

# Rowing Oar Strain Measurement System

## Geometry, Materials, and Specifications

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## Contents

|          |  |          |
|----------|--|----------|
| <b>1</b> | <b>Overview</b>                                      | <b>3</b> |
| <b>2</b> | <b>Coordinate System and Sign Conventions</b>        | <b>3</b> |
| 2.1      | Coordinate System . . . . .                          | 3        |
| 2.2      | Sign Conventions . . . . .                           | 3        |
| <b>3</b> | <b>Oar Geometry</b>                                  | <b>4</b> |
| 3.1      | Overall Dimensions . . . . .                         | 4        |
| 3.2      | Blade Geometry . . . . .                             | 4        |
| 3.3      | Shaft Geometry . . . . .                             | 4        |
| 3.4      | Sleeve Geometry . . . . .                            | 4        |
| 3.5      | Collar Geometry . . . . .                            | 4        |
| 3.6      | Oarlock Geometry . . . . .                           | 4        |
| 3.7      | Handle Geometry . . . . .                            | 4        |
| <b>4</b> | <b>Measurement Beam Geometry</b>                     | <b>4</b> |
| 4.1      | Beam Dimensions and Position . . . . .               | 5        |
| 4.2      | Beam Clamp Positions . . . . .                       | 5        |
| 4.3      | Clamp Rigidity and Assembly Tolerances . . . . .     | 5        |
| 4.3.1    | System Misalignment . . . . .                        | 5        |
| 4.3.2    | Initial Beam Twist . . . . .                         | 5        |
| <b>5</b> | <b>Material Properties</b>                           | <b>6</b> |
| 5.1      | Shaft Material: Carbon Fiber Composite . . . . .     | 6        |
| 5.2      | Beam Material: Aluminum 1050 . . . . .               | 6        |
| <b>6</b> | <b>Strain Gauge Specifications</b>                   | <b>6</b> |
| 6.1      | Gauge Type and Configuration . . . . .               | 6        |
| 6.2      | Bridge Configuration . . . . .                       | 6        |
| 6.3      | Expected Strain Range . . . . .                      | 7        |
| <b>7</b> | <b>Operating Conditions</b>                          | <b>8</b> |
| 7.1      | Temperature Range . . . . .                          | 8        |
| 7.2      | Mechanical Loading . . . . .                         | 8        |
| <b>8</b> | <b>Theoretical Framework</b>                         | <b>8</b> |
| 8.1      | Strain Decomposition and Measurement Model . . . . . | 8        |
| 8.2      | Beam Bending Theory . . . . .                        | 9        |
| 8.2.1    | Governing Equation . . . . .                         | 9        |
| 8.2.2    | Moment-Curvature Relation . . . . .                  | 9        |
| 8.2.3    | Strain-Displacement Relation . . . . .               | 9        |
| 8.2.4    | Second Moment of Area . . . . .                      | 9        |
| 8.2.5    | Boundary Conditions . . . . .                        | 10       |
| 8.3      | Thermal Effects . . . . .                            | 10       |
| 8.3.1    | Differential Thermal Expansion . . . . .             | 10       |
| 8.3.2    | Clamp-Induced Assembly Strain . . . . .              | 10       |

|           |   |           |
|-----------|---|-----------|
| 8.3.3     | Thermal Amplification . . . . .                                 | 10        |
| 8.4       | Stability and Buckling Considerations . . . . .                 | 11        |
| <b>9</b>  | <b>Calculation of Mechanical Strains</b>                        | <b>11</b> |
| 9.1       | Theory 1: Clamped Oarlock and Local Surface Strain . . . . .    | 12        |
| 9.1.1     | Shaft model and boundary conditions . . . . .                   | 12        |
| 9.1.2     | Bending moment, curvature and deflection of the shaft . . . . . | 12        |
| 9.1.3     | Mechanical strain at an arbitrary shaft fibre . . . . .         | 12        |
| 9.1.4     | Gauge locations and effective radius . . . . .                  | 13        |
| 9.1.5     | Mechanical strain at the gauge locations . . . . .              | 13        |
| 9.1.6     | Bridge response (mechanical contribution) . . . . .             | 13        |
| 9.1.7     | Numerical example . . . . .                                     | 14        |
| 9.2       | Theory 2: Pin Support Boundary Conditions . . . . .             | 14        |
| 9.2.1     | Boundary Conditions . . . . .                                   | 14        |
| 9.2.2     | Deflection Solution . . . . .                                   | 14        |
| 9.2.3     | Moment Distribution . . . . .                                   | 14        |
| 9.2.4     | Reaction Forces . . . . .                                       | 15        |
| 9.2.5     | Strain at Beam Location . . . . .                               | 15        |
| 9.2.6     | Numerical Example . . . . .                                     | 15        |
| <b>10</b> | <b>Analysis and Calibration Results</b>                         | <b>15</b> |
| 10.1      | Mechanical Calibration . . . . .                                | 15        |
| 10.2      | Thermal Characterization . . . . .                              | 15        |
| 10.3      | Joint Temperature and Load Influence . . . . .                  | 15        |
| 10.4      | Discussion . . . . .  | 16        |
| <b>11</b> | <b>Open Questions and Future Work</b>                           | <b>16</b> |
| <b>12</b> | <b>Nomenclature and Subscript Conventions</b>                   | <b>16</b> |
| 12.1      | Subscript Conventions . . . . .                                 | 16        |
| 12.2      | Nomenclature . . . . .  | 17        |
| <b>13</b> | <b>References</b>   | <b>17</b> |

# 1 Overview

This document provides a complete specification of the geometry, materials, and measurement system for strain measurement on a sculling oar. The primary objective is to evaluate whether the measurement system requires specific considerations for:

- Amplification of strain gauge measurements
- Effects of thermal expansion
- Misalignment of the device on the rowing oar shaft
- Manufacturing imprecision (e.g., beam curvature)

The measurement system consists of a small aluminum beam attached to the carbon fiber oar shaft, instrumented with strain gauges to measure bending during rowing.

## 2 Coordinate System and Sign Conventions

### 2.1 Coordinate System

The oar is represented in a top view and described in a right-handed Cartesian coordinate system ( $x, y, z$ ):

- **Origin ( $x = 0$ )**: Located at the oarlock position
- **$x$ -axis**: Along the oar longitudinal axis
  - Positive direction: toward the handle (inboard)
  - Negative direction: toward the blade (outboard)
- **$y$ -axis**: Perpendicular to the oar, in the plane of bending
  - This is the direction of boat travel
  - During the drive phase, the boat travels in the  $-y$  direction
  - The blade pushes the water in the  $+x$  direction
- **$z$ -axis**: Perpendicular to both  $x$  and  $y$  (vertical direction)
  - Positive direction: away from the water (upward)
  - Negative direction: toward the water (downward)

**Note:** In the top view representation, bending occurs in the  $x-y$  plane, with deflection  $w(x)$  in the  $y$ -direction. During rowing, the force at the handle is in the  $-y$  direction during the drive phase, causing the oar to bend horizontally.

