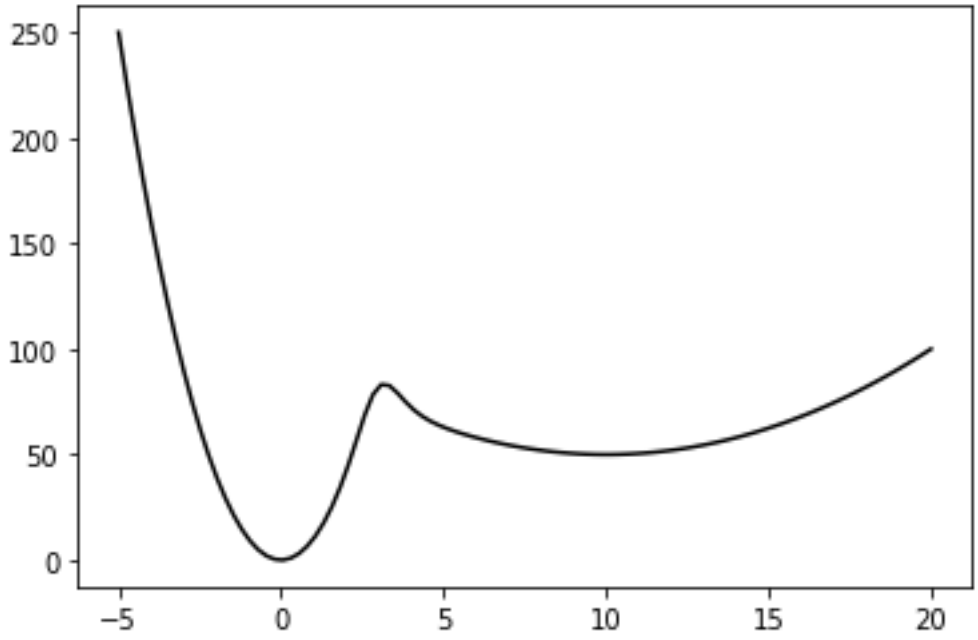
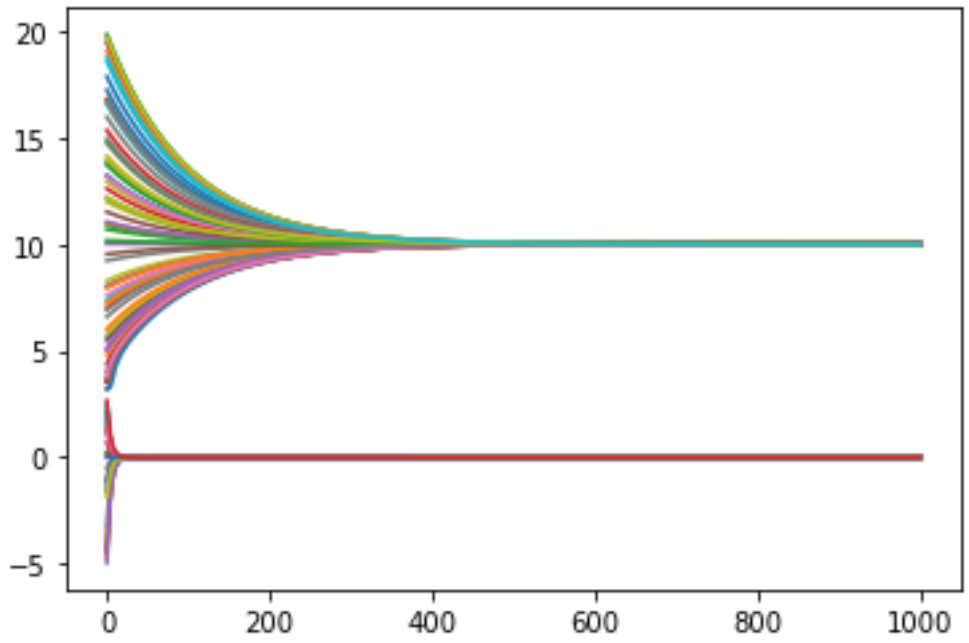
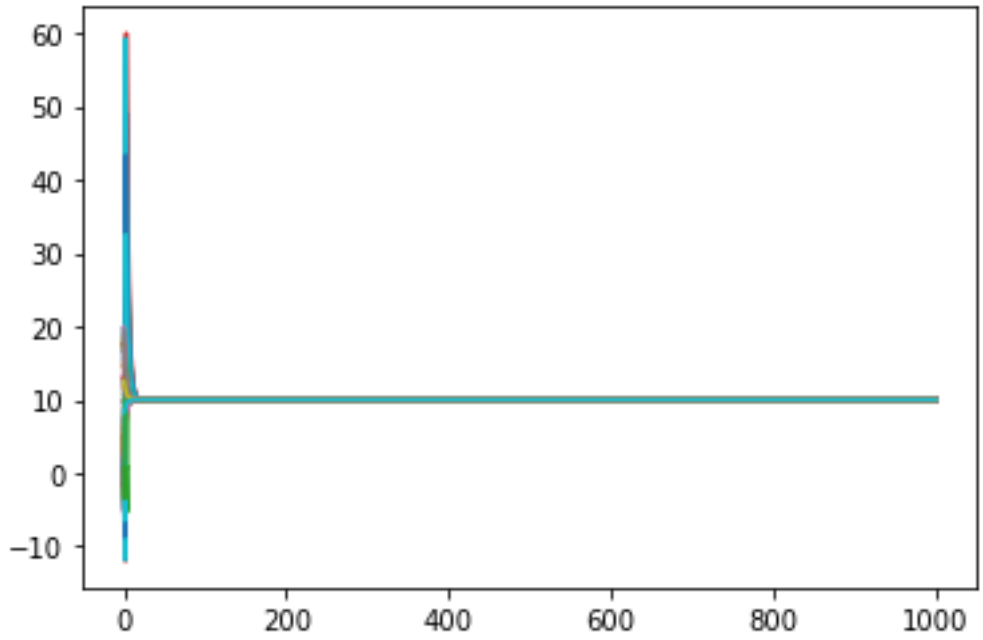
#4.

