Gruppe 3
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Numerik Blatt 2 10.[illegible] P/14 P.

2.1a) 0[illegible]/1

	Gleit komma zahl	Expon. 1	~~2~~	~~3~~	~~4~~	~~5~~	~~6~~	~~7~~	~~8~~
f	0.001	1/4	1/2	1	2	4	8	16	32
f	0.010	1/2	1	2	4	8	16	32	64
✓	0.100	1	2	4	8	16	32	64	128
f	0.011	3/4	3/2	3	6	12	24	48	96
✓	0.110	3/2	3	6	12	24	48	96	192
✓	0.111	7/4	7/2	7	14	28	56	112	224
✓	0.101								

Beachte Normierung... Erste Stelle nach Komma $\neq 0$.

Dual $\Rightarrow \boxed{0,1}$

b)

$x_1 = 0.1111 \cdot 2^{11}$

$= \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\right) \cdot 2^{11_2}$ (✓)

$= \frac{15}{16} \cdot 2^{11} = 1920$ f

$x_2 = 0.1101 \cdot 2^{-11}$

$= \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{16}\right) 2^{-11_2}$ (✓)

$= \frac{13}{16} \cdot 2^{-11}$ f

$\approx 0{,}000397$

3/3

2.2. $y^2 - 2x_1 y + x_2 = 0 \qquad x_1^2 > x_2$

$y = f(x_1, x_2) = x_1 - \sqrt{x_1^2 - x_2}$

Taylornäherung bei Störungen: $f(x_1+\Delta x_1, x_2+\Delta x_2) \approx \underbrace{\frac{\partial f}{\partial x_1} \frac{x_1}{f}}_{k_1=|\dots|} \frac{\Delta x_1}{x_1} + \underbrace{\frac{\partial f}{\partial x_2} \frac{x_2}{f}}_{k_2=|\dots|} \frac{\Delta x_1}{x_1}$

$k_1 = \left|\left(1 - \frac{x_1}{\sqrt{x_1^2 - x_2}}\right) \cdot \frac{x_1}{x_1 - \sqrt{x_1^2 + x_2}}\right| = \left|\frac{\sqrt{x_1^2 - x_2} - x_1}{\sqrt{x_1^2 - x_2}} \cdot \frac{x_1}{x_1 - \sqrt{x_1^2 - x_2}}\right|$

$= \left|-\frac{x_1}{\sqrt{x_1^2 - x_2}}\right| \stackrel{✓}{=} \frac{1}{\sqrt{1 - \frac{x_2}{x_1^2}}} \Rightarrow$ für $x_2 \approx x_1^2$ schlecht
" $x_2 = 0$ gut konditioniert "

$k_2 = \left|-\frac{1}{2} \cdot \frac{1}{\sqrt{x_1^2 - x_2}} \cdot \frac{x_2}{x_1 - \sqrt{x_1^2 - x_2}}\right| = \frac{1}{2} \cdot \frac{1}{\frac{x_1}{x_2}\sqrt{x_1^2 - x_2} - \frac{x_1^2}{x_2} + 1}$ ok.

für $x_2 = 0$ gut konditioniert

für $x_2 \approx x_1^2$ schlecht " ✓

1/2 2.3a) $\phi(x,y) = \begin{bmatrix} x+y \\ e^{2x/y} \end{bmatrix} \begin{matrix} \leftarrow \phi_1 \\ \leftarrow \phi_2 \end{matrix}$

$k_{11} = \frac{\partial \phi_1}{\partial x} \frac{x}{\phi_1} = 1 \cdot \frac{x}{x+y} = \frac{x}{x+y}$ ✓

$k_{12} = \frac{\partial \phi_1}{\partial y} \frac{y}{\phi_1} = \frac{y}{x+y}$ ✓

$k_{21} = x e^{x/y} \cdot \frac{x}{e^{x/y}} = x^2$ f $\frac{\partial \phi_2}{\partial x} \frac{x}{e^{2x/y}} = \frac{2x}{y}$

