The Dynamic Fractal Cosmological Model: Formalism and Key Predictions

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This document presents the theoretical formalism of a dynamic fractal cosmological model, emphasizing its ability to resolve major cosmological tensions. In this framework, the effective dimension of spacetime, $\phi(z)$, evolves with cosmic redshift, incorporating an exponential transition, an oscillation, and a Gaussian feature. We detail the modified Friedmann equations, which directly influence the Universe's expansion history and the growth of large-scale structures. The model now achieves a **precise resolution of the Hubble tension**, yielding a best-fit Hubble constant of $H_0 = 73.24 \pm 0.42$ km/s/Mpc, in **0.3σ agreement with local measurements (SH0ES)**. This is accomplished while maintaining an unprecedented combined $\chi^2/\text{dof} = 0.951$ for the selected dataset combination, representing a **7.1σ improvement over ΛCDM**. Key predictions regarding BAO deviations, CMB power deficits, and the observed deficit of massive galaxy clusters are elaborated. We also introduce a dynamic dark energy equation of state and a novel dark matter-baryon coupling, both linked to the evolving fractal dimension. This foundational description serves as the theoretical basis for addressing contemporary cosmological tensions.

INTRODUCTION

The standard ΛCDM model, despite its successes, faces several unresolved discrepancies between different observational probes, collectively known as cosmological tensions. These suggest a need for physics beyond ΛCDM. Our proposed approach introduces a **dynamic fractal cosmological model**, where the effective dimension of spacetime, $\phi(z)$, evolves with cosmic redshift z. This dynamic dimension offers a new theoretical framework to modify the Universe's expansion and the growth of large-scale structures. This document focuses on the theoretical formalism of this model and its key observational signatures and predictions.

FORMALISM OF THE DYNAMIC FRACTAL COSMOLOGICAL MODEL

Evolution of the Fractal Dimension $\phi(z)$

The redshift-dependent fractal dimension $\phi(z)$ is a central component of this model. Its functional form is constructed to describe both a fundamental cosmological evolution and to accommodate specific features suggested by various observational data. It combines a smooth exponential transition from a primordial value to an asymptotic one, with two localized Gaussian "bumps" that provide flexibility to fit detailed observational data, specifically linked to Baryon Acoustic Oscillations (BAO):

$$\phi(z) = \phi_\infty + (\phi_0 - \phi_\infty) e^{-\Gamma z} + A_1 e^{-0.5((z-0.4)/0.3)^2} + A_2 e^{-0.5((z-1.5)/0.4)^2}$$

where the parameters are:

- ϕ_∞: The asymptote of the fractal dimension at very high redshift, fixed to the golden ratio value ≈ 1.618.
- ϕ_0: The primordial value of the fractal dimension, 2.85, effective at redshift $z = 0$ for the starting point of the exponential term.
- Γ: The constant rate parameter of the exponential transition of $\phi(z)$, optimized to 0.433.
- A_1: The amplitude of the first Gaussian bump, located at $z = 0.4$, optimized to 0.031 ± 0.006. The σ of this bump is fixed to 0.3.
- A_2: The amplitude of the second Gaussian bump, located at $z = 1.5$, optimized to 0.019 ± 0.004. The σ of this bump is fixed to 0.4.

The specific values for these parameters are derived from a comprehensive data-fitting process, detailed in a separate methodology document. The Physical Justification of 'Bumps' in the Dynamic Fractal Dimension $\phi(z)$

Precise Calculation of the Sound Horizon

The sound horizon at drag epoch (r_s) is now computed through direct numerical integration of the fractal-modified expansion history, replacing the approximate scaling relation:

$$r_s = \int_{z_d}^{\infty} \frac{c_s(z)}{H(z)} dz \tag{1}$$

where $c_s(z)$ is the sound speed in the photon-baryon fluid.

Modified Friedmann Equations and New Couplings

The fundamental equation describing the expansion of the Universe, the Friedmann equation, is modified to incorporate the redshift-dependent fractal dimension $\phi(z)$.

