Quantum Information Engines: Assessing Time, Cost and Performance Criteria

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Information engine (IE, sometime nicknamed Maxwell Demon engines) uses information acquired by measurement to direct engine operation. Here, using a general von-Neumann measurement model, we investigate the crucial role of the time and energy cost of information acquisition in determining the performance, in particular the efficiency and power output, of these devices. We illustrate for an exemplary IE model that the information gain and subsequently the extracted work can increase with measurement time (t_m), however, there is a corresponding increase in the energetic cost. The efficiency of converting information into free energy diminishes as t_m approaches both 0 and infinity, peaking at an intermediate measurement time.

Introduction- A prominent example of engine conversion devices are heat engines which operate between reservoirs at different temperatures. Alternatively, a single heat bath may be used as the energy source in feedback controlled devices [1–17], referred to below as information engines (IEs), in which information about the system's state is obtained by some "Maxwell demon" and used to control the engine's operation [11, 18]. The second law is accounted for by the entropy increase during the demon's restoration to its initial state, also implying a minimal added operation cost, Landauer's erasure work [19]. In the quantum version of such devices the demon's acquisition of information is often described as a quantum measurement process with a prescribed action on the system, often utilizing positive operator-valued measures (POVM) with Kraus operators while disregarding the actual physical nature of information acquisition [See, e.g. [20–22]]. Such an approach makes it possible to investigate important thermodynamics characteristics of information engines (such as the aforementioned Landauer lower bound on the unavoidable dissipation, which recent studies have shown to be compatible with fluctuation theorems of stochastic thermodynamics [2, 3, 23]). However, the physical process of information acquisition and the subsequent resetting of measurement devices delivering this information are typically finite time processes, aspects that have not received widespread attention in the literature (see Refs. [10, 24–26] as noteworthy early exceptions). Recognizing and quantifying the intrinsic temporal and energetic expenditures of these processes are essential, as they are intrinsically linked and needed for evaluating performance measures such as efficiency and operational power. This letter addresses these issues for the first time. Although these matters are examined within a specific IE model, their implications are general as become clear below.

IE model- In our IE model (Fig. 1) the working entity (SYS) is a 2-state system (2SS) with the energy of its lower eigenstate $|0\rangle$ set as zero and the upper state $|1\rangle$ at energy ΔE. This system is monitored by coupling it (the interaction $V(t)$ in Fig. 1) to a meter M modeled as an otherwise free particle. As a consequence of this coupling, SYS and M becomes correlated during their interaction such that the state of M contains information about SYS. The Hamiltonian of this combined 2SS-M-system reads $\hat{H} = \Delta E\,|1\rangle\langle 1| + \frac{\hat{p}^2}{2} + \hat{V}(t)$ where $\hat{p}$ is the mass weighted momentum operator. In the present analysis we take $\hat{V}(t) \equiv \hat{V} = g \cdot \hat{x} \otimes |1\rangle\langle 1|$, for $0 \leq t \leq t_m$, with coupling constant g and position operator $\hat{x}$, and $\hat{V}(t) = 0$ otherwise, so that during the measurement time t_m the meter responds to the system only if the latter is in state 1. The system and meter are coupled to their thermal environments of temperature T_S and T_M, respectively (following [27] we use $T_M = 0$). Further, the meter state is monitored by a "Maxwell demon" (M1) and the information so obtained is used to control energy extraction from the excited 2SS as described below. The detailed IE engine

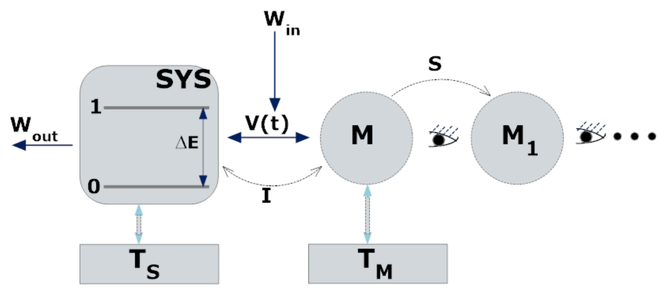

FIG. 1. General schematics of an IE model. A system (SYS) and a meter (M), each coupled to their own thermal bath of temperatures T_S and T_M, respectively, are entangled by an interaction $V(t)$. The state of M is projectively monitored by another meter M_1 accompanied by an entropy flow S between M and M_1. Information I on the state of the system, obtained by the meter, is used to extract energy W_{out} form the system bath. The measurement time and its energy cost W_{in} are computed and used to calculate the energy efficiency and operating power.

