TCS

nächstes mal zu zweit bitte

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Aufgabe 11

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$$\frac{dp}{p} = x \frac{dT}{T} \Rightarrow \int \frac{1}{p} dp = \int \frac{x}{T} dT$$

$$\Rightarrow \ln(p) = x \ln(T) + C$$

$$p = e^{x\ln(T)+C} = e^{\ln(T^x)} \cdot C_2 = T^x \cdot C_2$$ ✓

nehmt doch lieber die schönen Konstanten p_0, T_0

b) ideales Gas: $pV = Nk_bT \iff V = \frac{N}{p} k_b T = \frac{Nk_b}{C_2} \cdot T^{-x+1}$ ✓

c) $C_x = \frac{dQ}{dT}\Big|_x \qquad dQ = C_V dT + \left(\frac{\partial U}{\partial V}\Big|_T + p\right) dV$

$$\Rightarrow C_x = C_V + \left(\frac{\partial U}{\partial V}\Big|_T + p\right) \cdot -(x-1) \cdot \frac{Nk_b}{C_2} \cdot T^{-x}$$

$\frac{\partial U}{\partial V}\Big|_T \overset{*}{=} -p + T\frac{\partial p}{\partial T}\Big|_V = 0$! (siehe A4) $\qquad \frac{\partial p}{\partial T}\Big|_V = x \cdot T^{x-1} \cdot C_2$

$$\Rightarrow \frac{\partial U}{\partial V}\Big|_T = x \cdot T^x - p$$

$$\Rightarrow C_x = C_V + x \cdot T^x \cdot -\frac{(x-1)}{C_2} Nk_b T^{-x} = C_V - x \cdot (x-1)/C_2 \cdot Nk_b$$

f

* 1) $U = \frac{3}{2} Nk_BT$ (ideales Gas) $\to \frac{\partial U}{\partial V}\Big|_T = 0$

2) $T\frac{\partial p}{\partial T}\Big|_V = T\frac{Nk_B}{V} = p$

also erneut die Bestätigung, dass $\frac{\partial U}{\partial V}\Big|_T = 0$ ist

d) isochor: $\frac{p}{T} = \text{const} \quad ; C_3 = \frac{T^x}{T} \cdot C_2 \Rightarrow x = 1$ (✓)

isobar: $\frac{V}{T} = \text{const} = Nk_b/C_2 \; \frac{T^{-x+1}}{T} \Rightarrow x = 0$ (✓)

$x = 1: \; C_x = C_V$

$x = 0: \; C_x = C_V$?

C_p

A1	A2	A3	A4	Ges
3	4	3	3.5	13.5

Aufgabe 3 (3)

$$dU = TdS - pdV \iff dS = \frac{1}{T}dU + \frac{p}{T}dV$$

$$\frac{\partial^2 S}{\partial U \partial V} = \frac{\partial^2 S}{\partial V \partial U} \quad ✓$$

$$\Rightarrow \frac{\partial\left(\left[\frac{U}{V}\right]^{2/3} \cdot \frac{1}{8}\right)}{\partial V}\Bigg|_U = \frac{\alpha}{8} \cdot \frac{d\left(\left[\frac{U}{V}\right]^{-1/3}\right)}{\partial U}\Bigg|_V$$

$$\Rightarrow \frac{\partial\left(\left[\frac{V}{U}\right]^{2/3}\right)}{\partial V}\Bigg|_U = \alpha \frac{d\left(\left[\frac{U}{V}\right]^{-1/3}\right)}{dU}\Bigg|_V$$

$$\Rightarrow \frac{2}{3} \cdot \frac{V^{-1/3}}{U^{2/3}} = \alpha \cdot \frac{1}{3} \frac{U^{-2/3}}{V^{-1/3}}$$

$$\Rightarrow \alpha = 2, \; 8 \neq 0 \quad ✓$$

b)

$$dS = \frac{1}{T}dU + \frac{p}{T}dV$$

$$\Rightarrow S = \frac{1}{8}\int\left(\frac{V}{U}\right)^{2/3} dU + \frac{2}{8}\int\left(\frac{U}{V}\right)^{1/3} dV$$

$$= \frac{6V^{2/3}U^{1/3}}{8} + C \quad f$$

so ist das nicht richtig → $\int$ nur ausführen, wenn auf jeder Seite ein 'd'

$$\rightarrow \frac{\partial S}{\partial U}\Big|_V = \frac{1}{T}$$

$$\iff S = \int_{U_0}^{U} \frac{1}{T} dU'$$

$$\& \quad \frac{\partial S}{\partial V}\Big|_U = \frac{p}{T} \iff \ldots$$

$$S(0,0) \overset{!}{=} 0 \quad \Rightarrow C = 0 \quad (✓)$$

c)

$$F = U - TS = U - 8\left(\frac{U}{V}\right)^{2/3} \cdot 6 \frac{V^{2/3}U^{1/3}}{8} = -5U$$

$$T = 8\left(\frac{U}{V}\right)^{2/3} \iff \frac{T}{8} \cdot V^{2/3} = U^{2/3} \quad |(\ldots)^{\frac{3}{2}}$$

$$\Rightarrow \left(\frac{T}{8}\right)^{\frac{3}{2}} V = U$$

$$\Rightarrow F = -5\left(\frac{T}{8}\right)^{3/2} V \quad ff.$$

BRUNNEN

2a) $F = U - TS \quad , \quad dU = T\,dS - p\,dV$

$dF = dU - T\,dS - S\,dT$

$= -(S\,dT + p\,dV) \Rightarrow \quad F = F(T,V)$

b) $F = \frac{3}{10}\frac{\pi^2}{a}\frac{\hbar^2}{m} N \left(\frac{N}{V}\right)^{2/3} - \frac{a}{2}\frac{m}{\hbar^2} N (k_B T)^2 \left(\frac{V}{N}\right)^{2/3}$

aus dF:

$\left.\frac{\partial F}{\partial V}\right|_T = -p \quad , \quad \left.\frac{\partial F}{\partial T}\right|_V = -S$

$\Rightarrow S = \frac{a\,m}{\hbar^2} k_B^2 T N \left(\frac{V}{N}\right)^{2/3}$

c) $p = \frac{1}{5}\frac{\pi^2}{a}\frac{\hbar^2}{m}\left(\frac{N}{V}\right)^{5/3} + \frac{a}{3}\frac{m}{\hbar^2}(k_B T)^2\left(\frac{N}{V}\right)^{1/3}$

d) $U = F + TS = \frac{3}{10}\frac{\pi^2}{a}\frac{\hbar^2}{m} N \left(\frac{N}{V}\right)^{2/3} + \frac{a}{2}\frac{m}{\hbar^2} N (k_B T)^2 \left(\frac{V}{N}\right)^{2/3}$

e) $C_V = \left.\frac{\partial U}{\partial T}\right|_V = S = \frac{a\,m}{\hbar^2} k_B^2 T N \left(\frac{V}{N}\right)^{2/3}$

4a) $\left.\frac{\partial p}{\partial T}\right|_V = \frac{\alpha}{\kappa_T} = -\frac{\left.\frac{\partial V}{\partial T}\right|_p}{\left.\frac{\partial V}{\partial p}\right|_T} = -\frac{\frac{\partial(V,p)}{\partial(T,p)}}{\frac{\partial(V,T)}{\partial(p,T)}} = \frac{\frac{\partial(V,p)}{\partial(T,p)}}{\frac{\partial(V,T)}{\partial(T,p)}} = \frac{\partial(V,p)}{\partial(V,T)} = \left.\frac{\partial p}{\partial T}\right|_V \quad \square$

b) $\left.\frac{\partial T}{\partial V}\right|_U = \frac{1}{C_V}\left(p - T\left.\frac{\partial p}{\partial T}\right|_V\right) \Leftrightarrow C_V \left.\frac{\partial T}{\partial V}\right|_U = p - T\left.\frac{\partial p}{\partial T}\right|_V$

linke Seite: $C_V = \left.\frac{\partial U}{\partial T}\right|_V \Rightarrow \left.\frac{\partial U}{\partial T}\right|_V \left.\frac{\partial T}{\partial V}\right|_U = \frac{\partial(U,V)}{\partial(T,V)}\frac{\partial(T,U)}{\partial(V,U)} = -\frac{\partial(T,U)}{\partial(T,V)} = -\left.\frac{\partial U}{\partial V}\right|_T$

NR: (1): $F = U - TS \quad dF = -S\,dT - p\,dV$ totales Dif $\xRightarrow{\text{Schwarz}} \left.\frac{\partial S}{\partial V}\right|_T = \left.\frac{\partial p}{\partial T}\right|_V$

$\frac{\partial(1)}{\partial V}: \quad -p = \left.\frac{\partial U}{\partial V}\right|_S - T\left.\frac{\partial S}{\partial V}\right|_U \qquad \left.\frac{\partial F}{\partial V}\right|_T = -p$

? was passiert hier ?

was bleibt konstant ?

$\Rightarrow$? $\left.\frac{\partial U}{\partial V}\right|_T = -p + T\left.\frac{\partial p}{\partial T}\right|_V$

$-\left.\frac{\partial U}{\partial V}\right|_T = p - T\left.\frac{\partial p}{\partial T}\right|_V \quad \square$

(1) $\cdot$ 4m

so könnte man vorgehen $F = U - TS$

$\hookrightarrow \left.\frac{\partial F}{\partial V}\right|_T = \left.\frac{\partial U}{\partial V}\right|_T + \left.\frac{\partial(-TS)}{\partial V}\right|_T$

$\Leftrightarrow -p = \left.\frac{\partial U}{\partial V}\right|_T - T\left.\frac{\partial S}{\partial V}\right|_T$

Vorlesung:

$$c_p - c_V = p \left.\frac{\partial V}{\partial T}\right|_p + \left.\frac{\partial U}{\partial V}\right|_T \frac{\partial V}{\partial T}$$

c) $$c_p - c_V \overset{VT}{=} \frac{\alpha^2}{\kappa_T} = -T \frac{\left(\left.\frac{\partial V}{\partial T}\right|_p\right)^2}{\left.\frac{\partial V}{\partial p}\right|_T}$$

$$= -T \frac{\frac{\partial(V,p)}{\partial(T,p)} \frac{\partial(V,p)}{\partial(T,p)}}{\frac{\partial(V,T)}{\partial(p,T)}} = T \frac{\partial(V,p)}{\partial(T,p)} \frac{\partial(V,p)}{\partial(V,T)}$$

$$= T \left.\frac{\partial V}{\partial T}\right|_p \left.\frac{\partial p}{\partial T}\right|_V$$

aus b) $T \left.\frac{\partial p}{\partial T}\right|_V = -c_V \left.\frac{\partial T}{\partial V}\right|_U + p$

$$= \left.\frac{\partial V}{\partial T}\right|_p \left(p - \left.\frac{\partial U}{\partial T}\right|_V \left.\frac{\partial T}{\partial V}\right|_U\right)$$

$$= \left.\frac{\partial V}{\partial T}\right|_p \left(p - \frac{\partial(U,V)}{\partial(T,V)} \frac{\partial(T,U)}{\partial(V,U)}\right)$$

$$= \left.\frac{\partial V}{\partial T}\right|_p \left(p + \left.\frac{\partial U}{\partial V}\right|_T\right) = c_p - c_V \quad \square$$

ok