### 2.2 Sign Conventions

- **Forces**: Positive in the positive  $y$  direction
- **Moments**: Positive according to right-hand rule about  $z$ -axis
- **Deflection**:  $w(x) > 0$  indicates outward deflection (in  $+y$  direction)
- **Rotation about  $z$ -axis**:  $\theta(x) > 0$  indicates rotation about  $z$ -axis according to right-hand rule (bending rotation)
- **Rotation about  $x$ -axis**:  $\phi(x) > 0$  indicates rotation about  $x$ -axis according to right-hand rule (torsional rotation/twist)
- **Strain**:  $\varepsilon > 0$  indicates tension,  $\varepsilon < 0$  indicates compression

**Note:** During the drive phase, the force at the handle is in the  $-y$  direction, causing the oar to bend in the horizontal ( $x - y$ ) plane. Vertical bending (along  $z$ ) is neglected in this model.

Table 1: Overall oar dimensions

| Symbol             | Description                                  | Value   |
|--------------------|--|---------|
| $L_{\text{out}}$   | Total outboard length (blade tip to oarlock) | 2000 mm |
| $L_{\text{in}}$    | Inboard length (oarlock to handle end)       | 900 mm  |
| $L_{\text{total}}$ | Total oar length                             | 2900 mm |

## 3 Oar Geometry

### 3.1 Overall Dimensions

The sculling oar consists of outboard (blade side) and inboard (handle side) sections, separated by the oarlock at  $x = 0$ .

### 3.2 Blade Geometry

Table 2: Blade dimensions

| Symbol             | Description                        | Value                        |
|--------------------|------------------------------------|------------------------------|
| $L_{\text{blade}}$ | Blade length                       | 430 mm                       |
| $w_{\text{blade}}$ | Blade maximum width                | 150 mm                       |
| $t_{\text{blade}}$ | Blade thickness                    | 5 mm                         |
| $h_{\text{bow}}$   | Blade bow height (spoon curvature) | 40 mm                        |
| $x_{\text{blade}}$ | Blade tip position                 | $-L_{\text{out}} = -2000$ mm |

### 3.3 Shaft Geometry

The shaft is a hollow circular tube with constant cross-section.

Table 3: Shaft dimensions

| Symbol                 | Description           | Value   |
|------------------------|-----------------------|---------|
| $D_{o,s}$              | Shaft outer diameter  | 38 mm   |
| $D_{i,s}$              | Shaft inner diameter  | 32 mm   |
| $t_s$                  | Shaft wall thickness  | 3 mm    |
| $L_{\text{shaft,out}}$ | Outboard shaft length | 1570 mm |
| $L_{\text{shaft,in}}$  | Inboard shaft length  | 700 mm  |

### 3.4 Sleeve Geometry

The sleeve is a cylindrical component made of ABS plastic that provides reinforcement around the oarlock region.

### 3.5 Collar Geometry

The collar prevents the oar from sliding through the oarlock.

### 3.6 Oarlock Geometry

### 3.7 Handle Geometry

The handle consists of a taper section transitioning from shaft diameter to grip diameter, followed by the grip section.

## 4 Measurement Beam Geometry

The measurement beam is a rectangular beam attached to the shaft by means of two rigid clamps at specified positions.

Table 4: Sleeve dimensions

| Symbol                    | Description                                     | Value   |
|---------------------------|---|---------|
| $L_{\text{sleeve}}$       | Total sleeve length                             | 300 mm  |
| $L_{\text{sleeve},-x}$    | Sleeve extension in $-x$ direction from oarlock | 200 mm  |
| $L_{\text{sleeve},+x}$    | Sleeve extension in $+x$ direction from oarlock | 100 mm  |
| $D_{\text{sleeve}}$       | Sleeve outer diameter                           | 60 mm   |
| $x_{\text{sleeve,start}}$ | Sleeve start position                           | -200 mm |
| $x_{\text{sleeve,end}}$   | Sleeve end position                             | 100 mm  |

Table 5: Collar dimensions

| Symbol              | Description              | Value  |
|---------------------|--------------------------|--------|
| $D_{\text{collar}}$ | Collar diameter          | 120 mm |
| $t_{\text{collar}}$ | Collar thickness (axial) | 20 mm  |
| $x_{\text{collar}}$ | Collar position          | 20 mm  |

## 4.1 Beam Dimensions and Position

**Beam neutral axis position:** The beam neutral axis is located at  $y_b$  measured from the global coordinate origin (shaft centerline at  $y = 0$ ). This position is calculated as:

$$y_b = \frac{D_{o,s}}{2} + e_b \quad (1)$$

where  $D_{o,s}/2$  is the shaft outer radius and  $e_b$  is the distance from the shaft outer surface to the beam neutral axis. The beam extends from the shaft surface outward (in the  $+y$  direction) to provide clearance for mounting strain gauges on both top and bottom surfaces.

## 4.2 Beam Clamp Positions

The beam is attached to the shaft at two locations:

$$x_{b,1} = x_b = 200 \text{ mm} \quad (\text{root clamp}) \quad (2)$$

$$x_{b,2} = x_b + L_b = 300 \text{ mm} \quad (\text{tip clamp}) \quad (3)$$

## 4.3 Clamp Rigidity and Assembly Tolerances

The clamps are assumed to be **rigid connections** that prevent relative motion between the beam and shaft at the attachment points. However, two manufacturing and assembly phenomena must be considered:

### 4.3.1 System Misalignment

The entire beam assembly may be rotated around the shaft ( $x$ -axis) during installation. This misalignment angle, denoted  $\phi_{\text{mis}}$ , is expected to be within  $\pm 1^\circ$ . This creates a **static angular offset** of the entire measurement system, which may result in:

- Apparent strain offset in gauge readings
- Coupling between bending and the misaligned measurement axes

[TBD: Quantification of offset effect on strain measurements]

### 4.3.2 Initial Beam Twist

The two clamps may not be perfectly aligned with each other during assembly, creating an initial twist in the beam. This twist angle, denoted  $\phi_0$ , represents the relative rotation between the root clamp (at  $x = x_b$ ) and the tip clamp (at  $x = x_b + L_b$ ) around the  $x$ -axis.

**Thermal amplification:** The differential thermal expansion between the aluminum beam and carbon shaft can amplify this initial twist. Over the operating temperature range ( $\Delta T = 85$  K), the mismatch in thermal expansion creates additional torsional strain that compounds with  $\phi_0$ .

**Simplification:** The analysis assumes that twist-induced strains in the strain gauges are **negligible** compared to bending strains. This assumption should be verified for the expected values of  $\phi_{\text{mis}}$  and  $\phi_0$ .

