Rowing Oar Strain Measurement System
Geometry, Materials, and Specifications

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Contents

1 Overview

This document provides a complete specification of the geometry, materials, and measurement system for strain measurement on a sculling oar. The primary objective is to evaluate whether the measurement system requires specific considerations for:

- Amplification of strain gauge measurements
- Effects of thermal expansion
- Misalignment of the device on the rowing oar shaft
- Manufacturing imprecision (e.g., beam curvature)

The measurement system consists of a small aluminum beam attached to the carbon fiber oar shaft, instrumented with strain gauges to measure bending during rowing.

2 Coordinate System and Sign Conventions

2.1 Coordinate System

The oar is represented in a top view and described in a right-handed Cartesian coordinate system (x, y, z):

- **Origin ($x = 0$):** Located at the oarlock position
- **x-axis:** Along the oar longitudinal axis
  - Positive direction: toward the handle (inboard)
  - Negative direction: toward the blade (outboard)
- **y-axis:** Perpendicular to the oar, in the plane of bending
  - This is the direction of boat travel
  - During the drive phase, the boat travels in the $-y$ direction
  - The blade pushes the water in the $+x$ direction
- **z-axis:** Perpendicular to both x and y (vertical direction)
  - Positive direction: away from the water (upward)
  - Negative direction: toward the water (downward)

Note: In the top view representation, bending occurs in the x-y plane, with deflection $w(x)$ in the y-direction. During rowing, the force at the handle is in the $-y$ direction during the drive phase, causing the oar to bend horizontally.

2.2 Sign Conventions

- **Forces:** Positive in the positive y direction
- **Moments:** Positive according to right-hand rule about z-axis
- **Deflection:** $w(x) > 0$ indicates outward deflection (in $+y$ direction)
- **Rotation about z-axis:** $\theta(x) > 0$ indicates rotation about z-axis according to right-hand rule (bending rotation)
- **Rotation about x-axis:** $\phi(x) > 0$ indicates rotation about x-axis according to right-hand rule (torsional rotation/twist)
- **Strain:** $\varepsilon > 0$ indicates tension, $\varepsilon < 0$ indicates compression

Note: During the drive phase, the force at the handle is in the $-y$ direction, causing the oar to bend in the horizontal $(x - y)$ plane. Vertical bending (along z) is neglected in this model.

Table 1: Overall oar dimensions

Symbol	Description	Value
L_{out}	Total outboard length (blade tip to oarlock)	2000 mm
L_{in}	Inboard length (oarlock to handle end)	900 mm
L_{total}	Total oar length	2900 mm

3 Oar Geometry

3.1 Overall Dimensions

The sculling oar consists of outboard (blade side) and inboard (handle side) sections, separated by the oarlock at $x = 0$.

3.2 Blade Geometry

Table 2: Blade dimensions

Symbol	Description	Value
L_{blade}	Blade length	430 mm
w_{blade}	Blade maximum width	150 mm
t_{blade}	Blade thickness	5 mm
h_{bow}	Blade bow height (spoon curvature)	40 mm
x_{blade}	Blade tip position	$-L_{\text{out}} = -2000$ mm

3.3 Shaft Geometry

The shaft is a hollow circular tube with constant cross-section.

Table 3: Shaft dimensions

Symbol	Description	Value
$D_{o,s}$	Shaft outer diameter	38 mm
$D_{i,s}$	Shaft inner diameter	32 mm
t_s	Shaft wall thickness	3 mm
$L_{\text{shaft,out}}$	Outboard shaft length	1570 mm
$L_{\text{shaft,in}}$	Inboard shaft length	700 mm

3.4 Sleeve Geometry

The sleeve is a cylindrical component made of ABS plastic that provides reinforcement around the oarlock region.

3.5 Collar Geometry

The collar prevents the oar from sliding through the oarlock.

3.6 Oarlock Geometry

3.7 Handle Geometry

The handle consists of a taper section transitioning from shaft diameter to grip diameter, followed by the grip section.

4 Measurement Beam Geometry

The measurement beam is a rectangular beam attached to the shaft by means of two rigid clamps at specified positions.

Table 4: Sleeve dimensions

Symbol	Description	Value
L_{sleeve}	Total sleeve length	300 mm
$L_{\text{sleeve,-x}}$	Sleeve extension in $-x$ direction from oarlock	200 mm
$L_{\text{sleeve,+x}}$	Sleeve extension in $+x$ direction from oarlock	100 mm
D_{sleeve}	Sleeve outer diameter	60 mm
$x_{\text{sleeve,start}}$	Sleeve start position	−200 mm
$x_{\text{sleeve,end}}$	Sleeve end position	100 mm

Table 5: Collar dimensions

Symbol	Description	Value
D_{collar}	Collar diameter	120 mm
t_{collar}	Collar thickness (axial)	20 mm
x_{collar}	Collar position	20 mm

4.1 Beam Dimensions and Position

Beam neutral axis position: The beam neutral axis is located at y_b measured from the global coordinate origin (shaft centerline at $y = 0$). This position is calculated as:

$$y_b = \frac{D_{o,s}}{2} + e_b \tag{1}$$

where $D_{o,s}/2$ is the shaft outer radius and e_b is the distance from the shaft outer surface to the beam neutral axis. The beam extends from the shaft surface outward (in the $+y$ direction) to provide clearance for mounting strain gauges on both top and bottom surfaces.

4.2 Beam Clamp Positions

The beam is attached to the shaft at two locations:

$$x_{b,1} = x_b = 200\,\text{mm} \quad \text{(root clamp)} \tag{2}$$

$$x_{b,2} = x_b + L_b = 300\,\text{mm} \quad \text{(tip clamp)} \tag{3}$$

4.3 Clamp Rigidity and Assembly Tolerances

The clamps are assumed to be **rigid connections** that prevent relative motion between the beam and shaft at the attachment points. However, two manufacturing and assembly phenomena must be considered:

4.3.1 System Misalignment

The entire beam assembly may be rotated around the shaft (x-axis) during installation. This misalignment angle, denoted ϕ_{mis}, is expected to be within ±1°. This creates a **static angular offset** of the entire measurement system, which may result in:

- Apparent strain offset in gauge readings
- Coupling between bending and the misaligned measurement axes

[TBD: Quantification of offset effect on strain measurements]

4.3.2 Initial Beam Twist

The two clamps may not be perfectly aligned with each other during assembly, creating an initial twist in the beam. This twist angle, denoted ϕ_0, represents the relative rotation between the root clamp (at $x = x_b$) and the tip clamp (at $x = x_b + L_b$) around the x-axis.

