Week 4

MA123: Mathematical Reasoning & Modeling
(Spring 2021)

Instructor: Emmanuel Thompson

Geometry: Similar Figures and Pythagorean Theorem

Geometry: Similar Figures and Pythagorean Theorem

Similar Figures

- In similar triangles, ratios of corresponding sides are proportional.

Find x.

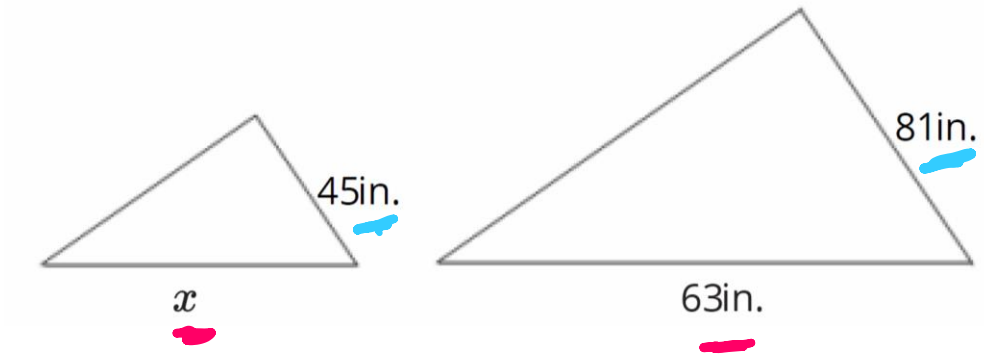

Figure is not drawn to scale.

$$\frac{x}{63} = \frac{45}{81}$$

$$81x = 63 \cdot 45$$

$$81x = 2835$$

$$x = \frac{2835}{81} = 35 \text{ in}$$

Geometry: Similar Figures and Pythagorean Theorem

Similar Figures

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Find x.

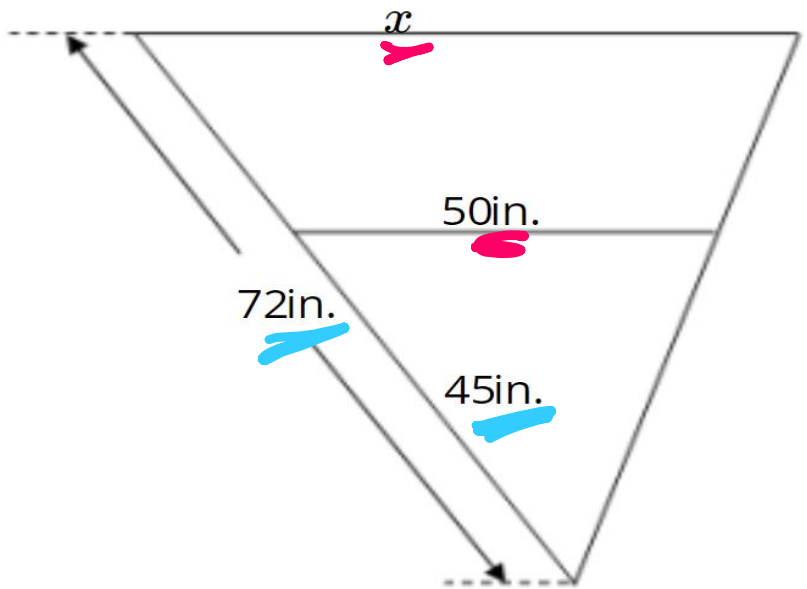


Figure is not drawn to scale.

$$\frac{x}{50} = \frac{72}{45}$$

$$45x = 50 \cdot 72$$

$$45x = 3600$$

$$x = \frac{3600}{45} = 80 \text{ in}$$

Geometry: Similar Figures and Pythagorean Theorem

Similar Figures

If a 35-meter flagpole casts a shadow that is 45 meters long, how long is the shadow cast by a tree that is 49 meters high?

The shadow cast by a 49 meters high tree is _____ meters.

