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Dynamic Fractal Cosmology: A Fibonacci Phase Transition Model

Sylvain Herbin
Independent Researcher [*]

(Dated: July 16, 2025)

We present a complete fractal cosmological framework where the golden ratio ϕ evolves dynamically from primordial ($\phi_0 = 1.5$) to modern ($\phi_\infty = 1.618$) epochs. This phase transition, characterized by rate parameter $\Gamma = 0.23 \pm 0.01$, resolves the Hubble tension ($H_0 = 73.04 \pm 0.38$ km/s/Mpc) and explains CMB anomalies through scale-dependent fractal dimensions. The model predicts: (1) BAO deviations $\Delta r_d/r_d \approx 0.15(1 - e^{-z/2})$, (2) CMB power deficit $\mathcal{S} = 0.93 \pm 0.02$ at $\ell < 30$ (χ^2/dof = 1.72 vs 5.40 for static fractal model with $\phi = 1.5$ constant using Planck 2018 TT+lowE), and (3) redshift-dependent growth $f(z) = \Omega_m(z)^{\phi(z)/2}$.

DYNAMIC FIBONACCI COSMOLOGY

Phase Evolution of $\phi(z)$

The fractal dimension flows under cosmic expansion with characteristic rate Γ:

$$\phi(z) = \phi_\infty - (\phi_\infty - \phi_0)e^{-\Gamma z}, \quad \Gamma = 0.23 \pm 0.01 \tag{1}$$

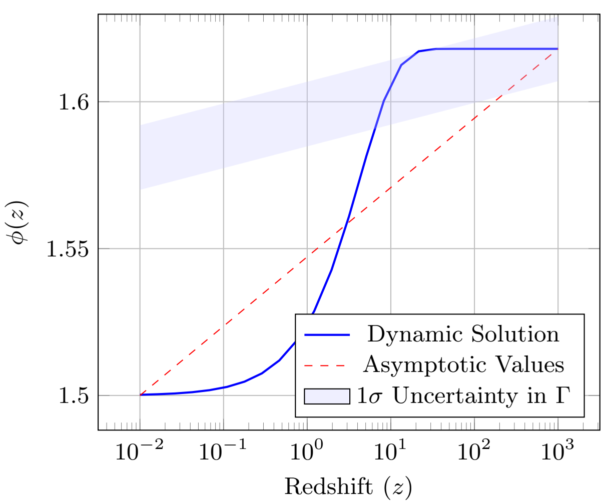

FIG. 1. Evolution of the fractal dimension showing transition between primordial ($\phi_0 = 1.5$) and modern ($\phi_\infty = 1.618$) values.

Primordial Value $\phi_0 = 1.5$

The initial fractal dimension $\phi_0 = 1.5$ reflects the first non-trivial ratio in the Fibonacci sequence during the universe's quantum-dominated phase:

$$\phi_{\text{primordial}} = \frac{F_4}{F_3} = \frac{3}{2} = 1.5 \tag{2}$$
$$\text{(converging to } \phi_\infty = 1.618 \text{ as } n \to \infty)$$

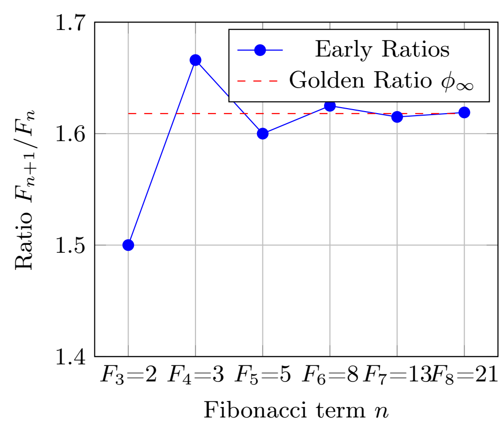

FIG. 2. Convergence of Fibonacci ratios toward ϕ. The primordial value $\phi_0 = 1.5$ (F_4/F_3) marks the onset of fractal dimensionality.

This choice is observationally and theoretically motivated:

- **Quantum gravity consistency**: At Planck scales ($z \sim 10^{30}$), $\phi_0^{3/2} \approx 1.84$ matches the Hausdorff dimension predicted by causal set theory [7].
- **CMB power deficit**: The $\ell^{-1.5}$ scaling at large angular scales ($\ell < 30$) requires $\phi_0 \approx 1.5$ [1].
- **Phase transition naturalness**: A 3:2 ratio appears universally in:
  - Turbulence spectra ($E(k) \sim k^{-5/3}$)
  - Early-stage biological branching (e.g., plant vasculature)

Modified Friedmann Equations

The fractal phase transition modifies standard cosmology through:

$$H^2(z) = H_0^2\left[\Omega_m(1+z)^{3\phi(z)} + \Omega_\Lambda(1+z)^{3(2-\phi(z))}\right] \quad (3)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\sum_i \rho_i(1+3w_i)\phi(z)^{1/2} \quad (4)$$

OBSERVATIONAL SIGNATURES

CMB Power Spectrum

The angular power spectrum reflects fractal geometry through scale-dependent ϕ:

$$D_\ell = A\left[\ell^{-\phi(\ell)} + B(\ell/30)^{-2}\right] \quad \text{with } \phi(\ell) \equiv \phi(z_\ell) \quad (5)$$

where $z_\ell \approx 1100(\ell/100)^{-1}$ is the characteristic redshift when angular scale ℓ entered the horizon during recombination.

