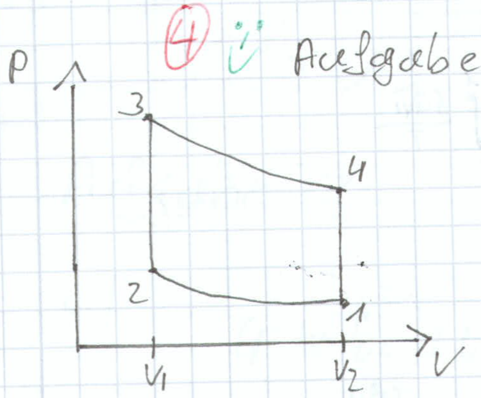
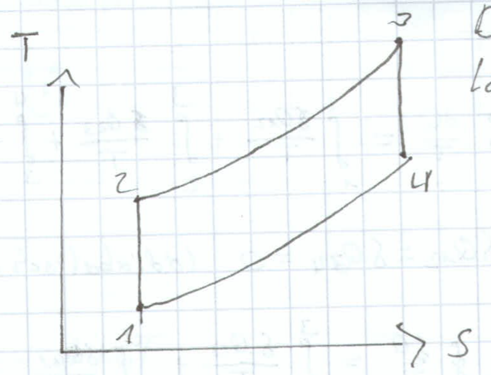
④ Aufgabe 1

David Roß
Lars Roll





b) $\eta = \frac{|W|}{Q} = \frac{|W_{23}|}{Q_{23}}$?

$dU = dQ - p\,dV$

Q_{23} : $V = \text{const}$, $dV = 0$

$\Rightarrow dU_{23} = Q_{23}$ ✓

Q_{23}: Wärmemenge, die zwischen P_2 & P_3 aufg. wurde

$\Rightarrow U_3 - U_2 = Q_{23}$

ideales Gas: $p_2 V = N k_B T_2$

$p_3 V = N k_B T_3$

$\Rightarrow \frac{p_2}{p_3} = \frac{T_2}{T_3}$

$dV = 0 \Rightarrow dU = c_V dT \Rightarrow U = c_V T + c$, $c = 0$ setzen

super :)

$\Rightarrow Q_{23} = c_V (T_3 - T_2)$ analog $Q_{41} = c_V (T_1 - T_4)$

$Q_{12} = Q_{34} = 0$ da adiabatisch

A1	A2	A3	A4	Ges
4	4	4	3.5	15.5

$W_{Ges} = \sum_i W_i = -\sum_i Q_i$

$|W_{Ges}| = \sum Q_i = Q_{23} + Q_{41} = c_V (T_1 - T_2 + T_3 - T_4)$

$\Rightarrow \eta = \frac{T_1 - T_2 + T_3 - T_4}{T_3 - T_2} = 1 + \frac{T_1 - T_4}{T_3 - T_2}$ ✓

$1 \to 2$ ist adiabatisch: $p_2 V_2^{\kappa} = p_1 V_1^{\kappa} = p V^{\kappa} = \text{const}$ $\kappa = \frac{c_p}{c_V}$

$\left.\begin{matrix} p_2 V_2 = N k_B T_2 \\ p_1 V_1 = N k_B T_1 \end{matrix}\right\} \frac{p_1}{p_2} = \frac{T_1 V_1}{T_2 V_2} \overset{!}{=} \left(\frac{V_1}{V_2}\right)^{\kappa} \Leftrightarrow \frac{T_1}{T_2} = \left(\frac{V_1}{V_2}\right)^{\kappa - 1}$ ✓

analog $3 \to 4$: $\frac{T_3}{T_4} = \left(\frac{V_3}{V_4}\right)^{\kappa-1} = \left(\frac{V_2}{V_1}\right)^{\kappa-1} \Rightarrow \frac{T_1}{T_2} = \frac{T_4}{T_3} \Leftrightarrow T_4 = \frac{T_1 T_3}{T_2}$

$\eta = 1 + \frac{T_1 \left(1 - \frac{T_3}{T_2}\right)}{T_3 - T_2} = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{V_1}{V_2}\right)^{\kappa - 1}$ (✓)

schon richtig, aber Rechnung etwas umständlich und unübersichtlich

c) $\oint \frac{\partial Q}{T} = \int_1^2 \frac{\delta Q_{12}}{T} + \int_2^3 \frac{\delta Q_{23}}{T} + \int_3^4 \frac{\delta Q_{34}}{T} + \int_4^1 \frac{\delta Q_{41}}{T}$

