TUS

Nr. 1a) ④ 2

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$df = y\,dx + x\,dy$

α) $\frac{\partial y}{\partial y} = 1 = \frac{\partial x}{\partial x} \to$ totales Dif. ✓

$\int y\,dx = xy + C(y)$

$\int x\,dy = xy + C(x) \to f(x,y) = xy + c$ ✓

β) $\frac{\partial y}{\partial y} = 1 \quad \frac{\partial(-x)}{\partial x} = -1$ ↯ kein vollst. Dif ✓

γ) $\frac{\partial(xy)}{\partial y} = x = \frac{\partial\left(\frac{1}{2}(x^2+y^2)\right)}{\partial x} \to$ vollst. ✓

$\int xy\,dx = \frac{x^2}{2}y + C(y)$

$\int \frac{x^2+y^2}{2}\,dy = \frac{x^2 y}{2} + \frac{y^3}{6} + C(x)$

$\to f = \frac{x^2 y}{2} + \frac{y^3}{6} + c$ ✓

δ)

$$\frac{\partial\left(\frac{\sin(2x)}{y^2+\sin^2 x+1}\right)}{\partial y} = -\frac{2\sin(2x)\,y}{(y^2+\sin^2 x+1)^2} = -\frac{4y\sin x\cos x}{(y^2+\sin^2 x+1)^2}$$ ✓

$$\frac{\partial\left(\frac{2y}{y^2+\sin^2 x+1}\right)}{\partial x} = -\frac{4y\sin x\cos x}{(y^2+\sin^2 x+1)^2} \to \text{vollst.}$$

Sehr schön!

$f = \int \frac{2\sin x\cos x}{y^2+\sin^2 x+1}\,dx \quad \Big|\; u = y^2+\sin^2 x+1, \; du = \frac{1}{2\sin x\cos x}\,dx$

$= \int \frac{1}{u}\,du = \ln(y^2+\sin^2 x+1) + C$ ✓

b) 2

$df = hy\,dx - hx\,dy$

$\frac{\partial(hy)}{\partial y} = -\frac{\partial(hx)}{\partial x}$

$h_y y + h = -h_x x - h$

$h = -\frac{1}{2}(h_x x + h_y y) \to h = \frac{1}{x^a}\frac{1}{y^b}\cdot c$

$\therefore\; c\frac{1}{x^a y^b} = -\frac{1}{2}\left(-\frac{a}{x^a y^b} - \frac{b}{x^a y^b}\right)c \Leftrightarrow a+b = 2 \quad (I)$ ✓

h ist nicht eindeutig, da sämtliche Kombinationen von a und b zu einem vollständigen Dif. führen, die (I) erfüllen. Außerdem kann h auch um einen konstanten Faktor c abweichen. ✓ es gibt sogar noch weitere Lösungen als die genannten

2 α) $\int df = \int y\,dx + x\,dy = \int \begin{pmatrix} y \\ x \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix} = \int \vec{g}\, d\vec{r}$

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wieso das VZ ?

i) $\vec{r}(t) = \begin{pmatrix} x_0 \cos(-t) \\ y_0 \sin(-t) \end{pmatrix}$ $\quad \dot{\vec{r}}(t) = \begin{pmatrix} x_0 \sin(-t) \\ -y_0 \cos(-t) \end{pmatrix}$ $\quad t \in [0, \pi/2]$

$\Rightarrow \int_0^{\pi/2} \begin{pmatrix} y_0 \sin(-t) \\ x_0 \cos(-t) \end{pmatrix} \begin{pmatrix} x_0 \sin(-t) \\ -y_0 \cos(-t) \end{pmatrix} dt = x_0 y_0 \int_0^{\pi/2} \sin^2(-t) - \cos^2(-t)\, dt$

$= 0$ (✓)

ii) $\vec{r}(t) = \begin{pmatrix} x_0 - \frac{x_0}{y_0} t \\ t \end{pmatrix}$ ✓ $\quad t \in [0, y_0]$

$\dot{\vec{r}}(t) = \begin{pmatrix} -\frac{x_0}{y_0} \\ 1 \end{pmatrix}$

$\Rightarrow \int_0^{y_0} \begin{pmatrix} t \\ x_0 - \frac{x_0}{y_0} t \end{pmatrix} \begin{pmatrix} -\frac{x_0}{y_0} \\ 1 \end{pmatrix} dt = \int_0^{y_0} x_0 - 2 \frac{x_0}{y_0} t \, dt = \quad 0$ ✓

iii) $\vec{c}_1(t) = \begin{pmatrix} x_0 - x_0 t \\ 0 \end{pmatrix}$ ✓ $\quad \vec{c}_2(t) = \begin{pmatrix} 0 \\ y_0 t \end{pmatrix}$ ✓

$\dot{\vec{c}}_1(t) = \begin{pmatrix} -x_0 \\ 0 \end{pmatrix}$ $\quad \dot{\vec{c}}_2(t) = \begin{pmatrix} 0 \\ y_0 \end{pmatrix}$ $\quad t \in [0, 1]$

