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1) $S(U,V,N) = \frac{N}{N_0} s_0 + N k_B \ln\left[\left(\frac{U}{U_0}\right)^{3/2} \frac{V}{V_0} \left(\frac{N}{N_0}\right)^{-5/2}\right]$

2.5

$\left.\frac{\partial S}{\partial U}\right|_V = \frac{1}{T} = \frac{3}{2} N k_B \frac{1}{U} \Leftrightarrow U = \frac{3}{2} N k_B T \;, \; U_0 = \frac{3}{2} N_0 k_B T_0$

$\Rightarrow S(U,V,N) = S(T,V,N) = \frac{N}{N_0} s_0 + N k_B \ln\left[\left(\frac{T}{T_0}\right)^{3/2} \left(\frac{V}{V_0}\right) \left(\frac{N}{N_0}\right)^{-4}\right]$

$dF = -S\,dT - p\,dV$

$F = -\int S\,dT = -\left(\frac{N}{N_0} s_0 T + N k_B \int \frac{3}{2}\ln T + \ln\left[\frac{V}{T_0^{3/2} V_0}\left(\frac{N}{N_0}\right)^{-4}\right] dT\right)$

$= -\left(\frac{N}{N_0} s_0 T + N k_B T \left(\ln\left[\left(\frac{T}{T_0}\right)^{3/2} \frac{V}{V_0}\left(\frac{N}{N_0}\right)^{-4}\right] - 3/2\right)\right) + C(V)$

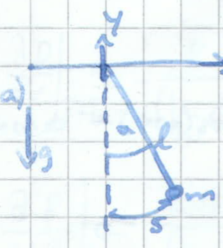
$F = -\int p\,dV = -\int N k_B T \frac{dV}{V} = -N k_B T \ln\left(\frac{V}{V_0}\right) + C(T)$

$\Rightarrow F = -\left(\frac{N}{N_0} s_0 T + N k_B T \left(\ln\left[\left(\frac{T}{T_0}\right)^{3/2} \frac{V}{V_0}\left(\frac{N}{N_0}\right)^{-4}\right] - \frac{3}{2}\right)\right)$ wenn da -1 steht stimmt es :-) ✓

+ Zustandsgleichung verifizieren

4

4) a)



$\alpha l = s =: q$

$F = -mg \sin\alpha = -mg \sin\left(\frac{q}{l}\right)$ ✓ kleine Winkel: $\sin x \approx x$ ✓

$\Rightarrow \; = -mg/l \; q = -m\omega^2 q$

konserv: $F = -\nabla_s V$

$\Rightarrow \frac{dV}{ds} = -F = m\omega^2 q \Leftrightarrow V = \frac{m\omega^2 q^2}{2}$ ✓

$T = \frac{p^2}{2m} \qquad H = T + V = \frac{p^2}{2m} + \frac{m\omega^2}{2} q^2$

b) $\dot q = \frac{\partial H}{\partial p} = p/m \qquad \dot p = -\frac{\partial H}{\partial q} = -m\omega^2 q$ ✓

$\begin{pmatrix} \dot q \\ \dot p \end{pmatrix} = \begin{pmatrix} p/m \\ -m\omega^2 q \end{pmatrix} \qquad \ddot q = \dot p/m \Rightarrow \ddot q = -\omega^2 q \qquad q(t) = Ae^{i\omega t} + Be^{-i\omega t}$

$p(t) = m\dot q(t) = i\omega m \left(Ae^{i\omega t} - Be^{-i\omega t}\right)$

$q(0) = q_0 = A + B$

$p(0) = p_0 = i\omega m (A - B) \Leftrightarrow -\frac{i p_0}{\omega m} = A - B$

$\Rightarrow \frac{1}{2}\left(q_0 - \frac{i p_0}{\omega m}\right) = A \Rightarrow B = \frac{1}{2}\left(q_0 + \frac{i p_0}{\omega m}\right)$

(Q–)

A1	A2	A3	A4	Ges
2.5	4	4	4	14.5

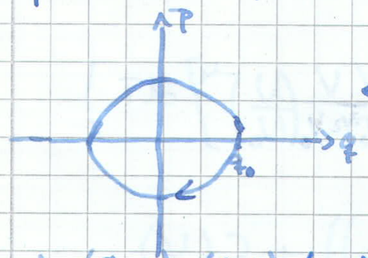
Blatt 7

$$\Rightarrow q(t) = \frac{1}{2}\left(\left(q_0 - \frac{ip_0}{m\omega}\right)e^{i\omega t} + \left(q_0 + \frac{ip_0}{m\omega}\right)e^{-i\omega t}\right)$$

$$= q_0 \cos(\omega t) + \frac{p_0}{m\omega}\sin(\omega t)$$ ✓ ← wesentlich schöner :-)

analog: $p(t) = p_0 \cos(\omega t) - q_0 m\omega \sin(\omega t)$ ✓

$p_0 = 0 \Rightarrow q(t) = q_0 \cos(\omega t)$, $p(t) = -q_0 m\omega \sin(\omega t)$



