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Placement of Access Points in an Ultra-Dense 5G Network with Optimum Power and Bandwidth

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***Abstract*—We address the problem of optimally placing the access points in an ultra-dense 5G network to cover a given rectangular region so that every point in the region will be covered by at least one access point (AP) with minimal sum of total transmission power used by all APs. We analyze the wastage of transmission power for different types of placement of the APs with touching and intersecting circles for the full coverage of the rectangular region. We show that the minimum wastages in the inner bulk region with touching and intersecting circles are 81.4% and 20.77%, respectively. We then propose a systematic scheme to place the intersecting circles and show that for moderate size of the rectangle, the wastage due to the spill over of the circles at the boundary can be much higher that 20.77%, but when the side(s) of the rectangle is(are) very large compared to the radius r of the circles, the wastage asymptotically tends to around 20.86%. Also, we show that use of a few circles of smaller radius $r' = 0.62r$ would significantly reduce the wastage even for a rectangle of small size and that would not require any additional bandwidth compared to the scheme for using all circles of the same radius r. The proposed placement scheme is very simple, elegant and effective in minimizing wastage of transmission power of APs as well as optimizing the bandwidth requirement through reuse of channels avoiding co-channel and adjacent channel interferences.**

***Index Terms*—5G technology, Access points, Placement problem, Transmission power, Channel interference, Bandwidth.**

I. Introduction

Driven by the choice of millimeter-wave technology and the goal of order of magnitude higher throughput in 5G, the number of cells will increase dramatically in 5Gs service domain, thus leading to an ultra-dense deployment of cells of very small size, such as picocells or femtocells. Due to drastic reduction in cell size, this extreme densification of the cellular network deployment in 5G will enable a small-cell base station and the mobile terminals within its coverage area to communicate with significantly lower transmission powers than is possible in todays 4G networks. Compared to a density of about $4-5$ BSs/Km^2 in 3G and about $8-10$ BSs/Km^2 in the fourth generation (4G) cellular networks (e.g., Long Term Evolution-Advanced or LTE-A), the density of 5G BSs is highly anticipated to come up to $40-50$ BSs/Km^2 due to the following two technology choices of 5G: i) The use of massive MIMO implies that when the 5G base station (BS) transmission power is constrained to the same level of 4G BS transmission power, each antennas transmission power at a 5G BS has to be decreased $10-20$ times compared to each antennas transmission power at a 4G BS, ii) and utilization of the unused, high frequency millimeter-wave spectrum band, ranging from 30 GHz-300 GHz would restrict communication ranges to $200m$ for most parts, due to the propagation characteristics of millimeter-wave spectrum band [1]–[3], [5].

While reducing the cell sizes would result in significant reduction in transmission power and more opportunity for spectrum reuse, the reality may be far from the ideal scenario. As network densification will increase, the problem of interference is also expected to significantly exacerbate due to proximity of transmitting pairs using the same frequency, thereby offsetting the gains from shorter links to BSs. In real networks, over-deployment and uncoordinated, arbitrary placement of base stations or access points (APs) have been shown to increase interference due to co-channel and adjacent channel interferences [2], [3] i.e., channels of transmitting pairs close to each other becoming spatially correlated [3]. Several studies on network ultra-densification have shown that this problem will significantly exacerbate in 5G networks [2]–[4], unless the placement of the base stations in an ultra-dense deployment mode is systematically done for optimizing the total bandwidth requirement coming from all cells. Unfortunately, both the placement problem of base stations to provide coverage of a given area and the channel allocation problem (CAP) that concerns minimizing the total bandwidth requirement for a given deployment of base stations are known to be NP-Hard [8], [9], [12], [14], [15]. Moreover, none of the numerous existing works on the placement problem or CAP have been designed to deal with the scenario of a huge number of cells that will be present in 5G networks. Therefore, fast and efficient algorithms for the positioning of the small

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cell base stations that simultaneously addresses the twin issues of transmission power and bandwidth requirements will be a critical concern for truly reaping the benefits of an ultra-dense deployment of small cells in 5G networks.

Driven by the above motivations for efficient placement of small-cell APs in an ultra-dense 5G network, we present in this paper a new approach to the problem of appropriately placing the 5G small cell APs such that there are no uncovered regions or *holes* in the service area while jointly minimizing transmission power and bandwidth requirement.

