

Week 1

MA123: Mathematical Reasoning & Modeling
(Spring 2021)

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Proportions

A true proportion is an equation that states that two ratios are equal.

Example 1: Is the proportion True/False

$$\frac{60 \text{ miles}}{12 \text{ gallons}} = \frac{5 \text{ miles}}{1 \text{ gallon}}$$

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i) $\frac{60 \text{ miles}}{12 \text{ gallons}} = \frac{5 \text{ miles}}{1 \text{ gallon}}$ ✓

$$\frac{60 \div 12}{12 \div 12} = \frac{5}{1}$$

True

Example 2:

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Another office has 30 printers for 105 computers.
Is the ratio of printers to computers the same in these two offices?

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$$\frac{4 \text{ printers}}{32 \text{ Computers}}$$

?

$$\frac{30 \text{ printers}}{105 \text{ Computers}}$$

Simplify each fraction and check if they are equivalent.

$$\text{LHS} = \frac{4 \div 4}{32 \div 4} = \frac{1}{8}$$

$$\text{RHS} = \frac{30 \div 15}{105 \div 15} = \frac{2}{7} \neq \frac{1}{8} \neq \frac{2}{7}$$

The ratio of printers to computers is not the same in these two offices.

Example 3:

Determine if the proportion is true

$$\frac{10}{30} = \frac{5}{15}$$

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$$\frac{10}{30} = \frac{5}{15}$$

$$LHS = \frac{10}{30} = \frac{10 \div 10}{30 \div 10} = \frac{1}{3}$$

$$RHS = \frac{5}{15} = \frac{5 \div 5}{15 \div 5} = \frac{1}{3}$$

OR: cross-multiply:

$$\frac{10}{30} = \frac{5}{15}$$

$$\frac{1}{3} = \frac{1}{3} \text{ (True)}$$

$$10 \cdot 15 = 30 \cdot 5$$
$$150 = 150$$

(True)

Example 4:

Determine if the proportion is true.

$$\frac{\frac{3}{5}}{\frac{4}{5}} = \frac{\frac{61}{60}}{\frac{4}{3}}$$

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$$\frac{\frac{3}{5}}{\frac{4}{5}} = \frac{\frac{61}{60}}{\frac{4}{3}}$$

$$\frac{3}{5} \cdot \frac{4}{3} \stackrel{?}{=} \frac{4}{5} \cdot \frac{61}{60}$$

$$\frac{12}{15} = \frac{244}{300}$$

$$12 \cdot 300 = 15 \cdot 244$$

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$$3600 \neq 3660$$

The proportion
is not true.

Example 5:

Solve for the unknown quantity, n

$$\frac{15}{n} = \frac{6}{10}$$

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Solve for the unknown quantity, n

$$\frac{15}{n} = \frac{6}{10}$$

$$15 \cdot 10 = 6 \cdot n$$

$$150 = 6n$$

$$\frac{150}{6} = \frac{6n}{6}$$

$$25 = n \quad \text{same as } n = 25$$

Example 6:

An 8ft length of 4-inch-wide crown molding costs \$14. How much will it cost to buy 50 ft of crown molding?

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Example 6:

For a given 4-inch-wide crown molding:

$$8 \text{ ft} : \$14 = \frac{8 \text{ ft}}{\$14}$$

For a given 4-inch-wide crown molding

$$50 \text{ ft} : \$x = \frac{50 \text{ ft}}{\$x}$$

$$\frac{8}{14} = \frac{50}{x}$$

;

$$8x = 14 \cdot 50$$

$$8x = 700$$

$$x = \frac{700}{8} = \$87.5$$

Example 7:

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$$3 \text{ rolls} : \$2 = \frac{3 \text{ rolls}}{\$2}$$

$$13 \text{ rolls} : \$x = \frac{13 \text{ rolls}}{\$x}$$

$$\frac{3}{2} = \frac{13}{x}$$

$$3x = 26$$

$$x = 26/3 = \$8.67$$

Example 8:

It takes a hose 3 minutes to fill a rectangular aquarium 10 inches long, 13 inches wide, and 14 inches tall. How long will it take the same hose to fill an aquarium measuring 26 inches by 28 inches by 31 inches?

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It takes a hose 3 minutes to fill a rectangular aquarium 10 inches long, 13 inches wide, and 14 inches tall. How long will it take the same hose to fill an aquarium measuring 26 inches by 28 inches by 31 inches?

$$10 \cdot 13 \cdot 14 = 1820 \text{ cubic inches}$$

$$\frac{1820}{3} \text{ Cubic inches per minute}$$

$$26 \cdot 28 \cdot 31 = 22568 \text{ Cubic inches}$$

Let x be the unknown time taken by the same hose to obtain 22568 Cubic inches

$$\frac{22568}{x} \text{ Cubic inches per minute}$$

$$\frac{1820}{3} = \frac{22568}{x}$$

$$1820x = 67704$$

$$x = \frac{67704}{1820}$$

$$= 37.2$$

$$x \approx \underline{\underline{37 \text{ minutes}}}$$

Example 9

When you knit something using a pattern, each line of the pattern contains a stitch count. Depending on the weight of the yarn you want to use to make the project, you can use the number of stitches to calculate how much yarn you need to finish your project.

The table below gives the number of inches per stitch for the basic weights of yarn you will find at a store:

Weight	Inches Per Stitch
Fingering	1 in.
DK	1.5 in.
Worsted	1.8 in.
Bulky1	2.5in.
Bulky2	2.75 in.
Super Bulky	7.5 in.

Given that you want to use Worsted weight of yarn with 1.8 inches per stitch, and your pattern calls for 575 stitches, answer the following questions:

- 1) What is the number of inches of yarn that you will need?

The store you are at only has yarn lengths given in yards, and there are 36 in/ yd

- 2) What is the number of yards of yarn you will need for your project?

Example 9

$$(1) \frac{1.8 \text{ in}}{1 \text{ stitch}} \cdot 575 \text{ stitches} =$$

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1) What is the number of inches of yarn that you will need?

$$\frac{1.8 \text{ in}}{1 \text{ stitch}} \cdot 575 \text{ stitches} = 1035 \text{ in}$$

The store you are at only has yarn lengths given in yards, and there are 36 in/ yd

2) What is the number of yards of yarn you will need for your project?

$$\frac{36 \text{ in}}{1 \text{ yard}} = \frac{1035 \text{ in}}{x \text{ yards}} \Rightarrow x = \frac{1}{36} \cdot 1035 = 28.75 \text{ yards}$$