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ON ENTROPY REDUCTION IN A THERMODYNAMIC SYSTEM BY  
INTERFERENCE BY INTELLIGENT SUBJECTS

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ON ENTROPY REDUCTION IN A THERMODYNAMIC SYSTEM BY  
INTERFERENCE BY INTELLIGENT SUBJECTS

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in Berlin

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We investigate what conditions cause one to construe an apparent perpetual motion of the second type if one allows an intelligent subject to influence a thermodynamic system. By having subjects make such measurements, they produce a behavior of the system which is clearly different from an isolated mechanical system. We show that a type of recall ability can offer an occasion to a continuous reduction of entropy. This recall is distinguished in a system in which measurements occur, and would lead to a contradiction of the second law if the measurements themselves did not necessarily proceed under entropy production. At first this entropy production is calculated universally from the requirement that it represents a complete compensation in the sense of the second law (Equation 1). Using an apparatus which is able to implement continuous measurements (under continuous entropy production), the corresponding amount of entropy is calculated. We found that it is just as large as is necessary for complete compensation: the actual entropy production on measurement, therefore, does not have to be larger than that determined by Equation (1).

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\* Numbers in margin indicate pagination in original foreign text.

There is already an historical objection to the general validity of the second law of thermodynamics which actually makes a quite disturbing impression. This is the objection of the Maxwellian Demon which still comes up today in different forms (and perhaps with some justification, inasmuch as quantitative relationships seem to be hidden behind the precise questions which have not yet been clarified). The objection is, in its original formulation one which operates with a demon which catches the fast molecules and allows the slow ones to pass. One can refute this by the fact that we humans cannot, in principle, discover the value of the thermal, varying parameters; but it cannot be denied that we could measure the value of such oscillating parameters very well, and could then surely obtain data on the costs of the warmth by arranging our concepts appropriately to the result of the measurements. Of course, it remains to be decided whether we do not make an error if we do not consider the interfering subject himself, in the system, along with his life processes.

Except for this point, it is known today that in an isolated 1841 system, in spite of the occurrence of oscillation, no perpetual motion, second type, can be effective, i.e., no machine using automatic heat of lowest temperature of continuous, finite capability, can be effective. This would be a machine whose effectiveness would have the result, even over long periods of time, that a weight would be raised at the cost of heat in a heat reservoir.

Expressed in another way, one can also say that, if we want to use the occurrences of variations in order to get data on the costs of the heat, we are in the same position as in a game of chance at which we can win certain amounts from time to time, but at which the expected value (mathematical hope) of winnings is zero or negative. The same is true for a system which is not

isolated, but at which the external interferences are highly periodic, as by machines moving periodically. We consider this statement as proven [1] and will only consider the difficulties which occur when subjects interfere in the system, and to try to recognize the quantitative relationships occurring in this.

Smoluchowski writes [2]: "As far as our present knowledge reaches, there is no automatic, continuously effective perpetual motion in spite of molecular vibrations, but such an apparatus could function regularly if it were operated by intelligent subjects in a suitable manner. ...

"Thus, a perpetual motion is possible if one considers the experimenting person, according to the usual methods of physics, as a kind of "Deus ex machina" which is continuously and exactly informed of the instantaneous condition of nature. This "Deus ex machina" can set in motion or interrupt the macroscopic natural processes at any moment. Thus, he does not at all need to possess, like the Maxwellian demon does, the ability to catch individual molecules, but would be completely different in the above points from real persons. The production of any kind of physical effect, by operation of the sensor or motor nervous system, is always connected with depreciation of energy, except that its entire existence is connected to a continuous dissipation of same.

"It is doubtful that real persons could produce continuous or at least uniform, work at the cost of heat at the lowest temperature, considering these circumstances, even though our lack of knowledge of life processes precludes a definitive answer. However, the questions just touched upon lead beyond actual physics ...."