Thermal amplification: The differential thermal expansion between the aluminum beam and carbon shaft can amplify this initial twist. Over the operating temperature range ($\Delta T = 85$ K), the mismatch in thermal expansion creates additional torsional strain that compounds with ϕ_0.

Simplification: The analysis assumes that twist-induced strains in the strain gauges are **negligible** compared to bending strains. This assumption should be verified for the expected values of ϕ_{mis} and ϕ_0.

Table 6: Oarlock dimensions

Symbol	Description	Value
t_{oarlock}	Oarlock thickness (axial)	20 mm
h_{oarlock}	Oarlock height (radial extent)	160 mm

Table 7: Handle dimensions

Symbol	Description	Value
L_{handle}	Total handle length	200 mm
L_{taper}	Taper length	50 mm
$D_{\text{taper,start}}$	Taper start diameter	38 mm
$D_{\text{taper,end}}$	Taper end diameter	30 mm
L_{grip}	Grip length	150 mm
D_{grip}	Grip diameter	35 mm
x_F	Handle end position (force application point)	$L_{\text{in}} = 900$ mm

5 Material Properties

5.1 Shaft Material: Carbon Fiber Composite

The shaft is constructed from carbon fiber composite with fibers aligned predominantly in the x (longitudinal) direction. The properties below are approximations for unidirectional carbon fiber/epoxy composite.

Note: Carbon fiber composites exhibit highly anisotropic behavior. The longitudinal modulus (fiber direction) is much higher than the transverse modulus. The negative coefficient of thermal expansion in the fiber direction is characteristic of carbon fibers. These values are approximations and can vary significantly depending on fiber type, volume fraction, and layup.

5.2 Beam Material: Aluminum 1050

The measurement beam is constructed from Aluminum 1050, a commercially pure aluminum alloy.

6 Strain Gauge Specifications

6.1 Gauge Type and Configuration

The measurement system uses four linear strain gauges arranged in a full Wheatstone bridge configuration.

All four strain gauges are located at the beam midpoint along the x-axis:

$$x_{\text{gauge}} = x_b + \frac{L_b}{2} \tag{4}$$

6.2 Bridge Configuration

During the drive phase, the rower applies a horizontal force at the handle in the $-y$ direction. This creates a bending moment that makes the measurement beam *convex toward* $+y$. In this condition, the top surface is in tension and the bottom surface is in compression.

The four strain gauges form a full Wheatstone bridge configured for differential bending measurement:

- R_1 & R_3 **(Top surface):** located at $y = y_{\text{top}}$, $x = x_{\text{gauge}}$
  - Experience **positive strain** (tension) during the drive phase.
- R_2 & R_4 **(Bottom surface):** located at $y = y_{\text{bottom}}$, $x = x_{\text{gauge}}$
  - Experience **negative strain** (compression) during the drive phase.

To ensure sensitivity to bending while rejecting common-mode strain (e.g., uniform axial or thermal strain), each half-bridge contains one top and one bottom gauge:

- Left voltage divider: R_1 (top) over R_2 (bottom)
- Right voltage divider: R_3 (bottom) over R_4 (top)

Table 8: Beam dimensions

Symbol	Description	Value
L_b	Beam length (between clamp centers)	100 mm
h_b	Beam height (bending direction, y-direction)	2 mm
b	Beam width (z-direction)	12 mm
x_b	Beam root position (first clamp)	200 mm
e_b	Beam neutral axis eccentricity from shaft centerline	20 mm
y_b	Beam neutral axis y-coordinate	$D_{o,s}/2 + e_b$

Table 9: Carbon fiber composite properties (longitudinal direction)

Property	Symbol	Value
Young's modulus (longitudinal)	E_s	140 GPa (approximate)
Poisson's ratio	ν_s	0.30 (approximate)
Coefficient of thermal expansion (longitudinal)	α_s	$-0.5 \times 10^{-6}\,\mathrm{K}^{-1}$ (approximate)
Density	ρ_s	$1600\,\mathrm{kg/m}^3$ (approximate)

The bridge output voltage is:

$$V_{\text{out}} = V_{\text{ex}} \left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \right) \tag{5}$$

with strain-dependent resistances:

$$R_1 = R_g\big(1 + GF \cdot \varepsilon_{\text{top}}\big) \tag{6}$$

$$R_2 = R_g\big(1 + GF \cdot \varepsilon_{\text{bottom}}\big) \tag{7}$$

$$R_3 = R_g\big(1 + GF \cdot \varepsilon_{\text{bottom}}\big) \tag{8}$$

$$R_4 = R_g\big(1 + GF \cdot \varepsilon_{\text{top}}\big) \tag{9}$$

For small strains ($GF \cdot \varepsilon \ll 1$), this simplifies to the familiar linearized form:

$$\frac{V_{\text{out}}}{V_{\text{ex}}} = \frac{GF}{2} \left(\varepsilon_{\text{top}} - \varepsilon_{\text{bottom}}\right) \tag{10}$$

where:

- V_{out} = bridge output voltage
- V_{ex} = bridge excitation voltage (here 3.3 V)
- ε_{top} = average strain on top surface (gauges R_1 & R_3)
- $\varepsilon_{\text{bottom}}$ = average strain on bottom surface (gauges R_2 & R_4)

Drive-phase sign convention:

- $\varepsilon_{\text{top}} > 0$ (tension)
- $\varepsilon_{\text{bottom}} < 0$ (compression)
- $V_{\text{out}} > 0$ indicates bending consistent with the drive-phase loading.

Unless otherwise stated, ε_{top} and $\varepsilon_{\text{bottom}}$ are understood to be the *total* axial strains at the corresponding gauge locations. In terms of the decomposition introduced in Eq. (11), they correspond to $\varepsilon_{\text{total}}(T, F)$ evaluated at $y = y_{\text{top}}$ and $y = y_{\text{bottom}}$, respectively. Positive bridge output therefore corresponds directly to positive drive-phase force.

6.3 Expected Strain Range

Based on preliminary beam theory estimates (see Section 8.2), the expected mechanical strain range at the gauge locations is on the order of:

- $\varepsilon_{\text{mech}} \in [-500\,\mu\varepsilon, 500\,\mu\varepsilon]$ for typical rowing forces during the drive phase.

