**CS 353 Fall 2018**
**Homework 5 Solutions**
**Due:** 7 December, Friday till 17:00

**Q.1** Consider the set F of functional dependencies on the relation schema r(A, B, C, D, E, F) given as F={AB → C, B → D, F → CE, DE → F}
Determine whether each of the following functional dependencies can be derived from F:
**a**) BE → FC
**b**) AF → D
**c**) CE →B
**d**) FD → A

**Solution**

a) $(BE)^+$ = BEDFC and hence YES
b) $(AF)^+$ = AFCE and hence NO
c) $(CE)^+$ = CE and hence NO
d) $(FD)^+$ = FDCE and hence NO

**Q.2** Consider the relation r and the set of functional dependencies as given in Q1. Show that the BE → C hold on r using ONLY Armstrong's Axioms. In each step of the proof, specify the axiom you are using.

**Solution**

| | |
|---|---|
| B → D | given in the F |
| BE → DE | augmentation |
| BE → F | transitivity |
| BE → CE | transitivity |
| CE → C | reflexivity |
| BE → C | transitivity |

**Q.3** Given relation r(A, B, C, D, E, F), for each of the following set of functional dependencies, determine whether the decomposition of R into r1(A,B,E,D) and r2(A,B,C,F) is a lossless-join decomposition or not:
**a**) F1 = {AB → C, B →D}
**b**) F2 = {F → CE, DE →F}

**Solution**

a) $(r1 \cap r2)^+$ with respect to F1 = ABCD and ABCD is not a super key for r1 nor r2 and hence the decomposition is NOT lossless-join with respect to F1
b) $(r1 \cap r2)^+$ with respect to F2 = AB and AB is not a super key for r1 nor r2 and hence the decomposition is NOT lossless-join with respect to F2

**Q.4** Consider the following set F of functional dependencies on the relation schema r(A, B, C, D, E, F)

F={ $A \rightarrow BCF$, $B \rightarrow DE$, $D \rightarrow A$}

**a)** Compute candidate keys for r

**b)** Given a canonical cover for F

**c)** Provide a 3NF decomposition of r

**Solution**

a) We start with sets of single attributes:

| | |
|---|---|
| $A^+ = ABCDEF$ | A is a candidate key |
| $B^+ = ABCDEF$ | B is a candidate key |
| $C^+ = C$ | C is NOT a candidate key |
| $D^+ = ABCDEF$ | D is a candidate key |
| $E^+ = E$ | E is NOT a candidate key |
| $F^+ = F$ | F is NOT a candidate key |

Potential candidate keys as sets of two attributes may not contain A, B and D attributes and hence we only need to check CE, CF and EF

| | |
|---|---|
| $(CE)^+ = CE$ | CE is NOT a candidate key |
| $(CF)^+ = CF$ | CF is NOT a candidate key |
| $(EF^+) = EF$ | EF is NOT a candidate key |

Potential candidate keys as sets of three attributes may not contain A, B and D attributes and hence we only need to check ECF:

| | |
|---|---|
| $(CEF)^+ = CEF$ | CEF is NOT a candidate key |

Hence, the candidate keys are A, B and D

b) No pairs of functional dependencies can be combined using the union rule

- Check if B is extraneous in $A \rightarrow BCF$:
  $F'$={ $A \rightarrow CF$, $B \rightarrow DE$, $D \rightarrow A$ }
  $A^+$ under $F'$ = ACF and $A^+$ does not include B, hence B is NOT extraneous
- Check if C is extraneous in $A \rightarrow BCF$:
  $F'$={ $A \rightarrow BF$, $B \rightarrow DE$, $D \rightarrow A$ }
  $A^+$ under $F'$ = ABDEF and $A^+$ does not include C, hence C is NOT extraneous
- Check if F is extraneous in $A \rightarrow BCF$:
  $F'$={ $A \rightarrow BC$, $B \rightarrow DE$, $D \rightarrow A$ }
  $A^+$ under $F'$ = ABCDE and $A^+$ does not include F, hence F is NOT extraneous
- Check if D is extraneous in $B \rightarrow DE$:
  $F'$={ $A \rightarrow BCF$, $B \rightarrow E$, $D \rightarrow A$ }
  $B^+$ under $F'$ = BE and $B^+$ does not include D, hence D is NOT extraneous
- Check if E is extraneous in $B \rightarrow DE$:
  $F'$={ $A \rightarrow BCF$, $B \rightarrow D$, $D \rightarrow A$ }
  $B^+$ under $F'$ = BDACF and $B^+$ does not include E, hence E is NOT extraneous

As a result, the original set F={ $A \rightarrow BCF$, $B \rightarrow DE$, $D \rightarrow A$} is a canonical cover

c) r1= (A,B,C,F)
   r2= (B,D,E)
   r3= (D,A)

**Q.5** Given relation r as r = (A, B, C, D, E) and the set F of functional dependencies as F = {A → BE, C → D, B → C}, give a BCNF decomposition of r using the functional dependencies only in F.