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It seems to us that the lack of knowledge of life processes should not be disturbing for recognizing that with which our work is concerned. Because we may be certain that one can replace the subjects (inasmuch as concerns their interference in a thermodynamic system) by any device whose "life processes" can be followed. Thus, one could determine whether a compensation actually occurs for the entropy reduction which is produced by the interference of the device in the system.

At first, we want to try to recognize what causes the entropy reduction on interference of intelligent subjects in a thermodynamic system. We shall see that it depends on a coupling of a special type between various parameters of the system. We shall observe an especially simple type of this threatening coupling. We will shortly speak of a "measurement" whenever we succeed in connecting the value of one parameter "y" (e.g., the distance coordinate of the pointer of a measuring instrument) at one instant, with the momentary value of a varying parameter "x" of the system. Thus, one can draw conclusions on the value of "x" at the moment of measurement from the value of "y". Then, "x" and "y" are separated again after the measurement so that "x" can change, whereas "y" retains its value a little while. Such measurements are no harmless interference; a system in which such measurements occur indicates a type of remembrance ability, in the sense that one can recognize what kind of value a different parameter, "x", had at an earlier time by the condition parameter "y". And we shall see that such ability of recollection would damage the second law if the process of measurement would take place without compensation. We will then see that the second law is not so greatly threatened by this entropy reduction as one might originally think. We recognize that the entropy reduction caused by the interference would be completely compensated if the implementation of such measurements were universally dependent on an entropy production of  $k \cdot \log 2$ ,

for example. It would then be possible to prescribe a somewhat general entropy law which is based universally on all measurements. In conclusion, a very simple apparatus (not arbitrarily chosen) is under consideration. This device is in the position to undertake continuous "measurements", and we may easily follow its "life processes". By direct calculation, one actually finds a continuous entropy production in the amount which is required by the mentioned, general entropy law which is derived from the validity of the second law.

The first example, which we consider typical, is the following: a standing hollow cylinder which is closed top and bottom, can be divided into two unequal halves of volumes  $V_1$  and  $V_2$  by pushing in a solid disc from the side. This disc forms a "piston" which can be pushed up and down inside the cylinder. An infinite heat reservoir of some temperature  $T$  insures that a gas present in the cylinder performs an isothermic expansion on movement of the piston. Now, this gas should consist of one single molecule which roams around in the whole cylinder according to its thermal motion, as long as the piston is not shoved into the cylinder.

Let's now imagine a subject who shoved the piston into the cylinder at some point in time, and simultaneously determines (somehow) whether the molecule is in the upper or lower half of the cylinder, in volume  $V_1$  or  $V_2$ . If he finds that the former is the case, then he pushes the piston slowly downward, until the base of the cylinder is reached. During this slow pushing of the piston, the molecule naturally remains above the piston at all times, but does not remain in the top half of the cylinder, but rather strikes the top of the moving piston many times. In this manner, the molecule does a certain amount of work on the piston. This is the work which corresponds to an isothermic expansion of an ideal gas consisting of one molecule, performed from volume  $V_1$  on volume  $V_1 + V_2$ . After a time, when the piston has reached the

base of the container, the entire volume  $V_1 + V_2$  is again available to the molecule, and the piston is then pulled out. The process can be repeated as often as desired, where the subject pushes the piston up or down depending on whether the molecule is in the upper or lower half. That is, the subject is so coupled to a weight which is to be raised by a mechanical transfer of force, that the weight is pushed up by the piston. In this manner, the potential energy of the weight certainly increases continuously. (The transfer of force to the weight is so represented, that the force transferred from the weight to the piston, at any position of the piston balances the average pressure of the gas). It is clear that continuous work can be obtained at the cost of heat, inasmuch as the life processes of the interfering subject are not considered in the calculation.

