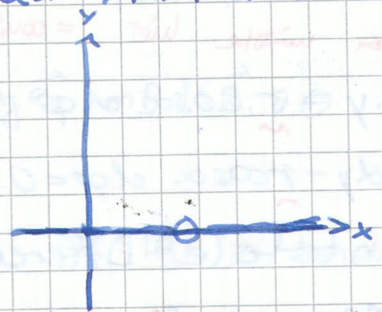
Lars
Jonah
David

5a) Draht mit Perle:



Draht auf der x-Achse ✓

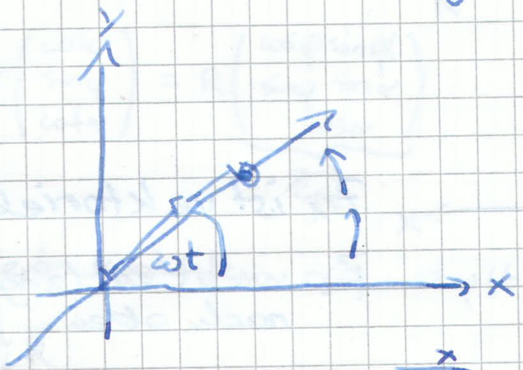
$\Rightarrow y = 0, \; z = 0$

$\Rightarrow 3 - 2 = 1$ Freiheitsgrad(e) ✓

$q_1 = x$? holonome, skleronome ✓ Zwangsbedingungen

X	5	6	7	8	Σ
	4	5	5	5	19/2

b)



$x = r\cos\omega t \Leftrightarrow r = \frac{x}{\cos\omega t}$ gleichsetzen:

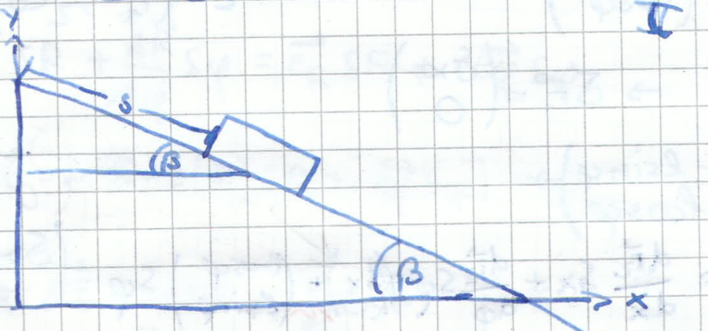
$y = r\sin\omega t \Leftrightarrow r = \frac{y}{\sin\omega t}$

$$\frac{y}{\sin\omega t} = x \frac{1}{\cos\omega t}$$

$\Leftrightarrow$ I $y = x \tan\omega t$ } beide holonom, ✓

II $z = 0$ } I ist rheo- und II skleronom

c)



I $z = 0$

$x = s\cos\beta$ } II $y = x\tan\beta$

$y = s\sin\beta$

~~holonom~~, skleronom ✓

"oberhalb"

d) Da der Körper rollt, gilt $v = r \cdot \dot{\varphi}$ und da er beliebig in der xy-Ebene beweglich ist muss v-Komponentenweise betrachtet werden. α: Winkel der Rollbahn zur y-Achse

$v_x = \dot{x} = -\dot{r}\sin\alpha \;,\; v_y = \dot{y} = r\cos\alpha$ ✓

differentiell geschrieben

r ~~...~~ ~~...~~ müsste hier = const sein...

