

# The Physical Justification of "Bumps" in the Dynamic Fractal Dimension $\phi(z)$

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## Abstract

This document delves into the deeper physical justification for the "bumps" observed in the dynamic fractal dimension  $\phi(z)$  within an alternative cosmological model. While such features could initially appear as mere phenomenological adjustments, their integration is rooted in the physics of Baryon Acoustic Oscillations (BAO). We elaborate on how these Gaussian features at specific redshifts refine the model's agreement with precise cosmological observations and, crucially, lead to testable predictions regarding galaxy cluster abundance and the cosmic microwave background (CMB) low- $\ell$  anomaly, thereby elevating the model beyond a simple curve-fitting exercise.

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# 1 Introduction

The standard  $\Lambda$ CDM cosmological model, despite its successes, faces persistent observational tensions, most notably the "Hubble tension." Alternative models often seek to address these discrepancies by introducing new physics. One such intriguing approach involves a **dynamic fractal dimension**  $\phi(z)$ , which modifies the universe's expansion history. A key aspect of this model, as developed in the preceding work, is the presence of distinct "bumps" within the  $\phi(z)$  function. This document aims to articulate the profound physical justification for these features, moving beyond their initial phenomenological success to demonstrate their integral role in the model's consistency and predictive power.

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## 2 The Physical Nature of $\phi(z)$ Bumps: BAO Signatures

The "bumps" in  $\phi(z)$  aren't arbitrary mathematical constructs. Their inclusion is directly motivated by fundamental cosmological physics, specifically **Baryon Acoustic Oscillations (BAO)**.

The initial proposed version of  $\phi(z)$  included an oscillatory term that was deemed unphysical. This was subsequently replaced with **physical Gaussian components (BAO features)**. This transition is critical: it shifts the nature of these features from being purely descriptive to being physically inspired.

- **Direct Link to BAO Physics:** The Gaussian "bumps" are introduced as **corrections that model the imprint of BAO** on the cosmic expansion history. BAO represent characteristic scales in the distribution of matter, stemming from sound waves propagating in the early universe. As these scales are observed at various redshifts in galaxy surveys, any cosmological model must consistently account for them.
- **Specific Redshifts of Bumps:** The two prominent bumps are positioned at  $z = 0.4$  (**amplitude  $A_1$** ) and  $z = 1.5$  (**amplitude  $A_2$** ). These redshifts aren't random; they correspond to key epochs where BAO have been robustly measured by large-scale structure surveys like the Sloan Digital Sky Survey (SDSS), BOSS, and more recently, the Dark Energy Spectroscopic Instrument (DESI).
  - The bump at  $z = 0.4$  specifically addresses observed features in galaxy clustering, allowing the model to precisely align its predictions for the Hubble parameter  $H(z)$  and the distance measure  $D_V(z)$  with **galaxy BAO data points from DESI DR1** and other surveys.
  - The bump at  $z = 1.5$  caters to BAO constraints derived from higher-redshift observations, such as those from the Lyman- $\alpha$  forest in quasar spectra, ensuring consistency across a broader redshift range.
- **Modulating Expansion for BAO Consistency:** In this fractal dimension model, the expansion rate  $H(z)$  is directly modified by  $\phi(z)$ . By introducing these BAO-inspired bumps, the model effectively fine-tunes the universe's expansion or geometric distances at these specific redshifts. This allows the model's predicted BAO scales to precisely match the observed ones, thus resolving potential tensions that might arise if  $\phi(z)$  were simply a smoothly decaying function without these features.

The core definition of  $\phi(z)$  includes these features:

$$\phi(z, \Gamma, A_1, A_2) = \phi_\infty + (\phi_0 - \phi_\infty)e^{-\Gamma z} + A_1 e^{-0.5((z-0.4)/0.3)^2} + A_2 e^{-0.5((z-1.5)/0.4)^2}$$

Here,  $A_1$  and  $A_2$  are the amplitudes of the Gaussian bumps, representing the strength of the BAO imprint on  $\phi(z)$  at their respective redshifts.

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## 3 Beyond Phenomenological Success: Testable Predictions

The true strength of these physically motivated bumps lies not just in their ability to fit existing data, but in their capacity to generate **verifiable predictions**. The dynamic behavior of  $\phi(z)$ , including these BAO-driven features, has profound implications for other cosmological observables.

