Week 6

MA123: Mathematical Reasoning & Modeling
(Spring 2021)

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Exponential Growth Models

Exponential Growth Models

Exponential Growth Model in Recursive Form:

Example 1:
Suppose a school presently has 2500 students attending. Suppose the number of students is increasing by 5% each year. How many students will attend in 5 years.

Let
P_0 = Initial Population
or
Population at time 0

5% is called the growth rate

P_n = Population in year n
P_1 = Population in year 1
P_2 = Population in year 2
⋮
P_{n-1} = Population in year n-1
P_n = Population in year n

Recursive Formula:

$P_0 = 2500$; $P_n = P_{n-1} + 0.05 P_{n-1}$

Exponential Growth Models

Exponential Growth Model in Recursive Form:

Recursive Formula:

$P_0 = 2500$; $P_n = P_{n-1} + 0.05 P_{n-1}$

$P_n = P_{n-1} + 0.05 P_{n-1}$

$P_n = P_{n-1}(1 + 0.05)$

$= 1.05 P_{n-1}$

1.05 is called the Growth multiplier

$P_0 = 2500$

$P_1 = 1.05 P_0 = 1.05(2500)$
$= 2625$

$P_2 = 1.05 P_1 = 1.05(2625)$
$= 2756.25$

$P_3 = 1.05 P_2 = 1.05(2756.25)$
$= 2894.062$

$P_4 = 1.05 P_3 = 1.05(2894.062)$
$= 3038.765$

$P_5 = 1.05 P_4 = 3190.703$

Exponential Growth Models

Exponential Growth Model in Recursive Form:

Note

Recursive Formula is

$$P_n = (1+r)\,P_{n-1}$$

Where r = Growth Rate

$1+r$ = Growth multiplier or Common Ratio

Exponential Growth Model in Explicit Form:

From Example 1:

$P_0 = 2500$; $P_n = 1.05\,P_{n-1}$

$P_1 = 1.05\,P_0$

$P_2 = 1.05\,P_1 = (1.05)(1.05\,P_0)$

$= (1.05)^2 P_0$

$P_3 = 1.05\,P_2 = (1.05)(1.05)^2 P_0$

$= (1.05)^3 P_0$

Therefore: $P_n = (1.05)^n P_0$

$P_n = (1+r)^n P_0$

Exponential Growth Models

Exponential Growth Model in Explicit Form:

Note

Explicit Formula

$P_n = (1+r)^n P_0$

r = Growth rate

$1+r$ = Growth multiplier

or

Common Ratio

Exponential Growth Model in Explicit Form:

- From recursive formula:

1) $r = \frac{P_n - P_{n-1}}{P_{n-1}}$

2) $1 + r = \frac{P_n}{P_{n-1}}$

Exponential Growth Models

Exponential Growth Model in Explicit Form:

A population grows according to an exponential growth model. The initial population is $P_0 = 15$, and the growth rate is $r = 0.25$.

Then:

P_1 = 18.75

P_2 = 23.4375

Use $P_n = (1+r)P_{n-1}$

Find an explicit formula for P_n. Your formula should involve n.

P_n = $15(1+0.25)^n$ $P_n = (1+r)^n P_0$

Use your formula to find P_9

P_9 = 111.8 $P_9 = (1+r)^9 P_0$

Give all answers accurate to at least one decimal place

Given the exponential growth function $f(x) = 77(1.01)^x$

What is the initial value of the function? 77 $P_n = (1+r)^n P_0$

What is the growth factor, or growth rate of the function (as a percent)? 1%

%

Exponential Growth Model in Explicit Form:

The rabbit population at the city park increases by 7% per year. If there are intially 222 rabbits in the city park. $P_n = C(1+r)^n P_0$

a) Write a model for the population (y) in terms of years (t). y = $222(1.07)^t$

b) Find the rabbit population in 20 years. (Round to the nearest whole rabbit)

859 $P_{20} = 222(1.07)^{20}$

x	1	2	3	4	5	6
y	1024	1457	2287	3289	4676	7062

Use regression to find an exponential equation that best fits the data above. The equation has form $y = ab^x$ where:

a =

b =

Click here to watch the video on exponential regression