Table 6: Oarlock dimensions

| Symbol               | Description                    | Value  |
|----------------------|--------------------------------|--------|
| $t_{\text{oarlock}}$ | Oarlock thickness (axial)      | 20 mm  |
| $h_{\text{oarlock}}$ | Oarlock height (radial extent) | 160 mm |

Table 7: Handle dimensions

| Symbol                   | Description                                   | Value                            |
|--------------------------|---|----------------------------------|
| $L_{\text{handle}}$      | Total handle length                           | 200 mm                           |
| $L_{\text{taper}}$       | Taper length                                  | 50 mm                            |
| $D_{\text{taper,start}}$ | Taper start diameter                          | 38 mm                            |
| $D_{\text{taper,end}}$   | Taper end diameter                            | 30 mm                            |
| $L_{\text{grip}}$        | Grip length                                   | 150 mm                           |
| $D_{\text{grip}}$        | Grip diameter                                 | 35 mm                            |
| $x_F$                    | Handle end position (force application point) | $L_{\text{in}} = 900 \text{ mm}$ |

## 5 Material Properties

### 5.1 Shaft Material: Carbon Fiber Composite

The shaft is constructed from carbon fiber composite with fibers aligned predominantly in the  $x$  (longitudinal) direction. The properties below are approximations for unidirectional carbon fiber/epoxy composite.

**Note:** Carbon fiber composites exhibit highly anisotropic behavior. The longitudinal modulus (fiber direction) is much higher than the transverse modulus. The negative coefficient of thermal expansion in the fiber direction is characteristic of carbon fibers. These values are approximations and can vary significantly depending on fiber type, volume fraction, and layup.

### 5.2 Beam Material: Aluminum 1050

The measurement beam is constructed from Aluminum 1050, a commercially pure aluminum alloy.

## 6 Strain Gauge Specifications

### 6.1 Gauge Type and Configuration

The measurement system uses four linear strain gauges arranged in a full Wheatstone bridge configuration.

All four strain gauges are located at the beam midpoint along the  $x$ -axis:

$$x_{\text{gauge}} = x_b + \frac{L_b}{2} \quad (4)$$

### 6.2 Bridge Configuration

During the drive phase, the rower applies a horizontal force at the handle in the  $-y$  direction. This creates a bending moment that makes the measurement beam *convex toward  $+y$* . In this condition, the top surface is in tension and the bottom surface is in compression.

The four strain gauges form a full Wheatstone bridge configured for differential bending measurement:

- **$R_1$  &  $R_3$  (Top surface):** located at  $y = y_{\text{top}}$ ,  $x = x_{\text{gauge}}$ 
  - Experience **positive strain** (tension) during the drive phase.
- **$R_2$  &  $R_4$  (Bottom surface):** located at  $y = y_{\text{bottom}}$ ,  $x = x_{\text{gauge}}$ 
  - Experience **negative strain** (compression) during the drive phase.

To ensure sensitivity to bending while rejecting common-mode strain (e.g., uniform axial or thermal strain), each half-bridge contains one top and one bottom gauge:

- Left voltage divider:  $R_1$  (top) over  $R_2$  (bottom)
- Right voltage divider:  $R_3$  (bottom) over  $R_4$  (top)

Table 8: Beam dimensions

| Symbol | Description  | Value             |
|--------|--|-------------------|
| $L_b$  | Beam length (between clamp centers)                  | 100 mm            |
| $h_b$  | Beam height (bending direction, $y$ -direction)      | 2 mm              |
| $b$    | Beam width ( $z$ -direction)                         | 12 mm             |
| $x_b$  | Beam root position (first clamp)                     | 200 mm            |
| $e_b$  | Beam neutral axis eccentricity from shaft centerline | 20 mm             |
| $y_b$  | Beam neutral axis $y$ -coordinate                    | $D_{o,s}/2 + e_b$ |

Table 9: Carbon fiber composite properties (longitudinal direction)

| Property  | Symbol     | Value  |
|---|------------|--|
| Young's modulus (longitudinal)                  | $E_s$      | 140 GPa (approximate)                              |
| Poisson's ratio                                 | $\nu_s$    | 0.30 (approximate)                                 |
| Coefficient of thermal expansion (longitudinal) | $\alpha_s$ | $-0.5 \times 10^{-6} \text{ K}^{-1}$ (approximate) |
| Density   | $\rho_s$   | 1600 kg/m <sup>3</sup> (approximate)               |

The bridge output voltage is:

$$V_{\text{out}} = V_{\text{ex}} \left( \frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \right) \quad (5)$$

with strain-dependent resistances:

$$R_1 = R_g (1 + GF \cdot \varepsilon_{\text{top}}) \quad (6)$$

$$R_2 = R_g (1 + GF \cdot \varepsilon_{\text{bottom}}) \quad (7)$$

$$R_3 = R_g (1 + GF \cdot \varepsilon_{\text{bottom}}) \quad (8)$$

$$R_4 = R_g (1 + GF \cdot \varepsilon_{\text{top}}) \quad (9)$$

For small strains ( $GF \cdot \varepsilon \ll 1$ ), this simplifies to the familiar linearized form:

$$\frac{V_{\text{out}}}{V_{\text{ex}}} = \frac{GF}{2} (\varepsilon_{\text{top}} - \varepsilon_{\text{bottom}}) \quad (10)$$

where:

- $V_{\text{out}}$  = bridge output voltage
- $V_{\text{ex}}$  = bridge excitation voltage (here 3.3 V)
- $\varepsilon_{\text{top}}$  = average strain on top surface (gauges  $R_1$  &  $R_3$ )
- $\varepsilon_{\text{bottom}}$  = average strain on bottom surface (gauges  $R_2$  &  $R_4$ )

#### Drive-phase sign convention:

- $\varepsilon_{\text{top}} > 0$  (tension)
- $\varepsilon_{\text{bottom}} < 0$  (compression)
- $V_{\text{out}} > 0$  indicates bending consistent with the drive-phase loading.

Unless otherwise stated,  $\varepsilon_{\text{top}}$  and  $\varepsilon_{\text{bottom}}$  are understood to be the *total* axial strains at the corresponding gauge locations. In terms of the decomposition introduced in Eq. (11), they correspond to  $\varepsilon_{\text{total}}(T, F)$  evaluated at  $y = y_{\text{top}}$  and  $y = y_{\text{bottom}}$ , respectively. Positive bridge output therefore corresponds directly to positive drive-phase force.

### 6.3 Expected Strain Range

Based on preliminary beam theory estimates (see Section 8.2), the expected mechanical strain range at the gauge locations is on the order of:

- $\varepsilon_{\text{mech}} \in [-500 \mu\varepsilon, 500 \mu\varepsilon]$  for typical rowing forces during the drive phase.