Thermal and offset contributions $\varepsilon_{\text{th}}(T)$ and $\varepsilon_{\text{off}}(T)$ may add bias and apparent gain variations, as discussed in Section 8.3.3.

Table 10: Aluminum 1050 properties

Property	Symbol	Value
Young's modulus	E_b	$69\,\mathrm{GPa}$
Poisson's ratio	ν_b	0.33
Coefficient of thermal expansion	α_b	$23.6 \times 10^{-6}\,\mathrm{K}^{-1}$
Density	ρ_b	$2710\,\mathrm{kg/m^3}$
Yield strength	$\sigma_{y,b}$	$34\,\mathrm{MPa}$ (annealed)

Table 11: Strain gauge specifications

Parameter	Symbol	Value
Nominal resistance	R_g	$1000\,\Omega \pm 3\,\Omega$
Gauge factor	GF	2.15
Maximum strain	–	$2000\,\mu\varepsilon$
Temperature coefficient of resistance	–	$20\,\mathrm{ppm/K}$
Backing material	–	Polyimide

7 Operating Conditions

7.1 Temperature Range

The measurement system is expected to operate over the following temperature range:

Table 12: Operating temperature range

Condition	Temperature
Minimum (cold water, early morning)	$-5\,^\circ\mathrm{C}$
Maximum (solar heating of carbon shaft)	$80\,^\circ\mathrm{C}$
Temperature excursion	$\Delta T = 85\,\mathrm{K}$

Note: The maximum temperature assumption of $80\,^\circ\mathrm{C}$ is based on solar radiation heating the black carbon fiber shaft. This value should be verified through:

- Thermal modeling of solar heating on carbon shaft
- Experimental measurements under various environmental conditions
- Measurement or estimation of solar absorption coefficient of carbon shaft surface [TBD]

7.2 Mechanical Loading

During the drive phase, the athlete applies a horizontal force at the handle in the $-y$ direction, which generates a bending moment that makes the measurement beam convex toward $+y$. Vertical bending (z-direction) is not considered.

The expected force range is:

Note: Only horizontal bending in the $x - y$ plane is considered; vertical (z-direction) forces are neglected in this specification.

8 Theoretical Framework

This section presents the theoretical foundations for analyzing the measurement system, including mechanical bending, thermal effects, geometric imperfections, and stability considerations.

8.1 Strain Decomposition and Measurement Model

At each strain gauge location, the total axial strain is decomposed into three contributions:

$$\varepsilon_{\text{total}}(T, F) = \varepsilon_{\text{mech}}(T, F) + \varepsilon_{\text{th}}(T) + \varepsilon_{\text{off}}(T), \tag{11}$$

where:

Table 13: Expected force range at handle

Condition	**Force**
Minimum	0 N
Maximum (peak during power stroke)	200 kg = 1962 N

- $\varepsilon_{\mathrm{mech}}(T, F)$ is the *mechanical* bending strain due to the applied load F; in the absence of bending ($F = 0$) this term is taken to be zero,
- $\varepsilon_{\mathrm{th}}(T)$ is the *thermal* strain caused by constrained differential expansion between the aluminum beam and the carbon shaft,
- $\varepsilon_{\mathrm{off}}(T)$ is an *offset* strain term that groups additional effects such as assembly pre-stress, imperfect bonding, twist misalignment, and electronics zero-shift.

The strain inferred from the bridge output after calibration is denoted by $\varepsilon_{\mathrm{meas}}(T, F)$. In general it does not coincide exactly with the purely mechanical component, because temperature influences both the mechanical response and the measurement system. Sections 8.2–8.3 derive $\varepsilon_{\mathrm{mech}}(T, F)$ and $\varepsilon_{\mathrm{th}}(T)$, while Section 8.3.3 introduces a parametric model for $\varepsilon_{\mathrm{meas}}(T, F)$ and $\varepsilon_{\mathrm{off}}(T)$.

8.2 Beam Bending Theory

The measurement system relies on Euler-Bernoulli beam theory to relate applied forces to measurable strains.

8.2.1 Governing Equation

For a beam subject to transverse loading:

$$EI\frac{d^4w}{dx^4} = q(x) \tag{12}$$

where:

- E is the Young's modulus,
- I is the second moment of area,
- $w(x)$ is the transverse deflection,
- $q(x)$ is the distributed transverse load.

Under typical rowing operation, the dominant loading is due to the handle force applied at the inboard end of the oar, resulting in bending of both the carbon shaft and the aluminum measurement beam.

8.2.2 Moment-Curvature Relation

For small deflections and linear elastic behavior, the bending moment $M(x)$ is related to curvature:

$$M(x) = -EI\frac{d^2w}{dx^2} \tag{13}$$

8.2.3 Strain-Displacement Relation

For a beam in pure bending, the longitudinal *mechanical* strain at distance y from the neutral axis is:

$$\varepsilon_{\mathrm{mech}}(x, y) = -y\frac{d^2w}{dx^2} = \frac{M(x) \cdot y}{EI} \tag{14}$$

8.2.4 Second Moment of Area

For a rectangular cross-section (beam):

$$I_b = \frac{bh_b^3}{12} \tag{15}$$

where b is the beam width and h_b is the beam height.

For a hollow circular cross-section (shaft):

$$I_s = \frac{\pi}{64}\left(D_{o,s}^4 - D_{i,s}^4\right) \tag{16}$$

8.2.5 Boundary Conditions

The exact form of $w(x)$ depends on the boundary conditions at the oarlock and at the handle. In this specification, two theoretical boundary conditions are considered:

- **Theory 1:** Clamped boundary condition at the oarlock, with the handle modeled as a point load.
- **Theory 2:** Pin support boundary conditions at the oarlock and handle, approximating more flexible support conditions.

The detailed solutions for $w(x)$ for each theory are derived in Section 9.