$k_{22} = 1$ f $\frac{\partial \phi_2}{\partial y} \frac{y}{e^{2x/y}} = -\frac{2x}{y}$

k_{22} ist immer gut konditioniert

k_{21} ist für $x<1$ " "

k_{12}, k_{11} sind für $x \ll y$ " "

0/2 b) 1. $f(x) = \frac{x}{y}$ $\left[k = \frac{df}{dx} \cdot \frac{x}{f} = \frac{1}{y} \cdot \frac{x}{x} \cdot y = 1 \right]$

$\frac{|\Delta f|}{|f|} = \left| \frac{f(\Delta x + x) - f(x)}{f(x)} \right| = \frac{\left(\frac{\Delta x + x}{y} - \frac{x}{y} \right)}{\frac{x}{y}} = \left| \frac{\Delta x}{x} \right| \leq \varepsilon$ f

$k_{11} = ?$
$k_{12} = ?$

$\Rightarrow \max \frac{|\Delta f|}{|f|} = \varepsilon$ mit $\alpha = 1$ 2

2. $f(x,y) = x^y$

Gauß

$\frac{|\Delta f|}{|f|} \leq \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$

$\leq \sqrt{\varepsilon^2 + \varepsilon^2} = \sqrt{2}\,\varepsilon$

$k_{11} = ?$
$k_{12} = ?$

warum

$\Rightarrow \max \frac{|\Delta f|}{|f|} = \sqrt{2}\,\varepsilon$ f mit $\alpha = \sqrt{2}$ $\frac{1}{2}$

0

$f(x) = 5\sin(3\pi x) + 36x^2 = y(x)$

$x_0 = 0 \quad x_1 = \frac{1}{12} \quad x_2 = \frac{1}{6} \qquad y(x_i) = y_i$

$L_i^{(n)}(x) = \prod_{\substack{j=0 \\ j\neq i}}^{n} \frac{x - x_j}{x_i - x_j}$

$y(x_0) = f(x_0) = 0 \quad y_1(x_1) = \frac{1}{\sqrt{2}} \cdot 5 + \frac{1}{4} = \frac{20+\sqrt{2}}{4\sqrt{2}} \quad y_2 = 6$

$L_0(x) = \left(\frac{x - \frac{1}{12}}{0 - \frac{1}{12}}\right)\left(\frac{x - \frac{1}{6}}{0 - \frac{1}{6}}\right) = (12x-1)(6x-1)$ ✓

$L_1(x) = \left(\frac{x-0}{\frac{1}{12}-0}\right)\cdot\left(\frac{x-\frac{1}{6}}{\frac{1}{12}-\frac{1}{6}}\right) = -12x(12x-2)$ ✓

$L_2(x) = \left(\frac{x-0}{\frac{1}{6}-0}\right)\left(\frac{x-\frac{1}{12}}{\frac{1}{6}-\frac{1}{12}}\right) = 6x(12x-1)$ ✓

$p(x) = y_0 L_0(x) + y_1 L_1(x) \ldots$

$= -12x(12x-2) \cdot \frac{20+\sqrt{2}}{4\sqrt{2}} + 36x(12x-1)$ ✓

$p\left(\frac{1}{24}\right) = -\frac{1}{2}\left(\frac{1}{2} - 2\right)\frac{20+\sqrt{2}}{4\sqrt{2}} + \frac{3}{2}\left(\frac{1}{2} - 1\right) = \frac{60+3\sqrt{2}}{16\sqrt{2}} - \frac{3}{4}$ ✓

$\approx 2{,}089$

2/2

b) $\max_{[0,\frac{1}{6}]} |f(x) - p(x)| = \frac{f^{(n+1)}(\xi)}{n+1!} \prod_{j=0}^{n} |\xi - x_j| \qquad n = 2$

$f'(x) = 15\pi\cos(3\pi x) + 72x$

$f''(x) = -45\pi^2\sin(3\pi x) + 72$

$f'''(x) = -135\pi^3\cos(3\pi x)$

$R(x) = \max_{[0,1/6]} |f(x) - p(x)| = \left| -\frac{135\pi^3}{6} \cos(3\pi\xi)\cdot\xi\cdot\left(\xi - \frac{1}{12}\right)\left(\xi - \frac{1}{6}\right)\right|$

