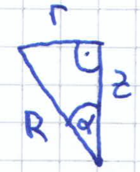
Lars
David
Jonah

7a) $\sum_{i=0}^{N} (F_i^{(a)} - m\ddot{\vec{R}}_i)\,\delta\vec{R}_i = 0$

$\Rightarrow (-mg\vec{e_z} - m\ddot{\vec{R}})\,\delta\vec{R}_i = 0 \quad , \ddot{\vec{R}} = \begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} \quad , \delta\vec{R}_i = \begin{pmatrix} \delta x \\ \delta y \\ \delta z \end{pmatrix}$

b) 

$R = |\vec{R}| \Rightarrow \tan\alpha = \frac{r}{z} \Rightarrow z = r\cot\alpha$

$\sin\alpha = \frac{r}{R} \Rightarrow r = R\sin\alpha$

holonom, skleronom

c) $\vec{R}(t) = r\begin{pmatrix} \cos\varphi \\ \sin\varphi \\ \cot\alpha \end{pmatrix} = R\underbrace{\begin{pmatrix} \cos\varphi\sin\alpha \\ \sin\varphi\sin\alpha \\ \cos\alpha \end{pmatrix}}_{\vec{e_R}}$

generalisierte Koordinaten: $r(t)$ und $\varphi(t)$

$\Rightarrow \ddot{\vec{R}}(t) = \frac{d}{dt}\dot{\vec{R}}$

$\dot{\vec{R}} = \dot{r}\begin{pmatrix} \cos\varphi \\ \sin\varphi \\ \cot\alpha \end{pmatrix} + r\begin{pmatrix} -\sin\varphi \\ \cos\varphi \\ 0\,\cancel{\cot\alpha} \end{pmatrix} = \dot{r}\begin{pmatrix} \cos\varphi \\ \sin\varphi \\ \cot\alpha \end{pmatrix} + r\vec{e_\varphi}\,\dot{\varphi}$

$\Rightarrow \ddot{\vec{R}} = \ddot{r}\begin{pmatrix} \cos\varphi \\ \sin\varphi \\ \cot\alpha \end{pmatrix} + \dot{r}\begin{pmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{pmatrix}\dot{\varphi} + \dot{r}\dot{\varphi}\vec{e_\varphi} + r\ddot{\varphi}\vec{e_\varphi} + r\dot{\varphi}^2\begin{pmatrix} -\cos\varphi \\ -\sin\varphi \\ 0 \end{pmatrix}$

$= \frac{\ddot{r}}{\sin\alpha}\vec{e_R} + 2\dot{r}\dot{\varphi}\vec{e_\varphi} + r\ddot{\varphi}\vec{e_\varphi} - r\dot{\varphi}^2\vec{e_r} = \frac{\ddot{r}}{\sin\alpha}\vec{e_R} + (2\dot{r}\dot{\varphi} + r\ddot{\varphi})\vec{e_\varphi} - r\dot{\varphi}^2\vec{e_r}$

$\delta\vec{R} = \frac{\partial\vec{R}}{\partial R}\delta R + \frac{\partial\vec{R}}{\partial\varphi}\delta\varphi = \vec{e_R}\,\delta R + r\vec{e_\varphi}\,\delta\varphi$

$\Rightarrow (-mg\vec{e_z} - m\ddot{\vec{R}})(\vec{e_R}\,\delta R + r\vec{e_\varphi}\,\delta\varphi) = 0$

$\vec{e_R}\cdot\vec{e_z} = \cos\alpha \quad , \quad \vec{e_R}\cdot\vec{e_r} = \sin\alpha$

$\Rightarrow \left(-mg\cos\alpha - m\left(\frac{\ddot{r}}{\sin\alpha} - r\dot{\varphi}^2\sin\alpha\right)\right)\delta R - m(2\dot{r}\dot{\varphi} + r\ddot{\varphi})\,\delta\varphi = 0$

$\Rightarrow -g\cos\alpha - \frac{\ddot{r}}{\sin\alpha} + r\dot{\varphi}^2\sin\alpha = 0 \;\wedge\; r(2\dot{r}\dot{\varphi} + r\ddot{\varphi}) = 0$

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