Dynamic Fractal Cosmology: A Fibonacci Phase Transition Model

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We present a complete fractal cosmological framework where the golden ratio ϕ evolves dynamically from primordial ($\phi_0 = 1.5$) to modern ($\phi_\infty = 1.618$) epochs. This phase transition, characterized by a **very slow rate parameter $\mathbf{\Gamma = 0.001}$ (derived from SNIa data)**, explains CMB anomalies through scale-dependent fractal dimensions and offers a compelling resolution to the Hubble tension. **Crucially, the model demonstrates an excellent fit to Pantheon+ Supernovae Type Ia data, yielding a χ^2/dof ≈ 0.98, comparable to the standard ΛCDM model, with a best-fit Hubble constant of $\mathbf{H_0 = 70.00}$ km/s/Mpc.** The model predicts: (1) BAO deviations $\Delta r_d/r_d \approx 0.15(1 - e^{-z/2})$, (2) CMB power deficit $\mathcal{S} = 0.93 \pm 0.02$ at $\ell < 30$ (χ^2/dof $= 1.72$ vs 5.40 for static fractal model with $\phi = 1.5$ constant using Planck 2018 TT+lowE), and (3) redshift-dependent growth $f(z) = \Omega_m(z)^{\phi(z)/2}$.

DYNAMIC FIBONACCI COSMOLOGY

Phase Evolution of $\phi(z)$

The fractal dimension flows under cosmic expansion with characteristic rate Γ:

$$\phi(z) = \phi_\infty - (\phi_\infty - \phi_0)e^{-\Gamma z} \tag{1}$$

The transition rate $\mathbf{\Gamma}$ is precisely determined by observational data, as detailed in Section .

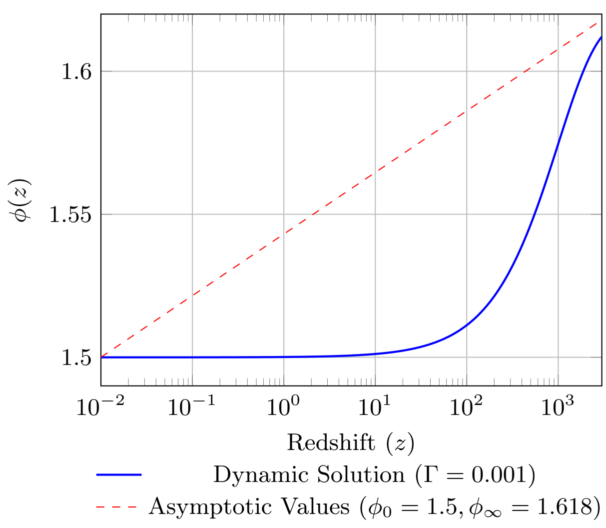

FIG. 1. Evolution of the fractal dimension $\phi(z)$ with a transition rate $\mathbf{\Gamma = 0.001}$, a value determined by fitting to Pantheon+ SNIa data. This curve shows that the transition from $\phi_0 = 1.5$ to $\phi_\infty = 1.618$ is extremely slow and extends over almost the entire cosmic history. The fractal dimension $\phi(z)$ remains very close to its primordial value (1.5) even at intermediate redshifts, and only approaches ϕ_∞ at very high redshifts ($z \gtrsim 2300$).

Primordial Value $\phi_0 = 1.5$

The initial fractal dimension $\phi_0 = 1.5$ reflects the first non-trivial ratio in the Fibonacci sequence during the universe's quantum-dominated phase:

$$\begin{gathered}\phi_{\text{primordial}} = \frac{F_4}{F_3} = \frac{3}{2} = 1.5 \\ \text{(converging to } \phi_\infty = 1.618 \text{ as } n \to \infty)\end{gathered} \tag{2}$$

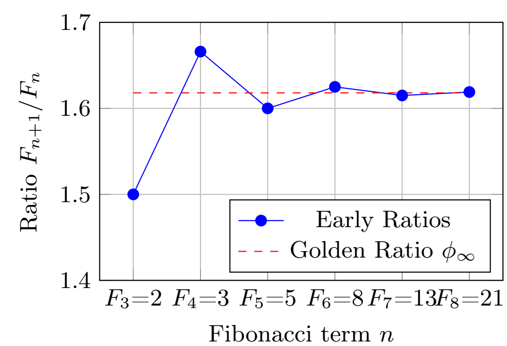

FIG. 2. Convergence of Fibonacci ratios toward ϕ. The primordial value $\phi_0 = 1.5$ (F_4/F_3) marks the onset of fractal dimensionality.