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cycle is described by the following steps:

(i) *Initial state:* The 2SS is taken to be initially in thermal equilibrium with a bath at temperature T_S. The meter is taken to start at a wavepacket state of the form

$$D(p) = \langle p|D\rangle = \left(\frac{2}{\pi\hbar^2 B}\right)^{1/4} e^{-p^2/\hbar^2 B} \quad (1)$$

in the momentum representation, whose width B represents a lower bound on its inverse size (in mass weighted length units). This defines the initial density matrix of the combined system as

$$\hat{\rho}(t=0) = \hat{\rho}_S(t=0) \otimes \hat{\rho}_M(t=0), \quad (2)$$

with

$$\hat{\rho}_S(t=0) = a\,|0\rangle\,\langle 0| + b\,|1\rangle\,\langle 1|\,;\ \hat{\rho}_M(t=0) = |D\rangle\,\langle D|\,, \quad (3)$$

where a and b are real positive numbers satisfying $T_S = \Delta E/k_B[\ln(a/b)]^{-1}$ and $a+b=1$.

(ii) *Entangling evolution:* During the time interval $(0, t_m)$, the system and meter evolve under the Hamiltonian $\hat{H}$, leading to an entangled state described by the density matrix $\hat{\rho}(t_m) = e^{-i\hat{H}t_m/\hbar}\hat{\rho}(0)e^{i\hat{H}t_m/\hbar}$.

(iii) *Projective measurement & information gain:* Following the entangling evolution, the state of the meter is projectively determined to be the eigenstate $|p\rangle$ of the momentum operator. It is assumed that this process is instantaneous and involves no energy cost [28]. Note that, in principle, a sequence of meters can be visualised, each determines the state of the preceding one [29], however here we take the simpler approach cutting the measurement process after the meter M (see Fig. 1), essentially assuming that the entity that observes it is classical. The conditional probability of the 2SS to be in state $i = 0; 1$ given the meter outcome p is thus determined by $P_i(t > t_m|p) = \langle i|\,\hat{P}(p, t_m)\,|i\rangle\,/Q(p, t_m)$ where $\hat{P}(p, t_m) \equiv \langle p|\hat{\rho(t_m)}|p\rangle$ and $Q(p, t_m) = \sum_{i=0}^{1} \langle i|\,\hat{P}(p, t_m)\,|i\rangle$. $\hat{P}(p, t_m)$ is the joint system-meter density operator to be in state $i = 0; 1$ and measuring the meter outcome p, which reads

$$\hat{P}(p, t > t_m) = P_0(p, t)\,|0\rangle\,\langle 0| + P_1(p, t)\,|1\rangle\,\langle 1|\,, \quad (4)$$

where for our IE model, $P_0(p, t) = \sqrt{\frac{2}{\pi\hbar^2 B}}\,a e^{-\frac{2p^2}{\hbar^2 B}}$ and $P_1(p, t) = \sqrt{\frac{2}{\pi\hbar^2 B}}\,b e^{-\frac{2(p+gt)^2}{\hbar^2 B}}$ [30].

The information gain, $I(t_m)$, in this measurement process can be quantified by averaging the conditional system entropy $S(t_m|p) = -k_B \sum_{i=0}^{1} P_i(t_m|p) \ln P_i(t_m|p)$ over an ensemble of identical measurements, $S(t_m) = \int dp Q(p, t_m) S(t_m|p)$, leading to [31, 32]

$$I(t_m) \equiv S(0) - S(t_m), \quad (5)$$

where $S(t_m) = -k_B \int dp \sum_{i=0}^{1} P_i(p, t_m) \ln P_i(t_m|p)$ and $S(0) = -k_B(a \ln a + b \ln b)$.

