21 May 2022 15:50

Session - 2

1. Probability -
2. Random Variable -

Expt - Match between A & B

Sample Space - { ~~A wins~~, B wins, ~~Draw~~ }

Event - 1. A doesn't lose

Probability - A number associated with each sample point.

1. ≥ 0
2. ≤ 1
3.

$$S = \{ s_1, s_2, s_3, --- ---- - , s_n \}$$
$$p_1 \quad p_2 \quad p_3, \quad , p_n$$

A: $\{ s_1, s_2, s_3 \}$

$P(A) = p_1 + p_2 + p_3$

B: $\{ s_3, s_4, s_8 \}$

$P(B) = p_3 + p_4 + p_8$

$C = \underbrace{A \cup B}$ → "Union" ; $P(C) \cancel{=} P(A) + P(B)$

$$P(C) = p_1 + p_2 + p_3 + p_4 + p_8 = P(A) + P(B) - p_3$$
$$= P(A) + P(B) - \underbrace{P(A \cap B)}_{\text{"intersection"}}$$

$D = A \cap B$

"Independent Events"

Dice: {1, 2, 3, 4, 5, 6,}

A: odd number comes $\quad P(A) = 1/2$

B = even number comes. $\quad P(B) = \frac{1}{2}$

Card Example

A: queen comes

$P(A) = \frac{4}{52} = \frac{1}{13}$

B = "Spade comes"

$P(B) = \frac{13}{52} = \frac{1}{4}$

$P(A|B) = \frac{1}{13}$ $\qquad$ $P(B|A) = \frac{1}{4}$

$\boxed{P(A \cap B) = P(A) \cdot P(B)}$ — For independent events.

$P(B|A) = \frac{P(B \cap A)}{P(A)} \Rightarrow P(B) = \frac{P(B \cap A)}{P(A)} \Rightarrow P(B \cap A) = P(B) \cdot P(A)$

---

* 27 days remaining $\qquad$ 25 days are required.

(1.) If he decides to go to B'lore (26 days are remaining)

Good Day — (G)
Bad Day — (B)

(1, 2, 3, 4, ... , 27)

(1, 2, 3, 4, _ _ _ 26)

P(he meets his target)
= P(atleast 25 good days)
= P(exactly 25 good days) + P(exactly 26 good days)

Rain	Haδtal
$P(R) = \frac{1}{30}$	$P(H) = \frac{14}{730}$

$P(B) = P(R \cup H)$
$= P(R) + P(H) - P(R \cap H)$
$= P(R) + P(H) - P(R) \cdot P(H)$
$= 0.05$

B_1 = D_1 is bad
B_2 = D_2 is bad
⋮
B_{26} = D_{26} is bad

G_1 = D_1 is good
G_2 = D_2 is good
⋮
G_{26} = D_{26} is good

$P(G_1) = 0.95$
$= P(G_2) = \ldots = P(G_{26})$

$P(G_1 \cap G_2) =$

$P(G_1 \cap G_2 \cap G_3 \ldots \cap G_{26})$

$= (0.95)^{26}$

1) $B_1 \cap G_2 \cap G_3 \ldots \cap G_{26}$ $\quad | \quad (0.95)^{25}(0.05)$

2.) $G_1 \cap B_2 \cap G_3 \ldots \cap G_{26}$ $\quad | \quad (0.95)(0.05)(0.95)^{24} = (0.95)^{25}(0.05)$

3.) $G_1 \cap G_2 \cap B_3 \ldots \cap G_{26}$

⋮

26.) $G_1 \cap G_2 \cap G_3 \ldots \quad \cap B_{26}$ $\qquad (0.95)^{25}(0.05)$

$26 \times (0.95)^{25} \times (0.05)$

P (meets his target)

$= 26 \times (0.95)^{25} \times (0.05) + (0.95)^{26}$

$\simeq$ "0.62"

---

② If he decides not to go to B'lore

$P(\text{meeting the target}) = P(\text{exactly 25 days}) + P(\text{exactly 26 days}) + P(\text{exactly 27 days})$

