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$$\begin{aligned}
 a) \quad s+t+u &= (p_1+p_2)^2 + (p_1-p_3)^2 + (p_1-p_4)^2 \\
 &= (p_1+p_2)^2 + (p_4-p_2)^2 + (p_3-p_2)^2 \\
 &= p_1^2 + p_2^2 + p_3^2 + p_4^2 + 2p_1p_2 + 2p_1p_4 - 2p_2p_4 - 2p_2p_3 \\
 &= m_1^2 + m_2^2 + m_3^2 + m_4^2 + 2p_2^2 + 2p_1p_2 - 2p_2p_4 - 2p_2p_3
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow 2p_2^2 &= 2p_2p_4 + 2p_2p_3 - 2p_1p_2 \\
 p_2^2 &= p_2(p_4 + p_3 - p_1) = p_2^2
 \end{aligned}$$

$$\Rightarrow s+t+u = \sum_{i=1}^4 m_i^2$$

$$b) \quad s = p_1^2 + 2p_1p_2 + p_2^2 = p_1^2 + 2(E_1E_2 - \vec{p}_1\vec{p}_2) + p_2^2 \quad (p_i = \begin{pmatrix} E_i \\ \vec{p}_i \end{pmatrix})$$

$$p_1^2 = E_1^2 - \vec{p}_1^2 \Rightarrow \vec{p}_1^2 = E_1^2 - p_1^2 = -\vec{p}_1\vec{p}_2$$

$$s = p_1^2 + 2(E_1E_2 + E_1^2 - p_1^2) + p_2^2$$

$$p_2^2 = E_2^2 - \vec{p}_2^2 \Rightarrow E_2 = \sqrt{p_2^2 + E_1^2 - p_1^2}$$

$$s = p_1^2 + p_2^2 + 2(E_1 \sqrt{p_2^2 + E_1^2 - p_1^2} + E_1^2 - p_1^2)$$

$$s + p_1^2 - p_2^2 - 2E_1^2 = 2E_1 \sqrt{p_2^2 + E_1^2 - p_1^2}$$

$$(s + p_1^2 - p_2^2)^2 - 4E_1^2(s + p_1^2 - p_2^2) + 4E_1^2 = 4E_1^2(p_2^2 + E_1^2 - p_1^2)$$

$$(s + p_1^2 - p_2^2)^2 = 4E_1^2[(s + p_1^2 - p_2^2) + (p_2^2 - p_1^2)] = 4E_1^2 s$$

$$\Rightarrow E_1^2 = \frac{(s + p_1^2 - p_2^2)}{4s} \quad \text{in massen? } \checkmark \quad 1.5/2p$$

$$c) \quad E_{ges,sp} = E_{1sp} + E_{2sp}$$

$$s = (p_1 + p_2)^2 = (E_{1sp} + E_{2sp})^2$$

$$\Rightarrow E_{ges,sp} = \sqrt{s} \quad \checkmark \quad 32$$

$$d) \quad \text{Elast. Streuung: } m_1 = m_3 \quad m_2 = m_4$$

$$s = (p_1 + p_2)^2 = (E_{1sp} + E_{2sp})^2$$

$$\begin{aligned}
 t &= (p_1 - p_3)^2 = -(\vec{p}_1 - \vec{p}_3)^2 = -\vec{p}_1^2 - \vec{p}_3^2 + 2\vec{p}_1\vec{p}_3 \\
 &= -2p_1^2(1 - \cos\theta) \\
 \Rightarrow \cos\theta &= 1 + \frac{t}{2p_1^2}
 \end{aligned}$$

e)

$$\text{Schwellenergie: } E_s = 2(m_L - m_p) = 2(113 - 0,938) \text{ GeV}$$

$$= 344,124 \text{ GeV} \quad \text{fop}$$

$$b) S = (\vec{p}_1 + \vec{p}_2)^2 = (E_1 + E_2)^2$$

$\xrightarrow{SP: \vec{p}_1 = -\vec{p}_2 = \vec{p}}$

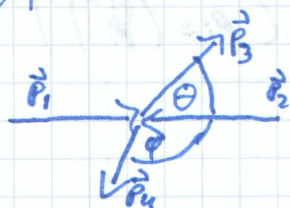
$$I) \vec{E}_2 = \vec{S} - E_1$$

$$II) E_2 = \sqrt{m_2^2 + \vec{p}_2^2} = \sqrt{m_2^2 + E_1^2 - m_1^2}$$

$$L) S - 2\sqrt{S}E_1 + E_1^2 = m_2^2 + E_1^2 - m_1^2$$

$$\Rightarrow E_1 = \frac{S + m_1^2 - m_2^2}{2\sqrt{S}}$$

c)



$$\vec{p}_1 = -\vec{p}_2 \Rightarrow \vec{p}_1 = -\vec{p}_2 \Rightarrow |\vec{p}_1| = |\vec{p}_2| \rightarrow E_1 = E_2$$

