Physik IV 15.5.18

Aufgabe 18

$\lambda_1 = \langle 1|\Phi\rangle = \frac{1}{3\sqrt{10}}(2+2-2) = \frac{2}{3\sqrt{10}}$

$\lambda_2 = \langle 2|\Phi\rangle = \frac{2}{\sqrt{10}}$

$\lambda_3 = \langle 3|\Phi\rangle = \frac{1}{6\sqrt{5}}$

$\lambda_4 = \langle 4|\Phi\rangle$

$$\hat{P}_1 = |1\rangle\langle 1| = \frac{1}{9}\begin{pmatrix}2\\1\\-1\\0\end{pmatrix}(2\ 1\ -1\ 0) = \frac{1}{9}\begin{pmatrix}4 & 2 & -4 & 0\\ 2 & 1 & -2 & 0\\ -4 & -2 & 4 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}$$

$\hat{P}_1^2 = P_1 \qquad U_1 = \text{span}\{|1\rangle\} = \{\alpha|1\rangle,\ \alpha \in \mathbb{C}\}$

c) $R_{nm} = \langle n|\hat{R}|m\rangle = m\langle n|m\rangle = m\,\delta_{nm} = \begin{cases} m & n = m \\ 0 & \text{sonst}\end{cases}$

$$\hat{R} = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 2 & 0 & 0\\ 0 & 0 & 3 & 0\\ 0 & 0 & 0 & 4\end{pmatrix} \qquad \hat{R}^2 = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 4 & 0 & 0\\ 0 & 0 & 9 & 0\\ 0 & 0 & 0 & 16\end{pmatrix}$$

Aufgabe 19

a) $|\psi\rangle = \sum c_i|a_i\rangle \Leftrightarrow \langle a_j|\psi\rangle = \sum_i \langle a_j|a_i\rangle c_i$

$= \sum \delta_{ij} c_i = c_j \Rightarrow c_j = \langle a_j|\psi\rangle$

$\Rightarrow |\psi\rangle = \sum_i \langle a_i|\psi\rangle |a_i\rangle$

b) $\langle\psi|\psi\rangle = (\sum_i a_i|i\rangle)^{+}(\sum a_i|i\rangle) = \sum\langle i|i\rangle a_i^* a_i$

$= \sum a_i^* a_i$

c) $\langle\psi|\psi\rangle = \int_{\mathbb{R}} \psi(x)^{+}\psi(x)\,dx = \int \underbrace{\langle\psi|x\rangle\langle x|}_{1 \text{ (mit Integral!)}}\psi\rangle\,dx$

$= \int (\sum a_i\langle i|)\,|x\rangle\langle x|\,\sum a_j|j\rangle\,dx$

$$= \int \sum \alpha_i^* \alpha_j \langle i|x\rangle\langle x|j\rangle = \sum_{ij} \alpha_i^* \alpha_j \delta_{ij} = \sum \alpha_i^* \alpha_i$$

d) $$\hat{Q} = \mathbb{1}\hat{Q}\mathbb{1} = \sum_{ij} |i\rangle\underbrace{\langle i|\hat{Q}|j\rangle}_{Q_{ij}}\langle j|$$

[Bsp.: $(0\,1\,0)\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = d = Q_{21}$]

z.Z.: $(\hat{Q}|\alpha\rangle)^+ = \langle\alpha|Q^+$

$$(\hat{Q}|\alpha\rangle)^+ = \left(\sum_{ij} Q_{ij}|i\rangle\langle j|\alpha\rangle\right)^+ = \sum_{ij} Q_{ij}^* \langle\alpha|(|i\rangle\langle j|)^+$$

$$= \sum_{ij} Q_{ij}^* \langle\alpha|j\rangle\langle i| = \langle\alpha|\underbrace{\sum |j\rangle Q_{ij}^* \langle i|}_{Q^+}$$

e) $$T_r\hat{A} = S_p\hat{A} = \sum_n \langle \alpha_n|\underset{\mathbb{1}}{\uparrow}\hat{A}\underset{\mathbb{1}}{\uparrow}|\alpha_n\rangle = \sum_n\sum_{\mu\nu} \langle\alpha_n|B_\nu\rangle\langle B_\nu|A|B_\mu\rangle\langle B_\mu|\alpha_n\rangle$$

$$= \sum_n\sum_{\mu\nu}\langle B_\mu|\alpha_n\rangle\langle\alpha_n|B_\nu\rangle\langle B_\nu|\hat{A}|B_\mu\rangle$$

$$= \sum_{\mu\nu}\underbrace{\langle B_\mu|B_\nu\rangle}_{\delta_{\mu\nu}}\langle B_\nu|\hat{A}|B_\mu\rangle = \sum_\gamma \langle B_\gamma|\hat{A}|B_\gamma\rangle$$

f) $$(\hat{A}\hat{B})^+ \overset{?}{=} \hat{B}^+\hat{A}^+$$

$$(\hat{A}\hat{B})^+ = (\mathbb{1}\hat{A}\mathbb{1}\mathbb{1}\hat{B}\mathbb{1})^+ = \left(\sum_{abcd}|a\rangle\underbrace{\langle a|\hat{A}|b\rangle}_{A_{ab}}\langle b|c\rangle\underbrace{\langle c|\hat{B}|d\rangle}_{B_{cd}}\langle d|\right)^+$$

$$= \left(\sum_{abcd}|a\rangle\underbrace{A_{ab}\langle b|c\rangle B_{cd}}_{\in\mathbb{C}}\langle d|\right)^+$$

$$= \sum_{abcd}|d\rangle B_{cd}^*\langle c|b\rangle A_{ab}^*\langle a| = B^+A^+$$