$dx = -r\sin\alpha\, d\varphi \quad , \quad dy = r\cos\alpha\, d\varphi$

$\Leftrightarrow \; dx + r\sin\alpha\, d\varphi = 0 \quad , \Leftrightarrow \; dy - r\cos\alpha\, d\varphi = 0$

differentiell, aber kein totales Differential

$\Rightarrow$ nicht-holonom, differentiell, skleronom ✓

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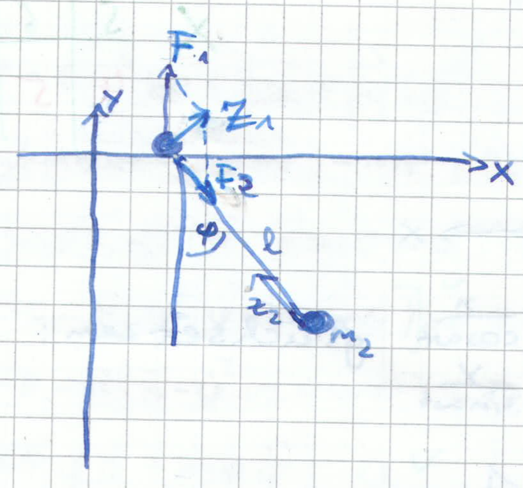
5a)



Z_1 ist vektorielle Addition von F_1 und F_2

Z_2 wirkt entlang des Seils nach oben

b) $\vec{F}_1 = F_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ ✓ $\quad \vec{Z}_2 = \begin{pmatrix} -\sin\varphi \\ \cos\varphi \end{pmatrix} \cdot Z_2$ ✓

$\vec{F}_2 = F_2 \begin{pmatrix} \sin\varphi \\ -\cos\varphi \end{pmatrix} \quad \vec{Z}_1 = \vec{F}_1 + \vec{F}_2 = \begin{pmatrix} F_2 \sin\varphi \\ F_1 - F_2\cos\varphi \end{pmatrix}$

$\vec{r}_1 = \begin{pmatrix} x \\ 0 \end{pmatrix} \rightarrow \delta\vec{r}_1 = \begin{pmatrix} \delta x \\ 0 \end{pmatrix}$

$\vec{r}_2 = \begin{pmatrix} x + l\sin\varphi \\ -l\cos\varphi \end{pmatrix} \rightarrow$

da steht $\begin{pmatrix} \delta x \\ 0 \end{pmatrix}$

$\delta\vec{r}_2 = \frac{d\vec{r}_2}{dx}\delta x + \frac{d\vec{r}_2}{d\varphi}\delta\varphi = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} l\cos\varphi \\ l\sin\varphi \end{pmatrix}\delta\varphi = \begin{pmatrix} \delta x + l\cos\varphi\,\delta\varphi \\ l\sin\varphi\,\delta\varphi \end{pmatrix}$ ✓

$\delta W_1 = \vec{Z}_1 \delta\vec{r}_1 = \begin{pmatrix} \delta x \\ 0 \end{pmatrix}\begin{pmatrix} F_2\sin\varphi \\ F_1 - F_2\sin\varphi \end{pmatrix} = F_2 \sin\varphi\, \delta x \neq 0$ ✓

$\delta W_2 = \vec{Z}_2 \delta\vec{r}_2 = Z_2 \begin{pmatrix} -\sin\varphi \\ \cos\varphi \end{pmatrix}\begin{pmatrix} \delta x + l\cos\varphi\,\delta\varphi \\ l\sin\varphi\,\delta\varphi \end{pmatrix}$

$= Z_2(-\sin\varphi\,\delta x - l\sin\varphi\cos\varphi\,\delta\varphi + l\sin\varphi\cdot\cos\varphi\,\delta\varphi)$

$= -Z_2 \sin\varphi\,\delta x \qquad \text{da } |\vec{F}_2| = |\vec{Z}_2|$

$= -F_2 \sin\varphi\,\delta x$ ✓

c) $\delta W_1 + \delta W_2 = F_2\sin\varphi\,\delta x - F_2\sin\varphi\,\delta x = 0$ ✓

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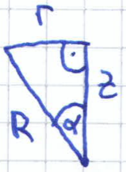
Lars
David
Jonah

