Annuities

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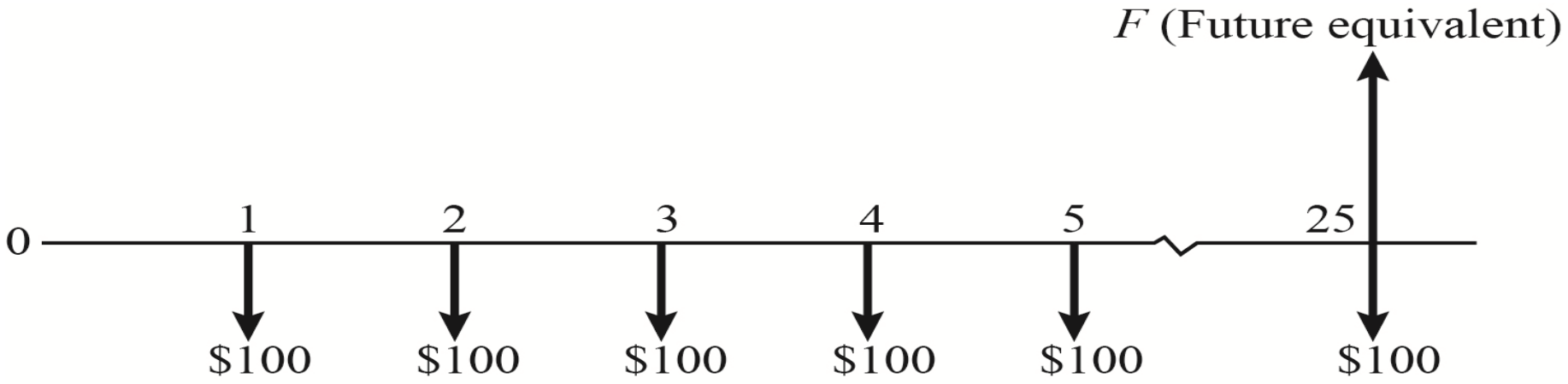
Annuities

- Equal, regular deposits are made into an account earning interest.

or

- Sequence of payments made at regular time intervals.



- For most of us, we can't put a large sum of money in the bank today. Instead, we save for the future by depositing a smaller amount of money from each paycheck into the bank.
  - This idea is called a savings annuity.
  - Most retirement plans like 401k plans or IRA plans are examples of savings annuities.

Annuities

Value of an Annuity with Compound Interest

r = annual nominal interest rate
t = number of years
n = number of compounds per year
P = amount of each deposit
A = value of annuity

$$A = \frac{P[(1 + r/n)^{nt} - 1]}{(r/n)}$$

For simple interest, n = 1

Note:

- If the compounding is done annually (once a year), then $\boldsymbol{n = 1}$.
- If the compounding is done quarterly, $\boldsymbol{n = 4}$.
- If the compounding is done monthly, $\boldsymbol{n = 12}$.
- If the compounding is done weekly, $\boldsymbol{n = 52}$.
- If the compounding is done daily, $\boldsymbol{k = 365}$.