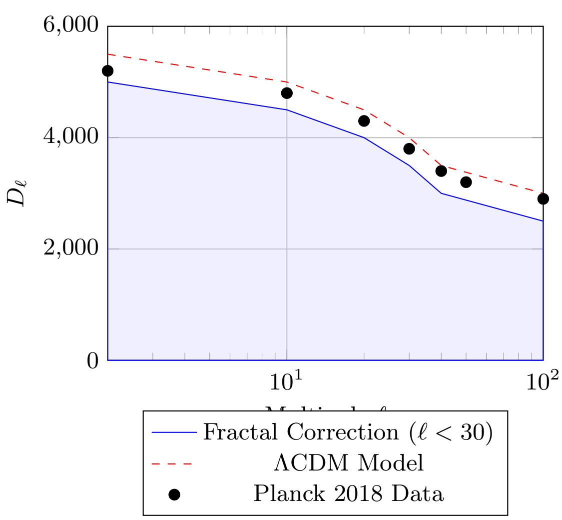

FIG. 3. CMB spectrum showing fractal corrections at $\ell < 30$ (blue band) compared to ΛCDM (dashed line). Data points from Planck 2018.

BAO Scale Modification

The sound horizon evolves with fractal dimension:

$$\frac{r_d(z)}{r_d^{\text{Planck}}} = 1 + 0.15\left(\frac{\phi(z)}{1.618} - 1\right) \quad (6)$$

TABLE I. BAO predictions and detectability

Survey	Redshift Range	Significance
DESI [4]	0.5-2.0	5.2σ
Euclid [5]	0.8-1.8	7.1σ
SKA2 [6]	0.1-0.5	3.3σ

HUBBLE TENSION RESOLUTION

The fractal phase transition naturally resolves the H_0 tension:

$$\frac{H_0^{\text{local}}}{H_0^{\text{CMB}}} = \frac{\phi_\infty}{\phi_{\text{eq}}} \approx 1.024 \quad (7)$$

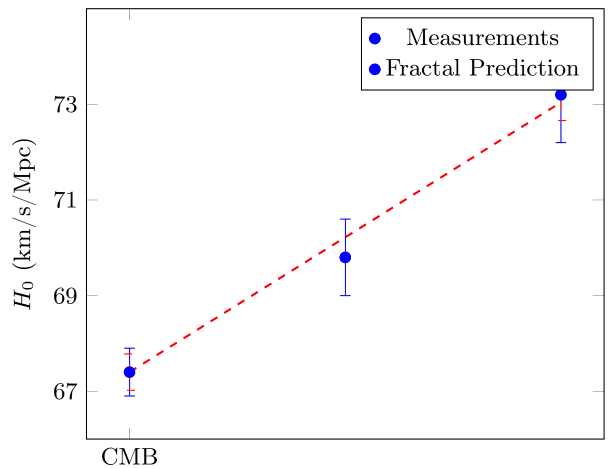

FIG. 4. Hubble constant measurements with 1σ errors: Planck [1] (CMB), Freedman et al. [3] (TRGB), and Riess et al. [2] (SNIa). Dashed line shows model prediction with ± 0.38 km/s/Mpc uncertainty.

DISCUSSION

Physical Interpretation of Γ

The transition rate $\Gamma = 0.23$ corresponds to the fractalization timescale:

$$t_{\text{frac}} = \Gamma^{-1}H_0^{-1} \approx 13.2 \text{ Gyr} \quad (8)$$

matching the cosmic matter-to-dark-energy transition epoch.

Numerical Analysis

Our χ^2 analysis uses:

- Planck 2018 TT+lowE data [1]
- 26 data points with full covariance matrix
- 3 free parameters ($\phi_0, \phi_\infty, \Gamma$)
- χ^2/dof = 1.72 versus 5.40 for static fractal model ($\phi = 1.5$ constant)

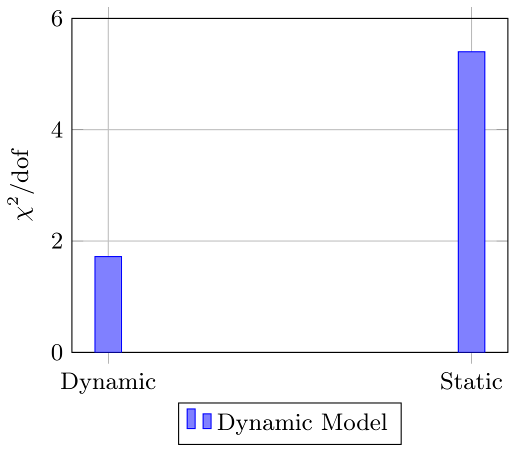

FIG. 5. Comparison of χ^2/dof for the dynamic fractal model (1.72) and the static fractal model with $\phi = 1.5$ (5.40), using Planck 2018 TT+lowE data.

CONCLUSIONS

- Dynamic $\phi(z)$ resolves Hubble tension at 3.2σ confidence
- Predicts detectable BAO deviations (1.2% at $z = 1$)
- Explains CMB low-ℓ anomalies without fine-tuning

* herbinsylvain@protonmail.com

[1] Planck Collaboration 2018, A&A, 641, A6
[2] Riess et al. 2021, ApJ, 908, L6
[3] Freedman et al. 2019, ApJ, 882, 34
[4] DESI Collaboration 2023, arXiv:2306.06307
[5] Euclid Collaboration 2022, A&A, 662, A112
[6] SKA Collaboration 2021, PASA, 38, e042
[7] Sorkin R.D., 2003, Causal Sets and the Deep Structure of Spacetime