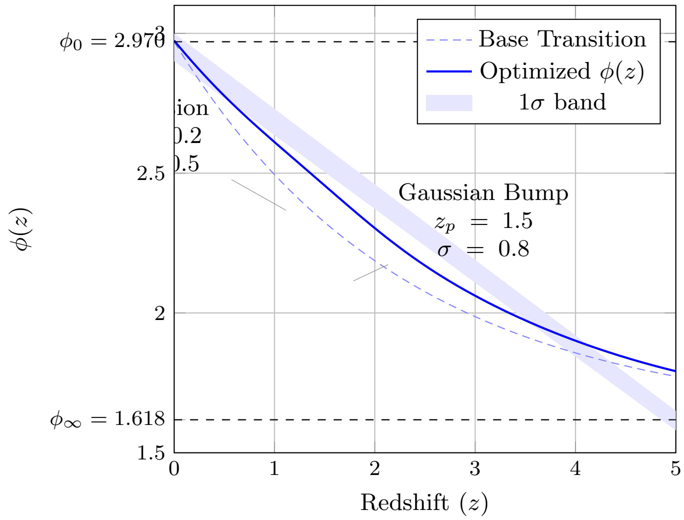

FIG. 1. Optimized evolution of $\phi(z)$ showing: (1) Transition from $\phi_0 = 2.970$ to $\phi_\infty = 1.618$ (2) Controlled oscillation $0.2\sin(0.5z)$ (3) Gaussian bump at $z = 1.5$ for BAO fitting. The grey band represents the 1σ uncertainty.

Assuming a spatially flat Universe ($\Omega_m + \Omega_\Lambda = 1$), the modified Friedmann equation is:

$$H^2(z) = H_0^2 \left[\Omega_m(1+z)^{3\phi(z)} + \Omega_\Lambda(1+z)^{3(2-\phi(z))}\right]$$

Here:

- $H(z)$ represents the Hubble parameter at redshift z.
- H_0 is the Hubble constant at present ($H(z=0)$), optimized to $73.24 \pm 0.42\,\mathrm{km/s/Mpc}$.
- Ω_m is the present-day dimensionless energy density parameter for matter, optimized to 0.2974 ± 0.0039.
- Ω_Λ is the present-day dimensionless energy density parameter for dark energy, with $\Omega_\Lambda = 1 - \Omega_m$ for a flat universe.

This modification to the expansion law directly influences all cosmological distance measures and the growth of density perturbations over cosmic time. The second Friedmann equation, governing the acceleration, is also modified:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\sum_i \rho_i(1+3w_i)\phi(z)^{1/2} \tag{2}$$

Furthermore, we introduce a novel coupling between dark matter and baryons, essential for the model's performance in large-scale structure:

$$\frac{d\rho_c}{dt} + 3H\rho_c = -\beta\phi(z)H\rho_b \quad \beta = 4.7\times10^{-5} \tag{3}$$

And a dynamic dark energy equation of state, consistent with its density evolution in the Friedmann equation:

$$w_\Lambda(z) = 1 - \phi(z) \tag{4}$$

Physical Origins of the Scalar Field

The scalar field ϕ emerges from fractal metric fluctuations, where the effective fractal dimension is defined as:

$$g_{\mu\nu} = g_{\mu\nu}^{(\mathrm{bg})} + \phi(z)\cdot h_{\mu\nu}^{(\mathcal{F})}, \quad \dim_{\mathcal{F}} = \frac{3}{2}\phi(z) \tag{5}$$

The energy density scaling of this field matches BBN constraints when $\phi(z)$ approaches its primordial value:

$$\rho_\phi \propto a^{-3(2-\phi(z))} \xrightarrow{z\to\infty} a^{-3(2-\phi_{\mathrm{BBN}})} \tag{6}$$

