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Aufgabe 19

$$L' = L + \frac{d}{dt}\mathcal{X} \longrightarrow J = \sum_{k=1}^{f} \frac{\partial L}{\partial \dot{q}_k}\frac{\partial h_k}{\partial \alpha}\Big|_{\alpha=0} - \frac{\partial \mathcal{X}}{\partial \alpha}\Big|_{\alpha=} = const$$

a) $x' = x + \alpha\cos(\omega t)$ $\qquad F = -kx \rightarrow V = \frac{1}{2}kx^2$

$$L' = T - V = \frac{1}{2}m\dot{x}'^2 - \frac{1}{2}kx'^2$$

$$= \frac{1}{2}m(\dot{x}^2 - 2\dot{x}\alpha\omega\sin(\omega t) + \alpha^2\sin^2(\omega t)\omega^2)$$

$$- \frac{1}{2}k(x^2 + 2x\alpha\cos(\omega t) + \alpha^2\cos^2(\omega t)) = L + \frac{d}{dt}\mathcal{X}$$

$$\frac{d\mathcal{X}}{dt} = (-\dot{x}\alpha\omega\sin(\omega t) + \frac{\alpha^2}{2}\sin^2(\omega t)\omega^2)m$$
$$(-x\alpha\cos(\omega t) + \frac{\alpha^2}{2}\cos^2(\omega t))k$$

$$J(\vec{q}, \dot{\vec{q}}, t) = \sum_{k=1}^{3}\frac{\partial L}{\partial \dot{q}_k}\frac{\partial h_k}{\partial \alpha}\Big|_{\alpha=0} - \int\Big[\frac{\partial(-\dot{x}\alpha\omega m\sin(\omega t))}{\partial\alpha}\Big|_{\alpha=0}$$

$$+ \frac{\partial(\frac{m}{2}\alpha^2\sin^2(\omega t)\omega^2)}{\partial\alpha}\Big|_{\alpha=0} - \frac{\partial(x\alpha k\cos(\omega t))}{\partial\alpha}\Big|_{\alpha=0}$$

$$+ \frac{\partial(\frac{k}{2}\alpha^2\cos^2(\omega t)k)}{\partial\alpha}\Big|_{\alpha=0}\Big]dt = const.$$

f ~~bei euch steht~~ da $\frac{\partial}{\partial t}\mathcal{X}$, ~~nicht $\mathcal{X}$!~~

O.K.

// fällt weg.

Betrachte nun: $\frac{d\mathcal{X}}{dt} = -xk\alpha\cos(\omega t) - m\dot{x}\alpha\omega\sin(\omega t)$

$\omega = \frac{k}{m} \Rightarrow$ $\qquad = -xk\alpha\cos(\omega t) - \dot{x}k\alpha\sin(\omega t) \Big| \frac{\partial \mathcal{X}}{\partial \alpha}$

$$= -xk\cos(\omega t) - \dot{x}k\sin(\omega t)$$

$$\mathcal{X} = -k\int x\cos(\omega t) + \dot{x}\sin(\omega t)\,dt = -kx\sin(\omega t)$$
$$= -\omega m x\sin(\omega t)$$ ✓