$\delta Q_{12} = \delta Q_{34} = 0$ (adiabatisch)

$\Rightarrow \oint \frac{\delta Q}{T} = \int_2^3 \frac{\delta Q_{23}}{T} + \int_4^1 \frac{\delta Q_{41}}{T}$

Prozess 2->3 & 4-1 sind isochor $\rightarrow V = V_1 = const$
bzw $V = V_2 = const$

$\rightarrow \delta Q = c_V dT$

$\Rightarrow \int_{T_2}^{T_3} c_V \frac{dT}{T} + \int_{T_4}^{T_1} c_V \frac{dT}{T} = c_V \left(\ln\left(\frac{T_3}{T_2}\right) + \ln\left(\frac{T_1}{T_4}\right) \right)$

$= 0 \quad \left(\frac{T_2}{T_3} = \frac{T_1}{T_4} \right)$

Das Integral $\int_A^B \frac{\delta Q}{T}$ von Punkt A zu Punkt B beschreibt die Entropieänderung. Kreisprozess mit $B = A \Rightarrow \oint \frac{\delta Q}{T} = 0$

d)

- isochore Druckänderung (2->3) nicht möglich warum?
- Wärme durch Reibungsverluste $\rightarrow$ Ausdehnung des Motorblockes ok

TUS (4) :)

Aufgabe 2

a)

$$\left(p+\alpha\frac{N^2}{V^2}\right)(V-bN)=Nk_bT$$

$$\Leftrightarrow p=\frac{Nk_bT}{(V-bN)}-\alpha\frac{N^2}{V^2}$$

$$dS=\frac{\partial S}{\partial T}\Big|_V dT+\frac{\partial S}{\partial V}dV$$

grenzenlose Integrale!

$$\Rightarrow S(T)=\int\frac{\partial S}{\partial T'}dT'=\int_{T_0}^{T}\frac{C_V}{T'}dT'==C_V\ln\left(\left|\frac{T}{T_0}\right|\right)+C_1(V)$$

$$=\frac{3}{2}Nk_b\ln\left(\left|\frac{T}{T_0}\right|\right)+C_1(V)$$

$$S(V)=\int\frac{\partial S}{\partial V}dV'=\int\frac{\partial p}{\partial T}dV'=\int_{V_0}^{V_0}\frac{Nk_b}{(V'-bN)}dV'=Nk_b\ln\left(\left|\frac{V-Nb}{V_0-Nb}\right|\right)+C_2(T)$$

$$\Rightarrow S(T,V)=Nk_b\left(\frac{3}{2}\ln\left(\left|\frac{T}{T_0}\right|\right)+\ln\left(\left|\frac{V-Nb}{V_0-Nb}\right|\right)+C\right)\ (\checkmark)$$

b)

$$f(T,V,N)=\text{const}$$

adiabatisch: $dQ=0\Rightarrow dU=-p\,dV$

$$\Rightarrow dU=C_VdT+\left(T\frac{\partial p}{\partial T}-p\right)dV=-p\,dV$$

$$\Rightarrow C_V\,dT'+T\frac{\partial p}{\partial T'}dV=0\ \hat{=}\ dS=0$$

$$\Leftrightarrow\frac{C_V}{T'}dT'=-\frac{Nk_b}{V-bN}dV$$

$$\Leftrightarrow\frac{C_V}{Nk_b}\cdot\frac{1}{T'}dT'=\frac{1}{bN-V}dV$$

$$\Rightarrow\frac{C_V}{Nk_b}\cdot\ln\left(\frac{T}{T_0}\right)=\ln\left(\left|\frac{V_0-bN}{V-bN}\right|\right)+C$$

$$\Rightarrow\left(\left|\frac{T}{T_0}\right|\right)^{C_V/(Nk_b)}=\left|\frac{V_0-bN}{V-bN}\right|\cdot C$$

$$\Rightarrow\left|\frac{T}{T_0}\right|^{\frac{C_V}{Nk_b}}\cdot\left|\frac{V-bN}{V_0-bN}\right|=C=\text{const}$$

$$=\left|\frac{T}{\cancel{T_0}}\right|^{3/2}\cdot\left|\frac{V-bN}{\cancel{V_0}-bN}\right|=C\ (\checkmark)$$

okay!