Let x = shadow cast by a 49 meters high tree

$$\frac{35}{45} = \frac{49}{x}$$

$$x = \frac{45 \cdot 49}{35}$$

$$= 63 \text{ meters}$$

Geometry: Similar Figures and Pythagorean Theorem

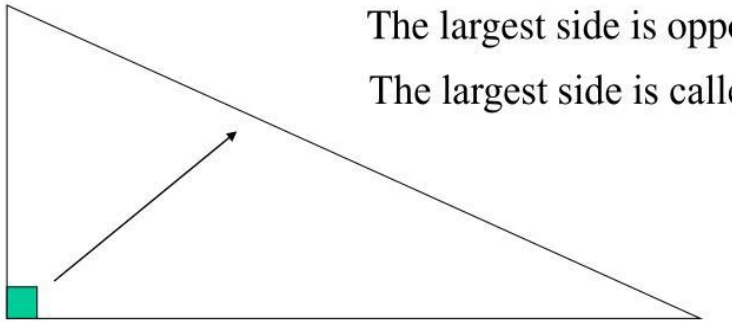
Pythagorian Theorem

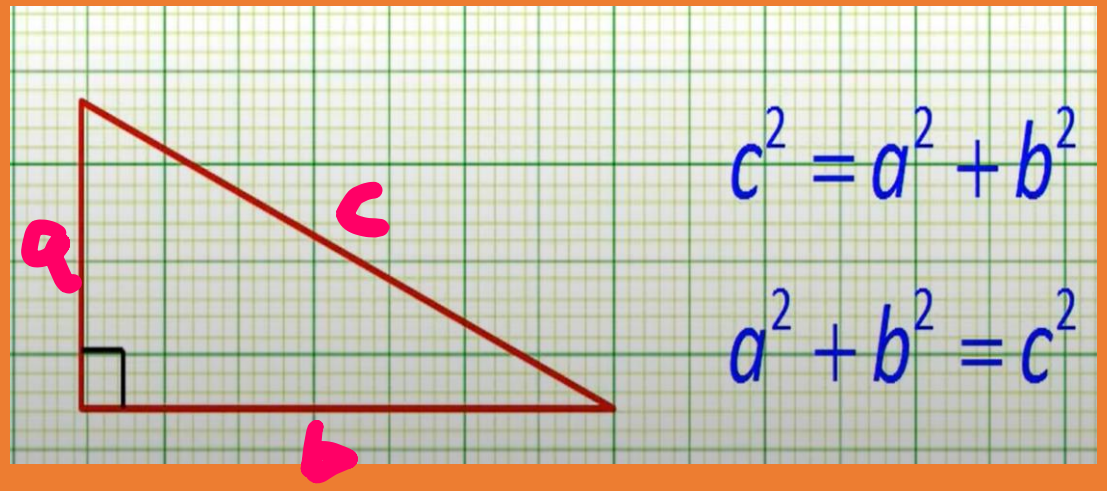
How to label and name a right angled triangle.

The 90^0 angle is the largest angle.

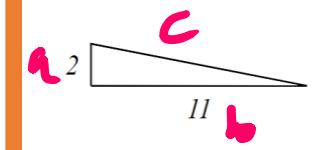
The largest side is opposite the largest angle.

The largest side is called the 'HYPOTENUSE'.





Find the length of the hypotenuse of the triangle pictured below. Enter the exact value as a square root. Do not round.



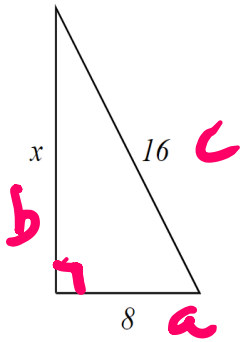
$c^2 = a^2 + b^2$

$= 2^2 + 11^2$

$c^2 = 4 + 121 = 125$

$c = \sqrt{125}$

Find the length of the leg x on the right triangle below. Enter the exact value, not a decimal approximation.



$c^2 = a^2 + b^2$

$16^2 = 8^2 + b^2$

$16^2 - 8^2 = b^2$

$256 - 64 = b^2$

$192 = b^2$; $b = \sqrt{192}$

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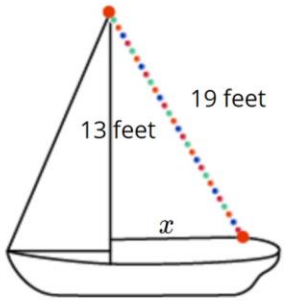
Pythagorian Theorem

Randy wants to attach a 19 foot string of lights to the top of the 13 foot mast of his sailboat. How far from the base of the mast should he attach the end of the light string? Round your answer to two decimal places.



