Lecture 5: Model-Free Control

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Outline

- 1 Introduction
- 2 On-Policy Monte-Carlo Control
- 3 On-Policy Temporal-Difference Learning
- 4 Off-Policy Learning
- 5 Summary

Model-Free Reinforcement Learning

- Last lecture:
 - Model-free prediction
 - Estimate the value function of an unknown MDP
- This lecture:
 - Model-free control
 - Optimise the value function of an unknown MDP

Uses of Model-Free Control

Some example problems that can be modelled as MDPs

- Elevator
- Parallel Parking
- Ship Steering
- Bioreactor
- Helicopter
- Aeroplane Logistics

- Robocup Soccer
- Quake
- Portfolio management
- Protein Folding
- Robot walking
- Game of Go

For most of these problems, either:

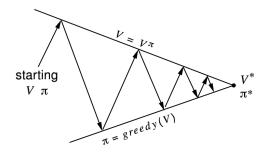
- MDP model is unknown, but experience can be sampled
- MDP model is known, but is too big to use, except by samples

Model-free control can solve these problems

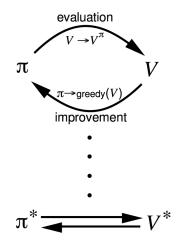
On and Off-Policy Learning

- On-policy learning
 - "Learn on the job"
 - \blacksquare Learn about policy π from experience sampled from π
- Off-policy learning
 - "Look over someone's shoulder"
 - Learn about policy π from experience sampled from μ

Generalised Policy Iteration (Refresher)

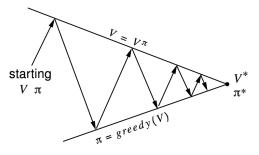


Policy evaluation Estimate v_{π} e.g. Iterative policy evaluation Policy improvement Generate $\pi' \geq \pi$ e.g. Greedy policy improvement



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On-Policy Monte-Carlo Control
Generalised Policy Iteration

Generalised Policy Iteration With Monte-Carlo Evaluation



Policy evaluation Monte-Carlo policy evaluation, $V = v_{\pi}$? Policy improvement Greedy policy improvement?

Model-Free Policy Iteration Using Action-Value Function

• Greedy policy improvement over V(s) requires model of MDP

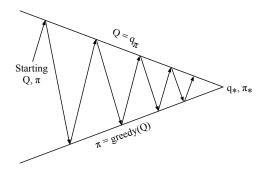
$$\pi'(s) = \operatorname*{argmax}_{s \in \mathcal{A}} \mathcal{R}^{a}_{s} + \mathcal{P}^{a}_{ss'} V(s')$$

• Greedy policy improvement over Q(s, a) is model-free

$$\pi'(s) = \underset{a \in \mathcal{A}}{\operatorname{argmax}} Q(s, a)$$

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On-Policy Monte-Carlo Control
Generalised Policy Iteration

Generalised Policy Iteration with Action-Value Function



Policy evaluation Monte-Carlo policy evaluation, $Q = q_{\pi}$ Policy improvement Greedy policy improvement?

Example of Greedy Action Selection



"Behind one door is tenure - behind the other is flipping burgers at McDonald's."

- There are two doors in front of you.
- You open the left door and get reward 0
 V(left) = 0
- You open the right door and get reward +1V(right) = +1
- You open the right door and get reward +3V(right) = +2
- You open the right door and get reward +2V(right) = +2

:

Are you sure you've chosen the best door?

ϵ -Greedy Exploration

- Simplest idea for ensuring continual exploration
- All m actions are tried with non-zero probability
- lacksquare With probability $1-\epsilon$ choose the greedy action
- lacktriangle With probability ϵ choose an action at random

$$\pi(a|s) = \left\{ egin{array}{ll} \epsilon/m + 1 - \epsilon & ext{if } a^* = rgmax \ Q(s,a) \ \epsilon/m & ext{otherwise} \end{array}
ight.$$