Aufgabe 4 (3.5)

a) $\left(p+\frac{N^2}{V^2}a\right)(V-Nb) = Nk_bT$

$\Leftrightarrow p = \frac{Nk_bT}{V-Nb} - \frac{N^2}{V^2}a \quad \Leftrightarrow T = \left(p+\frac{N^2}{V^2}a\right)\frac{V-Nb}{Nk_b}$

$\frac{\partial T}{\partial p}\Big|_H \overset{!}{=} 0 \quad \Leftrightarrow \quad T = V\frac{\partial T}{\partial V}\Big|_p$ ✓

$\frac{\partial T}{\partial V}\Big|_p = \frac{p}{Nk_b} - \frac{Na}{k_b}\frac{1}{V^2} + \frac{2N^2ab}{k_b}\frac{1}{V^3}$

T in p: $\quad p = \frac{Nk_bV}{V-Nb}\left(\frac{p}{Nk_b} - \frac{Na}{k_b}\frac{1}{V^2} + \frac{2N^2ab}{k_b}\frac{1}{V^3}\right) - \frac{N^2}{V^2}a$

$\Leftrightarrow p\left(1-\frac{V}{V-Nb}\right) = \frac{N^2a}{V}\left(\frac{2Nb}{V(V-Nb)} - \frac{1}{V-Nb} - \frac{1}{V}\right)$

$\Leftrightarrow p = -\frac{V-Nb}{Nb}\,\frac{N^2a}{V}\left(\frac{2Nb-V-V+Nb}{V(V-Nb)}\right)$

$= -\frac{Na}{b}\,\frac{3Nb-2V}{V^2} = \frac{Na}{b}\left(\frac{2}{V}-\frac{3Nb}{V^2}\right)$ ✓

b) siehe Plot ✓

c) $p = \frac{Nk_bT}{V-Nb} - \frac{N^2a}{V^2} \approx \frac{Nk_bT}{V}\left(\frac{1}{1+\frac{Nb}{V}} - \frac{Na}{k_bT\,V}\right)$ (Taylor: $\frac{1}{1-x} = 1+x+c$

$= \frac{Nk_bT}{V}\left(1+\frac{Nb}{V} - \frac{N^2a}{Nk_bTV}\right)$ $\qquad V$ groß: $V \gg Nb$, $V \gg \frac{N^2a}{Nk_bT}$ ✓

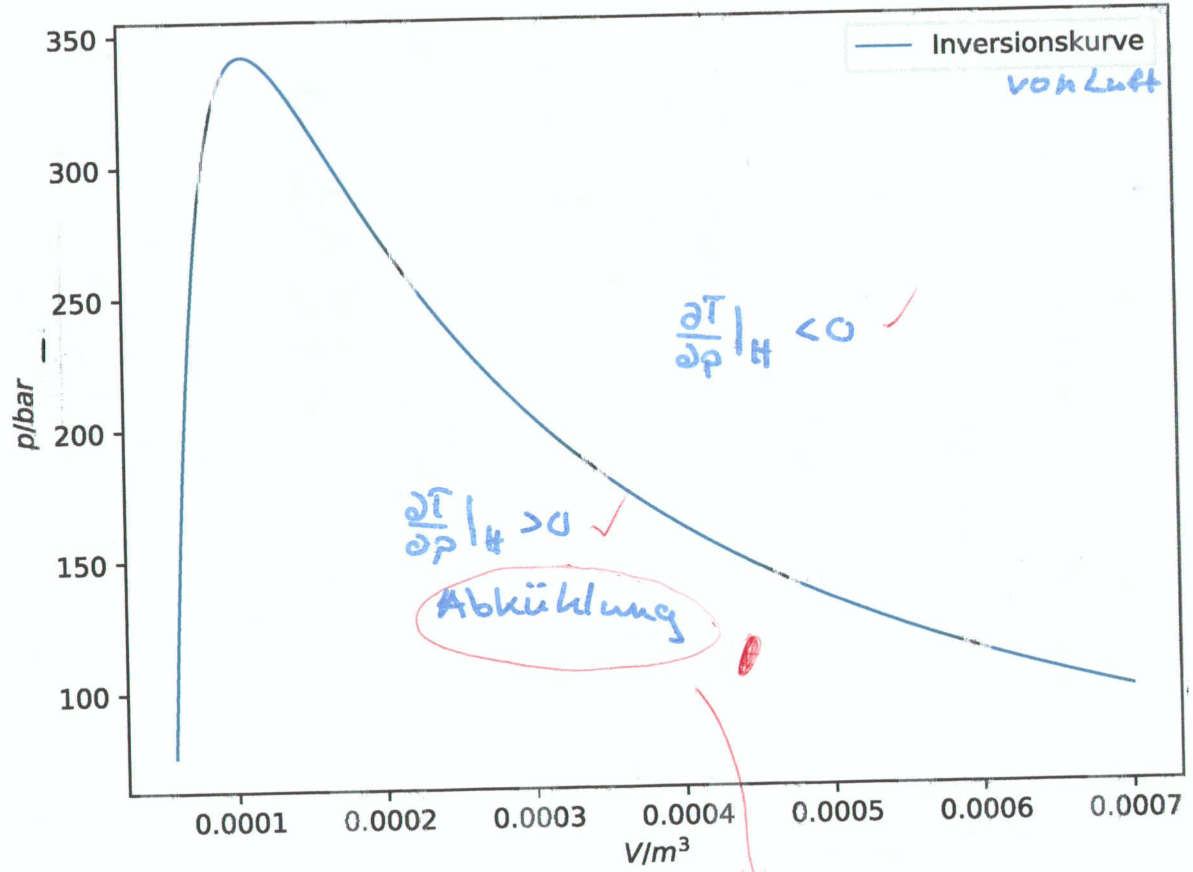
~~das vielleicht ihr doch weg, oder?~~ sorry