In order to recognize what the subject actually performs for the system here, one should imagine that the motion of the piston is done mechanically and that the whole activity of the subject is exhausted in determining the height of the molecule and in adjusting a lever left or right (which guides the motion of the piston) according to which motion of the piston, upwards or downwards, is required by the position of the molecule. That is, the interference of the subject consists in the coupling of two position coordinates, namely an "x" coordinate which determined the altitude of the molecule, with another coordinate "y", which defines the position of the lever and which is thus decisive in giving the piston an up or down motion. One should imagine the mass of the piston to be large and its speed to be sufficiently high so that the thermal agitation, which the piston has at the operating temperature, can be neglected.

We shall differentiate two time periods in the processes of the typical example under consideration here, namely:

1. The time period of measurement, at which the piston is just shoved into the middle of the cylinder and the molecule is shut into either the upper or lower half. Thus, we limit the  $x$ -coordinate to the interval  $x > 0$  or  $x < 0$  by choosing an appropriate coordinate origin.

2. The time period of utilization of the measurement, during which the up or down motion of the piston is accomplished, the "time period of entropy reduction". During this period, the  $x$ -coordinate of the molecule is by no means limited to the original  $x > 0$  or  $x < 0$ . If the molecule, at the time of measurement is in the upper half of the cylinder so that  $x > 0$ , then it must strike the piston which is now moving down if it is to contribute to this work. That is, the  $x$ -coordinate of the molecule must enter the interval  $x < 0$ . However, the lever will retain its setting corresponding to the downward motion during the entire time period. If this right position of the lever is determined by  $y = 1$  (and the corresponding left-position  $y = -1$ ), we see that the measurement of the position  $x > 0$  is paired with  $y = 1$  during the time interval. We also note that later,  $y = 1$  remains, even after  $x$  has passed into the other interval  $x < 0$ . One sees that on utilization of the measurement, the coupling of the two parameters,  $x$  and  $y$ , is lost. /845

We will speak quite generally about this, that a parameter " $y$ " measures the value of a parameter " $x$ ", which is varying by a probability law, whenever the value of " $y$ " changes to the value of the parameter " $x$ " at a definite point in time. Taking this measurement is the underlying principle of entropy reduction by interference by intelligent subjects.

It is now suggested to assume that taking the measurement is principally connected with a quite certain average entropy production. By this, agreement with the second law is again

restored. The amount of entropy occurring on measurement may always be larger, but not smaller. More exactly, two quantities of entropy must be differentiated here, the first,  $\bar{S}_1$ , is produced if "y" has the value of 1 on measurement, and the other,  $\bar{S}_2$ , when  $y = 1$ . We can not expect to learn anything, generally, about  $\bar{S}_1$  or  $\bar{S}_2$ , individually. But we shall indeed see from the assumption that the entropy produced by this "measurement" must be compensated by the utilization of the resultant entropy reduction of the system (in the sense of the second law), that the quite general relation

$$e^{-\frac{\bar{S}_1}{k}} + e^{-\frac{\bar{S}_2}{k}} \leq 1 \quad (1)$$

follows.

From this formula, one sees that we can make one of the quantities,  $\bar{S}_1$ , as small as desired, but that the other quantity,  $\bar{S}_2$ , becomes correspondingly large. It is also striking that it is not at all dependent on the size of the interval observed. /846  
Of course, one can easily see that it could not possibly be otherwise.

We can say the converse: as soon as the entropies  $\bar{S}_1$  and  $\bar{S}_2$  (occurring by the measurement) satisfy inequality (1), we may rest assured that the attainable entropy reduction is already completely compensated by the measurement.