II. Our Contribution

In this paper, we consider the placement of access points (APs) at suitable locations for a 5G ultra-dense deployment of small cells in order to completely cover an area of a given geometry such that there is no uncovered area or hole in the given region to be covered, while simultaneously minimizing the total power of all transmitting APs and the total bandwidth requirement of the cellular network. We assume that the transmission region of an AP has an omnidirectional radiation pattern which can be abstracted by a circular region in two-dimension. The radius r of this circle is assumed to be adjustable within some minimum and maximum values r_{min} and r_{max}, respectively, i.e., $r_{min} \leq r \leq r_{max}$, by controlling the transmitting power of the APs, where r_{min} will be governed by the minimum SINR value of the signal to maintain the desired bit-error rate (BER) in presence of noise and interferences from other neighboring nodes, and r_{max} will be determined by the maximum signal power that may be used by an AP from the viewpoint of energy conservation.

Towards the above goal, we first define a performance index in terms of power wastage of the APs which occurs when power of an AP is unnecessarily used to cover a zone already covered by some other AP, or it is being used to cover some zone which is not required at all to be covered. We next analyze the values of this power wastage by geometric techniques and show that the wastage in covering the inner bulk region (excluding the situations at the boundary) by touching circles is 81.4% and that by intersecting circles is 20.77%, which are independent of r and also the geometry of the boundary as well as its dimension.

We next consider the effect of the region boundary on the power wastage using intersecting circles to cover the region and propose an elegant scheme for systematically placing the circles to cover a given rectangular region. With such placement, we find that for a rectangular area with sides a and b, the power wastage will be a function of $\frac{a}{r}$ and $\frac{b}{r}$ values which asymptotically tends to about 20.86% when $ab >> r^2$. For low values of $\frac{a}{r}$ and $\frac{b}{r}$, the power wastage can be much more than that in the inner bulk region, i.e., 20.77% due to the out-of-region coverage (the coverage area spilling out of the rectangular region) which can, however, be reduced in an elegant way by using a few circles of smaller radius r' with $r_{min} \leq r' < r \leq r_{max}$ at the boundary region only. We also find geometrically the optimum value of r' as $0.62r$ to considerably reduce this wastage due to the out-of-region coverage at the boundary. We then estimate the bandwidth requirement for the use of such smaller circles in covering the region, taking into account the channel interferences arising out of calls in the same cell (a cell is the regular hexagonal region inscribed in a circle), cells at distances one and two apart, respectively in a 2-band buffering system where the interference will not extend beyond two cell distances. While estimating the bandwidth requirement, we consider the smaller transmission power of the APs placed at the centers of the circles of smaller radius r' than those at the centers of circles of radius r and also the variation of power spectral density of adjacent APs with distance, based on the signal propagation model in [19]. Assuming a uniform call density over all the cells, we show that the required bandwidth will always be less than that if all circles used for coverage were of the same radius r. Thus, the proposed coverage scheme using circles of radius r and $r' = 0.62r$ leads to minimizing the wastage of transmission power of the APs with no additional bandwidth requirement than that with all APs having the same transmission power.

III. Related Works

The problem of covering a given region by using some basic units of a specific geometrical shape like polygons or circles has been well studied in the literature by various authors [6], [7], [10], [11]. Deterministic node deployment using random and coordinated coverage technique for large-scale wireless networks have been proposed in [12], [13]. In [14], [15], random node deployment with virtual partitioning is proposed by decomposing the specific region into square grid blocks and investigating the coverage problem of each block. Given a random node deployment over a 2-dimensional region, authors in [20] described a simple method to cover the region by the minimum number of active nodes such that the maximum displacement of nodes is minimized to save energy in individual nodes to achieve longer life of the network. Covering problem of a region using circles has been proposed in [16]. The optimal k-connectivity deployment patterns and the multiple k-coverage problem) have been studied for 2-dimensional networks in [17]. Their proposed framework successfully identified the optimal deployment patterns for $4 \leq k \leq 9$ and significantly reduced the number of nodes to be deployed. Given a set of unit disks in the Euclidean plane, authors in [18] have investigated the discrete unit disk cover (DUDC) and the rectangular region cover (RRC) problems.

IV. Placement of Access Points for Minimum Power Wastage

We assume that every access point has an omni-directional radiation pattern whose projection on a 2-dimensional plane is a circle. As already explained in section II, the radius r of such a circle may be varied within some minimum and maximum values r_{min} and r_{max}, respectively.

