1	2	3	4
3.5	5	5	3

16,5/20

Jonas
David
Lois

Aufgabe 8

a) $\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} r\cos\varphi \\ r\sin\varphi \\ r\cot\alpha \end{pmatrix}$ also

$x = r\cos\varphi$ ✓
$y = r\sin\varphi$ ✓
$z = r\cot\alpha$ ← nur da ist Zwangsbedingung (✓)

b) $L = T - V$ ✓ $\quad T = \frac{1}{2} m \dot{\vec{r}}^2$ ✓ $\quad V = mgr\cot\alpha$ ✓

$$\dot{\vec{r}} = \begin{pmatrix} \dot{r}\cos\varphi - r\dot{\varphi}\sin\varphi \\ \dot{r}\sin\varphi + r\dot{\varphi}\cos\varphi \\ \dot{r}\cot\alpha \end{pmatrix} \checkmark$$

$$\dot{\vec{r}}^2 = \dot{r}^2\cos^2\varphi - 2\dot{r}r\dot{\varphi}\cos\varphi\sin\varphi + r^2\dot{\varphi}^2\sin^2\varphi + \dot{r}^2\sin^2\varphi + 2\dot{r}r\dot{\varphi}\cos\varphi\sin\varphi + r^2\dot{\varphi}^2\cos^2\varphi + \dot{r}^2\cot^2\alpha$$

$$= (1+\cot^2\alpha)\dot{r}^2 + r^2\dot{\varphi}^2 \checkmark$$

$$\Rightarrow L = \frac{1}{2} m \left[(1+\cot^2\alpha)\dot{r}^2 + r^2\dot{\varphi}^2\right] - mgr\cot\alpha \checkmark$$

1. DGL: $\frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{\varphi}} - \frac{\partial L}{\partial \varphi} = 0$

$$= \frac{\partial}{\partial t}\left[mr^2\dot{\varphi}^2\right] = mr(2\dot{r}\dot{\varphi} + r\ddot{\varphi}) = 0 \checkmark$$

2. DGL: $\frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{r}} - \frac{\partial L}{\partial r} = 0$

$$= \frac{\partial}{\partial t}\left[m(1+\cot^2\alpha)\dot{r}\right] - \left[mr\dot{\varphi}^2 - mg\cot\alpha\right]$$

$$= m(1+\cot^2\alpha)\ddot{r} - mr\dot{\varphi}^2 + mg\cot\alpha = 0 \quad (\checkmark)$$