7a) $\sum_{i=0}^{N} (\vec{F}_i^{(a)} - m\ddot{\vec{R}}_i)\delta\vec{R}_i = 0$

$\Rightarrow (-mg\vec{e_z} - m\ddot{\vec{R}})\delta\vec{R}_i = 0 \quad , \ddot{\vec{R}} = \begin{pmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{pmatrix} \quad , \delta\vec{R}_i = \begin{pmatrix}\delta x\\ \delta y\\ \delta z\end{pmatrix}$

b) 

$R = |\vec{R}| \Rightarrow \tan\alpha = \frac{r}{z} \Rightarrow z = r\cot\alpha$

$\sin\alpha = \frac{r}{R} \Rightarrow r = R\sin\alpha$

} holonom, skleronom

c) $\vec{R}(t) = r\begin{pmatrix}\cos\varphi\\ \sin\varphi\\ \cot\alpha\end{pmatrix} = R\underbrace{\begin{pmatrix}\cos\varphi\sin\alpha\\ \sin\varphi\sin\alpha\\ \cos\alpha\end{pmatrix}}_{\vec{e_R}}$

generalisierte Koordinaten: $r(t)$ und $\varphi(t)$

$\Rightarrow \ddot{\vec{R}}(t) = \frac{d}{dt}\dot{\vec{R}}$

$\dot{\vec{R}} = \dot{r}\begin{pmatrix}\cos\varphi\\ \sin\varphi\\ \cot\alpha\end{pmatrix} + r\begin{pmatrix}-\sin\varphi\\ \cos\varphi\\ 0\,\cancel{\cot\alpha}\end{pmatrix} = \dot{r}\begin{pmatrix}\cos\varphi\\ \sin\varphi\\ \cot\alpha\end{pmatrix} + r\vec{e_\varphi}\dot{\varphi}$

$\Rightarrow \ddot{\vec{R}} = \ddot{r}\begin{pmatrix}\cos\varphi\\ \sin\varphi\\ \cot\alpha\end{pmatrix} + \dot{r}\begin{pmatrix}-\sin\varphi\\ \cos\varphi\\ 0\end{pmatrix}\dot{\varphi} + \dot{r}\dot{\varphi}\vec{e_\varphi} + r\ddot{\varphi}\vec{e_\varphi} + r\dot{\varphi}^2\begin{pmatrix}-\cos\varphi\\ -\sin\varphi\\ 0\end{pmatrix}$

$= \frac{\ddot{r}}{\sin\alpha}\vec{e_R} + 2\dot{r}\dot{\varphi}\vec{e_\varphi} + r\ddot{\varphi}\vec{e_\varphi} - r\dot{\varphi}^2\vec{e_r} = \frac{\ddot{r}}{\sin\alpha}\vec{e_R} + (2\dot{r}\dot{\varphi} + r\ddot{\varphi})\vec{e_\varphi} - r\dot{\varphi}^2\vec{e_r}$

$\delta\vec{R} = \frac{\partial\vec{R}}{\partial R}\delta R + \frac{\partial\vec{R}}{\partial\varphi}\delta\varphi = \vec{e_R}\delta R + r\vec{e_\varphi}\delta\varphi$

$\Rightarrow (-mg\vec{e_z} - m\ddot{\vec{R}})(\vec{e_R}\delta R + r\vec{e_\varphi}\delta\varphi) = 0$

$\vec{e_R}\cdot\vec{e_z} = \cos\alpha \quad , \quad \vec{e_R}\cdot\vec{e_r} = \sin\alpha$

$\Rightarrow \left(-mg\cos\alpha - m\left(\frac{\ddot{r}}{\sin\alpha} - r\dot{\varphi}^2\sin\alpha\right)\right)\delta R - m(2\dot{r}\dot{\varphi} + r\ddot{\varphi})\delta\varphi = 0$

$\Rightarrow -g\cos\alpha - \frac{\ddot{r}}{\sin\alpha} + r\dot{\varphi}^2\sin\alpha = 0 \ \wedge \ r(2\dot{r}\dot{\varphi} + r\ddot{\varphi}) = 0$

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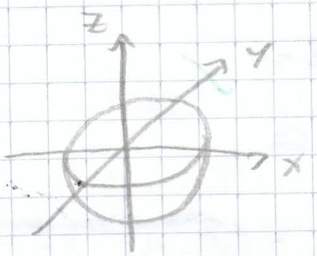
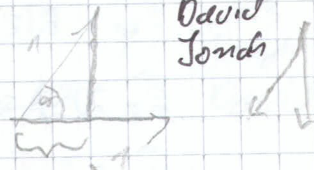
kein Bleistift!