**Solution**

For A → BE: A is a super key, hence no violation of BCNF
For C → D: C is NOT a super key of r, hence
r1=(CD) is in BCNF
r2=(ABCE)
For B → C: B is NOT a super key of r2, hence
r21=(BC) is in BCNF
r22=(ABE) is in BCNF
The BCNF decomposition of r is (C,D), (B,C), (A,B,E)

**Q.6** Considering the relation r(A,B,C,D,E,F) and the set of functional dependencies as F ={C→AE, B → C, D → ECB, A → F}
Check if the decomposition of r as below is dependency preserving:
r1(A,C) r2(B,C,F) r3(C,D,E)

**Solution**

- **for C→AE we have:**

result=C

using r1:
$(\text{result} \cap r1)^+ = C^+ = CAEF$
$t=(\text{result} \cap r1)^+ \cap r1 = CAEF \cap r1 = AC$
$\text{result} = \text{result} \cup t = AC$

using r2:
$(\text{result} \cap r2)^+ = (AC \cap BCF)^+ = C^+ = CAEF$
$t=(\text{result} \cap r2)^+ \cap r2 = CAEF \cap BCF = CF$
$\text{result} = \text{result} \cup t = ACF$

using r3:
$(\text{result} \cap r3)^+ = (ACF \cap CDE)^+ = C^+ = CAEF$
$t=(\text{result} \cap r3)^+ \cap r3 = CAEF \cap CDE = CE$
$\text{result} = \text{result} \cup t = ACFE$
no need to repeat over since the result includes rhs of the functional dependency C→AE

- **for B → C we have:**

result=B

using r1:

$(result \cap r1)^+ = \emptyset$

$t=(result \cap r1)^+ \cap r1 = \emptyset$

$result = result \cup t = B$

using r2:

$(result \cap r2)^+ = B^+ = BCAEF$

$t=(result \cap r2)^+ \cap r2 = BCAEF \cap BCF = BCF$

$result = result \cup t = B \cup BCF = BCF$

no need to proceed as the result includes C

- **for D → ECB we have:**

result = D

using r1:

$(result \cap r1)^+ = \emptyset$

$t=(result \cap r1)^+ \cap r1 = \emptyset$

$result = result \cup t = D$

using r2:

$(result \cap r2)^+ = \emptyset$

$t=(result \cap r2)^+ \cap r2 = \emptyset$

$result = result \cup t = D$

using r3:

$(result \cap r3)^+ = D^+ = DECBAF$

$t=(result \cap r3)^+ \cap r3 = CDE$

$result = result \cup t = CDE$

using r1:

$(result \cap r1)^+ = C^+ = CAEF$

$t=(result \cap r1)^+ \cap r1 = AC$

$result = result \cup t = CDE \cup AC = CDEA$

using r2:

$(result \cap r2)^+ = (CDEA \cap BCF)^+ = (C)^+ = CAEF$

$t=(result \cap r2)^+ \cap r2 = CAEF \cap BCF = CF$

$result = result \cup t = CDEAF$

using r3:

$(result \cap r3)^+ = (CDEAF \cap CDE)^+ = (CDE)^+ = ABCDEF$

$t=(result \cap r3)^{+} \cap r3 = ABCDEF \cap CDE = CDE$
$result = result \cup t = CDEAF$

using r1:
$(result \cap r1)^{+} = (CDEAF \cap AC)^{+} = (AC)^{+} = ACFE$
$t=(result \cap r1)^{+} \cap r1 = ACFE \cap AC = AC$
$result = result \cup t = CDEAF \cup AC = CDEAF$
since the result does not change, we stop here
As the rhs of functional dependency $D \rightarrow ECB$ is not included in the result, we conclude that the decomposition is not dependency preserving.