Aufgabe 28

$$\mathcal{L} = \frac{1}{2}\mu(x)\left(\frac{\partial u(x,t)}{\partial t}\right)^2 - \frac{1}{2}F_0\left(\frac{\partial u(x,t)}{\partial x}\right)^2$$

$$\mathcal{H} = \frac{1}{2}\mu(x)\left(\frac{\partial u(x,t)}{\partial t}\right)^2 + \frac{1}{2}F_0\left(\frac{\partial u(x,t)}{\partial x}\right)^2$$

a) Zeitliche Änderung der Energiedichte:

$$\frac{\partial \mathcal{H}}{\partial t} = \frac{\partial u(x,t)}{\partial t}\mu(x)\frac{\partial^2 u(x,t)}{\partial t^2} + F_0\frac{\partial}{\partial t}\left(\frac{\partial u(x,t)}{\partial x}\right)\frac{\partial u(x,t)}{\partial x}$$

Wellengleichung aus Lagrangedichte: $u(x,t)=u$; $\mu(x)=\mu$

$$\frac{\partial\mathcal{L}}{\partial u}=0 \qquad \frac{\partial\mathcal{L}}{\partial\frac{\partial\phi}{\partial x}} = -E\frac{\partial\phi}{\partial x} \qquad \frac{\partial\mathcal{L}}{\partial\frac{\partial u}{\partial t}} = \mu\frac{\partial\mathcal{U}}{\partial t}$$

?

müsste ein u. sein ...

$$\Rightarrow F_0\frac{\partial^2 u}{\partial x^2} - \mu\frac{\partial^2 u}{\partial t^2} = 0 \iff F_0\frac{\partial^2 u}{\partial x^2} = \mu\frac{\partial^2 u}{\partial t^2}$$

Einsetzen in $\frac{\partial\mathcal{H}}{\partial t}$:

$$\frac{\partial\mathcal{H}}{\partial t} = \frac{\partial u}{\partial t}F_0\frac{\partial^2 u}{\partial x^2} + F_0\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right)\frac{\partial u}{\partial x}$$

$$\iff \frac{\partial\mathcal{H}}{\partial t} - F_0\left[\frac{\partial u}{\partial t}\frac{\partial^2 u}{\partial x^2} + \frac{\partial}{\partial t}\left[\frac{\partial u}{\partial x}\right]\frac{\partial u}{\partial x}\right] = 0$$

$$\frac{\partial\mathcal{H}}{\partial t} + \underbrace{\left(-F_0\left[\frac{\partial u}{\partial t}\frac{\partial^2 u}{\partial x^2} + \frac{\partial}{\partial t}\left[\frac{\partial u}{\partial x}\right]\frac{\partial u}{\partial x}\right]\right)}_{\frac{\partial j}{\partial x}} = 0$$

$$\Rightarrow j(x) = -\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial t}\frac{\partial u}{\partial x}\right)F_0$$

b) $$F_0\frac{\partial^2 u}{\partial x^2} - \mu\frac{\partial^2 u}{\partial t^2} = 0 = F_0\frac{\partial^2 u}{\partial(x-c_1t)^2}\frac{\partial(x-c_1t)^2}{\partial x^2} - \mu\frac{\partial^2 u}{\partial(x-c_1t)^2}\frac{\partial(x-c_1t)^2}{\partial t^2}$$

$$= u''\left[F_0 - \mu c_1^2\right] = 0 \iff c_1 = \pm\sqrt{\frac{F_0}{\mu}}$$

BRUNNEN