(4) 1a) $Q_N(T,N,V) = \frac{1}{h^{3N}N!} \underbrace{\int d^{3N}q}_{V^N} \int d^{3N}p \; e^{-\beta H(p,q)}$ $\quad H = \sum_{i=1}^{3N} \frac{p_i^2}{2m}$

$\to H = p^2$ da e^H aufteilbar

das verstehe ich nicht

ja :)

$= \frac{V^N}{h^{3N}N!} \left[\int dp \; e^{-\frac{\beta}{2m}p^2}\right]^{3N}$ ✓

Lars
Jonah
Yascha
David

$= \frac{V^N}{N!} \left(\frac{2\pi m}{h^2 \beta}\right)^{3\frac{N}{2}} = \frac{V^N}{N!} \left(\frac{2\pi m k_B T}{h^2}\right)^{\frac{3}{2}N}$ ✓

b) $U(T,V,N) = -\frac{\partial}{\partial\beta}\left(\ln(Q_N)\right) = -\frac{\partial}{\partial\beta}\left(\ln\left(\frac{V^N}{N!}\right) + \frac{3N}{2}\left(\ln\left(\frac{2\pi m}{h^2}\right) - \ln\beta\right)\right)$

$= \frac{3}{2}N \cdot \frac{1}{\beta} = \frac{3}{2}Nk_BT$ ✓ $\quad c_V = \left.\frac{\partial U}{\partial T}\right|_{V,N} = \frac{3}{2}Nk_B$ ✓

c) $F = -\frac{1}{\beta}\ln(Q_N) = -\frac{1}{\beta}\left(N\ln(V) - \ln(N!) + \frac{3N}{2}\ln\left(\frac{2\pi m}{h^2}\right) - \frac{3N}{2}\ln\beta\right)$

$\ln(N!) \approx N\ln N - N$

$= -\frac{N}{\beta}\left(1 - \ln N + \ln V + \frac{3}{2}\ln\left(\frac{2\pi m}{h^2}\right) - \frac{3}{2}\ln\beta\right)$

$= -Nk_BT \ln\left[\frac{V}{N} e \left(\frac{2\pi m k_B T}{h^2}\right)^{3/2}\right]$ ✓

$\left.\frac{\partial F}{\partial V}\right|_{T,N} = -p = -\frac{Nk_BT}{V} \Leftrightarrow pV = Nk_BT$ ✓

d) $S = -\left.\frac{\partial F}{\partial T}\right|_{V,N} = Nk_B\left(\ln\left[\frac{V}{N} e \left(\frac{2\pi m k_B T}{h^2}\right)^{3/2}\right] + \frac{3}{2}\right)$ ✓

gut

2) $\langle H^2\rangle - \langle H\rangle^2 = \frac{\partial^2}{\partial\beta^2}\left(\ln Q_N\right) = \frac{\partial}{\partial\beta}\left(-\frac{3}{2}\frac{N}{\beta}\right) = \frac{3}{2}\frac{N}{\beta^2} = \frac{3}{2}Nk_B^2T^2 = k_BT^2c_V$

(1)

woher?

gilt i.A. nicht

18

A1	A2	A3	A4	Ges
4	1	2	1,5	8,5

Blatt 9

3) $Z_K = \left(\int\limits_{\Gamma\text{-Raum}} \frac{dp\,dq}{h^{3N} N!} \right) e^{-\frac{H(p,q)}{k_B T}}$

Zustände nicht abhängig von p q
Phasenraum wird von θ_i und φ_i aufgespannt
$\Rightarrow$ Integral wird zu $\int d\varphi \int d\theta_i \sin\theta_i$

$H = -h\mu \sum_{i=1}^{N} \cos\theta_i$

Impulse $=$ const. $= 0$ ✓

$\rightarrow$ Impulsteil im Γ-Raum:

$\delta(p_i)\, \underset{\text{Impulseinheit}}{p_0}$ für jeden Impuls

$N!$ entfällt, da magnetisches moment wird als unterscheidbar betrachtet

a) $\Rightarrow Z_K = \int\limits_{\Gamma\text{-Raum}\,p} \frac{dp}{h^{3N} \cancel{N!}} \int\limits_{\Gamma\text{-Raum}\,q} dq\; e^{-\frac{H(q)}{k_B T}} = \cancel{\left(\frac{p_0}{h^3}\right)^{3N}} \frac{1}{\cancel{N!}} \int\limits_{\Gamma\text{-Raum}\,q} \exp\left(\frac{h\mu}{k_B T} \sum_i \cos\theta_i\right) dq$

$= \cancel{\left(\frac{p_0}{h}\right)^{3N}} \frac{1}{\cancel{N!}} \prod_{i=1}^{N} \int\limits_{V_i} d\varphi_i\, \cancel{dr_i}\, d\theta_i\, \cancel{r_i}^2 \sin\theta_i \exp\left(\frac{h\mu}{k_B T}\cos\theta_i\right)$

$= \prod_{i=1}^{N} \int_0^{2\pi} d\varphi_i \cancel{\int_0^R r_i^2\, dr_i} \int_0^{\pi} \sin\theta_i \exp\left(\frac{h\mu}{k_B T}\cos\theta_i\right) d\theta_i$

$= \prod_{i=1}^{N} \frac{V_i}{2} \left[-\frac{2\pi k_B T}{h\mu} \exp\left(\frac{h\mu}{k_B T}\cos\theta_i\right) \right]_0^{\pi}$