ϵ-Greedy Policy Improvement

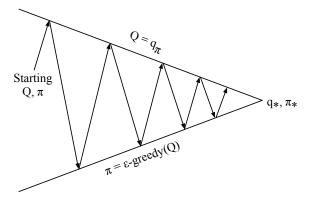
Theorem

For any ϵ -greedy policy π , the ϵ -greedy policy π' with respect to q_{π} is an improvement, $v_{\pi'}(s) \geq v_{\pi}(s)$

$$egin{aligned} q_\pi(s,\pi'(s)) &= \sum_{a \in \mathcal{A}} \pi'(a|s) q_\pi(s,a) \ &= \epsilon/m \sum_{a \in \mathcal{A}} q_\pi(s,a) + (1-\epsilon) \max_{a \in \mathcal{A}} q_\pi(s,a) \ &\geq \epsilon/m \sum_{a \in \mathcal{A}} q_\pi(s,a) + (1-\epsilon) \sum_{a \in \mathcal{A}} rac{\pi(a|s) - \epsilon/m}{1-\epsilon} q_\pi(s,a) \ &= \sum_{a \in \mathcal{A}} \pi(a|s) q_\pi(s,a) = v_\pi(s) \end{aligned}$$

Therefore from policy improvement theorem, $v_{\pi'}(s) \geq v_{\pi}(s)$

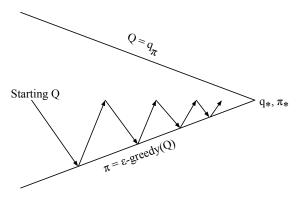
Monte-Carlo Policy Iteration



Policy evaluation Monte-Carlo policy evaluation, $Q=q_\pi$ Policy improvement ϵ -greedy policy improvement

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On-Policy Monte-Carlo Control
Exploration

Monte-Carlo Control



Every episode:

Policy evaluation Monte-Carlo policy evaluation, $Q \approx q_{\pi}$ Policy improvement ϵ -greedy policy improvement

GLIE

Definition

Greedy in the Limit with Infinite Exploration (GLIE)

All state-action pairs are explored infinitely many times,

$$\lim_{k\to\infty}\,N_k(s,a)=\infty$$

The policy converges on a greedy policy,

$$\lim_{k \to \infty} \pi_k(a|s) = \mathbf{1}(a = \operatorname*{argmax}_{a' \in \mathcal{A}} Q_k(s, a'))$$

■ For example, ϵ -greedy is GLIE if ϵ reduces to zero at $\epsilon_k = \frac{1}{k}$

GLIE Monte-Carlo Control

- Sample *k*th episode using π : $\{S_1, A_1, R_2, ..., S_T\} \sim \pi$
- For each state S_t and action A_t in the episode,

$$egin{aligned} N(S_t, A_t) &\leftarrow N(S_t, A_t) + 1 \ Q(S_t, A_t) &\leftarrow Q(S_t, A_t) + rac{1}{N(S_t, A_t)} \left(G_t - Q(S_t, A_t)
ight) \end{aligned}$$

Improve policy based on new action-value function

$$\epsilon \leftarrow 1/k$$
 $\pi \leftarrow \epsilon$ -greedy(Q)

Theorem

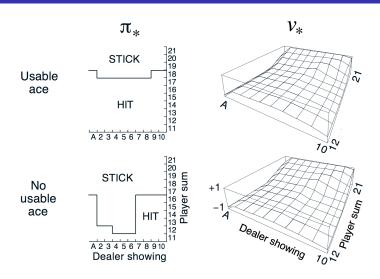
GLIE Monte-Carlo control converges to the optimal action-value function, $Q(s,a) o q_*(s,a)$

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On-Policy Monte-Carlo Control
Blackjack Example

Back to the Blackjack Example



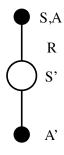
Monte-Carlo Control in Blackjack



MC vs. TD Control

- Temporal-difference (TD) learning has several advantages over Monte-Carlo (MC)
 - Lower variance
 - Online
 - Incomplete sequences
- Natural idea: use TD instead of MC in our control loop
 - \blacksquare Apply TD to Q(S, A)
 - Use ϵ -greedy policy improvement
 - Update every time-step

Updating Action-Value Functions with Sarsa

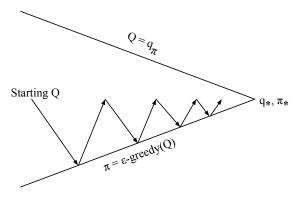


$$Q(S, A) \leftarrow Q(S, A) + \alpha \left(R + \gamma Q(S', A') - Q(S, A)\right)$$