There are two possible ways in which the transmission power of the APs may be wasted: i) if the circle corresponding to the transmission range of an AP covers some zone which is already covered by a circle corresponding to some other AP (overlapped covering), and ii) if the circle corresponding to the transmission range of an AP covers some area outside the region R to be covered (out-of-region covering). Based on these facts, we define the performance index of our covering technique in terms of power wastage as follows.

Definition IV.1. *Power wastage ω in covering a region R with n circles $C_1, C_2, \cdots, C_n$ is defined as*
$\omega = \frac{\sum_{i=1}^{n} A(C_i) - A(R)}{A(R)}$
where $A(C_i)$ is the area of the circle $C_i, 1 \leq i \leq n$, and $A(R)$ is the area of the region R.

When all circles have the same radius r, the value of ω reduces to $\omega = \frac{n\pi r^2 - A(R)}{A(R)}$.

We would first use all circles of uniform radius r, $r_{min} < r < r_{max}$ for full coverage with the objective of minimizing ω, and then will consider circles of different radii to further reduce the amount of power wastage, if possible. We try to place first the minimum number of either touching circles or intersecting circles of radius r and then use additional circles to cover the holes created within these circles. Two possible cases with touching and intersecting circles are considered below to estimate the power wastage in the inner bulk region, disregarding the effect at the boundary for the time being.

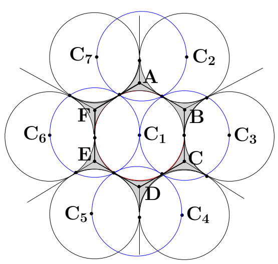

Fig. 1. Touching circles to cover a big region

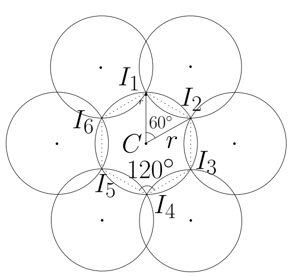

Fig. 2. Region filled with intersecting circles

A. Case 1: Touching circles used for coverage

When we use a large number of touching circles to cover a big region, the region to be covered may be thought of being divided into adjacent regular hexagonal cellular regions as shown in Fig. 1, where the hexagon $ABCDEF$ is formed with its vertices within six shaded regions around the circle at the central position circumscribing the hexagon. The area of the hexagon $ABCDEF$ is equal to $6\sqrt{3}(\frac{2r}{\sqrt{3}})^2/4 = 2\sqrt{3}r^2$. The area of the overlapped region within the central circle is $\pi r^2 - 0.324r^2 = 2.82r^2$. Hence, $\omega = \frac{2.82}{2\sqrt{3}} \approx 81.4\%$.

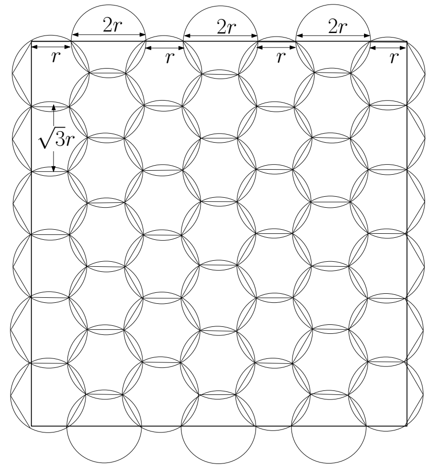

Fig. 3. Rectangular region covered with intersecting circles

B. Case 2: Intersecting circles used for coverage

Consider seven intersecting circles of uniform radius r as shown in Fig. 2 with no holes in between. Every circle in this arrangement is evenly intersected by its six adjacent circles (only one such intersected circle is shown Fig. 2). When every two consecutive points of intersection on the circumferences of the circles are joined, the resultant pattern gives rise to a region of hexagonal cells ($I_1 I_2 I_3 I_4 I_5 I_6$ is one such hexagon). The sides of these hexagonal cells divide the overlapped areas into two equal parts. One half of six overlapped areas around the perimeter (shown as shaded areas) of a hexagonal cell falls within the area of each cell. The sum of these six shaded areas falling within a hexagonal cell contributes to the wastage in each hexagonal cell which also gives the overall wastage in the inner bulk region. The area of six shaded areas (half of an overlap) = $6 \times 0.09r^2 = 0.54r^2$. The area of a hexagonal cell is equal to $\frac{6\sqrt{3}}{4}r^2 = 2.6r^2$. Hence, the wastage in the inner bulk region is equal to $\frac{0.54}{2.6} = 20.77\%$ which is independent of the radius r and much less than that ($\approx 81.4\%$) of touching circles. Hence, in our later part of discussion, we consider only intersecting circles for coverage.

