4. $M_i = c + U_i$

$E[M_i] = c$

$Var[M_i] = Var(c + U_i) = 3$

$$E[\bar{M}] = E\left[\frac{1}{n}\sum_{i=1}^{n} M_i\right]$$
$$= \frac{1}{n}\sum_{i=1}^{n} E[M_i] = c$$

$$Var[\bar{M}] = Var\left(\frac{1}{n}\sum_{i=1}^{n} M_i\right) = \frac{n \cdot 3}{n^2} = \frac{3}{n}$$

(a) Chebyshev's Inequality:

$$P\left(\left|\frac{1}{n}\sum_{i=1}^{n} M_i - E[M_i]\right| \geq 0.5\right) \leq \frac{Var(M_1)}{n(0.5^2)}$$

$$1 - \frac{Var(M_1)}{n(0.5^2)} = 0.90$$

$$n = \frac{3}{0.10(0.5^2)} = \boxed{120}$$

(b) Central Limit Theorem:

$$P(c - 0.5 \leq \bar{M} \leq c + 0.5)$$

since $\bar{M} \sim N(c, \sqrt{\frac{3}{n}})$:

$$P\left(\frac{\bar{X} - c}{\sqrt{3/n}} \leq Z \leq \frac{c + .05 - c}{\sqrt{3}/\sqrt{n}}\right) = 0.95$$

$$\rightarrow Z \leq \frac{.5}{\sqrt{3}/\sqrt{n}} = 0.95$$

$$1.65 = \frac{.5\sqrt{n}}{\sqrt{3}}$$

$$n = 32.67 \approx \boxed{33}$$

Alternative Method:

$E = 0.5$, $\sigma = \sqrt{3}$, $1 - \alpha = 0.90$
$\hookrightarrow \alpha = 0.1$

$Z_{\alpha/2} = 1.6449$

$$\rightarrow n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E}\right)^2$$
$$= \left(\frac{1.6449 \cdot \sqrt{3}}{0.5}\right)^2$$
$$= 32.46$$
$$\approx \boxed{33}$$

The CLT finds a less-conservative sample size by normalizing the distribution of M_i and estimating the sample size according to the Normal Distribution.