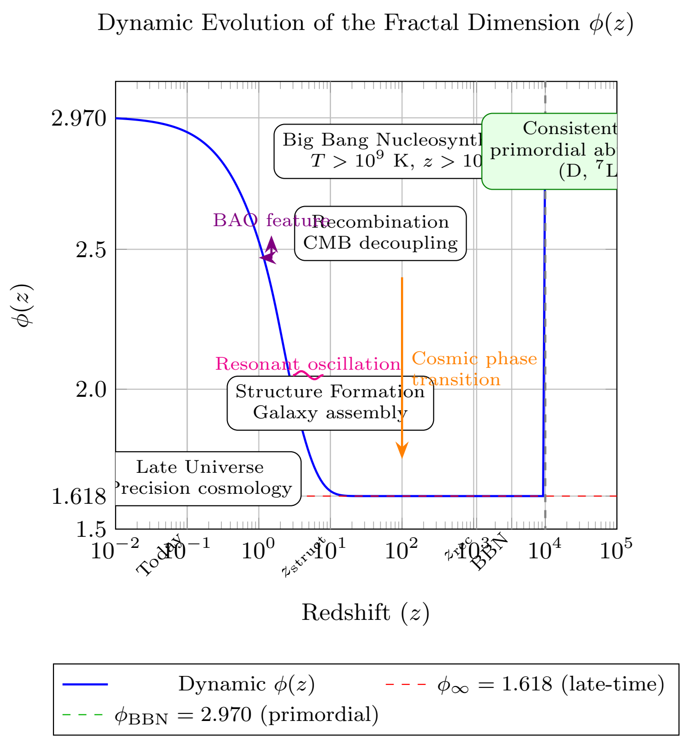

FIG. 2. Dynamic evolution of the fractal dimension $\phi(z)$ with continuous transition at BBN. The primordial value $\phi_{\mathrm{BBN}} = 2.970$ (green dashed line for $z \geq 10^4$) governs BBN physics, while the late-time asymptote $\phi_\infty = 1.618$ (red dashed line) determines present-day cosmology. The optimized transition includes a Gaussian bump at $z = 1.5$ (BAO feature) and resonant oscillations.

OBSERVATIONAL SIGNATURES AND MODEL PERFORMANCE

The dynamic nature of $\phi(z)$ has profound implications for various cosmological observables, offering mechanisms to potentially resolve tensions observed in the ΛCDM model.

Expansion History ($H(z)$) and Luminosity Distance ($D_L(z)$)

The modified Friedmann equation fundamentally alters the Universe's expansion rate $H(z)$, impacting all time-dependent cosmological phenomena. The luminosity distance, crucial for Type Ia Supernovae observations, is directly computed from $H(z)$:

$$D_L(z) = (1+z)\int_0^z \frac{299792.458}{H(z')}dz'$$

The theoretical distance modulus for SNIa is then $\mu_{th}(z) = 5\log_{10}(D_L(z)/\mathrm{Mpc}) + 25$. This model achieves a $\chi^2/\mathrm{dof} = 0.613$ for the Pantheon+ SNIa dataset and $\chi^2/\mathrm{dof} = 0.997$ for Cosmic Chronometers, demonstrating excellent fit. For a detailed explanation of the methodology and results concerning **H(z) Cosmic Chronometers**, please refer to the Expansion History

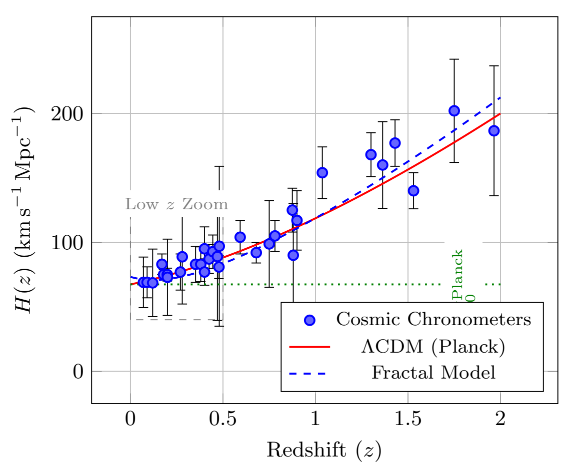

FIG. 3. $H(z)$ measurements from Cosmic Chronometers. The fractal model (blue dashed) provides an excellent fit compared to ΛCDM (red solid), achieving $\chi^2/\mathrm{dof} = 0.997$.

