22 May 2022 18:07

Sample should be "representative" of the population

① "Cluster Sampling"

V_1 V_2 V_3 V_4 … V_{500}

② Stratified Sampling

① "Middle level manager"	"50,000"	500
② Top level manager	"10,000"	100
③ Executives	"2,000"	20

Mean — (Population Mean)

$A_1, A_2, \ldots, A_N$ — Population

$X_1, X_2, \ldots, X_n$ — Sample

$$\boxed{\mu = \frac{A_1 + A_2 + \ldots + A_N}{N}}$$

$$\bar{X} = \frac{X_1 + X_2 + \ldots + X_n}{n}$$

X_1: A_1 — $1/N$; A_2 — $1/N$; … ; A_N — $1/N$

$E(X_1) = \mu$; $E(X_2) = \mu$ . . $E(X_i) = \mu$

$$\boxed{\bar{X} = \frac{X_1 + X_2 + \ldots + X_n}{n}}$$

$$E(\bar{X}) = \frac{E(X_1) + E(X_2) + \ldots + E(X_n)}{n} = \frac{\mu + \mu + \ldots + \mu}{n} = \frac{n\mu}{n} = \mu$$

$$\bar{X}_{\text{...}} = \frac{X_1 + X_2}{...}$$

$$E(\bar{X}_{\text{...}}) = \frac{E(X_1) + E(X_2)}{2} = \frac{\mu + \mu}{2} = \mu$$

$$\boxed{\bar{X}_{\text{initial}} = \frac{X_1 + X_2}{2}}$$

$$E(\bar{X}_{\text{initial}}) = \frac{E(X_1) + E(X_2)}{2} = \frac{\mu + \mu}{2} = \mu$$

$$Var(X_1) = E(X_1^2) - [E(X_1)]^2$$

$$= \frac{A_1^2 + A_2^2 + \dots + A_N^2}{N} - \left(\frac{A_1 + A_2 + \dots A_N}{N}\right)^2$$

$$= \sigma^2$$

$A_1^2 - \frac{1}{N}$

$A_2^2 - \frac{1}{N}$

$\vdots$

$A_N^2 - \frac{1}{N}$

$$Var(\bar{X}) = Var\left(\frac{X_1 + X_2 + \dots X_n}{n}\right)$$

$$= \frac{1}{n^2}[Var(X_1) + Var(X_2) + \dots Var(X_n)]$$

$$= \frac{1}{n^2}[\sigma^2 + \sigma^2 + \dots \sigma^2] = \frac{\sigma^2}{n} \leftarrow ?????$$

$Var(X) = 10$

$Var(5X) = 5^2 \cdot 10$

$= 250$

$X_1, X_2, \vdots, X_n$

① μ: $\bar{X} = \frac{X_1 + X_2 + \dots X_n}{n}$

② σ^2: $\left[S_X^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n} \times \left(\frac{n}{n-1}\right) = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1} \right]$

$$E(S_X^2) = \frac{\sigma^2(n-1)}{n}$$

$$\bar{X} = \frac{X_1 + X_2 + X_3 + \dots + X_n}{n} \quad ; n > 100$$

$\bar{X}$ approximately follows a normal distribution with

$$\left[\begin{array}{l} \text{mean} = \mu \\ \text{Variance} = \frac{\sigma^2}{} \end{array}\right]$$

$$\boxed{\begin{aligned} \text{mean} &= \mu \\ \text{Variance} &= \frac{\sigma^2}{n} \end{aligned}}$$

$$P\left(\mu - 1.96\frac{\sigma}{\sqrt{n}} \leq \bar{X} \leq \mu + 1.96\frac{\sigma}{\sqrt{n}}\right) = 95\%$$

(1) $\bar{X} \geq \mu - 1.96\frac{\sigma}{\sqrt{n}}$

and

(2) $\bar{X} \leq \mu + 1.96\frac{\sigma}{\sqrt{n}}$

(3) $\bar{X} + 1.96\frac{\sigma}{\sqrt{n}} \geq \mu$

and

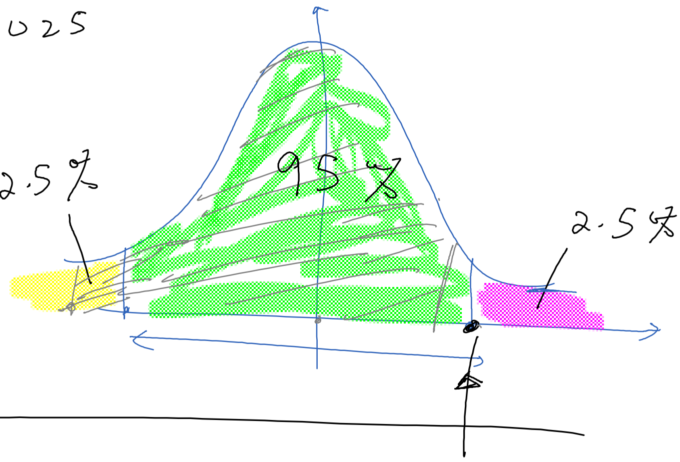
(4) $\bar{X} - 1.96\frac{\sigma}{\sqrt{n}} \leq \mu$

$$\boxed{P\left(\bar{X} - 1.96\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X} + 1.96\frac{\sigma}{\sqrt{n}}\right) = 0.95}$$

let $\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$

$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$



For sample of size 100, $\frac{\sigma^2}{n}$ $\frac{\sigma}{\sqrt{n}}$

$\frac{\sigma}{\sqrt{100}} = \frac{\sigma}{10}$

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$$\begin{vmatrix} X_1 \\ X_2 \\ \vdots \\ X_{100} \end{vmatrix}$$

$$\underline{\left(\bar{X} - 1.96 \frac{s}{\sqrt{n}} \; , \; \bar{X} + 1.96 \frac{s}{\sqrt{n}} \right)}$$