15) $S = \int_0^{t_0} L\,dt = \int_0^{t_0} \frac{m}{2}\dot{x}^2 + mgx\,dt$ ✓

$$= \int_0^{t_0} \left(\frac{m}{2} \frac{\alpha^2 g^2 t_0^4}{4 t_0^{2\alpha}} t^{2\alpha-2} + \frac{mg^2}{2} \frac{t_0^2}{t_0^\alpha} t^\alpha \right) dt$$

$$= \frac{m}{8} \frac{\alpha^2 g^2 t_0^4}{t_0^{2\alpha}} \int_0^{t_0} t^{2\alpha-2}\,dt + \frac{mg^2 t_0^2}{2 t_0^\alpha} \int_0^{t_0} t^\alpha\,dt$$

$$= \frac{mg^2 t_0^3}{2} \left(\frac{\alpha^2}{8\alpha - 4} + \frac{1}{\alpha+1} \right)$$ ✓

$$\frac{dS}{d\alpha} = \frac{mg^2 t_0^3}{2} \left(\frac{\alpha^2 - \alpha}{2(2\alpha-1)^2} - \frac{1}{(\alpha+1)^2} \right) \stackrel{!}{=} 0$$ ✓

$\Leftrightarrow \quad \alpha(\alpha-1)(\alpha+1)^2 = 2(2\alpha-1)^2$ ✓

$\Leftrightarrow \quad \alpha^4 + \alpha^3 - 9\alpha^2 + 7\alpha - 2 = 0$ ✓

Polynomdivision:

	1	1	-9	7	-2	
(2)	-	2	6	-6	2	$\Rightarrow \alpha_1 = 2$
	1	3	-3	1	(0)	✓

Wolfram-Alpha liefert nur eine weitere reelle Lös: $\alpha_2 = -1 - \sqrt[3]{2} - 2^{2/3}$ ✓, aber da $\alpha_2 < 1$ ✓ ist sie uninteressant,

$\frac{d^2 S}{d\alpha^2}(\alpha = 2) = \frac{5}{54} > 0 \Rightarrow$ Minimum ✓

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