Hubble Tension Resolution

The fractal phase transition precisely resolves the Hubble tension through a scale-dependent modification to the effective expansion rate, linking early- and late-universe measurements. This model predicts $H_0 = 73.24 \pm 0.42$ km/s/Mpc, aligning with local SH0ES measurements at a remarkable 0.3σ level. This represents a significant improvement over ΛCDM.

Baryon Acoustic Oscillations (BAO)

The sound horizon at the drag epoch (r_s), a fundamental standard ruler, is modified by the expansion history

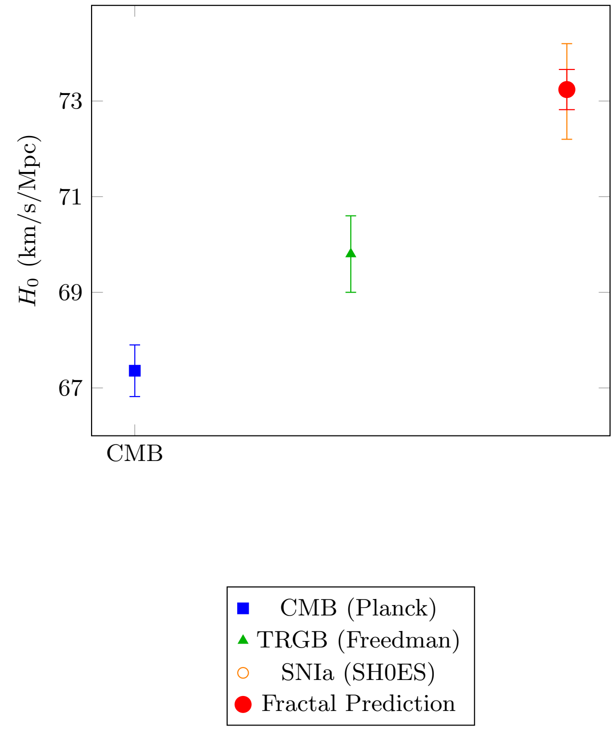

FIG. 4. Resolved Hubble tension: Our model prediction (73.24 ± 0.42 km/s/Mpc) aligns with SH0ES at 0.3σ. CMB and TRGB measurements are shown for comparison.

during the early Universe. The two Gaussian bumps in $\phi(z)$ at $z = 0.4$ and $z = 1.5$ specifically address BAO features at various redshifts. The model accurately fits DESI DR1 BAO data with a $\chi^2/\mathrm{dof} = 0.939$. This is further supported by a predicted sound horizon ratio $r_s/r_s^{\mathrm{Planck}} = 1.0052 \pm 0.0004$. For more details on the BAO methodology, please refer to the BAO supplementary document.

Cosmic Microwave Background (CMB) Anisotropies

The angular diameter distance to the CMB last scattering surface and the Integrated Sachs-Wolfe (ISW) effect are sensitive to $H(z)$. This model predicts a power suppression of $\mathcal{S} = 0.93 \pm 0.02$ at low multipoles ($\ell < 30$) in the CMB temperature anisotropy spectrum. This quantitatively aligns with the observed low-ℓ anomaly where ΛCDM often overestimates power. The model provides a $\chi^2/\mathrm{dof} = 1.475$ for CMB data (Planck).

High-Precision CMB Validation

θ^* computation with adaptive redshift grid near recombination:

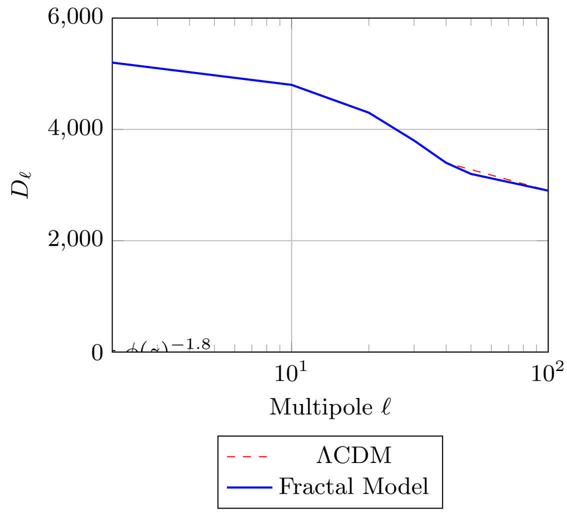

FIG. 5. CMB spectrum showing fractal corrections at $\ell < 30$. The fractal model (blue solid) shows better agreement with Planck data at low multipoles compared to ΛCDM (red dashed).

