Oar Measurement System Specification

Your Name

December 28, 2025

Contents

1 Theoretical Framework

This section presents the theoretical foundations for analyzing the measurement system, including mechanical bending, thermal effects, geometric imperfections, and stability considerations.

1.1 Beam Bending Theory

The measurement system relies on Euler-Bernoulli beam theory to relate applied forces to measurable strains.

1.1.1 Governing Equation

For a beam subject to transverse loading:

$$EI\frac{d^4w}{dx^4} = q(x) \tag{1}$$

where:

- E = Young's modulus
- I = second moment of area
- $w(x)$ = transverse deflection
- $q(x)$ = distributed load per unit length

1.1.2 Moment-Curvature Relation

$$M(x) = -EI\frac{d^2w}{dx^2} \tag{2}$$

1.1.3 Strain-Displacement Relation

For a beam in pure bending, the longitudinal strain at distance y from the neutral axis is:

$$\varepsilon(x, y) = -y\frac{d^2w}{dx^2} = \frac{M(x) \cdot y}{EI} \tag{3}$$

Note the sign convention: positive M causes compression on the top surface ($y > 0$).

1.1.4 Second Moment of Area

For a rectangular cross-section (beam) with height h_b (in bending direction, y) and width b (perpendicular to bending, z):

$$I_b = \frac{bh_b^3}{12} \tag{4}$$

For a hollow circular cross-section (shaft):

$$I_s = \frac{\pi}{64}\left(D_{o,s}^4 - D_{i,s}^4\right) \tag{5}$$

1.2 Thermal Effects

1.2.1 Differential Thermal Expansion

The aluminum beam and carbon shaft have significantly different thermal expansion coefficients:

$$\alpha_b = 23.6 \times 10^{-6}\,\mathrm{K}^{-1} \quad \text{(Aluminum 1050)} \tag{6}$$

$$\alpha_s = -0.5 \times 10^{-6}\,\mathrm{K}^{-1} \quad \text{(Carbon fiber, approximate)} \tag{7}$$

The differential thermal expansion coefficient is:

$$\Delta\alpha = \alpha_b - \alpha_s \tag{8}$$

The differential thermal strain over a temperature change ΔT is:

$$\varepsilon_{\text{thermal,differential}} = \Delta\alpha \cdot \Delta T \tag{9}$$

[TBD: Analysis of thermal stress induced by constrained differential expansion]

1.2.2 Solar Heating Assessment

[TBD: Calculation or citation to validate maximum temperature assumption of $80\,^\circ\mathrm{C}$**]**

1.3 Beam Misalignment Effects

System misalignment (ϕ_{mis}) creates a static angular offset that affects strain measurements through:

- Projection error in measured strain
- Coupling between bending moments and misaligned measurement axes

[TBD: Quantitative analysis of misalignment effects on bridge output]

1.4 Beam Twist Due to Clamps

Initial twist (ϕ_0) between clamps creates torsional pre-stress that:

- Induces initial strain in the beam
- Can be amplified by differential thermal expansion
- May couple with bending to create cross-sensitivity

[**TBD: Analysis of twist-induced strains and coupling effects**]

1.5 Beam Imperfections

Manufacturing imperfections in the beam (curvature, thickness variations, surface roughness) affect:

- Initial strain distribution
- Stress concentration locations
- Calibration accuracy

[**TBD: Sensitivity analysis for manufacturing tolerances**]

1.6 Buckling Analysis

For a beam under compressive loading, buckling stability must be evaluated. Two types of buckling are considered: mechanical buckling and thermal buckling.

1.6.1 Mechanical Buckling

The critical buckling load for a beam is given by the Euler buckling formula:

$$N_{cr} = \frac{\pi^2 E_b I_b}{(K L_b)^2} \tag{10}$$

where K is the effective length factor that depends on the boundary conditions:

For the measurement beam, the clamps are assumed to provide **pinned boundary conditions** at both ends (preventing translation but allowing rotation). This is more realistic than assuming perfect fixity, as the clamps may not completely prevent rotation. Therefore, $K = 1.0$, and the critical buckling load becomes:

Table 1: Effective length factor K for different boundary conditions

Boundary Condition	**K**	**Relative Strength**
Fixed–Fixed	0.50	Strongest
Fixed–Pinned	0.70	Very strong
Pinned–Pinned	1.00	Baseline
Fixed–Free (cantilever)	2.00	Weakest

$$N_{cr} = \frac{\pi^2 E_b I_b}{(1.0 L_b)^2} = \frac{\pi^2 E_b I_b}{L_b^2} \tag{11}$$

Substituting the beam second moment of area from Eq. (**??**):

$$N_{cr} = \frac{\pi^2 E_b b h_b^3}{12 L_b^2} \tag{12}$$

where:

- E_b = Young's modulus of beam (aluminum)
- b = beam width (z-direction)
- h_b = beam height (y-direction, bending direction)
- L_b = beam length between clamps
- $K = 1.0$ = effective length factor for pinned–pinned boundary conditions

Note: The pinned–pinned assumption ($K = 1.0$) is conservative. If the clamps provide partial rotational restraint, the actual K would be between 0.5 (fully fixed) and 1.0 (pinned), resulting in a higher critical buckling load than calculated here.

1.6.2 Thermal Buckling

Temperature changes induce compressive axial stress in the beam due to constrained differential thermal expansion. The thermal compressive load is:

$$N_{th} = E_b A_b \varepsilon_{th} \tag{13}$$

where $A_b = bh_b$ is the beam cross-sectional area, and ε_{th} is the constrained thermal strain:

$$\varepsilon_{th} = \Delta\alpha \cdot \Delta T = (\alpha_b - \alpha_s)\Delta T \tag{14}$$

Combining Eqs. (**??**) and (**??**):

$$N_{th} = E_b b h_b (\alpha_b - \alpha_s)\Delta T \tag{15}$$

The critical temperature change for thermal buckling occurs when $N_{th} = N_{cr}$. From Eqs. (**??**) and (**??**):

$$\Delta T_{cr} = \frac{\pi^2 E_b b h_b^3}{12 L_b^2 E_b b h_b (\alpha_b - \alpha_s)} = \frac{\pi^2 h_b^2}{12 L_b^2 (\alpha_b - \alpha_s)} \tag{16}$$

1.6.3 Total Compressive Load

The total axial compressive load N in the beam is the sum of:

$$N = N_{th} + N_{assy} \tag{17}$$

where:

- N_{th} = thermal compressive load from Eq. (**??**)
- N_{assy} = pre-stress from assembly (clamp tightening) [TBD]

Note: The bending of the beam does not contribute to axial compression (for small deflections in classical beam theory). Bending creates moments $M_b(x)$ and shear forces $V_b(x)$, which must be checked separately against yield criteria.

1.6.4 Stability Criteria

The beam remains stable if the total load is below the critical buckling load:

$$N < N_{cr} \tag{18}$$

For design safety, accounting for imperfections and uncertainties:

$$N < 0.5 N_{cr} \tag{19}$$

[TBD: Estimation of assembly pre-stress N_{assy} from clamp tightening]

[TBD: Verification that $N = N_{th} + N_{assy} < 0.5N_{cr}$ for operating temperature range]

[TBD: Verification of stability criteria for operating temperature range]

1.7 Thermal Amplification

Temperature changes amplify several effects:

- Initial twist (ϕ_0) increases due to differential expansion
- Beam pre-stress changes with temperature
- Compressive load approaches critical buckling load

[**TBD: Comprehensive thermal amplification analysis**]