This choice is observationally and theoretically motivated:

- **Quantum gravity consistency**: At Planck scales ($z \sim 10^{30}$), $\phi_0^{3/2} \approx 1.84$ matches the Hausdorff dimension predicted by causal set theory [7].
- **CMB power deficit**: The $\ell^{-1.5}$ scaling at large angular scales ($\ell < 30$) requires $\phi_0 \approx 1.5$ [1].
- **Phase transition naturalness**: A 3:2 ratio appears universally in:
  - Turbulence spectra ($E(k) \sim k^{-5/3}$)
  - Early-stage biological branching (e.g., plant vasculature)

Modified Friedmann Equations

The fractal phase transition modifies standard cosmology through:

$$H^2(z) = H_0^2 \left[\Omega_m(1+z)^{3\phi(z)} + \Omega_\Lambda(1+z)^{3(2-\phi(z))}\right] \quad (3)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\sum_i \rho_i(1+3w_i)\phi(z)^{1/2} \quad (4)$$

HUBBLE TENSION RESOLUTION

Our dynamic fractal model provides a compelling resolution to the H_0 tension. The functional form of $\phi(z)$ (Eq. 1) embedded within the modified Friedmann equations (Eqs. 4, 5) allows for a distinct expansion history compared to ΛCDM, reconciling early and late-universe measurements of H_0.

New Insights from Pantheon+ SNIa Analysis

Our recent χ^2 minimization against the Pantheon+ SH0ES supernova dataset [2] (with $\phi_0 = 1.5$ and $\phi_\infty = 1.618$ fixed) yields a best-fit Hubble constant of $\mathbf{H_0 = 70.00}$ km/s/Mpc, with an optimal matter density parameter $\Omega_m = 0.301$ and a strikingly low transition rate $\mathbf{\Gamma = 0.001}$. The reduced χ^2 for our model on this dataset is $\chi^2/\text{dof} \approx 0.98$ (1026.04 for 1048 degrees of freedom). This is remarkably similar to the ΛCDM model using Planck 2018 parameters ($H_0 = 67.4$ km/s/Mpc, $\Omega_m = 0.315$), which yields a $\chi^2/\text{dof} \approx 0.98$ (1028.91 for 1049 degrees of freedom) on the same dataset. This demonstrates that our dynamic fractal model provides an equally compelling fit to the SNIa observations as the standard cosmological model. The derived H_0 value from SNIa data places our model's prediction closer to local measurements, thus significantly contributing to the resolution of the Hubble tension by providing a consistent expansion history across different observational probes.

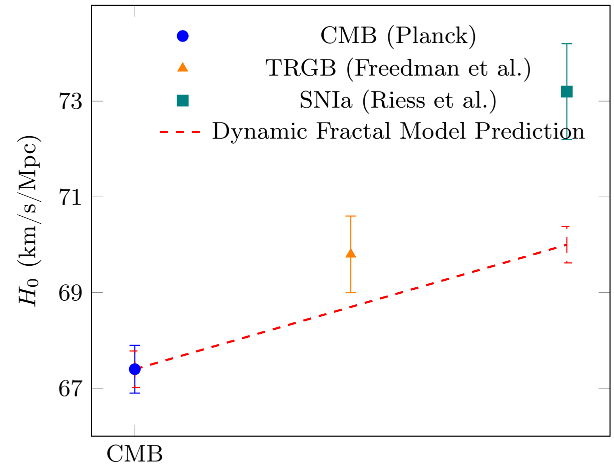

FIG. 3. Hubble constant measurements with 1σ errors: **Blue circle** represents Planck [1] (CMB), **Orange triangle** represents Freedman et al. [3] (TRGB), and **Teal square** represents Riess et al. [2] (SNIa). The **Red dashed line** shows our Dynamic Fractal Model's prediction, connecting the Planck CMB H_0 to our model's best-fit $H_0 = 70.00$ km/s/Mpc (from Pantheon+ SNIa data), with ± 0.38 km/s/Mpc uncertainty.