Thermal and offset contributions  $\varepsilon_{\text{th}}(T)$  and  $\varepsilon_{\text{off}}(T)$  may add bias and apparent gain variations, as discussed in Section 8.3.3.

Table 10: Aluminum 1050 properties

| Property                         | Symbol         | Value                                |
|----------------------------------|----------------|--------------------------------------|
| Young's modulus                  | $E_b$          | 69 GPa                               |
| Poisson's ratio                  | $\nu_b$        | 0.33                                 |
| Coefficient of thermal expansion | $\alpha_b$     | $23.6 \times 10^{-6} \text{ K}^{-1}$ |
| Density                          | $\rho_b$       | $2710 \text{ kg/m}^3$                |
| Yield strength                   | $\sigma_{y,b}$ | 34 MPa (annealed)                    |

Table 11: Strain gauge specifications

| Parameter                             | Symbol | Value                      |
|---------------------------------------|--------|----------------------------|
| Nominal resistance                    | $R_g$  | $1000 \Omega \pm 3 \Omega$ |
| Gauge factor                          | $GF$   | 2.15                       |
| Maximum strain                        | —      | $2000 \mu\epsilon$         |
| Temperature coefficient of resistance | —      | 20 ppm/K                   |
| Backing material                      | —      | Polyimide                  |

## 7 Operating Conditions

### 7.1 Temperature Range

The measurement system is expected to operate over the following temperature range:

Table 12: Operating temperature range

| Condition                               | Temperature               |
|---|---------------------------|
| Minimum (cold water, early morning)     | $-5^\circ\text{C}$        |
| Maximum (solar heating of carbon shaft) | $80^\circ\text{C}$        |
| Temperature excursion                   | $\Delta T = 85 \text{ K}$ |

**Note:** The maximum temperature assumption of  $80^\circ\text{C}$  is based on solar radiation heating the black carbon fiber shaft. This value should be verified through:

- Thermal modeling of solar heating on carbon shaft
- Experimental measurements under various environmental conditions
- Measurement or estimation of solar absorption coefficient of carbon shaft surface [TBD]

### 7.2 Mechanical Loading

During the drive phase, the athlete applies a horizontal force at the handle in the  $-y$  direction, which generates a bending moment that makes the measurement beam convex toward  $+y$ . Vertical bending ( $z$ -direction) is not considered.

The expected force range is:

**Note:** Only horizontal bending in the  $x - y$  plane is considered; vertical ( $z$ -direction) forces are neglected in this specification.

## 8 Theoretical Framework

This section presents the theoretical foundations for analyzing the measurement system, including mechanical bending, thermal effects, geometric imperfections, and stability considerations.

### 8.1 Strain Decomposition and Measurement Model

At each strain gauge location, the total axial strain is decomposed into three contributions:

$$\varepsilon_{\text{total}}(T, F) = \varepsilon_{\text{mech}}(T, F) + \varepsilon_{\text{th}}(T) + \varepsilon_{\text{off}}(T), \quad (11)$$

where:

Table 13: Expected force range at handle

| Condition                          | Force           |
|------------------------------------|-----------------|
| Minimum                            | 0 N             |
| Maximum (peak during power stroke) | 200 kg = 1962 N |

- $\varepsilon_{\text{mech}}(T, F)$  is the *mechanical* bending strain due to the applied load  $F$ ; in the absence of bending ( $F = 0$ ) this term is taken to be zero,
- $\varepsilon_{\text{th}}(T)$  is the *thermal* strain caused by constrained differential expansion between the aluminum beam and the carbon shaft,
- $\varepsilon_{\text{off}}(T)$  is an *offset* strain term that groups additional effects such as assembly pre-stress, imperfect bonding, twist misalignment, and electronics zero-shift.

The strain inferred from the bridge output after calibration is denoted by  $\varepsilon_{\text{meas}}(T, F)$ . In general it does not coincide exactly with the purely mechanical component, because temperature influences both the mechanical response and the measurement system. Sections 8.2–8.3 derive  $\varepsilon_{\text{mech}}(T, F)$  and  $\varepsilon_{\text{th}}(T)$ , while Section 8.3.3 introduces a parametric model for  $\varepsilon_{\text{meas}}(T, F)$  and  $\varepsilon_{\text{off}}(T)$ .

## 8.2 Beam Bending Theory

The measurement system relies on Euler-Bernoulli beam theory to relate applied forces to measurable strains.

### 8.2.1 Governing Equation

For a beam subject to transverse loading:

$$EI \frac{d^4 w}{dx^4} = q(x) \quad (12)$$

where:

- $E$  is the Young's modulus,
- $I$  is the second moment of area,
- $w(x)$  is the transverse deflection,
- $q(x)$  is the distributed transverse load.

Under typical rowing operation, the dominant loading is due to the handle force applied at the inboard end of the oar, resulting in bending of both the carbon shaft and the aluminum measurement beam.

### 8.2.2 Moment-Curvature Relation

For small deflections and linear elastic behavior, the bending moment  $M(x)$  is related to curvature:

$$M(x) = -EI \frac{d^2 w}{dx^2} \quad (13)$$

### 8.2.3 Strain-Displacement Relation

For a beam in pure bending, the longitudinal *mechanical* strain at distance  $y$  from the neutral axis is:

$$\varepsilon_{\text{mech}}(x, y) = -y \frac{d^2 w}{dx^2} = \frac{M(x) \cdot y}{EI} \quad (14)$$

### 8.2.4 Second Moment of Area

For a rectangular cross-section (beam):

$$I_b = \frac{bh_b^3}{12} \quad (15)$$

where  $b$  is the beam width and  $h_b$  is the beam height.

For a hollow circular cross-section (shaft):

$$I_s = \frac{\pi}{64} (D_{o,s}^4 - D_{i,s}^4) \quad (16)$$

### 8.2.5 Boundary Conditions

The exact form of  $w(x)$  depends on the boundary conditions at the oarlock and at the handle. In this specification, two theoretical boundary conditions are considered:

- **Theory 1:** Clamped boundary condition at the oarlock, with the handle modeled as a point load.
- **Theory 2:** Pin support boundary conditions at the oarlock and handle, approximating more flexible support conditions.

The detailed solutions for  $w(x)$  for each theory are derived in Section 9.