← Das ist eine Ellipse. ✓

c) $(0,0), (0,b), (a,0), (a,b)$

$q_0 = p_0 = 0$ $(0,0)$: $q(t) = 0$, $p(t) = 0 \Rightarrow$ Der Punkt bewegt sich nicht ✓

$(0,b)$: $q(t) = \frac{b}{m\omega}\sin(\omega t)$ $p(t) = b\cos(\omega t)$

$(0,b) \xrightarrow{t=\frac{\pi}{2\omega}} \left(\frac{b}{m\omega}, 0\right)$

$(a,0)$: $q(t) = a\cos(\omega t)$ $p(t) = -am\omega \sin(\omega t)$

$(a,0) \to (0, -am\omega)$

(a,b): $q(t) = a\cos(\omega t) + \frac{b}{m\omega}\sin(\omega t)$ $p(t) = b\cos(\omega t) - a\omega m \sin(\omega t)$

$(a,b) \to (b/\omega m, -a\omega m)$

Volumen vorher $V = a \cdot b$ ✓

nachher $V = \frac{b}{m\omega} \cdot am\omega = a \cdot b \to$ Liouville
$V = \text{const}$ für beliebiege t

gut!

d) $\frac{H}{E_0} = \frac{1}{2} Q^* Q = \frac{1}{2E_0}\left(\frac{p^2}{m} + m\omega^2 q^2\right)$

$Q = a + ib$ $Q^*Q = (a+ib)(a-ib) = a^2 + b^2$

$\Rightarrow \frac{p^2}{mE_0} = a^2$, $\frac{m\omega^2 q^2}{E_0} = b^2$ oder andersherum

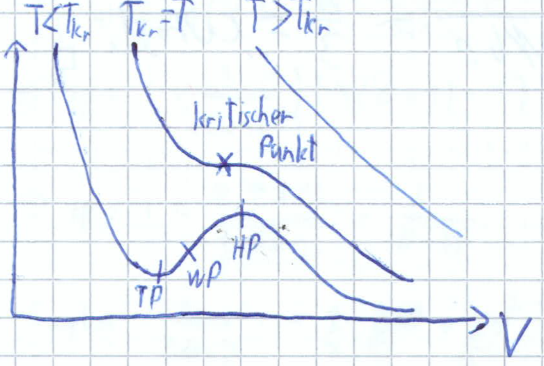
$a = \pm \frac{p}{\sqrt{mE_0}}$, $b = \pm \omega q \sqrt{\frac{m}{E_0}}$

$\Rightarrow Q = \pm\left(\frac{p}{\sqrt{mE_0}} + \omega q \sqrt{\frac{m}{E}}\, i\right)$ oder $Q = \pm\left(\omega q \sqrt{\frac{m}{E_0}} + \frac{p}{\sqrt{mE_0}}\, i\right)$

✓

2) a)

(4)



bei $T<T_{kr}$ → Tiefpunkt, Wendepunkt, Hochpunkt ⇒ Umschwung ⇒ mehrere Lsg.

bei $T=T_{kr}$ → TP + WP + HP in einem Punkt vereint
⇒ kritischer Punkt ≙ ~~Wend~~ Sattelpunkt ⇒ eine Lsg.

~~b~~ bei $T>T_{kr}$ → keine Extrempunkte mehr ⇒ eine Lsg.

⇒ Suche Punkt mit $\left.\frac{\partial p}{\partial V}\right|_{T=T_{kr}}=0$ & $\left.\frac{\partial^2 p}{\partial V^2}\right|_{T=T_{kr}}=0$

b) $\left(p+\frac{N^2a}{V^2}\right)\left(\frac{V}{N}-b\right)=k_BT$

$\Leftrightarrow p=\frac{Nk_BT}{V-Nb}-\frac{N^2a}{V^2}$ (*)

(1) $\left.\frac{\partial p}{\partial V}\right|_{T=T_{kr}}=-\frac{Nk_BT_{krit}}{(V_{kr}-Nb)^2}+\frac{2N^2a}{V_{kr}^3}=0 \Leftrightarrow T_{kr}=\frac{2Na}{k_B}\frac{(V_{kr}-Nb)^2}{V_{kr}^3}$

(2) $\left.\frac{\partial^2 p}{\partial V^2}\right|_{T=T_{kr}}=\frac{2Nk_BT}{(V_{kr}-Nb)^3}-\frac{6N^2a}{V_{kr}^4}=0$

(1) in (2)

$\frac{4N^2a}{(V_{kr}-Nb)V_{kr}^3}=\frac{6N^2a}{V^4} \quad |\cdot(V-Nb)V^4$

$4N^2aV_{kr}=6N^2a(V_{kr}-Nb)=6N^2aV_{kr}-6N^3ab$

$6N^3b=(6-4)N^2V_{kr}$

$3Nb=V_{kr}$

V_{kr} in (1)

