TUS

3) $\rho(p,q) = \begin{cases} \frac{1}{c} & H(p,q) \leq E \\ 0 & \text{sonst} \end{cases}$ | Nicht identische, nicht ww. Teilchen $H(p,q) = \sum_{i=1}^{n} \frac{p_i^2}{2m} = \sum_{i=1}^{3n} \frac{p_i}{2m}$

Wir gehen über in 3N dim. Kugelkoordinaten

$\sum_{i=1}^{3N} p_i^2 = p^2$ (unabh. von den 3N-1 Winkeln)

vgl. 3dim Kugelkoord.: $x^2+y^2+z^2=r^2$

Transformation des Integrals

$$\int d^{3N}p\, f(\Sigma p_i^2) = \int d\Omega_{3N} \int_0^\infty dp\, p^{3N-1} f(p) = \underbrace{\Omega_{3N}}_{\frac{2\pi^{\frac{3N}{2}}}{\Gamma(\frac{3N}{2})}} \int_0^\infty dp\, p^{3N-1} f(p)$$

vgl. 3D Kugelkoordinaten: $\int dx\,dy\,dz\, f(x^2+y^2+z^2) = \int d\Omega_3 \int_0^\infty dr\, r^2 f(r)$

$d\Omega_3 = \int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta \sin\vartheta = 4\pi$

a)

Normierung

$$\frac{1}{N!(2\pi\hbar)^{3N}} \int d^{3N}q \int d^{3N}p\, \rho(p,q) \overset{!}{=} 1$$

$H(p) = \frac{p^2}{2m}$

$\frac{p^2}{2m} \leq E \Leftrightarrow p \leq \sqrt{2mE}$

Teilchen identisch

$$= \frac{1}{N!(2\pi\hbar)^{3N}} \underbrace{\int d^3q_1}_{V} \underbrace{\int d^3q_2}_{V} \dots \int d^3q_N \int_{H(p,q)\leq E} dp\, p^{3N-1} \Omega_{3N} \frac{1}{c}$$

Jedes Teilchen kann sich überall im Volumen befinden.

$$= \frac{V^N \Omega_{3N}}{N!(2\pi\hbar)^{3N} \cdot c} \int_0^{\sqrt{2mE}} dp\, p^{3N-1} \overset{!}{=} 1 \quad \Longleftrightarrow c = \frac{1}{h^{3N} N!} V^N \Omega_{3N} \frac{\sqrt{2mE}^{3N}}{3N}$$

$$\frac{\langle H \rangle}{N} \underset{4}{=} \frac{1}{(2\pi\hbar)^{3N} N!} \cdot \frac{\Omega_{3N}}{cN} \underbrace{\int d^{3N}q}_{V^N} \int_0^{\sqrt{2mE}} dp\, p^{3N-1} \underbrace{H(p)}_{\frac{p^2}{2m}}$$

analog

$$= \frac{V^N}{(2\pi\hbar)^{3N} N!} \frac{\Omega_{3N}}{2mcN} \int_0^{\sqrt{2mE}} p^{3N+1}\, dp = \frac{3E}{3N+2}$$

$$\frac{1}{N^2}\left(\langle H^2 \rangle - \langle H \rangle^2\right) = \left[\frac{\Omega_{3N} V^N}{(2\pi\hbar)^{3N} N! N^2 c} \int_0^{\sqrt{2mE}} dp\, p^{3N-1} H^2(p)\right] - \left(\frac{3E}{3N+2}\right)^2$$

$$= 3E^2 \left[\frac{1}{N(3N+4)} - \frac{3}{(3N+2)^2}\right]$$

b)

mikrokan. Ensemble: Phasenraumdichte ≙ Deltafunktion $\delta(H(p,q)-E)$

vgl. a) $\rho(p,q)$:

$\to$ kan. Ensemble: $(T, V, N) = const$

$$\frac{\langle H\rangle_{mikro}}{N} = \frac{E}{N} \to \frac{1}{N^2}\left(\langle H^2\rangle - \langle H\rangle^2\right) = 0$$

Varianz d. Energie verschwindet

c) Limes N groß (in der a)) $\frac{1}{N^2}\left(\langle H^2\rangle - \langle H\rangle^2\right)_{kan} \to 0$

$$\frac{\langle H\rangle_{kan}}{N} \to \frac{E}{N}$$