We now consider the coverage in the inner bulk region as well as the boundary of the given region R. Let us consider only rectangular regions to be covered. An example is given in Fig. 3 to cover a rectangle of vertical and horizontal sides as a and b, respectively. Each circle covers a hexagonal cell inscribed in it whose each side will be equal to the radius r of the circle and distance between two parallel sides of the hexagon will be $\sqrt{3}r$. With $a = 6\sqrt{3}r$ and $b = 10r$, we thus use 45 intersecting circles as shown in Fig. 3, leaving no uncovered area in between. It is seen that apart from wastage due to overlapped areas in the inner bulk region, there is additional wastage due to spilled out areas of the circles placed at the boundary of the rectangle. Due to this out-of-region covering at the boundary, the overall wastage comes about 36% (area of 45 circles $= 45\pi r^2 = 141.37r^2$, area of the rectangle $= 6\sqrt{3} \times 10r^2 = 103.92r^2$, the wastage $= \frac{141.37-103.92}{103.92} \approx 36\%$). If the length a of the vertical side is gradually reduced from the bottom by, say $0.4\sqrt{3}r$ (just a little less than $\frac{\sqrt{3}r}{2}$), then the three circles at the bottommost row will have more spilled out areas and it may be checked that the wastage would increase to 45.75%. But, if the length a of the vertical side is reduced by just a little more than $\frac{\sqrt{3}r}{2}$, these three circles at the bottommost row will no longer be required and the wastage with 42 circles will come down to 38.44%, which is, however, higher than the wastage of 36% with 45 circles.

Following Fig. 3, it turns out that for $\frac{a}{\sqrt{3}r} = m$, m being an integer, the circles used for covering are arranged in successive rows and columns such that starting with m circles in the leftmost column, successive columns will have alternately $m+1$ and m circles, and centers of successive circles in a row will be separated by $3r$. In fact, for $\sqrt{3}r(m-\frac{1}{2}) < a \le \sqrt{3}rm$, the number of circles in the columns will alternate between m and $m+1$, while for $\sqrt{3}r(m-1) < a \le \sqrt{3}r(m-\frac{1}{2})$, the number of circles in all the columns will be equal to m.

Let $\lceil \frac{b}{3r} \rceil = k$. Then the number n of columns in the arrangement is given by

$$n = \begin{cases} 2k-1, & \text{if } 3k-3 < \frac{b}{r} \le 3k-2 \\ 2k, & \text{if } 3k-2 < \frac{b}{r} \le 3k-\frac{1}{2} \\ 2k+1, & \text{if } 3k-\frac{1}{2} < \frac{b}{r} \le 3k. \end{cases}$$

Considering the lowest value of b in each of the above three ranges, the value of w for $\frac{a}{\sqrt{3}r} = m$ is given as follows:

$$\omega \le \begin{cases} \frac{\pi r^2\{mk+(m+1)(k-1)\}}{a(3k-3)r} - 1, & \text{if } 3k-3 < \frac{b}{r} \le 3k-2 \\ \frac{\pi r^2\{mk+(m+1)k\}}{a(3k-2)r} - 1, & \text{if } 3k-2 < \frac{b}{r} \le 3k-\frac{1}{2} \\ \frac{\pi r^2\{m(k+1)+(m+1)k\}}{a(3k-\frac{1}{2})r} - 1, & \text{if } 3k-\frac{1}{2} < \frac{b}{r} \le 3k. \end{cases}$$

When a is also varied, the above expression for ω will be applicable for $\sqrt{3}(m-\frac{1}{2}) < a \le \sqrt{3}rm$, while for $\sqrt{3}(m-1) < a \le \sqrt{3}r(m-\frac{1}{2})$, all appearances of $(m+1)$ will be replaced by m in the above expression for ω.