$T_{kr}=\frac{2Na}{k_B}\frac{(3Nb-Nb)^2}{27N^3b^3}=\frac{8}{k_B27}\frac{a}{b}$

V_{kr} & T_{kr} in (*)

$p_{kr}=N\frac{8}{27}\frac{a}{b}\frac{1}{2Nb}-\frac{a}{9b^2}=\frac{4}{27}\frac{a}{b^2}-\frac{1}{9}\frac{a}{b^2}=\frac{1}{27}\frac{a}{b^2}$

$$Z_{kr} = \left(\frac{N k_B T_{kr}}{V_{kr}\, p_{kr}}\right)^{-1} = \left(\frac{N 8a\, 27b^2}{27 b\, 3N b\, a}\right)^{-1} = \left(\frac{8}{3}\right)^{-1} = \text{const.} \quad \square$$

(✓)

c) $p \to \bar{p} \cdot \frac{a}{27b^2}$

$V \to \bar{V} \cdot 3Nb$

$T \to \bar{T} \cdot \frac{8a}{27 k_B b}$

in VdW-Gleichung

$$\left(\frac{a}{27b^2}\bar{p} + \frac{a}{9b^2}\frac{1}{\bar{V}^2}\right)(3b\bar{V} - b) = \frac{8a}{27b}\bar{T}$$

$$\frac{a}{9b^2}\left(\frac{\bar{p}}{3} + \frac{1}{\bar{V}^2}\right) b\,(3\bar{V} - 1) = \frac{8}{27}\frac{a}{b}\bar{T}$$

$$\left(\frac{\bar{p}}{3} + \frac{1}{\bar{V}^2}\right)(3\bar{V} - 1) = \frac{8}{3}\bar{T} \checkmark$$

schön!

T45 3) $\left(\bar{p} + \frac{3}{\bar{v}^2}\right)(3\bar{v}-1) = 8\bar{T}$

4

$$\Rightarrow \left(\varphi + 1 + \frac{3}{(\theta+1)^2}\right)(3(\theta+1)-1) = 8(\tau+1)$$

$$\Rightarrow \varphi + 1 = \frac{8(\tau+1)}{3\theta+2} - \frac{3}{(\theta+1)^2}$$

Taylor: $\frac{1}{3\theta+2} \sim \frac{1}{2} - \frac{3}{4}\theta + \frac{9}{8}\theta^2 - \frac{27}{16}\theta^3$ ja, kann man machen

$$\frac{1}{(\theta+1)^2} \sim 1 - 2\theta + 3\theta^2 - 4\theta^3$$

$$\Rightarrow \varphi + 1 \sim 8(\tau+1)\left(\frac{1}{2} - \frac{3}{4}\theta + \frac{9}{8}\theta^2 - \frac{27}{16}\theta^3\right) - 3\left(1 - 2\theta + 3\theta^2 - 4\theta^3\right)$$

$\theta^3\tau \approx 0 \rightarrow$

$$\approx 4\tau + 4 - 6\theta\tau - 6\theta + 9\theta^2\tau + 9\theta^2 - \frac{27}{2}\theta^3 - 3 + 6\theta - 9\theta^2 + 12\theta^3$$

$$= 4\tau - 6\theta\tau + 9\theta^2\tau - \frac{3}{2}\theta^3 + 1$$

$$\Rightarrow \varphi \sim 4\tau - 6\theta\tau + 9\theta^2\tau - \frac{3}{2}\theta^3$$

a) $K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T \sim \left(\frac{\partial\theta}{\partial\varphi}\right)_{\tau,\theta=0} \sim \frac{1}{\left(\frac{\partial\varphi}{\partial\theta}\right)_{\tau,\theta=0}}$

$\left(\frac{\partial\varphi}{\partial\theta}\right)_{\tau,\theta=0} = -6\tau \Rightarrow K_T \sim \frac{1}{\tau} = \tau^{-1} \Rightarrow \gamma = 1$

b) $\varphi \sim \theta^3 \quad (\tau = 0) \Rightarrow \delta = 3$ aufpassen hier ($\mathrm{sgn}(\vartheta)$)

c) $\varphi(\theta_{fl}, t) = \varphi(\theta_g, t)$, $\int_{fl}^{g} \bar{v}\, dp = 0$

$$\Rightarrow \frac{\partial\varphi}{\partial\theta} \overset{!}{=} 0 = -6\tau + 18\tau\theta - \frac{9}{2}\theta^2$$

$$\Rightarrow \theta^2 - 4\theta\tau + \frac{4}{3}\tau = 0 \Rightarrow \theta_{1,2} = 2\tau \pm \sqrt{4\tau^2 - \frac{4}{3}\tau}$$

$$\Rightarrow \Delta\theta = 2\left(-\frac{4}{3}\tau + O(\tau^2)\right)^{\frac{1}{2}} \sim \tau^{1/2} \quad (\tau \text{ klein})$$

$$\Rightarrow \beta = \frac{1}{2}$$

okay