Lecture 5: Model-Free Control

☐ On-Policy Temporal-Difference Learning
☐ Sarsa(λ)

On-Policy Control With Sarsa



Every time-step:

Policy evaluation Sarsa, $Q pprox q_{\pi}$

Policy improvement ϵ -greedy policy improvement

Sarsa Algorithm for On-Policy Control

```
Initialize Q(s,a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s), arbitrarily, and Q(terminal\text{-}state, \cdot) = 0 Repeat (for each episode):

Initialize S
Choose A from S using policy derived from Q (e.g., \varepsilon\text{-}greedy)
Repeat (for each step of episode):

Take action A, observe R, S'
Choose A' from S' using policy derived from Q (e.g., \varepsilon\text{-}greedy)
Q(S,A) \leftarrow Q(S,A) + \alpha \left[R + \gamma Q(S',A') - Q(S,A)\right]
S \leftarrow S'; A \leftarrow A';
until S is terminal
```

Convergence of Sarsa

Theorem

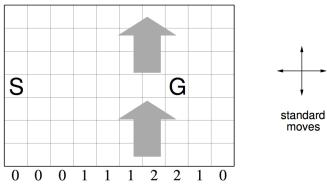
Sarsa converges to the optimal action-value function, $Q(s, a) \rightarrow q_*(s, a)$, under the following conditions:

- GLIE sequence of policies $\pi_t(a|s)$
- **Robbins-Monro** sequence of step-sizes α_t

$$\sum_{t=1}^{\infty} \alpha_t = \infty$$

$$\sum_{t=1}^{\infty} \alpha_t^2 < \infty$$

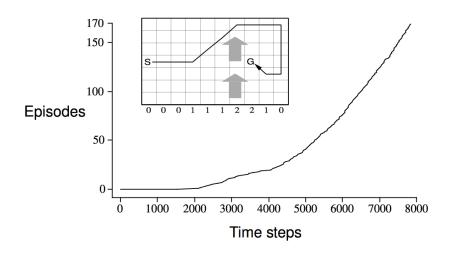
Windy Gridworld Example



standard king's moves

- Reward = -1 per time-step until reaching goal
- Undiscounted

Sarsa on the Windy Gridworld



n-Step Sarsa

■ Consider the following *n*-step returns for $n = 1, 2, \infty$:

$$\begin{array}{ll} \textit{n} = 1 & \textit{(Sarsa)} & q_t^{(1)} = R_{t+1} + \gamma Q(S_{t+1}) \\ \textit{n} = 2 & q_t^{(2)} = R_{t+1} + \gamma R_{t+2} + \gamma^2 Q(S_{t+2}) \\ \vdots & \vdots & \vdots \\ \textit{n} = \infty & \textit{(MC)} & q_t^{(\infty)} = R_{t+1} + \gamma R_{t+2} + ... + \gamma^{T-1} R_T \end{array}$$

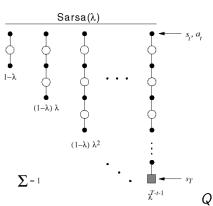
■ Define the *n*-step Q-return

$$q_t^{(n)} = R_{t+1} + \gamma R_{t+2} + \dots + \gamma^{n-1} R_{t+n} + \gamma^n Q(S_{t+n})$$

■ n-step Sarsa updates Q(s, a) towards the n-step Q-return

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left(q_t^{(n)} - Q(S_t, A_t)\right)$$

Forward View Sarsa(λ)



- The q^{λ} return combines all *n*-step Q-returns $a_{+}^{(n)}$
- Using weight $(1 \lambda)\lambda^{n-1}$

$$q_t^{\lambda} = (1 - \lambda) \sum_{n=1}^{\infty} \lambda^{n-1} q_t^{(n)}$$

Forward-view Sarsa (λ)

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left(q_t^{\lambda} - Q(S_t, A_t)\right)$$

Backward View Sarsa(λ)