Example 1:

Let $m = 20$ and $b = 21r$. Hence $k = 7$ with $3k - \frac{1}{2} < \frac{b}{r} \le 3k$, leading to $n = 2k+1 = 15$. The number of circles $= 8m+7(m+1) = 307$. Thus, the wastage $\omega = \frac{307\pi r^2 - ab}{ab} \approx 26\%$.

When there will be a large number of circles for coverage, the overall power wastage ω tends to $(\frac{2\pi}{3\sqrt{3}} - 1) \approx 20.86\%$, which is pretty close to the power wastage of 20.77% in the inner bulk region. Thus, we get the following result.

Theorem 1: The arrangement of placing the circles of uniform radius r as shown in Fig. 3 with their centers separated by $\sqrt{3}r$ along any column and $3r$ along any row in covering a rectangular area will asymptotically lead to the power wastage of 20.86% when the sides of the rectangle will become very large compared to r.

V. Reduction of Out-of-Region Wastage

For low values of $\frac{a}{r}$ and $\frac{b}{r}$, the increase in ω from the value of 20.77% due to out-of-region coverage can be very high. Referring to Fig. 3, three circles on each of the topmost and bottommost rows have half of their areas outside the boundary of the rectangular region causing considerable amount of wastage. This wastage due to out-of-region coverage can be significantly reduced if we replace each of these circles by two smaller circles of radius r' leaving no hole in between, as shown in Fig. 4. We redraw the portion of the region covered by these smaller circles as in Fig. 5 where PC = $(\sqrt{3}-1)r$, AP = r. Hence, AC = $\sqrt{(r^2 + (\sqrt{3}-1)^2 r^2)} \approx 1.24r$. The center of the left smaller circle is placed at the mid-point of line AC with its radius r' as $r' = \frac{1.24}{2}r = 0.62r$. The position of the right smaller circle will also be similarly identified. Thus, if $0.62r \ge r_{min}$, we can use such an arrangement which would lead to a reduction in spilled out area. As an example, the arrangement of Fig. 4 corresponds to a power wastage of 33% compared to 36% wastage with the arrangement of Fig. 3 using circles of radius r only.

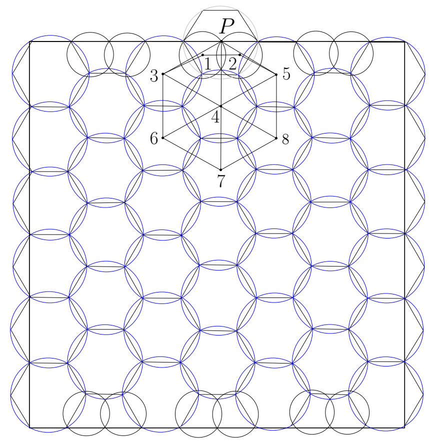

Fig. 4. Coverage with circles of different radii at the boundary

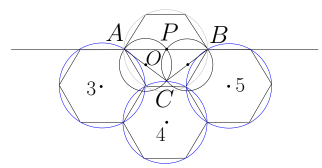

Fig. 5. Portion of the region covered by smaller circles of radius r'

A. Effect of Smaller Circles on Bandwidth Requirement

To estimate the minimum bandwidth requirement with the above placement scheme, we note that the centers of two smaller circles of radius r' along with the adjacent six circles of radius r will correspond to an eight-node subgraph, as shown in Fig. 6 where only points 1 and 2 correspond to circles of smaller radius r'. We assume a 2-band buffering system [8] in which the interference will not extend to cells more than two cell distance apart. Every node in this subgraph is within cell distance two from each other. So, no frequency reuse is possible within this subgraph. Thus, the bandwidth required to assign channels to the nodes of this subgraph gives an estimate of the minimum bandwidth requirement for the boundary region.

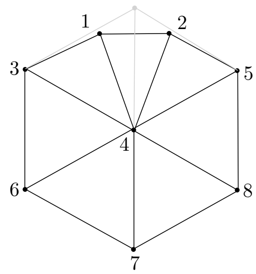

Fig. 6. Eight-node subgraph

Since the assignment bandwidth depends on the minimum frequency separations (which we call as s-parameters) to be satisfied for avoiding interference between the assigned channels, an estimate of the relative values of these s-parameters due to placement of circles of different radii will give an understanding about how the variation of these parameters affect the overall bandwidth requirement of the placement scheme using circles of different radii.