(iv) *Work extraction:* We assume, in an idealized setup, that if the 2SS is known to be in its excited state, this excitation energy can be fully extracted (eg., by stimulated emission [7, 9, 10]). Additionally, any attempt to extract this energy involves unknown costs. The need to consider such costs may be circumvented by focusing on the excess energy gain, which for the meter outcome p is given by [33]

$$G(p, t_m) = \Delta E[P_1(t_m|p) - P_1(0|p)] = \Delta E[P_1(t_m|p) - b]. \quad (6)$$

A productive use of the information engine is achieved by restricting the photon extraction attempts to events for which the measurement outcome p indicates that the 2SS probability to be in the excited state 1 is large enough relative to its thermal value. For our model these are events in which the meter outcome is smaller than some bound $p < p'$ where $-\infty < p' < 0$ (see Fig. 2 (A) and discussion below and in [30]). The average useful energy extracted per cycle is therefore

$$W_{out}(t_m, p') = \int_{-\infty}^{p'} dp Q(p, t_m) G(p, t_m). \quad (7)$$

(v) *Restoration:* Following the measurement-informed extraction of useful energy, the engine cycle is closed by restoring the 2SS and meter to their initial states. The former is brought back to equilibrium with its thermal environment at temperature T_S. Because the photon extraction step leaves the 2SS in its lower (ground) state $|0\rangle$, restoring thermal equilibration involves heat transfer from the bath which on the the average must be equal to the gain (7). For the meter, a route for preparing it in the initial state $|D\rangle\,\langle D|$ (Eq. (1)) may start by bringing it to equilibrium with a zero temperature bath, followed by adiabatically confining it in a harmonic potential for which Eq. (1) is the ground state and then suddenly releasing the confinement leaving a free particle in the state $|D\rangle\,\langle D|$ (see energetic discussion below).

Consider next the energy cost of the cycle described above which is the sum of two contributions:

(a) The energy needed to create the system-meter entanglement is henceforth referred to as the measurement energy, W_{meas}. Because energy is conserved during the unitary evolution of the interacting 2SS and meter, the cost of this process is associated with switching the system-meter interaction on and off, which is given by

$$W_{meas}(t_m) \equiv \mathrm{tr}[\hat{\rho}(0)\hat{V}] - \mathrm{tr}[\hat{\rho}(t_m)\hat{V}], \quad (8)$$

where $\mathrm{tr}[\dots] \equiv \int dp \sum_{i=0,1} \langle p|\,\langle i|\dots|i\rangle\,|p\rangle$. This energy may be thought of as the work done by the agent who switches the interaction on and off (more generally, who affects the time-dependence of $\hat{V}$). In writing Eq. (8) we have assumed that this switching is instantaneous. For our choice of initial states and system-meter interaction $\mathrm{tr}[\hat{\rho}(0)\hat{V}] = 0$, namely switching on the interaction costs no energy.

(b) The energy needed to restore the meter to its initial state (1). Because $T_M = 0$, the Landauer entropic cost $T_M S$ associated with erasure of Demon memory can be disregarded, however energy is needed to reconstruct state (1) and is estimated as follows; (a) Return the particle to its ground state by contacting it with a zero-temperature bath. (b) Adiabatically confine the particle in a harmonic potential of frequency $\Omega = \hbar B/2$ for which the wavepacket (meter state in Eq. (3)) is the ground state. This step incurs an investment equal to the zero-point energy $\hbar\Omega/2$. (c) Suddenly remove the harmonic confining potential, a step that provides no energy gain. This set the wavepacket preparation energy cost as $W_{prep} = \hbar\Omega/2 = \hbar^2 B/4$ and the total energy for the measurement as

$$W_{in}(t_m) = W_{meas}(t_m) + W_{prep}. \tag{9}$$

To show the dependence of these quantities on the measurement time t_m it is convenient to express time in terms of a timescale defined by the coupling constant and the parameters of the initial meter wavepacket Eq. (1). Henceforth, time is represented in terms of the reduced quantity $\bar{t} = t/\tau^*$ where $\tau^* = 2b\frac{\sqrt{\langle\delta\hat{p}^2(t=0)\rangle}}{|d\langle\hat{p}(t)\rangle/dt|_{t=0}} = \frac{\sqrt{\hbar^2 B}}{g}$, and where $|d\langle\hat{p}(t)\rangle/dt|_{t=0}$ is the initial change rate of the meter momentum (see [34]).