$\Rightarrow \int_0^1 \begin{pmatrix} 0 \\ x_0 - x_0 t \end{pmatrix} \begin{pmatrix} -x_0 \\ 0 \end{pmatrix} dt + \int_0^1 \begin{pmatrix} y_0 t \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ y_0 \end{pmatrix} dt = 0$ ✓

β) $\vec{g} = \begin{pmatrix} y \\ -x \end{pmatrix} \Rightarrow$ i) $\int_0^{\pi/2} \begin{pmatrix} y_0 \sin(-t) \\ -x_0 \cos(-t) \end{pmatrix} \begin{pmatrix} x_0 \sin(-t) \\ -y_0 \cos(-t) \end{pmatrix} dt = \frac{x_0 y_0 \pi}{2} \cdot (-1)$

ich glaube, dieses Vorzeichen ist falsch

ii) $\int_0^{y_0} \begin{pmatrix} t \\ -x_0 + \frac{x_0 t}{y_0} \end{pmatrix} \begin{pmatrix} -\frac{x_0}{y_0} \\ 1 \end{pmatrix} dt = -x_0 y_0$ ✓

iii) $\int_0^1 \begin{pmatrix} 0 \\ -x_0 + x_0 t \end{pmatrix} \begin{pmatrix} -x_0 \\ 0 \end{pmatrix} dt + \int_0^1 \begin{pmatrix} y_0 t \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ y_0 \end{pmatrix} dt = 0$ ✓

schön!

Nr. 3 (4)

a) $$\frac{d}{dx}\psi(x,y,z)\Big|_{\psi,z} = \frac{\partial\psi}{\partial x}\Big|_{y,z} + \frac{\partial y}{\partial x}\Big|_{\psi,z}\frac{\partial\psi}{\partial y}\Big|_{x,z} + \underbrace{\frac{\partial z}{\partial x}\Big|_{\psi,z\to\psi,y}\frac{\partial\psi}{\partial z}\Big|_{x,y}}_{\text{verschwindet bei festgehaltenem } z} = 0$$

$$\Rightarrow 0 = \frac{\partial\psi}{\partial x}\Big|_{y,z} + \frac{\partial y}{\partial x}\Big|_{\psi,z}\frac{\partial\psi}{\partial y}\Big|_{x,z}$$

$$\Rightarrow \frac{\partial y}{\partial x}\Big|_{\psi,z} = \frac{-\frac{\partial\psi}{\partial x}\big|_{y,z}}{\frac{\partial\psi}{\partial y}\big|_{x,z}} \quad \square$$

b) analog mit $\frac{d}{dy}\psi(x,y,z) \Rightarrow 0 = \frac{\partial\psi}{\partial y}\Big|_{x,z} + \frac{\partial x}{\partial y}\Big|_{\psi,z}\frac{\partial\psi}{\partial x}\Big|_{y,z}$

$$\Rightarrow \frac{\partial x}{\partial y}\Big|_{\psi,z} = \frac{-\frac{\partial\psi}{\partial y}\big|_{x,z}}{\frac{\partial\psi}{\partial x}\big|_{y,z}} = \frac{1}{\frac{\partial y}{\partial x}\big|_{\psi,z}} \quad \square$$

c) aus a) mit festgehaltenem y: $\frac{\partial z}{\partial x}\Big|_{\psi,y} = \frac{-\frac{\partial\psi}{\partial x}\big|_{y,z}}{\frac{\partial\psi}{\partial z}\big|_{x,y}}$

Analog mit $\frac{d}{dz}\psi(x,y,z) \Rightarrow \frac{\partial y}{\partial z}\Big|_{\psi,x} = \frac{-\frac{\partial\psi}{\partial z}\big|_{x,y}}{\frac{\partial\psi}{\partial y}\big|_{x,z}}$

(festgehaltenes x)

siehe b)

$$\Rightarrow \frac{\partial x}{\partial y}\Big|_{\psi,z}\frac{\partial y}{\partial z}\Big|_{\psi,x}\frac{\partial z}{\partial x}\Big|_{\psi,y} = \frac{\partial x}{\partial y}\Big|_{\psi,z}\cdot\frac{\frac{\partial\psi}{\partial x}\big|_{y,z}}{\frac{\partial\psi}{\partial y}\big|_{x,z}} = \frac{\partial x}{\partial y}\Big|_{\psi,z}\cdot\frac{-1}{\frac{\partial x}{\partial y}\big|_{\psi,z}}$$

$$= -1 \quad \square$$

gut!

A1	A2	A3	Ges
4	4	4	12

3a) Alternativ: $d\psi = \frac{\partial\psi}{\partial x}\Big|_{y,z}dx + \frac{\partial\psi}{\partial y}\Big|_{x,z}dy + \frac{\partial\psi}{\partial z}\Big|_{x,y}dz$

$$\to \frac{d\psi}{dx}$$