$$S = (\vec{p}_1 + \vec{p}_2)^2 = \left( \begin{pmatrix} E_1 + E_2 \\ \vec{p}_1 + \vec{p}_2 \end{pmatrix} \right)^2 = (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2 = 4E_1^2 - 4(m^2 + p^2)$$

$$t = \dots = -2|\vec{p}_1|^2(1 - \cos\theta)$$

$$u = -2|\vec{p}_1| = (1 + \cos\theta)$$

e)

$$S = (\vec{p}_1 + \vec{p}_2)^2 = (2m_t)^2$$

$$\left( \begin{pmatrix} E \\ \vec{p} \end{pmatrix} + \begin{pmatrix} m_p \\ 0 \end{pmatrix} \right)^2 = (E + m_p)^2 - \vec{p}^2 = 4m_t^2$$

$$\Rightarrow E = \frac{2m_t}{m_p} - m_p \approx 60 \text{ TeV}$$



2a) Ruhesystem:  $\vec{p}_z = \vec{0}$ ,  $\vec{p}_+ = -\vec{p}_- \rightarrow |\vec{p}_+| = |\vec{p}_-| = p_z$

$$\Rightarrow \begin{pmatrix} m_z \\ 0 \end{pmatrix} = \begin{pmatrix} E_+ + E_- \\ \vec{p}_+ - \vec{p}_- \end{pmatrix} \quad | \quad ( )^2$$

$E_+ = E_- = E_z$ : Die Energie wird gleichmäßig verteilt, da sie dieselbe Masse  $m_+ = m_- = m_z$  besitzen ✓

$$E^2 = p^2 + m^2$$

$$m_z^2 = (E_+ + E_-)^2 = E_+^2 + E_-^2 + 2E_+E_-$$

$$= 4E_z^2 = 4(p_z^2 + m_z^2) \quad \checkmark$$

$$\Leftrightarrow p_z = \frac{1}{2} \sqrt{m_z^2 - 4m_z^2} \quad \checkmark = 45,559 \text{ GeV} \quad \checkmark \quad 2P$$

b)  $|\vec{p}_z| \neq 0$



$$\vec{p}_+ + \vec{p}_- = \vec{p}_z$$

Beide im Winkel  $\frac{\theta}{2}$   
 $\theta/2 \rightarrow$  Energie gleichverteilt.  
 Impulse gleich groß (Im Betrag) ✓

$$(|\vec{p}_+| + |\vec{p}_-|) \cos(\theta/2) = |\vec{p}_z|$$

$$2p_z \cos(\theta/2) = p_z$$

$$\begin{pmatrix} E_z \\ \vec{p}_z \end{pmatrix} = \begin{pmatrix} 2E_z \\ \vec{p}_+ + \vec{p}_- \end{pmatrix} \quad | \quad ( )^2$$

$$\Leftrightarrow p_z = \frac{p_z}{2 \cos(\theta/2)} \quad \checkmark$$

$$E_z^2 - p_z^2 = 4E_z^2 - |\vec{p}_+|^2 - |\vec{p}_-|^2 - 2\vec{p}_+ \cdot \vec{p}_- \quad \checkmark$$

$$\Leftrightarrow m_z^2 = 2p_z^2 + 4m_z^2 - 2p_z^2 \cos \theta$$

$$|\vec{p}_+ \cdot \vec{p}_-| = |\vec{p}_+| |\vec{p}_-| \cos \theta = p_z^2 \cos \theta \quad \checkmark$$

$$\Leftrightarrow m_z^2 = \frac{p_z^2}{2 \cos^2(\theta/2)} + 4m_z^2 - \frac{p_z^2 \cos \theta}{2 \cos^2(\theta/2)} \quad \text{Kosinus 1} \quad 2 \cos^2(\theta/2) = \cos \theta + 1$$

$$\Leftrightarrow m_z^2 - 4m_z^2 - p_z^2 = \cos \theta (4m_z^2 - m_z^2 - p_z^2)$$

$$\Leftrightarrow \theta = \arccos \left( - \frac{m_z^2 - 4m_z^2 - p_z^2}{m_z^2 - 4m_z^2 + p_z^2} \right) \quad \checkmark$$

$$|\vec{p}_z| = 0 \text{ GeV} \quad \theta = 180^\circ \quad \checkmark$$

$$|\vec{p}_z| = 100 \text{ GeV} \quad \theta = 84,678^\circ \quad \checkmark$$

$$|\vec{p}_z| = 1 \text{ TeV} \quad \theta = 10,413^\circ \quad \checkmark$$

5P

c) Die  $z$ -Masse 92,187 GeV ist viel kleiner als

die von  $t\bar{t}$  344,88 GeV.  $\rightarrow$  Ein Zerfall eines ruhenden  $z^0$  in  $t\bar{t}$  ist nicht möglich.

1P

und wie wäre das bei einem beschleunigten  $z$ .

Σ 8P

$$\Rightarrow 8P + 5P = 13P$$