Time-reversal invariant Chalker-Coddington model and the real-space renormalisation group

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University of Warwick

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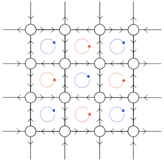
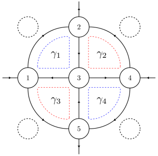
"Real-space renormalisation approach to the Chalker-Coddington model revisited: improved statistics"
S. Shaw, R. A. Römer Physica E 165, 116073 (2025)

Outline

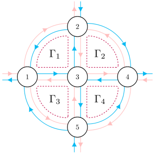
1. Chalker-Coddington model



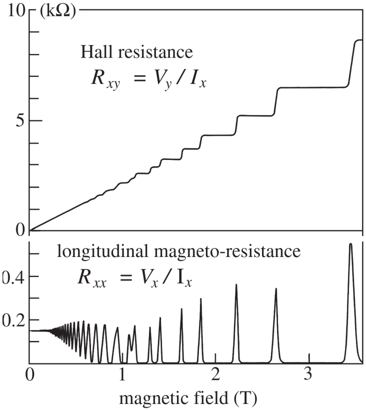
2. Real-space renormalisation group (RSRG)



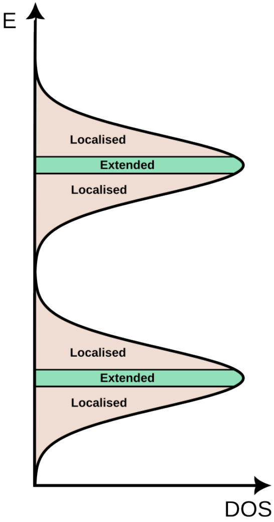
3. Time-reversal invariance



Quantum Hall effect and localisation



Weis and von Klitzing, Phil. Trans. R. Soc. A(2011) 369, 3954-3974



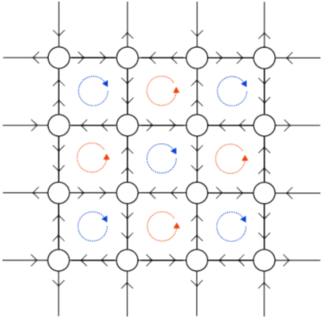
$$\xi \propto \frac{1}{|x - x_x|^{\nu}}$$

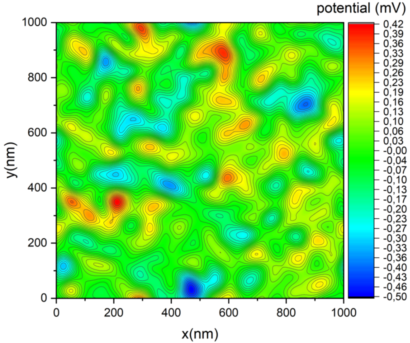
ν	model	method	reference
∞	short-range impurities	self-consistent perturbation	[21]
≈ 2	Peierls tight binding	transfer matrix scaling	[22]
≈ 2.0	short-range impurities	recursive Green function	[23,24]
2.35(3)	random Landau matrix	recursive Green function	[19,20]
2.3(1)	random Landau matrix	recursive Green function	[25]
2.4(2)	random Landau matrix	recursive Green function	[26]
2.4(1)	finite range impurities	Chern number scaling	[27]
≈ 2.3	spin orbit scattering	Thouless number scaling	[28]
≈ 2	double layer system	Thouless number scaling	[29]
≈ 2	random matrix model	scaling of level statistics	[30]
2.5(5)	Chalker-Coddington	transfer matrix scaling	[31]
2.4(2)	random saddle points	transfer matrix scaling	[32]
2.5(5)	Chalker-Coddington type	real space renormalization	[33]
2.39(1)	Chalker-Coddington type	real space renormalization	[34,35,36]
2.5(4)	super spin chain	density matrix renormalization	obtained from [37]
2.33(3)	counter-propagating chiral Fermions	Monte Carlo	[38]

Kramer, Ohtsuki, Kettemann, Phys. Rep.(2005) 417, 221-342

Chalker-Coddington Model

1. 2D Disordered potential landscape
2. Semi-classical





Percolation, quantum tunnelling and the integer Hall effect

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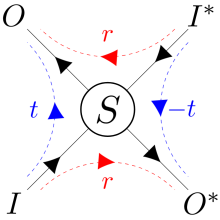
Received 16 October 1987

Römer and Oswald, Ann. Phys.(2021) 435

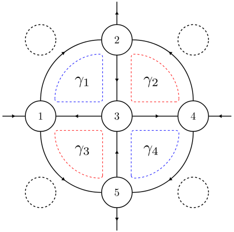
$$S = \begin{pmatrix} t & r \\ r & -t \end{pmatrix}$$