Information gain and energetic cost- Fig. 2 (A) illustrates the conditional probability $P_{i=0;1}(t_m|p)$ to be in the excited state given the meter outcome p. Obviously, $P_{i=0;1}(t_m = 0|p) = a, b$ is independent of p. For $t_m > 0$, the evolution of these probabilities may be written as $a \to a - \delta$ and $b \to b + \delta$, where, if g is chosen positive, $\delta > 0$ if the meter outcome is negative ($p < 0$), and $\delta < 0$ when $p > 0$, indicating a higher or lower likelihood that the 2SS is in the excited state, respectively. The information gain $I(t_m)$ (Eq. (5)) and measurement cost $W_{meas}(t)$ (Eq. (9) first term) are depicted in Fig. 2 (B) as function of t_m for different initial 2SS states defined by $b/a = \exp[-\Delta E/k_B T_S]$. Three observations are notable:

(i) The information gain is a monotonously increasing function of t_m that approaches the entropy of the initial state, $-k_B(a\ln a + b\ln b)$, as $t_m \to \infty$. This stands in contrast to the model of Ref. [10] where, because of the discrete nature of the meter (another two state system), the dependence on t_m reflects the intrinsic Rabi-oscillation in the system-meter dynamics.

(ii) The rate of information gain, given by the slope $dI(\bar{t}_m)/d\bar{t}_m$ of $I(\bar{t}_m)$ in Fig. 2 (B), is maximal near $t_m = \tau^*$, a time determined by the width of the initial meter wavepacket and the system-meter coupling.

(iii) As measurement time t_m increases the information gain I approaches its maximal value. However, in the model considered, the measurement energy cost W_{meas} (first component in Eq. (9)) increases indefinitely, (see dotted lines in Fig. 2 (B)) resulting in a decreasing trend of the information gain to energy ratio. As already stated, in realistic settings t_m may be controlled by the range of system-meter interaction.

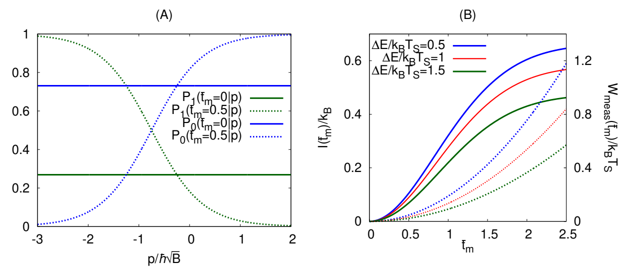

FIG. 2. (A) The conditional probability $P_{i=0;1}(\bar{t}_m|p)$ that a 2SS is in state 0 or 1 given that the meter outcome is p. The 2SS is initially in thermal equilibrium with T_S. The parameters used are $T_S = 300$K, $\Delta E = k_B T_S$ and the initial meter state is given by Eq. (1) with $\hbar^2 B = 25.85$meV (This choice of B corresponds to $W_{prep} = k_B T/4$ with $T = 300$K). The horizontal lines represent the system's initial state, and the dotted lines are for $\bar{t}_m = 0.5$. (B) The information gain $I(\bar{t}_m)$ (solid lines, left axis) and the measurement energy cost $W_{meas}(\bar{t}_m)$ (dotted lines, right axis) plotted against measurement time $\bar{t}_m$ for different choices of $\Delta E/k_B T_S$ with $\hbar^2 B = 25.85$meV and $\Delta E = 25.85$meV (which corresponds to $\Delta E = k_B T$ with $T = 300$K).

Efficiency and Power output- We define the IE's efficiency of work extraction by [27, 35, 36] $\eta(t_m, p') = W_{out}(t_m, p')(Q_{in}(t_m, p') + W_{in}(t_m))^{-1}$, where $Q_{in}(t_m, p')$ is the average heat per cycle taken from the system thermal bath in order to return the 2SS back to thermal state following work (photon) extraction. This average heat is equal to the average work extracted per cycle $Q_{in}(t_m, p') \equiv W_{out}(t_m, p')$. Therefore

$$\eta(t_m, p') = \frac{1}{1 + W_{in}(t_m)/W_{out}(t_m, p')}. \tag{10}$$

It is important to note that our idealized model disregards other physical processes in which part of this energy input might be lost, such as non-radiative decay of the 2SS. Eq. (10) should therefore be regarded as an upper bound to the efficiency of a realistic engine. Also, the average power output of the engine per measurement cycle [37], is obtained from

$$\Pi(t_m, p') = \frac{W_{out}(t_m, p')}{t_m}. \tag{11}$$

Fig. 3 shows these efficiency $\eta(t_m, p')$ and power output $\Pi(t_m, p')$, against the measurement time t_m for different bounds p' for triggering photon extraction attempt. Several observations follow:

(i) Choosing $p' \to \infty$, that is disregarding the measurement outcome in proceeding with photon extraction attempts, leads to vanishing efficiency and power, that is zero gain in the IE operation. Conversely, for $p' \to -\infty$, the 2SS is determined to be in its excited state with probability approaching 1, so photon extraction attempt is assured. However, in this limit $Q(p, t_m)$ in Eq. (7) vanishes, therefore $W_{out}(t_m, p' \to -\infty) = 0$, implying $\eta(t_m, p' \to -\infty) = 0$.