$\Rightarrow V = \frac{Nk_bT}{p} + \cancel{Nb} - \cancel{\frac{Na}{k_bT}}$ ✓ $\qquad \frac{\partial V}{\partial T}\Big|_p = \frac{Nk_b}{p} + \frac{Na}{k_bT^2}$

$\frac{\partial T}{\partial p}\Big|_H \overset{!}{=} 0 \quad \Leftrightarrow \quad T\frac{\partial V}{\partial T}\Big|_p - V = 0 = \frac{2Na}{k_bT} - Nb$

$\Leftrightarrow T = \frac{2a}{bk_b} = \frac{27}{4}T_{krit}$ ✓ schön!

VZ von $\frac{dT}{dp}$ bei $T > T_{inv}$?



Abkühlung bei Druckerhöhung :-)

Aufgabe 3 4

a) $W = \int p\,dV = p\Delta V$ 4 (p auf beiden Seiten = const wird angenommen) (1)

$pV = nRT \Rightarrow 1\,\text{mol} \Rightarrow p\Delta V = RT$ Das Gas muss nicht ideal sein

b) Zufuhr von Arbeit $p_1 \Delta V_1$ auf der linken Seite, Abgabe der Arbeit $p_2 \Delta V_2$ auf der rechten Seite

$\Rightarrow U_2 = U_1 + p_1 \Delta V_1 - p_2 \Delta V_2$

$\Rightarrow U_1 + p_1 \Delta V_1 = U_2 + \Delta V_2 \Rightarrow H_1 = H_2 = \text{const}$

c) $H = U + pV \Rightarrow dH = d(pV + U) = dU + V\,dp + p\,dV$

$dU = T\,dS - p\,dV \Rightarrow dH = T\,dS \cancel{- p\,dV} + V\,dp$

$dS(p,T) = \frac{\partial S}{\partial p}\Big|_T dp + \frac{\partial S}{\partial T}\Big|_p dT$

$\Rightarrow dH = T\left(\frac{\partial S}{\partial T}\Big|_p\right) dT + T\left(\frac{\partial S}{\partial p}\Big|_T\right) dp + V\,dp$

$dG = -S\,dT + V\,dp = \frac{\partial G}{\partial T}\Big|_p dT + \frac{\partial G}{\partial p}\Big|_T dp$

$\frac{\partial^2 G}{\partial T \partial p} = \frac{\partial^2 G}{\partial p \partial T} \Rightarrow -\frac{\partial S}{\partial p}\Big|_T = \frac{\partial V}{\partial T}\Big|_p$

$\Rightarrow dH = c_p\,dT + \left(V - T\frac{\partial V}{\partial T}\Big|_p\right) dp \overset{!}{=} 0$

$\Rightarrow \frac{dT}{dp}\Big|_H = \frac{1}{c_p}\left(T\frac{\partial V}{\partial T}\Big|_p - V\right)$