wideMinima (2).py

```python
import numpy as np
import matplotlib.pyplot as plt


np.seterr(invalid='ignore', over='ignore')  # suppress warning caused by division by inf

def f(x):
    return 1/(1 + np.exp(3*(x-3))) * 10 * x**2  + 1 / (1 + np.exp(-3*(x-3))) * (0.5*(x-10)**2 + 50)

def fprime(x):
    return 1 / (1 + np.exp((-3)*(x-3))) * (x-10) + 1/(1 + np.exp(3*(x-3))) * 20 * x + (3* np.exp(9))/(np.exp(9-1.5*x) + np.exp(1.5*x))**2 * ((0.5*(x-10)**2 + 50) - 10 * x**2)

x = np.linspace(-5,20,100)
plt.plot(x,f(x), 'k')
plt.show()



def test(lr, test_num, iter_num):

    for _ in range(test_num):
        # n one gradient descent
        x = np.random.rand() * 25 -5
        ylist = [x]
        xlist = np.arange(iter_num+1)
        for rep in range(iter_num):
            x = x - lr  * fprime(x)
            ylist.append(x)

        plt.plot(xlist, ylist)
    plt.show()



test(0.01, 100, 1000)
test(0.3, 100, 1000)
test(4, 100 , 1000)
```




20
15
10
5
0
−5
0
200
400
600
800
1000


60
50
40
30
20
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0
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200
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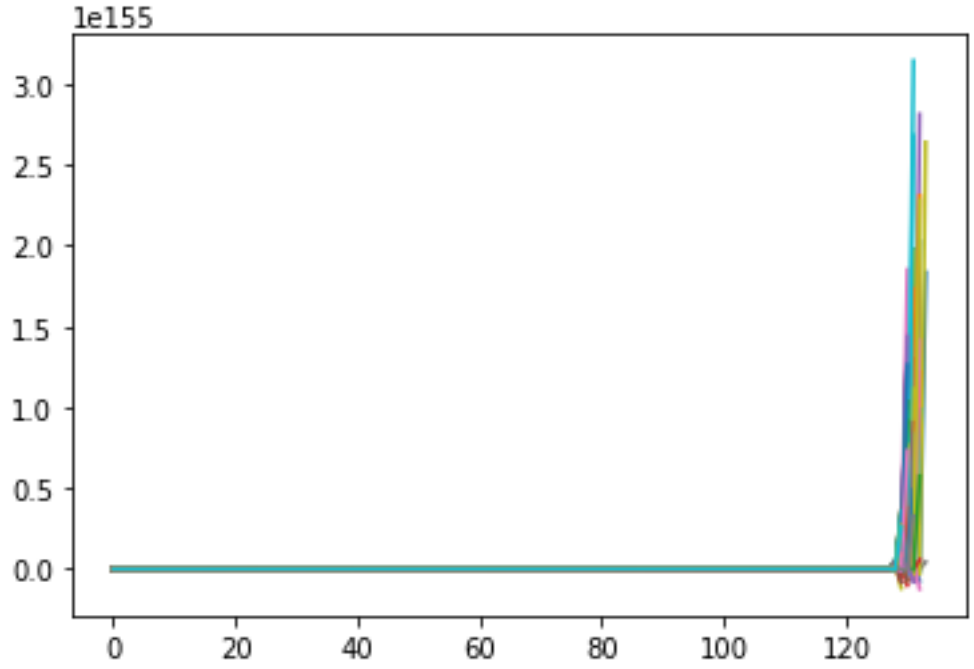
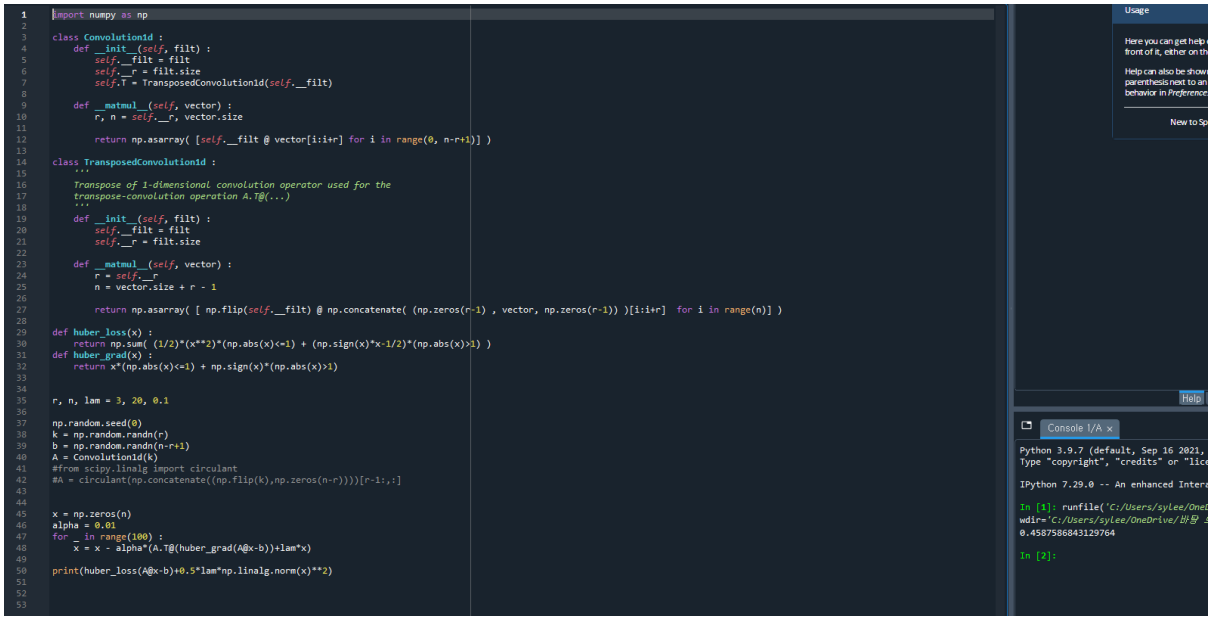



