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Aufgabe 1

a)
$$\cos(qa) = \frac{t^2 - r^2}{2t} e^{ika} \quad (7) \qquad \varepsilon = \frac{\hbar^2 k^2}{2m}$$

$+\frac{1}{2t} e^{-ika}$

hatten wir in der letzten Übung vorgestellt...

$$t = |t| e^{i\delta} \quad (8)$$
$$1 = |t|^2 + |r|^2 \quad (9)$$
$$r = \pm i\, |r| e^{i\delta} \quad (12)$$

$$\cos(qa) = \frac{|t|^2 e^{i2\delta} + |r|^2 e^{i2\delta}}{2|t| e^{i\delta}} e^{ika}$$
$$= \frac{|t|^2 + |r|^2}{|t|} \frac{e^{i(ka+\delta)}}{2}$$
$$= \frac{1}{2|t|} e^{i(ka+\delta)}$$
$$= \frac{1}{2|t|} \left(\cos(ka+\delta) + i\sin(ka+\delta)\right)$$ ff. => -1 => 1P

b)
$$\varepsilon_{\text{Lücke}} \to |\cos(ka+\delta)| / |t| \geq 1$$ ✓

$$|t| = \sqrt{1-|r|^2}$$

$$\frac{|\cos(ka+\delta)|}{\sqrt{1-|r|^2}} \geq 1$$ ✓ (*)

$$|\cos(ka+\delta)| \geq \sqrt{1-|r|^2} \approx 1 - \frac{|r|^2}{2}$$

$$1 - \frac{(ka+\delta-\pi)^2}{2} \geq 1 - \frac{|r|^2}{2} \qquad |\sqrt{1-|r|^2}|$$

$$|ka+\delta| \leq |r| \qquad |\sqrt{1-\sqrt{1-|r|^2}}|$$

$$\left|\sqrt{\frac{2Em}{\hbar^2}}a - n\pi\right| \leq |r| \qquad |\sqrt{1-\sqrt{1-|r|^2}}|$$

$$\sqrt{\frac{2Em}{\hbar^2}}a - n\pi \approx |r|$$

$$E \approx \left(\frac{|r| + n\pi}{a}\right)^2 \frac{\hbar^2}{2m} \approx (n^2\pi^2 + 2\pi|r| \ldots) \frac{\hbar^2}{2ma^2}$$

Kommt ja nicht so das richtige raus.
Idee ist, dass ihr in bei (*) den cos taylort

Aber hey... -1P => 1P

c)

$$|\cos(Ka+\delta)|/|t| < 1$$

$$|\cos(Ka+\delta)| < |t|$$

$$|Ka+\delta| < |t|$$

$$\left|\sqrt{\frac{2Em}{\hbar^2}}+\delta\right| < |t|$$

$$\sqrt{\frac{2Em}{\hbar^2}}+\delta < |t|$$

$$\sqrt{\frac{2Em}{\hbar^2}} < |t|-\delta$$

$$\frac{2Em}{\hbar^2} < (|t|-\delta)^2$$

$$E < (|t|-\delta)^2\frac{\hbar^2}{2m}$$

$$E < (|t|^2-2|t|\delta+\delta^2)\frac{\hbar^2}{2m}$$

Nope... -2

$\Rightarrow 0p$

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Aufgabe 2

b)

$$0 < \Delta < 2|U_2|$$

$$\mathcal{E} = \mathcal{E}^0_{K/2} + \frac{\hbar^2}{2m} k^2 \pm \left(4\mathcal{E}^0_{K/2} \frac{\hbar}{2m} K_\parallel^2 + |U_2|^2\right)^{1/2}$$

wieso? hinschreiben

$$= \mathcal{E}^0_{K/2} + \frac{\hbar^2}{2m} k^2 \pm |U_2|$$

$$\mathcal{E} = \mathcal{E}_F \Rightarrow \mathcal{E}_F = \mathcal{E}^0_{K/2} + \frac{\hbar^2 \rho_1^2}{2m} - |U_2|$$ Weshalb fällt "+" weg? -0.5 Δ wäre zu groß

$$\Rightarrow \mathcal{E}^0_{K/2} - |U_2| + \Delta = \mathcal{E}^0_{K/2} + \frac{\hbar^2 \rho_1^2}{2m} - |U_2|$$

$$\Rightarrow \Delta = \frac{\hbar^2 \rho_1^2}{2m}$$

$$\Rightarrow \rho_1 = \sqrt{\frac{2m\Delta}{\hbar^2}}$$ ✓ → 0.5P

c)

$$\Delta \geq 2|U_2|$$

$$\mathcal{E} = \mathcal{E}^0_{K/2} + \frac{\hbar^2}{2m} k^2 \pm \left(4\mathcal{E}^0_{K/2} \frac{\hbar^2}{2m} k_\parallel^2 + |U_2|^2\right)^{1/2}$$

$$\mathcal{E} = \mathcal{E}_f$$

$$\Rightarrow \mathcal{E}_F = \mathcal{E}^0_{K/2} + \frac{\hbar^2}{2m} k_f^2 \pm \left(4\mathcal{E}^0_{K/2} \frac{\hbar^2}{2m} K_\parallel^2 + |U_2|^2\right)^{1/2}$$

$$\Rightarrow \mathcal{E}_f - \mathcal{E}^0_{K/2} = \frac{\hbar^2}{2m} k_f^2 \pm |U_2|$$

$$k_f = \rho_1: \mathcal{E}_f - \mathcal{E}_{K/2} = \frac{\hbar^2}{2m} \rho_1^2 \pm |U_2|$$

$$k_f = \rho_2: \mathcal{E}_f - \mathcal{E}^0_{K/2} = \frac{\hbar^2}{2m} \rho_2^2 + |U_2|$$

$$\Rightarrow \frac{\hbar^2}{2m} \rho_1^2 + |U_2| = \frac{\hbar^2}{2m} = \frac{\hbar^2}{2m} \rho_2^2 - |U_2|$$

$$\Leftrightarrow \frac{\hbar^2}{2m} (\rho_1^2 - \rho_2^2) = -2|U_2|$$

$$\Leftrightarrow (\rho_2^2 - \rho_1^2) = \frac{4m}{\hbar^2} |U_2|$$

$$\Rightarrow (\rho_2^2 - \rho_1^2) \cdot \pi = \frac{4m\pi}{\hbar} |U_2|$$ ✓ → 1P

BRUNNEN