

Q b. Result coordinate:

$$(0,0) (1,0) (2,2) (5,3) (6,5) (9,0) (10,0)$$

(omitted all coordinate if x or y == "float")

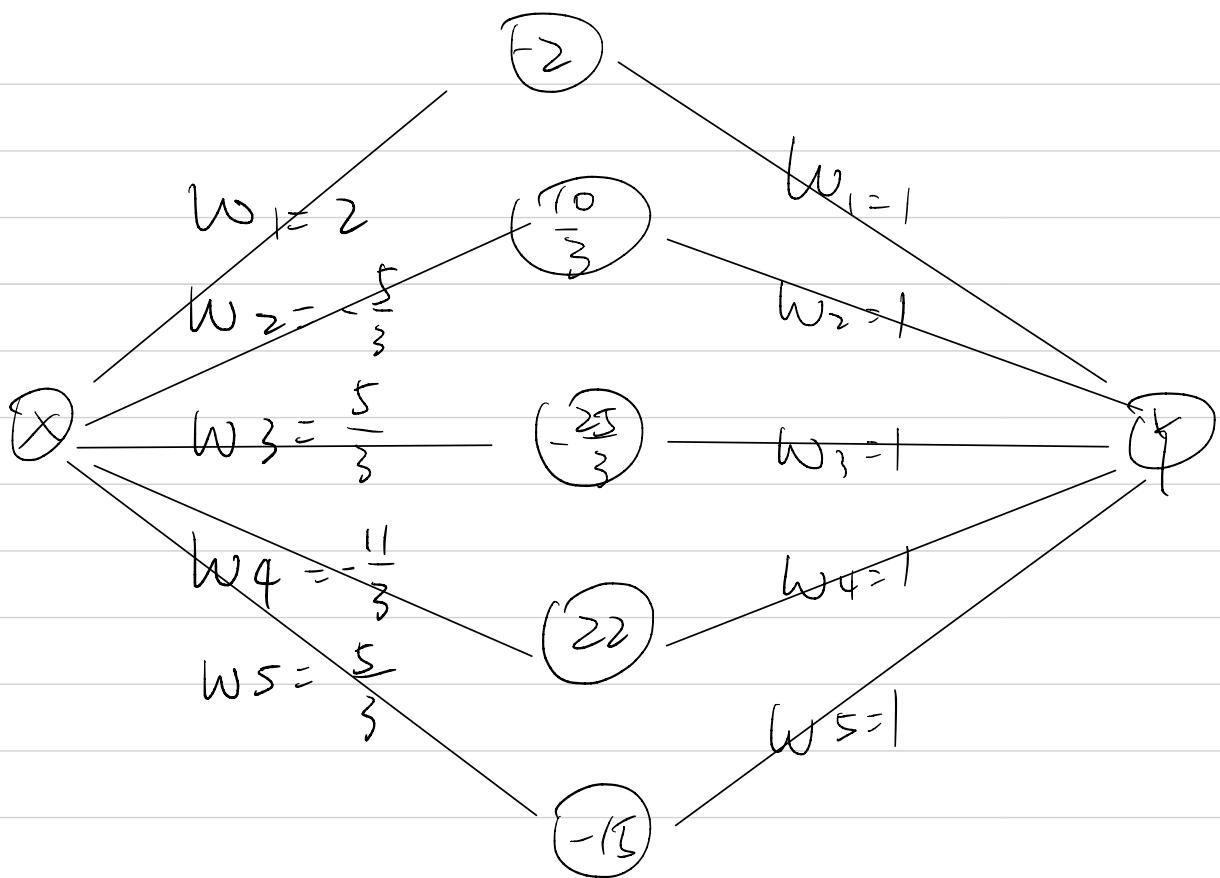
$$\Rightarrow y = \begin{cases} 0 & \text{for } x \in [0,1] \\ 2x-2 & \text{for } x \in [1,2] \\ \frac{1}{3}x + \frac{4}{3} & \text{for } x \in [2,5] \\ 2x-7 & \text{for } x \in [5,6] \\ -\frac{5}{3}x + 15 & \text{for } x \in [6,9] \\ 0 & \text{for } x \in [9,10] \end{cases} = \sigma(w_i Y_{i-1}^T + \beta_i)$$

$$\text{Thus } (2-0) \cdot \text{relu}(x-1) + \left(\frac{1}{3}-2\right) \text{relu}(x-2) \\ (2-\frac{1}{3}) \text{relu}(x-5) + \left(-\frac{5}{3}-2\right) \text{relu}(x-6) \\ + \left(0+\frac{5}{3}\right) \text{relu}(x-9)$$

$$\Rightarrow \begin{cases} w_1=2, & f_1=-2, \\ w_2=-\frac{5}{3}, & \beta_2=\frac{4}{3}+2=\frac{10}{3} \\ w_3=\frac{5}{3}, & \beta_3=-7-\frac{4}{3}=-\frac{25}{3} \\ w_4=-\frac{11}{3}, & \beta_4=15+7=22 \\ w_5=\frac{5}{3}, & \beta_5=0-15=-15 \end{cases}$$

Thus: There are 1 layer with 5 units.

The graph is shown below.



Q3.

P_i be included for 1 data $\geq \frac{1}{n}$

P (Not be included 1 data) $= 1 - \frac{1}{n}$

P (Not be included for n data) $= \left(1 - \frac{1}{n}\right)^n$

When $n \rightarrow \infty$,

$Exp (Not be included for n data)$

$= P$ (Not be included for n data)

$$= \lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^n = \frac{1}{e} = 0.3679$$