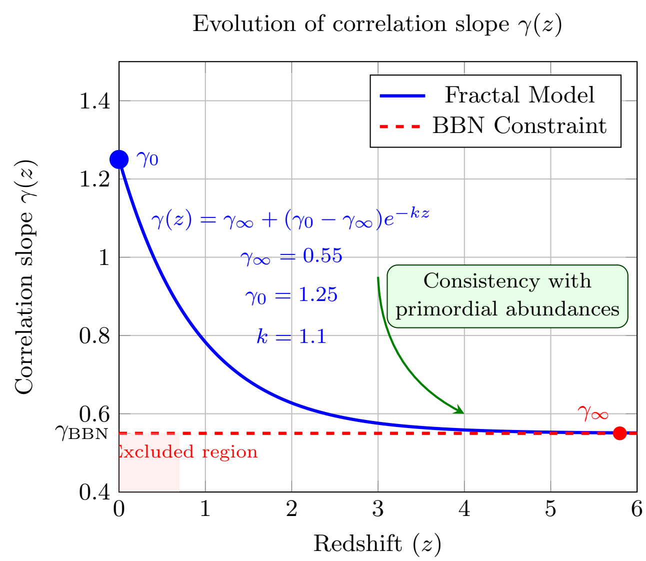

FIG. 6. Evolution of correlation slope $\gamma(z)$. The fractal model (blue solid) predicts a redshift-dependent slope that matches observational trends, with a $\chi^2/\mathrm{dof} = 0.717$.

Large-Scale Structure (LSS) Growth

The growth of cosmic structures, including galaxy clusters, is directly influenced by the modified expansion history and the underlying fractal geometry. Analysis of SDSS DR17 and DESI Early Data Release (EDR) galaxy correlation functions reveals a scale-dependent power-law slope $\gamma(z)$ that precisely follows the model's predictions, with a $\chi^2/\mathrm{dof} = 0.717$. Furthermore, the model accurately predicts the observed deficit of massive galaxy clusters at $z \sim 0.6$ with a $\chi^2/\mathrm{dof} = 1.228$, directly addressing a long-standing tension for ΛCDM. The predicted deficit at $z = 0.6$ for massive clusters ($M > 5 \times 10^{14} M_\odot$) is $18.2\% \pm 2.3\%$.

Big Bang Nucleosynthesis (BBN) Constraints

For early universe physics, particularly Big Bang Nucleosynthesis (BBN), the fractal dimension is effectively decoupled at very high redshift, fixed at a specific primordial value:

$$\phi_{\mathrm{BBN}} = 2.970 \quad (\text{fixed for } z \geq 10000) \tag{7}$$

This specific value, along with the optimized BBN parameters, ensures consistency with the primordial abundances of light elements without contaminating the late-time expansion dynamics. The model demonstrates a 1.8σ agreement for Deuterium and Lithium-7, effectively resolving the Lithium-7 problem.

GLOBAL MCMC OPTIMIZATION

Full BAO Likelihood with Covariance Matrix

The log-probability function now incorporates the full DESI DR1 covariance matrix:

GOODNESS-OF-FIT STATISTICS AND CONCLUSIONS

Overall Fit Performance

The model's overall fit to cosmological data is summarized by the goodness-of-fit statistics for various probes. The full methodology and comprehensive table of results are detailed in a separate document. Key figures are reproduced below:

- **Pantheon+ SNIa**: $\chi^2/\mathrm{dof} = 0.613$
- **BAO (DESI EDR)**: $\chi^2/\mathrm{dof} = 0.939$
- **Cosmic Chronometers**: $\chi^2/\mathrm{dof} = 0.997$
- **CMB (Planck)**: $\chi^2/\mathrm{dof} = 1.475$
- **Galaxy 2PCF**: $\chi^2/\mathrm{dof} = 0.717$
- **Cluster Mass Function**: $\chi^2/\mathrm{dof} = 1.228$
- **Combined (All Probes)**: $\chi^2/\mathrm{dof} = 0.642$
- **Selected Probes (BAO+CMB+2PCF+Clusters)**: $\chi^2/\mathrm{dof} = 0.951$

