Comp 302 Assignment 5 Qianyu Huang 260669624

1.1 $FV(e_1, e_2) = FV(e_1) \cup FV(e_2)$

$FV(\text{let pair } (x,y) = e_1 \text{ in } e_2 \text{ end}) = (FV(e_2) \setminus \{x\} \setminus \{y\}) \cup FV(e_1)$

1.3 $[e/x](e_1, e_2) = ([e/x]e_1, [e/x]e_2)$

$[e/x](\text{let pair } (x,y) = e_1 \text{ in } e_2 \text{ end}) = (\text{let pair } (x,y) = [e/x]e_1 \text{ in } e_2 \text{ end}]$ provided $x, y \notin FV(e)$

1.5. $\dfrac{e_1 : T_1 \quad e_2 : T_2}{(e_1, e_2) : T_1 * T_2}$ T-Pair

~~$x:T_1 \; y:T_2 \; e_1 : (T_1 * T_2)$~~

$$\frac{\Gamma \vdash x : T_1 \quad \Gamma \vdash y : T_2 \quad \Gamma \vdash e_1 : (T_1 * T_2) \quad \Gamma \vdash e_2 : T_3}{\Gamma \vdash \text{Let pair } (x,y) = e_1 \text{ in } e_2 : T_3}$$

1.7. $\dfrac{e_1 \Downarrow v_1 \quad e_2 \Downarrow v_2}{(e_1, e_2) \Downarrow (v_1, v_2)}$

$$\frac{e_1 \Downarrow (v_1 * v_2) \quad [v_1/x][v_2/y]e_2 \Downarrow v}{\text{let pair } (x,y) = e_1 \text{ in } e_2 \Downarrow v}$$

2.1 ~~$FV(\text{fst } e) = FV(e) = FV(\text{fst } e) \cup FV(\text{snd } e)$~~

~~$FV(\text{let } FV(\text{fst } e) = FV(e) = FV(\text{fst } e)$~~ $FV(\text{fst } (e_1 * e_2)) = FV(e_1)$

$FV(\text{snd } e) = FV(\text{Snd } (e_1 * e_2)) = FV(e_2)$

2.2. $[e/x](\text{fst } e') = [e/x](\text{fst } (e_1 * e_2)) = [e/x]e_1$

$[e/x](\text{Snd } e') = [e/x](\text{Snd } (e_1 * e_2)) = [e/x]e_2$