볼 수 있듯이, learning rate(lr)이 0.01 일때에는 sharp/wide minimum 중에 하나로 수렴하는 모습이 보여졌으며, lr이 0.3 일때는 wide minimum 으로 수렴하는 모습이 보여졌고, 마지막으로 lr이 4 일때는 발산하는 경향이 보여졌다.

#5.



```python
import numpy as np

class Convolution1d :
    def __init__(self, filt) :
        self.__filt = filt
        self.__r = filt.size
        self.T = TransposedConvolution1d(self.__filt)

    def __matmul__(self, vector) :
        r, n = self.__r, vector.size

        return np.asarray( [self.__filt @ vector[i:i+r] for i in range(0, n-r+1)] )

class TransposedConvolution1d :
    '''
    Transpose of 1-dimensional convolution operator used for the
    transpose-convolution operation A.T@(...)
    '''
    def __init__(self, filt) :
        self.__filt = filt
        self.__r = filt.size

    def __matmul__(self, vector) :
        r = self.__r
        n = vector.size + r - 1

        return np.asarray( [ np.flip(self.__filt) @ np.concatenate( (np.zeros(r-1) , vector, np.zeros(r-1)) )[i:i+r]  for i in range(n)] )

def huber_loss(x) :
    return np.sum( (1/2)*(x**2)*(np.abs(x)<=1) + (np.sign(x)*x-1/2)*(np.abs(x)>1) )
def huber_grad(x) :
    return x*(np.abs(x)<=1) + np.sign(x)*(np.abs(x)>1)


r, n, lam = 3, 20, 0.1

np.random.seed(0)
k = np.random.randn(r)
b = np.random.randn(n-r+1)
A = Convolution1d(k)
#from scipy.linalg import circulant
#A = circulant(np.concatenate((np.flip(k),np.zeros(n-r))))[r-1:,:]


x = np.zeros(n)
alpha = 0.01
for _ in range(100) :
    x = x - alpha*(A.T@(huber_grad(A@x-b))+lam*x)

print(huber_loss(A@x-b)+0.5*lam*np.linalg.norm(x)**2)
```

```
Usage

Here you can get help
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Help can also be show
parenthesis next to an
behavior in Preference

New to Sp

Help

Console 1/A

Python 3.9.7 (default, Sep 16 2021,
Type "copyright", "credits" or "lic

IPython 7.29.0 -- An enhanced Inter

In [1]: runfile('C:/Users/sylee/One
wdir='C:/Users/sylee/OneDrive/바탕 
0.4587586843129764

In [2]:
```

위와 같이 slicing 과 concatenation을 적절히 사용하면 처리 가능하다. 출력으로 0.4587586.... 이 나오는 것을 확인할 수 있다. 코드를 살펴보면, Convolution1d의 경우 0을 하나도 사용하지 않고 처리한 것을 볼 수 있으며 Transpose의 경우 0을 $O(nr)$개 추가로 사용했음을 볼 수 있다. 이는 원래 행렬의 0의 개수인 $O(n^2)$ 개보다 적은 값이며 따라서 보다 효율적이다.