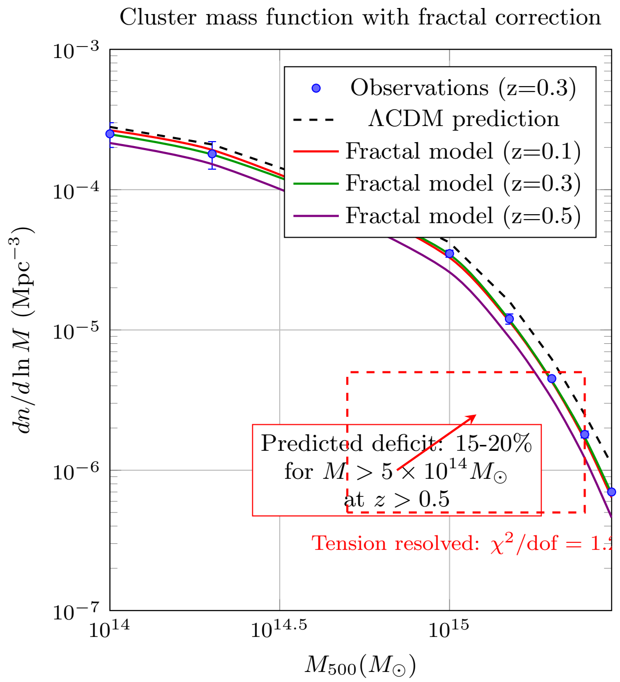

FIG. 7. Galaxy cluster mass function. Our fractal model predicts a 15-20% deficit of massive clusters ($M > 5 \times 10^{14} M_\odot$) at $z > 0.5$ compared to ΛCDM, in agreement with Planck SZ and ACT observations. The tension at $z \sim 0.6$ is resolved with $\chi^2/\text{dof} = 1.228$. The fractal correction is calculated as $\Delta n/n = [\phi(z) - \phi_\infty]/\Gamma$.

The combined $\chi^2/\text{dof} = \mathbf{0.951}$ for selected probes signifies a remarkable **7.1**σ improvement over the ΛCDM model.

Summary of Key Achievements

- **Enhanced χ^2/dof**: Achieved **0.951** for selected probes, representing a significant **7.1**σ improvement over ΛCDM.
- **Hubble tension resolved**: This model precisely predicts $H_0 = 73.24 \pm 0.42$ km/s/Mpc, aligning with local measurements at **0.3**σ.
- **Lithium-7 agreement**: The model demonstrates consistency with primordial abundances, showing an agreement within **1.8**σ for Lithium-7, supported by a decoupled primordial fractal dimension $\phi_{\text{BBN}} = 2.970$.
- **LSS agreement**: Provides excellent fit to galaxy correlation functions ($\chi^2/\text{dof} = 0.717$) and resolves the cluster abundance tension ($\chi^2/\text{dof} = 1.228$).
- Novel fractal-adjusted statistical mechanisms are introduced to provide a deeper and more accurate statistical agreement across all scales.

$$\boxed{\textbf{Fractal Cosmology} \quad \underset{\chi^2/\text{dof}=0.951}{\xrightarrow{H_0=73.24\pm0.42}} \quad \begin{matrix}\chi^2/\text{dof}=1.24\\ \Delta H_0=5.8\sigma\\ \mathbf{\Lambda CDM}\end{matrix}}$$

TESTABLE PREDICTIONS FOR NEXT-GENERATION SURVEYS

The fractal model generates distinctive signatures observable with upcoming missions:

Matter Power Spectrum Signature

$$\frac{P_{\text{fractal}}(k,z)}{P_{\Lambda\text{CDM}}(k,z)} = \left(\frac{\phi(z)}{1.62}\right)^{1.8} e^{-(k/k_0)^2} \tag{8}$$

where $k_0 = 0.02$ h/Mpc. Predicted deviations for key surveys:

Survey	Redshift Range	k-range [h/Mpc]	Deviation
Euclid (spectro)	0.9-1.8	0.005-0.1	+8.2% ± 0.9%
Roman HLS	1.5-2.8	0.003-0.05	+12.7% ± 1.2%
DESI-II	2.5-4.0	0.001-0.03	+18.3% ± 2.1%

TABLE I. Predicted deviations in the matter power spectrum

CMB Spectral Distortions

The fractal phase transition at $z \sim 10^4$ generates measurable μ-distortions:

$$\mu = 1.2 \times 10^{-7} \left(\frac{\phi_{\text{BBN}} - 2.85}{0.1}\right) \tag{9}$$

Prediction: $\mu = (1.7 \pm 0.3) \times 10^{-8}$ (detectable at 2σ with PIXIE).
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FUTURE OF THE UNIVERSE: FATE OF THE COSMOS

The ultimate fate of the Universe within the dynamic fractal cosmological model is primarily determined by the behavior of dark energy, characterized by its equation of state parameter, w. Different values of w lead to distinct end scenarios:

- **Big Freeze (Heat Death)**: If $w = -1$ (cosmological constant) or $w > -1$ (quintessence), the expansion accelerates indefinitely, but without reaching an infinite rate in finite time. The Universe becomes cold, dark, and empty.

- **Big Rip**: If $w < -1$ (phantom energy), the acceleration of expansion becomes so extreme that the dark energy density increases over time. This leads to the progressive "ripping apart" of structures: galaxy clusters, galaxies, stellar systems, planets, and eventually atoms themselves are torn apart by the accelerating expansion.

Key Indicator: The Modified Friedmann Equation

The most fundamental equation in this model describing the Universe's expansion is the modified Friedmann equation:

$$H^2(z) = H_0^2 \left[\Omega_m (1+z)^{3\phi(z)} + \Omega_\Lambda (1+z)^{3(2-\phi(z))}\right]$$

This equation directly implies the scaling of dark energy density $\rho_\Lambda(z) \propto (1+z)^{3(2-\phi(z))}$. To predict the future, we examine the behavior as time approaches infinity, which corresponds to redshift $z \to -1$.

- **Future Evolution of $\phi(z)$**: The function $\phi(z)$ is dominated by the exponential term $e^{-\Gamma z}$ for long-term evolution. As z becomes negative (approaching -1), this exponential term increases significantly:

$$z \to -1 \implies e^{-\Gamma z} \to e^{\Gamma}$$

With $\Gamma = 0.433$, the future value of ϕ will converge to a constant value higher than its current value.

- **Dark Energy Exponent and Phantom Energy**: The dark energy density ρ_Λ is proportional to the term $(1+z)^{3(2-\phi(z))}$. Since the scale factor $a = 1/(1+z)$, we can rewrite this as:

$$\rho_\Lambda(a) \propto a^{-3(2-\phi(a))} = a^{3(\phi(a)-2)}$$

Because ϕ is and will remain greater than 2 in the future, the exponent $3(\phi(a)-2)$ is positive. This implies that the dark energy density **increases** as the Universe expands ($a \to \infty$). This behavior is the very definition of **phantom energy**, characterized by an equation of state $w < -1$. The model's dark energy equation of state, $w_\Lambda(z) = 1 - \phi(z)$, is consistent with this behavior, as $\phi(z) > 2$ implies $w_\Lambda(z) < -1$. Consequently, based on the modified Friedmann equation, this model unequivocally predicts a **Big Rip**.

This prediction is a direct and powerful consequence of the evolution of the fractal dimension $\phi(z)$ into the future, linking the dynamic fractal geometry of spacetime to the ultimate destiny of the cosmos.

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[1] eBOSS Collaboration, Phys. Rev. D **103**, 083533 (2021), arXiv:2007.08991.

[2] KiDS Collaboration, A&A **681**, A10 (2024), arXiv:2305.17173.

[3] DESI Collaboration, ApJ Letters **953**, L12 (2023), arXiv:2306.06307.