- Just like $TD(\lambda)$, we use eligibility traces in an online algorithm
- But Sarsa(λ) has one eligibility trace for each state-action pair

$$E_0(s, a) = 0$$

 $E_t(s, a) = \gamma \lambda E_{t-1}(s, a) + \mathbf{1}(S_t = s, A_t = a)$

- ullet Q(s,a) is updated for every state s and action a
- In proportion to TD-error δ_t and eligibility trace $E_t(s, a)$

$$\delta_t = R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t)$$

$$Q(s, a) \leftarrow Q(s, a) + \alpha \delta_t E_t(s, a)$$

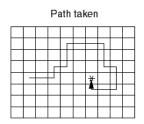
$Sarsa(\lambda)$ Algorithm

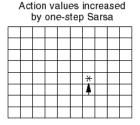
 \sqsubseteq Sarsa(λ)

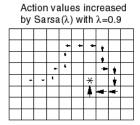
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Initialize Q(s, a) arbitrarily, for all s \in S, a \in A(s)
Repeat (for each episode):
   E(s, a) = 0, for all s \in S, a \in A(s)
   Initialize S, A
   Repeat (for each step of episode):
       Take action A, observe R, S'
       Choose A' from S' using policy derived from Q (e.g., \varepsilon-greedy)
       \delta \leftarrow R + \gamma Q(S', A') - Q(S, A)
       E(S,A) \leftarrow E(S,A) + 1
       For all s \in S, a \in A(s):
           Q(s,a) \leftarrow Q(s,a) + \alpha \delta E(s,a)
           E(s,a) \leftarrow \gamma \lambda E(s,a)
       S \leftarrow S' \colon A \leftarrow A'
   until S is terminal
```

igspace On-Policy Temporal-Difference Learning igspace Sarsa (λ)

$Sarsa(\lambda)$ Gridworld Example







Off-Policy Learning

- Evaluate target policy $\pi(a|s)$ to compute $v_{\pi}(s)$ or $q_{\pi}(s,a)$
- While following behaviour policy $\mu(a|s)$

$$\{S_1, A_1, R_2, ..., S_T\} \sim \mu$$

- Why is this important?
- Learn from observing humans or other agents
- Re-use experience generated from old policies $\pi_1, \pi_2, ..., \pi_{t-1}$
- Learn about optimal policy while following exploratory policy
- Learn about multiple policies while following one policy

Importance Sampling

■ Estimate the expectation of a different distribution

$$\mathbb{E}_{X \sim P}[f(X)] = \sum_{X \sim P} P(X)f(X)$$

$$= \sum_{X \sim Q} Q(X) \frac{P(X)}{Q(X)} f(X)$$

$$= \mathbb{E}_{X \sim Q} \left[\frac{P(X)}{Q(X)} f(X) \right]$$

Importance Sampling for Off-Policy Monte-Carlo

- Use returns generated from μ to evaluate π
- Weight return G_t according to similarity between policies
- Multiply importance sampling corrections along whole episode

$$G_t^{\pi/\mu} = \frac{\pi(A_t|S_t)}{\mu(A_t|S_t)} \frac{\pi(A_{t+1}|S_{t+1})}{\mu(A_{t+1}|S_{t+1})} \dots \frac{\pi(A_T|S_T)}{\mu(A_T|S_T)} G_t$$

Update value towards corrected return

$$V(S_t) \leftarrow V(S_t) + \alpha \left(\frac{G_t^{\pi/\mu}}{V(S_t)} - V(S_t) \right)$$

- lacksquare Cannot use if μ is zero when π is non-zero
- Importance sampling can dramatically increase variance

Importance Sampling for Off-Policy TD

- $lue{}$ Use TD targets generated from μ to evaluate π
- Weight TD target $R + \gamma V(S')$ by importance sampling
- Only need a single importance sampling correction

$$V(S_t) \leftarrow V(S_t) + \alpha \left(\frac{\pi(A_t|S_t)}{\mu(A_t|S_t)} (R_{t+1} + \gamma V(S_{t+1})) - V(S_t) \right)$$

- Much lower variance than Monte-Carlo importance sampling
- Policies only need to be similar over a single step

Q-Learning

- We now consider off-policy learning of action-values Q(s, a)
- No importance sampling is required
- Next action is chosen using behaviour policy $A_{t+1} \sim \mu(\cdot|S_t)$
- But we consider alternative successor action $A' \sim \pi(\cdot|S_t)$
- And update $Q(S_t, A_t)$ towards value of alternative action

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left(R_{t+1} + \gamma Q(S_{t+1}, A') - Q(S_t, A_t) \right)$$

Off-Policy Control with Q-Learning

- We now allow both behaviour and target policies to improve
- The target policy π is greedy w.r.t. Q(s, a)

$$\pi(S_{t+1}) = \operatorname*{argmax}_{a'} Q(S_{t+1}, a')$$

- The behaviour policy μ is e.g. ϵ -greedy w.r.t. Q(s,a)
- The Q-learning target then simplifies:

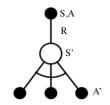
$$R_{t+1} + \gamma Q(S_{t+1}, A')$$

$$= R_{t+1} + \gamma Q(S_{t+1}, \underset{a'}{\operatorname{argmax}} Q(S_{t+1}, a'))$$

$$= R_{t+1} + \max_{a'} \gamma Q(S_{t+1}, a')$$

Q-Learning

Q-Learning Control Algorithm



$$Q(S,A) \leftarrow Q(S,A) + \alpha \left(R + \gamma \max_{a'} Q(S',a') - Q(S,A)\right)$$

Theorem

Q-learning control converges to the optimal action-value function, $Q(s,a) o q_*(s,a)$

Q-Learning Algorithm for Off-Policy Control

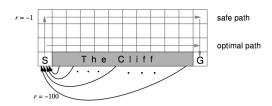
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Repeat (for each episode):
Initialize S
Repeat (for each step of episode):
Choose A from S using policy derived from Q (e.g., \varepsilon\text{-}greedy)
Take action A, observe R, S'
Q(S,A) \leftarrow Q(S,A) + \alpha \left[R + \gamma \max_a Q(S',a) - Q(S,A)\right]
S \leftarrow S';
until S is terminal
```

