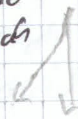
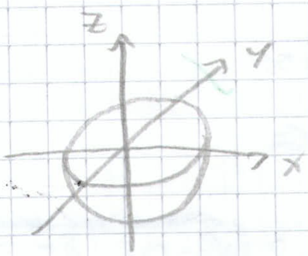
kein Bleistift!

Lars David Jonda



…l, Nr. 8

a)



$\vec{a}\cos\alpha\, |\vec{b}|$

$\cos\alpha = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

$\cos\theta = \frac{Z}{mg}$

Zwangsbedingung: $x^2+y^2+z^2=R^2$ bzw.: $r=R$ ✓

$$\vec{r} = \begin{pmatrix} R\cos\varphi\sin\theta \\ R\sin\varphi\sin\theta \\ R\cos\theta \end{pmatrix} = R\underbrace{\begin{pmatrix} \cos\varphi\sin\theta \\ \sin\varphi\sin\theta \\ \cos\theta \end{pmatrix}}_{\vec{e_r}}$$

$$\vec{e_\varphi} = \begin{pmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{pmatrix} \quad \vec{e_\theta} = \begin{pmatrix} \cos\varphi\cos\theta \\ \sin\varphi\cos\theta \\ -\sin\theta \end{pmatrix} ✓$$

$\dot{\vec{e_r}}(\varphi,\theta) = \sin\theta\dot\varphi\vec{e_\varphi} + \dot\theta\vec{e_\theta}$ ✓

b) $\vec{F} = \vec{K}+\vec{Z} \Rightarrow m\ddot{\vec{r}} = -mg\vec{e_z} - \lambda\vec{e_r}$

$m\ddot{\vec{r}} = -mg\vec{e_z} - \lambda\vec{e_r} \Rightarrow \ddot{\vec{r}} = g\vec{e_z} - \tilde\lambda\vec{e_r}$ mit $\tilde\lambda = \frac{\lambda}{m}$ ✓

c) $$\dot{\vec{e_r}} = \begin{pmatrix} -\sin\varphi\dot\varphi\sin\theta + \cos\theta\dot\theta\cos\varphi \\ \cos\varphi\dot\varphi\sin\theta + \sin\varphi\cos\theta\dot\theta \\ -\sin\theta\dot\theta \end{pmatrix} = \sin\theta\dot\varphi\vec{e_\varphi} + \dot\theta\vec{e_\theta}$$

$\ddot{\vec{e_r}} = (\cos\theta\dot\varphi\dot\theta + \ddot\varphi\sin\theta)\vec{e_\varphi} + (\sin\theta\dot\varphi)\dot{\vec{e_\varphi}} + \ddot\theta\vec{e_\theta} + \dot\theta\dot{\vec{e_\theta}}$ ✓

$$\dot{\vec{e_\varphi}} = \begin{pmatrix} -\cos\varphi\dot\varphi \\ -\sin\varphi\dot\varphi \\ 0 \end{pmatrix} = -\dot\varphi\begin{pmatrix} \cos\varphi \\ \sin\varphi \\ 0 \end{pmatrix} = -\dot\varphi(\sin\theta\vec{e_r} + \cos\theta\vec{e_\theta})$$ ✓

$$\dot{\vec{e_\theta}} = \begin{pmatrix} -\sin\varphi\dot\varphi\cos\theta - \sin\theta\dot\theta\cos\varphi \\ \cos\varphi\dot\varphi\cos\theta - \sin\theta\dot\theta\sin\varphi \\ -\cos\theta\dot\theta \end{pmatrix} ✓$$

$= \dot\varphi\cos\theta\vec{e_\varphi} - \dot\theta\vec{e_r}$

$\Rightarrow \ddot{\vec{e_r}} = (\cos\theta\dot\varphi\dot\theta + \ddot\varphi\sin\theta)\vec{e_\varphi} + (\sin\theta\dot\varphi) - \dot\varphi(\sin\theta\vec{e_r} + \cos\theta\vec{e_\theta}) + \ddot\theta\vec{e_\theta} + \dot\theta\dot\varphi\cos\theta\vec{e_\varphi} - \dot\theta^2\vec{e_r}$

$= \underbrace{(\cos\theta\dot\varphi\dot\theta + \ddot\varphi\sin\theta + \dot\theta\dot\varphi\cos\theta)}_{2\cos\theta\dot\varphi\dot\theta + \sin\theta\ddot\varphi}\vec{e_\varphi} - (\sin^2\theta\dot\varphi^2 + \dot\theta^2)\vec{e_r} + (\ddot\theta - \dot\varphi^2\cos\theta\sin\theta)\vec{e_\theta}$

$\vec{e_z} = \cos\theta\vec{e_r} - \sin\theta\vec{e_\theta}$

$\ddot{\vec{r}} = R\ddot{\vec{e_r}}$