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Aufgabe 1

KET Gruppe 1

kosmische Myonen: $m = 105{,}7\ \frac{MeV}{c^2}$ $\tau = 2{,}197 \cdot 10^{-6}\,s$

$E = 10^9\,eV$, $h = 10^4\,m$

1 a) ~~$m = 105{,}7\ \frac{MeV}{c^2}$~~

$E = \sqrt{m_0^2c^4 + c^2p^2} \Leftrightarrow p = \frac{\sqrt{E^2 - m_0^2c^4}}{c} = 994\ \frac{MeV}{c}$ ✓ ✓

1 b) $\gamma v = \frac{p}{m}$ ✓ $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ ✓

$\Rightarrow v^2 = \frac{p^2}{m^2}\left(1 - \frac{v^2}{c^2}\right) \Leftrightarrow v^2\left(1 + \frac{p^2}{m^2c^2}\right) = \frac{p^2}{m^2}$

$\Leftrightarrow v = \frac{p}{\sqrt{m^2 + \frac{p^2}{c^2}}} = 0{,}994c$ ✓

1 c) ~~δt~~

$v = \frac{h}{t} \Leftrightarrow \frac{h}{v} = t = 3{,}35\ 10^{-5}\,s$

$p(t) = e^{-\frac{t}{\tau}} \Rightarrow p(3{,}35 \cdot 10^{-5}\,s) = 2{,}34 \cdot 10^{-5}$ ~~s~~ ✓

0,5 d) Zeitdilatation $t' = \frac{t}{\gamma}$ oder ?

1 e) $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 9{,}47$ ✓

1 f) $\beta = \frac{v}{c} = 0{,}994$ ✓

1 g) $p(t') = 0{,}199$ ✓

0,5 h) Masse ist invariant zu Geschwindigkeit

0,5 i) falsch, siehe h. ?

Diskussion

7,5/9

Aufgabe 2

a) $x^\mu x'_\mu = x^\mu g_{\mu\nu} x'^\nu = (ct, x, y, z)(ct', -x', -y', -z')$

$= c^2tt' - x'x - y'y - zz'$ ✓ 1P

b) $x^{\mu'} = \Lambda^\mu_\nu x^\nu = (\gamma ct - \gamma\beta x, \gamma x - \gamma\beta c, y, z)$ ✓ ✓

$$\Lambda^\mu_\nu = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \Lambda^{\mu^{-1}}_\nu : \beta \to -\beta$$ ✓

$$\Rightarrow \Lambda^{\mu^{-1}}_\nu \Lambda^\mu_\nu = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \gamma^2 - \gamma^2\beta^2 & 0 & 0 & 0 \\ 0 & \gamma^2 - \gamma^2\beta^2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \mathbb{1}$$

3P

NR: $\gamma^2 - \gamma^2\beta^2 = \gamma^2(1-\beta^2) = \frac{1-\beta^2}{1-\beta^2} = 1$ ✓

$x^\mu \stackrel{!}{=} \Lambda^{-1\nu}_\nu \Lambda^\mu_\nu x^{\mu'} = \Lambda^{\mu^{-1}}_\nu \Lambda^\mu_\nu x^\mu = x^\mu$

c)

$m^2 = p^\mu p_\mu$

$p^{\mu'} = (\gamma \frac{E}{c} - \gamma\beta p_x, -\gamma\beta\frac{E}{c} + \gamma p_x, p_y, p_z)$

$p_\mu' = (\gamma\frac{E}{c} - \gamma\beta p_x, \gamma\beta\frac{E}{c} - \gamma p_x, -p_y, -p_z)$

$p^{\mu'} p_\mu' = \gamma^2\frac{E^2}{c^2} + \gamma^2\beta^2 p_x^2 - 2\gamma^2\beta\frac{E}{c}p_x - \gamma^2\beta^2\frac{E^2}{c^2} + 2\gamma^2\beta\frac{E}{c}p_x$

$- \gamma^2 p_x^2 - p_y^2 - p_z^2$

$= \frac{E^2}{c^2}(\gamma^2 - \gamma^2\beta^2) - (\gamma^2 - \beta^2\gamma^2)p_x^2 - p_y^2 - p_z^2$ 2P

$= \frac{E^2}{c^2} - \vec{p}^{\,2} = p^\mu p_\mu$ 6P

das ist nicht allg., sondern nur für X

$\sum 7{,}5 + 6 = 13{,}5$ P