INTEGRALS

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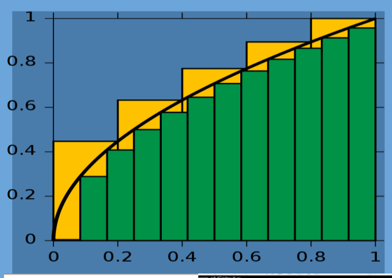

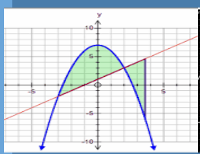

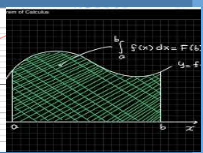

ABSTRACT

INTEGRAL=AREA UNDER GRAPH

Integrals are generally of two types Indefinite and definite integrals. This is the inverse operation of differentials .

Fundamental Theorem of calculus

Part 1: Let f be a continuous real-valued function defined on a closed interval [a, b]. Let F be the function defined, for all x in [a, b], by:

F(x)=∫axf(t)dt

Then F is uniformly continuous on [a, b] and differentiable on the open interval (a, b), and

Here, the F'(x) is a derivative function of F (x).

Part 2:if the function "f" is continuous on the closed interval [a, b], and F is an indefinite integral of a function "f" on [a, b], then the second fundamental theorem of calculus is defined as:

F(b)- F(a) = a∫b f(x) dx

Here R.H.S. of the equation indicates the integral of f(x) with respect to x.

f(x) is the integrand.

dx is the integrating

THEOREMS

Theorem 1: If f is integrable on [a,b],

$$\int_a^b f(x)\,dx = \lim_{n \to +\infty} S_n$$

$$= \lim_{n \to +\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

$$= \lim_{\Delta x \to 0} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

Theorem 2: Net change Theorem:

The integral of a rate of change us net change,

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

Substitution Rule: If u=g(x) is a differentiable function whose range is interval I and f is continuous on I , then

$$\int f(g(x))g'(x)dx = \int f(u)du = F(u) + C$$

where $u = g(x)$ $du = g'(x)$

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INDEFINITE INTEGRALS

Indefinite Integrals are the integrals that can be calculated by the reverse process of differentiation and are referred to as the antiderivatives of functions. For a function f(x), if the derivative is represented by f'(x), the integration of the resultant f'(x) gives back the initial function f(x).

So, d/dx f(x) = f'(x) then ∫ f'(x) dx = f(x) + C

EXAMPLES:

$$\begin{aligned} F(-\pi) &= \int_0^{-\pi} (\sin t)dt \\ &= [-\cos t]\Big|_0^{-\pi} \\ &= [-\cos(-\pi)] - [-\cos 0] \\ &= [1] - [-1] \\ &= 2. \end{aligned}$$

Integration Formulas

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$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ n \neq 1$

$\int dx = x + C$

$\int \cos x\,dx = \sin x + C$

$\int \sin x\,dx = -\cos x + C$

$\int \sec^2 x\,dx = \tan x + C$

$\int \mathrm{cosec}^2 x\,dx = -\cot x + C$

$\int \sec x \tan x\,dx = \sec x + C$

$\int \mathrm{cosec}\,x \cot x\,dx = -\mathrm{cosec}\,x + C$

$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$

$\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1} x + C$

$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$

$\int \frac{dx}{1+x^2} = -\cot^{-1} x + C$

$\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C$

$\int \frac{dx}{x\sqrt{x^2-1}} = -\mathrm{cosec}^{-1} x + C$

$\int e^x\,dx = e^x + C$

$\int \frac{dx}{x} = \log|x| + C$

$\int a^x\,dx = \frac{a^x}{\ln a} + C$