## 8.3 Thermal Effects

### 8.3.1 Differential Thermal Expansion

The aluminum beam and carbon shaft have significantly different thermal expansion coefficients:

$$\alpha_b = 23.6 \times 10^{-6} \text{ K}^{-1} \quad (\text{Aluminum 1050}) \quad (17)$$

$$\alpha_s = -0.5 \times 10^{-6} \text{ K}^{-1} \quad (\text{Carbon fiber, approximate}) \quad (18)$$

When constrained together by the clamping system, differential thermal expansion generates internal forces and moments. The free thermal elongation of beam and shaft over length  $L_b$  would be:

$$\Delta L_b = \alpha_b L_b \Delta T \quad (19)$$

$$\Delta L_s = \alpha_s L_b \Delta T \quad (20)$$

The differential free expansion is:

$$\Delta L_{\text{thermal}} = (\alpha_b - \alpha_s) L_b \Delta T = \Delta \alpha L_b \Delta T \quad (21)$$

with:

$$\Delta \alpha = \alpha_b - \alpha_s \quad (22)$$

If the beam and shaft are perfectly constrained, the differential expansion is converted into internal axial force  $N_{\text{th}}$  and bending moments, resulting in a thermal strain component  $\varepsilon_{\text{th}}(T)$  at the gauge location. A simplified axial thermal strain (if fully constrained) would be:

$$\varepsilon_{\text{thermal}} = \Delta \alpha \Delta T \quad (23)$$

which, in the context of Eq. (11), represents the thermal component  $\varepsilon_{\text{th}}(T)$ .

### 8.3.2 Clamp-Induced Assembly Strain

Clamp tightening can introduce additional pre-stress and twist. These effects are lumped into the offset strain term  $\varepsilon_{\text{off}}(T)$  in Eq. (11). While some of these effects may be approximately temperature-independent, differential thermal expansion can amplify twist and pre-stress, causing  $\varepsilon_{\text{off}}(T)$  to vary with  $T$ .

### 8.3.3 Thermal Amplification

Temperature variations do not only induce direct thermal expansion; they also modify material properties, clamp stiffness, and the effective geometry of the assembly. These effects can be captured phenomenologically as an effective change in both the measurement gain and offset.

Building on the decomposition in Eq. (11), the measured strain at temperature  $T$  and load  $F$  is expressed as:

$$\varepsilon_{\text{meas}}(T, F) = A_T(T) \varepsilon_{\text{mech}}(T, F) + \varepsilon_{\text{th}}(T) + \varepsilon_{\text{off}}(T) \quad (24)$$

where:

- $\varepsilon_{\text{mech}}(T, F)$  is the mechanical bending strain component defined in Eq. (11),
- $A_T(T)$  is a dimensionless thermal amplification factor,
- $\varepsilon_{\text{th}}(T)$  is the thermal strain component due to differential expansion,
- $\varepsilon_{\text{off}}(T)$  is a temperature-dependent offset caused by assembly pre-stress, twist amplification, and residual electronics effects.

For small excursions around the reference temperature  $T_0$ , the amplification factor may be linearized as:

$$A_T(T) \approx 1 + k_T (T - T_0) \quad (25)$$

where  $k_T$  [K<sup>-1</sup>] is an effective thermal amplification coefficient to be obtained from calibration.

The linear approximation in Eq. (25) assumes that higher-order terms in  $(T - T_0)$  are negligible, which is appropriate for modest temperature excursions where:

$$|T - T_0| \ll \frac{1}{|k_T|} \quad (26)$$

In the context of this system, the validity of the linear model must be verified experimentally during thermal calibration. If necessary, additional higher-order terms in temperature may be introduced.

Substituting Eq. (25) into Eq. (24) gives:

$$\varepsilon_{\text{meas}}(T, F) \approx [1 + k_T (T - T_0)] \varepsilon_{\text{mech}}(T, F) + \varepsilon_{\text{th}}(T) + \varepsilon_{\text{off}}(T) \quad (27)$$

Equation (27) shows that thermal effects contribute an effective gain variation on the mechanical strain measurement in addition to an additive temperature-dependent bias. Both  $k_T$  and the behavior of  $\varepsilon_{\text{off}}(T)$  must be determined experimentally during calibration.

## 8.4 Stability and Buckling Considerations

Under compression, the aluminum beam may approach Euler buckling limits. The classical critical load for a prismatic column is:

$$N_{\text{cr}} = \frac{\pi^2 E_b I_b}{(KL_b)^2} \quad (28)$$

where:

- $N_{\text{cr}}$  is the critical buckling load,
- $E_b$  is the Young's modulus of the beam,
- $I_b$  is the second moment of area of the beam,
- $L_b$  is the effective length of the beam,
- $K$  is the effective length factor (depends on end conditions).

In this system, the beam is clamped at both ends via the clamping system. A conservative estimate is to take  $K \approx 0.7\text{--}1.0$  depending on the rotational stiffness of the clamps.

The combined thermal and assembly loads must remain well below  $N_{\text{cr}}$  across the full operating temperature range:

$$N = N_{\text{th}} + N_{\text{assy}} < \eta N_{\text{cr}} \quad (29)$$

with a safety factor  $\eta$  (e.g.,  $\eta = 0.5$ ).

[TBD: Verification that  $N = N_{\text{th}} + N_{\text{assy}} < 0.5N_{\text{cr}}$  for operating temperature range]  
[TBD: Verification of stability criteria for operating temperature range]

## 9 Calculation of Mechanical Strains

This section presents detailed calculations of the mechanical bending strains  $\varepsilon_{\text{mech}}(x, y)$  for the measurement beam under two different boundary condition assumptions. These calculations provide the theoretical basis for interpreting strain gauge measurements and understanding the relationship between applied forces and measured strains.

The mechanical strain component  $\varepsilon_{\text{mech}}(T, F)$  defined in Eq. (11) is derived from classical beam theory applied to the specific geometry and loading conditions of the rowing oar system. Two theoretical models are considered to bracket the range of expected behavior:

- **Theory 1 (Section 9.1):** Assumes a clamped boundary condition at the oarlock, representing a rigid constraint.
- **Theory 2 (Section 9.2):** Assumes pin support boundary conditions at both the oarlock and at the end of the outboard shaft, representing a more flexible support system.

Both theories use the same fundamental beam equations (Section 8.2) but differ in their boundary conditions, leading to different deflection profiles  $w(x)$  and consequently different strain distributions along the beam.

## 9.1 Theory 1: Clamped Oarlock and Local Surface Strain

Theory 1 models the oar shaft as a uniform Euler–Bernoulli beam of length  $x_F$  (clamped at the oarlock and free at the handle) with a transverse point load  $F$  applied at  $x = x_F$  during the drive phase. The aluminum measurement beam is treated purely as a kinematic attachment that places the strain gauges at a known distance from the shaft neutral axis. All strains derived in this section are therefore *purely mechanical bending strains* arising from the global bending of the shaft.

Thermal and assembly contributions are handled separately in Section 8.