Amount of Each Deposit with Compound Interest

$$P = \frac{A(r/n)}{[(1 + r/n)^{nt} - 1]}$$

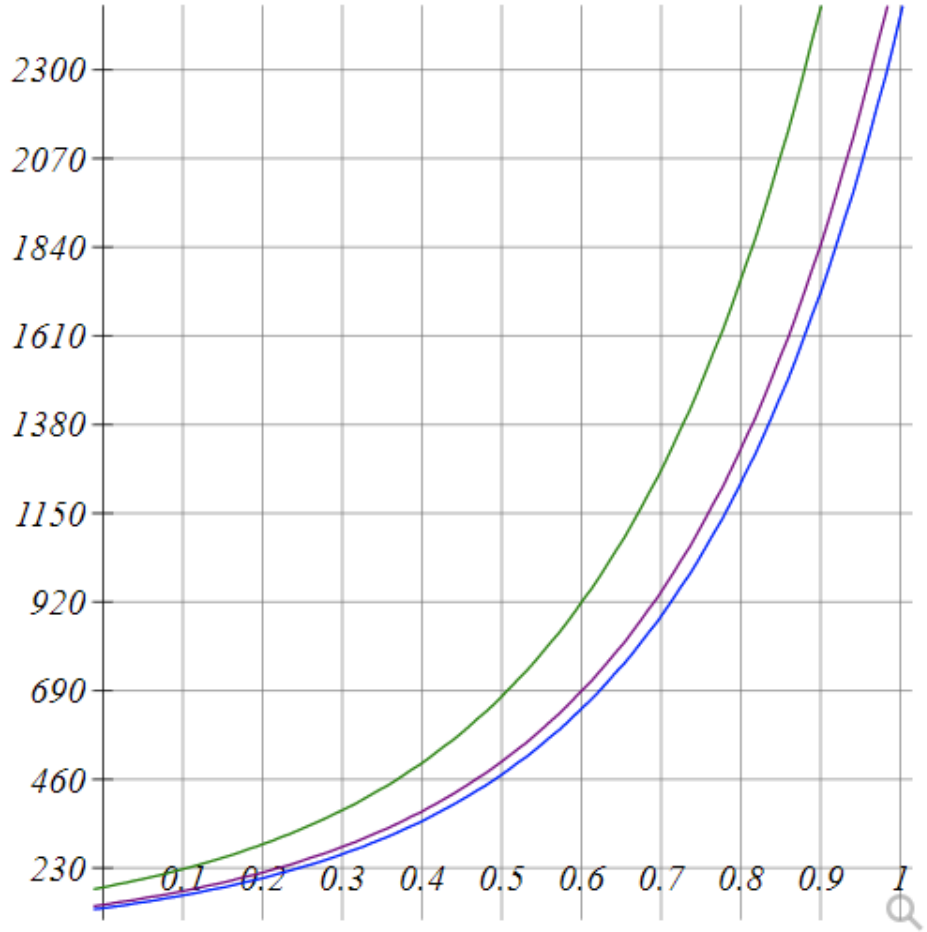
Note:

- If the compounding is done annually (once a year), then $\boldsymbol{n = 1}$.
- If the compounding is done quarterly, $\boldsymbol{n = 4}$.
- If the compounding is done monthly, $\boldsymbol{n = 12}$.
- If the compounding is done weekly, $\boldsymbol{n = 52}$.
- If the compounding is done daily, $\boldsymbol{k = 365}$.

Annuities

Q1

The future value in thousands of dollars of an annuity of 450 months and a monthly payment of $280, $300 and $400 over a number of monthly percentage rates follows.



Identify the monthly payment with the color of its graph.