Before we go into the proof of inequality (1), we can see from the observed mechanical example how this all fits together. For the produced entropys  $\bar{S}_1$  and  $\bar{S}_2$ , we write the special formula:

$$\bar{S}_1 = \bar{S}_2 = k \log 2.$$

This statement obeys inequality (1), and the average value of measurement-produced entropy is:

$$\bar{s} = k \log 2$$

(in this special case, it is naturally independent of the ratio of frequency  $w_1, w_2$  of both occurrences). In the observed example, one achieves entropy reduction\* by isothermal expansion:

$$-s_1 = -k \log \frac{V_1}{V_1 + V_2}; \quad -s_2 = -k \log \frac{V_2}{V_1 + V_2},$$

regardless of whether the molecule is encountered in volume  $V_1$  or  $V_2$  when the piston is pushed in. (Entropy reduction equals the quotient of the amount of heat withdrawn from the heat reservoir by isothermal expansion, and the temperature of the respective heat reservoir.) Since in the present case, the frequency  $w_1, w_2$  of both occurrences is in the ratio as the volume  $V_1, V_2$ , then the average amount of produced entropy (negative number) is:

$$s = w_1 \cdot (-s_1) + w_2 \cdot (-s_2) = \frac{V_1}{V_1 + V_2} k \log \frac{V_1}{V_1 + V_2} + \frac{V_2}{V_1 + V_2} k \log \frac{V_2}{V_1 + V_2}.$$

One sees that actually

$$\frac{V_1}{V_1 + V_2} k \log \frac{V_1}{V_1 + V_2} + \frac{V_2}{V_1 + V_2} k \log \frac{V_2}{V_1 + V_2} + k \log 2 \geq 0$$

and thus  $\bar{s} + \bar{s} \geq 0$ .

Thus, in the present special case, there would be a full compensation for the attained entropy reduction on utilization of the measurement. /847

Let us now observe other special examples and try to derive formula (1) and define the above relationships with the aid of a general observation. Let us now think of the entire system as

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\* The entropy produced is designated  $\bar{s}_1, \bar{s}_2$ .

divided into several samples, which are all enclosed in a common box. In this system, "x" is subject of some kind of thermal oscillations, and can be measured by "y" in the manner described. Each individual section can be moved independently, so that the individual systems can be considered as the molecules of an ideal gas which wander around in the common box as a consequence of thermal agitation. These molecules exert a pressure on the walls of the box according to the temperature. We now consider two of these molecules as chemically different, and separable by a semi-permeable membrane in case the x-coordinate of one molecule is in a prominent interval, and is outside of the interval for the other molecule. We will likewise consider them chemically different if they differ only by having values of the y-coordinate of 1 and -1, respectively.

Let us shape the "box" as a hollow cylinder, in which four pistons are placed (see Figure 1). "A" and "A'" are fixed, the two others, B and B' are moveable so that the distance BB' is always equal to the distance AA', as is indicated in the figure by the brackets. A', the base, and B, the top of the whole container, are impenetrable by all molecules, however, A and B' are semi-permeable; A is only permeable for those molecules for which the parameter "X" lies in the prominent interval  $x_1 x_2$ , B' is only permeable for the others.

At first, piston B is at A, and thus B' is at A' and all molecules are located in the space between. For a certain fraction of the molecules, "x" is found in the prominent interval. The probability of this for any given molecule is designated  $w_1$ , the probability of "x" being outside the interval is  $w_2$ .



Figure 1.

Then:

$$w_1 + w_2 = 1$$

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The distribution of the  $y$ -parameter may be in any ratio of the two values 1 and -1, but is independent of the  $x$ -values in any case. We now imagine an interference by a subject, by which "y" receives the value 1 in all molecules, "x" falls in the prominent interval at the time concerned, and corresponds to the value of -1 in the opposite case. If, as a consequence of thermal variation, the parameter "x" leaves the prominent interval for any molecule, or, as we can express it here, if the molecule suffers a monomolecular chemical reaction with respect to "x", then the parameter "y" retains its value of 1 at first so that the molecule is able to "remember" during the entire later process that "x" originally fell in the prominent interval. The monomolecular chemical reaction causes the molecule, which was permeable to piston A, to become impermeable. We shall see shortly what role this "remembrance" can play. We now push in the pistons as described above so that we separate the two kinds of molecules from each other without doing any work. Two containers are then present, of these, one gets one fraction, the other gets the other fraction. Now, each fraction alone takes up the same volume as the mixture did before. In one of these containers, an equilibrium distribution with respect to the two "modifications in 'x'", is not present. The quantity ratio of the two modifications (fractions) is still  $w_1:w_2$ . If we let this equilibrium distribution occur by itself at constant volume and temperature in each vessel, then the entropy of the system definitely increases during the process. Because the total heat production is 0, since the ratio of both "modifications in x",  $w_1:w_2$  does not change. If we lead the adjustment of the equilibrium distribution in both containers by reversible paths, then the entropy of the surroundings would decrease by the same amount. Thus, the entropy increases by a negative amount, and the value of entropy increase per molecule is:

$$\bar{s} = k(w_1 \log w_1 + w_2 \log w_2). \quad (2)$$

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(The entropy constants which we have assigned to the two "modifications in 'x'" do not explicitly occur here, because the process which belongs to one or the other modification leaves the total number of molecules unchanged.)

Naturally, we could return the two gases to their original volumes without doing any more work by simply pushing back the piston, since the volume determined by the piston BB' contains such molecules whose x-coordinates fall outside of the prominent interval and for whom piston A is no longer permeable. So we see that the entropy reduction found (2) does not mean a contradiction to the second law. So long as we do not use the fact that the molecules in vessel BB' of coordinate "y" can "remember" that the x-coordinate of the molecules in this container originally lay in the prominent interval, then complete compensation is actually present for the calculated entropy reduction; the partial pressures in both vessels are now smaller than in the original mixture.

But we can use the fact that all molecules in vessel BB' have a y-coordinate of 1 and those in the other vessel have a y-coordinate of -1, in order to return all molecules to the original volume. For this, we merely replace semipermeable wall A by wall A\*, which is semipermeable with respect to "y" and not to "x", so that it is permeable for molecules with y-coordinate 1, and impermeable for the others. Conversely, we replace B' by a piston B'\* which is impermeable for molecules with y = -1 and permeable for the others. Then both vessels can be pushed together again without expending work. The distribution of the y-coordinates to 1 and -1 has become statistically independent of the x-values and, moreover, we can produce the original distribution to 1 and -1: thus we pass through a complete circular

process. The only change that we have to register is the attained entropy reduction of the amount (2),

$$\bar{s} = k(w_1 \log w_1 + w_2 \log w_2).$$

Thus, if we do not want to admit that the second law is violated, then we must conclude that the interference which causes the coupling between "x" and "y", the measurement of "x" by "y", is insolubly connected to the entropy production. If a definite path for production of this coupling is given, and if the necessary amounts of entropy produced are designated  $S_1$  and  $S_2$ , then the average produced entropy is given by

$$w_1 S_1 + w_2 S_2 = S.$$

Here  $S_1$  is the average entropy increase which occurs when "y" has the value 1, and  $S_2$  is that which occurs when "y" has the value -1. In order for the second law to be consistent, this amount of entropy must be larger than the entropy reduction  $\bar{s}$  which is attainable by the evaluation of the measurement according to (2). Thus

$$\begin{aligned} \bar{s} + s &\geq 0, \\ w_1 S_1 + w_2 S_2 + k(w_1 \log w_1 + w_2 \log w_2) &\geq 0. \end{aligned} \quad (3)$$

must be valid. This inequality must be true for any values of  $w_1$  and  $w_2$ \*. Naturally the peripheral condition  $w_1 + w_2 = 1$  must be retained. In particular, we are asking for those values of  $w_1$  and  $w_2$  for which the expression has a minimum for given  $S$ -values. For this pair of values,  $w_1$  and  $w_2$ , inequality (3) must still hold. Under the mentioned peripheral conditions, the minimum occurs when

$$\frac{S_1}{k} + \log w_1 = \frac{S_2}{k} + \log w_2 \quad (4)$$

\* The increase in entropy can only be dependent on the type of measurement and its result, and not on how many systems of one type or another are present.

