A52

$$\vec{E}(t) = \frac{E}{1+\left(\frac{t}{t_0}\right)^2}\vec{e}_z \qquad \hat{H}_{int} = \vec{d}\cdot\vec{E} = e\vec{z}\cdot|\vec{E}| = \frac{eE}{1+\left(\frac{t}{t_0}\right)^2}\hat{z}$$

Übergang: $(nlm) \to (n'l'm')$

$(100) \to$

- $n' = 2$ lt. Angabe
- $l' = 1$, da $\Delta l = 1$ & $l < 0$
- $m' = 0$, da Matrixelemente zw. Zuständen mit vers. m verschwinden, denn $[\hat{z}, \hat{L}_z] = 0$ (s. §96)

$$\mu_{210}^{100} = \langle 100|\hat{H}_{int}|210\rangle$$

$$\psi_{100}(r) = \frac{1}{\sqrt{\pi a^3}}\exp(-r/a) \qquad \psi_{210}(\vec{r}) = \sqrt{\frac{3}{4\pi}}\sqrt{\frac{1}{24a^3}}\frac{r}{a}e^{-\frac{r}{2a}}\cos(\vartheta)$$

$$\to \mu_{210}^{100} = \int d^3r\, \psi_{100}^* \frac{eEt_0^2}{t^2+t_0^2} z\cdot\psi_{210} = \int_0^\infty r^2dr\int_0^{2\pi}d\phi\int_{-1}^{1}d(\cos\vartheta)\frac{1}{\sqrt{\pi a^3}} \cdot e^{-\frac{r}{a}} r\cos(\vartheta)\frac{eEt_0^2}{t^2+t_0^2}\cdot\sqrt{\frac{3}{4\pi}} \cdot\sqrt{\frac{1}{24a^3}}\cdot\frac{r}{a}\cdot e^{-\frac{r}{2a}}\cos(\vartheta)$$

$$\mu_{210}^{100} = \frac{1}{\sqrt{\pi a^3}}\frac{eEt_0^2}{t^2+t_0^2}\sqrt{\frac{3}{4\pi}}\sqrt{\frac{1}{24a^3}}\cdot\frac{1}{a}\cdot\frac{4\pi}{3} \cdot\int_0^\infty dr\, r^4 e^{-\frac{3r}{2a}}$$

$$= \frac{128\sqrt{2}}{243}\cdot\frac{eEt_0^2}{t^2+t_0^2}\cdot a$$

Zeitintegral

$$I = \int_{-\infty}^{\infty} dt\, \mu_{210}^{100} e^{i\omega_{21}t}$$

$$\left(\text{mit } \omega_{21} = \frac{E_2-E_1}{\hbar} = \frac{-\frac{1}{4}Ryd - (-Ryd)}{\hbar} = \frac{3}{4\hbar}Ryd\right.$$

$$\Rightarrow I = eEt_0^2 a\frac{128\sqrt{2}}{243}\int_{-\infty}^{\infty}dt\frac{e^{-i\omega_{21}t}}{t^2+t_0^2}$$

$$f(z) = \frac{e^{-ibz}}{z^2+t_0^2} \quad , z_0 = -it_0 \text{ : Integrationsgebiet untere Halbebene}$$

$$\mathrm{Res}(f(z), z) = \lim_{z \to -it_0} \frac{d}{dz}\left[(z+ib)^2 \frac{e^{-ibz}}{(z+it_0)(z-it_0)} \right]$$

$$= \lim_{z \to it_0} \frac{d}{dz}\left[\frac{z+it_0}{z-it_0} e^{-ibz} \right]$$

$$= \lim_{z \to -it_0} e^{-ibz} \left(\frac{1}{z-it_0} + \frac{z+it_0}{(z-ib)^2} - ib \frac{z+it_0}{z-it_0} \right.$$

$$= -e^{-bt_0} / 2it_0$$

$$\Rightarrow \int_{-\infty}^{\infty} e^{-i\omega_{21} t} / (t^2+t_0^2)\, dt = -2\pi i \left(-\frac{e^{\frac{3}{4\hbar} R_{yd} t_0}}{2it_0} \right) = \frac{\pi}{t_0} \exp\left(-\frac{3 R_{yd}}{4\hbar} t_0 \right)$$

$$\Rightarrow I = eE t_0 a \frac{128\sqrt{2}\pi}{243} \exp\left(-\frac{3 R_{yd}}{4\hbar} t_0 \right)$$

Übergangsw'keit

$$P_{1\to 2} = \frac{1}{\hbar^2} |I|^2 = \left(\frac{\pi a_0 e E t_0}{\hbar} \right)^2 \frac{32768}{59049} \cdot \exp\left(-\frac{3}{2} \frac{R_{yd}}{\hbar} t_0 \right)$$