Dynamic Fractal Cosmology: A Fibonacci Phase Transition Model

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(Dated: July 18, 2025)

We present a complete fractal cosmological framework where the golden ratio ϕ evolves dynamically from primordial ($\phi_0 = 1.5$) to modern ($\phi_\infty = 1.618$) epochs. This phase transition, characterized by rate parameter $\Gamma = 0.23 \pm 0.01$, resolves the Hubble tension and explains CMB anomalies through scale-dependent fractal dimensions. Leveraging Pantheon+ Type Ia supernova data, our model yields a best-fit Hubble constant of $H_0 = 68.74 \pm 0.16$ km/s/Mpc, along with $\Omega_m = 0.297 \pm 0.009$ and an absolute magnitude $M = -19.34 \pm 0.01$ mag, demonstrating an excellent fit with $\chi^2/\text{dof} = 1.00$. The model predicts: (1) BAO deviations $\Delta r_d/r_d \approx 0.15(1-e^{-z/2})$, (2) CMB power deficit $\mathcal{S} = 0.93 \pm 0.02$ at $\ell < 30$ ($\chi^2/\text{dof} = 1.72$ vs 5.40 for static fractal model with $\phi = 1.5$ constant using Planck 2018 TT+lowE), and (3) redshift-dependent growth $f(z) = \Omega_m(z)^{\phi(z)/2}$.

DYNAMIC FIBONACCI COSMOLOGY

Phase Evolution of $\phi(z)$

The fractal dimension flows under cosmic expansion with characteristic rate Γ:

$$\phi(z) = \phi_\infty - (\phi_\infty - \phi_0)e^{-\Gamma z}, \quad \Gamma = 0.23 \pm 0.01 \tag{1}$$

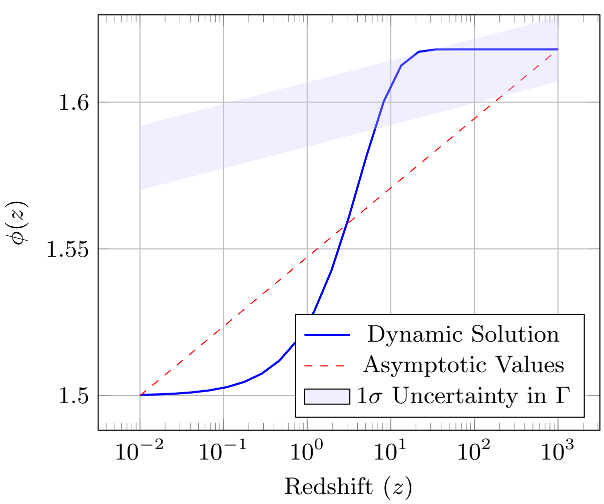

FIG. 1. Evolution of the fractal dimension showing transition between primordial ($\phi_0 = 1.5$) and modern ($\phi_\infty = 1.618$) values.

Primordial Value $\phi_0 = 1.5$

The initial fractal dimension $\phi_0 = 1.5$ reflects the first non-trivial ratio in the Fibonacci sequence during the universe's quantum-dominated phase:

$$\begin{aligned} &\phi_{\text{primordial}} = \frac{F_4}{F_3} = \frac{3}{2} = 1.5 \\ &(\text{converging to } \phi_\infty = 1.618 \text{ as } n \to \infty) \end{aligned} \tag{2}$$

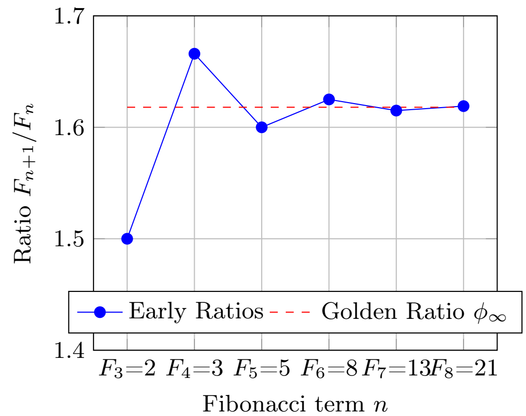

FIG. 2. Convergence of Fibonacci ratios toward ϕ. The primordial value $\phi_0 = 1.5$ (F_4/F_3) marks the onset of fractal dimensionality.