is true. But then

$$e^{-\frac{S_1}{k}} + e^{-\frac{S_2}{k}} \leq 1.$$

This can be seen very easily if one introduces the equation

$$\frac{S_1}{k} + \log w_1 = \frac{S_2}{k} + \log w_2 = \lambda$$

Then

$$w_1 = e^{\lambda - \frac{S_1}{k}}; \quad w_2 = e^{\lambda - \frac{S_2}{k}}. \quad (5)$$

If we substitute these values into inequality (3), there results

$$\lambda e^{\lambda} \left( e^{-\frac{S_1}{k}} + e^{-\frac{S_2}{k}} \right) \geq 0.$$

Thus  $\lambda \geq 0$  holds. But if we substitute the values for  $w_1$  and  $w_2$  from (5) in the equation  $w_1 + w_2 = 1$ , we obtain:

$$e^{-\frac{S_1}{k}} + e^{-\frac{S_2}{k}} = e^{-\lambda}.$$

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And because  $\lambda \geq 0$ ,

$$e^{-\frac{S_1}{k}} + e^{-\frac{S_2}{k}} \leq 1. \quad (6)$$

is also true. This formula must be generally true if the thermodynamics are not to be violated.

As long as we let the interference be done by intelligent subjects, a direct check must be excluded. However, we can try to give simple, specific devices which cause such a coupling, and observe whether entropy is actually produced and in what quantity. Not until we have recognized that it depends on a definite characteristic kind of coupling, a "measurement" will be needed to construct some complicated model which significantly imitates the interference of the subject. Thus, for the present, we can be satisfied with the creation of this special coupling equipped with memory.

In this example, the position coordinates of a moving pointer are "measured" by the energy content of a body K. At first, the pointer will purely mechanically effect that the body K be measured by its energy content, according to the position of the pointer with one of two distances, A or B. The pointer works by heat so that the body is connected to A as long as the coordinates determined by the pointer fall in a certain prominent interval "a" of arbitrary length. The opposite holds with B in interval "b". Both distances are in thermal contact with a heat reservoir of temperature  $T_0$  up to the moment of "measurement". At this moment, distance A is cooled to temperature  $T_A$  by a periodically functioning mechanical device. This is done by successive contact with heat reservoirs of intermediate temperatures in contact with a heat reservoir of temperature  $T_A$ . Simultaneously, B is warmed to temperature  $T_B$  in the same manner. Then, the distances are again isolated from the heat reservoirs.

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We assume that the position of the pointer changes so slowly that all operations sketched here proceed at one and the same pointer position. If the position coordinate of the pointer in the prominent interval fell, then the body would be connected to interval A during the mentioned operation, and would be consequently cooled to temperature  $T_A$ . Conversely, the body is now warmed to temperature  $T_B$ . Its energy content is thus, according to the position of the pointer at the moment of "measurement", small corresponding to temperature  $T_A$  or large corresponding to temperature  $T_B$ . It retains its value even when the pointer leaves the prominent interval in the course of time. After a time when the pointer carries out its further oscillations, no more conclusions can be made on the instantaneous position of the pointer from the energy content of body K, rather, a sure conclusion on the position of the pointer at the time of measurement. Then the measurement is completed.

The mentioned periodically functioning mechanical device which makes the interference for us should bring the now thermally isolated segments A and B into direct contact with heat reservoir  $T_o$  after completion of the measurement. This has the purpose of bringing body K (now in contact with one of the two intervals) back to its original condition as it was before the "measurement". The direct connection of the intervals A and B, and of the cooled or heated body K with the reservoir  $T_o$ , necessarily causes an increase in entropy. This cannot be avoided, because it would be useless to try to warm the interval A by successive contact with reservoirs of intermediate temperatures to temperature  $T_o$ , or likewise to try to cool B to  $T_o$ . After the completed measurement we do not know which of the two intervals is now in contact with body K, and likewise, we do not know whether body K was last in contact with  $T_A$  or  $T_B$ . We also do not know whether we should use intermediate temperatures between  $T_A$  and  $T_o$  or between  $T_o$  and  $T_B$ .