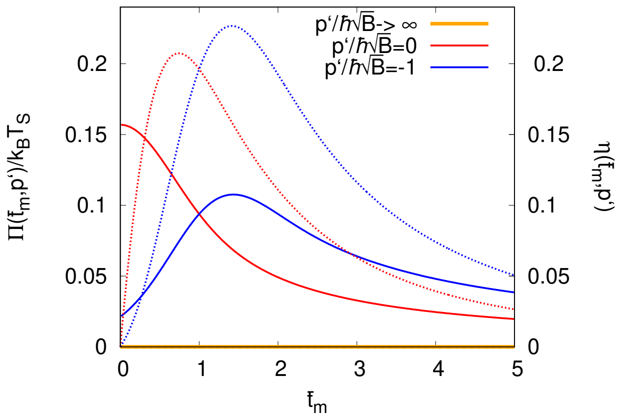

FIG. 3. Output power $\Pi(\bar{t}_m, p')$ (solid lines, left axis) and engine's efficiency $\eta(\bar{t}_m, p')$ (dotted lines, right axis) shown as functions of system-meter interaction time $\bar{t}_m$ for different values of threshold parameter p' to attempt photon extraction by stimulated emission. Parameters are the same as those used in Fig. 2 (A).

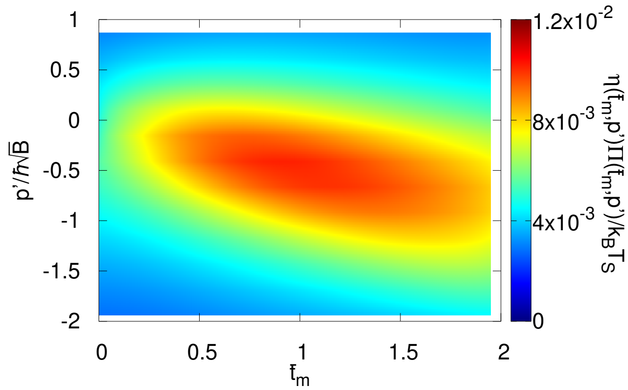

FIG. 4. Product of engine power and efficiency $\eta(\bar{t}_m, p')\Pi(\bar{t}_m, p')$ as color plot in dependence of upper bound of p' where we send a photon in for potentially stimulating a photon emission and various system-meter interaction time $\bar{t}_m$. Parameters are the same as those used in Fig. 2 (A).

(ii) For a given finite p', the IE efficiency increases as t_m increases from zero (see Fig. 3), indicating a finite measurement time is needed for the IE operation.

(iii) For the IE model considered, in which the system-meter interactions remains constant until cutoff at time t_m, the saturation in the time of information gain (see Fig. 2(B)) and increasing energy cost imply that the efficiency vanishes at $t_m \to \infty$. Consequently, $\eta(t_m, p')$ exhibits a peak at some intermediate measurement time.

(iv) In the limit $\bar{t}_m \to 0$ the power output is given by $\Pi(\bar{t}_m \to 0, p') = ab\Delta E\sqrt{\frac{2}{\pi}}e^{\frac{-2p'^2}{\hbar^2 B}}$ [38]. For $p' \geq 0$, this is the maximal value of Π, which is a monotonously decreasing function of $t_,$ as exemplified by $p' = 0$ in Fig. (3).

(v) For $p' < 0$, the average power output goes through a maximum as function of t_m. Its initial increase with t_m reflects again the fact that the that the system and the meter has to interact for enough time to affect a useful work output. The decrease at long time results from the fact that the IE is extracting at most energy ΔE per cycle whose duration increases as $t_m \to \infty$ while information gain saturates.

As performance quantifiers, the efficiency η and power Π provide complementary views of machine operations [39]. Their product, $\eta(\bar{t}_m, p')\Pi(\bar{t}_m, p')$, may be used as a balanced quantifier, see [40]. The heat map in Fig. 4 displays this product against the measurement time $\bar{t}_m$ and the threshold parameter p'. Importantly, a finite measurement time is required for optimal operation. Note that both the efficiency and power trend to zero as $t_m \to \infty$ because of the runaway energy cost and saturation in work extraction and information gain in this limit.