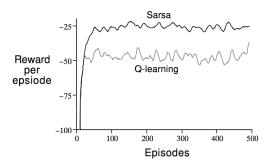
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Lecture 5: Model-Free Control
Off-Policy Learning
LQ-Learning
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Q-Learning Demo

Q-Learning Demo

Cliff Walking Example





Relationship Between DP and TD

	Full Backup (DP)	Sample Backup (TD)
Bellman Expectation	$v_{\sigma}(s) \leftarrow s$ r $v_{\sigma}(s') \leftarrow s'$	
Equation for $v_{\pi}(s)$	Iterative Policy Evaluation	TD Learning
Bellman Expectation Equation for $q_{\pi}(s, a)$	$q_r(s,a) \leftrightarrow s,a$ $q_r(s',a') \leftrightarrow a'$ Q-Policy Iteration	s s A R S S S S S S S S S S S S S S S S S S
Equation for $q_{\pi}(s, a)$	Q-Folicy Iteration	Jarsa
Bellman Optimality Equation for $q_*(s, a)$	$q_*(s,a) \leftrightarrow s,a$ $q_*(s',a') \leftrightarrow a'$ Q-Value Iteration	Q-Learning

Relationship Between DP and TD (2)

Full Backup (DP)	Sample Backup (TD)	
Iterative Policy Evaluation	TD Learning	
$V(s) \leftarrow \mathbb{E}\left[R + \gamma V(S') \mid s\right]$	$V(S) \stackrel{\alpha}{\leftarrow} R + \gamma V(S')$	
Q-Policy Iteration	Sarsa	
$Q(s, a) \leftarrow \mathbb{E}\left[R + \gamma Q(S', A') \mid s, a\right]$	$Q(S,A) \stackrel{\alpha}{\leftarrow} R + \gamma Q(S',A')$	
Q-Value Iteration	Q-Learning	
$Q(s, a) \leftarrow \mathbb{E}\left[R + \gamma \max_{a' \in \mathcal{A}} Q(S', a') \mid s, a\right]$	$Q(S,A) \stackrel{\alpha}{\leftarrow} R + \gamma \max_{a' \in A} Q(S',a')$	

where $x \stackrel{\alpha}{\leftarrow} y \equiv x \leftarrow x + \alpha(y - x)$

Summary

Questions?