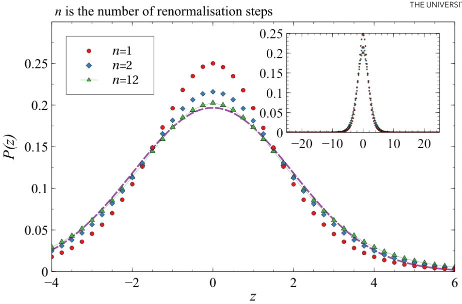
Real-space renormalisation group method

Single saddle point



Super saddle point





$$S = \begin{pmatrix} t & r \\ r & -t \end{pmatrix} \quad \text{or} \quad S = \begin{pmatrix} i\cos\theta & \sin\theta \\ \sin\theta & i\cos\theta \end{pmatrix}$$

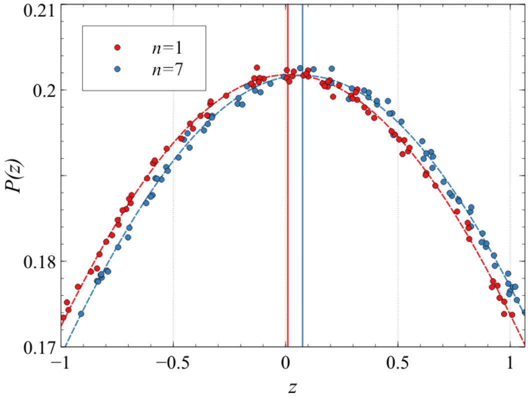
$$|t|^2 + |r|^2 = 1$$

$$t^2 = \frac{1}{1+e^z}$$

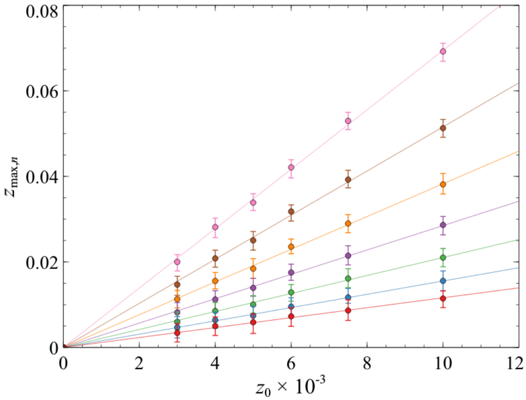
z is proportional to saddle point height

$$t' = \left| \frac{e^{i(\phi_1+\phi_4-\phi_2)} r_1 r_3 r_5 t_2 t_4 + e^{i(\phi_1+\phi_4)} t_2 t_4 - e^{i\phi_4} t_1 t_3 t_4 + e^{i\phi_3} r_2 r_3 r_4 t_1 t_5 - e^{i\phi_1} t_2 t_3 t_5}{-1 - e^{i\phi_3} r_2 r_3 r_4 + e^{i\phi_2} r_1 r_3 r_5 + e^{i(\phi_2+\phi_3)} r_1 r_2 r_4 r_5 + e^{i\phi_1} t_1 t_2 t_3 - e^{i(\phi_1+\phi_4)} t_1 t_2 t_4 t_5 + e^{i\phi_4} t_3 t_4 t_5} \right|$$

Critical exponent of localisation

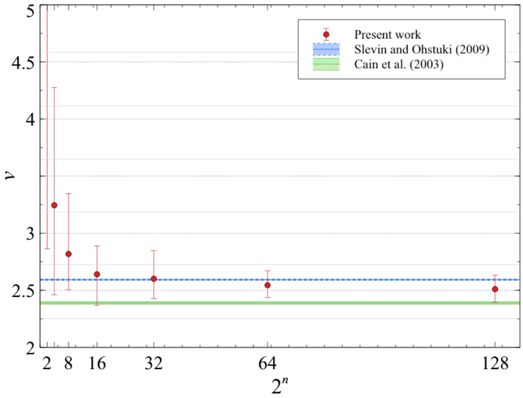


Plot has reduced no. of points for clarity
In practise, each iteration uses distribution of
5×10^9 samples



$$\nu = \frac{\ln 2^n}{\ln \left[z_{\max}(n) / z_{\max}(0) \right]}$$

Revised estimates of exponent




Real-space renormalisation approach to the Chalker–Coddington model revisited: Improved statistics

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Current estimate using RSRG:	$\nu \approx 2.51$
Previous estimate using RSRG:	$\nu \approx 2.39$

Time-reversal invariance and the quantum spin Hall effect

1. QHE breaks TRI via magnetic field
2. No magnetic field
3. Spin-orbit coupling causes non-trivial topology
4. Quantum spin Hall insulator