While not necessarily the only way, the simplest meter resetting involves equilibration with a thermal bath [19]. In such case the IE operates in principle between two (not necessarily different) temperatures and it is interesting to compare its performance to that of a standard heat engine. First, unlike a heat engine, part of the energy input, W_{in} of Eq. (9), needed for the IE operation is 'useful' work, rather than heat. Therefore, $W_{out}/W_{in} > 1$, is a performance requirement which in our model translates into $\eta > 1/2$ (see Eq. (10)). While examples in Fig. 3 fall below this limit, this criterion can be satisfied by increasing ΔE and T_S while keeping their ratio $\Delta E/k_B T_S$ constant (see Fig. (S1) in [30]). Second, the IE efficiency might exceed the Carnot limit $\eta_C = 1 - T_M/T_S$ only if T_M is larger than some threshold value. We leave detailed considerations of this issue to future work.

In summary, we have analyzed an information engine (IE) model which highlights the importance of considering measurement time and the energy cost associated with its operation. Although details of these IE characteristics depend on the process used for information acquisition, their determination for the information acquisition protocol used for any IE model is essential for estimating standard performance quantifiers such as engine efficiency and power. Our findings suggest that there is an optimal time for information acquisition beyond which increasing energy cost lead to diminishing returns. Using the product of efficiency and power as a performance quantifier we are able to identify the regime of optimal engine performance. In comparison to standard heat engine we find that our 2SS IE model is advantageous at large system energy gap and high system temperature, provided that its efficiency exceeds 0.5.

We emphasize that our general considerations, in particular the need to address the energy and time involved in the measurement process and meter resetting, apply to any IE model. Moving forward, examination of other

IE models with different working and measurement protocols are needed to delve further into the complexities of the measurement process, including entropic costs at finite temperatures of the measurement channel. This study lays the groundwork for investigating energy and entropy costs in realistic information-based engines and processes, particularly in quantum optic set-ups as proposed in previous research studies (e.g., [5]).

ACKNOWLEDGMENTS

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the entropic part of the free energy loss.

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[30] *Supplemental Material.*

[31] *Note that "ensemble average" here refers to average over the meter outcome, in contrast to the standard meaning as average over initial conditions.*

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[33] *This assumpltion has to be applied with caution: In thermal equilibrium no net gain can be achieved, implying that only the excess probability following measurement (relative to the thermal one) that the molecule is in the excited state must be associated with this net gain.*

[34] *See Eqs. (S.5) and (S.6) in [30].*

[35] *The definition of the efficiency for the process described here needs some scrutiny: The process converts heat* $Q_{in}(t_m, p')$ *extracted from the system thermal bath into useful photon energy* $W_{out}(t_m, p')$ *- both are given equal and given by Eq.* (7)*. This comes as with an additional energy cost by* $W_{in}(t_m, p')$*, Eq.* (9)*. One may choose to subtract the latter energy from the gain, leading to an efficiency expression of the from* $(Q_{in}(t_m, p') - W_{in}(t_m))/Q_{in}(t_m, p')$ *(reminiscent to the expression* $1 - Q_{low}/Q_{high}$ *of a Carnot engine). Another choice that we have opted for is to regard* $W_{in}(t)$ *as an additional investment, leading to Eq.* (10)*. See also discussion in [36].*

[36] S. Seah, S. Nimmrichter, and V. Scarani, Maxwell's lesser demon: A quantum engine driven by pointer measurements, Phys. Rev. Lett. **124**, 100603 (2020).

[37] *In making this identification we assume that the time needed for the photon extraction itself as well as the relaxation time associated with the restoration step discussed below can be disregarded. Otherwise these times should be added to* t_m.

[38] *See Eq. (S.8) in [30].*

[39] *The product of efficiency and power is a technique to analyze the performance of a device. To give an example: In the context of the Carnot cycle, this product is an important performance quantifier. Remind that a classical heat engine operating at Carnot efficiency must function adiabatically slow with an infinite cycle time, and thus with zero power.*

[40] C. Y. Wang, H. and H. Zhang, Characteristic analysis of new hybrid compensation topology for wireless charging circuits., J. Power Electron. **21**, 1309–1321 (2021).