

Week 4

MA123: Mathematical Reasoning & Modeling
(Spring 2021)

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Measurement

Rates and Unit Rates

- Rate
 - Ratio that compares two quantities with different units. As a result, the units must always be included.

Example 1:

- If you drive 342 miles on 15.4 gallons of gas, the ratio of miles to gallons is(rate):

$$\frac{342 \text{ miles}}{15.4 \text{ gallons}}$$

- Unit rate
 - Rate when the denominator is 1. Unit rates are used in business, sports, budgeting, science etc.

Example 2:

- If you drive 342 miles on 15.4 gallons of gas, the ratio of miles to gallons in terms of miles per gallon (unit rate) is:

$$\frac{342 \text{ miles}}{15.4 \text{ gallons}} = \frac{342}{15.4} = 22.2 \text{ miles/gallon} (22.2 \text{ mpg})$$

Rates and Unit Rates

Example 3

Dan completed a 9 miles race in 63 minutes, what is the rate of minutes per mile?

$$\frac{63 \text{ minutes}}{9 \text{ miles}} = 7 \text{ minutes/mile}$$

Example 4: Determine the best buy

Compare the cost per ounce to determine which brand is cheaper:

- Brand A coffee: \$6.25 for 18 ounces
- Brand B coffee: \$5.15 for 14 ounces

Compare the unit rate.

$$\$6.25/15 \text{ ounces} = \$0.347/\text{1 ounce.}$$

$$\$5.15/14 \text{ ounces} = \$0.368/\text{1 ounce.}$$

Brand A is cheaper.

American Unit Conversions

United States Customary Unit System (American Units System or Standard Units System)

- The most commonly used system of measurement in the united states.
- The United States is the only industrialized nation that does not use the metric system in its commercial and standard activities although the metric system is universally used.

Conversion of: Length, Weight, Capacity, Time

Length	Weight
1 foot (ft) = 12 inches (in)	1 pound (lb) = 16 ounces (oz)
1 yard (yd) = 3 feet (ft)	1 ton (T) = 2000 pound (lbs)
1 mile (mi) = 5280 feet (ft)	
Capacity	Time
1 cup (c) = 8 fluid ounces (fl oz)	1 minute (min) = 60 seconds
1 pint (pt) = 2 cups (c)	1 hour (hr) = 60 minutes (min)
1 quart (qt) = 2 pints (pt)	1 day = 24 hours (hr)
1 gallon (gal) = 4 quarts (qt)	1 week (wk) = 7 days

Converting among Measurement Units

- Method 1: Multiplying or Dividing
 1. Multiply when converting from a larger unit to a smaller unit.
 2. Divide when converting from a smaller unit to a larger unit.
- Method 2: Using Unit Fractions
 1. Multiply by unit fractions. A unit fraction is a fraction equal to 1.
 2. Each unit fraction will have different units in the numerator and denominator formed by using a known conversion.

American Unit Conversions: Length

Example 5: Complete each length conversion:

1. $45 \text{ in} = \underline{\hspace{2cm}} \text{ ft.}$

$$\frac{45 \text{ in}}{1} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = 3.75 \text{ ft}$$

2. $6 \text{ mi} = \underline{\hspace{2cm}} \text{ yd.}$

$$\frac{6 \text{ mi}}{1} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ yd}}{3 \text{ ft}} = \frac{6 \cdot 5280}{3} = 10560 \text{ yd}$$

3. $9.6 \text{ yd} = \underline{\hspace{2cm}} \text{ ft.}$

$$\frac{9.6 \text{ yd}}{1} \cdot \frac{3 \text{ ft}}{1 \text{ yd}} = 28.8 \text{ ft}$$

Conversion of: Length, Weight, Capacity, Time

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American Unit Conversions: Weight

Example 6: Complete each weight conversion:

1. $6.3\text{T} = \underline{\hspace{2cm}}\text{lb.}$

$$\frac{6.3\text{T}}{1} \cdot \frac{2000\text{lb}}{1\text{T}} = 12600\text{lb}$$

2. $86\text{oz} = \underline{\hspace{2cm}}\text{lb.}$

$$\frac{86\text{oz}}{1} \cdot \frac{1\text{lb}}{16\text{oz}} = 5.375\text{lb}$$

3. $6400\text{ oz} = \underline{\hspace{2cm}}\text{T.}$

$$\frac{6400\text{oz}}{1} \cdot \frac{1\text{lb}}{16\text{oz}} \cdot \frac{1\text{t}}{2000\text{lb}} = 0.2\bar{t}$$