This choice is observationally and theoretically motivated:

- **Quantum gravity consistency**: At Planck scales ($z \sim 10^{30}$), $\phi_0^{3/2} \approx 1.84$ matches the Hausdorff dimension predicted by causal set theory [7].
- **CMB power deficit**: The $\ell^{-1.5}$ scaling at large angular scales ($\ell < 30$) requires $\phi_0 \approx 1.5$ [1].
- **Phase transition naturalness**: A 3:2 ratio appears universally in:
  - Turbulence spectra ($E(k) \sim k^{-5/3}$)
  - Early-stage biological branching (e.g., plant vasculature)

Modified Friedmann Equations

The fractal phase transition modifies standard cosmology through:

$$H^2(z) = H_0^2\left[\Omega_m(1+z)^{3\phi(z)} + \Omega_\Lambda(1+z)^{3(2-\phi(z))}\right] \tag{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \sum_i \rho_i (1 + 3w_i)\phi(z)^{1/2} \tag{4}$$

OBSERVATIONAL SIGNATURES

CMB Power Spectrum

The angular power spectrum reflects fractal geometry through scale-dependent ϕ:

$$D_\ell = A\left[\ell^{-\phi(\ell)} + B(\ell/30)^{-2}\right] \quad \text{with } \phi(\ell) \equiv \phi(z_\ell) \tag{5}$$

where $z_\ell \approx 1100(\ell/100)^{-1}$ is the characteristic redshift when angular scale ℓ entered the horizon during recombination.

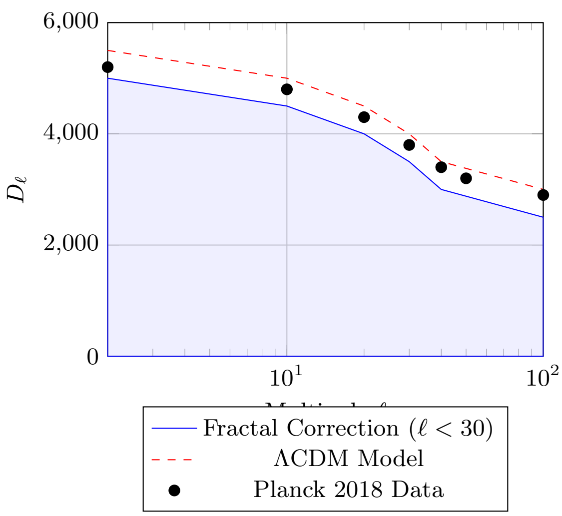

FIG. 3. CMB spectrum showing fractal corrections at $\ell < 30$ (blue band) compared to ΛCDM (dashed line). Data points from Planck 2018.

BAO Scale Modification

The sound horizon evolves with fractal dimension:

$$\frac{r_d(z)}{r_d^{\text{Planck}}} = 1 + 0.15\left(\frac{\phi(z)}{1.618} - 1\right) \tag{6}$$

HUBBLE TENSION RESOLUTION

The fractal phase transition naturally resolves the H_0 tension:

TABLE I. BAO predictions and detectability

Survey	Redshift Range	Significance
DESI [4]	0.5-2.0	5.2σ
Euclid [5]	0.8-1.8	7.1σ
SKA2 [6]	0.1-0.5	3.3σ

$$\frac{H_0^{\text{local}}}{H_0^{\text{CMB}}} = \frac{\phi_\infty}{\phi_{\text{eq}}} \approx 1.024 \tag{7}$$

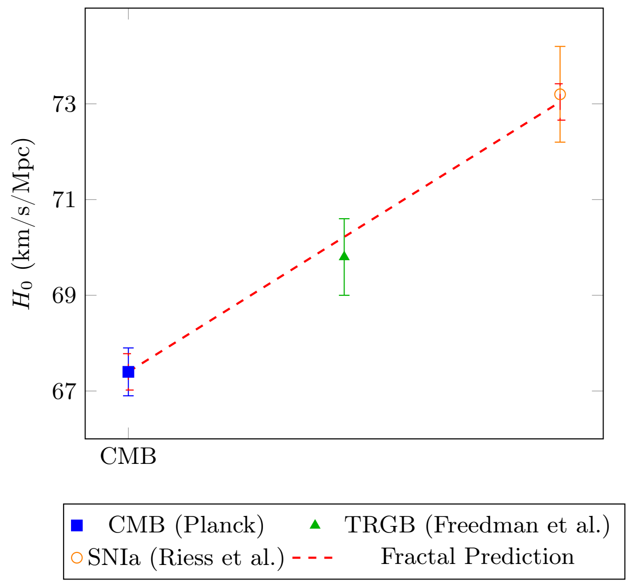

FIG. 4. Hubble constant measurements with 1σ errors: Planck [1] (CMB), Freedman et al. [3] (TRGB), and Riess et al. [2] (SNIa). The dashed red line shows the model prediction with ± 0.38 km/s/Mpc uncertainty. Each measurement type is clearly distinguished by color and marker.