Lars David Jonda

Nr. 8

a)



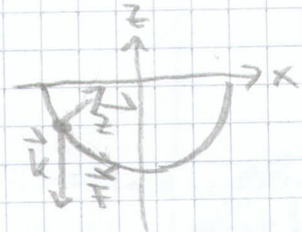


$|\vec a|\cos\varphi\,|\vec b|$

$\cos\varphi = \frac{\vec a\cdot\vec b}{|\vec a||\vec b|}$

$\cos\theta = \frac{F}{mg}$

Zwangsbedingung: $x^2+y^2+z^2=R^2$ bzw.: $r=R$



$$\vec r = \begin{pmatrix} R\cos\varphi\sin\theta \\ R\sin\varphi\sin\theta \\ R\cos\theta \end{pmatrix} = R\underbrace{\begin{pmatrix} \cos\varphi\sin\theta \\ \sin\varphi\sin\theta \\ \cos\theta \end{pmatrix}}_{\vec e_r}$$

$$\vec e_\varphi = \begin{pmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{pmatrix} \qquad \vec e_\theta = \begin{pmatrix} \cos\varphi\cos\theta \\ \sin\varphi\cos\theta \\ -\sin\theta \end{pmatrix}$$

$$\dot{\vec e}_r(\varphi,\theta) = \sin\theta\,\dot\varphi\,\vec e_\varphi + \dot\theta\,\vec e_\theta$$

b) $\vec F = \vec K + \vec Z \Rightarrow m\ddot{\vec r} = -mg\vec e_z - \lambda\vec e_r$

$$m\ddot{\vec r} = -mg\vec e_z - \lambda\vec e_r \Rightarrow \ddot{\vec r} = g\vec e_z - \tilde\lambda\vec e_r \quad \text{mit } \tilde\lambda = \frac{\lambda}{m}$$

c) $$\dot{\vec e}_r = \begin{pmatrix} -\sin\varphi\dot\varphi\sin\theta + \cos\theta\dot\theta\cos\varphi \\ \cos\varphi\dot\varphi\sin\theta + \sin\varphi\cos\theta\dot\theta \\ -\sin\theta\dot\theta \end{pmatrix} = \sin\theta\dot\varphi\vec e_\varphi + \dot\theta\vec e_\theta$$

$$\ddot{\vec e}_r = (\cos\theta\dot\varphi\dot\theta + \ddot\varphi\sin\theta)\vec e_\varphi + (\sin\theta\dot\varphi)\dot{\vec e}_\varphi + \ddot\theta\vec e_\theta + \dot\theta\dot{\vec e}_\theta$$

$$\dot{\vec e}_\varphi = \begin{pmatrix} -\cos\varphi\dot\varphi \\ -\sin\varphi\dot\varphi \\ 0 \end{pmatrix} = -\dot\varphi\begin{pmatrix} \cos\varphi \\ \sin\varphi \\ 0 \end{pmatrix} = -\dot\varphi(\sin\theta\vec e_r + \cos\theta\vec e_\theta)$$

$$\dot{\vec e}_\theta = \begin{pmatrix} -\sin\varphi\dot\varphi\cos\theta - \sin\theta\dot\theta\cos\varphi \\ \cos\varphi\dot\varphi\cos\theta - \sin\theta\dot\theta\sin\varphi \\ -\cos\theta\dot\theta \end{pmatrix} = \dot\varphi\cos\theta\vec e_\varphi - \dot\theta\vec e_r$$