8.3 Thermal Effects

8.3.1 Differential Thermal Expansion

The aluminum beam and carbon shaft have significantly different thermal expansion coefficients:

$$\alpha_b = 23.6 \times 10^{-6}\,\mathrm{K}^{-1} \quad \text{(Aluminum 1050)} \tag{17}$$

$$\alpha_s = -0.5 \times 10^{-6}\,\mathrm{K}^{-1} \quad \text{(Carbon fiber, approximate)} \tag{18}$$

When constrained together by the clamping system, differential thermal expansion generates internal forces and moments. The free thermal elongation of beam and shaft over length L_b would be:

$$\Delta L_b = \alpha_b L_b \Delta T \tag{19}$$

$$\Delta L_s = \alpha_s L_b \Delta T \tag{20}$$

The differential free expansion is:

$$\Delta L_{\mathrm{thermal}} = (\alpha_b - \alpha_s) L_b \Delta T = \Delta\alpha \, L_b \Delta T \tag{21}$$

with:

$$\Delta\alpha = \alpha_b - \alpha_s \tag{22}$$

If the beam and shaft are perfectly constrained, the differential expansion is converted into internal axial force N_{th} and bending moments, resulting in a thermal strain component $\varepsilon_{\mathrm{th}}(T)$ at the gauge location. A simplified axial thermal strain (if fully constrained) would be:

$$\varepsilon_{\mathrm{thermal}} = \Delta\alpha \, \Delta T \tag{23}$$

which, in the context of Eq. (11), represents the thermal component $\varepsilon_{\mathrm{th}}(T)$.

8.3.2 Clamp-Induced Assembly Strain

Clamp tightening can introduce additional pre-stress and twist. These effects are lumped into the offset strain term $\varepsilon_{\mathrm{off}}(T)$ in Eq. (11). While some of these effects may be approximately temperature-independent, differential thermal expansion can amplify twist and pre-stress, causing $\varepsilon_{\mathrm{off}}(T)$ to vary with T.

8.3.3 Thermal Amplification

Temperature variations do not only induce direct thermal expansion; they also modify material properties, clamp stiffness, and the effective geometry of the assembly. These effects can be captured phenomenologically as an effective change in both the measurement gain and offset.

Building on the decomposition in Eq. (11), the measured strain at temperature T and load F is expressed as:

$$\varepsilon_{\mathrm{meas}}(T, F) = A_T(T)\, \varepsilon_{\mathrm{mech}}(T, F) + \varepsilon_{\mathrm{th}}(T) + \varepsilon_{\mathrm{off}}(T) \tag{24}$$

where:

- $\varepsilon_{\mathrm{mech}}(T, F)$ is the mechanical bending strain component defined in Eq. (11),
- $A_T(T)$ is a dimensionless thermal amplification factor,
- $\varepsilon_{\mathrm{th}}(T)$ is the thermal strain component due to differential expansion,
- $\varepsilon_{\mathrm{off}}(T)$ is a temperature-dependent offset caused by assembly pre-stress, twist amplification, and residual electronics effects.

For small excursions around the reference temperature T_0, the amplification factor may be linearized as:

$$A_T(T) \approx 1 + k_T\,(T - T_0) \tag{25}$$

where k_T [K^{-1}] is an effective thermal amplification coefficient to be obtained from calibration.

The linear approximation in Eq. (25) assumes that higher-order terms in $(T - T_0)$ are negligible, which is appropriate for modest temperature excursions where:

$$|T - T_0| \ll \frac{1}{|k_T|} \tag{26}$$

In the context of this system, the validity of the linear model must be verified experimentally during thermal calibration. If necessary, additional higher-order terms in temperature may be introduced.

Substituting Eq. (25) into Eq. (24) gives:

$$\varepsilon_{\mathrm{meas}}(T, F) \approx [1 + k_T\,(T - T_0)]\,\varepsilon_{\mathrm{mech}}(T, F) + \varepsilon_{\mathrm{th}}(T) + \varepsilon_{\mathrm{off}}(T) \tag{27}$$

Equation (27) shows that thermal effects contribute an effective gain variation on the mechanical strain measurement in addition to an additive temperature-dependent bias. Both k_T and the behavior of $\varepsilon_{\mathrm{off}}(T)$ must be determined experimentally during calibration.

8.4 Stability and Buckling Considerations

Under compression, the aluminum beam may approach Euler buckling limits. The classical critical load for a prismatic column is:

$$N_{\mathrm{cr}} = \frac{\pi^2 E_b I_b}{(K L_b)^2} \tag{28}$$

where:

- N_{cr} is the critical buckling load,
- E_b is the Young's modulus of the beam,
- I_b is the second moment of area of the beam,
- L_b is the effective length of the beam,
- K is the effective length factor (depends on end conditions).

In this system, the beam is clamped at both ends via the clamping system. A conservative estimate is to take $K \approx 0.7$–1.0 depending on the rotational stiffness of the clamps.

The combined thermal and assembly loads must remain well below N_{cr} across the full operating temperature range:

$$N = N_{\mathrm{th}} + N_{\mathrm{assy}} < \eta\, N_{\mathrm{cr}} \tag{29}$$

with a safety factor η (e.g., $\eta = 0.5$).

[TBD: Verification that $N = N_{\mathbf{th}} + N_{\mathbf{assy}} < 0.5 N_{\mathbf{cr}}$ for operating temperature range]

[TBD: Verification of stability criteria for operating temperature range]

9 Calculation of Mechanical Strains

This section presents detailed calculations of the mechanical bending strains $\varepsilon_{\mathrm{mech}}(x, y)$ for the measurement beam under two different boundary condition assumptions. These calculations provide the theoretical basis for interpreting strain gauge measurements and understanding the relationship between applied forces and measured strains.

The mechanical strain component $\varepsilon_{\mathrm{mech}}(T, F)$ defined in Eq. (11) is derived from classical beam theory applied to the specific geometry and loading conditions of the rowing oar system. Two theoretical models are considered to bracket the range of expected behavior:

- **Theory 1 (Section 9.1):** Assumes a clamped boundary condition at the oarlock, representing a rigid constraint.
- **Theory 2 (Section 9.2):** Assumes pin support boundary conditions at both the oarlock and at the end of the outboard shaft, representing a more flexible support system.

Both theories use the same fundamental beam equations (Section 8.2) but differ in their boundary conditions, leading to different deflection profiles $w(x)$ and consequently different strain distributions along the beam.

9.1 Theory 1: Clamped Oarlock and Local Surface Strain

Theory 1 models the oar shaft as a uniform Euler–Bernoulli beam of length x_F (clamped at the oarlock and free at the handle) with a transverse point load F applied at $x = x_F$ during the drive phase. The aluminum measurement beam is treated purely as a kinematic attachment that places the strain gauges at a known distance from the shaft neutral axis. All strains derived in this section are therefore *purely mechanical bending strains* arising from the global bending of the shaft.

Thermal and assembly contributions are handled separately in Section 8.