### 9.1.1 Shaft model and boundary conditions

We consider the inboard shaft segment  $0 \leq x \leq x_F$  with the following boundary conditions at the oarlock and handle:

- Clamped at the oarlock ( $x = 0$ ):

$$w_s(0) = 0 \quad (\text{zero transverse displacement}), \quad (30)$$

$$\theta_s(0) = \left. \frac{dw_s}{dx} \right|_{x=0} = 0 \quad (\text{zero rotation}). \quad (31)$$

- Free at the handle ( $x = x_F$ ), with a transverse force of magnitude  $F$ :

$$V_s(x_F) = F \quad (\text{shear force}), \quad (32)$$

$$M_s(x_F) = 0 \quad (\text{zero bending moment}). \quad (33)$$

The shaft has bending stiffness  $E_s I_s$ , with  $E_s$  and  $I_s$  as defined in Table 16 and Eq. (16).

### 9.1.2 Bending moment, curvature and deflection of the shaft

For a prismatic cantilever of length  $x_F$  with a tip load  $F$ , the internal bending moment distribution is

$$M_s(x) = F(x_F - x), \quad 0 \leq x \leq x_F. \quad (34)$$

The maximum moment occurs at the clamped oarlock:

$$M_s(0) = Fx_F. \quad (35)$$

The curvature of the shaft in the  $x$ - $y$  bending plane follows directly from the Euler–Bernoulli relation

$$\kappa_s(x) = \frac{d^2 w_s}{dx^2} = \frac{M_s(x)}{E_s I_s} = \frac{F(x_F - x)}{E_s I_s}. \quad (36)$$

Integrating twice with the clamped boundary conditions at  $x = 0$  yields the standard cantilever expressions

$$\theta_s(x) = \frac{dw_s}{dx} = -\frac{F}{2E_s I_s} (2x_F x - x^2), \quad (37)$$

$$w_s(x) = -\frac{F}{6E_s I_s} x^2 (3x_F - x). \quad (38)$$

The maximum deflection at the handle is therefore

$$w_s(x_F) = -\frac{Fx_F^3}{3E_s I_s}. \quad (39)$$

### 9.1.3 Mechanical strain at an arbitrary shaft fibre

For small strains, the mechanical bending strain at a material fibre located a distance  $y$  from the shaft neutral axis (positive in the  $+y$  direction) is

$$\varepsilon_{\text{mech},s}(x, y) = \kappa_s(x) y = \frac{M_s(x) y}{E_s I_s} = \frac{F(x_F - x) y}{E_s I_s}. \quad (40)$$

This expression is the core result: once the gauge position  $(x, y)$  is known relative to the shaft centreline, the local mechanical strain due to an applied handle force  $F$  follows directly.

#### 9.1.4 Gauge locations and effective radius

The aluminum measurement beam is attached to the shaft by two clamps located at

$$x_{b,1} = x_b, \quad (41)$$

$$x_{b,2} = x_b + L_b, \quad (42)$$

as defined in Table 4.1. The four strain gauges are located at midspan of the beam,

$$x_{\text{gauge}} = x_b + \frac{L_b}{2}, \quad (43)$$

so the relevant shaft curvature is  $\kappa_s(x_{\text{gauge}})$ .

In the radial ( $y$ ) direction, the beam neutral axis lies at

$$y_b = \frac{D_{o,s}}{2} + e_b, \quad (44)$$

measured from the shaft centreline (see Eq. (1)). The top and bottom gauge fibres are located symmetrically about this neutral axis at

$$y_{\text{top}} = y_b + \frac{h_b}{2}, \quad (45)$$

$$y_{\text{bottom}} = y_b - \frac{h_b}{2}. \quad (46)$$

#### 9.1.5 Mechanical strain at the gauge locations

Evaluating Eq. (40) at the gauge station  $x = x_{\text{gauge}}$  gives

$$\kappa_s(x_{\text{gauge}}) = \frac{F(x_F - x_{\text{gauge}})}{E_s I_s}. \quad (47)$$

The purely mechanical bending strains at the top and bottom gauge fibres are then

$$\varepsilon_{\text{top}}(F) = \kappa_s(x_{\text{gauge}}) y_{\text{top}} = \frac{F(x_F - x_{\text{gauge}}) y_{\text{top}}}{E_s I_s}, \quad (48)$$

$$\varepsilon_{\text{bottom}}(F) = \kappa_s(x_{\text{gauge}}) y_{\text{bottom}} = \frac{F(x_F - x_{\text{gauge}}) y_{\text{bottom}}}{E_s I_s}. \quad (49)$$

The bridge differential strain component is the difference between these two:

$$\Delta\varepsilon(F) = \varepsilon_{\text{top}}(F) - \varepsilon_{\text{bottom}}(F) = \kappa_s(x_{\text{gauge}}) (y_{\text{top}} - y_{\text{bottom}}) = \kappa_s(x_{\text{gauge}}) h_b. \quad (50)$$

Notably, the *differential* mechanical strain driving the full bridge output is *independent* of the beam eccentricity  $y_b$ ; it depends only on the local curvature and beam thickness  $h_b$ .

In contrast, the *absolute* (common-mode) strains at the gauge locations scale with the eccentricity  $y_b$ :

$$\varepsilon_{\text{cm}}(F) \approx \kappa_s(x_{\text{gauge}}) y_b, \quad (51)$$

which must remain within the strain gauge specification.

#### 9.1.6 Bridge response (mechanical contribution)

Using the full-bridge configuration defined in Section 6.2, and for small strains, the normalized bridge output is

$$\frac{V_{\text{out}}}{V_{\text{ex}}} = \frac{GF}{2} (\varepsilon_{\text{top}} - \varepsilon_{\text{bottom}}) = \frac{GF}{2} \Delta\varepsilon(F), \quad (52)$$

with  $\Delta\varepsilon(F)$  given by Eq. (50). Combining Eqs. (36) and (50), the purely mechanical bridge strain is therefore

$$\Delta\varepsilon(F) = \frac{F(x_F - x_{\text{gauge}}) h_b}{E_s I_s}. \quad (53)$$

### 9.1.7 Numerical example

Using the geometric and material parameters in Tables 3.3–4.1:

$$x_F = 900 \text{ mm}, \quad x_b = 200 \text{ mm}, \quad L_b = 100 \text{ mm}, \quad (54)$$

$$h_b = 2 \text{ mm}, \quad D_{o,s} = 38 \text{ mm}, \quad e_b = 20 \text{ mm}, \quad (55)$$

$$E_s = 140 \text{ GPa}, \quad I_s = 50956 \text{ mm}^4, \quad (56)$$

and the peak handle force  $F = 1962 \text{ N}$ , we have

$$x_{\text{gauge}} = x_b + \frac{L_b}{2} = 250 \text{ mm}, \quad (57)$$

$$y_b = \frac{D_{o,s}}{2} + e_b = 39 \text{ mm}, \quad (58)$$

$$y_{\text{top}} = 40 \text{ mm}, \quad y_{\text{bottom}} = 38 \text{ mm}. \quad (59)$$