$^{27}C_2 \times (0.95)^{25}(0.05)^2 + 27(0.95)^{26}(0.05) + (0.95)^{27}$

$\simeq 0.81$

B_1	G_1	① $B_1 B_2 G_3 G_4 \ldots G_{27}$
B_2	G_2	② $B_1 G_2 B_3 G_4 \ldots G_{27}$
B_{27}	G_{27}	

$^{27}C_2 = \frac{27 \times 26}{1 \times 2}$

$(0.95)^{25}(0.05)^2$

"Random variable"

1 Spade	1
2 Spade	1
⋮	
13 Spade	1
1 Heart	0
⋮	0
13 Heart	0
1 Club	0
⋮	
13 Club	0
1 Diamond	−1
⋮	
13 Diamond	−1

E

$\{ s_1, s_2, s_3, s_4, _ $

$x_1 \quad x_2 \quad x_3 \quad x_4 \ldots$

"Random variable"

Discrete Random Variables

1. Bernoulli
2. Binomial
3. Poisson

Continuous Random Variables

1. Uniform
2. Normal

D_1: "Bernoulli Random Variable"

Expt : { Success, Failure }

$p_s \qquad 1-p_s$

RV : 1 0

Values Probability

$$X = \begin{cases} 1, & p_s \\ 0, & 1-p_s \end{cases}$$

$E(X) = p_s$

2. Binomial Random Variable

Do an expt "n" times, each trial can result in success with probability "p".

X = "number of successes"

$$\boxed{E(X) = n \cdot p}$$

Values Probability

$$X = \begin{cases} 0, & (1-p)^n \\ 1, & n\,p\,(1-p)^{n-1} \\ 2, & \vdots \\ \vdots \\ r \longrightarrow & {}^nC_r\, p^r (1-p)^{n-r} \\ n, & p^n \end{cases}$$

$p = \frac{1}{6}$

"10 times"

$P($"⚅" comes $\frac{10}{10}$ times$) = \left(\frac{1}{6}\right)^{10}$

Expected Value of a Random Variable

"$E(X) = p_1x_1 + p_2x_2 + \ldots + p_nx_n$"

Expected Value of a Random Variable

$E(X) = p_1x_1 + p_2x_2 + \dots + p_nx_n$

X	Prob
1	0.1
2	0.1
3	0.1
4	0.1
5	0.1
6	0.5

"3.5" – Expected Value

"6 lakh times"

$$\frac{1\cdot(60k) + 2\cdot(60k) + 3\cdot(60k) + \dots + 6\,(3\text{ lakh})}{6 \text{ lakh}}$$

$$= 1\cdot(0.1) + 2\cdot(0.1) + 3(0.1) + 4(0.1) + 5(0.1) + 6(0.5)$$

X	X_1	X_2	. –	X_n
	1	1		1
	0	0		0

$X = X_1 + X_2 + \dots + X_n$

$E(X) = E(X_1) + E(X_2) + \dots + E(X_n)$

$= p + p + \dots + p = n.p$

A LAB doing Covid testing gets 1000 samples to test everyday. However, due to the positivity rate drop in cases of Covid samples, the LAB is contemplating if it is better to mix the samples to get the result in lesser number of tests. Assuming 3% positivity rate, what is the number of samples that should be pooled together?

Scheme:
1. Pool "k" samples together
2. Test all the mixtures
3. For the mixtures which give a "+ve" report, test all the samples separately.

Minimize: E("Total no. of tests") → Some sort of average or expected value.

(3%)

"k" $E($ "1000" $+ N_p \cdot k)$

1, 2, ... k

$\boxed{"k"}$ $E\left(\underbrace{\frac{1000}{k}}_{\text{Phase 1}} + \underbrace{N_p \cdot k}_{\text{Phase 2}}\right)$

1, 2, ... k

$P(\text{mixture is +ve}) = "p"$

$= 1 - P(\text{mixture is -ve})$

$= 1 - (0.97)^k$

$(0.97) \cdot (0.97) \ldots (0.97)$

$\underbrace{"\left(\frac{1000}{k}\right)"}_{n}$ $\quad \underbrace{1 - (0.97)^k}_{p}$

$= \frac{1000}{k} + E(N_p) \cdot k$

$= \frac{1000}{k} + \left[\left(\frac{1000}{k}\right) \cdot \left(1 - 0.97\right)^k\right] k$