Conversion of: Length, Weight, Capacity, Time

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American Unit Conversions: Capacity

Example 7: Complete each capacity conversion:

1. $3 \text{ qt} = \underline{\hspace{2cm}} \text{ c.}$

$$\frac{3 \text{ qt}}{1} \cdot \frac{2 \text{ pt}}{1 \text{ qt}} \cdot \frac{2 \text{ c}}{1 \text{ pt}} = 12 \text{ c}$$

2. $72 \text{ fl oz} = \underline{\hspace{2cm}} \text{ pt.}$

$$\frac{72 \text{ fl oz}}{1} \cdot \frac{1 \text{ pt}}{8 \text{ fl oz}} \cdot \frac{1 \text{ pt}}{2 \text{ c}} = 4.5 \text{ pt}$$

3. $3.5 \text{ gal} = \underline{\hspace{2cm}} \text{ fl oz.}$

$$\frac{3.5 \text{ gal}}{1} \cdot \frac{4 \text{ qt}}{1 \text{ gal}} \cdot \frac{2 \text{ pt}}{1 \text{ qt}} \cdot \frac{2 \text{ c}}{1 \text{ pt}} \cdot \frac{8 \text{ fl oz}}{1 \text{ c}} = 3.5 \cdot 4 \cdot 2 \cdot 2 \cdot 8 = 448 \text{ fl oz}$$

Conversion of: Length, Weight, Capacity, Time

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Metric Unit Conversion

- The **metric system** is widely used by most of the world because it is based on powers of 10.
- The United States uses the standard system, the use of metric system has increased.

Basic units of measure of length

- The basic unit of length is the meter (m).
 - It is just over 1 yard.

Basic units of measure for mass

- The basic unit of mass is the gram (g).
 - One gram is the mass of 1 cubic centimeter (1 cm^3 or mL) of water.

Basic units of measure of capacity

- The basic unit of capacity or volume is the liter.
 - A liter equals 1000 cubic centimeters.

Metric Conversion Table

	meter							
	liter							
Prefix	kilo	hecto-	deka	gram	deci	centi	milli	
Meaning	1000	100	10	1	0.1	0.01	0.001	
Length	km	hm	dam	m	dm	cm	mm	
Capacity	kL	hL	daL	L	dL	cL	mL	
Weight	kg	hg	dag	g	dg	cg	mg	

Metric Unit Conversion

Example 8: Complete each length conversion

1. $6.3 \text{ km} = \underline{\hspace{2cm}} \text{ m}$.

$$6.3 \cdot 1000 = 6300 \text{ m}$$

2. $62,300 \text{ cm} = \underline{\hspace{2cm}} \text{ hm}$.

$$\frac{62300}{10000} = 6.23 \text{ hm}$$

3. $37,200 \text{ g} = \underline{\hspace{2cm}} \text{ kg}$.

$$\frac{37200}{1000} = 37.2 \text{ kg}$$

4. $1.73 \text{ hg} = \underline{\hspace{2cm}} \text{ dg}$.

$$1.73 \cdot 1000 = 1730 \text{ dg}$$

5. $93.7 \text{ L} = \underline{\hspace{2cm}} \text{ ml}$.

$$93.7 \cdot 1000 = 93700 \text{ ml}$$

Metric Conversion Table

	meter							
	liter							
Prefix	kilo	hecto-	deka	gram	deci	centi	milli	
Meaning	1000	100	10	1	0.1	0.01	0.001	
Length	km	hm	dam	m	dm	cm	mm	
Capacity	kL	hL	daL	L	dL	cL	mL	
Weight	kg	hg	dag	g	dg	cg	mg	

6. $62300 \text{ cL} = \underline{\hspace{2cm}} \text{ kL}$.

$$\frac{62300}{100000} = 0.623 \text{ kL}$$

Metric Unit Conversion

Useful Comparisons of Units

American	Metric
1 in.	2.540 cm
1 ft	0.305 m
1 yd	0.914 m
0.621 mi	1 km
1.094 yd	1 m
3.281 ft	1 m
39.370 in.	1 m

American	Metric
1 quart	0.9464 L
1 gallon	3.785 L
1 fl oz	29.57 mL
1.057 qt	1 L
0.2642 gal	1 L