Referring to Fig. 6, there are three possible situations to be considered at the boundary :

1. There are three intersecting circles each of radius r, e.g., the circles corresponding to the points 3, 4 and 5 in Fig. 6. Let s_0, s_1 and s_2 denote the required minimum separations between two frequencies when they are assigned to the same cell of this type, two cells at a distance one (e.g., cells corresponding to the points 3 and 4, or 4 and 5 in Fig. 6) and two cells at distance two apart, (e.g., cells corresponding to the points 3 and 5 in Fig. 6), respectively.
2. There are three intersecting circles with only one of them having radius r', which is placed at one end, e.g., the cells corresponding to the points 6, 3 and 1. Let s'_0 denote the minimum frequency separation between two frequencies when both are assigned to the cell of radius r' (e.g., the point 1 in Fig. 6), and s'_2 be the minimum separation between two frequencies when one is assigned to the cell of radius r at one end and the other assigned to the cell of radius r' at the other end (e.g., points 6 and 1 in Fig. 6).
3. There are three intersecting circles with only one of them having radius r, which is placed at one end (e.g., points 3, 1 and 2 in Fig. 6). Let s'_1 denote the minimum frequency separation when two frequencies are assigned to the two cells - one of radius r and the other of radius r' adjacent to it (e.g., points 3 and 1 in Fig. 6), and and s''_2 be the minimum frequency separation when one is assigned to the cell of radius r and the other to the cell of radius r' at the other end (e.g. points 3 and 2 in Fig. 6).

We note that an AP corresponding to a circle with the smaller radius r' will have transmission power less than that of an AP corresponding to a circle with radius r. Further, distances between two points in Fig. 6 are functions of r and r'. For example, distance between points 6 and 3 is $\sqrt{3}r$ while that between points 1 and 2 is $\sqrt{3}r'$, between 3 and 1 is $\frac{\sqrt{3}(r+r')}{2}$, and so on. Considering the power spectral density of the APs placed at centers of circles of radii r and r' in the above three cases and also the variation of power with distance following the propagation model given in [19], it can be verified that $s_0 \geq s_1 \geq s_2$, $s_0 \geq s'_0$, $s_1 \geq s'_1$, $s_2 \geq s'_2$, $s''_2 \geq s'_2$ and $(s_1 - s'_1) \geq (s_2 - s'_2)$. Detailed derivation of these inequalities are omitted due to brevity.

Referring to Fig. 6, let w_i be the call demand of node $i, 1 \leq i \leq 8$), with $w_T = \sum_{i=1}^{8} w_i$ as the total call demand of all eight nodes. Note that w_T will be equal to the total demands in all seven nodes of the corresponding subgraph had the cell at the boundary with radius r been not replaced by the two smaller cells 1 and 2. We term node 4 as the central node and nodes $1, 2, 3, 5, 6, 7$ and 8 as the peripheral nodes. We now follow the approach similar to that in [9] to derive the minimum bandwidth requirement of the eight-node subgraph. The frequencies assigned to any two nodes of the eight-node subgraph must be separated by a bandgap of at least s'_2. Similarly, the frequencies assigned to the central node and any of the peripheral nodes must be separated by a bandgap of at least s'_1. For minimum bandwidth assignment, the least frequency channel number, say channel 0, will be assigned to the central node [9]. After assigning the channel number 0 to the central node 4, there will be only $w_T - 1$ demands to be assigned further for which we need $w_T - 1$ number of successive bandgaps, each of length at least s'_2 between two consecutive assignments. Thus, the bandgaps required to satisfy all the demands of the subgraph will be at least $(w_T - 1)s'_2$. However, the required minimum bandwidth will actually be more than this value when we go to assign the successive channels to the call demands at different nodes in the subgraph. Two assignment schemes as given below are possible for the minimum bandwidth assignment.

Scheme I: After assigning the minimum frequency to the central node, we complete the assignments of all the demands of the peripheral nodes and then assign successively the remaining demands at the central node.

Scheme II: After assigning the minimum frequency to the central node, we assign a frequency to a demand at a peripheral node and then assign the next demand at the central node, and so on.

