David
Lars
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Gruppe 3

6/6

(6/14P)
6/6P

10.1) $\frac{1}{2}\vec{x}^T A \vec{x} - \vec{b}^T \vec{x} = f(\vec{x})$ $\quad \vec{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\quad A = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$

$= \frac{3}{2}x^2 + xy + y^2 - x - y$ $\qquad \vec{x} := \begin{pmatrix} x \\ y \end{pmatrix}$

$\vec{\nabla} f(x,y) = \begin{pmatrix} 3x + y - 1 \\ x + 2y - 1 \end{pmatrix}$ ok. $\qquad x_{k+1} = x_k - \alpha_k \vec{\nabla} f(\vec{x})$

$\alpha_k = \frac{\langle A\vec{x}_k - \vec{b}, \vec{\nabla} f(\vec{x}_k) \rangle}{\langle A \vec{\nabla} f(\vec{x}_k), \vec{\nabla} f(\vec{x}_k) \rangle}$ ✓

$x_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \alpha_0 = \frac{\langle -\begin{pmatrix} 1 \\ 1 \end{pmatrix}, -\begin{pmatrix} 1 \\ 1 \end{pmatrix} \rangle}{\langle -\begin{pmatrix} 4 \\ 3 \end{pmatrix}, -\begin{pmatrix} 1 \\ 1 \end{pmatrix} \rangle} = \frac{2}{7}$ ✓

$x_1 = \frac{2}{7}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow \alpha_1 = \frac{\langle \frac{1}{7}\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \frac{1}{7}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \rangle}{\langle \frac{1}{7}\begin{pmatrix} 2 \\ -1 \end{pmatrix}, \frac{1}{7}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \rangle} = \frac{2}{3}$ ✓

$x_2 = \frac{2}{7}\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \frac{2}{3} \cdot \frac{1}{7}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{4}{21}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ ✓ $\quad \alpha_2 = \frac{\frac{1}{21^2}\langle -\begin{pmatrix} 1 \\ 1 \end{pmatrix}, -\begin{pmatrix} 1 \\ 1 \end{pmatrix} \rangle}{\frac{1}{21^2}\langle -\begin{pmatrix} 4 \\ 3 \end{pmatrix}, -\begin{pmatrix} 1 \\ 1 \end{pmatrix} \rangle} = \frac{2}{7}$ ✓

$x_3 = \frac{4}{21}\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \frac{2}{7} \frac{1}{21}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{2}{21}\begin{pmatrix} 2 + 1/7 \\ 4 + 1/7 \end{pmatrix} = \frac{1}{147}\begin{pmatrix} 15 \\ 29 \end{pmatrix}$ ✓