2. Conditional Probability

(a) $P(W=2, B=3)$

- \# Total Outcomes: $\binom{14}{6}$
- $W=2$: $\binom{3}{2}$
- $B=3$: $\binom{5}{3}$
- $R=1$: $\binom{6}{1}$

$$\rightarrow P(W=2, B=3) = \frac{\binom{3}{2}\cdot\binom{5}{3}\cdot\binom{6}{1}}{\binom{14}{6}}$$

$$= \frac{3\cdot 10\cdot 6}{3003} = \boxed{0.05994}$$

(b) PMF of W given $B=3$:

$$P(W|B=3) = \frac{P(W, B=3)}{P(B=3)}$$

$$= \frac{\binom{5}{3}\binom{3}{w}\binom{6}{3-w}}{\binom{14}{6}} \cdot \frac{\binom{14}{6}}{\binom{5}{3}\binom{9}{3}}$$

$$= \boxed{\frac{\binom{3}{w}\binom{6}{3-w}}{\binom{9}{3}}}$$, where w = # White Selected, $w = \{0, 1, 2, 3\}$

$$P(B=3) = \frac{\binom{5}{3}\binom{9}{3}}{\binom{14}{6}}$$

$$P(W, B=3) = \frac{\binom{5}{3}\binom{3}{w}\binom{6}{3-w}}{\binom{14}{6}}$$

(c) PMF of W, with Replacement:

$$P(B=3) = \left(\frac{5}{14}\right)^3\left(\frac{9}{14}\right)^3\binom{6}{3}$$

$$P(W, B=3) = \left(\frac{5}{14}\right)^3\left(\frac{3}{14}\right)^w\left(\frac{6}{14}\right)^{3-w}\binom{6}{3}\binom{3}{w}$$

$$\hookrightarrow P(W|B=3) = \boxed{\frac{\left(\frac{3}{14}\right)^w\left(\frac{6}{14}\right)^{3-w}\binom{3}{w}}{\left(\frac{9}{14}\right)^3}}$$, $w = \{0, 1, 2, 3\}$