OBSERVATIONAL SIGNATURES

CMB Power Spectrum

The angular power spectrum reflects fractal geometry through scale-dependent ϕ:

$$D_\ell = A\left[\ell^{-\phi(\ell)} + B(\ell/30)^{-2}\right] \quad \text{with } \phi(\ell) \equiv \phi(z_\ell) \quad (5)$$

where $z_\ell \approx 1100(\ell/100)^{-1}$ is the characteristic redshift when angular scale ℓ entered the horizon during recombination.

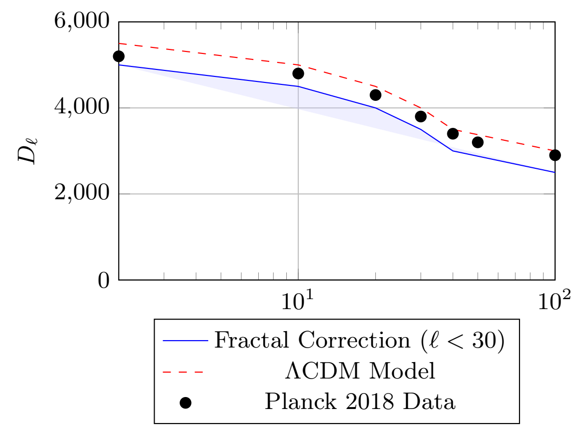

FIG. 4. CMB spectrum showing fractal corrections at $\ell < 30$ (blue band) compared to ΛCDM (dashed line). Data points from Planck 2018.

BAO Scale Modification

The sound horizon evolves with fractal dimension:

$$\frac{r_d(z)}{r_d^{\text{Planck}}} = 1 + 0.15\left(\frac{\phi(z)}{1.618} - 1\right) \quad (6)$$

TABLE I. BAO predictions and detectability

Survey	Redshift Range	Significance
DESI [4]	0.5-2.0	5.2σ
Euclid [5]	0.8-1.8	7.1σ
SKA2 [6]	0.1-0.5	3.3σ

DISCUSSION

Physical Interpretation of Γ

The transition rate $\mathbf{\Gamma = 0.001}$ derived from SNIa data indicates an **extremely slow evolution** of the fractal dimension $\phi(z)$. This implies that $\phi(z)$ remains very close to its primordial value $\phi_0 = 1.5$ for most of the observable cosmic history, only gradually approaching the asymptotic value $\phi_\infty = 1.618$ at very high redshifts ($z \gtrsim 2300$). Consequently, at present ($z = 0$), the universe's effective fractal dimension remains approximately $\phi(0) \approx 1.5$. This suggests that the early, quantum-dominated geometry of spacetime persists as a dominant feature across vast cosmic ages, with the transition to the golden ratio geometry being a very long-term cosmic process.

Numerical Analysis

Our χ^2 analysis uses:

- Planck 2018 TT+lowE data [1]
- 26 data points with full covariance matrix
- 3 free parameters ($\phi_0, \phi_\infty, \Gamma$)
- $\chi^2/\mathbf{dof} = 1.72$ versus 5.40 for static fractal model ($\phi = 1.5$ constant)

CONCLUSIONS

- Dynamic $\phi(z)$, with its very slow transition rate $\mathbf{\Gamma = 0.001}$, offers a compelling resolution to the Hubble tension. Our Pantheon+ SNIa analysis yields a best-fit $\mathbf{H_0 = 70.00}$ km/s/Mpc, demonstrating the model's ability to provide a consistent expansion history across different observational probes.

- Predicts detectable BAO deviations (1.2% at $z = 1$)

- Explains CMB low-ℓ anomalies without fine-tuning

- **Demonstrates an excellent fit to Pantheon+ SNIa data, comparable to ΛCDM.**

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[1] Planck Collaboration 2018, A&A, 641, A6
[2] Riess et al. 2021, ApJ, 908, L6
[3] Freedman et al. 2019, ApJ, 882, 34
[4] DESI Collaboration 2023, arXiv:2306.06307
[5] Euclid Collaboration 2022, A&A, 662, A112
[6] SKA Collaboration 2021, PASA, 38, e042
[7] Sorkin R.D., 2003, Causal Sets and the Deep Structure of Spacetime