9.1.1 Shaft model and boundary conditions

We consider the inboard shaft segment $0 \le x \le x_F$ with the following boundary conditions at the oarlock and handle:

- Clamped at the oarlock ($x = 0$):

$$w_s(0) = 0 \qquad \text{(zero transverse displacement)}, \tag{30}$$

$$\theta_s(0) = \left.\frac{\mathrm{d}w_s}{\mathrm{d}x}\right|_{x=0} = 0 \qquad \text{(zero rotation)}. \tag{31}$$

- Free at the handle ($x = x_F$), with a transverse force of magnitude F:

$$V_s(x_F) = F \qquad \text{(shear force)}, \tag{32}$$

$$M_s(x_F) = 0 \qquad \text{(zero bending moment)}. \tag{33}$$

The shaft has bending stiffness E_sI_s, with E_s and I_s as defined in Table 16 and Eq. (16).

9.1.2 Bending moment, curvature and deflection of the shaft

For a prismatic cantilever of length x_F with a tip load F, the internal bending moment distribution is

$$M_s(x) = F\,(x_F - x), \qquad 0 \le x \le x_F. \tag{34}$$

The maximum moment occurs at the clamped oarlock:

$$M_s(0) = F x_F. \tag{35}$$

The curvature of the shaft in the x–y bending plane follows directly from the Euler–Bernoulli relation

$$\kappa_s(x) = \frac{\mathrm{d}^2 w_s}{\mathrm{d}x^2} = \frac{M_s(x)}{E_s I_s} = \frac{F\,(x_F - x)}{E_s I_s}. \tag{36}$$

Integrating twice with the clamped boundary conditions at $x = 0$ yields the standard cantilever expressions

$$\theta_s(x) = \frac{\mathrm{d}w_s}{\mathrm{d}x} = -\frac{F}{2E_sI_s}\left(2x_F x - x^2\right), \tag{37}$$

$$w_s(x) = -\frac{F}{6E_sI_s}\,x^2\left(3x_F - x\right). \tag{38}$$

The maximum deflection at the handle is therefore

$$w_s(x_F) = -\frac{F x_F^3}{3E_sI_s}. \tag{39}$$

9.1.3 Mechanical strain at an arbitrary shaft fibre

For small strains, the mechanical bending strain at a material fibre located a distance y from the shaft neutral axis (positive in the $+y$ direction) is

$$\varepsilon_{\mathrm{mech},s}(x, y) = \kappa_s(x)\,y = \frac{M_s(x)\,y}{E_sI_s} = \frac{F\,(x_F - x)\,y}{E_sI_s}. \tag{40}$$

This expression is the core result: once the gauge position (x, y) is known relative to the shaft centreline, the local mechanical strain due to an applied handle force F follows directly.

9.1.4 Gauge locations and effective radius

The aluminum measurement beam is attached to the shaft by two clamps located at

$$x_{b,1} = x_b, \tag{41}$$

$$x_{b,2} = x_b + L_b, \tag{42}$$

as defined in Table 4.1. The four strain gauges are located at midspan of the beam,

$$x_{\text{gauge}} = x_b + \frac{L_b}{2}, \tag{43}$$

so the relevant shaft curvature is $\kappa_s(x_{\text{gauge}})$.

In the radial (y) direction, the beam neutral axis lies at

$$y_b = \frac{D_{o,s}}{2} + e_b, \tag{44}$$

measured from the shaft centreline (see Eq. (1)). The top and bottom gauge fibres are located symmetrically about this neutral axis at

$$y_{\text{top}} = y_b + \frac{h_b}{2}, \tag{45}$$

$$y_{\text{bottom}} = y_b - \frac{h_b}{2}. \tag{46}$$

9.1.5 Mechanical strain at the gauge locations

Evaluating Eq. (40) at the gauge station $x = x_{\text{gauge}}$ gives

$$\kappa_s(x_{\text{gauge}}) = \frac{F\,(x_F - x_{\text{gauge}})}{E_s I_s}. \tag{47}$$

The purely mechanical bending strains at the top and bottom gauge fibres are then

$$\varepsilon_{\text{top}}(F) = \kappa_s(x_{\text{gauge}})\, y_{\text{top}} = \frac{F\,(x_F - x_{\text{gauge}})\, y_{\text{top}}}{E_s I_s}, \tag{48}$$

$$\varepsilon_{\text{bottom}}(F) = \kappa_s(x_{\text{gauge}})\, y_{\text{bottom}} = \frac{F\,(x_F - x_{\text{gauge}})\, y_{\text{bottom}}}{E_s I_s}. \tag{49}$$

The bridge differential strain component is the difference between these two:

$$\Delta\varepsilon(F) = \varepsilon_{\text{top}}(F) - \varepsilon_{\text{bottom}}(F) = \kappa_s(x_{\text{gauge}})\,(y_{\text{top}} - y_{\text{bottom}}) = \kappa_s(x_{\text{gauge}})\, h_b. \tag{50}$$

Notably, the *differential* mechanical strain driving the full bridge output is *independent* of the beam eccentricity y_b; it depends only on the local curvature and beam thickness h_b.

In contrast, the *absolute* (common-mode) strains at the gauge locations scale with the eccentricity y_b:

$$\varepsilon_{\text{cm}}(F) \approx \kappa_s(x_{\text{gauge}})\, y_b, \tag{51}$$

which must remain within the strain gauge specification.

9.1.6 Bridge response (mechanical contribution)

Using the full-bridge configuration defined in Section 6.2, and for small strains, the normalized bridge output is

$$\frac{V_{\text{out}}}{V_{\text{ex}}} = \frac{GF}{2}\left(\varepsilon_{\text{top}} - \varepsilon_{\text{bottom}}\right) = \frac{GF}{2}\,\Delta\varepsilon(F), \tag{52}$$

with $\Delta\varepsilon(F)$ given by Eq. (50). Combining Eqs. (36) and (50), the purely mechanical bridge strain is therefore

$$\Delta\varepsilon(F) = \frac{F\,(x_F - x_{\text{gauge}})\, h_b}{E_s I_s}. \tag{53}$$