b - 300

c - 400

a - 280

a. blue

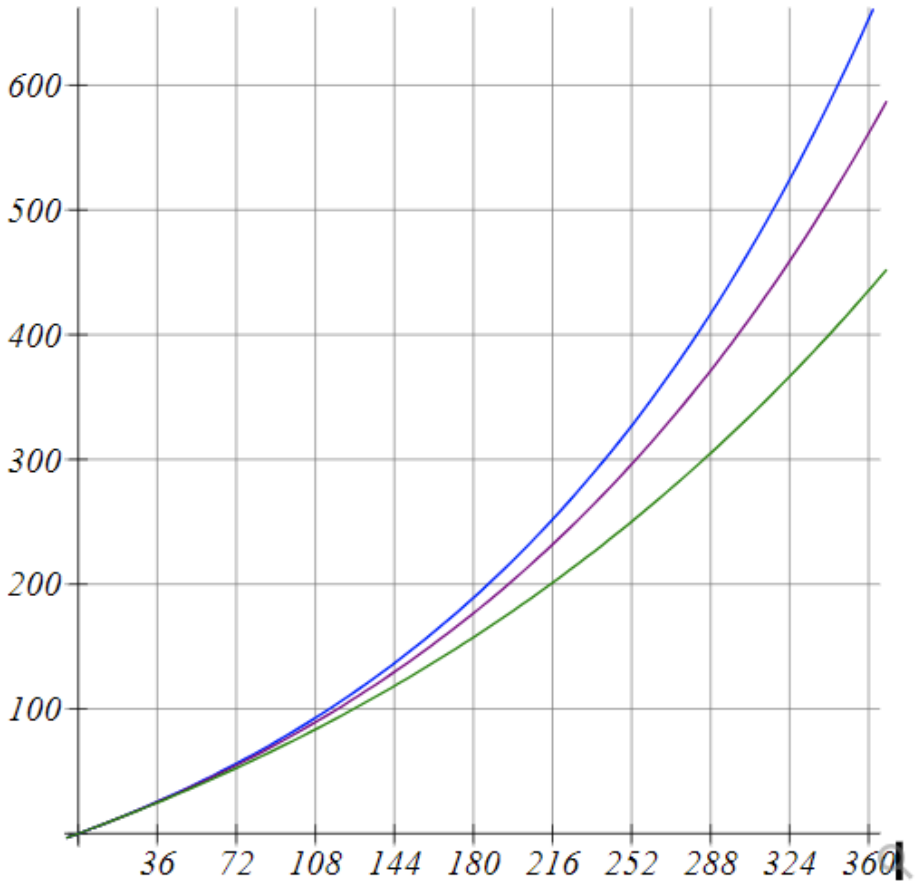
b. purple

c. green

Annuities

Q2

The future value in thousands of dollars of an annuity with a monthly payment of $650 and an annual returns of 6%, 5.2% and 3.8% over a number of months follows.



Match the rate with its color.

b - 3.8

a - 5.2

c - 6

a. purple

b. green

c. blue

Annuities

$$A = \frac{P[(1 + r/n)^{nt} - 1]}{(r/n)}$$

For simple interest, n = 1

Q3: You deposit $1000 each year into an account earning 3% interest compounded annually. How much will you have in the account in 30 years?

$P = 1000, r = 0.03, n = 1, t = 30, A = ?$

$$A = \frac{1000[(1 + 0.03/1)^{1 \cdot 30} - 1]}{\left(\frac{0.03}{1}\right)} \approx 47,575.42$$

$$\frac{1000 \cdot \left(\left(1+\frac{0.03}{1}\right)^{1 \cdot 30} - 1\right)}{\left(\frac{0.03}{1}\right)} = 47575.41570632$$

Link to TI 84 Online Calculator

(	)	|	[	]	√	∛	≥
x	7	8	9	▪/▪	▪▪	▪▪	≤
y	4	5	6	/	^	×	>
z	1	2	3	−	+	÷	<

Annuities

$$A = \frac{P[(1 + r/n)^{nt} - 1]}{(r/n)}$$

For simple interest, n = 1

Q4: You deposit $200 each month into an account earning 4% interest compounded monthly.

a) How much will you have in the account in 25 years?

b) How much total money will you put into the account?

c) How much total interest will you earn?

a) $P = 200, r = 0.04, n = 12, t = 25, A = ?$

$$A = \frac{200\left[\left(1 + \frac{0.04}{12}\right)^{12 \cdot 25} - 1\right]}{\left(\frac{0.04}{12}\right)} = 102{,}825.91$$

$$\frac{200 \cdot \left(\left(1 + \frac{0.04}{12}\right)^{12 \cdot 25} - 1\right)}{\left(\frac{0.04}{12}\right)} = 102{,}825.9094766$$

Link to TI 84 Online Calculator

(	)	|	[	]	√	∛	≥
x	7	8	9	■/■	■²	■□	≤
y	4	5	6	/	^	×	>
z	1	2	3	−	+	÷	<
abc	,	0	.	%	␣		=

b) $200 \cdot 12 \cdot 25$

$= \$60{,}000$

c) $102{,}825.91 - 60{,}000$

$= \$42{,}825.91$

Annuities

Q5: You would like to have $950,000 when you retire in 25 years. How much should you invest each quarter if you can earn a rate of 7.1% compounded quarterly?

