Aufgabe 51

$$P_{if} = \frac{1}{\hbar^2} \left| \int_{-\infty}^{\infty} \omega_{fi}(t)\, e^{i(E_f - E_i)t/\hbar}\, dt \right|^2$$

$$\omega_{fi}(t) = \langle \varphi_f | \hat{H}_{int}(t) | \varphi_i \rangle \qquad \varphi_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2} \quad \varphi_1 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} 2\sqrt{\frac{m\omega}{\hbar}}\, x \cdot e^{-\frac{m\omega}{2\hbar}x^2} \cdot \frac{1}{\sqrt{2}}$$

$$\hat{H}_{int}(t) = \phi(t) q \qquad -\vec{\nabla}\phi(t) = \vec{E}(t) \Rightarrow \phi(t) = E_0 e^{-\alpha t^2} x$$

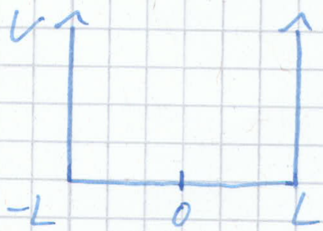
$$W_{10} = \int_{-\infty}^{\infty} \frac{m\omega}{\sqrt{\pi}\hbar} \sqrt{2}\, E_0 q\, e^{-\alpha t^2} x\, e^{-\frac{m\omega}{\hbar}x^2} dx$$

$$= \sqrt{\frac{2}{\pi}} \frac{m\omega}{\hbar} E_0 q \cdot e^{-\alpha t^2} \frac{\sqrt{\pi}}{2\left(\frac{m\omega}{\hbar}\right)^{3/2}} = \sqrt{\frac{\hbar}{2m\omega}} E_0 q e^{-\alpha t^2}$$

$$P_{01}^{(1)} = \frac{1}{\hbar^2} \left| \int_{-\infty}^{\infty} \sqrt{\frac{\hbar}{2m\omega}} E_0 q e^{-\alpha t^2} e^{i\left(\frac{\hbar\omega}{\hbar}t\right)} dt \right|^2$$

$$= \frac{E_0^2 q^2}{\hbar^2} \frac{\hbar}{2m\omega} \left| \int_{-\infty}^{\infty} e^{i\omega t - \alpha t^2} dt \right|^2 = \frac{E_0^2 q^2}{2\hbar m\omega} \frac{\pi^2}{\alpha^2} e^{-\frac{\omega^2}{\alpha}}$$

Aufgabe 53



$$\varphi_n = N \cdot \begin{cases} \sin\left(\frac{n\pi}{2L}x\right) & n \text{ gerade} \\ \cos\left(\frac{n\pi}{2L}x\right) & n \text{ ungerade} \\ 0 & \text{sonst} \end{cases}$$

(1) Fermionen:

Spin $\frac{1}{2}$ $\quad E = 2E_1 + 1E_2 = \frac{3\hbar^2\alpha^3}{4mL^2}$

Spin $-\frac{3}{2}$ $\quad E = 3E_1 = \frac{3}{8}\frac{\hbar^2\pi^2}{mL^2}$

Boson

Spin 0 : $\quad E = 3E_1 = \frac{3}{8}\frac{\hbar^2\pi^2}{mL^2}$

(2)

Boson : $\varphi(x_1, x_2, x_3) = N \cdot \cos\left(\frac{\pi x_1}{2L}\right) \cdot \cos\left(\frac{\pi x_2}{2L}\right) \cdot \cos\left(\frac{\pi x_3}{2L}\right)$

Spin-$\frac{1}{2}$-Fermionen: $\Psi = \frac{1}{\sqrt{N!}} \begin{vmatrix} \phi_1(1) & \cdots & \phi_N(1) \\ \vdots & & \vdots \\ \phi_1(N) & \cdots & \phi_N(N) \end{vmatrix}$ Slater Determinante

- Fermion Grundzustand mit $|\uparrow\rangle$: $N \cdot \cos\left(\frac{\pi x}{2L}\right)_\uparrow$
- " " " $|\downarrow\rangle$: $N \cdot \cos\left(\frac{\pi x}{2L}\right)_\downarrow$
- " 1. Ang. Zustand mit indifferentem Spin: $N \cdot \sin\left(\frac{\pi x}{L}\right)$

$$\Psi(x_1, x_2, x_3) = \frac{1}{\sqrt{6}} \cdot N^3 \begin{vmatrix} \cos(x_1)_\uparrow & \cos(x_2)_\uparrow & \cos(x_3)_\uparrow \\ \cos(x_1)_\downarrow & \cos(x_2)_\downarrow & \cos(x_3)_\downarrow \\ \sin(x_1) & \sin(x_2) & \sin(x_3) \end{vmatrix} = \ldots\ldots$$