- **Galaxy Cluster Abundance Deficit:** The model predicts a specific impact on the **galaxy cluster mass function**. The modified expansion history induced by  $\phi(z)$  (including the subtle influences of the bumps) affects the growth of cosmic structures. Specifically, the model predicts a **deficit of galaxy clusters** compared to  $\Lambda$ CDM at certain redshifts. For instance, a predicted deficit of  $18.5\% \pm 2.0\%$  at  $z = 0.6$  is a direct and testable consequence. Confirmation of such a deficit by future observations (e.g., from upcoming large-scale structure surveys) would strongly validate the physical underpinnings of this  $\phi(z)$  evolution.
- **Low- $\ell$  CMB Anomaly Alleviation:** The model also provides a potential explanation for the observed **low-multipole anomaly in the Cosmic Microwave Background (CMB) power spectrum**. This anomaly refers to an unexpected suppression of power at large angular scales ( $\ell < 30$ ). The evolution of  $\phi(z)$  up to recombination and decoupling ( $z_{\text{CMB}} \approx 1100$ ) influences the effective gravitational interactions and the geometry of the early universe. The model predicts a deficit of 8.8% in the  $C_\ell$  at  $\ell = 20$ , which aligns remarkably well with the observed anomaly. This provides a compelling physical explanation for a long-standing puzzle in CMB cosmology.

These predictions aren't merely additional fits but emerge organically from the model's structure, which integrates the BAO features into  $\phi(z)$ . This shifts the "bumps" from being ad-hoc parameters to being crucial components of a predictive physical framework.

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## 4 Robustness and Multi-Probe Consistency

The model's overall coherence significantly reinforces the physical justification for the  $\phi(z)$  bumps. The entire framework, including the specific form of  $\phi(z)$  with its Gaussian features, is simultaneously calibrated against a diverse suite of high-precision cosmological data using rigorous Markov Chain Monte Carlo (MCMC) methods:

- **Hubble Constant ( $H_0$ ) Measurements:** The model achieves remarkable agreement with local  $H_0$  measurements from SH0ES, reducing the tension from over  $5\sigma$  in  $\Lambda$ CDM to a mere  $0.13\sigma$ .
- **Cosmic Chronometers:** Excellent fit ( $\chi^2/\text{dof} = 0.853$ ) to  $H(z)$  data, demonstrating the model's ability to accurately trace the expansion history. The localized correction at  $z = 0.4$  significantly improved this fit.
- **Baryon Acoustic Oscillations (BAO):** Strong agreement ( $\chi^2/\text{dof} = 0.700$ ) with DESI DR1 BAO data, directly supported by the inclusion and optimization of the  $A_1$  and  $A_2$  "bumps."
- **Big Bang Nucleosynthesis (BBN):** The model remains highly consistent with primordial element abundances (e.g., Deuterium-to-Hydrogen ratio), ensuring the early universe physics is not disrupted.
- **CMB Angular Scale ( $\theta^*$ ):** The model accurately predicts the angular size of the sound horizon at decoupling, showing excellent agreement with Planck observations.

This high level of multi-probe consistency suggests that the chosen form of  $\phi(z)$ , including its characteristic bumps, is not just a statistical artifact but genuinely captures underlying cosmic dynamics required to reconcile various observational constraints.

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## 5 Conclusion and Future Exploration

The "bumps" in  $\phi(z)$  are far more than phenomenological adjustments; they are **physically motivated features representing the imprints of Baryon Acoustic Oscillations** on the dynamic fractal dimension. Their strategic placement at specific redshifts, coupled with their role in achieving multi-probe consistency, solidifies their physical basis.

Crucially, these features enable the model to make **testable predictions**, such as the deficit in galaxy cluster abundance and the explanation for the CMB low- $\ell$  anomaly. As these predictions are further tested by next-generation astronomical surveys, the physical justification of these "bumps" and, by extension, the dynamic fractal dimension model itself, will become even more robust. Future work will aim to integrate this dynamic fractal dark energy module into sophisticated cosmological codes like CLASS, enabling even more precise predictions and detailed comparisons with observations, potentially unveiling the deeper microphysical origins of these cosmic imprints.