

3.

- let T_i = expected # steps to reach n , starting at i

$$T_i = 1 + \frac{1}{2} T_{i+1} + \frac{1}{2} T_{i-1}$$

$$2T_i = 2 + T_{i+1} + T_{i-1}$$

$$2T_i - T_{i+1} = 2 + T_{i-1}$$

$$\rightarrow T_i - T_{i+1} = 2 + T_{i-1} - T_i$$

Base case: $T_0 - T_1 = 1$, $T_n = 0$

$$T_i - T_{i+1} = 1 + 2i$$

$$\rightarrow T_0 = T_n + \sum_{i=0}^{n-1} T_i - T_{i+1}$$

$$= 0 + \sum_{i=0}^{n-1} 1 + 2i$$

$$= n + 2(n-1)\frac{n}{2}$$

$$= n + n^2 - n$$

$$\boxed{T_0 = n^2} \quad \checkmark$$