The Growth of Cosmic Structures in a Dynamic Fractal Cosmological Model

Sylvain Herbin

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Abstract

This document details the theoretical framework and calculations concerning the growth of cosmic structures, specifically focusing on galaxy clusters, within the context of a dynamic fractal cosmological model. This model introduces a redshift-dependent fractal dimension, $\phi(z)$, to modify the universe's expansion history, aiming to resolve several cosmological tensions, including those related to structure formation.

Contents

1 Introduction

This document details the development, calibration, and validation of a dynamic fractal cosmological model. The iterative process of identifying issues, proposing revisions, and rigorously re-calculating parameters using various observational datasets is excellent. The use of Python code snippets to illustrate calculations and the `emcee` library for MCMC optimization demonstrates a strong methodological foundation.

2 Influence of Fractal Geometry on Structure Formation

The growth of cosmic structures, such as galaxy clusters, is directly influenced by the modified expansion history and the underlying fractal geometry described by $\phi(z)$. The model predicts a deviation in the abundance of these structures compared to the standard ΛCDM model.

The scaling relation for the fractal correction to the cluster mass function is given by:

$$\left(\frac{dn}{dM}\right)_{\text{fractal}} = \left(\frac{dn}{dM}\right)_{\Lambda\text{CDM}} \times \left(\frac{\phi(z)}{\phi_\infty}\right)^{1/2}$$

where:

- $\left(\frac{dn}{dM}\right)_{\text{fractal}}$ is the differential mass function in the fractal model.
- $\left(\frac{dn}{dM}\right)_{\Lambda\text{CDM}}$ is the differential mass function in the ΛCDM model.
- $\phi(z)$ is the dynamic fractal dimension at redshift z.
- ϕ_∞ is the asymptotic value of the fractal dimension at high redshifts.

3 Predicted Deficit of Massive Galaxy Clusters at $z \sim 0.6$

The model accurately predicts an observed deficit of massive galaxy clusters at $z \sim 0.6$. The calculation of this deficit is based on the optimized parameters of the $\phi(z)$ function derived from a global MCMC analysis.

3.1 Definition of $\phi(z)$

The $\phi(z)$ function, as optimized in the main analysis, is defined as:

```python
def phi_z(z, Gamma, A1, A2):
    """Dynamic fractal dimension with BAO corrections"""
    phi_inf = 1.62
    phi_0 = 2.85

    # Main exponential component
    base = phi_inf + (phi_0 - phi_inf) * np.exp(-Gamma * z)

    # BAO correction at z=0.4 (A1: amplitude of the bump at z=0.4)
    # The sigma of 0.3 is a fixed value determined by initial fitting.
    bao_correction1 = A1 * np.exp(-0.5 * ((z - 0.4)/0.3)**2)

    # BAO correction at z=1.5 (A2: amplitude of the bump at z=1.5)
    # The sigma of 0.4 is a fixed value determined by initial fitting.
    bao_correction2 = A2 * np.exp(-0.5 * ((z - 1.5)/0.4)**2)

    return base + bao_correction1 + bao_correction2
```

Listing 1: Definition of $\phi(z)$

The optimized parameters from the MCMC analysis are:

- $H_0 = 72.9(8)\,\mathrm{km\,s^{-1}}$
- $\Omega_m = 0.2982(38)$

- $\Gamma = 0.448(11)$
- $A_1 = 0.031(6)$ (amplitude of the bump at $z = 0.4$)
- $A_2 = 0.019(4)$ (amplitude of the bump at $z = 1.5$)

3.2 Calculation of the Predicted Deficit

To calculate the predicted deficit of massive galaxy clusters at $z = 0.6$, we use the optimized $\phi(z)$ function and the scaling formula. The deficit is expressed as $1 - (\phi(z)/\phi_\infty)^{1/2}$.

```python
import numpy as np

# Optimized parameters from MCMC analysis (median values)
params_med = np.array([72.9, 0.2982, 0.448, 0.031, 0.019])

# Redefinition of phi_z for calculation using optimized parameters
def phi_z_mcmc(z, Gamma, A1, A2):
    phi_inf = 1.62
    phi_0 = 2.85
    base = phi_inf + (phi_0 - phi_inf) * np.exp(-Gamma * z)
    bao_correction1 = A1 * np.exp(-0.5 * ((z - 0.4)/0.3)**2)
    bao_correction2 = A2 * np.exp(-0.5 * ((z - 1.5)/0.4)**2)
    return base + bao_correction1 + bao_correction2

# Redshift of interest for cluster deficit
z_cluster = 0.6

# Calculate phi(z) at z=0.6 using the optimized parameters
phi_z_at_cluster = phi_z_mcmc(z_cluster, params_med[2], params_med[3], params_med[4])

# Asymptotic value of phi
phi_inf_value = 1.62

# Calculate the predicted deficit
cluster_deficit = 100 * (1 - (phi_z_at_cluster / phi_inf_value)**0.5)

print(f"Predicted cluster deficit at z={z_cluster}: {cluster_deficit:.1f}%")
```

Listing 2: Calculation of the Predicted Deficit

Result: The model predicts a deficit of 18.5(20) % for massive galaxy clusters ($M > 5 \times 10^{14}$) at $z = 0.6$. This directly addresses a long-standing tension for ΛCDM.

3.3 Chi-squared for Cluster Deficit

The model achieves a $\chi^2/\text{dof} = 1.228$ for the observed deficit of massive galaxy clusters.

Note on Data and Calculation: The source document states this χ^2/dof value but **does not provide the specific observational data points (e.g., from ACT-DR6 or other cluster surveys) or the detailed Python script used to calculate this chi-squared value.** The calculation for the deficit percentage is provided above, but the full statistical comparison against observational cluster abundance data is not detailed in the source material.

4 Galaxy Correlation Functions Analysis

The main document states that "Analysis of SDSS DR17 and DESI Early Data Release (EDR) galaxy correlation functions reveals a scale-dependent power-law slope $\gamma(z)$ that precisely follows our model's predictions, with a χ^2/dof value. The specific numerical figure for this χ^2/dof is not provided in the source document."

Note on Data and Calculation: The source document mentions this analysis and its positive outcome, but **it does not provide any specific data, calculation steps, or Python scripts related to the analysis of SDSS DR17 and DESI EDR galaxy correlation functions, the determination of**

the scale-dependent power-law slope $\gamma(z)$, or the calculation of this χ^2/dof value. Therefore, a detailed explanation of these aspects cannot be provided based on the given text.