22 May 2022 16:02

Random Variables

Discrete

(1.) Bernoulli

$$X = \begin{cases} 1, & p \\ 0, & 1-p \end{cases}$$

$$\boxed{\begin{aligned} E(X) &= p \\ Var(X) &= p(1-p) \end{aligned}}$$

(2.) Binomial $[n, p]$

$$X: \begin{array}{ll} 0, & (1-p)^n \\ 1, & \\ 2, & \\ \vdots & \\ r - & {}^nC_r\, p^r (1-p)^{n-r} \\ \vdots & \\ n, & p^n \end{array}$$

$$\boxed{\begin{aligned} E(X) &= np \\ Var(X) &= np(1-p) \end{aligned}}$$

(1.) "Expectation" of a Random Variable — [Average, Mean]

(2.) Variance of a Random Variable

$$\cancel{\frac{\sum_{i=1}^{n} (X_i - \mu)^2}{n}}$$

$$= \sum_{i=1}^{n} p_i \left(X_i - E(X)\right)^2$$

$$= E\left(\left(X - E(X)\right)^2\right)$$

$$= E\left(X^2 - 2X \cdot E(X) + (E(X))^2\right)$$

$$= E(X^2) - 2E(X) \cdot E(X) + (E(X))^2 = E(X^2) - (E(X))^2$$

X	Y	(I) Prob	(II) Prob
0	9.8	0.2	0.01
5	9.9	0.2	0.01
10	10	0.2	0.96
15	10.1	0.2	0.01
20	10.2	0.2	0.01

$$E(X) = \sum_{i=1}^{n} x_i p_i$$

X	$P(X = x_i)$	$(X - E(X))^2 = Y$
x_1	p_1	$(x_1 - E(X))^2$
x_2	p_2	$(x_2 - E(X))^2$
$\vdots$	$\vdots$	$\vdots$
x_n	p_n	$(x_n - E(X))^2$

$$\boxed{\sum_{i=1}^{n} p_i \left(x_i - E(X)\right)^2} = E(Y)$$

x_n	p_n	$(x_n - E(X))^2$

$$Var(X) = E\left(\left(X - E(X)\right)^2\right) = E(X^2) - \left(E(X)\right)^2$$

X	p	$(X-E(X))^2$	X^2
1	1/6		1
2	1/6		4
3	1/6		9
4	1/6		16
5	1/6		25
6	1/6		36

X	Prob	X^2
0	0.1	0
2	0.9	4

$E(X) = 1.8$

$(E(X))^2 = 3.24$

$E(X^2) = 3.6$

X	Prob	X^2
0	$1-p$	0
1	p	1

$$Var(X) = E(X^2) - (E(X))^2$$
$$= p - p^2$$
$$= p(1-p)$$

$$X = X_1 + X_2 + X_3 + \dots + X_n$$
$$Var(X) = Var(X_1) + Var(X_2) + \dots + Var(X_n) \quad \text{[Caution]}$$
$$= np(1-p)$$

$$\boxed{\begin{aligned} X &= X_1 + X_2 + X_3 \\ E(X) &= E(X_1) + E(X_2) + E(X_3) \\ Var(X) &= Var(X_1) + Var(X_2) + Var(X_3) + 2\left[Cov(X_1,X_3) + Cov(X_2,X_3) + Cov(X_1,X_3)\right] \end{aligned}}$$

$= 0$, If Random Variables are independent.

③ Poisson Random Variables

$$P(X = x) = e^{-\lambda}\frac{\lambda^x}{x!}, \quad e = 2.7183\dots$$

λ = average no. of customers

$\lambda = 5$, $P(X=2) = e^{-5}\dfrac{5^2}{2!} = \left(e^{-5}\dfrac{25}{2}\right)$

$E(X) = \lambda$, $Var(X) = \lambda$

Continuous Random Variable

150–160 – 1/3
160–170 – 1/3
170–180 – 1/3

① "Uniform Random Variable" "(150, 180)"

"Height" "(a, b)"

$E(X) = \dfrac{a+b}{2}$

$P(\text{height} \le 160) =$

$P(159 \le \text{height} \le 161) =$

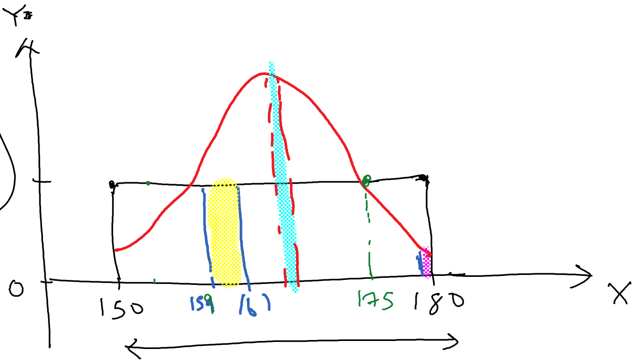
$E(X) = \int x f(x)\,dx$

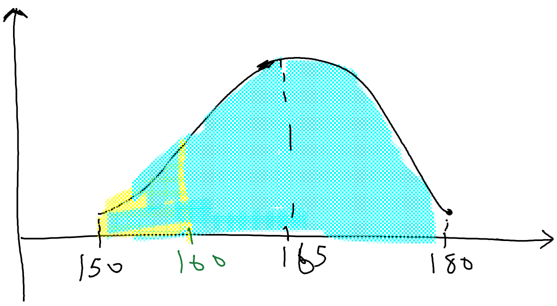
$Var(X) = E(X^2) - (E(x))^2$
$= \int x^2 f(x)\,dx -$

"Probability Density"

$$f(x) = \begin{cases} 0; & x \le 150 \\ \frac{1}{30}; & 150 \le x \le 180 \\ 0; & x > 180 \end{cases}$$

$\left(\frac{1}{30}\right)$



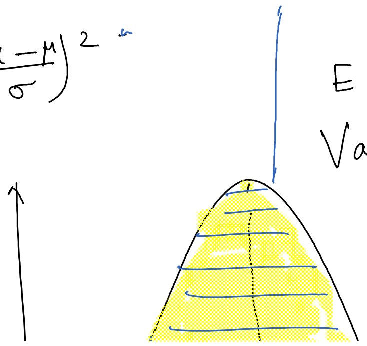


② Normal Random Variable

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

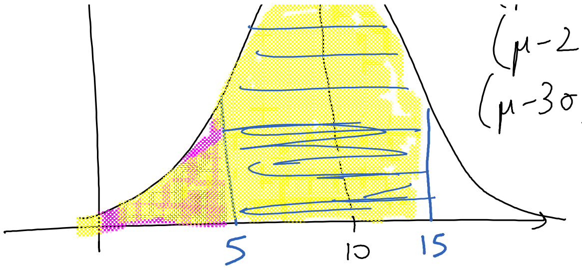
$E(X) = \mu$
$Var(X) = \sigma^2$

$\mu = 10$
$\sigma = 5$



$(\mu-\sigma, \mu+\sigma) \approx 68\%$
$(\mu-2\sigma, \mu+2\sigma) \approx 95.44\%$
$(\mu-3\sigma, \mu+3\sigma) \approx 99.73\%$

$\sigma = 5$



$(\mu - 2\sigma, \mu + 2\sigma) = 95.44\%$

$(\mu - 3\sigma, \mu + 3\sigma) = 99.73\%$