9.1.7 Numerical example

Using the geometric and material parameters in Tables 3.3–4.1:

$$x_F = 900 \text{ mm}, \qquad x_b = 200 \text{ mm}, \qquad L_b = 100 \text{ mm}, \tag{54}$$
$$h_b = 2 \text{ mm}, \qquad D_{o,s} = 38 \text{ mm}, \qquad e_b = 20 \text{ mm}, \tag{55}$$
$$E_s = 140 \text{ GPa}, \qquad I_s = 50\,956 \text{ mm}^4, \tag{56}$$

and the peak handle force $F = 1962$ N, we have

$$x_{\text{gauge}} = x_b + \frac{L_b}{2} = 250 \text{ mm}, \tag{57}$$
$$y_b = \frac{D_{o,s}}{2} + e_b = 39 \text{ mm}, \tag{58}$$
$$y_{\text{top}} = 40 \text{ mm}, \qquad y_{\text{bottom}} = 38 \text{ mm}. \tag{59}$$

The shaft curvature at the gauge station is

$$\kappa_s(x_{\text{gauge}}) = \frac{F\,(x_F - x_{\text{gauge}})}{E_s I_s} \approx 0.179 \text{ m}^{-1}, \tag{60}$$

yielding the mechanical strains

$$\varepsilon_{\text{top}} \approx 7.2 \times 10^{-3} = 7\,160\ \mu\varepsilon, \tag{61}$$
$$\varepsilon_{\text{bottom}} \approx 6.8 \times 10^{-3} = 6\,800\ \mu\varepsilon, \tag{62}$$
$$\Delta\varepsilon = \varepsilon_{\text{top}} - \varepsilon_{\text{bottom}} \approx 3.6 \times 10^{-4} = 360\ \mu\varepsilon. \tag{63}$$

In this formulation, the large absolute strains at the gauge locations are recognized as *true mechanical bending strains* at the eccentric radius y_b, while the full-bridge output is driven by the differential component $\Delta\varepsilon(F)$ given by Eq. (53). Comparison of these theoretical values with the calibrated sensitivity in Section 10.1 provides a direct check on the effective gauge radius and clamp stiffness.

9.2 Theory 2: Pin Support Boundary Conditions

Theory 2 explores an alternative boundary condition model in which the oar is supported by two pin supports rather than a single clamp. This represents a more compliant constraint system that may better approximate the actual rowing conditions where the oarlock and blade-water interface provide vertical support but permit some rotational freedom.

9.2.1 Boundary conditions and structural model

The inboard and outboard shaft are modeled as a continuous Euler–Bernoulli beam with pin supports at two locations:

- **Pin support at oarlock** ($x = 0$):

$$w_s(0) = 0 \qquad \text{(zero transverse displacement)}, \tag{64}$$
$$M_s(0) = 0 \qquad \text{(zero moment; pin permits rotation)}. \tag{65}$$

- **Pin support at blade-shaft junction** ($x = x_{\text{pin,L}} = -(L_{\text{out}} - L_{\text{blade}}) = -1570\,\text{mm}$):

$$w_s(x_{\text{pin,L}}) = 0 \qquad \text{(zero transverse displacement)}, \tag{66}$$
$$M_s(x_{\text{pin,L}}) = 0 \qquad \text{(zero moment; pin permits rotation)}. \tag{67}$$

- **Free end at handle** ($x = x_F = 900\,\text{mm}$):

$$V_s(x_F) = F \qquad \text{(applied shear force)}, \tag{68}$$
$$M_s(x_F) = 0 \qquad \text{(free end; no moment)}. \tag{69}$$

The span between the two pin supports is $L_{\text{span}} = 0 - x_{\text{pin,L}} = 1570\,\text{mm}$, and the handle extends as an overhang of length $x_F = 900\,\text{mm}$ beyond the right support.

9.2.2 Reaction forces

The vertical reaction forces at the two pins are determined from global equilibrium. Taking vertical force equilibrium:

$$R_L + R_R = F, \tag{70}$$

and moment equilibrium about the right pin at $x = 0$:

$$R_L \cdot L_{\text{span}} - F \cdot x_F = 0. \tag{71}$$

Solving for the reactions:

$$R_L = \frac{F \cdot x_F}{L_{\text{span}}} = \frac{F \times 900}{1570} = 0.573\,F, \tag{72}$$

$$R_R = F - R_L = 0.427\,F. \tag{73}$$

Substituting numerical values with $F = 1962\,\text{N}$:

$$R_L \approx 1124\,\text{N}, \tag{74}$$

$$R_R \approx 838\,\text{N}. \tag{75}$$

Note on reaction directions: Both reactions are positive (upward, in the $+y$ direction) as expected for a beam supporting a downward load in the overhang region. This contrasts with cantilever configurations where support moments would be required.

9.2.3 Bending moment distribution

The bending moment varies across different regions of the beam. We focus on the overhang region $0 \le x \le x_F$ where the measurement beam is located.

Overhang region ($0 \le x \le x_F$): Cutting the beam at position x in the overhang and considering the right portion (from x to x_F), the only external force is the applied load F at $x = x_F$. The internal bending moment at the cut is found from moment equilibrium:

$$M_s(x) = F(x_F - x), \qquad 0 \le x \le x_F. \tag{76}$$

This expression satisfies the boundary conditions:

$$M_s(x_F) = 0 \quad \checkmark \quad \text{(free end)}, \tag{77}$$

$$M_s(0) = F \cdot x_F = 900F \quad \text{(to be reconciled with pin condition)}. \tag{78}$$

Apparent inconsistency at $x = 0$: Equation (76) gives $M_s(0) = 900F \neq 0$, which appears to violate the pin support condition $M_s(0) = 0$. This reflects the physical reality that the overhang creates a substantial bending moment at the support location. In practice, the oarlock provides a combination of vertical support and partial rotational constraint, making it neither a perfect pin nor a perfect clamp. The pin support model represents an idealization that captures the vertical support while the calculated moment $M_s(0)$ indicates the degree of rotational constraint actually required. For the purposes of strain calculation at the measurement location ($x = x_{\text{gauge}} = 250\,\text{mm}$), the moment distribution in the overhang region given by Eq. (76) provides a reasonable working model.

Supported span ($x_{\mathbf{pin,L}} \le x < 0$): The bending moment distribution in this region depends on both pin reactions. For completeness:

$$M_s(x) = R_L(x - x_{\text{pin,L}}), \qquad x_{\text{pin,L}} \le x < 0, \tag{79}$$

which satisfies $M_s(x_{\text{pin,L}}) = 0$ and provides continuity considerations at the right support.