Quantum Spin Hall Effect

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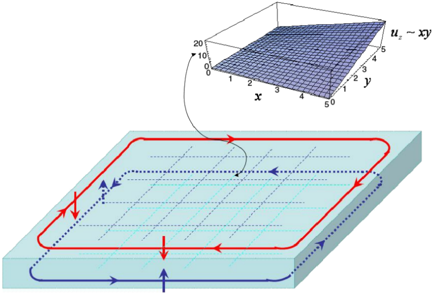
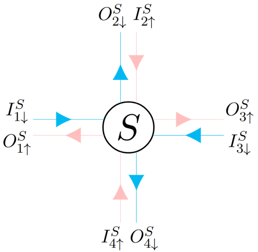
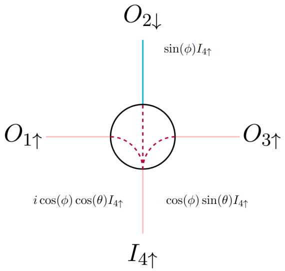
Department of Physics, Stanford University, Stanford, California 94305, USA
(Received 18 April 2005; published 14 March 2006)



Time-reversal invariant Chalker-Coddington model





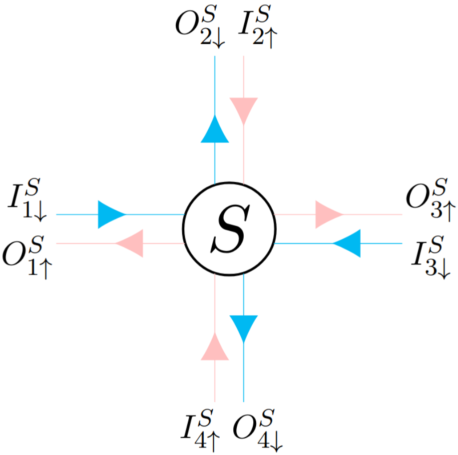
$$S = \begin{pmatrix} 0 & i\cos\phi\cos\theta & \cos\phi\sin\theta & \sin\phi \\ i\cos\phi\cos\theta & 0 & -\sin\phi & \cos\phi\sin\theta \\ \cos\phi\sin\theta & \sin\phi & 0 & i\cos\phi\cos\theta \\ -\sin\phi & \cos\phi\sin\theta & i\cos\phi\cos\theta & 0 \end{pmatrix}$$

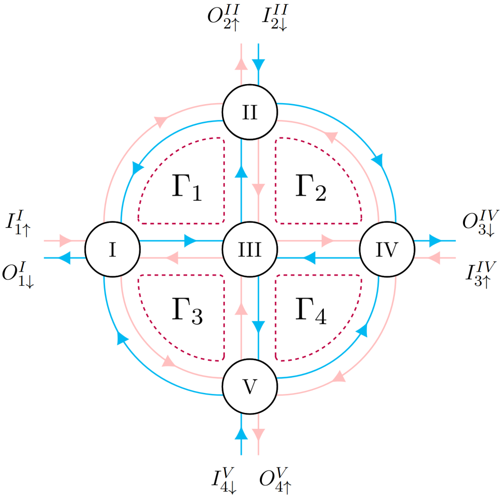
$$\cos(\theta) = t$$

$\theta =$ scattering angle

$\phi =$ spin mixing angle

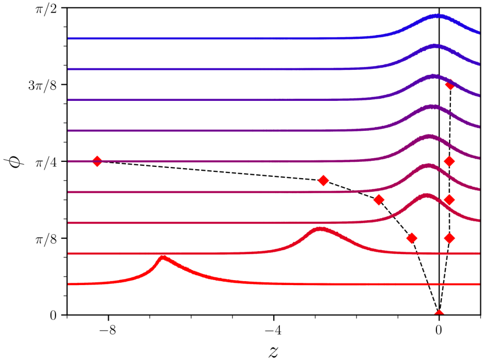
Time-reversal invariant RSRG

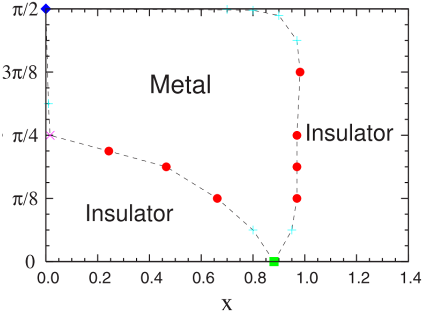




Current findings

Preliminary phase diagram





Obuse, Furusaki, Ryu, Mudry, Phys. Rev. B 76, 075301

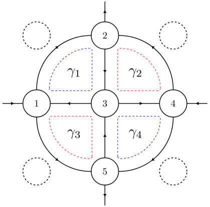
Further work and Conclusion

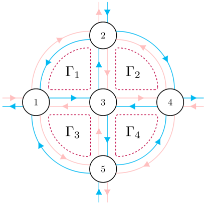
What we have done

1. RSRG for CC model
2. Renewed calculation for localisation critical exponent ν
3. Extension to time-reversal invariant model

What we want to do

1. Critical exponent of topological transition
2. 3 dimensional Chalker-Coddington model





Thank you for your time!