|a^\dagger b|2,1\rangle\right)$$
$$= 5\underbrace{8\left(\frac{\hbar}{2m\omega}\right)}_{\alpha}$$

$$\langle 1,1|H_1|1,1\rangle = \alpha\langle 1,1|a^\dagger a b^\dagger b + a^\dagger a b b^\dagger + a a^\dagger b^\dagger b + a a^\dagger b b^\dagger|1,1\rangle$$
$$= \alpha\Big(\underbrace{\langle 0,1|ab^\dagger|1,0\rangle}_{1} + \underbrace{\sqrt{2}\left(\langle 0,1|ab|1,2\rangle + \langle 2,1|a^\dagger b^\dagger|1,0\rangle\right)}_{4}$$
$$+ 2\underbrace{\langle 2,1|a^\dagger b|1,2\rangle}_{4}$$
$$= 9\alpha$$

$$\langle 0,2|H_1|2,0\rangle = 8\left(\frac{\hbar}{2m\omega}\right)^2 \langle 0,2|a^2 b^{\dagger 2}|2,0\rangle = 2\alpha$$

$$V = \begin{pmatrix} \langle 2,0|H_1|2,0\rangle & \langle 2,0|H_1|1,1\rangle & \langle 2,0|H_1|0,2\rangle \\ * & \langle 1,1|H_1|1,1\rangle & \langle 1,1|H_1|0,2\rangle \\ * & * & \langle 0,2|H_1|0,2\rangle \end{pmatrix}$$

$$= \alpha\begin{pmatrix} 5 & 0 & 2 \\ 0 & 9 & 0 \\ 2 & 0 & 5 \end{pmatrix}$$

$E_{21}^{(1)} = 3\alpha$

$E_{22}^{(1)} = 7\alpha$

$E_{23}^{(1)} = 9\alpha$