

Week 2

MA123: Mathematical Reasoning & Modeling
(Spring 2021)

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Solving Percent Applications

- Percents have a wide variety of applications to everyday life.
- They show up often in:
 - Taxes
 - Discounts
 - Markups
 - Interest rates

Example 1:

Jeff has a coupon at the Guitar Store for 15% off any purchase of \$100 or more. He wants to buy a used guitar that has a price tag of \$220 on it. Jeff wonders how much money the coupon will take off of the \$220 original price.

Coupon take off is 15% of 220

$$0.15 \times 220 = \$33$$

Example 2:

The U.S. Weather Bureau has a station on Mauna Loa in Hawaii that has measured carbon dioxide levels since 1959. At that time, there were 319 parts per million of carbon dioxide in the atmosphere. In 2005, the figure was 372 parts per million.

a) Find the increase in carbon dioxide levels.

$$\text{Increase} = \text{Level in 2005} - \text{Level in 1959} = 372 - 319 = 53$$

b) Find the percent of increase, to two decimal places.

$$A = P \times B$$

$$372 = P \times 319$$

$$P = \frac{372}{319} \times 100 = 116.6\% \approx 117\%$$

The increase is 17%

Example 2:

Another way
to solve b)

The U.S. Weather Bureau has a station on Mauna Loa in Hawaii that has measured carbon dioxide levels since 1959. At that time, there were 319 parts per million of carbon dioxide in the atmosphere. In 2005, the figure was 372 parts per million.

a) Find the increase in carbon dioxide levels.

$$\text{Increase} = \underset{\text{in 2005}}{\text{Level}} - \underset{\text{in 1959}}{\text{Level}} = 372 - 319 = 53$$

b) Find the percent of increase, to two decimal places.

$$\frac{\text{Increase}}{\text{Level in 1959}} \times 100 = \frac{53}{319} \times 100 = 16.6\% \approx 17\%$$

Example 3:

Susana worked 20 hours at her job last week. This week, she worked 35 hours. In terms of a percent, how much more did she work this week than last week?

$$A = P \times B$$

$$35 = P \times 20$$

$$P = \frac{35}{20} \times 100 = 175\%$$

Susana worked 75% more this week than she did last week.

Example 3:

Susana worked 20 hours at her job last week. This week, she worked 35 hours. In terms of a percent, how much more did she work this week than last week?

Another way to answer the question

$$\begin{aligned}\text{Increase} &= \text{Hours this week} - \text{Hours last week} \\ &= 35 - 20 = 15\end{aligned}$$

$$\begin{aligned}\text{Percent Increase} &= \frac{\text{Increase}}{\text{Hours last week}} \times 100 = \frac{15}{20} \times 100 = 75\%\end{aligned}$$

Susana worked 75% more this week than she did last week.

Example 4:

The population of Range Town is currently 2000 and expected to grow by 4.4% over the next year. What will its population be by then? **2088**

$$\begin{aligned}\text{Growth} &= 4.4\% \text{ of } 2000 \\ &= 0.044 \times 2000 = 88\end{aligned}$$

$$\begin{aligned}\text{The population next year} \\ &= \text{Current Population} + \text{Growth} \\ &= 2000 + 88 = 2088\end{aligned}$$

Example 5:

The population of Dullsville, on the other hand, is currently 3000 and expected to decrease by 5% over the next year. What will its population be by then?

$$\begin{aligned}\text{Decrease} &= 5\% \text{ of } 3000 \\ &= 0.05 \times 3000 = 150\end{aligned}$$

$$\begin{aligned}\text{The population next year} \\ &= \text{Current Population} - \text{Growth} \\ &= 3000 - 150 = 2850\end{aligned}$$

Note

Decrease = - Growth

Example 6:

A pair of trousers is on sale for \$30. If the sales tax rate is 6.7%:

a) What is the amount of sales tax?

$$6.7\% \text{ of } \$30 = 0.067 \times 30 = \$2.01$$

b) What is the total cost of the trousers?

$$\$30 + \$2.01 = \$32.01$$

Example 7:

A hand-bag is on sale for \$8.15 and has been discounted 40%. What is the original price of the hand-bag?

Let $P = \text{Original Price}$

$D = \text{Discount Amount}$

$$D = 0.4 \times P = 0.4P$$

$$P - 0.4P = 8.15$$

$$0.6P = 8.15$$

$$P = \frac{8.15}{0.6} = \$13.58$$

Simple Interest:

- **Interest** is a fee or charge for borrowing money.
 - It is usually expressed as a percent rate charged per year.
 - We can compute simple interest by finding the interest rate percentage of the amount borrowed, then multiply by the number of years interest is earned.

Simple Interest Equation

$$I = p \cdot r \cdot t$$

Where: I is the interest paid.

p is the principal – the original amount of money borrowed.

r is the interest rate, a per-year rate, written as a decimal

t is the time of the loan, expressed in years or portions of a year

Example 8:

Treasury Notes (T-notes) are bonds issued by the federal government to cover its expenses. Suppose you obtain a \$1,000 T-note with a 4% annual rate, with a maturity in 4 years. How much interest will you earn?

$$I = p \cdot r \cdot t$$

$$p = \$1000, r = 0.04$$

$$t = 4 \text{ years}$$

$$\begin{aligned} I &= 1000 \cdot 0.04 \cdot 4 \\ &= \$160 \end{aligned}$$

You will earn \$160 interest total over the four years.

Example 9:

A friend asks to borrow \$240, offering to repay you \$250 in 1 month.

What annual interest rate is this equivalent to?

$$I = p \cdot r \cdot t$$

$$I = 250 - 240 = 10$$

$$p = 240, r = ?, t = \frac{1}{12}$$

$$10 = 240 \cdot r \cdot \frac{1}{12}$$

$$r = \frac{10 \cdot 12}{240} = \frac{120}{240} = 0.5$$

$$r = 0.5 \times 100\%$$

This is equivalent to a 50% annual interest rate. $= 50\%$

Commission

Example 10:

A salesman receives \$35 in commission on sales amounting to \$700. What is the salesman's commission rate in this situation?

$$\text{Commission Rate} = \frac{35}{700} \cdot 100\% \\ = 5\%$$

Example 11:

A salesman works for a base salary of \$410 a month plus 8% commission on all the merchandise he sells beyond \$6,000. If he sells \$8,500 worth of merchandise in one month, what will his total salary be for the month?

Commission

$$\begin{aligned}\text{Base Salary per month} &= \$410 \\ \text{Commission on sales} \\ \text{beyond } \$6000 &= 0.08 \times \$2500 \\ &= \$200\end{aligned}$$

$$\begin{aligned}\text{Total Salary for the month} \\ &= \$410 + \$200 \\ &= \$610\end{aligned}$$

Example 12:

How many shirts will a salesperson need to sell to make \$35.25 in commission if the commission rate is 5% and the shirts sell for \$15 apiece?

Commission

$$\text{Commission per shirt} = 0.05 \cdot 15 = \$0.75$$

$$\frac{\$0.75}{1} = \frac{\$35.25}{x}$$

$$x = \frac{\$35.25}{\$0.75} = 47$$

47 shirts need to be sold to make \$35.25 in commission.