9.2.4 Mechanical strain at gauge locations

Following the same approach as Theory 1, the curvature at the gauge station $x = x_{\text{gauge}} = x_b + L_b/2 = 250\,\text{mm}$ is:

$$\kappa_s(x_{\text{gauge}}) = \frac{M_s(x_{\text{gauge}})}{E_s I_s} = \frac{F(x_F - x_{\text{gauge}})}{E_s I_s}. \tag{80}$$

The mechanical bending strains at the top and bottom gauge fibers are:

$$\varepsilon_{\text{top}}(F) = \kappa_s(x_{\text{gauge}})\, y_{\text{top}} = \frac{F(x_F - x_{\text{gauge}})\, y_{\text{top}}}{E_s I_s}, \tag{81}$$

$$\varepsilon_{\text{bottom}}(F) = \kappa_s(x_{\text{gauge}})\, y_{\text{bottom}} = \frac{F(x_F - x_{\text{gauge}})\, y_{\text{bottom}}}{E_s I_s}. \tag{82}$$

The differential strain driving the bridge output is:

$$\Delta\varepsilon(F) = \varepsilon_{\text{top}}(F) - \varepsilon_{\text{bottom}}(F) = \kappa_s(x_{\text{gauge}})\, h_b = \frac{F(x_F - x_{\text{gauge}})\, h_b}{E_s I_s}. \tag{83}$$

As in Theory 1, the differential mechanical strain is independent of the beam eccentricity y_b and depends only on the local curvature and beam thickness h_b.

9.2.5 Numerical example

Using the same geometric and material parameters as Theory 1:

$$x_F - x_{\text{gauge}} = 900 - 250 = 650 \text{ mm}, \tag{84}$$

$$h_b = 2 \text{ mm}, \tag{85}$$

$$E_s = 140 \text{ GPa}, \tag{86}$$

$$I_s = 50\,956 \text{ mm}^4, \tag{87}$$

and the peak handle force $F = 1962\,\text{N}$, the shaft curvature at the gauge station is:

$$\kappa_s(x_{\text{gauge}}) = \frac{1962 \times 650}{140 \times 10^3 \times 50\,956} \approx 0.179 \text{ m}^{-1}. \tag{88}$$

Note that this curvature is identical to Theory 1, as both theories place the gauge at $x = 250\,\text{mm}$ where the moment arm to the load is the same ($x_F - x_{\text{gauge}} = 650\,\text{mm}$). The mechanical strains are therefore:

$$\varepsilon_{\text{top}} \approx 7.2 \times 10^{-3} = 7\,160\ \mu\varepsilon, \tag{89}$$

$$\varepsilon_{\text{bottom}} \approx 6.8 \times 10^{-3} = 6\,800\ \mu\varepsilon, \tag{90}$$

$$\Delta\varepsilon = \varepsilon_{\text{top}} - \varepsilon_{\text{bottom}} \approx 3.6 \times 10^{-4} = 360\ \mu\varepsilon. \tag{91}$$

9.2.6 Comparison with Theory 1

Both theories predict *identical* mechanical strains at the gauge location for the same applied handle force. This equivalence arises because:

1. The moment arm from the load to the gauge position is the same in both models: $x_F - x_{\text{gauge}} = 650\,\text{mm}$.
2. The local bending moment $M_s(x_{\text{gauge}}) = F(x_F - x_{\text{gauge}})$ depends only on this moment arm and the applied force, regardless of the support conditions elsewhere on the beam.
3. The beam properties (E_s, I_s) and gauge geometry (y_{top}, y_{bottom}, h_b) are identical.

The key difference between the theories lies in the *global deflection pattern* and the *reaction forces at the oarlock*:

- **Theory 1 (clamped):** Single reaction at oarlock, zero rotation, larger deflections in outboard region.
- **Theory 2 (pinned):** Two reaction forces, rotational freedom at supports, reduced deflections due to additional support at blade-shaft junction.

However, these global differences do not affect the local strain measurement at $x = x_{\text{gauge}}$, confirming that the measurement system is sensitive primarily to the *local bending moment* rather than the specific boundary condition model.

10 Analysis and Calibration Results

This section presents the calibration results obtained from mechanical loading and thermal characterization experiments. The goal is to determine the coefficients that relate the measured strain $\varepsilon_{\text{meas}}(T, F)$ to the mechanical strain $\varepsilon_{\text{mech}}(T, F)$ while accounting for thermal and offset contributions as defined in Eq. (11).

10.1 Mechanical Calibration

During mechanical calibration, the oar was held at approximately constant temperature $T \approx T_0$, such that:

$$\varepsilon_{\text{th}}(T_0) + \varepsilon_{\text{off}}(T_0) \approx 0 \tag{92}$$

Thus, Eq. (27) simplifies to:

$$\varepsilon_{\text{meas}}(T_0, F) \approx \varepsilon_{\text{mech}}(T_0, F) \tag{93}$$

The calibration curve $\varepsilon_{\text{meas}}$ vs. applied force F exhibits a strong linear correlation. A first-order model was fitted:

$$\varepsilon_{\text{meas}}(T_0, F) = C_F \cdot F \tag{94}$$

with calibration factor C_F $[\mu\varepsilon\,\mathrm{N}^{-1}]$ representing the mechanical sensitivity at T_0.

This establishes the baseline mechanical response contribution to Eq. (27).

10.2 Thermal Characterization

With the oar unloaded ($F = 0$), Eq. (27) becomes:

$$\varepsilon_{\text{meas}}(T, 0) = \varepsilon_{\text{th}}(T) + \varepsilon_{\text{off}}(T) \tag{95}$$

The measured output over temperature therefore reveals a combination of thermal expansion effects (predictable) and assembly/electronics offsets (less predictable). A linear model was fitted to $\varepsilon_{\text{meas}}(T, 0)$ to estimate the dominant temperature-dependent trend:

$$\varepsilon_{\text{meas}}(T, 0) \approx k_0 + k_{\text{off}}(T - T_0) \tag{96}$$

where:

- k_0 is the residual strain at the reference temperature T_0,
- k_{off} captures the dominant linear temperature dependence of the offset.

These coefficients contribute to modeling $\varepsilon_{\text{off}}(T)$ in Eq. (27).

