496 Mathematics for Machine Learning and Inference

Problem Sheet for Tutorial 3

Problem 1
Let matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ -2 & -3 & 1 \end{bmatrix}$$

perform QR decomposition on $\mathbf{A}$.

Problem 2
Prove the following:

- (1) A triangular matrix $\mathbf{A}^{n \times n}$ is normal (i.e. $\mathbf{A}^\mathsf{T}\mathbf{A} = \mathbf{A}\mathbf{A}^\mathsf{T}$) iff it is diagonal (just prove for an upper triangular).
- (2) If a triangular matrix $\mathbf{A}^{n \times n}$ is unitary and $a_{ii} > 0$, $i = 1, \ldots, n$ then $\mathbf{A} = \mathbf{I}$.
- (3) Show that the inverse of an upper triangular matrix is upper triangular and that the product of two upper triangular matrices is upper triangular.
- (4) Based on (1, 2, 3) prove that the QR decomposition of a matrix $\mathbf{A}^{n \times n}$ (i.e. $\mathbf{A} = \mathbf{QR}$) with $r_{ii} > 0$, $\forall\, i = 1, \ldots, n$ is unique.

Problem 3

- (1) Show that the absolute value of the determinant of a unitary matrix equals 1.
- (2) Prove that the determinant of a upper triangular square matrix is equal to the product of its diagonal elements.
- (3) Using (3) and (3), show that the absolute value of the determinant of an $n \times n$-matrix $\mathbf{A}$ with QR decomposition $\mathbf{A} = \mathbf{QR}$ equals to the product of diagonal elements of $\mathbf{R}$, i.e. $|\det\mathbf{A}| = \prod_{i=1}^{n} r_{ii}$