a) How much should you deposit each quarter?

b) How much total money will you put into the account?

c) How much total interest will you earn?

a) $P = ?$, $r = 0.071$, $n = 4$, $t = 25$, $A = 950{,}000$

$$P = \frac{A\left(r/n\right)}{\left[\left(1+r/n\right)^{nt}-1\right]}$$

$$P = \frac{950{,}000\left(\frac{0.071}{4}\right)}{\left[\left(1+\frac{0.071}{4}\right)^{4\cdot 25}-1\right]} \approx 3506.35$$

$$\frac{950000\cdot\left(\frac{0.071}{4}\right)}{\left(\left(1+\frac{0.071}{4}\right)^{4\cdot 25}-1\right)} = 3506.35136634$$

Link to TI 84 Online Calculator

b) $3506.35 \cdot 4 \cdot 25 = 350{,}635$

c) $950{,}000 - 350{,}635 = 599{,}635$

Annuity

Q6: Suppose you want to have $600,000 for retirement in 25 years. Your account earns 8% interest.

a) How much would you need to deposit in the account each month?

b) How much interest will you earn?

$$P = \frac{A\left(r/n\right)}{\left[\left(1+r/n\right)^{nt}-1\right]}$$

a) $P = ?,\ r = 0.08,\ n = 12,\ t = 25,$

$A = 600{,}000$

$$P = \frac{600{,}000\left(\frac{0.08}{12}\right)}{\left[\left(1+\frac{0.08}{12}\right)^{12\cdot 25}-1\right]} \approx 630.90$$

$$\frac{600000\cdot\left(\frac{0.08}{12}\right)}{\left(\left(1+\frac{0.08}{12}\right)^{12\cdot 25}-1\right)} = 630.89731623$$

Link to TI 84 Online Calculator

b) $600{,}000 - [630.90 \cdot 12 \cdot 25]$

$600{,}000 - 189{,}270 = 410{,}730$

Find the future value for each of the following scenarios, where *m* is the periodic deposit and *r* is the interest rate.

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Trial

$$A = \frac{P[(1 + r/n)^{nt} - 1]}{(r/n)}$$

For simple interest, n = 1

m	*r*	compounding frequency	time in years	future value	interest earned
$450	7%	annually (n=1)	13	$ [ ] $9,063.29	$ [ ] $3,213.29
$450	6.6%	semiannually (n=2)	8	$ [ ] $9,288.34	$ [ ] $2,088.34
$400	7.2%	quarterly (n=4)	9	$ [ ] $20,016.18	$ [ ] $5,616.18
$300	2%	monthly (n=12)	15	$ [ ] $62,913.92	$ [ ] $8,913.92
$150	7.3%	weekly (n=52)	10	$ [ ] $114,758.15	$ [ ] $36,758.15

Note

m = P

Find the future value for each of the following scenarios, where m is the periodic deposit and r is the interest rate.

Link to TI 84 Online Calculator

Trial

$$P = \frac{A\left(r/n\right)}{\left[\left(1+r/n\right)^{nt}-1\right]}$$

Future Value	r	compounding frequency	time in years	periodic deposit (m)	interest earned
$275,000	4.8%	annually	7	$ [ ] $33,981.56	$ [ ] $37,129.07
$250,000	7.4%	semiannually	15	$ [ ] $4,685.56	$ [ ] $109,433.09
$75,000	5.6%	quarterly	9	$ [ ] $1,616.50	$ [ ] $16,806.17
$175,000	5.5%	monthly	12	$ [ ] $860.72	$ [ ] $51,056.61
$50,000	2.9%	weekly	6	$ [ ] $146.76	$ [ ] $4,210.01

Note

$P = m$

This is a link to a video on Annuities (watch it!)

This is another link to a video on Annuities (watch it!)