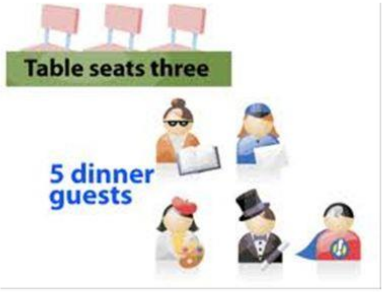
Permutations



Counting Methods-Permutations

Instructor:

Emmanuel Thompson

Counting Methods-Permutations

Fundamental Counting Principle

- The fundamental counting principle states that:
  - If there is a sequence of independent events that can occur $a_1, a_2, a_3, \cdots, a_n$ ways, then the number of ways all the events can occur is:

$$a_1 \cdot a_2 \cdot a_3 \cdot \cdots \cdot a_n$$

Factorials

Factorial Notation

If n is a positive integer, n factorial denoted by $n!$ is a product of all positive integers less than or equal to n. It is defined by

$$n! = n(n-1)(n-2)\ldots(2)(1)$$

As a special case: $0! = 1$

Note: $n! = n(n-1)!$

Examples:

$5! = 5\times4\times3\times2\times1$
$= 120$

$7! = 7\times6\times5\times4\times3\times2\times1$
$= 5040$

$\frac{8!}{6!} = \frac{8\times7\times6!}{6!}$
$= 8\times7$
$= 56$

$\frac{n!}{(n-2)!} = \frac{n(n-1)(n-2)!}{(n-2)!}$
$= n(n-1)$

Counting Methods-Permutations

Permutation (Repetition Not Allowed, Order Matters)

- Permutations apply the Fundamental Counting Principle to determine the number of ways you can arrange members of a group.
- The permutation formula calculates the number of arrangements of *n* objects taken *r* at a time:

$$P(n,r) = \frac{n!}{(n-r)!}$$

Counting Methods-Permutations

Permutations with Repeated Items

The number of different permutations of n objects, where there are n_1 indistinguishable items, n_2 indistinguishable items, . . . , and n_k indistinguishable items, is

$$\frac{n!}{n_1! \cdot n_2! \cdot \cdots \cdot n_k!}$$

Counting Methods-Permutations

Q1:

A boy owns 7 pairs of pants, 4 shirts, 1 ties, and 8 jackets. How many different outfits can he wear to school if he must wear one of each item?

He can wear 224 different outfits.

$a_1 = 7 \quad a_4 = 8$

$a_2 = 4$

$a_3 = 1$

The fundamental counting principle

$a_1 \cdot a_2 \cdot a_3 \cdot a_4$

$= 7 \cdot 4 \cdot 1 \cdot 8$

$= 224$

Q2:

$\frac{15!}{(15-2)!} = 210$

Suppose a designer has a palette of 15 colors to work with, and wants to design a flag with 2 vertical stripes, all of different colors.

How many possible flags can be created?

210

You can also use a Calculator

This is the link to an online calculator

$n = 15, \; r = 2$

$P(15, 2) = \frac{15!}{(15-2)!} = \frac{15!}{13!} = \frac{15 \cdot 14 \cdot \cancel{13!}}{\cancel{13!}} = 210$

Counting Methods-Permutations

You can also use a Calculator

Q3:

This is the link to an online calculator

There are 8 people at a meeting and there are 5 different door prizes to give out. How many ways can we select 5 people to receive the door prizes?

6720

$n=8,\ r=5$

$$P(n,r)=\frac{n!}{(n-r)!}$$

$$\frac{8!}{(8-5)!} = 6720$$

$$P(8,5)=\frac{8!}{(8-5)!}=\frac{8!}{3!}=\frac{8\cdot7\cdot6\cdot5\cdot4\cdot\cancel{3!}}{\cancel{3!}}=6720$$

Q4:

Carrie has 14 bulbs to make a string of holiday lights. How many distinct arrangements can she make if she has 3 red bulbs, 3 green bulbs, 3 blue bulbs, and 5 yellow bulbs?

3363360

$$\frac{n!}{n_1!\cdot n_2!\cdot\cdots\cdot n_k!}$$

This is the link to an online calculator

$n=14$, $n_1=3$, $n_2=3$, $n_3=3$, $n_4=5$

$$\frac{14!}{3!\cdot3!\cdot3!\cdot5!}=3363360$$

$$\frac{14!}{3!\cdot3!\cdot3!\cdot5!} = 3363360$$

Counting Methods-Permutations

Q5:

A pianist plans to play 4 pieces at a recital from her repertoire of 22 pieces. How many different recital programs are possible?

175560

$$P(n,r) = \frac{n!}{(n-r)!}$$

This is the link to an online calculator

$n = 22$

$r = 4$

$P(22,4) = \frac{22!}{(22-4)!} = 175560$

Q6:

Find the number of distinguishable permutations of the given letters "AAABCDD".

Your answer is :

$n = 7$ $n_2 = 1$

$n_1 = 3$ $n_3 = 2$

$\frac{7!}{3! \cdot 1! \cdot 2!} = 420$

This is the link to an online calculator

Counting Methods-Permutations

Q7:

How many 7-letter words can be made from the letters WASTEFUL if

repetition of letters is allowed? 2097152 $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 = 2097152$

repetition of letters is *not* allowed? 40320 $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 40320$

Q8:

A computer password is required to be 5 characters long. How many passwords are possible if the password requires 2 letter(s) followed by 3 digits (numbers 0-9), where no repetition of any letter or digit is allowed?

There are 468000 possible passwords.

$26 \cdot 25 \cdot 10 \cdot 9 \cdot 8 = 468000$

Counting Methods-Permutations

Q9:

Eight bands are to perform at a weekend festival. How many different ways are there to schedule their appearances?

40320 $8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40320$

Q10:

There are seven books in the Harry Potter series. In how many ways can you arrange the books on your shelf?

5040 $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$

Counting Methods-Permutations

Q11:

Find the number of distinguishable permutations of the letters in the word **GRAPES**.

Your answer is : 720

$$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$
$$= 720$$

Q12:

Find the number of distinguishable permutations of the letters in the word

ILLINOIS

This is the link to an online calculator

$I = 3 = n_1$
$L = 2 = n_2$
$N = 1 = n_3$
$O = 1 = n_4$
$S = 1 = n_5$
$n = 8$

$$\frac{8!}{3! \cdot 2! \cdot 1! \cdot 1! \cdot 1!}$$
$$= 3360$$

Your answer is : 3360

Counting Methods-Permutations

Q13:

How many three-letter "words" can be made from 6 letters "FGHIJK" if repetition of letters

(a) is allowed?

Your answer is : 216

$6 \cdot 6 \cdot 6 = 216$

(b) is not allowed?

Your answer is : 120

$6 \cdot 5 \cdot 4 = 120$

Counting Methods-Permutations

Permutations of n items taken r at a time $P(n,r)$ is: (No Repetition, Order Matters)

$$P(n,r) = {}_nP_r = \frac{n!}{(n-r)!}$$

$$P(E) = \frac{\text{favorable number of outcomes}}{\text{total number of outcomes}}$$

Q14:

You pick 5 digits (0-9) at random without replacement, and write them in the order picked.

What is the probability that you have written the first 5 digits of your phone number? Assume there are no repeats of digits in your phone number.

Give your answer as a fraction.

$\frac{1}{30240}$

$$n(E) = 1$$

$$n(S) = P(10,5) = \frac{10!}{(10-5)!} = 30240$$

$$= 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 30240$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{30240}$$