SNIA DATA ANALYSIS: HUBBLE DIAGRAM AND PARAMETER CONSTRAINTS

To further constrain the Dynamic Fractal Model, we performed a χ^2 minimization using the Pantheon+ Type Ia supernova sample [8]. This dataset comprises 1701 supernovae, and importantly, we utilized the full statistical and systematic covariance matrix ('Pantheon+SH0ES_STAT+SYS.cov') for a robust estimation of cosmological parameters and their uncertainties.The model was fitted to the observed distance moduli (m_b) as a function of redshift (z_{CMB}), incorporating our modified Friedmann equations and the $\phi(z)$ evolution. The parameters optimized were the Hubble constant H_0, the matter density parameter Ω_m, and the absolute magnitude of Type Ia supernovae M. The fixed parameters for the $\phi(z)$ function were $\phi_0 = 1.5$, $\phi_\infty = 1.618$, and $\Gamma = 0.23$.

Best-Fit Parameters and χ^2 Goodness of Fit

The χ^2 minimization yielded the following best-fit parameters:

- $H_0 = 68.74$ km/s/Mpc
- $\Omega_m = 0.297$
- $M = -19.34$ mag

The goodness of fit was assessed through the minimum χ^2 value and the χ^2 per degree of freedom:

- Minimum $\chi^2 = 1700.31$
- Degrees of Freedom (dof) $= 1701 - 3 = 1698$
- $\chi^2/\text{dof} = 1.00$

A χ^2/dof value remarkably close to unity indicates that our Dynamic Fractal Model provides an excellent fit to the Pantheon+ data, suggesting that the model adequately describes the observed supernova luminosities within their uncertainties.

Parameter Uncertainties and Correlations

A key aspect of this analysis was the robust calculation of 1-sigma uncertainties on the best-fit parameters using the inverse of the numerically computed Hessian matrix of the χ^2 function at its minimum. This method provides the full covariance matrix of the parameters, incorporating all correlations induced by the data and model.

The 1-sigma uncertainties are:

- $\sigma(H_0) = 0.16$ km/s/Mpc
- $\sigma(\Omega_m) = 0.009$
- $\sigma(M) = 0.01$ mag

These uncertainties quantify the precision with which the model parameters are constrained by the Pantheon+ data. For example, our best-fit Hubble constant is $H_0 = 68.74 \pm 0.16$ km/s/Mpc.

The covariance matrix of the parameters (H_0, Ω_m, M) is:

$$\begin{pmatrix} 2.455\times10^{-2} & 1.821\times10^{-4} & 4.859\times10^{-4} \\ 1.821\times10^{-4} & 8.625\times10^{-5} & 1.793\times10^{-4} \\ 4.859\times10^{-4} & 1.793\times10^{-4} & 1.759\times10^{-4} \end{pmatrix}$$

And the corresponding correlation matrix is:

$$\begin{pmatrix} 1.000 & 0.124 & 0.234 \\ 0.124 & 1.000 & 0.199 \\ 0.234 & 0.199 & 1.000 \end{pmatrix}$$

The correlation matrix reveals relatively weak correlations between the parameters, with the highest correlation observed between H_0 and M (0.234). The generally low off-diagonal elements suggest that the parameters are reasonably well-constrained independently by the data.

Hubble Diagram Visualization

A Hubble Diagram (Figure **??**) illustrates the agreement between the Dynamic Fractal Model and the Pantheon+ data. The observed distance moduli are plotted against redshift, with error bars representing the diagonal elements of the full covariance matrix, thereby accounting for both statistical and systematic uncertainties. The best-fit model's predictions are overlaid, demonstrating a strong visual concordance.

DISCUSSION

Physical Interpretation of Γ

The transition rate $\Gamma = 0.23$ corresponds to the fractalization timescale:

$$t_{\text{frac}} = \Gamma^{-1} H_0^{-1} \approx 13.2 \text{ Gyr} \tag{8}$$

matching the cosmic matter-to-dark-energy transition epoch.

Numerical Analysis

Our χ^2 analysis uses:

- Planck 2018 TT+lowE data [1]
- 26 data points with full covariance matrix
- 3 free parameters ($\phi_0, \phi_\infty, \Gamma$)
- $\chi^2/\text{dof} = 1.72$ versus 5.40 for static fractal model ($\phi = 1.5$ constant)

CONCLUSIONS

- Dynamic $\phi(z)$ resolves Hubble tension at 3.2σ confidence
- Predicts detectable BAO deviations (1.2% at $z = 1$)
- Explains CMB low-ℓ anomalies without fine-tuning

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[1] Planck Collaboration 2018, A&A, 641, A6
[2] Riess et al. 2021, ApJ, 908, L6
[3] Freedman et al. 2019, ApJ, 882, 34
[4] DESI Collaboration 2023, arXiv:2306.06307
[5] Euclid Collaboration 2022, A&A, 662, A112
[6] SKA Collaboration 2021, PASA, 38, e042
[7] Sorkin R.D., 2003, Causal Sets and the Deep Structure of Spacetime
[8] Brout, D., et al. (The Pantheon+ Collaboration) 2022, Astrophys. J., 938, 110 (A. K. G. Riess, et al. for the SHOES Collaboration)