11.12.18

$$C_V = \frac{\partial U}{\partial T}\Big|_{V,N} = -\frac{\partial}{\partial T}\Big|_V \frac{\partial}{\partial \beta}\Big|_V \ln(Q_N)$$

$$= -k_B \beta^2 \frac{\partial^2}{\partial^2 \beta^2} \ln(Q_N) = k_B \beta^2 \left(\frac{1}{Q_N} \frac{\partial^2 Q_N}{\partial \beta^2}\Big|_V - \frac{1}{Q_N^2}\left(\frac{\partial Q_N}{\partial \beta}\Big|_V\right)^2 \right)$$

$$Q_N = \frac{1}{h^{3N} N!} \iint d^{3N}q\, d^{3N}p\, \exp(-\beta H(p,q))$$

I) $$\frac{1}{Q_N}\frac{\partial^2 Q_N}{\partial \beta^2} = \frac{1}{Q_N}\frac{1}{h^{3N}N!}\iint d^{3N}p\, d^{3N}q \frac{\partial^2}{\partial \beta^2} \exp(-\beta H(p,q))$$

$$= \frac{1}{Q_N}\frac{1}{h^{3N}N!}\iint d^{3N}p\, d^{3N}q\, H^2(p,q) \exp(-\beta H(p,q))$$

$$\frac{1}{Q_N^2}\left(\frac{\partial^2 Q_N}{\partial \beta^2}\right) = \frac{1}{Q_N^2}\left(\frac{1}{h^{3N}N!}\iint d^{3N}p\, d^{3N}q\, H(p,q)\exp(-\beta H(p,q))\right)^2$$

$$\beta = \frac{1}{k_B T} \qquad \frac{d\beta}{dT} = -\frac{1}{k_B T^2} \qquad d\beta = \frac{1}{-k_B T^2}\, dT$$

$$\Rightarrow C_V = k_B \beta^2 \left(\frac{\iint d^{3N}p\, d^{3N}q\, H^2(p,q)\exp(-\beta H(p,q))}{\iint d^{3N}p\, d^{3N}q\, \exp(-\beta H(p,q))} - \left(\frac{\iint d^{3N}p\, d^{3N}q\, H(p,q)\exp(-\beta H(p,q))}{\iint d^{3N}p\, d^{3N}q\, \exp(-\beta H(p,q))}\right)^2 \right)$$

$$C_V = k_B^2 (\langle H^2\rangle - \langle H\rangle^2)$$

A3)

a) $$z_1 = \int_0^{2\pi}\int_0^{\pi} \sin\theta\, e^{\frac{1}{k_B T} h\mu\cos\theta}\, d\theta\, d\varphi \qquad u = \cos\theta$$

$$d\theta = -\frac{1}{\sin\theta} du$$

$$= 2\pi \int_{-1}^{1} e^{\beta h \mu u}\, du = 4\pi \frac{k_B T}{\mu h} \sinh\left(\frac{\mu h}{k_B T}\right)$$

$$\Rightarrow Z_K(T,h,N) = \left(4\pi \frac{k_B T}{\mu h}\sinh\left(\frac{\mu h}{k_B T}\right)\right)^N$$

b) $$F(T,h,N) = -k_B T \ln(Z_K(T,h,N)) = -k_B T N \ln\left(4\pi \frac{k_B T}{\mu h}\sinh\left(\frac{\mu h}{k_B T}\right)\right)$$

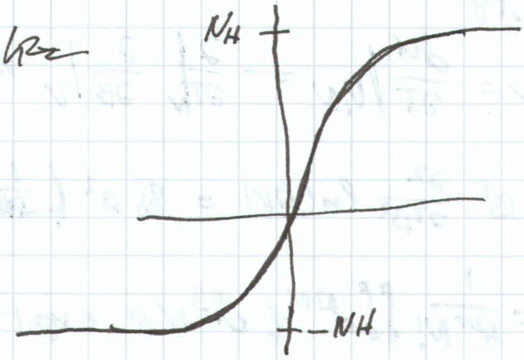
c) $$\langle O\rangle = -\frac{dF}{dh}\Big|_{T,N} = N k_B T \frac{d}{dh} \ln\left(4\pi \frac{\sinh(\mu h \beta)}{\mu h \beta}\right)$$

$$= N k_B T \left(\frac{\mu\beta \cosh(\mu h\beta)}{\sinh(h\mu\beta)} - \frac{1}{h} \right)$$

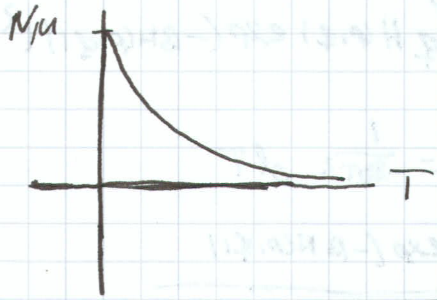
$$= N\mu L(\mu h \beta)$$

d) $\lim_{h\to\infty} \langle D\rangle = N\mu$

$\lim_{h\to -\infty} \langle D\rangle = -N\mu$



$\lim_{T\to 0} \langle D\rangle = N\mu$ $\qquad$ $\lim_{T\to\infty} \langle D\rangle = 0$



A4)

$c_1 + c_2 = 1$ $\qquad$ $\rho = \rho(0)\, e^{-\beta E}$ $\qquad$ $E = -\frac{m\omega^2}{2} r^2$

③ $q = \dfrac{c_2^{(2)}/c_1^{(2)}}{c_2^{(1)}/c_1^{(2)}}$ $\qquad$ $n \sim \rho$ $\qquad$ ② $n = n(0)\, e^{\frac{1}{2}\beta m\omega^2 r^2}$

$c_i^{(2)} = \dfrac{n_i(0)}{n_{ges}(0)}$ $\qquad$ $c_i^{(1)} = \dfrac{N_i}{N_{Ges}}$

$$N_i = \int_0^R dr \int_0^H dh \int_0^{2\pi} d\varphi\, r\, n_i(0)\, e^{\frac{1}{2}\beta m_i \omega^2 r^2}$$ $\qquad$ Subst.: $x = r^2$

$$= \pi H n_i(0) \int_0^{R^2} dx\, e^{\frac{1}{2}\beta m_i \omega^2 x} = \frac{H\, 2\pi\, n_i(0)}{\beta m_i \omega^2}\left(e^{\frac{1}{2}\beta m_i \omega^2 R^2} - 1\right)$$

$$\underbrace{\frac{c_2^{(1)}}{c_1^{(1)}} = \frac{N_2}{N_1}}_{①} = \frac{n_2(0) m_1}{n_1(0) m_2} \; \frac{\exp\left(\frac{1}{2}\beta m_2 \omega^2 R^2\right) - 1}{\exp\left(\frac{1}{2}\beta m_1 \omega^2 R^2\right) - 1}$$

① & ② in ③

$$\Longrightarrow q = \frac{m_1}{m_2} \; \frac{\exp\left(\frac{1}{2} m_1 \omega^2 R^2\right) - 1}{\exp\left(\frac{1}{2} m_2 \omega^2 R^2\right) - 1}$$

$^{238}UF_6 \rightarrow m_1 = (238 + 196)\, m_p = 352\, m_p$

$^{235}UF_6 \rightarrow m_1 = (235 + 196)\, m_p = 349\, m_p$

$$q \simeq \frac{m_2}{m_1}\, e^{\frac{\beta}{2}(m_1 - m_2) v^2}$$

$q(T = 300°) \simeq 1{,}07$

$q(T = 20°) = 1{,}16$