The average amounts of entropy produced in this manner,  $\bar{S}_1$  and  $\bar{S}_2$ , can be determined if the heat capacity as a function of the temperature  $\bar{u}(T)$  for body K is known, since the entropy is /853 to be calculated from the heat capacity. We naturally abstract from the heat capacity of the intervals, A and B. If the position coordinates of the pointer at the time of "measurement" were in the prominent interval, and if the body was in contact with interval A, then the entropy on successive cooling of the heat reservoirs is:

$$\int_{T_A}^{T_o} \frac{1}{T} \frac{du}{dT} dT$$

Thus the entropy of this heat reservoir on direct contact with heat reservoir  $T_o$  is:

$$\frac{\bar{u}(T_o) - \bar{u}(T_A)}{T_o}$$

Thus, the sum of the entropy is increased by:

$$S_A = \frac{\bar{u}(T_A) - \bar{u}(T_0)}{T_0} + \int_{T_0}^{T_A} \frac{1}{T} \frac{d\bar{u}}{dT} dT \quad (7)$$

Analogous to this, the entropy would increase by

$$S_B = \frac{\bar{u}(T_B) - \bar{u}(T_0)}{T_0} + \int_{T_0}^{T_B} \frac{1}{T} \frac{d\bar{u}}{dT} dT \quad (8)$$

when the body was in contact with interval B at the moment of "measurement".

Now let's evaluate these expressions for the special, simple case when the body possesses only two energy states, a lower and upper. If such a body with a heat reservoir of some temperature  $T$  is in thermal contact, then the probability of its being in the upper or lower state is given by

$$\left. \begin{aligned} p(T) &= \frac{1}{1 + g e^{-\frac{u}{kT}}} \\ q(T) &= \frac{g e^{-\frac{u}{kT}}}{1 + g e^{-\frac{u}{kT}}} \end{aligned} \right\} \quad (9)$$

or

Here, " $u$ " is the energy difference of the two states and "g" is the statistical weight. We can equate the energy of the lower state to zero, without this being a limitation. There results\* /854

$$\left. \begin{aligned} S_A &= q(T_A) k \log \frac{q(T_A) p(T_0)}{q(T_0) p(T_A)} + k \log \frac{p(T_A)}{p(T_0)} \\ S_B &= p(T_B) k \log \frac{q(T_0) p(T_B)}{q(T_B) p(T_0)} + k \log \frac{q(T_B)}{q(T_0)} \end{aligned} \right\} \quad (10)$$

or

\* See the appendix at the conclusion of the work.

Here  $q$  and  $p$  are the functions of  $T$  given by formula (9) which are to be taken here for the arguments  $T_0$ ,  $T_A$ , and  $T_B$ .

If we now want to determine that a certain conclusion regarding the position coordinate of the pointer is possible from the energy content of the body  $K$ , we must show that the body got into the lower state whenever it comes into contact with  $T_A$ , or in the higher state when in contact with  $T_B$ . This means that:

$$p(T_A) = 1, \quad q(T_A) = 0; \quad p(T_B) = 0, \quad q(T_B) = 1$$

This cannot be proven, but can be approximated if we let  $T_A$  go to absolute zero, and the statistical weight "g" go to infinity. [In going to the limit,  $T_0$  is so changed that  $p(T_0)$  and  $q(T_0)$  remain unchanged.] From the formula (10) results:

$$S_A = -k \log p(T_0); \quad S_B = -k \log q(T_0). \quad (11)$$

Now if we form the expression  $e^{-\frac{S_A}{k}} + e^{-\frac{S_B}{k}}$  we find that:

$$e^{-\frac{S_A}{k}} + e^{-\frac{S_B}{k}} = 1.$$

The limit can be realized by our previous conclusions; the use of semipermeable walls according to Figure 1 allows a complete utilization of the measurement: inequality (1) certainly cannot be intensified.

