Nr. 16 a) $\delta S = \delta \int_0^\ell E(q, q^{(n)}, x)\,dx = \int_0^\ell \delta E(q, q^{(n)}, x)\,dx$

$$\delta E = \frac{\partial E}{\partial q}\delta q + \frac{\partial E}{\partial q^{(n)}}\delta q^{(n)} + \underbrace{\frac{\partial E}{\partial x}\delta x}_{=0}$$

$$\Rightarrow \delta S = \int_0^\ell \left(\frac{\partial E}{\partial q}\varepsilon + \frac{\partial E}{\partial q^{(n)}}\varepsilon^{(n)}\right)dx$$

NR: $$\int_0^\ell \frac{\partial E}{\partial q^{(n)}}\varepsilon^{(n)}dx = \left[\frac{\partial E}{\partial q^{(n)}}\varepsilon^{(n-1)}\right]_0^\ell - \left[\frac{d}{dt}\left(\frac{\partial E}{\partial q^{(n)}}\right)\varepsilon^{(n-2)}\right]_0^\ell + \ldots + \left[(-1)^{n-1}\frac{d^{n-1}}{dx^{n-1}}\left(\frac{\partial E}{\partial q^{(n)}}\right)\varepsilon\right]_0^\ell$$

$$+(-1)^n\int_0^\ell \frac{d^n}{dx^n}\left(\frac{\partial E}{\partial q^{(n)}}\right)\varepsilon\,dx = \underbrace{\sum_{i=1}^{n}\left[\frac{d^{n-i}}{dx^{n-i}}\left(\frac{\partial E}{\partial q^{(n)}}\right)\varepsilon^{(i-1)}(-1)^{n-i}\right]_0^\ell}_{\overset{!}{=}\,0\ \text{(Bedingung)}}$$ ($\checkmark$)

$$+(-1)^n\int_0^\ell \frac{d^n}{dx^n}\left(\frac{\partial E}{\partial q^{(n)}}\right)\varepsilon\,dx$$

$$\Rightarrow \delta S = \int_0^\ell \left(\frac{\partial E}{\partial q}\varepsilon + (-1)^n\frac{d^n}{dx^n}\frac{\partial E}{\partial q^{(n)}}\varepsilon\right)dx \overset{!}{=} 0$$

da ε beliebig

$$\Rightarrow \frac{\partial E}{\partial q} + (-1)^n\frac{d^n}{dx^n}\frac{\partial E}{\partial q^{(n)}} = 0$$ ✓ (sorry, dachte das " wäre ein n)

abe[r] gauch cool das mal gesehen zu haben.

mit $n=2$: $$\frac{\partial E}{\partial q} + \frac{d^2}{dx^2}\frac{\partial E}{\partial q''} = 0$$ ✓

$$\wedge\ \varepsilon\frac{dx}{dt}\left(\frac{\partial E}{\partial q''}\right)\Big|_0^\ell = \varepsilon'\frac{\partial E}{\partial q''}\Big|_0^\ell$$ ✓

b) $E = -\frac{k}{2}q''^2 + q f(x) \Rightarrow \frac{\partial E}{\partial q} = f(x),\ \frac{\partial E}{\partial q''} = -kq'' \Rightarrow \frac{d^2}{dx^2}\frac{\partial E}{\partial q''} = -kq^{(4)}(x)$

$$\Rightarrow f(x) - kq^{(4)}(x) = 0 \Rightarrow q^{(4)}(x) = \frac{f(x)}{k}$$ ✓

$$\wedge\left[\varepsilon k q^{(3)}(x)\right]_0^\ell = \left[-\varepsilon' k q^{(2)}(x)\right]_0^\ell \Leftrightarrow \left[\varepsilon q^{(3)}(x)\right]_0^\ell = \left[\varepsilon' q^{(2)}(x)\right]_0^\ell$$ ✓

c) $f(x) = -\mathcal{S}g \Rightarrow q^{(4)}(x) = \underbrace{-\frac{\mathcal{S}g}{k}}_{\lambda} \Rightarrow q(x) = \frac{\lambda x^4}{24} + c_1x^3 + c_2x^2 + c_3x + c_4$ ✓

mit $\underbrace{q(0) = q'(0) = 0}_{\Rightarrow\ \varepsilon(0) = \varepsilon'(0) = 0} \Rightarrow q(x) = \frac{\lambda x^4}{24} + c_1x^3 + c_2x^2 \quad (c_3 = c_4 = 0)$ ✓

hiermit kann man noch c_1 und c_2 bestimmen!

4/5