$= \prod_{i=1}^{N} \cancel{V_i} \frac{4\pi k_B T}{h\mu} \sinh\left(\frac{h\mu}{k_B T}\right) = \left(\frac{4\pi k_B T}{h\mu} \sinh\left(\frac{h\mu}{k_B T}\right) \right)^N$

Annahme wegen gleicher Teilchen $\forall V_i = V$

$\left(\Rightarrow Z_K = \left(\frac{V p_0^3}{h^3}\right)^N \frac{1}{N!} \frac{k_B T}{h\mu} \sinh\left(\frac{h\mu}{k_B T}\right) \right)$

b) $F(T, h, N) = -k_B T \ln Z_K = -k_B T \left[N \ln\left(\frac{V p_0^3}{h^3}\right) - \ln N! + \ln\left(\frac{k_B T}{h\mu}\right) + \ln\left(\sinh\left(\frac{h\mu}{k_B T}\right)\right) \right]$ ff.

c) $\langle D \rangle = -\frac{\partial F}{\partial h}\Big|_{T,N} = \frac{\partial}{\partial h}\left[\ln\left(\frac{k_B T}{h\mu}\right) + \ln\left(\sinh\left(\frac{h\mu}{k_B T}\right)\right) \right]$

$= \left(\frac{\mu}{k_B T} \frac{1}{\tanh\left(\frac{\mu h}{k_B T}\right)} - \frac{1}{h} \right)$

mit Langevin-Funktion: $L(x) = \frac{1}{\tanh x} - \frac{1}{x}$

$\langle D \rangle = \frac{\mu}{k_B T} L\left(\frac{\mu h}{k_B T}\right)$ ff.

d) Schreibe $\alpha = \frac{\mu}{k_B T}$

$$\lim_{h\to\infty} \langle D \rangle = \alpha \lim_{h\to\infty} \left(\frac{e^{\alpha h} + e^{-\alpha h}}{e^{\alpha h} - e^{-\alpha h}} - \frac{1}{\alpha h} \right)$$

$$= \alpha \lim_{h\to\infty} \frac{e^{\alpha h}}{e^{\alpha h}} = \frac{\mu}{k_B T} \text{ ✓.}$$

$$\lim_{h\to-\infty} \langle D \rangle = \alpha \lim_{h\to-\infty} \left(\frac{e^{\alpha h} + e^{-\alpha h}}{e^{\alpha h} - e^{-\alpha h}} - \frac{1}{\alpha h} \right)$$

$$= \alpha \lim_{h\to-\infty} \frac{e^{-\alpha h}}{-e^{-\alpha h}} = -\frac{\mu}{k_B T} \text{ ✓.}$$

$\lim_{T\to\infty}$ $\lim_{T\to 0}$ Skizze?

2/4

Nr. 4 a) $F_{zp} = m\omega^2 r \Rightarrow a_{zp} = \omega^2 r$

$$\Rightarrow \frac{dp}{dr} = -\rho a_{zp} = -\rho\omega^2 r \quad , \rho = \frac{pM}{RT}$$

$$\Rightarrow \frac{dp}{dr} = -\frac{pM\omega^2 r}{RT} \Rightarrow \frac{dp}{p} = -\frac{M\omega^2 r}{RT}\,dr$$

$$\Rightarrow \ln\left(\frac{p}{p_0}\right) = -\frac{M\omega^2}{2RT} r^2 \Rightarrow p = p_0 \exp\left(-\frac{M\omega^2}{2RT} r^2\right) ✓$$

c ≙ Dichte ↯ c = Relative Konzentration

n_i ≙ Dichte

$$p = \frac{\rho RT}{M} = cRT \Rightarrow c = \frac{p}{RT} = c_0 \exp\left(-\frac{M\omega^2}{2RT} r^2\right)$$

$c_i^{(2)} = \frac{n_i(0)}{n_{ges}(0)}$

$c_i^{(1)} = \frac{N_i}{N_{ges}}$

$$\Rightarrow \frac{c_2^{(2)}}{c_1^{(2)}} = \frac{c_2^{(1)}}{c_1^{(1)}} \exp\left(-\frac{M_2\omega^2}{2RT} r^2\right) \cdot \exp\left(\frac{M_1\omega^2}{2RT} r^2\right)$$

$$= \frac{c_2^{(1)}}{c_1^{(1)}} \exp\left(\frac{M_1 - M_2}{2RT}\omega^2 r^2\right) = \frac{c_2^{(1)}}{c_1^{(1)}} \exp\left(\frac{M_1-M_2}{2RT} v^2\right)$$

$$\Rightarrow q = \exp\left(\frac{M_1-M_2}{2RT} v^2\right) \left[\cancel{\approx 1 + \frac{M_1-M_2}{RT} v^2} \; (\text{Taylor bis } \cancel{2. \text{ Ordnung in } r})\right]$$

$N_i = \int dr \int d\varphi\, r\, n_i(0) e^{\frac{1}{2}\beta m_i \omega^2 r^2}$

b) $v = 500 \frac{m}{s}$, $T_1 = 20°C$, $T_2 = 300°C$, $M_1 = 238 \frac{g}{Mol}$, $M_2 = 235 \frac{g}{Mol}$

$$\Rightarrow q(20°C) = q(293{,}15\,K) = \cancel{6{,}577 \cdot 10^{66}}\ 1{,}166 \; ?$$

$$q(300°C) = q(573{,}15\,K) = \cancel{1{,}497 \cdot 10^{34}}\ 1{,}082 \; ?$$

} irgendwie sehr groß?

Bei niedrigen Temperaturen ist die Trennung effektiver ✓

1,5/4