As we have seen from this example, a simple device can accomplish what interference by an intelligent subject would provide, with respect to that which is important. In this example, we have pursued the "life processes" of a device and we can see that the requisite amount of entropy is produced, just exactly as provided by thermodynamics.

## APPENDIX

In the case under consideration, when the frequency of both conditions depends on the temperature according to the formulas:

$$p(T) = \frac{1}{1 + g e^{-\frac{u}{kT}}}; \quad q(T) = \frac{g e^{-\frac{u}{kT}}}{1 + g e^{-\frac{u}{kT}}}$$

and thus the average energy of the body is given by

$$\bar{u}(T) = u q(T) = \frac{u g e^{-\frac{u}{kT}}}{1 + g e^{-\frac{u}{kT}}}$$

then the identity

$$\frac{1}{T} \frac{d\bar{u}}{dT} = \frac{d}{dT} \left\{ \frac{\bar{u}(T)}{T} + k \log \left( 1 + g e^{-\frac{u}{kT}} \right) \right\}.$$

is valid.

With this, we can write the expression

$$s_A = \frac{\bar{u}(T_A) - \bar{u}(T_0)}{T_0} + \int_{T_0}^{T_A} \frac{1}{T} \frac{d\bar{u}}{dT} dT$$

thus:

$$s_A = \frac{\bar{u}(T_A) - \bar{u}(T_0)}{T_0} + \left\{ \frac{\bar{u}(T)}{T} + k \log \left( 1 + g e^{-\frac{u}{kT}} \right) \right\}_{T_0}^{T_A},$$

and when we pass to the limits, we obtain

$$s_A = \bar{u}(T_A) \left( \frac{1}{T_0} - \frac{1}{T_A} \right) + k \log \frac{1 + g e^{-\frac{u}{kT_0}}}{1 + g e^{-\frac{u}{kT_A}}}.$$

If we now substitute in the last expression according to (9),

$$1 + g e^{-\frac{u}{kT}} = \frac{1}{p(T)}$$

for  $T_A$  and  $T_0$ , we obtain

$$s_A = \bar{u}(T_A) \left( \frac{1}{T_0} - \frac{1}{T_A} \right) + k \log \frac{p(T_A)}{p(T_0)}.$$

and if then, according to (12), we substitute  
 $\bar{u}(T_A) = uq(T_A)$

there results

$$S_A = q(T_A) \left( \frac{u}{T_0} - \frac{u}{T_A} \right) + k \log \frac{p(T_A)}{p(T_0)}.$$

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If we substitute according to (9)

$$\frac{u}{T'} = -k \log \frac{q(T)}{q_p(T)}$$

for  $T_A$  and  $T_0$ , we obtain

$$S_A = q(T_A) k \log \frac{p(T_0) q(T_A)}{q(T_0) p(T_A)} + k \log \frac{p(T_A)}{p(T_0)}.$$

We obtain the corresponding formula for  $S_B$  by exchanging Index A for B. Then we get:

$$S_B = q(T_B) k \log \frac{p(T_0) q(T_B)}{q(T_0) p(T_B)} + k \log \frac{p(T_B)}{p(T_0)}.$$

The former is the formula which is given in the text for  $S_A$ .

We can also express formula for  $S_B$  somewhat differently, by writing

$$q(T_B) = 1 - p(T_B)$$

multiplying out and adding the first and last member. It then becomes:

$$S_B = p(T_B) k \log \frac{q(T_0) p(T_B)}{p(T_0) q(T_B)} + k \log \frac{q(T_B)}{q(T_0)}.$$

This is the formula given in the text for  $S_B$ .

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