The shaft curvature at the gauge station is

$$\kappa_s(x_{\text{gauge}}) = \frac{F(x_F - x_{\text{gauge}})}{E_s I_s} \approx 0.179 \text{ m}^{-1}, \quad (60)$$

yielding the mechanical strains

$$\varepsilon_{\text{top}} \approx 7.2 \times 10^{-3} = 7160 \mu\varepsilon, \quad (61)$$

$$\varepsilon_{\text{bottom}} \approx 6.8 \times 10^{-3} = 6800 \mu\varepsilon, \quad (62)$$

$$\Delta\varepsilon = \varepsilon_{\text{top}} - \varepsilon_{\text{bottom}} \approx 3.6 \times 10^{-4} = 360 \mu\varepsilon. \quad (63)$$

In this formulation, the large absolute strains at the gauge locations are recognized as *true mechanical bending strains* at the eccentric radius  $y_b$ , while the full-bridge output is driven by the differential component  $\Delta\varepsilon(F)$  given by Eq. (53). Comparison of these theoretical values with the calibrated sensitivity in Section 10.1 provides a direct check on the effective gauge radius and clamp stiffness.

## 9.2 Theory 2: Pin Support Boundary Conditions

### 9.2.1 Boundary Conditions

Theory 2 assumes the following boundary conditions with pin supports at two locations:

- **At the oarlock ( $x = 0$ ):** Pin support prevents displacement but allows rotation:

$$w_s(0) = 0 \quad (\text{no transverse displacement}) \quad (64)$$

$$M_s(0) = 0 \quad (\text{no moment, pin allows rotation}) \quad (65)$$

- **At the end of outboard shaft ( $x = -(L_{\text{out}} - L_{\text{blade}}) = -1570 \text{ mm}$ ):** Pin support prevents displacement but allows rotation:

$$w_s(-(L_{\text{out}} - L_{\text{blade}})) = 0 \quad (\text{no transverse displacement}) \quad (66)$$

$$M_s(-(L_{\text{out}} - L_{\text{blade}})) = 0 \quad (\text{no moment, pin allows rotation}) \quad (67)$$

- **At the handle ( $x = x_F$ ):** A transverse force  $F$  is applied, with no moment:

$$V_s(x_F) = -F \quad (\text{applied shear force}) \quad (68)$$

$$M_s(x_F) = 0 \quad (\text{free end, no moment}) \quad (69)$$

See Figure ?? for a visual representation of the boundary conditions.

### 9.2.2 Deflection Solution

[TBD: Derivation of deflection  $w_s(x)$  for beam with two pin supports and point load]

### 9.2.3 Moment Distribution

[TBD: Calculation of bending moment  $M_s(x)$  along the shaft]

#### 9.2.4 Reaction Forces

[TBD: Calculation of reaction forces at the two pin support locations]

#### 9.2.5 Strain at Beam Location

[TBD: Calculation of mechanical strain  $\varepsilon_{\text{mech}}$  at the beam gauge locations]

#### 9.2.6 Numerical Example

[TBD: Numerical example with typical force values]

## 10 Analysis and Calibration Results

This section presents the calibration results obtained from mechanical loading and thermal characterization experiments. The goal is to determine the coefficients that relate the measured strain  $\varepsilon_{\text{meas}}(T, F)$  to the mechanical strain  $\varepsilon_{\text{mech}}(T, F)$  while accounting for thermal and offset contributions as defined in Eq. (11).

### 10.1 Mechanical Calibration

During mechanical calibration, the oar was held at approximately constant temperature  $T \approx T_0$ , such that:

$$\varepsilon_{\text{th}}(T_0) + \varepsilon_{\text{off}}(T_0) \approx 0 \quad (70)$$

Thus, Eq. (27) simplifies to:

$$\varepsilon_{\text{meas}}(T_0, F) \approx \varepsilon_{\text{mech}}(T_0, F) \quad (71)$$

The calibration curve  $\varepsilon_{\text{meas}}$  vs. applied force  $F$  exhibits a strong linear correlation. A first-order model was fitted:

$$\varepsilon_{\text{meas}}(T_0, F) = C_F \cdot F \quad (72)$$

with calibration factor  $C_F$  [ $\mu\varepsilon \text{ N}^{-1}$ ] representing the mechanical sensitivity at  $T_0$ .

This establishes the baseline mechanical response contribution to Eq. (27).

### 10.2 Thermal Characterization

With the oar unloaded ( $F = 0$ ), Eq. (27) becomes:

$$\varepsilon_{\text{meas}}(T, 0) = \varepsilon_{\text{th}}(T) + \varepsilon_{\text{off}}(T) \quad (73)$$

The measured output over temperature therefore reveals a combination of thermal expansion effects (predictable) and assembly/electronics offsets (less predictable). A linear model was fitted to  $\varepsilon_{\text{meas}}(T, 0)$  to estimate the dominant temperature-dependent trend:

$$\varepsilon_{\text{meas}}(T, 0) \approx k_0 + k_{\text{off}}(T - T_0) \quad (74)$$

where:

- $k_0$  is the residual strain at the reference temperature  $T_0$ ,
- $k_{\text{off}}$  captures the dominant linear temperature dependence of the offset.

These coefficients contribute to modeling  $\varepsilon_{\text{off}}(T)$  in Eq. (27).

### 10.3 Joint Temperature and Load Influence

When both temperature changes and mechanical loads occur simultaneously, the measured strain follows:

$$\varepsilon_{\text{meas}}(T, F) \approx [1 + k_T(T - T_0)] C_F F + \varepsilon_{\text{th}}(T) + \varepsilon_{\text{off}}(T) \quad (75)$$

The parameter  $k_T$  was extracted from controlled tests where load was applied at different temperatures. A linear regression on the gain variation yielded:

$$A_T(T) \approx 1 + k_T(T - T_0)$$

allowing the separation of:

- mechanical gain variation with temperature (via  $k_T$ )
- additive thermal and offset effects (via  $\varepsilon_{\text{th}}$  and  $\varepsilon_{\text{off}}$ )

## 10.4 Discussion

The results demonstrate that:

- The **mechanical calibration** is linear and stable near  $T_0$ .
- The **thermal characterization** reveals a temperature-dependent offset that must be compensated.
- A **joint thermo-mechanical model** is required to ensure accuracy across realistic operating temperatures.

To enable reliable performance during on-water operation, the coefficients  $C_F$ ,  $k_T$  and the characterization of  $\varepsilon_{\text{off}}(T)$  must be incorporated into a real-time compensation algorithm.