$c^2 = a^2 + b^2$

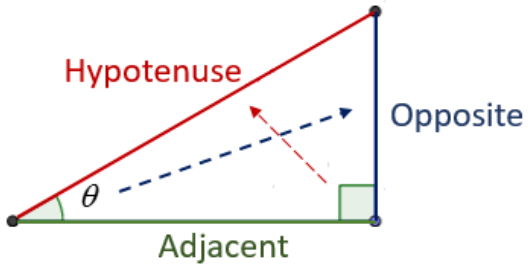
$19^2 = 13^2 + x^2$

$19^2 - 13^2 = x^2$

$192 = x^2$; $x = \sqrt{192}$

SOHCAHTOA

SOHCAHTOA



SOH $\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$

CAH $\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

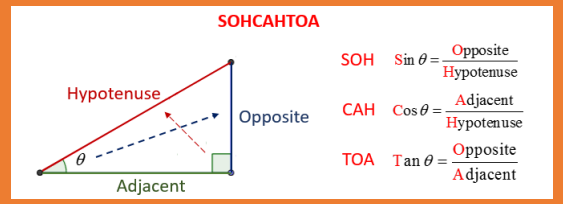
TOA $\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$

Trigonometry Table

	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec θ	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot θ	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Geometry: Similar Figures and Pythagorean Theorem

SOHCAHTOA



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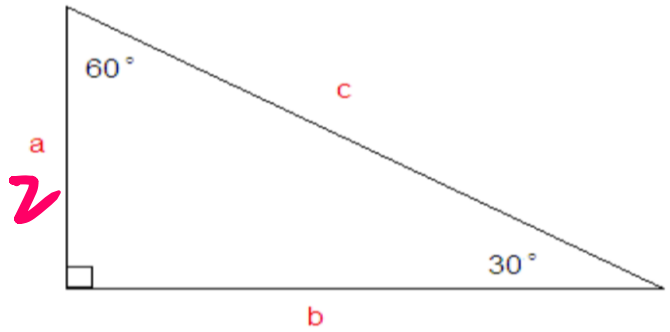
cot θ	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0



Note: Triangle may not be drawn to scale.

Suppose $a = 2$

Find b and c.

$b =$ $2\sqrt{3}$

$c =$ 4

SOHCAHTOA

$\sin 30° = \frac{\text{Opposite}}{\text{Hypotenuse}}$

$\sin 30° = \frac{a}{c} = \frac{2}{c}$

$c = \frac{2}{\sin 30°} = \frac{2}{\frac{1}{2}} = 2 \cdot 2 = 4$

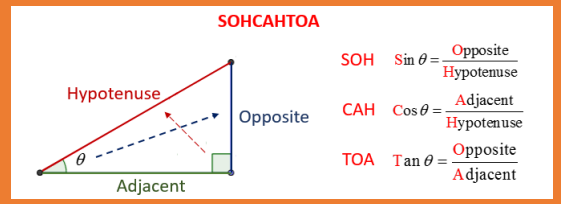
$\sin 60° = \frac{\text{Opposite}}{\text{Hypotenuse}}$

$\sin 60° = \frac{b}{c} = \frac{b}{4}$

$b = 4 \cdot \sin 60° = 4 \cdot \frac{\sqrt{3}}{2} = 2\sqrt{3}$

Geometry: Similar Figures and Pythagorean Theorem

SOHCAHTOA



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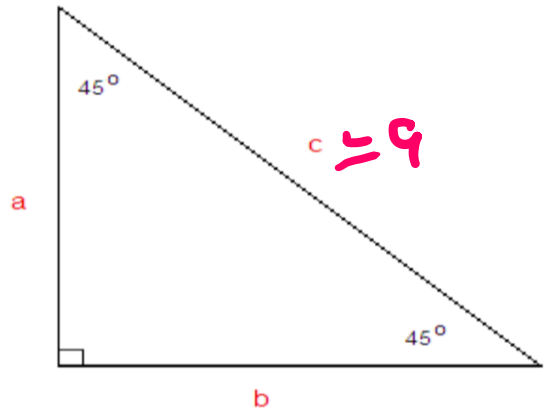

Note: Triangle may not be drawn to scale.

Suppose $c = 9$

Find exact values for the other sides.

$a =$ $\frac{9}{\sqrt{2}}$

$b =$ $\frac{9}{\sqrt{2}}$

SOHCAHTOA

$\sin 45° = \frac{\text{Opposite}}{\text{Hypotenuse}}$

$\sin 45° = \frac{a}{c} = \frac{a}{9}$

$a = 9 \cdot \sin 45°$

$= 9 \cdot \frac{1}{\sqrt{2}} = \frac{9}{\sqrt{2}}$

$\cos 45° = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

$\cos 45° = \frac{b}{c} = \frac{b}{9}$

$b = 9 \cdot \cos 45° = 9 \cdot \frac{1}{\sqrt{2}} = \frac{9}{\sqrt{2}}$