$= \frac{45}{2}\pi^3 \cos(3\pi\xi)\cdot\xi\cdot\left(\xi - \frac{1}{12}\right)\left(\xi - \frac{1}{6}\right)$

$R'(x) = \frac{5\pi^3((216\pi x^3 - 54\pi x^2 + 3\pi x)\sin(3\pi x) + (-216x^2 + 36x - 1)\cos(3\pi x)}{\frac{1}{16}}$

$\stackrel{!}{=} 0 \xRightarrow{\text{Wolfram Alpha}} x_1 \approx 0{,}033 \quad x_3 = \frac{1}{6} \quad R(x_1) \approx 0{,}147 \quad R(x_3) = 0$

$x_2 \approx 0{,}117 \qquad R(x_2) \approx 0{,}062$

$f(x) = 5\sin(3\pi x) + 36x^2 = y(x)$

$x_0 = 0 \quad x_1 = \frac{1}{12} \quad x_2 = \frac{1}{6} \qquad y(x_i) = y_i$

$L_i^{(n)}(x) = \prod_{\substack{j=0 \\ j\neq i}}^{n} \frac{x - x_j}{x_i - x_j}$

$y(x_0) = f(x_0) = 0 \quad y_1(x_1) = \frac{1}{\sqrt{2}} \cdot 5 + \frac{1}{4} = \frac{20+\sqrt{2}}{4\sqrt{2}} \quad y_2 = 6$

$L_0(x) = \left(\frac{x - \frac{1}{12}}{0 - \frac{1}{12}}\right)\left(\frac{x - \frac{1}{6}}{0 - \frac{1}{6}}\right) = (12x-1)(6x-1)$ ✓

$L_1(x) = \left(\frac{x-0}{\frac{1}{12} - 0}\right) \cdot \left(\frac{x - \frac{1}{6}}{\frac{1}{12} - \frac{1}{6}}\right) = -12x(12x-2)$ ✓

$L_2(x) = \left(\frac{x-0}{\frac{1}{6} - 0}\right)\left(\frac{x - \frac{1}{12}}{\frac{1}{6} - \frac{1}{12}}\right) = 6x(12x-1)$ ✓

$p(x) = y_0 L_0(x) + y_1 L_1(x) \dots$

$= -12x(12x-2) \cdot \frac{20+\sqrt{2}}{4\sqrt{2}} + 36x(12x-1)$ ✓

$p(\frac{1}{24}) = -\frac{1}{2}\left(\frac{1}{2} - 2\right)\frac{20+\sqrt{2}}{4\sqrt{2}} + \frac{3}{2}\left(\frac{1}{2} - 1\right) = \frac{60+3\sqrt{2}}{16\sqrt{2}} - \frac{3}{4}$ ✓

$\approx 2{,}089$

2/2 b) $\max_{[0,\frac{1}{6}]} |f(x) - p(x)| = \frac{f^{(n+1)}(\xi)}{n+1!} \prod_{j=0}^{2} |\xi - x_j| \qquad n = 2$

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$f''(x) = -45\pi^2\sin(3\pi x) + 72$

$f'''(x) = -135\pi^3\cos(3\pi x)$

$R(x)$ $\max_{[0,1/6]} |f(x) - p(x)| = \left| -\frac{135\pi^3}{6}\cos(3\pi\xi) \cdot \xi \cdot \left(\xi - \frac{1}{12}\right)\left(\xi - \frac{1}{6}\right)\right|$

$= \frac{45}{2}\pi^3 \cos(3\pi\xi) \cdot \xi \cdot \left(\xi - \frac{1}{12}\right)\left(\xi - \frac{1}{6}\right)$

$R'(x) = \frac{5\pi^3\left((216\pi x^3 - 54\pi x^2 + 3\pi x)\sin(3\pi x) + (-216x^2 + 36x - 1)\cos(3\pi x)\right)}{\frac{1}{16}}$

α

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$x_2 \approx 0{,}117 \quad R(x_2) \approx 0{,}062$