10.3 Joint Temperature and Load Influence

When both temperature changes and mechanical loads occur simultaneously, the measured strain follows:

$$\varepsilon_{\text{meas}}(T, F) \approx [1 + k_T\,(T - T_0)]\,C_F\,F + \varepsilon_{\text{th}}(T) + \varepsilon_{\text{off}}(T) \tag{97}$$

The parameter k_T was extracted from controlled tests where load was applied at different temperatures. A linear regression on the gain variation yielded:

$$A_T(T) \approx 1 + k_T(T - T_0)$$

allowing the separation of:

- mechanical gain variation with temperature (via k_T)
- additive thermal and offset effects (via ε_{th} and ε_{off})

10.4 Discussion

The results demonstrate that:

- The **mechanical calibration** is linear and stable near T_0.
- The **thermal characterization** reveals a temperature-dependent offset that must be compensated.
- A **joint thermo-mechanical model** is required to ensure accuracy across realistic operating temperatures.

To enable reliable performance during on-water operation, the coefficients C_F, k_T and the characterization of $\varepsilon_{\text{off}}(T)$ must be incorporated into a real-time compensation algorithm.

11 Open Questions and Future Work

The following items require additional information or analysis:

1. Validation of maximum temperature (80 °C) through thermal modeling or measurement
2. Solar absorption coefficient of carbon shaft surface
3. Characterization of system misalignment tolerance (ϕ_{mis})
4. Measurement of initial beam twist (ϕ_0) after assembly
5. Analysis of thermal amplification of initial twist
6. Verification of assumption that twist-induced strains are negligible
7. Thermal stress analysis from constrained differential expansion
8. Estimation of assembly pre-stress (N_{assy}) from clamp tightening
9. Verification that $N = N_{th} + N_{assy} < 0.5N_{cr}$ over operating temperature range
10. Sensitivity analysis: manufacturing tolerances, gauge positioning errors
11. Signal amplification requirements based on calculated strain levels
12. Experimental validation plan

12 Nomenclature and Subscript Conventions

12.1 Subscript Conventions

To maintain clarity and avoid ambiguity, the following subscript conventions are used throughout this document:

Table 14: Subscript conventions

Subscript	Meaning
s	Shaft (carbon fiber oar shaft)
b	Beam (aluminum measurement beam)
top	Top surface of beam (tension side under drive-phase handle load)
bottom	Bottom surface of beam (compression side under drive-phase handle load)
th	Thermal component (differential expansion)
mech	Mechanical component (bending)
off	Offset component (assembly and electronics)
meas	Measured quantity inferred from bridge/ADC
x, y, z	Cartesian coordinates / components

12.2 Nomenclature

13 References

[TBD: Add references for material properties, beam theory, strain gauge technology]

Table 15: Nomenclature - Geometric parameters

Symbol	Description	Units
x, y, z	Cartesian coordinates	mm
L_{out}	Total outboard length	mm
L_{in}	Total inboard length	mm
L_b	Beam length	mm
h_b	Beam height	mm
b	Beam width	mm
$D_{o,s}$	Shaft outer diameter	mm
$D_{i,s}$	Shaft inner diameter	mm
t_s	Shaft wall thickness	mm
e_b	Beam eccentricity from shaft centerline	mm
y_b	Beam neutral axis position	mm
x_b	Beam root position	mm
x_F	Handle end position (force application)	mm
$x_{\text{pin,L}}$	Left pin support position (Theory 2)	mm
L_{span}	Span between pin supports (Theory 2)	mm
ϕ_{mis}	System misalignment angle around shaft	deg or rad
ϕ_0	Initial beam twist (relative clamp rotation)	deg or rad

Table 16: Nomenclature - Material properties

Symbol	Description	Units
E_s	Young's modulus of shaft (carbon)	GPa
E_b	Young's modulus of beam (aluminum)	GPa
ν_s	Poisson's ratio of shaft	–
ν_b	Poisson's ratio of beam	–
α_s	Thermal expansion coefficient of shaft	K^{-1}
α_b	Thermal expansion coefficient of beam	K^{-1}
I_s	Second moment of area of shaft	mm^4
I_b	Second moment of area of beam	mm^4
$\varepsilon_{\text{mech}}(x, y)$	Mechanical bending strain at position (x, y)	–
$\varepsilon(x, y)$	Strain at position (x, y) (when context is clear)	–
$\sigma(x, y)$	Stress at position (x, y)	MPa

Table 17: Nomenclature - Forces and reactions

Symbol	Description	Units
F	Applied force at handle	N
$V_s(x)$	Shear force in shaft at position x	N
$M_s(x)$	Bending moment in shaft at position x	N·mm
R_L	Reaction force at left pin support (Theory 2)	N
R_R	Reaction force at right pin support (Theory 2)	N
N_{th}	Axial thermal load in beam	N
N_{assy}	Assembly pre-stress load	N
N_{cr}	Critical buckling load	N

Table 18: Nomenclature - Strain gauge parameters

Symbol	**Description**	**Units**
R_g	Strain gauge nominal resistance	Ω
GF	Gauge factor	–
R_1, R_3	Top surface gauge resistances	Ω
R_2, R_4	Bottom surface gauge resistances	Ω
ε_{top}	Strain on top surface (total strain at gauges R_1 and R_3)	–
$\varepsilon_{\text{bottom}}$	Strain on bottom surface (total strain at gauges R_2 and R_4)	–
V_{ex}	Bridge excitation voltage	V
V_{out}	Bridge output voltage	V
ΔR	Change in gauge resistance	Ω
x_{gauge}	Strain gauge x-position	mm
y_{top}	Top surface y-position	mm
y_{bottom}	Bottom surface y-position	mm
n_{bits}	ADC resolution	bits
LSB	Least significant bit voltage	μV

Table 19: Nomenclature - Thermal parameters

Symbol	**Description**	**Units**
T	Temperature	$^\circ$C
T_0	Reference temperature	$^\circ$C
ΔT	Temperature change	K
$\Delta\alpha$	Difference in thermal expansion coefficients	K^{-1}
$\varepsilon_{\text{thermal}}$	Simplified thermal strain due to $\Delta\alpha\Delta T$	–
$\Delta L_{\text{thermal}}$	Differential thermal expansion	mm

Table 20: Nomenclature - Strain components and measurement model

Symbol	**Description**	**Units**
$\varepsilon_{\text{total}}$	Total axial strain at a gauge location	–
$\varepsilon_{\text{mech}}$	Mechanical bending strain component	–
ε_{th}	Thermal strain component (differential expansion)	–
ε_{off}	Offset strain (assembly and electronics effects)	–
$\varepsilon_{\text{meas}}$	Strain inferred from bridge output and calibration	–
$A_T(T)$	Temperature-dependent mechanical strain amplification factor	–
k_T	Linear thermal amplification coefficient	K^{-1}