$$\Rightarrow \ddot{\vec e}_r = (\cos\theta\dot\varphi\dot\theta + \ddot\varphi\sin\theta)\vec e_\varphi + (\sin\theta\dot\varphi) - \dot\varphi(\sin\theta\vec e_r + \cos\theta\vec e_\theta) + \ddot\theta\vec e_\theta + \dot\theta\dot\varphi\cos\theta\vec e_\varphi - \dot\theta^2\vec e_r$$

$$= \underbrace{(\cos\theta\dot\varphi\dot\theta + \ddot\varphi\sin\theta + \dot\theta\dot\varphi\cos\theta)}_{2\cos\theta\dot\varphi\dot\theta + \sin\theta\ddot\varphi}\vec e_\varphi - (\sin^2\theta\dot\varphi^2 + \dot\theta^2)\vec e_r + (\ddot\theta - \dot\varphi^2\cos\theta\sin\theta)\vec e_\theta$$

$$\vec e_z = \cos\theta\vec e_r - \sin\theta\vec e_\theta$$

$$\ddot{\vec r} = R\ddot{\vec e}_r$$

$\Rightarrow \vec{e_\varphi}: R(2\cos\theta\dot\varphi\dot\theta + \sin\theta\ddot\varphi) = 0$

$\vec{e_r}: R(-\sin^2\theta\dot\varphi^2 - \dot\theta^2) = -\tilde\lambda - g\cos\theta$ ✓

$\vec{e_\theta}: R(\ddot\theta - \dot\varphi^2\cos\theta\sin\theta) = g\sin\theta$

d) $\sum_{i=0}^{N}(\vec{F}_i^{(a)} - m\ddot{\vec{r}}_i)\delta\vec{r}_i = 0$ ✓ 1 Teilchen $\Rightarrow$ $(m\vec{g} - m\ddot{\vec{r}})\delta\vec{r} = 0$ ✓ $\ddot{\vec{r}} = \begin{pmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{pmatrix}$

e) $\delta\vec{r} = \frac{\partial\vec{r}}{\partial\theta}\delta\theta + \frac{\partial\vec{r}}{\partial\varphi}\delta\varphi$ $\delta\vec{r} = \begin{pmatrix}\delta x\\ \delta y\\ \delta z\end{pmatrix}$

$\vec{x} = a\vec{e_x} + b\vec{e_y} + c\vec{e_z} = \begin{pmatrix}a\\ b\\ c\end{pmatrix}$

$\Rightarrow \delta\vec{r} = R\vec{e_\theta}\,\delta\theta + R\sin\theta\,\vec{e_\varphi}\,\delta\varphi$ ✓ $\delta\vec{r} = \begin{pmatrix}0\,\delta r\\ R\sin\theta\,\delta\varphi\\ R\delta\theta\end{pmatrix}$

$\Rightarrow (-mg(\cos\theta\vec{e_r} - \sin\theta\vec{e_\theta}) - m\ddot{\vec{r}})(R(\vec{e_\theta}\delta\theta + \sin\theta\vec{e_\varphi}\delta\varphi)) = 0$

$\Rightarrow (mg\sin\theta R - mR^2(\ddot\theta - \dot\varphi^2\cos\theta\sin\theta))\delta\theta - m\sin\theta R^2(2\cos\theta\dot\varphi\dot\theta + \sin\theta\ddot\varphi)\delta\varphi = 0$

$\Rightarrow [g\sin\theta - R(\ddot\theta - \dot\varphi^2\cos\theta\sin\theta)]\delta\theta$ ✓ $- \sin\theta R(2\cos\theta\dot\varphi\dot\theta + \sin\theta\ddot\varphi)\delta\varphi = 0$ ✓

$\Rightarrow g\sin\theta - R(\ddot\theta - \dot\varphi^2\cos\theta\sin\theta) = 0$ Top!

$\wedge\ 2\cos\theta\dot\varphi\dot\theta + \sin\theta\ddot\varphi = 0$ ✓ Vorrechnen?

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