Aufgabe 1.1.

$0.100 \cdot 2^0 = 0.5$
$0.101 \cdot 2^0 = 0{,}625$
$0.110 \cdot 2^0 = 0.75$
$0.111 \cdot 2^0 = 0{,}875$

$0.100 \cdot 2^1 = 1$
$0.101 \cdot 2^1 = 1{,}25$
$0.110 \cdot 2^1 = 1{,}5$
$0.111 \cdot 2^1 = 1{,}75$

b) $x_1 = 0.1111 \cdot 2^{11}$ $\qquad x_2 = 0{,}1101 \cdot 2^{-11}$

$x_1 = (2^{-1} + 2^{-2} + 2^{-3} + 2^{-4}) \cdot 2^{11_2}$
$= (0{,}5 + 0{,}25 + 0{,}125 + 0{,}0625) \cdot 2^3 = 7{,}5$

$x_2 = (\frac{1}{2} + \frac{1}{4} + \frac{1}{16}) \cdot 2^{-3} = 0{,}10$

2.3

b) $\varepsilon = \left|\frac{\Delta x}{x}\right|$ $\qquad \Delta f(x) = f(x + \Delta x) - f(x)$
$= f(x) - f(x) + \frac{\Delta x}{1!} \cdot \frac{\partial f}{\partial x} \ldots$

$\left|\frac{\Delta f(x)}{f}\right| = |K_1 \varepsilon + \ldots| \leq \varepsilon |K_1| + O(\varepsilon^2)$

$\Delta f(x,y) = f(x + \Delta x, y + \Delta y) - f(x,y)$

$\Rightarrow \left|\frac{\Delta f(x,y)}{f}\right| \leq \varepsilon \cdot |K_{11}| + \varepsilon |K_{22}| + O(\varepsilon^2)$

~~$\mathcal{H}_{11}$~~

$$f(x,y) = \frac{x}{y} \qquad y \neq 0$$

$$K_{11} = \frac{\partial f}{\partial x} \cdot \frac{x}{f(x,y)} = 1$$

Immer gut konditioniert

$$K_{12} = \frac{\partial f}{\partial y} \frac{y}{f} = -1$$

$$\Rightarrow \left|\frac{\Delta f}{f}\right| = \left|\sum_{ij} k_{ij} \varepsilon\right| \leq \varepsilon \cdot 1 + \varepsilon \cdot 1 + O(\varepsilon^2)$$
$$= 2\varepsilon + O(\varepsilon^2)$$

2) $f(x,y) = x^y$

$$K_{11} = \frac{\partial f}{\partial x} \frac{x}{x^y} = y \qquad K_{12} = \frac{\partial f}{\partial y} \frac{y}{x^y} = y \ln(x)$$

$$\left|\frac{\Delta f}{f}\right| \leq \varepsilon \cdot |K_{11}| + \varepsilon |K_{12}| + O(\varepsilon^2)$$
$$= |y|\varepsilon + |y \ln x|\varepsilon + O(\varepsilon^2)$$

Fehlerdämpfung für $|y| < 1$ & $e^{-\frac{1}{y}} < x < e^{\frac{1}{y}}$

schlecht Kond.: $|y| >> 1 \quad x > e^{\frac{1}{|y|}} \quad x < e^{-\frac{1}{y}}$

$y \to \frac{1}{2}$

$$K_1 = \frac{\partial f}{\partial x} \cdot \frac{x}{x^{1/2}} = \frac{1}{2} \Rightarrow \left|\frac{\Delta f}{f}\right| \leq \frac{1}{2}\varepsilon + O(\varepsilon^2$$