## 11 Open Questions and Future Work

The following items require additional information or analysis:

1. Validation of maximum temperature ( $80^\circ\text{C}$ ) through thermal modeling or measurement
2. Solar absorption coefficient of carbon shaft surface
3. Characterization of system misalignment tolerance ( $\phi_{\text{mis}}$ )
4. Measurement of initial beam twist ( $\phi_0$ ) after assembly
5. Analysis of thermal amplification of initial twist
6. Verification of assumption that twist-induced strains are negligible
7. Detailed boundary condition derivations for Theory 1 (clamped) and Theory 2 (pin supports)
8. Analytical solutions for deflection  $w(x)$  and strain  $\varepsilon(x)$  distributions
9. Thermal stress analysis from constrained differential expansion
10. Estimation of assembly pre-stress ( $N_{\text{assy}}$ ) from clamp tightening
11. Verification that  $N = N_{\text{th}} + N_{\text{assy}} < 0.5N_{\text{cr}}$  over operating temperature range
12. Sensitivity analysis: manufacturing tolerances, gauge positioning errors
13. Signal amplification requirements based on calculated strain levels
14. Experimental validation plan

## 12 Nomenclature and Subscript Conventions

### 12.1 Subscript Conventions

To maintain clarity and avoid ambiguity, the following subscript conventions are used throughout this document:

Table 14: Subscript conventions

| Subscript | Meaning   |
|-----------|---|
| $s$       | Shaft (carbon fiber oar shaft)  |
| $b$       | Beam (aluminum measurement beam)  |
| top       | Top surface of beam (tension side under drive-phase handle load)        |
| bottom    | Bottom surface of beam (compression side under drive-phase handle load) |
| th        | Thermal component (differential expansion)                              |
| mech      | Mechanical component (bending)  |
| off       | Offset component (assembly and electronics)                             |
| meas      | Measured quantity inferred from bridge/ADC                              |
| $x, y, z$ | Cartesian coordinates / components                                      |

Table 15: Nomenclature - Geometric parameters

| Symbol              | Description                                  | Units      |
|---------------------|--|------------|
| $x, y, z$           | Cartesian coordinates                        | mm         |
| $L_{\text{out}}$    | Total outboard length                        | mm         |
| $L_{\text{in}}$     | Total inboard length                         | mm         |
| $L_b$               | Beam length                                  | mm         |
| $h_b$               | Beam height                                  | mm         |
| $b$                 | Beam width                                   | mm         |
| $D_{o,s}$           | Shaft outer diameter                         | mm         |
| $D_{i,s}$           | Shaft inner diameter                         | mm         |
| $t_s$               | Shaft wall thickness                         | mm         |
| $e_b$               | Beam eccentricity from shaft centerline      | mm         |
| $y_b$               | Beam neutral axis position                   | mm         |
| $x_b$               | Beam root position                           | mm         |
| $x_F$               | Handle end position (force application)      | mm         |
| $\phi_{\text{mis}}$ | System misalignment angle around shaft       | deg or rad |
| $\phi_0$            | Initial beam twist (relative clamp rotation) | deg or rad |

Table 16: Nomenclature - Material properties

| Symbol                            | Description   | Units           |
|-----------------------------------|---|-----------------|
| $E_s$                             | Young's modulus of shaft (carbon)                   | GPa             |
| $E_b$                             | Young's modulus of beam (aluminum)                  | GPa             |
| $\nu_s$                           | Poisson's ratio of shaft                            | —               |
| $\nu_b$                           | Poisson's ratio of beam                             | —               |
| $\alpha_s$                        | Thermal expansion coefficient of shaft              | K <sup>-1</sup> |
| $\alpha_b$                        | Thermal expansion coefficient of beam               | K <sup>-1</sup> |
| $I_s$                             | Second moment of area of shaft                      | mm <sup>4</sup> |
| $I_b$                             | Second moment of area of beam                       | mm <sup>4</sup> |
| $\varepsilon_{\text{mech}}(x, y)$ | Mechanical bending strain at position $(x, y)$      | —               |
| $\varepsilon(x, y)$               | Strain at position $(x, y)$ (when context is clear) | —               |
| $\sigma(x, y)$                    | Stress at position $(x, y)$                         | MPa             |

## 12.2 Nomenclature

## 13 References

[TBD: Add references for material properties, beam theory, strain gauge technology]

Table 17: Nomenclature - Strain gauge parameters

| Symbol                        | Description  | Units         |
|-------------------------------|--|---------------|
| $R_g$                         | Strain gauge nominal resistance                                    | $\Omega$      |
| $GF$                          | Gauge factor   | —             |
| $R_1, R_3$                    | Top surface gauge resistances                                      | $\Omega$      |
| $R_2, R_4$                    | Bottom surface gauge resistances                                   | $\Omega$      |
| $\varepsilon_{\text{top}}$    | Strain on top surface (total strain at gauges $R_1$ and $R_3$ )    | —             |
| $\varepsilon_{\text{bottom}}$ | Strain on bottom surface (total strain at gauges $R_2$ and $R_4$ ) | —             |
| $V_{\text{ex}}$               | Bridge excitation voltage  | V             |
| $V_{\text{out}}$              | Bridge output voltage  | V             |
| $\Delta R$                    | Change in gauge resistance   | $\Omega$      |
| $x_{\text{gauge}}$            | Strain gauge $x$ -position   | mm            |
| $y_{\text{top}}$              | Top surface $y$ -position  | mm            |
| $y_{\text{bottom}}$           | Bottom surface $y$ -position                                       | mm            |
| $n_{\text{bits}}$             | ADC resolution   | bits          |
| LSB                           | Least significant bit voltage                                      | $\mu\text{V}$ |

Table 18: Nomenclature - Thermal parameters

| Symbol                         | Description   | Units              |
|--------------------------------|---|--------------------|
| $T$                            | Temperature   | $^{\circ}\text{C}$ |
| $T_0$                          | Reference temperature                                   | $^{\circ}\text{C}$ |
| $\Delta T$                     | Temperature change                                      | K                  |
| $\Delta\alpha$                 | Difference in thermal expansion coefficients            | $\text{K}^{-1}$    |
| $\varepsilon_{\text{thermal}}$ | Simplified thermal strain due to $\Delta\alpha\Delta T$ | —                  |
| $\Delta L_{\text{thermal}}$    | Differential thermal expansion                          | mm                 |

Table 19: Nomenclature - Strain components and measurement model

| Symbol                       | Description  | Units           |
|------------------------------|--|-----------------|
| $\varepsilon_{\text{total}}$ | Total axial strain at a gauge location                       | —               |
| $\varepsilon_{\text{mech}}$  | Mechanical bending strain component                          | —               |
| $\varepsilon_{\text{th}}$    | Thermal strain component (differential expansion)            | —               |
| $\varepsilon_{\text{off}}$   | Offset strain (assembly and electronics effects)             | —               |
| $\varepsilon_{\text{meas}}$  | Strain inferred from bridge output and calibration           | —               |
| $A_T(T)$                     | Temperature-dependent mechanical strain amplification factor | —               |
| $k_T$                        | Linear thermal amplification coefficient                     | $\text{K}^{-1}$ |