Q2. a) Let $x = \{0, 1\}$, $0$ = has no flu, $1$ = has flu.
$C$ = coughing, $S$ = sneezing, $C, S = \{0, 1\}$.

$$P(x=1 \mid C=1, S=1)$$
$$= \frac{P(x=1 \cap C=1 \cap S=1)}{P(C=1, S=1)}$$
$$= 80\% \times 4\% \ / \ 20\% \times 75\% + 80\% \times 4\%$$
$$= 17.582\%$$

b) $$P(X \mid C=1, S=1) = \frac{P(C=1, S=1 \mid X) \cdot P(X)}{P(C=1, S=1)}$$

$$P(C=1, S=1) = P(C=1, S=1 \mid X=0) \cdot P(X=0) + P(C=1, S=1 \mid X=1) \cdot P(X=1)$$
$$= P(C=1 \mid X=0) \cdot P(S=1 \mid X=0) \cdot P(X=0) + P(C=1 \mid X=1) P(S=1 \mid X=1) \cdot P(X=1)$$

C & S are conditional independent within same class

$$= (75\% + 5\%) \cdot (75\% + 5\%) \times 0.2 + (4\% + 1\%) \cdot (4\% + 1\%) \times 0.8$$
$$= 0.13$$

$$P(C=1, S=1 \mid X=0) \cdot P(X=0) = P(C=1 \mid X=0) \cdot P(S=1 \mid X=0) \cdot P=0$$
$$= (4\% + 1\%) \cdot (4\% + 1\%) \times 0.8$$
$$= 0.02$$

$$P(X=0 \mid C=1, S=1) = \frac{0.02}{0.13} = 1.538\%$$

c) The answers are significantly different, Since if we compare both assumptions..

① Generative model:

$$P(c, s \mid x) P(x) = P(x \mid c, s) \cdot P(c, s)$$

② Naive Bays assumption:

$P(c, s \mid x) = P(c \mid x) P(s \mid x)$

~ C & S are conditional independent within each category (i.e given $x=0$ or $X=1$).

The generative model is better here since it explicitly model the actual distribution of each class while the naive Bayes model is just use for classifier to find the $\arg\max_y P_\theta(x \mid y) P(y)$ so not as accurate as generative model.