Physik - Übung 3.5.18

Impulsdarstellung

$$\tilde{\varphi}(p) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \varphi(x) \exp\left(-\frac{i}{\hbar} p x\right) dx$$

Aufgabe

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{U}{x_0}\hat{x} \quad ,$$

Ortsdarstellung	Impulsdarstellung
$\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}$	$\hat{\tilde{p}} = p$
$\hat{x} = x$	$\hat{\tilde{x}} = i\hbar\frac{\partial}{\partial p}$

a) im Impulsdarstellung lösen

b) (versuche) in Ortsdarstellung zu lösen

a) $\hat{H}\tilde{\varphi}(p) = E\tilde{\varphi}(p)$

$$\Rightarrow \left(\frac{\hat{p}^2}{2m} + \frac{U}{x_0}\hat{x}\right)\tilde{\varphi}(p) = E\tilde{\varphi}(p)$$

$$\Rightarrow \left(\frac{p^2}{2m} + i\hbar^2\frac{U}{x_0}\frac{\partial}{\partial p} - E\right)\tilde{\varphi}(p) = 0$$

$$\Rightarrow \left[\frac{\partial\tilde{\varphi}(p)}{\partial p} = \frac{x_0}{i\hbar U}\left(E - \frac{p^2}{2m}\right)\tilde{\varphi}(p)\right]$$

$$\Rightarrow \hat{\varphi}(p) = N \cdot \exp\left(\frac{x_0}{i\hbar U}\left(E - \frac{p^3}{6m}\right)\right)$$

b) $$\hat{H}\varphi(x) = \left(\frac{\hat{p}^2}{2m} + \frac{U}{x_0}\hat{x}\right)\varphi(x) = E\varphi(x)$$
$$= \left(\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{U}{x_0}x\right)\varphi(x)$$

⇝ $x \cdot \varphi(x)$ - Term in DGL - 2. Ordnung

→ Airy-Funktion

→ hässlich!

Impulsoperator Noether-Theorem

$$T(\varepsilon)|\psi\rangle = \int_{-\infty}^{\infty} T(\varepsilon)|x\rangle\langle x|\psi\rangle$$

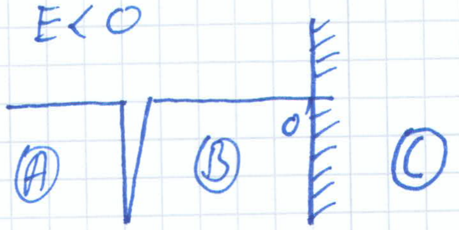
$$= \int_{-\infty}^{\infty} |x+\varepsilon\rangle\langle x|\psi\rangle = \int_{-\infty}^{\infty} |x\rangle\langle x-\varepsilon|\psi\rangle = \int_{-\infty}^{\infty} |x\rangle\psi(x-\varepsilon)$$

$$\psi(x-\varepsilon) = \psi(x) - \varepsilon\frac{\partial\psi}{\partial x} \Rightarrow \hat{T}(\varepsilon)\psi(x) = \left(1-\varepsilon\frac{\partial}{\partial x}\right)\psi(x)$$

$$\hat{T}(\varepsilon) = \left(1-\varepsilon\frac{d}{dx}\right) = 1 - \frac{i\varepsilon}{\hbar}\underbrace{\left(-i\hbar\frac{d}{dx}\right)}_{\hat{p}}$$

$$\boxed{\hat{T}(\varepsilon) = 1 - \frac{i}{\hbar}\varepsilon\hat{p}}$$ Generator

Delta-Peak vor einer Wand



a) Ansätze für A, B, C

C $\varphi_C = 0$

A $\varphi_A = A_1 e^{\kappa x} + A_2 e^{-\kappa x}$

B $\varphi_B = B_1 e^{\kappa x} + B_2 e^{-\kappa x}$

I $\varphi_C(0) = \varphi_B(0) = 0$

II Sprungbed. für $x = -x_0$

III $A_2 = 0$ (Normierbarkeit)

IV $-\frac{\hbar^2}{2m}\varphi'' - V_0\,\delta(x+x_0)\,\varphi(x) = E\,\varphi(x)$