B. Assignment order and required bandwidth for Scheme I:

In this scheme, we assign channel 0 to the central node, then go to a peripheral node assignment with a bandgap of at least s'_1, assign all demands of the peripheral nodes, and then return to the central node to assign its second demand with another bandgap of s'_1, i.e., an additional bandgap $2(s'_1 - s'_2)$ over $2s'_2$ is required to assign two demands at the central node. To assign the remaining $w_4 - 2$ demands of the central node successively, we have to maintain a bandgap of s_0 between two consecutive assignments. Thus, an additional bandgap of

$2(s'_1 - s'_2) + (s_0 - s'_2)(w_4 - 2)$ is required to assign all the demands of the central node. Further, the five peripheral nodes $3,\ 5,\ 6,\ 7,\ 8$ each of which represents a larger cell, have a total demand of $w_T - w_1 - w_2 - w_4$ and to satisfy those demands, $(w_T - w_1 - w_2 - w_4) - 1$ number of successive bandgaps, each of minimum length s_2, are required. Summing all these additional bandgaps along with the total number of minimum bandgaps $(w_T - 1)s'_2$, the minimum bandwidth LB_1 required for assigning frequencies to satisfy all the demands at all nodes of the eight-node subgraph will be $LB_1 = (w_T - 1)s'_2 + (s_0 - s'_2)(w_4 - 2) + 2(s'_1 - s'_2) + \{(w_T - w_1 - w_2 - w_4) - 1\}(s_2 - s'_2) = (w_T - 1)s_2 + (s_0 - s_2)(w_4 - 2) + 2(s'_1 - s_2) - (w_1 + w_2)(s_2 - s'_2)$. Had the larger circle at the boundary been not replaced by two smaller circles, the corresponding minimum bandwidth requirement for the seven-node subgraph following the results in [9] will be ${LB_0}^1 = (w_T - 1)s_2 + (s_0 - s_2)(w_4 - 2) + 2(s_1 - s_2)$ for $s_1 \leq s_0 \leq (2s_1 - s_2)$. Hence, it follows that $LB_1 \leq {LB_0}^1$ as $(s'_1 - s_2) \leq (s_1 - s_2)$ and $s_2 \geq s'_2$.

C. Assignment order and required bandwidth for Scheme II:

In this scheme, every time after assigning a channel at the central node, we go to a peripheral node for assigning a channel and again come back to the central node for its next assignment, we require a minimum bandgap of $2s'_1$, i.e., an additional bandgap of $2(s'_1 - s'_2)$ over $2s'_2$. The additional bandgap for assigning all the demands of the central node then comes as $2(s'_1 - s'_2)(w_4 - 1)$. Apart from this, the extra bandgap required to satisfy the total demand of $w_T - w_1 - w_2 - w_4$ of the five peripheral nodes $3,\ 5,\ 6,\ 7,\ 8$ comes as $\{w_T - w_1 - w_2 - w_4 - 1\}(s_2 - s'_2)$. Summing all these additional bandgaps along with the total number of minimum bandgaps $(w_T - 1)s'_2$, the minimum bandwidth LB_2 required for assigning frequencies to satisfy all the demands at each node of the eight-node subgraph comes as $LB_2 = (w_T - 1)s_2 + (2s'_1 - s_2 - s'_2)(w_4 - 1) - (w_1 + w_2 + 1)(s_2 - s'_2)$. Comparing it with the corresponding minimum bandwidth ${LB_0}^2$ for its equivalent seven-node subgraph in [9], we see that $LB_2 \leq {LB_0}^2$ since $(s_1 - s'_1) \geq (s_2 - s'_2)$ and $s_2 \geq s'_2$.

Based on all these discussions, we get the following result:

Theorem 2: The use of circles of radius r' for coverage at the boundary region reduces the transmission power wastage of access points without asking for any additional bandwidth, as compared to the circles of uniform radius r all throughout.

VI. Conclusions

We have proposed an elegant scheme to cover a given rectangular region by circles which can be very effectively used for placing access points (APs) in an ultra-dense 5G network where the center and radius of a circle will correspond to the location and transmission range of an access point. We show that using intersecting circles will lead to a smaller wastage of transmission power than using the touching circles to cover a given region. With intersecting circles the power wastage in the inner bulk region is 20.77% following our placement scheme. The power wastage increases from this value due to out-of-region-coverage near the boundary which, however, asymptotically tends to 20.86% for very large area of the rectangle compared to the area of the circle used for coverage. We further show that even for moderate size of the rectangle, the wastage at the boundary can be significantly reduced by replacing a few circles with circles of smaller (0.62 times) radius at the boundary, and this would not cause any additional bandwidth requirement over that using all circles of the same radius.

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