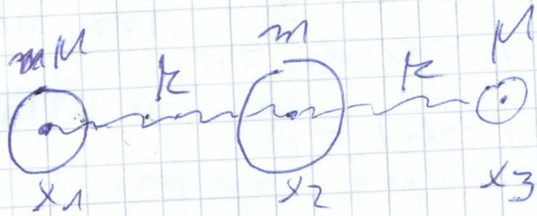
24. Aufgabe



$$V = \frac{1}{2} k \left[(x_1 - x_2)^2 + (x_3 - x_2)^2 \right]$$ ✓

$$L = T - V$$

$$T = \frac{1}{2} M (\dot{x}_1^2 + \dot{x}_3^2) + \frac{1}{2} m \dot{x}_2^2$$ ✓

$$L = \frac{1}{2} M (\dot{x}_1^2 + \dot{x}_3^2) + \frac{1}{2} m \dot{x}_2^2 - \frac{1}{2} k \left[(x_1 - x_2)^2 + (x_3 - x_2)^2 \right]$$

$$= \frac{1}{2M} (p_1^2 + p_3^2) + \frac{1}{2m} p_2^2 - \frac{1}{2} k \left[(x_1 - x_2)^2 + (x_3 - x_2)^2 \right],$$ ✓

da $p_i = \frac{\partial L}{\partial \dot{x}_i} = m \dot{x}_i \Leftrightarrow \dot{x}_i = \frac{p_i}{m_i}$

$$H = \sum_{i=1}^{3} \dot{x}_i p_i - L$$

$$= \frac{1}{2M} (p_1^2 + p_3^2) + \frac{1}{2m} p_2^2 + \frac{1}{2} k \left[(x_1 - x_2)^2 + (x_3 - x_2)^2 \right]$$ ✓

$$\dot{p}_i = -\frac{\partial H}{\partial x_i} : \quad \dot{p}_1 = -k (x_1 - x_2)$$

$$\dot{p}_2 = +k \left[(x_1 - x_2) + (x_3 - x_2) \right]$$

$$= +k x_1 + k x_3 - 2 k x_2 = +k \left[x_1 - 2 x_2 + x_3 \right]$$

$$\dot{p}_3 = -k (x_3 - x_2)$$

$$\dot{x}_i = \frac{\partial H}{\partial p_i} : \quad \dot{x}_1 = \frac{p_1}{m} ; \quad \dot{x}_2 = \frac{p_2}{m} ; \quad \dot{x}_3 = \frac{p_3}{m}$$

(Ihr hättet natürlich auch einfach direkt die Lagrangegleichung nehmen können)

Berechne $\ddot{x}_i$:

$$\ddot{x}_1 = \frac{\dot{p}_1}{m} = -\frac{k}{m} (x_1 - x_2)$$

$$\ddot{x}_2 = \frac{\dot{p}_2}{m} = +\frac{k}{m} (x_1 - 2x_2 + x_3)$$

$$\ddot{x}_3 = \frac{\dot{p}_3}{m} = -\frac{k}{m} (x_3 - x_2)$$ ✓